

Production of $\phi\Lambda$, $D_s^{*-}\Lambda_c^+$, and $J/\psi\Lambda$ in kaon-induced reactions off the nucleon

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We investigate the reaction mechanism of strangeness production in $K^-p \rightarrow \phi\Lambda$ within a hybrid Regge framework, taking into account t -channel K - and K^* -Reggeon exchanges. We present results for the total cross section, t -dependent differential cross sections, and spin-density matrix elements (SDMEs), and compare them with the available experimental data. We find that K^* -Reggeon exchange provides the dominant contribution, while K -Reggeon exchange remains non-negligible, particularly in describing the SDMEs. In contrast, the s -channel Λ and u -channel nucleon exchanges are negligible. To obtain reliable predictions for the open-charm production in $K^-p \rightarrow D_s^{*-}\Lambda_c^+$, we employ a quark-gluon-string-model-motivated Regge prescription that incorporates both pseudoscalar- and vector-Reggeon exchanges. Within this prescription, the Regge trajectories $\alpha(t)$ and energy-scale parameters s_0 are determined consistently, thereby constraining the model and reducing theoretical ambiguities. For the hidden-charm production in $K^-p \rightarrow J/\psi\Lambda$, we use the same quark-level diagrammatic correspondence. The total cross sections for $K^-p \rightarrow D_s^{*-}\Lambda_c^+$ and $K^-p \rightarrow J/\psi\Lambda$ are suppressed by approximately 4–6 and 8–9 orders of magnitude, respectively, compared with that for $K^-p \rightarrow \phi\Lambda$. We also examine possible s -channel contributions from the hidden-charm pentaquark states with strangeness, $P_{cs}(4338)^0$ and $P_{cs}(4459)^0$, to both $D_s^{*-}\Lambda_c^+$ and $J/\psi\Lambda$ production.

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I. INTRODUCTION

Open- and hidden-charm production constitute central topics in current hadron-physics programs at the PANDA experiment at the Facility for Antiproton and Ion Research [1] and at J-PARC (Japan Proton Accelerator Research Complex) [2]. At PANDA, the high-intensity antiproton beam with momenta up to 15 GeV/ c will enable systematic investigations of charm dynamics covering open-charm processes such as $\bar{p}p \rightarrow D\bar{D}^{(*)}$, $\Lambda_c\bar{\Lambda}_c^{(*)}$, $\Sigma_c\bar{\Lambda}_c^{(*)}$, as well as hidden-charm production channels like $\bar{p}p \rightarrow J/\psi X$, $\eta_c\gamma$ [3,4]. Corresponding theoretical studies have been performed within various frameworks, including the Jülich meson-baryon coupled-channel model [5,6], effective Lagrangian and Regge frameworks [7–9], nonrelativistic quantum chromodynamics (NRQCD) factorization [10], and hadronic pole models [11,12].

In parallel, J-PARC has developed complementary programs employing meson beams. The E50 spectrometer, for

example, is designed to detect charged open-charm hadrons produced in $\pi^-p \rightarrow D^{*-}Y_c^{*+}$ at an incident pion momentum of 20 GeV/ c , where Y_c^{*+} denotes excited charmed baryons, such as Λ_c^* 's and Σ_c^* 's [13]. With its wide angular coverage and high momentum resolution, this experiment is expected to provide valuable information on charmed-baryon spectroscopy [14–16]. More recently, a new proposal (P111) has also been submitted to study hidden-charm production near threshold via $\pi^-p \rightarrow J/\psi n$ [17]. Existing theoretical predictions for this cross section span orders of magnitude, ranging from $\sigma_{\text{tot}} \sim 0.1$ pb [18] to ~ 50 pb [19] and ~ 1.0 nb [20] depending strongly on the assumed mechanism and model inputs. Reliable and model-constrained predictions are therefore essential for assessing experimental feasibility and optimizing detector design.

In previous works, the author and collaborators investigated open-strangeness ($\pi^-p \rightarrow K^{(*)}\Lambda$) and open-charm ($\pi^-p \rightarrow D^{(*)}\Lambda_c^+$) reactions [15,21,22] within a unified framework based on the quark-gluon string model (QGSM) [23–26]. In this picture, the annihilation of a $q\bar{q}$ pair in the initial state leads to the formation of an intermediate string, which subsequently fragments into the observed hadrons via planar diagrams. A key advantage of the QGSM-based prescription is that the two essential ingredients, Regge trajectories $\alpha(t)$ and energy-scale parameters s_0 , can be determined consistently across both the strangeness and charm sectors. Using the similarity of

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quark diagrams, hidden-strangeness ($\pi^- p \rightarrow \phi n$) and hidden-charm ($\pi^- p \rightarrow J/\psi n$) reactions were also studied in a coherent manner [18].

Beyond pion-induced reactions, J-PARC has established an extensive program of kaon-induced reactions using the high-intensity K1.8 beam line [2,27,28]. Motivated by these developments, in the present work, we extend the QGSM-based prescription [23–26] to kaon-induced reactions off the nucleon. We first analyze the strangeness-production process $K^- p \rightarrow \phi \Lambda$, including both pseudoscalar K - and vector K^* -Reggeon exchanges in the t channel. We then provide predictions for the open-charm production in $K^- p \rightarrow D_s^{*-} \Lambda_c^+$ based on the same quark-level diagrammatic correspondence, where the parameters associated with D - and D^* -Reggeon exchanges are constrained within the QGSM framework. For the hidden-charm production in $K^- p \rightarrow J/\psi \Lambda$, we employ the same Reggeized K - and K^* -exchange structure, with the ϕ -meson couplings replaced by the corresponding J/ψ couplings. All the cutoff masses in the hadronic form factors are fixed by fitting the available $K^- p \rightarrow \phi \Lambda$ data [29–36], thereby reducing model ambiguities and allowing more reliable predictions for charm production. We present results for the total cross sections, t -dependent differential cross sections, and spin-density matrix elements (SDMEs). In particular, SDMEs encode the helicity structure of the reaction amplitude and thus provide a stringent test of the underlying production mechanism [22].

