

Productions of T_{cc} and its SU(3)-flavor symmetry and heavy quark spin symmetry partners in B_c decays

Yi Zhang *School of Physics, Beihang University, Beijing 102206, China*Ming-Zhu Liu **Frontiers Science Center for Rare isotopes, Lanzhou University, Lanzhou 730000, China
and School of Nuclear Science and Technology, Lanzhou University, Lanzhou 730000, China*Li-Sheng Geng †*School of Physics, Beihang University, Beijing 102206, China; China Sino-French Carbon Neutrality
Research Center, École Centrale de Pékin/School of General Engineering, Beihang University,
Beijing 100191, China; Peng Huanwu Collaborative Center for Research and Education, Beihang
University, Beijing 100191, China; Beijing Key Laboratory of Advanced Nuclear Materials and Physics,
Beihang University, Beijing 102206, China and Southern Center for Nuclear-Science Theory (SCNT),
Institute of Modern Physics, Chinese Academy of Sciences, Huizhou 516000, China*

(Received 15 October 2025; accepted 2 February 2026; published 24 February 2026)

Inspired by the observation of the doubly charmed tetraquark state T_{cc} at pp collisions in the inclusive processes, we systematically investigate the production of doubly charmed tetraquark states in exclusive B_c decays. In this work, we assume the T_{cc} as a DD^* bound state, and then predict the masses of its heavy quark spin symmetry partner D^*D^* (denoted by T_{cc}^*) as well as their SU(3)-flavor symmetry partners, i.e., $D_s D^*/D_s^* D$ (denoted by T_{ccs}^+ and T_{ccs}^{++}) and $D^* D_s^*$ (denoted by T_{ccs}^{*+} and T_{ccs}^{*++}), using the contact range effective field theory. Within the molecular picture, we compute their partial decay widths and production rates in B_c meson decays. We identify the decays $B_c \rightarrow D^0 D^0 \pi^+ \bar{D}^0$ and $B_c \rightarrow D_s^+ D^0 \pi^+ D^-$ as promising channels to observe the tetraquark states $T_{cc}^{(*)}$ and $T_{ccs}^{(*)++}$, respectively. Finally, by combining these results with B_c production cross sections, we estimate the expected event yields for these states in the upcoming LHC Runs 3 and 4. Our results indicate that the T_{ccs}^+ and T_{ccs}^{*++} states are likely to be observed in B_c decays, while it is quite difficult for the T_{cc} and T_{cc}^* states.

DOI: [10.1103/891b-jzpy](https://doi.org/10.1103/891b-jzpy)

I. INTRODUCTION

According to the conventional constituent quark model, hadrons are classified as mesons made of a quark and an antiquark, and baryons made of three quarks [1]. These conventional hadrons play a vital role in advancing our understanding of the strong interaction and hadron structure. A notable example is the study of charmonium spectroscopy using the Cornell potential [2], which has successfully validated both the long-range and short-range behaviors of the strong interaction [3]. Since 2003, an increasing number of states beyond the conventional quark

model have been discovered [4–6], providing new insights into hadron structure and the underlying dynamics of the strong interaction. Exotic states offer profound insights into the complexity of the strong interaction and the structure of hadronic systems. These include hadron-hadron interactions in hadronic molecules, QCD dynamics in compact multiquark states, and electromagnetic forces in hadro-charmonium systems. In addition, some exotic states are not genuine physical states but may arise from kinematical effects. Despite extensive experimental and theoretical investigations [7–19], the nature of these exotic states remains a subject of controversy—highlighting the intricate and challenging character of the nonperturbative strong interaction.

It is well established that hadron-hadron interactions play a pivotal role in excited or exotic hadron spectroscopy and in strong and weak decays. For the spectrum of exotic states, a prime example is the $X(3872)$ [20]. A detailed analysis of its invariant mass distributions supports the interpretation that the $X(3872)$ possesses a significant

*Contact author: liumz@lzu.edu.cn†Contact author: lisheng.geng@buaa.edu.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

molecular component [21–23], which reflects the role of the $\bar{D}^*D$ interaction. Theoretical predictions for the ratio of $\mathcal{B}[\psi(2S) \rightarrow \rho\pi]$ to $\mathcal{B}[\psi(1S) \rightarrow \rho\pi]$ show significant discrepancies with experimental results, a phenomenon known as the ρ – π puzzle [24]. One classical explanation is that final-state interactions (FSIs) play a vital role in addressing this long-standing puzzle [25–28]. FSIs play a significant role in the weak decays of heavy hadrons as well, substantially enhancing the branching fractions of non-factorizable decay channels [29–32], and giving rise to significant CP violation signals in these processes [33,34]. Therefore, investigations of hadron-hadron interactions are of great value. Information from lattice QCD scattering data [35], femtoscopy correlation functions [16], and invariant mass distributions in exclusive and inclusive decay processes [36,37] can all help improve our understanding of hadron-hadron interactions with increasing precision.

Doubly heavy tetraquark states fall into the category of exotic hadronic states. In 2021, the LHCb Collaboration first observed a doubly charmed tetraquark state T_{cc} in the inclusive process at pp collision, with a mass close to the DD^* mass threshold [38,39]. Unlike the case of hidden-charm tetraquarks, the T_{cc} state can only be of two exotic configurations, i.e., a hadronic molecule [40–50] or a compact tetraquark state [51–53]. While its mass and decay width are readily explained within the molecular picture, the compact tetraquark picture has difficulty explaining the decay width. Therefore, the discovery of T_{cc} motivates a lot of studies on the DD^* hadron interactions in lattice QCD [54–58], chiral effective field theory (ChEFT) [59–61], and one-boson-exchange model [62–66]. By treating the T_{cc}^+ as a DD^* molecular state, the DD^* interaction can be extracted. This interaction can then be used to investigate other phenomena related to the DD^* system, such as relevant three-body hadronic molecules DDD^* [67], $\bar{\Sigma}_c DD^*$ [68], and KDD^* [69], and heavy quark spin symmetry (HQSS) partner D^*D^* [42,44,46], SU(3)-flavor symmetry partners $D_s D^*$ and $D_s^* D^*$ [70–73], and correlation functions [74]. See, e.g., Ref. [16]. In addition, the DD^* interaction can be verified in the exclusive B_c decay [75] and heavy ion collisions [76,77].

In this work, we focus on the T_{cc} state from the DD^* molecular perspective, together with SU(3)-flavor symmetry partners $D^{*0}D_s^+/D^0D_s^{*+}$ and $D^{*+}D_s^+/D^+D_s^{*+}$, as well as their HQSS partners D^*D^* , $D^{*0}D_s^{*+}/D^{*0}D_s^{*+}$, and $D^{*+}D_s^{*+}/D^{*+}D_s^{*+}$. The multiplet hadronic molecule picture provides a nice approach to classifying exotic states. A notable example is the three pentaquark states $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$, which are well arranged into the $\bar{D}^{(*)}\Sigma_c^{(*)}$ HQSS multiplet molecular picture [78,79]. Moreover, the multiplet hadronic molecule picture is argued that experimental searches for their symmetry partners are helpful to verify the molecular nature of the exotic states. A notable example is that assuming $X(3872)$

as a $J^{PC} = 1^{++} \bar{D}^*D$ molecule, the experimental search for its HQSS partner $J^{PC} = 2^{++} \bar{D}^*D^*$ and heavy quark flavor symmetry partner $J^{PC} = 1^{++} \bar{B}^*B$ are another model independent approach to verifying its molecular nature [80,81]. Along the same line, the molecular picture of T_{cc} could also be tested by investigating its symmetry partners [16]. To guide future experimental observations, we further investigate the partial decay properties of the T_{cc} state and its symmetry partners, along with their production mechanisms in B_c decays.¹ By considering the branching fractions of their partial decays and their production rates in B_c decays, we identify the most promising channels for searching for doubly charmed tetraquark states. At last, using the detection efficiency and the B_c cross sections at the LHC, we estimate the event number of observing the doubly charmed tetraquark states.

This work is structured as follows. In Sec. II, we begin with a brief introduction to the contact-range potentials for the doubly charmed tetraquark system. We then present the tree-level Feynman diagrams responsible for the three-body partial decays of doubly charmed hadronic molecules, as well as the triangle diagrams describing the two-body partial decays of doubly charmed tetraquark molecules and their productions in B_c meson decays. The effective Lagrangian approach is also outlined in this section. Numerical results and corresponding discussions are provided in Sec. III. Finally, a summary is presented in the last section.

II. THEORETICAL FORMALISM

In this work, we focus on the doubly heavy charmed tetraquark states with the quark configuration $cc\bar{q}\bar{q}$, where the light quark constituents are constrained by SU(3)-flavor symmetry. Since charmed mesons can be denoted by the $\bar{3}$ representation of the SU(3) flavor symmetry, a pair of charmed mesons contains irreducible representations 3 and $\bar{6}$ [82], where the 3 representation includes three states with the flavor wave function $I = 0 \ DD^*$, $I = 1/2 \ \frac{1}{\sqrt{2}}(D_s D^* - D_s^* D)$, and the $\bar{6}$ representation includes six states with the flavor wave function $I = 1 \ DD^*$, $I = 1/2 \ \frac{1}{\sqrt{2}}(D_s D^* + D_s^* D)$, and $I = 0 \ D_s D_s^*$. From the product of two matrix elements, the potential of the 3 multiplet is attractive, but the potential of the $\bar{6}$ multiplet is repulsive [72]. The doubly charmed tetraquark state T_{cc} can be assigned as a bound state of $I = 0 \ DD^*$, and therefore we focus on the three states in the 3 representation: $I = 0 \ D^+ D^{*0}$, $I = 1/2 \ \frac{1}{\sqrt{2}}(D_s^+ D^{*0} - D_s^{*+} D^0)$, and $I = 1/2 \ \frac{1}{\sqrt{2}}(D_s^+ D^{*+} - D_s^{*+} D^+)$ (denoted by T_{cc} , T_{ccs}^+ , and T_{ccs}^{++}), as well as their HQSS partners $I = 0 \ D^{*+} D^{*0}$, $I = 1/2$

¹Different from the production of hidden-charm states in b -flavored hadron decays, doubly charmed tetraquarks can be produced through B_c decays.

$\frac{1}{\sqrt{2}}(D_s^{*+}D^{*0} - D_s^{*+}D^{*0})$, and $I = 1/2$ $\frac{1}{\sqrt{2}}(D_s^{*+}D^{*+} - D_s^{*+}D^{*+})$ (denoted by T_{cc}^* , T_{ccs}^{*+} , and T_{ccs}^{*++}). Taking into account the HQSS and SU(3)-flavor symmetry, the EFT contact potentials between a pair of heavy mesons can be parametrized as [42,71,83]

$$\begin{aligned} V(I=0, D_{(s)}D^*) &= C_a + C_b, \\ V(I=0, D_{(s)}^*D^*) &= C_a + C_b, \end{aligned} \quad (1)$$

where C_a and C_b are unknown low-energy constants. One can see that the contact range potentials between those six states are the same. Therefore, assuming the doubly charmed tetraquark state T_{cc} as a DD^* bound state, we can determine the contact potentials of $I=0$ DD^* , and then predict the masses of the other five states. In this work, we employ the contact range EFT approach to study the spectrum of these doubly charmed tetraquark states [84,85]. To search for bound states, we solve the Lippmann-Schwinger equation with contact-range potentials

$$T(\sqrt{s}) = [1 - VG(\sqrt{s})]^{-1}V, \quad (2)$$

where $G(\sqrt{s})$ is the loop function, and V is contact-range potential. To avoid the divergence of the loop function $G(\sqrt{s})$, we employ a regulator of Gaussian form e^{-2q^2/Λ^2} in the integral as

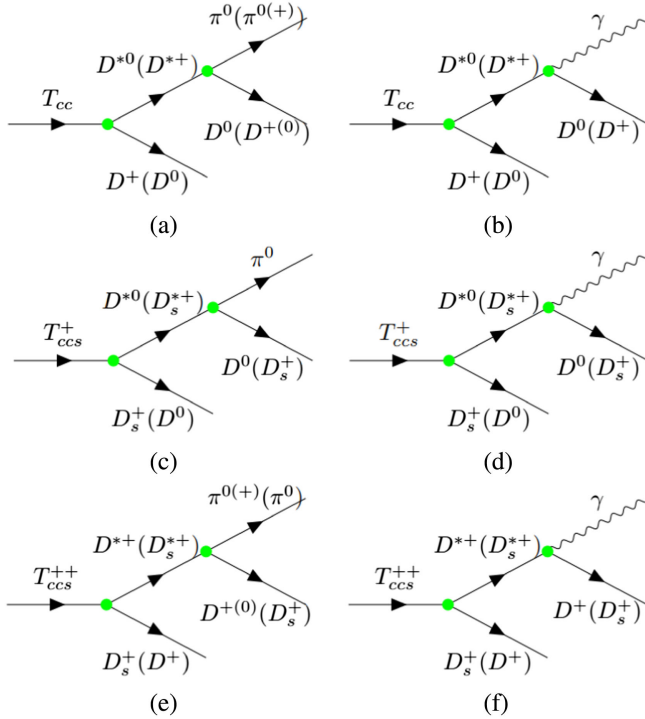


FIG. 1. Tree diagrams responsible for the three-body decays of the T_{cc} (a), (b), T_{ccs}^+ (c), (d), and T_{ccs}^{++} (e), (f) states.

$$G(\sqrt{s}) = \int \frac{d^3q}{(2\pi)^3} \frac{e^{-2q^2/\Lambda^2}}{\sqrt{s} - m_1 - m_2 - q^2/(2\mu_{12}) + i\epsilon}, \quad (3)$$

where $\sqrt{s}$ is the total energy in the center-of-mass frame of m_1 and m_2 , $\mu_{12} = m_1 m_2 / (m_1 + m_2)$ is the reduced mass, and Λ is the momentum cutoff. As shown later, in addition to T_{cc} , we predict five more doubly charmed tetraquark states. In the following, we investigate the decays and productions of these tetraquark states in B_c meson decays.