The $K^- p \rightarrow J/\psi \Lambda$ reaction is especially interesting because it offers direct access to possible hidden-charm pentaquark states with strangeness, P_{cs} , in the s -channel [37,38]. Recently, the LHCb Collaboration reported hidden-charm pentaquark candidates with quark content $udsc\bar{c}$. The $P_{cs}(4459)^0$ state was observed in the analysis of the $\Xi_b^- \rightarrow J/\psi \Lambda K^-$ decay [37], while the $P_{cs}(4338)^0$ state was found in the $B^- \rightarrow J/\psi \Lambda \bar{p}$ decay [38]. Their measured masses and widths are

$$\begin{aligned} M_{P_{cs}} &= 4458.8 \pm 2.9^{+4.7}_{-1.1}, & \Gamma_{P_{cs}} &= 17.3 \pm 6.5^{+8.0}_{-5.7}, \\ M_{P_{cs}} &= 4338.2 \pm 0.7 \pm 0.4, & \Gamma_{P_{cs}} &= 7.0 \pm 1.2 \pm 1.3, \end{aligned} \quad (1)$$

respectively, in units of MeV. The dominant open-charm decay modes of these P_{cs} states have been studied theoretically within meson-baryon molecular pictures [39–50], some of which predict a sizable (possibly dominant) branching ratio into the $D_s^* \Lambda_c$ channel [51,52]. This suggests that the $P_{cs}(4459)^0$ state may also be probed through the associated-production reaction $K^- p \rightarrow D_s^{*-} \Lambda_c^+$. Accordingly, we investigate possible s -channel contributions from the P_{cs} states to both the $K^- p \rightarrow D_s^{*-} \Lambda_c^+$ and $K^- p \rightarrow J/\psi \Lambda$ reactions.

This paper is organized as follows. In Sec. II, we present the formalism of the hybrid Regge framework and specify

the model ingredients, including coupling constants, Regge trajectories, energy-scale parameters, and form factors. Section III contains numerical results for the total and differential cross sections as well as SDMEs, followed by discussion. Finally, Sec. IV summarizes our findings and provides concluding remarks.

II. THEORETICAL FRAMEWORK

In this section, we introduce a hybrid Regge framework that combines an effective Lagrangian approach with Regge phenomenology. As illustrated in Fig. 1, the reaction $K^- p \rightarrow \phi \Lambda$ can be described by two quark diagrams. In Fig. 1(a), replacing the strange quark in the propagator with a charm quark ($s \rightarrow c$) leads to the open-charm process $K^- p \rightarrow D_s^{*-} \Lambda_c^+$. In Fig. 1(b), the corresponding replacement in the produced vector meson results in the hidden-charm process $K^- p \rightarrow J/\psi \Lambda$.

For the $K^- p \rightarrow \phi \Lambda$ and $K^- p \rightarrow J/\psi \Lambda$ reactions, the t -channel contributions are described by pseudoscalar K - and vector K^* -Reggeon exchanges [Fig. 2(a)]. For $K^- p \rightarrow D_s^{*-} \Lambda_c^+$, we include pseudoscalar D - and vector D^* -Reggeon exchanges in the t channel [Fig. 2(b)].

A. Strangeness production: $K^- p \rightarrow \phi \Lambda$

We begin with the t -channel process of the $K^- p \rightarrow \phi \Lambda$ reaction shown in Fig. 2. The four momenta of the initial K and proton are denoted by k_1 and p_1 , respectively, while those of the final ϕ and Λ are denoted by k_2 and p_2 . The effective Lagrangians for the $\phi K K$ and $\phi K^* K$ vertices are given by

$$\begin{aligned} \mathcal{L}_{\phi K K} &= -ig_{\phi K K}(K^- \partial_\mu K^+ - \partial_\mu K^- K^+) \phi^\mu, \\ \mathcal{L}_{\phi K^* K} &= g_{\phi K^* K} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \phi_\nu (\partial_\alpha K_\beta^{*-} K^+ + \partial_\alpha K_\beta^{*+} K^-), \end{aligned} \quad (2)$$

where K , K^* , and ϕ represent the fields of $K(494, 0^-)$, $K^*(892, 1^-)$, and $\phi(1020, 1^-)$ mesons, respectively. The coupling constant $g_{\phi K K} = 4.48$ is fixed from the branching ratio $\mathcal{B}(\phi \rightarrow K^+ K^-) = 49.9\%$ [53] using the total decay width of $\Gamma_\phi = 4.249$ MeV and the relation

$$\Gamma(\phi \rightarrow K^+ K^-) = \frac{p_K^3}{6\pi M_\phi^2} g_{\phi K K}^2, \quad (3)$$

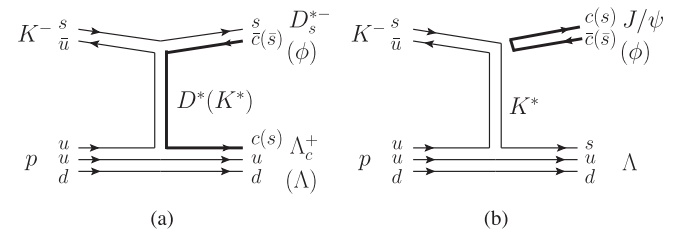


FIG. 1. Quark diagrams for (a) $K^- p \rightarrow \phi \Lambda, D_s^{*-} \Lambda_c^+$ and (b) $K^- p \rightarrow \phi \Lambda, J/\psi \Lambda$ in the t channel.

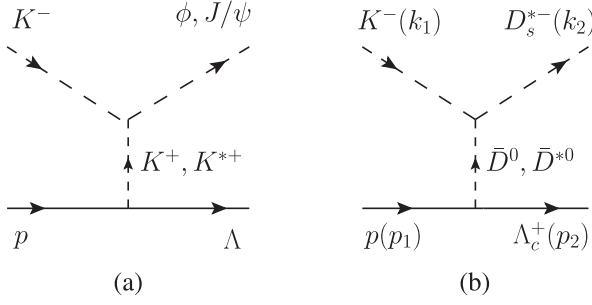


FIG. 2. Feynman diagrams for (a) t -channel K^- and K^* -Reggeon exchanges for $K^-p \rightarrow \phi\Lambda, J/\psi\Lambda$, and (b) t -channel D^- and D^* -Reggeon exchanges for $K^-p \rightarrow D_s^{*-}\Lambda_c^+$.

where $p_K = \sqrt{M_\phi^2 - 4M_K^2}/2$. To obtain the coupling constant $g_{\phi K^*K}$, we use the SU(3) flavor symmetry relation

$$g_{\phi K^*K} = g_{\rho\omega\pi}/\sqrt{2}, \quad (4)$$

where the coupling $g_{\rho\omega\pi}$ is taken from the hidden gauge approach,

$$g_{\rho\omega\pi} = \frac{N_c g_{\rho\pi\pi}^2}{8\pi^2 f_\pi} = 14.4 \text{ GeV}^{-1}, \quad (5)$$

with $N_c = 3$, $f_\pi = 93 \text{ MeV}$, and $g_{\rho\pi\pi} = 5.94$ [54,55]. $g_{\rho\pi\pi}$ is calculated from the branching ratio $\mathcal{B}(\rho \rightarrow \pi\pi) \sim 1$ [53].

The effective Lagrangians for the meson-baryon octet vertices are written as

$$\begin{aligned} \mathcal{L}_{KN\Lambda} &= \frac{g_{KN\Lambda}}{M_N + M_\Lambda} \bar{N} \gamma_\mu \gamma_5 \Lambda \partial^\mu K + \text{H.c.}, \\ \mathcal{L}_{K^*N\Lambda} &= -g_{K^*N\Lambda} \bar{N} \left[\gamma_\mu \Lambda - \frac{\kappa_{K^*N\Lambda}}{M_N + M_\Lambda} \sigma_{\mu\nu} \Lambda \partial^\nu \right] K^{*\mu} \\ &\quad + \text{H.c.}, \end{aligned} \quad (6)$$

where N and Λ stand for the nucleon and $\Lambda(1116)$ baryon fields, respectively. The coupling constants are taken from the Nijmegen soft-core model (NSC97a) [56,57]:

$$\begin{aligned} g_{KN\Lambda} &= -13.4, \\ g_{K^*N\Lambda} &= -4.26, \quad \kappa_{K^*N\Lambda} = 2.91. \end{aligned} \quad (7)$$

The t -channel Regge amplitudes are constructed within a hybrid framework by replacing the Feynman propagators for pseudoscalar K^- and vector K^* -meson exchanges with the corresponding Regge propagators associated with their Regge trajectories [7,15]:

$$\begin{aligned} T_K(s, t) &= \mathcal{M}_K(s, t) \left(\frac{s}{s_{KN:\phi\Lambda}} \right)^{\alpha_K(t)} \\ &\quad \times \Gamma(-\alpha_K(t)) \alpha'_K F_{PS}(t), \\ T_{K^*}(s, t) &= \mathcal{M}_{K^*}(s, t) \left(\frac{s}{s_{K^*:\phi\Lambda}} \right)^{\alpha_{K^*}(t)-1} \\ &\quad \times \Gamma(1 - \alpha_{K^*}(t)) \alpha'_{K^*} F_V(t), \end{aligned} \quad (8)$$

where the amplitudes $\mathcal{M}_K$ and $\mathcal{M}_{K^*}$ are derived from the Lagrangians in Eq. (2):

$$\begin{aligned} \mathcal{M}_K^\mu &= 2i \frac{g_{\phi K K} g_{KN\Lambda}}{M_N + M_\Lambda} \gamma_\nu \gamma_5 k_1^\mu (k_2 - k_1)^\nu, \\ \mathcal{M}_{K^*}^\mu &= g_{\phi K^* K} g_{K^*N\Lambda} \epsilon^{\mu\nu\alpha\beta} \\ &\quad \times \left[\gamma_\nu - \frac{i\kappa_{K^*N\Lambda}}{2M_N} \sigma_{\nu\lambda} (k_2 - k_1)^\lambda \right] k_{2\alpha} k_{1\beta}, \end{aligned} \quad (9)$$

with $\mathcal{M} = \epsilon_\mu^* \bar{u}_\Lambda \mathcal{M}^\mu u_N$. Here u_N and u_Λ denote the Dirac spinors of the initial nucleon and the final Λ , respectively, and are normalized to $\bar{u}_B u_B = 1$. The four-vector ϵ_μ stands for the polarization of the outgoing ϕ meson.

The form factor in Eq. (8) is introduced to dress the vertices in the diagrams. We adopt the following form:

$$F_{PS(V)}(t) = \left(\frac{n\Lambda_{PS(V)}^2}{n\Lambda_{PS(V)}^2 - t} \right)^n, \quad (10)$$

where the cutoff masses are determined by fitting the available data

$$\Lambda_{PS} = 0.3 \text{ GeV}, \quad \Lambda_V = 0.5 \text{ GeV}, \quad (11)$$

with $n = 1$.

The differential cross section $d\sigma/dt$ is expressed as

$$\frac{d\sigma}{dt} = \frac{M_N M_\Lambda}{16\pi(p_{\text{c.m.}})^2 s} \frac{1}{2} \sum_{\lambda_V, s_f, s_i} |T|^2, \quad (12)$$

where $p_{\text{c.m.}}$ denotes the kaon momentum in the center-of-mass (c.m.) frame. s_i , s_f , and λ_V label the helicity states of the nucleon, the Λ baryon, and the ϕ meson, respectively.

$d\sigma/dt$ satisfies the following asymptotic behavior:

$$\frac{d\sigma}{dt}(s \rightarrow \infty, t \rightarrow 0) \propto s^{2\alpha(0)-2}. \quad (13)$$

More specifically, the asymptotic behaviors of the squared amplitudes of Eq. (9) are derived as

$$\lim_{s \rightarrow \infty} \sum_{\lambda_V, s_f, s_i} |\mathcal{M}_K(s, t)|^2 \propto A,$$

$$\lim_{s \rightarrow \infty} \sum_{\lambda_V, s_f, s_i} |\mathcal{M}_{K^*}(s, t)|^2 \propto -Bs^2t + Cs^2t^2, \quad (14)$$

where A , B , and C are kinematics-independent constants, leading to distinct cross-section shapes at very forward angles. For K -Reggeon exchange, the cross section increases monotonically in this region, whereas for K^* -Reggeon exchange the presence of an antisymmetric tensor in the amplitude substantially reduces the cross section at very forward angles.

To determine the K - and K^* -Regge trajectories in Eq. (8), we follow Ref. [58], where the so-called “square-root” trajectory is adopted:

$$\alpha(t) = \alpha(0) + \gamma [\sqrt{T} - \sqrt{T-t}], \quad (15)$$

where γ is the universal slope and T is a scale parameter specific to each trajectory. In the limit $-t \ll T$, Eq. (15) can be approximated by a linear form:

$$\alpha(t) = \alpha(0) + \alpha' t, \quad (16)$$

with the slope $\alpha' = \gamma/(2\sqrt{T})$. In Ref. [58], the parameters $\sqrt{T}$ for the π - and ρ -Regge trajectories were determined to be

$$\begin{aligned} \sqrt{T_\pi} &= 2.82 \pm 0.05 \text{ GeV}, \\ \sqrt{T_\rho} &= 2.46 \pm 0.03 \text{ GeV}, \end{aligned} \quad (17)$$

with $\gamma = 3.65 \pm 0.05 \text{ GeV}^{-1}$. Following the same procedure, we extract the corresponding values of $\sqrt{T}$ for the K - and K^* -Regge trajectories.