For the T_{cc} , T_{ccs}^+ , and T_{ccs}^{++} states, two-body strong decays are forbidden. Instead, they can decay via the strong or radiative decays of the vector charmed mesons, proceeding via the tree diagrams as illustrated in Fig. 1. Since only the three-body decays are allowed, the widths of the T_{cc} , T_{ccs}^+ , and T_{ccs}^{++} states are expected to be less than the widths of the $D_{(s)}^*$ mesons. Similarly, the T_{cc}^* , T_{ccs}^{*+} , and T_{ccs}^{*++} states can proceed via the three-body decay modes as shown in Fig. 2. Moreover, the T_{cc}^* , T_{ccs}^{*+} , and T_{ccs}^{*++} states can undergo two-body decays through triangle diagrams, as illustrated in Fig. 3. In addition to the DD_s^* channel, these states may also decay into their HQSS partners, accompanied by a pion or a photon. These HQSS partners subsequently decay into three-body final states, as shown in Fig. 1, which effectively correspond to four-body decays of the T_{cc}^* , T_{ccs}^{*+} , and T_{ccs}^{*++} states. Since the two-body decays are allowed, the

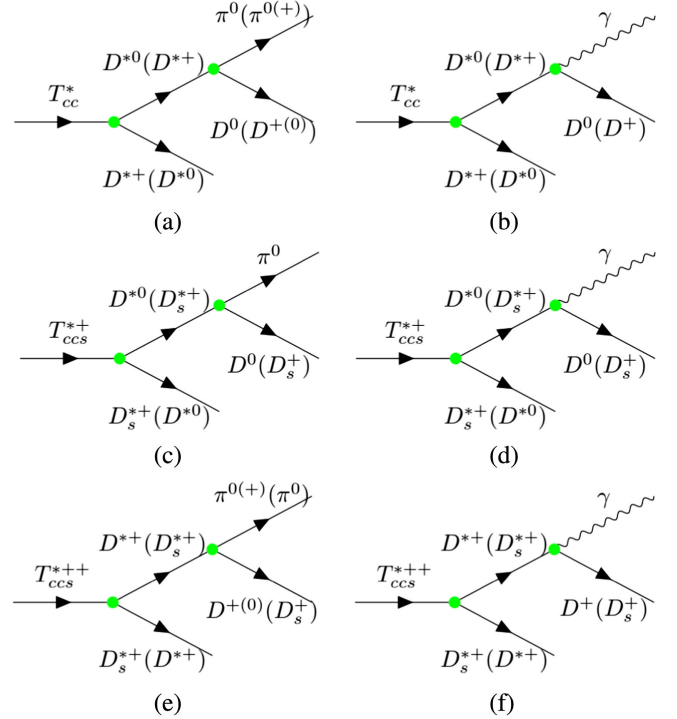


FIG. 2. Tree diagrams responsible for the three-body decays of the T_{cc}^* (a), (b), T_{ccs}^{*+} (c), (d), and T_{ccs}^{*++} (e), (f) states.

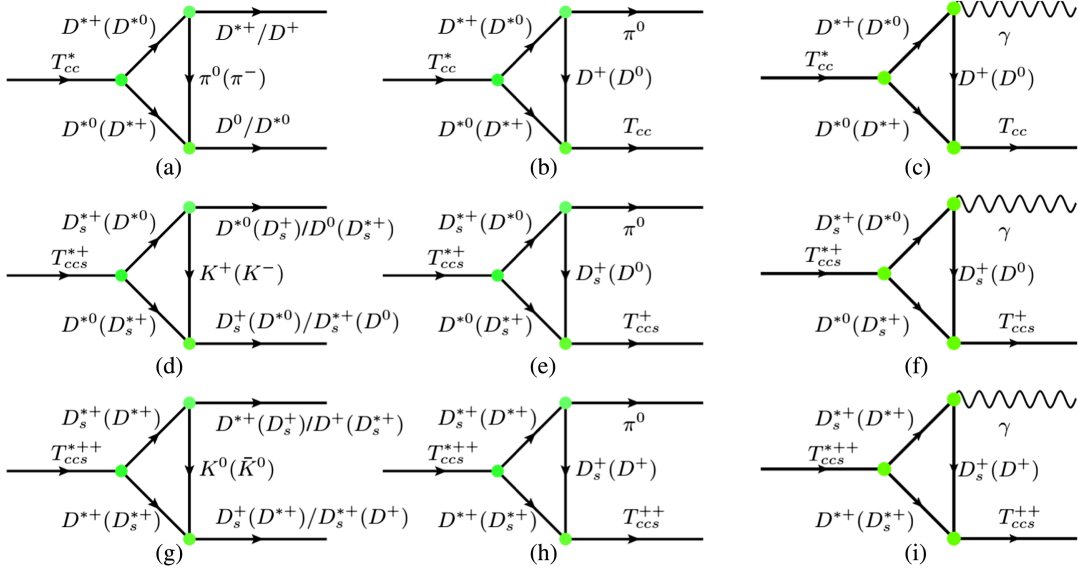


FIG. 3. Triangle diagrams responsible for the two-body decays of the T_{cc}^* (a)–(c), T_{ccs}^* (d)–(f), and T_{ccs}^{*++} (g)–(i) states.

widths of the T_{cc}^* , T_{ccs}^{*+} , and T_{ccs}^{*++} states are expected to be larger than their HQSS partners.

Unlike the production of T_{cc} in inclusive processes [38,39], we study the productions of the doubly charmed tetraquark states in B_c meson decays in this work. Heavy-flavor hadron decays are a primary way to produce exotic hadron states. For instance, many hidden-charm exotic states were produced in b -flavored hadron decays. The productions of these doubly charmed tetraquark states in B_c meson decays are difficult to calculate at the quark level, but can be calculated at the hadron level via the FSIs approach. Recent studies indicated that FSIs play an important role in producing exotic states [86–89]. As shown in Fig. 4, the B_c meson first weakly decays into charmonium states (J/ψ and η_c) and charmed mesons $D_{(s)}^{(*)}$, then the charmonium states scatter into a pair of charmed mesons $D^{(*)}\bar{D}^{(*)}$, and finally the doubly charmed tetraquark states are dynamically generated via the $D^{(*)}D_{(s)}^{(*)}$ interactions. In this work, we neglect the contributions of orbitally and radially excited charmonium states, which are relatively small due to the suppression of B_c meson transition to excited charmonium states and the couplings of the excited charmonium states to a pair of charmed mesons.

A. Effective Lagrangians

This section presents the effective Lagrangians that will be utilized in this work for computing the Feynman diagrams depicted in Figs. 1–4. Since the Lagrangians involving $D^{(*)}$ mesons are similar to those with $D_s^{(*)}$ mesons via SU(3)-flavor symmetry, we only present the Lagrangians for the charmed mesons $D^{(*)}$ here.

The first category of Lagrangians involves the interactions between the hadronic molecules and their constituents. The interactions of the T_{cc} state with DD^* and the T_{cc}^* state with D^*D^* are described by the following Lagrangian [87]:

$$\begin{aligned}\mathcal{L}_{T_{cc}D^*D} &= g_{T_{cc}D^*D}T_{cc}^\mu DD_\mu^*, \\ \mathcal{L}_{T_{cc}^*D^*D^*} &= ig_{T_{cc}^*D^*D^*}\epsilon_{\mu\nu\alpha\beta}\partial^\mu T_{cc}^{*\nu}D^{*\alpha}D^{*\beta},\end{aligned}\quad (4)$$

where g with a subscript denote the couplings between the hadronic molecules and their corresponding constituents. These couplings are determined by the contact range approach [85]. After obtaining the pole positions of these doubly charmed tetraquark states, we determined the couplings as $g_{T_{cc}D^{*+}D^0} = 4.12$ GeV, $g_{T_{cc}D^{*+}D^{*+}} = 3.67$ GeV, $g_{T_{ccs}^{*+}D_s^{*0}D_s^{*+}} = 4.87$ GeV, $g_{T_{ccs}^{*+}D_s^{*0}D_s^{*+}} = 4.28$ GeV, $g_{T_{ccs}^{*++}D_s^{*+}D_s^{*+}} = 4.51$ GeV, $g_{T_{ccs}^{*++}D_s^{*+}D_s^{*+}} = 3.66$ GeV, $g_{T_{cc}^*D^*D^*} = 1.92$, $g_{T_{ccs}^{*+}D_s^{*+}D_s^{*0}} = 2.09$, and $g_{T_{ccs}^{*++}D_s^{*+}D_s^{*+}} = 2.08$.

The Lagrangian describing the interactions between charmed mesons and π/γ is written as [90]

$$\begin{aligned}\mathcal{L}_{DD^*\pi} &= -ig_{DD^*\pi}(D\partial^\mu\pi D_\mu^{*\dagger} - D_\mu^*\partial^\mu\pi D^\dagger), \\ \mathcal{L}_{D^*D^*\pi} &= -g_{D^*D^*\pi}\epsilon_{\mu\nu\alpha\beta}\partial^\mu D^{*\nu}\partial^\alpha D^{*\beta\dagger}\pi, \\ \mathcal{L}_{DD^*\gamma} &= eg_{DD^*\gamma}\epsilon^{\mu\nu\alpha\beta}\partial_\mu A_\nu\partial_\alpha D_\beta^{*\dagger}D,\end{aligned}\quad (5)$$

where the relevant couplings are taken as $g_{D^{*+}D^0\pi^+} = 16.818$ and $g_{D^{*+}D^+\pi^0} = 11.940$, $g_{D^{*+}D^+\gamma} = 0.468$ and $g_{D^{*0}D^0\gamma} = 1.843$ GeV⁻¹ [41]. The electron charge e is derived by the relationship $\frac{e^2}{4\pi} = \frac{1}{137}$. The couplings $g_{D^*D^*\pi}$ are determined by the relationship $g_{D^*D^*\pi} = g_{D^*D\pi}/m_{\bar{D}}$, where $m_{\bar{D}}$ is the average mass of D and D^* [90,91].

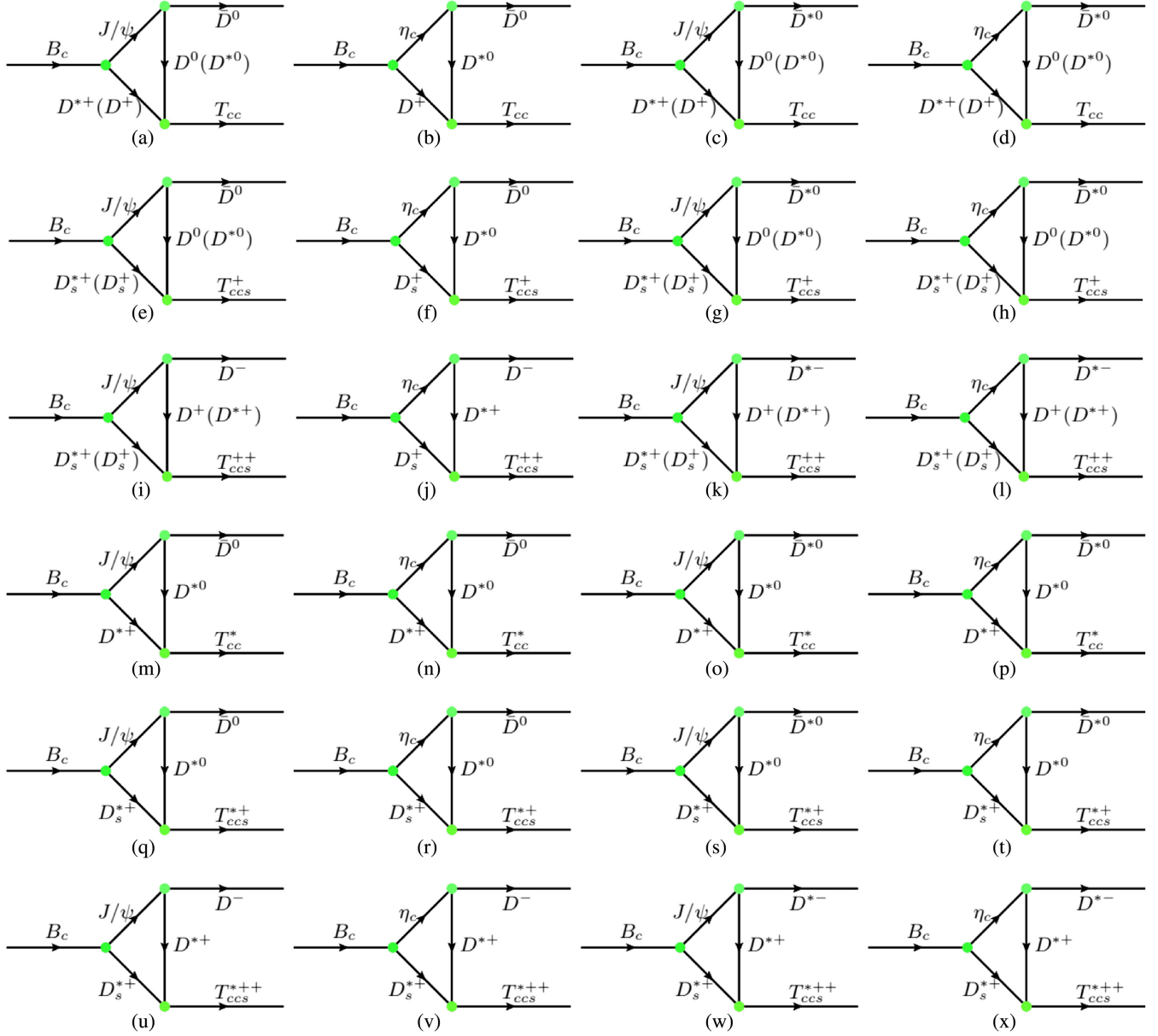


FIG. 4. Triangle diagrams responsible for the productions of the T_{cc} (a)–(d), T_{ccs}^+ (e)–(h), T_{ccs}^{++} (i)–(l), T_{cc}^* (m)–(p), T_{ccs}^{*+} (q)–(t), and T_{ccs}^{*++} (u)–(x) states in B_c meson decays.