The η_s - and ϕ -Regge trajectories are determined using the additivity relations of the intercepts and inverse slopes:

$$\begin{aligned} 2\alpha_{\bar{s}q}(0) &= \alpha_{\bar{q}q}(0) + \alpha_{\bar{s}s}(0), \\ 2/\alpha'_{\bar{s}q} &= 1/\alpha'_{\bar{q}q} + 1/\alpha'_{\bar{s}s}, \end{aligned} \quad (18)$$

where the $\alpha_{\bar{q}q}(t)$, $\alpha_{\bar{s}q}(t)$, and $\alpha_{\bar{s}s}(t)$ denote the trajectories corresponding to π , K , and η_s for pseudoscalar mesons, and to ρ , K^* , and ϕ for vector mesons, respectively. All Regge-trajectory parameters are summarized in Table I.

With the Regge trajectories determined, the energy-scale parameters $s_K^{KN:\phi\Lambda}$ and $s_{K^*}^{KN:\phi\Lambda}$ in Eq. (8) can be derived using the corresponding scale parameters for the diagonal transitions $KN \rightarrow KN(s^{KN})$ and $\phi\Lambda \rightarrow \phi\Lambda(s^{\phi\Lambda})$ as [24–26]

TABLE I. Pseudoscalar- and vector-meson trajectories in the strangeness sector [7,58].

	$\alpha(0)$	$\sqrt{T}$ (GeV)	$\alpha'(\text{GeV}^{-2})$
$\bar{q}q(\pi)$	-0.0118	2.82	0.647
$\bar{s}q(K)$	-0.151	2.96	0.617
$\bar{s}s(\eta_s)$	-0.291	3.10	0.589
$\bar{q}q(\rho)$	0.55	2.46	0.742
$\bar{s}q(K^*)$	0.414	2.58	0.707
$\bar{s}s(\phi)$	0.27	2.70	0.676

$$\begin{aligned} (s_K^{KN:\phi\Lambda})^{2(\alpha_K(0))} &= (s^{KN})^{\alpha_K(0)} \times (s^{\phi\Lambda})^{\alpha_{\eta_s}(0)}, \\ (s_{K^*}^{KN:\phi\Lambda})^{2(\alpha_{K^*}(0)-1)} &= (s^{KN})^{\alpha_{\rho}(0)-1} \times (s^{\phi\Lambda})^{\alpha_{\phi}(0)-1}, \end{aligned} \quad (19)$$

where s^{ab} is proportional to the total transverse masses of the constituent quarks in hadrons a and b :

$$s^{ab} = \left(\sum_i m_{\perp i} \right)_a \left(\sum_j m_{\perp j} \right)_b. \quad (20)$$

We use $m_{\perp q} \simeq 0.5 \text{ GeV}$, $m_{\perp s} \simeq 0.6 \text{ GeV}$, and $m_{\perp c} \simeq 1.6 \text{ GeV}$, which lead to

$$\begin{aligned} s^{KN} &= 1.65, & s^{\phi\Lambda} &= 1.92, \\ s_K^{KN:\phi\Lambda} &= 1.91, & s_{K^*}^{KN:\phi\Lambda} &= 1.82, \end{aligned} \quad (21)$$

in units of GeV^2 .

B. Charm production: $K^-p \rightarrow D_s^{*-}\Lambda_c^+, J/\psi\Lambda$

We extend the analysis of strangeness production in $K^-p \rightarrow \phi\Lambda$ to charm production by replacing the relevant strange hadrons with their charm counterparts.

1. $K^-p \rightarrow D_s^{*-}\Lambda_c^+$

The t -channel Regge amplitudes for the $K^-p \rightarrow D_s^{*-}\Lambda_c^+$ reaction are obtained by replacing $K \rightarrow D(1869, 0^-)$ and $K^* \rightarrow D^*(2007, 1^-)$ in the propagators, and $\phi \rightarrow D_s^*(2106, 1^-)$ and $\Lambda \rightarrow \Lambda_c(2286, 1/2^+)$ in the final states [Fig. 1(a)]:

$$\begin{aligned} T_D(s, t) &= \mathcal{M}_D(s, t) \left(\frac{s}{s_D^{KN:D_s^*\Lambda_c}} \right)^{\alpha_D(t)} \\ &\quad \times \Gamma(-\alpha_D(t)) \alpha'_D F_{PS}(t), \\ T_{D^*}(s, t) &= \mathcal{M}_{D^*}(s, t) \left(\frac{s}{s_{D^*}^{KN:D_s^*\Lambda_c}} \right)^{\alpha_{D^*}(t)-1} \\ &\quad \times \Gamma(1 - \alpha_{D^*}(t)) \alpha'_{D^*} F_V(t). \end{aligned} \quad (22)$$

The D - and D^* -Regge trajectories are determined as in the strangeness-production case. The relevant values are listed in Table II [58].

TABLE II. Pseudoscalar- and vector-meson trajectories in the charm sector [58].

	$\alpha(0)$	$\sqrt{T}[(\text{GeV})]$	$\alpha'(\text{GeV}^{-2})$
$\bar{q}q(\pi)$	-0.0118	2.82	0.647
$\bar{c}q(D)$	-1.61105	4.16	0.439
$\bar{c}c(\eta_c)$	-3.2103	5.49	0.332
$\bar{q}q(\rho)$	0.55	2.46	0.742
$\bar{c}q(D^*)$	-1.02	3.91	0.467
$\bar{c}c(J/\psi)$	-2.60	5.36	0.340

Equation (19) can be also modified as [24–26]

$$\begin{aligned} (s_D^{KN:D_s^*\Lambda_c})2\alpha_D(0) &= (s^{KN})\alpha_\pi(0) \times (s^{D_s^*\Lambda_c})\alpha_{\eta_c}(0), \\ (s_D^{KN:D_s^*\Lambda_c})2(\alpha_{D^*}(0)-1) &= (s^{KN})\alpha_{\rho}(0)-1 \times (s^{D_s^*\Lambda_c})\alpha_{J/\psi}(0)-1, \end{aligned} \quad (23)$$

where

$$\begin{aligned} s^{KN} &= 1.65, & s^{D_s^*\Lambda_c} &= 5.72, \\ s_D^{KN:D_s^*\Lambda_c} &= 5.69, & s_{D^*}^{KN:D_s^*\Lambda_c} &= 5.00, \end{aligned} \quad (24)$$

in units of GeV^2 .