The couplings of their SU(3)-flavor partners are determined as $g_{D^*D_s K} = 15.275$ and $g_{D^*D_s^* K} = 7.887 \text{ GeV}^{-1}$ [92]. If the partial decay width of $D_s^{*+} \rightarrow D_s^+ \gamma$ is chosen to be 0.066 keV from the lattice QCD calculation [93], we can determine the coupling $g_{D_s^{*+} D_s^+ \gamma} = 0.101 \text{ GeV}^{-1}$. According to the ratio of the partial decay width $\Gamma(D_s^{*+} \rightarrow D_s^+ \gamma)$ to that of $\Gamma(D_s^{*+} \rightarrow D_s^+ \pi^0)$ [94], we can determine the coupling $g_{D_s^{*+} D_s^+ \pi^0} = 0.111$.

The effective Lagrangians describing the interactions between charmonium states (J/ψ , η_c) and a pair of charmed mesons read [90,92,95,96]

$$\begin{aligned}
 \mathcal{L}_{\psi DD} &= i g_{\psi DD} \bar{\psi}_\mu (\partial^\mu D \bar{D} - D \partial^\mu \bar{D}), \\
 \mathcal{L}_{\psi DD^*} &= -g_{\psi DD^*} \epsilon^{\alpha\beta\mu\nu} \partial_\alpha \bar{\psi}_\beta (\partial_\mu D_\nu^* \bar{D} + D \partial_\mu \bar{D}_\nu^*), \\
 \mathcal{L}_{\psi D^* D^*} &= -i g_{\psi D^* D^*} [\bar{\psi}^\mu (\partial_\mu D_\nu^* \bar{D}^{*\nu} - D^{*\nu} \partial_\mu \bar{D}_\nu^*) \\
 &\quad + (\partial_\mu \bar{\psi}_\nu D^{*\nu} - \bar{\psi}_\nu \partial_\mu D^{*\nu}) \bar{D}^{*\mu} \\
 &\quad + D^{*\mu} (\bar{\psi}_\nu \partial_\mu \bar{D}_\nu^* - \partial_\mu \bar{\psi}_\nu \bar{D}^{*\nu})], \\
 \mathcal{L}_{\eta_c D^* D} &= i g_{\eta_c D^* D} [D^{*\mu} (\partial_\mu \eta_c \bar{D} - \eta_c \partial_\mu \bar{D}) \\
 &\quad - (\partial_\mu \eta_c D - \eta_c \partial_\mu D) \bar{D}^{*\mu}], \\
 \mathcal{L}_{\eta_c D^* D^*} &= -g_{\eta_c D^* D^*} \epsilon^{\mu\nu\alpha\beta} \partial_\mu D_\nu^* \partial_\alpha \bar{D}_\beta^* \eta_c,
 \end{aligned} \tag{6}$$

where $g_{\psi DD}$, $g_{\psi D^* D}$, $g_{\psi D^* D^*}$, $g_{\eta_c D^* D}$, and $g_{\eta_c D^* D^*}$ are the couplings of the charmonium mesons to the charmed mesons. The coupling constants are determined as follows: $g_{\psi DD} = g_{\psi D^* D} = m_\psi/f_\psi$, $g_{\psi D^* D} = \frac{1}{m_D} g_{\psi DD}$ [97,98], $g_{\eta_c D^* D} = g_{\eta_c D^* D^*} \sqrt{\frac{m_D}{m_{D^*}}} m_{\eta_c} = g_2 \sqrt{m_{\eta_c} m_D m_{D^*}}$ with $g_2 = \frac{\sqrt{m_\psi}}{2m_D f_\psi}$ [99].

The weak decays $B_c \rightarrow \eta_c D_{(s)}^{(*)}$ and $B_c \rightarrow J/\psi D_{(s)}^{(*)}$ proceed mainly via the external W -emission mechanism at the quark level, which belong to T -type diagrams and are the dominant contribution in terms of the topological classification of weak decays [100–103]. In the naive factorization ansatz [104], the amplitudes of the weak decays $B_c \rightarrow \eta_c D_{(s)}^{(*)}$ and $B_c \rightarrow J/\psi D_{(s)}^{(*)}$ can be expressed as products of two current matrix elements

$$\mathcal{A}(B_c \rightarrow \eta_c D_{(s)}) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cd(s)} a_1 \langle D_{(s)} | J_\mu | 0 \rangle \langle \eta_c | J^\mu | B_c \rangle, \quad (7)$$

$$\mathcal{A}(B_c \rightarrow \eta_c D_{(s)}^*) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cd(s)} a_1 \langle D_{(s)}^* | J_\mu | 0 \rangle \langle \eta_c | J^\mu | B_c \rangle, \quad (8)$$

$$\begin{aligned} \mathcal{A}(B_c \rightarrow J/\psi D_{(s)}) \\ = \frac{G_F}{\sqrt{2}} V_{cb} V_{cd(s)} a_1 \langle D_{(s)} | J_\mu | 0 \rangle \langle J/\psi | J^\mu | B_c \rangle, \end{aligned} \quad (9)$$

$$\begin{aligned} \mathcal{A}(B_c \rightarrow J/\psi D_{(s)}^*) \\ = \frac{G_F}{\sqrt{2}} V_{cb} V_{cd(s)} a_1 \langle D_{(s)}^* | J_\mu | 0 \rangle \langle J/\psi | J^\mu | B_c \rangle, \end{aligned} \quad (10)$$

where $J_\mu = \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$, G_F is the Fermi coupling constant, V_{cb} and $V_{cd(s)}$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, and a_1 is the Wilson coefficient depending on the renormalization scale [100,105,106]. In this work, we deal with B_c decays, and therefore extract the value of the effective Wilson coefficient at the energy scale $\mu = m_b$, i.e., $a_1 = 1.07$ [103], consistent with Ref. [107]. Moreover, the weak decays $B_c \rightarrow \eta_c D_{(s)}^{(*)}$ and $B_c \rightarrow J/\psi D_{(s)}^{(*)}$ proceed via the internal W -emission mechanism at the quark level, which belong to C -type diagrams. The amplitudes of the weak decays $B_c \rightarrow \eta_c D_{(s)}^{(*)}$ and $B_c \rightarrow J/\psi D_{(s)}^{(*)}$ belonging to C -type diagrams are color suppressed and neglected, and therefore only those of T -type diagrams are taken into account.

The former current matrix elements are written as

$$\begin{aligned} \langle 0 | \bar{q}_1 \gamma_\mu \gamma_5 q_2 | D_{(s)}(p_1) \rangle &= -i f_{D_{(s)}} p_{1\mu}, \\ \langle 0 | \bar{q}_1 \gamma_\mu q_2 | D_{(s)}^*(p_1, \epsilon) \rangle &= f_{D_{(s)}^*} \epsilon_\mu(p_1) m_{D_{(s)}^*}, \end{aligned} \quad (11)$$

where $f_{D_{(s)}}$ and $f_{D_{(s)}^*}$ are the decay constants of $D_{(s)}$ and $D_{(s)}^*$. In this work, we take $G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$, $V_{cb} = 0.041$, $V_{cs} = 0.973$, $V_{cd} = 0.226$, $a_1 = 1.07$, $f_D = 210.4 \text{ MeV}$, $f_{D^*} = 250 \text{ MeV}$, $f_{D_s} = 247.7 \text{ MeV}$, and $f_{D_s^*} = 282.5 \text{ MeV}$ [103,108–113].

The latter current matrix elements describing the transitions of $B_c \rightarrow \eta_c$ and $B_c \rightarrow J/\psi$ are characterized by a series of form factors [109,110]:

$$\begin{aligned} \langle \eta_c(p_{\eta_c}) | \bar{c} \gamma_\mu b | B_c(p_{B_c}) \rangle &= \left[P_\mu - \frac{m_{B_c}^2 - m_{\eta_c}^2}{q^2} q_\mu \right] f_+(q^2) \\ &+ \left[\frac{m_{B_c}^2 - m_{\eta_c}^2}{q^2} q_\mu \right] f_0(q^2), \end{aligned} \quad (12)$$

$$\begin{aligned} \langle J/\psi(p_{J/\psi}, \epsilon) | \bar{c} \gamma_\mu (1 - \gamma_5) b | B_c(p_{B_c}) \rangle \\ = i \epsilon_\alpha^* \left\{ -g^{\mu\alpha} (m_{B_c} + m_{J/\psi}) A_1(q^2) + P^\mu P^\alpha \frac{A_2(q^2)}{m_{B_c} + m_{J/\psi}} \right. \\ + i \epsilon^{\mu\alpha\beta\gamma} P_\beta q_\gamma \frac{V(q^2)}{m_{B_c} + m_{J/\psi}} + q^\mu P^\alpha \left[\frac{m_{B_c} + m_{J/\psi}}{q^2} A_1(q^2) \right. \\ \left. \left. - \frac{m_{B_c} - m_{J/\psi}}{q^2} A_2(q^2) - \frac{2m_{J/\psi}}{q^2} A_0(q^2) \right] \right\}, \end{aligned} \quad (13)$$

where ϵ is the polarization vector of the J/ψ meson, $q = p_{B_c} - p_{\eta_c(J/\psi)}$ is the momentum transfer, and $V(q^2)$, $A_0(q^2)$, $A_1(q^2)$, and $A_2(q^2)$ are the transition form factors. In general, the transition form factors extracted from the light cone sum rules can be extrapolated by using Bourrely-Caprini-Lellouch (BCL) parametrization [114] of the form factor series expansion in powers of a conformal mapping variable, which satisfies unitarity, analyticity, and perturbative QCD scaling. The BCL parametrization is based on a rapidly converging series in the parameter z as

$$\begin{aligned} f(t) &= \frac{1}{P(t)} \sum_{k=0} \alpha_k [z(t) - z(0)]^k, \\ z(t) &= \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}, \end{aligned} \quad (14)$$

weighted by a simple pole function $P(t) = 1 - t/m_R^2$ that accounts for low-lying resonances present below the threshold production of real $B_c - M$ pairs at $t_+ = (m_{B_c} + m_M)^2$. The parameter t_0 , $0 \leq t_0 \leq t_- = (m_{B_c} - m_M)^2$ is a free parameter that can be used to optimize the convergence of the series expansion. For the truncation to only two terms in expansion Eq. (14), it was shown that the optimized value of t_0 has the form [115]

$$t_{0|\text{opt}} = t_+ \left(1 - \sqrt{1 - \frac{t_-}{t_+}} \right), \quad (15)$$

and that the other choices of t_0 do not make a visible change in the form factor parametrization. The masses of resonances appearing in the fits are determined by the properties of the form factors. In Table I, we collect the values of the fitting parameters [116] in the form factors to be used in this work.

With Eqs. (11)–(14), we obtain the amplitudes for the decays $B_c(k_0) \rightarrow D^{(*)}(q_1)\eta_c/J/\psi(q_2)$,

TABLE I. Summary of the BCL fit for $B_c \rightarrow \eta_c$ and $B_c \rightarrow J/\psi$ transition form factors.

Form factor	J^P	m_R (GeV)	α_0	α_1
f_+	1^-	6.34	0.62	−6.13
f_0	0^+	6.71	0.63	−4.86
V	1^-	6.34	0.74	−8.66
A_0	0^-	6.28	0.54	−6.80
A_1	1^+	6.75	0.55	−4.67
A_2	1^+	6.75	0.35	−1.78

$$\mathcal{A}(B_c \rightarrow \eta_c D) = i \frac{G_F}{\sqrt{2}} V_{cb} V_{cd(s)} a_1 f_D (m_{B_c}^2 - m_{\eta_c}^2) f_0(q^2), \quad (16)$$

$$\mathcal{A}(B_c \rightarrow \eta_c D^*) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cd(s)} a_1 m_{D^*} f_{D^*}(k_0 + q_2) \cdot \epsilon(q_1) f_+(q^2), \quad (17)$$

$$\begin{aligned} \mathcal{A}(B_c \rightarrow J/\psi D) = & -\frac{G_F}{\sqrt{2}} V_{cb} V_{cd(s)} a_1 f_D \left\{ -q_1 \cdot \epsilon(q_2) (m_B + m_{J/\psi}) A_1(q^2) + (k_0 + q_2) \cdot \epsilon(q_2) q_1 \cdot (k_0 + q_2) \frac{A_2(q^2)}{m_B + m_{J/\psi}} \right. \\ & \left. + (k_0 + q_2) \cdot \epsilon(q_2) [(m_B + m_{J/\psi}) A_1(q^2) - (m_B - m_{J/\psi}) A_2(q^2) - 2m_{J/\psi} A_0(q^2)] \right\}, \end{aligned} \quad (18)$$

$$\begin{aligned} \mathcal{A}(B_c \rightarrow J/\psi D^*) = & i \frac{G_F}{\sqrt{2}} V_{cb} V_{cd(s)} a_1 m_{D^*} f_{D^*} \epsilon_\mu(q_1) \left[-g_{\mu\alpha} (m_B + m_{J/\psi}) A_1(q^2) + (k_0 + q_2)_\mu (k_0 + q_2)_\alpha \frac{A_2(q^2)}{m_B + m_{J/\psi}} \right. \\ & \left. + i \epsilon_{\mu\alpha\rho\sigma} (k_0 + q_2)^\rho q^\sigma \frac{V(q^2)}{m_B + m_{J/\psi}} \right] \epsilon^\alpha(q_2). \end{aligned} \quad (19)$$

B. Three-body decay amplitudes

With the above Lagrangians, the decay amplitudes of $T_{cc} \rightarrow DD\pi$ and $T_{cc} \rightarrow DD\gamma$ of Fig. 1 are written as

$$\mathcal{M}_{T_{cc} \rightarrow DD\pi}^a = i g_{T_{cc}} g_{DD^*\pi} p_{2\mu} \frac{-g^{\mu\nu} + k_1^\mu k_1^\nu / m_{D^*}^2}{k_1^2 - m_{D^*}^2 + i m_{D^*} \Gamma_{m_{D^*}}} \epsilon_\nu(p_0), \quad (20)$$

$$\mathcal{M}_{T_{cc} \rightarrow DD\gamma}^b = i g_{T_{cc}} g_{DD^*\gamma} \epsilon^{\mu\nu\alpha\beta} p_{2\mu} \epsilon_\nu(p_2) k_{1\alpha} \frac{-g_{\beta\sigma} + k_{1\beta} k_{1\sigma} / m_{D^*}^2}{k_1^2 - m_{D^*}^2 + i m_{D^*} \Gamma_{m_{D^*}}} \epsilon^\sigma(p_0), \quad (21)$$

where p_2 , k_1 , and p_0 are the momentum of $\pi(\gamma)$, D^* , and T_{cc} , respectively. Similarly, we express the decay amplitudes of $T_{cc}^* \rightarrow DD^*\pi/\gamma$ and $T_{ccs}^* \rightarrow D_s^* D\pi/\gamma$ of Fig. 2

$$\mathcal{M}_{T_{cc}^* \rightarrow D^* D\pi}^a = i g_{T_{cc}^* D^* D^*} g_{DD^*\pi} \epsilon_{\mu\nu\alpha\beta} p_0^\mu p_2^\lambda \frac{-g^{\alpha\lambda} + k_1^\alpha k_1^\lambda / m_{D^*}^2}{k_1^2 - m_{D^*}^2 + i m_{D^*} \Gamma_{m_{D^*}}} \epsilon_\nu(p_0) \epsilon_\beta(p_1), \quad (22)$$

$$\mathcal{M}_{T_{cc}^* \rightarrow D^* D\gamma}^b = i g_{T_{cc}^* D^* D^*} g_{DD^*\gamma} \epsilon^{\mu\nu\alpha\beta} p_{2\mu} \epsilon_\nu(p_2) k_{1\alpha} \frac{-g_{\beta\sigma} + k_{1\beta} k_{1\sigma} / m_{D^*}^2}{k_1^2 - m_{D^*}^2 + i m_{D^*} \Gamma_{m_{D^*}}} \epsilon_{uvab} k_0^\mu \epsilon^v(k_0) \epsilon^a(p_1), \quad (23)$$

where p_1 , p_2 , k_1 , p_3 and p_0 are the momentum of D , $\pi(\gamma)$, D^* , D^* , and T_{cc} , respectively. Here we note that the amplitudes of Figs. 1(a) and 2(a) are similar to those of Figs. 1(c)–1(e) and 2(c)–2(e), and 1(b) and 2(b) are similar to those of Figs. 1(d)–1(f) and 2(d)–2(f).

As for the three-body decays, the partial decay widths of $T_{cc(s)}^{(*)} \rightarrow D_{(s)} D_{(s)}^{(*)} \pi/\gamma$ as a function of m_{12}^2 and m_{23}^2 read

$$\Gamma = \frac{1}{(2\pi)^3} \frac{1}{2J+1} \int \int \frac{|\overline{\mathcal{M}}|^2}{32m_{T_{cc(s)}^{(*)}}^3} dm_{12}^2 dm_{23}^2, \quad (24)$$

with m_{12} the invariant mass of $D_{(s)} D_{(s)}^{(*)}$ and m_{23} the invariant mass of $D_{(s)}^{(*)} \pi$ or $D_{(s)}^{(*)} \gamma$ for the $T_{cc(s)}^{(*)} \rightarrow D_{(s)} D_{(s)}^{(*)} \pi/\gamma$ decays. Here m_0 , m_1 , m_2 , and m_3 represent the masses of $T_{cc(s)}^{(*)}$, $D_{(s)}$, $D_{(s)}^{(*)}$, and π/γ , respectively.

The integration limits for m_{12}^2 and m_{23}^2 are: m_{12}^2 ranges from lower limit $(m_1 + m_2)^2$ to upper limit $(m_0 - m_3)^2$, and m_{23}^2 ranges from the lower limit $(E_2^* + E_3^*)^2 - (\sqrt{E_2^{*2} - m_2^2} + \sqrt{E_3^{*2} - m_3^2})^2$ to the upper limit $(E_2^* + E_3^*)^2 - (\sqrt{E_2^{*2} - m_2^2} - \sqrt{E_3^{*2} - m_3^2})^2$, where $E_2^* = (m_{12}^2 - m_1^2 + m_2^2)/(2m_{12})$ and $E_3^* = (m_0^2 - m_{12}^2 - m_3^2)/(2m_{12})$.

C. Two-body decay amplitudes

The decay amplitudes of $T_{cc}^* \rightarrow DD^*$ and $T_{cc}^* \rightarrow T_{cc} \pi/\gamma$ of Fig. 3 are written as

$$\begin{aligned} i\mathcal{M}_{T_{cc}^* \rightarrow DD^*}^a &= g_{T_{cc}^* D^* D^*} g_{DD^* \pi} g_{D^* D^* \pi} \int \frac{d^4 q}{(2\pi)^4} \frac{-g^{\theta\alpha} + \frac{k_1^\theta k_1^\alpha}{k_1^2}}{k_1^2 - m_{D^*}^2} \frac{1}{q^2 - m_\pi^2} \frac{-g^{\beta b} + \frac{k_2^\beta k_2^b}{k_2^2}}{k_2^2 - m_{D^*}^2} \\ &\quad \times [-\varepsilon_{\mu\nu\alpha\beta} k_0^\mu \varepsilon^\nu(k_0)] [-q^\theta] [-\varepsilon_{uvab} p_2^\mu \varepsilon^v(p_2) k_2^a] \mathcal{F}^2(q^2, m_\pi^2), \\ i\mathcal{M}_{T_{cc}^* \rightarrow T_{cc} \pi^0}^b &= g_{T_{cc}^* D^* D^*} g_{DD^* \pi} g_{T_{cc} D^* D} \int \frac{d^4 q}{(2\pi)^4} \frac{-g^{\theta\alpha} + \frac{k_1^\theta k_1^\alpha}{k_1^2}}{k_1^2 - m_{D^*}^2} \frac{1}{q^2 - m_D^2} \frac{-g^{\beta a} + \frac{k_2^\beta k_2^a}{k_2^2}}{k_2^2 - m_{D^*}^2} \\ &\quad \times [-\varepsilon_{\mu\nu\alpha\beta} k_0^\mu \varepsilon^\nu(k_0)] [-p_1^\theta] [\varepsilon^a(p_2)] \mathcal{F}^2(q^2, m_D^2), \\ i\mathcal{M}_{T_{cc}^* \rightarrow T_{cc} \gamma}^c &= g_{T_{cc}^* D^* D^*} g_{DD^* \gamma} g_{T_{cc} D^* D} \int \frac{d^4 q}{(2\pi)^4} \frac{-g^{b\alpha} + \frac{k_1^b k_1^\alpha}{k_1^2}}{k_1^2 - m_{D^*}^2} \frac{1}{q^2 - m_D^2} \frac{-g^{\beta\theta} + \frac{k_2^\beta k_2^\theta}{k_2^2}}{k_2^2 - m_{D^*}^2} \\ &\quad \times [-\varepsilon_{\mu\nu\alpha\beta} k_0^\mu \varepsilon^\nu(k_0)] [e\varepsilon_{uvab} p_1^\mu \varepsilon^v(p_1) k_1^a] [\varepsilon^\theta(p_2)] \mathcal{F}^2(q^2, m_D^2), \end{aligned} \quad (25)$$

where q and p_1 represent the momenta for the D meson and the T_{cc} state, and ε_μ represents the polarization vector for the state of spin $S = 1$. Here one should note that the amplitudes of Figs. 3(a), 3(b), and 3(c)– are similar to those of Figs. 3(d), 3(g), 3(e), 3(h), and 3(f), 3(i), respectively.

Similarly, the amplitudes of the decays $B_c \rightarrow T_{cc(s)} \bar{D}^{(*)}$ in Fig. 4 are written as

$$\begin{aligned} \mathcal{M}_{a,e,i} &= i^2 \int \frac{d^4 q}{(2\pi)^4} [\mathcal{A}_{\mu\nu}(B_c \rightarrow J/\psi D_{(s)}^*)] [g_{T_{cc} D^* D} \varepsilon_\beta(p_2)] [g_{\psi \bar{D} D}(q - p_1)_\alpha] \\ &\quad \times \frac{-g^{\mu\alpha} + q_1^\mu q_1^\alpha/q_1^2 - g^{\nu\beta} + q_2^\nu q_2^\beta/q_2^2}{q_1^2 - m_1^2} \frac{1}{q^2 - m^2} \mathcal{F}^2(q^2, m^2), \end{aligned} \quad (26)$$

$$\begin{aligned} \mathcal{M}_{a,e,i} &= i^2 \int \frac{d^4 q}{(2\pi)^4} [\mathcal{A}_\mu(B_c \rightarrow J/\psi D_{(s)})] [g_{T_{cc} D^* D} \varepsilon_\beta(p_2)] [-g_{\psi \bar{D} D^*} \varepsilon^{\sigma\alpha\tau\nu} q_{1\sigma} q_\tau] \\ &\quad \times \frac{-g^{\mu\alpha} + q_1^\mu q_1^\alpha/q_1^2}{q_1^2 - m_1^2} \frac{1}{q_2^2 - m_2^2} \frac{-g^{\nu\beta} + q^\nu q^\beta/q^2}{q^2 - m^2} \mathcal{F}^2(q^2, m^2), \end{aligned} \quad (27)$$

$$\begin{aligned} \mathcal{M}_{b,f,j} &= i^2 \int \frac{d^4 q}{(2\pi)^4} [\mathcal{A}(B_c \rightarrow \eta_c D_{(s)})] [g_{T_{cc} D^* D} \varepsilon_\beta(p_2)] [-g_{\eta_c \bar{D} D^*}(q_1 + p_1)_\nu] \\ &\quad \times \frac{1}{q_1^2 - m_1^2} \frac{1}{q_2^2 - m_2^2} \frac{-g^{\nu\beta} + q^\nu q^\beta/q^2}{q^2 - m^2} \mathcal{F}^2(q^2, m^2), \end{aligned} \quad (28)$$

$$\begin{aligned} \mathcal{M}_{c,g,k} = & i^2 \int \frac{d^4 q}{(2\pi)^4} [\mathcal{A}_{\mu\nu}(B_c \rightarrow J/\psi D_{(s)}^*)][g_{T_{cc}D^*D}\varepsilon_\beta(p_2)][-g_{\psi\bar{D}D^*}\varepsilon^{\sigma\alpha\tau\nu}q_{1\sigma}p_{1\tau}\varepsilon_\nu(p_1)] \\ & \times \frac{-g^{\mu\alpha} + q_1^\mu q_1^\alpha/q_1^2}{q_1^2 - m_1^2} \frac{-g^{\nu\beta} + q_2^\nu q_2^\beta/q_2^2}{q_2^2 - m_2^2} \frac{1}{q^2 - m^2} \mathcal{F}^2(q^2, m^2), \end{aligned} \quad (29)$$