The coupling constants in Eq. (9) are replaced as $g_{\phi KK} \rightarrow g_{D_s^* DK}$ and $g_{\phi K^* K} \rightarrow g_{D_s^* D^* K}$. The coupling $g_{D_s^* DK}$ is fixed using SU(4) flavor symmetry as $g_{D_s^* DK} = g_{\rho\pi\pi}/\sqrt{2}$ [5], where $g_{\rho\pi\pi} = 5.94$. The strength of the $D_s^* D^* K$ vertex is estimated from heavy-quark spin symmetry, which relates it to $g_{D_s^* DK}$ through $g_{D_s^* D^* K} = -g_{D_s^* DK}/\sqrt{M_D M_{D^*}} = -1.53 \text{ GeV}^{-1}$ [59,60]. For the meson-baryon couplings, we assume SU(4) symmetry and take them to be identical to those in the strange sector, namely $g_{D N \Lambda_c} = g_{K N \Lambda}$, $g_{D^* N \Lambda_c} = g_{K^* N \Lambda}$, and $\kappa_{D^* N \Lambda_c} = \kappa_{K^* N \Lambda}$.

2. $K^- p \rightarrow J/\psi \Lambda$

The t -channel Regge amplitudes for the $K^- p \rightarrow J/\psi \Lambda$ reaction are obtained by replacing $\phi \rightarrow J/\psi(3096, 1^-)$ in the final state [Fig. 1(b)]:

$$\begin{aligned} T_K(s, t) &= \mathcal{M}_K(s, t) \left(\frac{s}{s_K^{KN:J/\psi\Lambda}} \right)^{\alpha_K(t)} \\ &\quad \times \Gamma(-\alpha_K(t)) \alpha'_K F_{PS}(t), \\ T_{K^*}(s, t) &= \mathcal{M}_{K^*}(s, t) \left(\frac{s}{s_{K^*}^{KN:J/\psi\Lambda}} \right)^{\alpha_{K^*}(t)-1} \\ &\quad \times \Gamma(1 - \alpha_{K^*}(t)) \alpha'_{K^*} F_V(t). \end{aligned} \quad (25)$$

The energy-scale parameters are assumed to be the same as those in the strange sector: $s_K^{KN:J/\psi\Lambda} = s_K^{KN:\phi\Lambda}$ and $s_{K^*}^{KN:J/\psi\Lambda} = s_{K^*}^{KN:\phi\Lambda}$.

The coupling constants in Eq. (9) are replaced as $g_{\phi KK} \rightarrow g_{J/\psi KK}$ and $g_{\phi K^* K} \rightarrow g_{J/\psi K^* K}$. Since the branching ratios for J/ψ decays into $K\bar{K}$ and $K\bar{K}^*$ are known experimentally as [53]

$$\begin{aligned} \mathcal{B}(J/\psi \rightarrow K^+ K^-) &= 0.0306\%, \\ \mathcal{B}(J/\psi \rightarrow K^+ K^{*-}) &= 0.60\%, \end{aligned} \quad (26)$$

we can obtain their coupling constants as

$$\begin{aligned} g_{J/\psi KK} &= 1.28 \times 10^{-3}, \\ g_{J/\psi K^* K} &= 2.85 \times 10^{-3} \text{ GeV}^{-1}, \end{aligned} \quad (27)$$

using the total decay width of $\Gamma_{J/\psi} = 92.6 \text{ keV}$ and the relations

$$\begin{aligned} \Gamma(J/\psi \rightarrow K^+ K^-) &= \frac{q_K^3}{6\pi M_{J/\psi}^2} g_{J/\psi KK}^2, \\ \Gamma(J/\psi \rightarrow K^+ K^{*-}) &= \frac{q_{K^*}^3}{12\pi} g_{J/\psi K^* K}^2, \end{aligned} \quad (28)$$

where $q_{K(K^*)}$ is the magnitude of the three-momentum of $K(K^*)$ in the rest frame of J/ψ .

C. s - and u -channel contributions to $K^- p \rightarrow \phi \Lambda$

The s -channel Λ [Fig. 3(a)] and u -channel nucleon [Fig. 3(b)] exchanges may contribute to the $K^- p \rightarrow \phi \Lambda$ reaction, and they are examined in this work.

The effective Lagrangians for the ϕ -meson-baryon interaction vertices are written as

$$\begin{aligned} \mathcal{L}_{\phi\Lambda\Lambda} &= -g_{\phi\Lambda\Lambda} \bar{\Lambda} \left[\gamma_\mu \Lambda - \frac{\kappa_{\phi\Lambda\Lambda}}{2M_N} \sigma_{\mu\nu} \Lambda \partial^\nu \right] \phi^\mu, \\ \mathcal{L}_{\phi NN} &= -g_{\phi NN} \bar{N} \left[\gamma_\mu N - \frac{\kappa_{\phi NN}}{2M_N} \sigma_{\mu\nu} N \partial^\nu \right] \phi^\mu, \end{aligned} \quad (29)$$

where the Nijmegen soft-core model (NSC97a) [56,57] is used to determine the coupling constants:

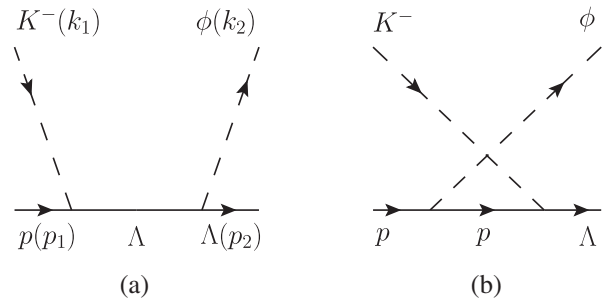


FIG. 3. Feynman diagrams for $K^- p \rightarrow \phi \Lambda$: (a) Λ exchange in the s channel and (b) nucleon exchange in the u channel.

$$\begin{aligned} g_{\phi\Lambda\Lambda} &= -3.80, & \kappa_{\phi\Lambda\Lambda} &= 1.78, \\ g_{\phi NN} &= -1.47, & \kappa_{\phi NN} &= -2.64. \end{aligned} \quad (30)$$