$$\begin{aligned} \mathcal{M}_{c,g,k} = & i^2 \int \frac{d^4 q}{(2\pi)^4} [\mathcal{A}_\mu(B_c \rightarrow J/\psi D_{(s)})][g_{T_{cc}D^*D}\varepsilon_\beta(p_2)] \\ & \times [-g_{\psi\bar{D}^*D^*}\varepsilon^\sigma(p_1)(g_{\nu\sigma}(p_1 - q)_\alpha + g_{a\nu}(q_1 + q)_\sigma - g_{a\sigma}(p_1 + q_1)_\nu)] \\ & \times \frac{-g^{\mu\alpha} + q_1^\mu q_1^\alpha/q_1^2}{q_1^2 - m_1^2} \frac{1}{q_2^2 - m_2^2} \frac{-g^{\nu\beta} + q^\nu q^\beta/q^2}{q^2 - m^2} \mathcal{F}^2(q^2, m^2), \end{aligned} \quad (30)$$

$$\begin{aligned} \mathcal{M}_{d,h,l} = & i^2 \int \frac{d^4 q}{(2\pi)^4} [\mathcal{A}_\nu(B_c \rightarrow \eta_c D_{(s)}^*)][g_{T_{cc}D^*D}\varepsilon_\beta(p_2)][g_{\eta_c\bar{D}^*D}(q_1 + q)_\sigma\varepsilon^\sigma(p_1)] \\ & \times \frac{1}{q_1^2 - m_1^2} \frac{-g^{\nu\beta} + q_2^\nu q_2^\beta/q_2^2}{q_2^2 - m_2^2} \frac{1}{q^2 - m^2} \mathcal{F}^2(q^2, m^2), \end{aligned} \quad (31)$$

$$\begin{aligned} \mathcal{M}_{d,h,l} = & i^2 \int \frac{d^4 q}{(2\pi)^4} [\mathcal{A}(B_c \rightarrow \eta_c D_{(s)})][g_{T_{cc}D^*D}\varepsilon_\beta(p_2)][g_{\eta_c\bar{D}^*D^*}\varepsilon_{\mu\nu\alpha\sigma}q^\mu p_1^\alpha\varepsilon^\sigma(p_1)] \\ & \times \frac{1}{q_1^2 - m_1^2} \frac{1}{q_2^2 - m_2^2} \frac{-g^{\nu\beta} + q^\nu q^\beta/q^2}{q^2 - m^2} \mathcal{F}^2(q^2, m^2), \end{aligned} \quad (32)$$

and the amplitudes of the decays $B_c \rightarrow T_{cc(s)}^* \bar{D}^{(*)}$ in Fig. 4 are written as

$$\begin{aligned} \mathcal{M}_{m,q,u} = & i^2 \int \frac{d^4 q}{(2\pi)^4} [\mathcal{A}_{\mu\nu}(B_c \rightarrow J/\psi D_{(s)}^*)][g_{T_{cc}D^*D^*}\varepsilon_{\phi\theta\beta\omega}p_2^\phi\varepsilon^\theta(p_2)][-g_{\psi\bar{D}D^*}\varepsilon^{\sigma\alpha\rho}q_{1\sigma}q_\tau] \\ & \times \frac{-g^{\mu\alpha} + q_1^\mu q_1^\alpha/q_1^2}{q_1^2 - m_1^2} \frac{-g^{\nu\beta} + q_2^\nu q_2^\beta/q_2^2}{q_2^2 - m_2^2} \frac{-g^{\rho\omega} + q^\rho q^\omega/q^2}{q^2 - m^2} \mathcal{F}^2(q^2, m^2), \end{aligned} \quad (33)$$

$$\begin{aligned} \mathcal{M}_{n,r,v} = & i^2 \int \frac{d^4 q}{(2\pi)^4} [\mathcal{A}_\nu(B_c \rightarrow \eta_c D_{(s)}^*)][g_{T_{cc}D^*D^*}\varepsilon_{\phi\theta\beta\omega}p_2^\phi\varepsilon^\theta(p_2)][-g_{\eta_c\bar{D}D^*}(q_1 + p_1)_\rho] \\ & \times \frac{1}{q_1^2 - m_1^2} \frac{-g^{\nu\beta} + q_2^\nu q_2^\beta/q_2^2}{q_2^2 - m_2^2} \frac{-g^{\rho\omega} + q^\rho q^\omega/q^2}{q^2 - m^2} \mathcal{F}^2(q^2, m^2), \end{aligned} \quad (34)$$

$$\begin{aligned} \mathcal{M}_{o,s,w} = & i^2 \int \frac{d^4 q}{(2\pi)^4} [\mathcal{A}_{\mu\nu}(B_c \rightarrow J/\psi D_{(s)}^*)][g_{T_{cc}D^*D^*}\varepsilon_{\phi\theta\beta\omega}p_2^\phi\varepsilon^\theta(p_2)] \\ & \times [-g_{\psi\bar{D}^*D^*}\varepsilon^\sigma(p_1)(g_{a\sigma}(p_1 - q)_\rho + g_{\rho\alpha}(q_1 + q)_\sigma - g_{\rho\sigma}(p_1 + q_1)_\alpha)] \\ & \times \frac{-g^{\mu\alpha} + q_1^\mu q_1^\alpha/q_1^2}{q_1^2 - m_1^2} \frac{-g^{\nu\beta} + q_2^\nu q_2^\beta/q_2^2}{q_2^2 - m_2^2} \frac{-g^{\rho\omega} + q^\rho q^\omega/q^2}{q^2 - m^2} \mathcal{F}^2(q^2, m^2), \end{aligned} \quad (35)$$

$$\begin{aligned} \mathcal{M}_{p,t,x} = & i^2 \int \frac{d^4 q}{(2\pi)^4} [\mathcal{A}_\nu(B_c \rightarrow \eta_c D_{(s)}^*)][g_{T_{cc}D^*D^*}\varepsilon_{\phi\theta\beta\omega}p_2^\phi\varepsilon^\theta(p_2)][g_{\eta_c\bar{D}^*D^*}\varepsilon^{\sigma\alpha\rho}p_{1\sigma}q_\tau\varepsilon_\alpha(p_1)] \\ & \times \frac{1}{q_1^2 - m_1^2} \frac{-g^{\nu\beta} + q_2^\nu q_2^\beta/q_2^2}{q_2^2 - m_2^2} \frac{-g^{\rho\omega} + q^\rho q^\omega/q^2}{q^2 - m^2} \mathcal{F}^2(q^2, m^2), \end{aligned} \quad (36)$$

where the $\mathcal{F}(q^2, m^2)$ is the form factor. The form factor not only reduces the divergence of loop integrals but also estimates the internal structures of hadron interaction vertices. The form factor is written as $\mathcal{F}(q^2, m^2) = \frac{\Lambda^2 - m^2}{\Lambda^2 - q^2}$, where the cutoff Λ is further parameterized as dimensionless parameter τ , e.g., $\Lambda = m + \tau\Lambda_{\text{QCD}}$.

TABLE II. Masses and quantum numbers of mesons relevant to the present work [118].

Meson	$I(J^P)$	M (MeV)	Meson	$I(J^P)$	M (MeV)	Meson	$I(J^P)$	M (MeV)
π^0	$1(0^-)$	134.977	$\pi^\pm$	$1(0^-)$	139.570	π	$1(0^-)$	138.039
K^0	$\frac{1}{2}(0^-)$	497.611	$K^\pm$	$\frac{1}{2}(0^-)$	493.677	K	$\frac{1}{2}(0^-)$	495.644
D^0	$\frac{1}{2}(0^-)$	1864.84	$D^\pm$	$\frac{1}{2}(0^-)$	1869.66	D	$\frac{1}{2}(0^-)$	1867.25
D^{*0}	$\frac{1}{2}(1^-)$	2006.85	$D^{*\pm}$	$\frac{1}{2}(1^-)$	2010.26	D^*	$\frac{1}{2}(1^-)$	2008.56
$D_s^\pm$	$0(0^-)$	1968.35	$D_s^{*\pm}$	$0(1^-)$	2112.2	T_{cc}	$0(1^+)$	3874.74
$B^\pm$	$\frac{1}{2}(0^-)$	5279.34	B^0	$\frac{1}{2}(0^-)$	5279.66	B_c	$0(0^-)$	6274.47
η_c	$0(0^-)$	2984.1	J/ψ	$0(1^-)$	3096.90	B_c	$0(0^-)$	6274.47

TABLE III. Masses of DD^* , D^*D^* , DD_s^* , and $D^*D_s^*$ with $J^P = 1^+$ for a cutoff of $\Lambda = 1$ and $\Lambda = 0.5$ GeV.

Molecule	I	J^P	m (MeV)	Experimental data (MeV)
DD^*	0	1^+	Input	3874.74
D^*D^*	0	1^+	4015.28–4015.70	...
D_sD^*	$\frac{1}{2}$	1^+	3978.23–3978.35/3974.55–3974.75	...
$D_s^*D^*$	$\frac{1}{2}$	1^+	4119.98–4120.78/4116.60–4117.38	...

With the amplitudes of two-body decays, one can calculate the corresponding partial decay widths as

$$\Gamma = \frac{1}{2J+1} \frac{1}{8\pi m^2} |\vec{p}| \overline{|\mathcal{M}|^2}, \quad (37)$$

where J is the total angular momentum of the initial state, the overline indicates the sum over the polarization vectors of final states, and $|\vec{p}|$ is the momentum of either final state in the rest frame of initial state.

III. RESULTS AND DISCUSSION

In Table II, we tabulate the masses and quantum numbers of relevant particles involved in this work. In the heavy quark and SU(3)-flavor symmetry limits, the contact-range potentials of $I(J^P) = 0(1^+) DD^*$, $I(J^P) = 0(1^+) D^*D^*$, $I(J^P) = 1/2(1^+) D_sD^*$, and $I(J^P) = 1/2(1^+) D_s^*D^*$ systems are identical. By treating the doubly charmed tetraquark state T_{cc} as a DD^* bound state, we first determine the potential value and subsequently predict the masses of the other five states as shown in Table III. Here we vary the

cutoff in the loop integral in Eq. (3) from 1 to 2 GeV, and the corresponding results indicate that the pole positions are weakly dependent on the cutoff. Our results show that there exist five additional doubly charmed tetraquark states, which are slightly more bound than the T_{cc} state. The majority of theoretical studies suggested the existence of an isoscalar $I(J^P) = 0(1^+) D^*D^*$ molecular bound state [40,44,58,70–72]. However, the existence of $I(J^P) = 1/2(1^+) D_sD^*$ and $I(J^P) = 1/2(1^+) D_s^*D^*$ bound states have large uncertainty [70–73].² If taking into account the uncertainty of SU(3)-flavor symmetry of 20% [85], we find that the $I(J^P) = 1/2(1^+) D_sD^*$ and $I(J^P) = 1/2(1^+) D_s^*D^*$ systems are hardly to bind for their weaker potentials. The SU(3)-flavor symmetry-breaking effects for hadron-hadron interactions require further experimental confirmation. In the following, we assume that there exist six hadronic molecules as shown in Table III, and then present a systematic investigation of their partial decay channels and production mechanism in B_c decays, offering additional insights for experimental measurements.

In Table IV, we present the partial decay widths of the DD^* and D_sD^* molecules. Our results indicate that the partial decay widths $T_{cc}^+ \rightarrow D^0D^0\pi^+$, $T_{cc}^+ \rightarrow D^+D^0\pi^0$, and $T_{cc}^+ \rightarrow D^+D^0\gamma$ are around 23.69, 19.82, and 9.40 keV,

TABLE IV. Partial decay widths of DD^* and D_sD^* molecules.

Molecule	$D^0D^0\pi^+$	$D^0D^+\pi^0$	$D^0D^+\gamma$	Total
DD^* (keV)	23.69	19.82	9.40	52.91
Molecule	$D_s^+D^0\pi^0$	$D_s^+D^0\gamma$		Total
D_sD^* (keV)	8.00	12.91		20.91
Molecule	$D_s^+D^0\pi^+$	$D_s^+D^+\pi^0$	$D_s^+D^0\gamma$	Total
D_sD^* (keV)	16.82	7.40	1.04	25.26

²A recent Lattice QCD study showed that the potentials of the $I(J^P) = 1/2(1^+) D_sD^*$ and $I(J^P) = 1/2(1^+) D_s^*D^*$ systems are too weak to form hadronic molecules [117]. It should be noted that the DD^* is viewed as a virtual state in similar lattice QCD group [54], which indicates that the DD^* interaction derived by lattice QCD is less attractive than other models. Therefore, the D^*D_s interaction simulated by lattice QCD may have similar situation to other models.

TABLE V. Partial decay widths of D^*D^* and $D^*D_s^*$ molecules.

Molecule	D^0D^{*+}	D^+D^{*0}	$T_{cc}\pi^0$	$T_{cc}\gamma$	$D^{*+}D^0\pi^0$	$D^{*0}D^0\pi^+$	$D^{*0}D^+\pi^0$	$D^{*0}D^+\gamma$	$D^{*+}D^0\gamma$	Total
D^*D^* (keV)	1215.76	1215.76	13.41	20.89	10.12	11.61	4.84	1.98	31.64	2526.01
Molecule	$D^0D_s^{*+}$	$D^{*0}D_s^+$	$T_{ccs}^+\pi^0$	$T_{ccs}^+\gamma$	$D_s^{*+}D^0\pi^0$	$D_s^+D^{*0}\pi^0$		$D^{*0}D_s^+\gamma$	$D_s^{*+}D^0\gamma$	Total
$D_s^*D^*$ (keV)	694.18	656.66	27.55	13.96	6.07	0.001		0.09	29.85	1428.36
Molecule	$D^+D_s^{*+}$	$D^{*+}D_s^+$	$T_{ccs}^{++}\pi^0$	$T_{ccs}^{++}\gamma$	$D_s^{*+}D^+\pi^0$	$D_s^+D^{*0}\pi^+$	$D^{*+}D_s^+\pi^0$	$D^{*+}D_s^+\gamma$	$D_s^{*+}D^+\gamma$	Total
$D_s^*D^*$ (keV)	687.78	654.28	20.19	1.12	2.36	5.88	0.001	0.09	1.85	1373.55

respectively. The total width of T_{cc} is around 52.91 keV, consistent with the experimental data [39], which supports the molecular nature of T_{cc} from the perspective of its decays. For the SU(3)-flavor partners of the DD^* molecules, the total widths of T_{ccs}^+ and T_{ccs}^{++} are around 20.91 and 25.26 keV, respectively, almost half of the width of T_{cc} . The smaller widths of T_{ccs}^+ and T_{ccs}^{++} are caused by the particularly weak couplings of $g_{D_s^*D_s\pi}$ and $g_{D_s^*D_s\gamma}$. The T_{ccs}^+ molecule mainly decays into $D_s^+D^0\pi^0$ and $D_s^+D^0\gamma$, which are difficult to observe by LHCb because of the low efficiency of observing π^0 and γ . In addition to decaying into the above channels, the T_{ccs}^{++} molecule dominantly decays into $D_s^+D^0\pi^+$, which is a favored channel for experimental observation with large branching fraction and high detection efficiency. Therefore, we suggest searching for the T_{ccs}^{++} molecule in $D_s^+D^0\pi^+$ final states.

The two-body decay modes of the T_{cc}^* molecule proceed via the triangle diagram mechanisms, where an unknown parameter τ in the form factors is introduced. The FSI approach is employed to study heavy hadron decays, where the parameter τ varies from 1 to 2 [31,119]. The meson exchange model is used to compute the masses of hadronic molecular states or molecular candidates of exotic states by treating them as bound states [16]. By reproducing the masses of the deuteron, $X(3872)$, and $P_c(4312)$, which are assumed to be bound states of NN , $\bar{D}^*D$, and $\bar{D}\Sigma_c$, the values of the parameter τ were found to be 1.361, 1.468, and 1.416, respectively. Therefore, we take the parameter τ from 1 to 2 in this work. In Table V, we present the partial decay widths of the D^*D^* and $D^*D_s^*$ molecules at $\tau = 1$, whose widths are much larger than their HQSS partners since two-body decays are allowed. We find that the total width of the T_{cc}^* molecule is around 2.5 MeV, consistent with the value calculated in Ref. [70]. As shown in Table V, the three-body partial decay widths of the T_{cc}^* molecule vary from several to tens keV, and the dominant three-body decay is $D^{*+}D^0\gamma$. Taking into account the branching fractions of $\mathcal{B}(T_{cc} \rightarrow D^0D^0\pi^+) = 0.45$, $\mathcal{B}(T_{cc} \rightarrow D^0D^+\pi^0) = 0.37$, and $\mathcal{B}(T_{cc} \rightarrow D^0D^+\gamma) = 0.18$, we can estimate the four-body partial widths of T_{cc}^* molecule, i.e., $\Gamma(T_{cc}^* \rightarrow D^0D^0\pi^+\pi^0) = 6.03$ keV, $\Gamma(T_{cc}^* \rightarrow D^0D^+\pi^0\pi^0) = 4.96$ keV, $\Gamma(T_{cc}^* \rightarrow D^0D^0\pi^+\gamma) = 9.40$ keV, $\Gamma(T_{cc}^* \rightarrow D^0D^+\pi^0\gamma) = 10.14$ keV, and $\Gamma(T_{cc}^* \rightarrow D^0D^+\gamma\gamma) = 3.76$ keV. In Refs. [120,121], the

three-body and four-body pionic and radiative decays of the T_{cc}^* molecule were investigated; their results are a bit larger than ours. The predicted binding energy of the T_{cc}^* state is about 0.5 MeV [120,121], compared to approximately 1 MeV in our work, which implies that the phase space available for three-body and four-body decays of T_{cc}^* is larger in Refs. [120,121] than in our calculation. Additionally, final-state rescattering effects of the tree-level diagrams have been included in those references, further enhancing the contributions from three- and four-body decay channels of T_{cc}^* . The branching fractions of the two-body decay $T_{cc}^* \rightarrow DD^*$ are around 96%, which indicates that the promising channel of experimental search for the T_{cc}^* molecule are D^0D^{*+} and D^+D^{*0} channels because the branching fractions of three-body and four-body decays are much smaller than those of two-body decays.

The $D_s^*D^*$ and $D^*D_s^*$ molecules have similar decay mechanisms, while the decay widths of the $D_s^*D^*$ molecules are around half of the D^*D^* molecule, as shown in Table V. For the two-body decays, the FSIs $D^*D^* \rightarrow DD^*$ and $D_s^*D^* \rightarrow D_sD^*/D_s^*D$ processes proceed by one pion exchange and one kaon exchange, respectively, where the heavier kaon mass suppresses the two-body partial decay widths of the $D^*D_s^*$ molecules. Moreover, the violation of isospin conservation in the strong decay $D_s^* \rightarrow D_s\pi$, along with the narrow width of the D_s^* state, naturally leads to small coupling constants for $g_{D_s^*D_s\pi}$ and $g_{D_s^*D_s\gamma}$. This, in turn, leads to very small partial widths for the $D_s^*D^*$ molecules decaying into $D^*D_s\pi$ and $D^*D_s\gamma$, as shown in Table V. However, the width of the decay $T_{ccs}^{++} \rightarrow D_s^{*+}D^0\gamma$ is similar to that of $T_{cc}^* \rightarrow D^{*+}D^0\gamma$ due to the same decay mechanism, which proceed via the decay of $D^{*0} \rightarrow D^0\gamma$. The branching fractions $\mathcal{B}(T_{ccs}^{++} \rightarrow D_sD^*/D_s^*D) = 95\%$ and $\mathcal{B}(T_{ccs}^{++} \rightarrow D_sD^*/D_s^*D) = 98\%$ indicate that the T_{ccs}^+ and T_{ccs}^{++} molecules dominantly decay into D_sD^*/D_s^*D , similar to the decays of T_{cc}^* . Therefore, the two-body channels are thus promising for experimental searches of the $D_s^*D^*$ molecules.

Unlike the decays of B mesons, the experimental observation of B_c meson decay modes remains very limited. In this work, we employ the naive factorization approach to predict the branching fractions of $B_c \rightarrow J/\psi(\eta_c)\bar{D}_s^{(*)}$ decays as shown in Table VI [104]. We find that the branching fractions for $B_c \rightarrow J/\psi(\eta_c)D^{(*)}$ are

TABLE VI. Branching fractions of the decays $B_c \rightarrow J/\psi \bar{D}_s^{(*)}$ and $B_c \rightarrow \eta_c \bar{D}_s^{(*)}$ as well as the branching fractions of the decays $B_c \rightarrow T_{cc}^{(*)} \bar{D}^{(*)}$ and $B_c \rightarrow T_{ccs}^{(*)} \bar{D}^{(*)}$.

Decay channels	$\eta_c D^+$	$\eta_c D^{*+}$	$J/\psi D^+$	$J/\psi D^{*+}$
Branching fraction (10^{-4})	2.30	2.02	1.09	4.92
Decay channels	$\eta_c D_s^+$	$\eta_c D_s^{*+}$	$J/\psi D_s^+$	$J/\psi D_s^{*+}$
Branching fraction (10^{-3})	5.91	4.53	2.62	12.34
Decay channels	$T_{cc} \bar{D}^0$	$T_{cc} \bar{D}^{*0}$	$T_{cc}^* \bar{D}^0$	$T_{cc}^* \bar{D}^{*0}$
Branching fraction (10^{-7})	0.82–2.07	5.92–14.18	8.48–20.61	3.12–9.28
Decay channels	$T_{ccs}^+ \bar{D}^0$	$T_{ccs}^+ \bar{D}^{*0}$	$T_{ccs}^{*+} \bar{D}^0$	$T_{ccs}^{*+} \bar{D}^{*0}$
Branching fraction (10^{-5})	0.55–1.24	1.52–3.70	1.98–4.88	0.56–1.80
Decay channels	$T_{ccs}^{++} D^-$	$T_{ccs}^{++} D^{*-}$	$T_{ccs}^{*++} D^-$	$T_{ccs}^{*++} D^{*-}$
Branching fraction (10^{-5})	0.53–1.18	1.18–2.87	1.87–4.62	0.53–1.72

typically of order 10^{-4} , while those for $B_c \rightarrow J/\psi(\eta_c)D_s^{(*)}$ are about 20 times larger, such a ratio is consistent with $V_{cs}^2/V_{cd}^2 \sim 18.5$. Then, we predict the branching fractions of the decays $B_c \rightarrow T_{cc(s)}^* \bar{D}_{(s)}^{(*)}$ as a function of the parameter τ varying from 1 to 2 as shown in Table VI. The branching fraction of the decay $B_c \rightarrow T_{cc} \bar{D}^0$ is $(0.82\text{--}2.07) \times 10^{-7}$, almost fifty times smaller than those of the decays $B_c \rightarrow T_{ccs}^+ \bar{D}^0$ and $B_c \rightarrow T_{ccs}^{++} D^-$, consistent with Ref. [75]. Similarly, the branching fractions of the decays of $B_c \rightarrow T_{ccs} \bar{D}^*$, $B_c \rightarrow T_{ccs}^* \bar{D}$, and $B_c \rightarrow T_{ccs}^* \bar{D}^*$ are 2 orders of magnitude larger than those of the decays of $B_c \rightarrow T_{cc} \bar{D}^*$, $B_c \rightarrow T_{cc}^* \bar{D}$, and $B_c \rightarrow T_{cc}^* \bar{D}^*$, respectively. Our findings indicate that the yields of doubly charmed tetraquarks with strangeness $S = -1$ are relatively high in B_c decays, primarily due to the large value of the CKM matrix element V_{cs} . The branching fraction of the decays $B_c \rightarrow T_{cc(s)} \bar{D}^*$ are similar to those of the decays

$B_c \rightarrow T_{cc(s)}^* \bar{D}$, larger than those of the decays $B_c \rightarrow T_{cc(s)}^* \bar{D}^*$ and $B_c \rightarrow T_{cc(s)} \bar{D}$. It should be noted that the branching fractions of the decay $B_c \rightarrow T_{cc(s)}^* \bar{D}^{(*)}$ decays are about 2 orders of magnitude smaller than those of their corresponding two-body weak decays $B_c \rightarrow \eta_c/J/\psi D_{(s)}^{(*)}$. This ratio is consistent with the observed pattern in the decays $B \rightarrow X(3872)K$ [$X(3872)$ as a $\bar{D}^* D$ molecule] relative to their two-body weak decays $B \rightarrow \bar{D}^{(*)} D_s^{(*)}$ [87].

For the experimental measurements, we must account for both the decay channels of the $T_{cc(s)}^{(*)}$ state and the detection efficiency of the experiment. Taking into account the dominant branching fraction of the T_{cc} state as shown in Table VII, we estimate the branching fractions of the decays $B_c \rightarrow (T_{cc} \rightarrow D^0 D^0 \pi^+) \bar{D}^0$ and $B_c \rightarrow (T_{cc} \rightarrow D^0 D^0 \pi^+) \bar{D}^{*0}$ being around $(0.4\text{--}1) \times 10^{-7}$ and $(3\text{--}7) \times 10^{-7}$, respectively. However, the latter decay

TABLE VII. Branching fractions of the B_c meson decays into the doubly charmed tetraquark states and charmed mesons, along with the decays of the doubly charmed tetraquark states.