The scattering amplitudes corresponding to the s -channel Λ and u -channel nucleon exchanges are, respectively, given by

$$\begin{aligned} \mathcal{M}_\Lambda^\mu &= i \frac{g_{\phi\Lambda\Lambda}}{s - M_\Lambda^2} \frac{g_{KN\Lambda}}{M_N + M_\Lambda} \left[\gamma^\mu - \frac{i\kappa_{\phi\Lambda\Lambda}}{2M_N} \sigma^{\mu\nu} k_{2\nu} \right] \\ &\quad \times (k_1 + \not{p}_1 + M_\Lambda) \gamma^\alpha \gamma_5 k_{1\alpha}, \\ \mathcal{M}_N^\mu &= i \frac{g_{\phi NN}}{u - M_N^2} \frac{g_{KN\Lambda}}{M_N + M_\Lambda} \gamma^\alpha \gamma_5 (\not{p}_2 - k_1 + M_N) \\ &\quad \times \left[\gamma^\mu - \frac{i\kappa_{\phi NN}}{2M_N} \sigma^{\mu\nu} k_{2\nu} \right] k_{1\alpha}, \end{aligned} \quad (31)$$

with $\mathcal{M} = \epsilon_\mu^* \bar{u}_\Lambda \mathcal{M}^\mu u_N$. The relevant hadrons are spatially extended, so we consider the following form factor for each vertex:

$$\begin{aligned} F_\Lambda(s) &= \frac{\Lambda_s^4}{\Lambda_s^4 + (s - M_\Lambda^2)^2}, \\ F_N(u) &= \frac{\Lambda_u^4}{\Lambda_u^4 + (u - M_N^2)^2}, \end{aligned} \quad (32)$$

for the s -channel and u -channel diagrams, respectively. The cutoff masses are determined to be $\Lambda_s = \Lambda_u = 0.83$ GeV to reproduce the backward-angle scattering data.

D. Pentaquark contributions to $K^- p \rightarrow D_s^{*-} \Lambda_c^+, J/\psi \Lambda$

We now examine the s -channel contributions from the hidden-charm pentaquark states with strangeness, $P_{cs}(4338)^0$ [38] and $P_{cs}(4459)^0$ [37], as shown in Fig. 4. Note that $P_{cs}(4338)^0$ exchange is allowed only for $K^- p \rightarrow J/\psi \Lambda$, because the $D_s^{*-} \Lambda_c^+$ production threshold lies above the $P_{cs}(4338)^0$ mass.

While the spin-parity of $P_{cs}(4338)^0$ has been determined to be $J^P = 1/2^-$, that of $P_{cs}(4459)^0$ is still unsettled. In a hadronic molecular interpretation, Ref. [40] favors $J^P = 3/2^-$ over $1/2^-$, which is also supported by Ref. [51]. In contrast, within a unitarized approach, $J^P = 1/2^-$ is preferred over $3/2^-$ [61]. Accordingly, we consider both $J^P = 1/2^-$ and $3/2^-$ assignments for $P_{cs}(4459)^0$ in our analysis. The branching ratios are taken as follows [52]:

$$\begin{aligned} \mathcal{B}(P_{cs}(4459, 1/2^-) \rightarrow \bar{D}_s^* \Lambda_c) &= 80.86\%, \\ \mathcal{B}(P_{cs}(4459, 3/2^-) \rightarrow \bar{D}_s^* \Lambda_c) &= 81.64\%, \\ \mathcal{B}(P_{cs}(4459, 1/2^-) \rightarrow J/\psi \Lambda) &= 3.31\%, \\ \mathcal{B}(P_{cs}(4459, 3/2^-) \rightarrow J/\psi \Lambda) &= 14.68\%. \end{aligned} \quad (33)$$

For the $P_{cs}(4338)^0$ state, the branching ratio for the $J/\psi \Lambda$ decay has not been determined explicitly within the same

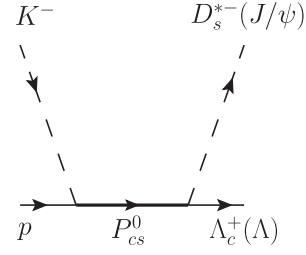


FIG. 4. Feynman diagram for the $K^- p \rightarrow D_s^{*-} \Lambda_c^+, J/\psi \Lambda$ reactions via s -channel P_{cs}^0 pentaquark exchange.

framework as that used for $P_{cs}(4459)^0$. We expect it to be of the same order of magnitude as that for $P_{cs}(4459)^0$ and therefore assume, for simplicity, $\mathcal{B}(P_{cs}(4338) \rightarrow J/\psi \Lambda) = \mathcal{B}(P_{cs}(4459, 1/2^-) \rightarrow J/\psi \Lambda) = 3.31\%$, from which the corresponding $P_{cs}(4338) \Lambda J/\psi$ coupling is determined. For comparison, Ref. [62] predicts a much larger branching ratio, 74%, within a QCD-sum-rule approach. Note that since no information is currently available for the $P_{cs} \rightarrow \bar{K} N$ branching ratios of $P_{cs}(4338)^0$ and $P_{cs}(4459)^0$, we treat them as free parameters in the present analysis.

The effective Lagrangians for the $P_{cs} NK$ vertex are given by [63]

$$\begin{aligned} \mathcal{L}_{PNK}^{1/2^\pm} &= -ig_{PNK} \bar{N} \Gamma^{(\pm)} P K + \text{H.c.}, \\ \mathcal{L}_{PNK}^{3/2^\pm} &= \frac{g_{PNK}}{M_K} \bar{N} \Gamma^{(\mp)} P^\mu \partial_\mu K + \text{H.c.} \end{aligned} \quad (34)$$

The off-shell terms of the Rarita-Schwinger fields are neglected because the resonances are considered near their mass shell. For the $K^- p \rightarrow D_s^{*-} \Lambda_c^+$ reaction, the effective Lagrangians for the $P_{cs} \Lambda_c D_s^*$ vertex can be expressed as [63]

$$\begin{aligned} \mathcal{L}_{P\Lambda_c D_s^*}^{1/2^\pm} &= -g_{P\Lambda_c D_s^*} \bar{\Lambda}_c \Gamma_\mu^{(\mp)} P D_s^{*\mu} + \text{H.c.}, \\ \mathcal{L}_{P\Lambda_c D_s^*}^{3/2^\pm} &= -\frac{ig_{P\Lambda_c D_s^*}}{2M_N} \bar{\Lambda}_c \Gamma_\nu^{(\pm)} P_\mu D_s^{*\mu\nu} + \text{H.c.}, \end{aligned} \quad (35)$$

where $D_s^{*\mu\nu} = \partial^\mu D_s^{*\nu} - \partial^\nu D_s^{*\mu}$ and P denotes the field of the pentaquark state P_{cs} . Since the heavy pentaquark states are expected to contribute mainly near threshold, additional terms are neglected.