Decay channels	$T_{cc} \rightarrow D^0 D^0 \pi^+$	$B_c \rightarrow (T_{cc} \rightarrow D^0 D^0 \pi^+) \bar{D}^0$	$B_c \rightarrow (T_{cc} \rightarrow D^0 D^0 \pi^+) \bar{D}^{*0}$
Branching fraction	0.45	$(0.4\text{--}1) \times 10^{-7}$	$(3\text{--}7) \times 10^{-7}$
Decay channels	$T_{cc}^* \rightarrow D^0 D^{*+}$	$B_c \rightarrow (T_{cc}^* \rightarrow D^0 D^{*+}) \bar{D}^0$	$B_c \rightarrow (T_{cc}^* \rightarrow D^0 D^{*+}) \bar{D}^{*0}$
Branching fraction	0.48	$(4\text{--}10) \times 10^{-7}$	$(2\text{--}5) \times 10^{-7}$
Decay channels	$T_{cc}^* \rightarrow D^{*0} D^+$	$B_c \rightarrow (T_{cc}^* \rightarrow D^{*0} D^+) \bar{D}^0$	$B_c \rightarrow (T_{cc}^* \rightarrow D^{*0} D^+) \bar{D}^{*0}$
Branching fraction	0.48	$(4\text{--}10) \times 10^{-7}$	$(2\text{--}5) \times 10^{-7}$
Decay channels	$T_{ccs}^+ \rightarrow D_s^+ D^0 \gamma$	$B_c \rightarrow (T_{ccs}^+ \rightarrow D_s^+ D^0 \gamma) \bar{D}^0$	$B_c \rightarrow (T_{ccs}^+ \rightarrow D_s^+ D^0 \gamma) \bar{D}^{*0}$
Branching fraction	0.62	$(3\text{--}8) \times 10^{-6}$	$(1\text{--}2) \times 10^{-5}$
Decay channels	$T_{ccs}^{++} \rightarrow D_s^+ D^0 \pi^+$	$B_c \rightarrow (T_{ccs}^{++} \rightarrow D_s^+ D^0 \pi^+) D^-$	$B_c \rightarrow (T_{ccs}^{++} \rightarrow D_s^+ D^0 \pi^+) D^{*-}$
Branching fraction	0.67	$(4\text{--}8) \times 10^{-6}$	$(1\text{--}2) \times 10^{-5}$
Decay channels	$T_{ccs}^{*+} \rightarrow D_s^+ D^{*0}$	$B_c \rightarrow (T_{ccs}^{*+} \rightarrow D_s^+ D^{*0}) \bar{D}^0$	$B_c \rightarrow (T_{ccs}^{*+} \rightarrow D_s^+ D^{*0}) \bar{D}^{*0}$
Branching fraction	0.49	$(1\text{--}2) \times 10^{-5}$	$(0.5\text{--}1) \times 10^{-5}$
Decay channels	$T_{ccs}^{*++} \rightarrow D_s^+ D^{*+}$	$B_c \rightarrow (T_{ccs}^{*++} \rightarrow D_s^+ D^{*+}) D^-$	$B_c \rightarrow (T_{ccs}^{*++} \rightarrow D_s^+ D^{*+}) D^{*-}$
Branching fraction	0.50	$(1\text{--}2) \times 10^{-5}$	$(0.5\text{--}1) \times 10^{-5}$

channel is not recommended experimentally, as the $\bar{D}^{*0}$ meson must be reconstructed via $\bar{D}^0$ and π^0 or γ , and the π^0 meson and γ photon have low detection efficiency at the LHCb. Therefore, we suggest observing the T_{cc} state in the decay $B_c \rightarrow D^0 D^0 \pi^+ \bar{D}^0$. Combining the dominant branching fractions of the T_{cc}^* state, we obtain the branching fractions of the decays $B_c \rightarrow (T_{cc}^* \rightarrow D^0 D^{*+}) \bar{D}^0$ and $B_c \rightarrow (T_{cc}^* \rightarrow D^{*0} D^+) \bar{D}^0$ being around $(4-10) \times 10^{-7}$, and those of the decays $B_c \rightarrow (T_{cc}^* \rightarrow D^{*0} D^+) \bar{D}^{*0}$ and $B_c \rightarrow (T_{cc}^* \rightarrow D^{*0} D^+) \bar{D}^{*0}$ being around $(2-5) \times 10^{-7}$. If neglecting the decay modes with the D^{*0} and $\bar{D}^{*0}$ mesons, we suggest searching for the T_{cc}^* in the decay $B_c \rightarrow D^0 D^{*+} \bar{D}^0$. As shown in Table VII, we estimate the branching fractions of the decays associated with T_{ccs}^+ and T_{ccs}^{++} . Our results suggest searching for the T_{ccs}^{++} state in the decay $B_c \rightarrow D_s^+ D^0 \pi^+ D^-$ or the decay $B_c \rightarrow D_s^+ D^0 \pi^+ D^{*-}$. However, the observation of T_{ccs}^+ in B_c decays for the LHCb experiment is challenged due to the final states with both π^0 meson and γ photon. For the corresponding results of T_{ccs}^+ and T_{ccs}^{++} in Table VII, it is likely to be observed T_{ccs}^{++} in the decay $B_c \rightarrow D_s^+ D^{*+} D^-$, while it is difficult to be observed T_{ccs}^{++} in B_c decay.

Taking into account the branching fraction $\mathcal{B}(D^{*+} \rightarrow D^0 \pi^+) = 0.677$, we obtain the branching fractions for B_c meson decaying into the doubly charmed tetraquark states T_{cc} , T_{cc}^* , T_{ccs}^{++} , and T_{ccs}^{+++} accompanied by several pseudoscalar mesons to be approximately $(0.4-1) \times 10^{-7}$, $(3-7) \times 10^{-7}$, $(4-8) \times 10^{-6}$, and $(0.7-1.4) \times 10^{-5}$, respectively. Furthermore, the D_s^+ meson, D^0 meson, and D^- meson are experimentally reconstructed by the decays of $D_s^+ \rightarrow K^+ K^- \pi^+$, $D^0 \rightarrow K^- \pi^+$, and $D^- \rightarrow K^+ \pi^- \pi^-$. Therefore, with the branching fractions $\mathcal{B}(D_s^+ \rightarrow K^+ K^- \pi^+) = 0.05$, $\mathcal{B}(D^0 \rightarrow K^- \pi^+) = 0.04$, and $\mathcal{B}(D^- \rightarrow K^+ \pi^- \pi^-) = 0.1$, we can estimate the production rates of T_{cc} , T_{cc}^* , T_{ccs}^{++} , and T_{ccs}^{+++} in B_c decays are around 10^{-12} , 10^{-11} , 10^{-9} , and 10^{-9} , respectively. The cross sections of the B_c meson is around $\sigma(B_c) \sim 100$ nb at the LHC [122], and the event number of the B_c meson would be expected to be around 10^{10} at run 3 and 10^{11} at high-luminosity LHC (HL LHC) [123]. Therefore, the T_{ccs}^{++} and T_{ccs}^{+++} states are expected to be observed in B_c meson decays during LHC Run 3 and the HL-LHC, whereas the experimental detection of T_{cc} and T_{cc}^* remains challenging.

IV. SUMMARY

The study of exotic states provides a pathway to understanding the nonperturbative strong interaction, thereby elucidating the mechanisms by which these states are formed. Up to now, the only experimentally observed exotic state with doubly charmed quarks is the T_{cc} discovered by the LHCb Collaboration. Recent studies have indicated that the properties (mass and width) of the T_{cc} state are consistent with it being a DD^* hadronic molecule, where the nonperturbative interaction, i.e., hadron-hadron interaction, plays a dominant role. In other

words, the discovery of T_{cc} provides the chance to study the DD^* interaction. In this work, we employed the contact-range EFT approach to predict the interactions in SU(3)-flavor and heavy-quark spin symmetry partners of DD^* , along with their corresponding pole positions. To better support experimental measurements for doubly charmed tetraquark states, we further investigated their partial decay widths and production mechanism in B_c decays.

Our results suggest the existence of three doubly charmed hadronic molecules, T_{cc} , T_{ccs}^+ , and T_{ccs}^{++} , which are linked by SU(3)-flavor symmetry, along with their HQSS partners, namely, T_{cc}^* , T_{ccs}^{++} , and T_{ccs}^{+++} . However, the existence of doubly charmed tetraquark states with strangeness is associated with a large uncertainty. Based on the molecular picture, the T_{cc} , T_{ccs}^+ , and T_{ccs}^{++} states decay predominantly into the three-body final states $D^0 D^0 \pi^+ / D^0 D^+ \pi^0$, $D_s^+ D^0 \gamma / D_s^+ D^0 \pi^0$, and $D_s^+ D^0 \pi^+ / D_s^+ D^+ \pi^0$, respectively. In contrast, the T_{cc}^* , T_{ccs}^{++} , and T_{ccs}^{+++} states decay mainly into the two-body final states $D^0 D^{*+} / D^+ D^{*0}$, $D^0 D_s^{*+} / D_s^+ D^{*0}$, and $D^+ D_s^{*+} / D_s^+ D^{*+}$, respectively. The production rates for T_{cc} and T_{ccs} in B_c decays are around 10^{-7} and 10^{-5} , respectively. For T_{cc}^* and T_{ccs}^+ , the corresponding values are around 10^{-6} and 10^{-5} . Combining the detection efficiency of final states, we estimated the branching fractions of $\mathcal{B}[B_c \rightarrow (T_{cc} \rightarrow D^0 D^0 \pi^+) \bar{D}^0]$, $\mathcal{B}[B_c \rightarrow (T_{cc}^* \rightarrow D^0 D^0 \pi^+) \bar{D}^0]$, $\mathcal{B}[B_c \rightarrow (T_{ccs}^{++} \rightarrow D_s^+ D^0 \pi^+) D^-]$, $\mathcal{B}[B_c \rightarrow (T_{ccs}^{+++} \rightarrow D_s^+ D^0 \pi^+) D^-]$ are around 10^{-12} , 10^{-11} , 10^{-9} , and 10^{-9} , respectively. Based on the cross section of the B_c meson at the LHC, a significant number of events is expected—around 10^{10} during Run 3 and 10^{11} at the HL-LHC. Therefore, our calculations suggest that the T_{ccs}^{++} and T_{ccs}^{+++} states could be observed in B_c decays in LHC Run 3 and at the HL-LHC, while the experimental discovery of T_{cc} and T_{cc}^* remains challenging. Such investigations can test the molecular nature of the T_{cc} and provide useful guidance for experimental searches for doubly charmed tetraquark states in B_c decays.

ACKNOWLEDGMENTS

This work is partly supported by the National Key R&D Program of China under Grant No. 2023YFA1606703 and the National Natural Science Foundation of China under Grants No. 12435007 and No. W2543006. M.Z.L. acknowledges support from the National Natural Science Foundation of China under Grant No. 12575086. Y.Z. acknowledges support from the National Natural Science Foundation of China under Grants No. 12347182 and the Project funded by China Postdoctoral Science Foundation 2023M740190.

DATA AVAILABILITY

The data that support the findings of this article are openly available [94,116].