For the $K^- p \rightarrow J/\psi \Lambda$ reaction, we replace the hadrons as $D_s^* \rightarrow J/\psi$ and $\Lambda_c \rightarrow \Lambda$.

We use the following notation:

$$\Gamma^{(\pm)} = \begin{pmatrix} \gamma_5 \\ \mathbf{1} \end{pmatrix}, \quad \Gamma_\mu^{(\pm)} = \begin{pmatrix} \gamma_\mu \gamma_5 \\ \gamma_\mu \end{pmatrix}. \quad (36)$$

The scattering amplitudes for the exchanges of the pentaquark states are written as

$$\begin{aligned}
\mathcal{M}_{P(1/2^+)}^\mu &= i \frac{g_{PNK} g_{P\Lambda_c D_s^*}}{s - M_P^2} \gamma^\mu (\not{q}_s + M_P) \gamma_5, \\
\mathcal{M}_{P(1/2^-)}^\mu &= i \frac{g_{PNK} g_{P\Lambda_c D_s^*}}{s - M_P^2} \gamma_5 \gamma^\mu (\not{q}_s + M_P), \\
\mathcal{M}_{P(3/2^+)}^\mu &= i \frac{g_{PNK} g_{P\Lambda_c D_s^*}}{M_K} \frac{1}{2M_N} \frac{1}{s - M_P^2} \gamma_5 \gamma_\nu \\
&\quad \times (k_2^\alpha g^{\mu\nu} - k_2^\nu g^{\mu\alpha}) \Delta_{\alpha\beta}(q_s, M_P) k_1^\beta, \\
\mathcal{M}_{P(3/2^-)}^\mu &= i \frac{g_{PNK} g_{P\Lambda_c D_s^*}}{M_K} \frac{1}{2M_N} \frac{1}{s - M_P^2} \gamma_\nu \\
&\quad \times (k_2^\alpha g^{\mu\nu} - k_2^\nu g^{\mu\alpha}) \Delta_{\alpha\beta}(q_s, M_P) k_1^\beta \gamma_5, \quad (37)
\end{aligned}$$

with $\mathcal{M} = \epsilon_\mu^* \bar{u}_\Lambda \mathcal{M}^\mu u_N$ and $q_s = k_1 + p_1$. The spin-3/2 projection operator is given by

$$\begin{aligned}
\Delta_{\alpha\beta}(p, M) &= (\not{p} + M) \left[-g_{\alpha\beta} + \frac{1}{3} \gamma_\alpha \gamma_\beta \right. \\
&\quad \left. + \frac{1}{3M} (\gamma_\alpha p_\beta - p_\alpha \gamma_\beta) + \frac{2}{3M^2} p_\alpha p_\beta \right]. \quad (38)
\end{aligned}$$

Given the total decay widths of the P_{cs} states, the propagators of the pentaquark states should be modified to $M_{P_{cs}} \rightarrow (M_{P_{cs}} - i\Gamma_{P_{cs}}/2)$.

While the Breit-Wigner parametrization is not necessarily optimal for near-threshold molecular states, where coupled-channel dynamics and threshold effects might be important [64], it is widely used in experimental analyses of pentaquark candidates. In this work, we therefore use the Breit-Wigner form as a phenomenological parametrization of the possible P_{cs} contributions, providing a practical baseline for comparison with future measurements.

Using the branching ratios given in Eq. (33), the coupling constants for the P_{cs} interactions are obtained and summarized in Table III [63].

We consider the form factor

$$F_{P_{cs}}(s) = \frac{\Lambda_P^4}{\Lambda_P^4 + (s - M_{P_{cs}}^2)^2}, \quad (39)$$

at each vertex, where the cutoff mass is chosen as $\Lambda_P = 1.0$ GeV. This cutoff mass does not play a crucial role in the present calculation, since the P_{cs} states lie close to the reaction threshold.

TABLE III. Coupling constants of $P_{cs}(4459)$ to $D_s^* \Lambda_c$ and $J/\psi \Lambda$ for each $J^P = 1/2^-$ and $3/2^-$ assignment, together with that of $P_{cs}(4338)$ to $J/\psi \Lambda$.

	$g_{P\Lambda_c D_s^*}$	$g_{P\Lambda J/\psi}$
$P_{cs}(4459, 1/2^-)$	0.382	0.0802
$P_{cs}(4459, 3/2^-)$	0.589	0.176
$P_{cs}(4338)$	(...)	0.0619

E. Spin-density matrix elements

The SDMEs, which characterize the polarization states of the relevant hadrons, provide key observables for understanding the reaction mechanism [22]. In this work, we focus on the polarization of the produced vector meson. For definiteness, we consider the decay channel $\phi \rightarrow K^+ K^-$ in the $K^- p \rightarrow \phi \Lambda$ reaction. The analysis of the outgoing K^+ in the vector-meson rest frame involves an ambiguity in the choice of the quantization axis. One possible choice is to take the axis antiparallel to the momentum of the outgoing hyperon Λ in the ϕ decay. Alternatively, it may be defined parallel to the momentum of the incoming meson, i.e., along the initial beam direction. Following the convention of Refs. [65,66], the former choice is referred to as the helicity (H) frame, while the latter is known as the Gottfried-Jackson (GJ) frame. The helicity frame is commonly used to test s -channel helicity conservation, whereas the GJ frame is suitable for investigating the t -channel exchange mechanism.

The decay probabilities are expressed in terms of the SDMEs $\rho_{\lambda\lambda'}$, where the vector-meson helicity λ_V is abbreviated as λ . These SDMEs are determined by the reaction amplitudes given by Eq. (8)

$$\rho_{\lambda\lambda'} = \frac{1}{\mathcal{N}^2} \sum_{s_f = \pm\frac{1}{2}, s_i = \pm\frac{1}{2}} T_{\lambda, s_f; s_i} T_{\lambda', s_f; s_i}^*, \quad (40)$$

with the normalization factor

$$\mathcal{N}^2 = \sum_{\lambda, s_f, s_i} |T_{\lambda, s_f; s_i}|^2. \quad (41)$$

We make use of the Hermitian conditions:

$$\rho_{1-1} = \rho_{-11}, \quad \rho_{10} = \rho_{01}, \quad \rho_{-10} = \rho_{0-1}. \quad (42)$$

In addition, the normalization condition $\rho_{00} + \rho_{11} + \rho_{-1-1} = 1$ and the symmetry conditions,

$$\begin{aligned}
\rho_{11} &= \rho_{-1-1}, & \rho_{\pm 10} &= \rho_{0\pm 1}, \\
\rho_{1-1} &= \rho_{-11}, & \rho_{\pm 10} &= -\rho_{0\mp 1},
\end{aligned} \quad (43)$$

are satisfied in our numerical calculations.