- [1] M. Gell-Mann, *Phys. Lett.* **8**, 214 (1964).
- [2] E. Eichten, K. Gottfried, T. Kinoshita, J. B. Kogut, K. D. Lane, and T.-M. Yan, *Phys. Rev. Lett.* **34**, 369 (1975); **36**, 1276(E) (1976).
- [3] G. S. Bali, *Phys. Rep.* **343**, 1 (2001).
- [4] N. Brambilla *et al.*, *Eur. Phys. J. C* **71**, 1534 (2011).
- [5] S. L. Olsen, T. Skwarnicki, and D. Zieminska, *Rev. Mod. Phys.* **90**, 015003 (2018).
- [6] N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C.-P. Shen, C. E. Thomas, A. Vairo, and C.-Z. Yuan, *Phys. Rep.* **873**, 1 (2020).
- [7] H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, *Phys. Rep.* **639**, 1 (2016).
- [8] R. F. Lebed, R. E. Mitchell, and E. S. Swanson, *Prog. Part. Nucl. Phys.* **93**, 143 (2017).
- [9] E. Oset *et al.*, *Int. J. Mod. Phys. E* **25**, 1630001 (2016).
- [10] A. Esposito, A. Pilloni, and A. D. Polosa, *Phys. Rep.* **668**, 1 (2017).
- [11] Y. Dong, A. Faessler, and V. E. Lyubovitskij, *Prog. Part. Nucl. Phys.* **94**, 282 (2017).
- [12] F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, and B.-S. Zou, *Rev. Mod. Phys.* **90**, 015004 (2018); **94**, 029901(E) (2022).
- [13] A. Ali, J. S. Lange, and S. Stone, *Prog. Part. Nucl. Phys.* **97**, 123 (2017).
- [14] M. Karliner, J. L. Rosner, and T. Skwarnicki, *Annu. Rev. Nucl. Part. Sci.* **68**, 17 (2018).
- [15] F.-K. Guo, X.-H. Liu, and S. Sakai, *Prog. Part. Nucl. Phys.* **112**, 103757 (2020).
- [16] M.-Z. Liu, Y.-W. Pan, Z.-W. Liu, T.-W. Wu, J.-X. Lu, and L.-S. Geng, *Phys. Rep.* **1108**, 1 (2025).
- [17] N. Hüsken, E. S. Norella, and I. Polyakov, *Mod. Phys. Lett. A* **40**, 2530002 (2025).
- [18] Z.-G. Wang, *Front. Phys.* **21**, 016300 (2026).
- [19] M. Döring, J. Haidenbauer, M. Mai, and T. Sato, *Prog. Part. Nucl. Phys.* **146**, 104213 (2026).
- [20] S. K. Choi *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **91**, 262001 (2003).
- [21] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. D* **102**, 092005 (2020).
- [22] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. Lett.* **132**, 151903 (2024).
- [23] T. Ji, X.-K. Dong, F.-K. Guo, C. Hanhart, and U.-G. Meißner, *arXiv:2502.04458*.
- [24] M. E. B. Franklin *et al.*, *Phys. Rev. Lett.* **51**, 963 (1983).
- [25] X.-Q. Li, D. V. Bugg, and B.-S. Zou, *Phys. Rev. D* **55**, 1421 (1997).
- [26] Q. Zhao, *Phys. Lett. B* **697**, 52 (2011).
- [27] Q. Zhao, G. Li, and C.-H. Chang, *Chin. Phys. C* **34**, 299 (2010).
- [28] Q. Wang, G. Li, and Q. Zhao, *Phys. Rev. D* **85**, 074015 (2012).
- [29] P. Colangelo, F. De Fazio, and T. N. Pham, *Phys. Lett. B* **542**, 71 (2002).
- [30] C.-P. Jia, H.-Y. Jiang, J.-P. Wang, and F.-S. Yu, *J. High Energy Phys.* **11** (2024) 072.
- [31] H.-Y. Cheng, C.-K. Chua, and A. Soni, *Phys. Rev. D* **71**, 014030 (2005).
- [32] X.-Z. Ling, M.-Z. Liu, J.-X. Lu, L.-S. Geng, and J.-J. Xie, *Phys. Rev. D* **103**, 116016 (2021).
- [33] Z.-D. Duan, J.-P. Wang, R.-H. Li, C.-D. Lü, and F.-S. Yu, *J. High Energy Phys.* **09** (2025) 160.
- [34] J.-P. Wang and F.-S. Yu, *Chin. Phys. C* **48**, 101001 (2024).
- [35] S. Aoki and T. Doi, *Front. Phys.* **8**, 307 (2020).
- [36] S. X. Nakamura, X. H. Li, H. P. Peng, Z. T. Sun, and X. R. Zhou, *Phys. Rev. D* **112**, 054027 (2025).
- [37] M.-L. Du, V. Baru, F.-K. Guo, C. Hanhart, U.-G. Meissner, J. A. Oller, and Q. Wang, *Phys. Rev. Lett.* **124**, 072001 (2020).
- [38] R. Aaij *et al.* (LHCb Collaboration), *Nat. Phys.* **18**, 751 (2022).
- [39] R. Aaij *et al.* (LHCb Collaboration), *Nat. Commun.* **13**, 3351 (2022).
- [40] X.-K. Dong, F.-K. Guo, and B.-S. Zou, *Commun. Theor. Phys.* **73**, 125201 (2021).
- [41] X.-Z. Ling, M.-Z. Liu, L.-S. Geng, E. Wang, and J.-J. Xie, *Phys. Lett. B* **826**, 136897 (2022).
- [42] M.-L. Du, V. Baru, X.-K. Dong, A. Filin, F.-K. Guo, C. Hanhart, A. Nefediev, J. Nieves, and Q. Wang, *Phys. Rev. D* **105**, 014024 (2022).
- [43] A. Feijoo, W. H. Liang, and E. Oset, *Phys. Rev. D* **104**, 114015 (2021).
- [44] M. Albaladejo, *Phys. Lett. B* **829**, 137052 (2022).
- [45] Q. Xin and Z.-G. Wang, *Eur. Phys. J. A* **58**, 110 (2022).
- [46] C. Deng and S.-L. Zhu, *Phys. Rev. D* **105**, 054015 (2022).
- [47] S. Fleming, R. Hodges, and T. Mehen, *Phys. Rev. D* **104**, 116010 (2021).
- [48] M.-J. Yan and M. P. Valderrama, *Phys. Rev. D* **105**, 014007 (2022).
- [49] H. Ren, F. Wu, and R. Zhu, *Adv. High Energy Phys.* **2022**, 9103031 (2022).
- [50] S. S. Agaev, K. Azizi, and H. Sundu, *J. High Energy Phys.* **06** (2022) 057.
- [51] S. S. Agaev, K. Azizi, and H. Sundu, *Nucl. Phys. B* **975**, 115650 (2022).
- [52] X.-Z. Weng, W.-Z. Deng, and S.-L. Zhu, *Chin. Phys. C* **46**, 013102 (2022).
- [53] T.-W. Wu and Y.-L. Ma, *Phys. Rev. D* **107**, L071501 (2023).
- [54] M. Padmanath and S. Prelovsek, *Phys. Rev. Lett.* **129**, 032002 (2022).
- [55] S. Chen, C. Shi, Y. Chen, M. Gong, Z. Liu, W. Sun, and R. Zhang, *Phys. Lett. B* **833**, 137391 (2022).
- [56] Y. Lyu, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, and J. Meng, *Phys. Rev. Lett.* **131**, 161901 (2023).
- [57] S. Collins, A. Nefediev, M. Padmanath, and S. Prelovsek, *Phys. Rev. D* **109**, 094509 (2024).
- [58] T. Whyte, D. J. Wilson, and C. E. Thomas (Hadron Spectrum Collaboration), *Phys. Rev. D* **111**, 034511 (2025).
- [59] B. Wang and L. Meng, *Phys. Rev. D* **107**, 094002 (2023).
- [60] Q.-Y. Zhai, M.-Z. Liu, J.-X. Lu, and L.-S. Geng, *Phys. Rev. D* **109**, 034015 (2024).
- [61] H. Xu and L. Ren, *Phys. Rev. D* **112**, 114003 (2025).
- [62] M. Sakai and Y. Yamaguchi, *Phys. Rev. D* **109**, 054016 (2024).
- [63] T. Asanuma, Y. Yamaguchi, and M. Harada, *Phys. Rev. D* **110**, 074030 (2024).
- [64] Z.-F. Sun, N. Li, and X. Liu, *Phys. Rev. D* **110**, 094025 (2024).

- [65] L. Qiu, C. Gong, and Q. Zhao, *Phys. Rev. D* **109**, 076016 (2024).
- [66] X. Zhang, *Phys. Rev. D* **109**, 094010 (2024).
- [67] T.-W. Wu, Y.-W. Pan, M.-Z. Liu, S.-Q. Luo, L.-S. Geng, and X. Liu, *Phys. Rev. D* **105**, L031505 (2022).
- [68] Y.-W. Pan, T.-W. Wu, M.-Z. Liu, and L.-S. Geng, *Phys. Rev. D* **105**, 114048 (2022).
- [69] Y.-W. Pan, J.-X. Lu, E. Hiyama, L.-S. Geng, and A. Hosaka, *Phys. Rev. D* **111**, 114006 (2025).
- [70] L. R. Dai, R. Molina, and E. Oset, *Phys. Rev. D* **105**, 016029 (2022); **106**, 099902(E) (2022).
- [71] F.-Z. Peng, M.-J. Yan, and M. Pavon Valderrama, *Phys. Rev. D* **108**, 114001 (2023).
- [72] K. Chen and B. Wang, *Phys. Rev. D* **110**, 116017 (2024).
- [73] X. Liu, D. Chen, H. Huang, and J. Ping, *Phys. Rev. D* **109**, 054021 (2024).
- [74] M. Albaladejo, A. Feijoo, I. Vidaña, J. Nieves, and E. Oset, *Eur. Phys. J. A* **61**, 187 (2025).
- [75] Y. Li, Y.-B. He, X.-H. Liu, B. Chen, and H.-W. Ke, *Eur. Phys. J. C* **83**, 258 (2023).
- [76] Y. Hu, J. Liao, E. Wang, Q. Wang, H. Xing, and H. Zhang, *Phys. Rev. D* **104**, L111502 (2021).
- [77] B. Chen, M. Yang, G. Chen, J. Zhao, and X.-h. Liu, *Phys. Rev. C* **109**, 064909 (2024).
- [78] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **122**, 222001 (2019).
- [79] M.-Z. Liu, Y.-W. Pan, F.-Z. Peng, M. Sánchez Sánchez, L.-S. Geng, A. Hosaka, and M. Pavon Valderrama, *Phys. Rev. Lett.* **122**, 242001 (2019).
- [80] J. Nieves and M. P. Valderrama, *Phys. Rev. D* **86**, 056004 (2012).
- [81] F.-K. Guo, C. Hidalgo-Duque, J. Nieves, and M. P. Valderrama, *Phys. Rev. D* **88**, 054007 (2013).
- [82] T. A. Kaeding, *At. Data Nucl. Data Tables* **61**, 233 (1995).
- [83] M.-Z. Liu, T.-W. Wu, M. Pavon Valderrama, J.-J. Xie, and L.-S. Geng, *Phys. Rev. D* **99**, 094018 (2019).
- [84] M.-Z. Liu, F.-Z. Peng, M. Sánchez Sánchez, and M. P. Valderrama, *Phys. Rev. D* **98**, 114030 (2018).
- [85] T.-W. Wu, M.-Z. Liu, and L.-S. Geng, *Phys. Rev. Lett.* **135**, 031902 (2025).
- [86] Q. Wu and D.-Y. Chen, *Phys. Rev. D* **100**, 114002 (2019).
- [87] Q. Wu, M.-Z. Liu, and L.-S. Geng, *Eur. Phys. J. C* **84**, 147 (2024).
- [88] Z. Yu, Q. Wu, and D.-Y. Chen, *Eur. Phys. J. C* **84**, 985 (2024).
- [89] Q.-W. Yuan, Q. Wu, and M.-Z. Liu, *Phys. Rev. D* **111**, 114035 (2025).
- [90] Y. Oh, T. Song, and S. H. Lee, *Phys. Rev. C* **63**, 034901 (2001).
- [91] S. Irfan, F. Akram, B. Masud, and B. Shafaq, *Phys. Rev. C* **100**, 064906 (2019).
- [92] P. Colangelo, F. De Fazio, and T. N. Pham, *Phys. Rev. D* **69**, 054023 (2004).
- [93] G. C. Donald, C. T. H. Davies, J. Koponen, and G. P. Lepage, *Phys. Rev. Lett.* **112**, 212002 (2014).
- [94] S. Navas *et al.* (Particle Data Group Collaboration), *Phys. Rev. D* **110**, 030001 (2024).
- [95] F.-K. Guo, C. Hanhart, Y. S. Kalashnikova, U.-G. Meißner, and A. V. Nefediev, *Phys. Lett. B* **742**, 394 (2015).
- [96] P.-P. Shi, J. M. Dias, and F.-K. Guo, *Phys. Lett. B* **843**, 137987 (2023).
- [97] A. Deandrea, G. Nardulli, and A. D. Polosa, *Phys. Rev. D* **68**, 034002 (2003).
- [98] Y. Oh, W. Liu, and C. M. Ko, *Phys. Rev. C* **75**, 064903 (2007).
- [99] Q. Wang, X.-H. Liu, and Q. Zhao, *Phys. Lett. B* **711**, 364 (2012).
- [100] L.-L. Chau and H.-Y. Cheng, *Phys. Rev. D* **36**, 137 (1987); **39**, 2788(A) (1989).
- [101] A. Ali, G. Kramer, and C.-D. Lu, *Phys. Rev. D* **58**, 094009 (1998).
- [102] A. Ali, G. Kramer, Y. Li, C.-D. Lu, Y.-L. Shen, W. Wang, and Y.-M. Wang, *Phys. Rev. D* **76**, 074018 (2007).
- [103] H. Li, C.-D. Lu, and F.-S. Yu, *Phys. Rev. D* **86**, 036012 (2012).
- [104] M. Bauer, B. Stech, and M. Wirbel, *Z. Phys. C* **34**, 103 (1987).
- [105] H.-Y. Cheng and B. Tseng, *Phys. Rev. D* **48**, 4188 (1993).
- [106] H.-Y. Cheng and C.-W. Chiang, *Phys. Rev. D* **81**, 074021 (2010).
- [107] C.-D. Lu, Y.-M. Wang, H. Zou, A. Ali, and G. Kramer, *Phys. Rev. D* **80**, 034011 (2009).
- [108] R. L. Workman *et al.* (Particle Data Group Collaboration), *Prog. Theor. Exp. Phys.* **2022**, 083C01 (2022).
- [109] R. C. Verma, *J. Phys. G* **39**, 025005 (2012).
- [110] Y. Aoki *et al.* (Flavour Lattice Averaging Group (FLAG) Collaboration), *Phys. Rev. D* **113**, 014508 (2026).
- [111] S. Narison, *Nucl. Part. Phys. Proc.* **270–272**, 143 (2016).
- [112] Y. Li, P. Maris, and J. P. Vary, *Phys. Rev. D* **96**, 016022 (2017).
- [113] Z.-G. Wang, *Eur. Phys. J. C* **75**, 427 (2015).
- [114] C. Bourrely, I. Caprini, and L. Lellouch, *Phys. Rev. D* **79**, 013008 (2009); **82**, 099902(E) (2010).
- [115] A. Bharucha, T. Feldmann, and M. Wick, *J. High Energy Phys.* **09** (2010) 090.
- [116] D. Leljak, B. Melic, and M. Patra, *J. High Energy Phys.* **05** (2019) 094.
- [117] T. Shrimal, S. Collins, P. Jana, M. Padmanath, and S. Prelovsek, *Phys. Rev. D* **112**, 054513 (2025).
- [118] P. A. Zyla *et al.* (Particle Data Group Collaboration), *Prog. Theor. Exp. Phys.* **2020**, 083C01 (2020).
- [119] F.-S. Yu, H.-Y. Jiang, R.-H. Li, C.-D. Lü, W. Wang, and Z.-X. Zhao, *Chin. Phys. C* **42**, 051001 (2018).
- [120] Z.-S. Jia, M.-J. Yan, Z.-H. Zhang, P.-P. Shi, G. Li, and F.-K. Guo, *Phys. Rev. D* **107**, 074029 (2023).
- [121] Z.-S. Jia, Z.-H. Zhang, G. Li, and F.-K. Guo, *Phys. Rev. D* **108**, 094038 (2023).
- [122] C.-H. Chang and X.-G. Wu, *Eur. Phys. J. C* **38**, 267 (2004).
- [123] A. Cerri *et al.*, *CERN Yellow Rep. Monogr.* **7**, 867 (2019).