The decay angular distributions can be expressed in terms of the SDMEs as

$$\begin{aligned}
W^0(\Omega_f) &= \frac{3}{4\pi} \left[\rho_{00} \cos^2 \Theta + \rho_{11} \sin^2 \Theta - \rho_{1-1} \sin^2 \Theta \cos 2\Phi \right. \\
&\quad \left. - \sqrt{2} \text{Re}(\rho_{10}) \sin 2\Theta \cos \Phi \right], \quad (44)
\end{aligned}$$

where Θ and Φ denote the polar and azimuthal angles of the outgoing pseudoscalar K^+ meson, respectively [22].

III. NUMERICAL RESULTS

A. Strangeness production: $K^-p \rightarrow \phi\Lambda$

We first present our numerical results for the $K^-p \rightarrow \phi\Lambda$ reaction. Figure 5 displays the total cross section as a function of the laboratory beam momentum, P_{Lab} . Both the K - and K^* -Reggeon exchanges exhibit a peak around $P_{\text{Lab}} \simeq 2.2$ GeV and then gradually decrease with increasing P_{Lab} . The K^* -Reggeon exchange dominates over the K -Reggeon exchange. The coherent sum of the two exchanges provides a good overall description of the experimental data [29–35]. The contributions from the Λ and nucleon exchanges to the total cross section are negligible and therefore not shown here.

We present the differential cross sections as functions of $-t' = -t + t_{\text{min}}$ at nine fixed beam momenta in Fig. 6. The K^* -Reggeon exchange reproduces the t -dependence of the experimental data well [31–35], thereby supporting the validity of our Regge formalism. Owing to the different intercepts of the Regge trajectories, $\alpha_K(0) < \alpha_{K^*}(0)$, the K -Reggeon exchange decreases more rapidly with increasing $-t'$ than the K^* -Reggeon one [see Eq. (13)]. Note that the t -dependence of the two Reggeon exchanges also depends on the order of the form factor in Eq. (10), n . If we use $n = 2$ instead of the value $n = 1$ adopted here, the calculated t -dependences become steeper and deviate from the experimental data. The two Reggeon exchanges exhibit qualitatively different behaviors at very forward angles, as discussed in Eq. (14). The K -Reggeon contribution enhances the cross sections in the region $-t' \leq 0.4$ GeV².

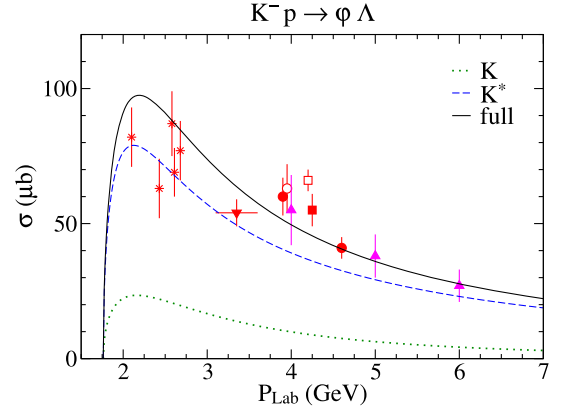


FIG. 5. Total cross section for the $K^-p \rightarrow \phi\Lambda$ reaction as a function of P_{Lab} . The green dotted and blue dashed curves denote the K - and K^* -Reggeon exchange contributions, respectively, while the black solid curve represents the full result. Experimental data are taken from Refs. [29–35]: stars, filled inverted triangles, filled circles, open circles, filled triangles, open squares, and filled squares.

As shown in Fig. 7, the u -channel nucleon contribution becomes significant in the backward-angle region [31,32]. The small deviations observed at $-t' \geq 4$ GeV² can be accounted for by including this contribution. In contrast, the s -channel Λ exchange interferes destructively with the t -channel contribution at backward angles and is therefore strongly suppressed.

The role of the individual contributions becomes evident in Fig. 8, where the SDMEs [22] are shown as functions of

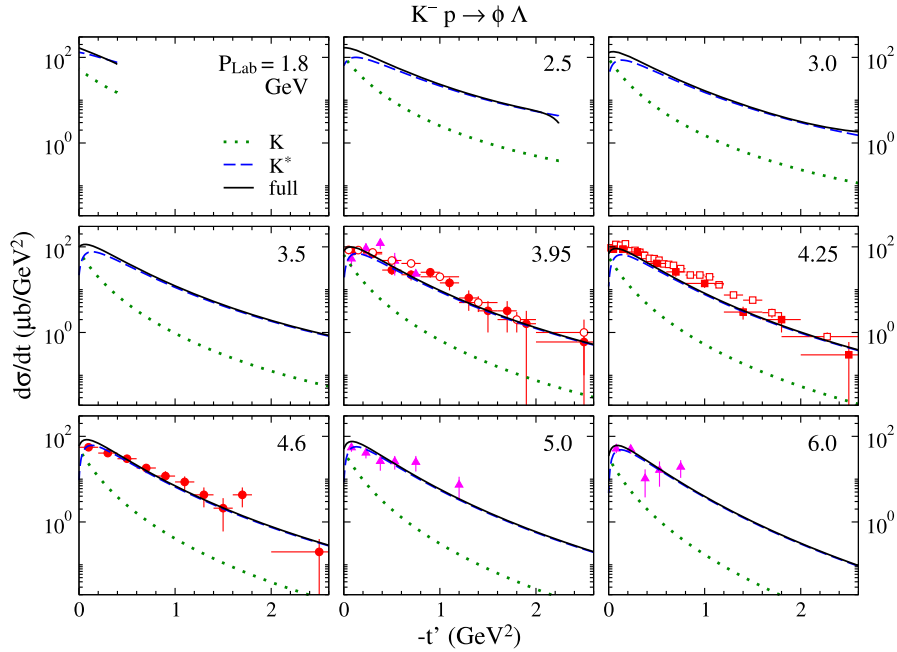


FIG. 6. t -dependent differential cross sections for the $K^-p \rightarrow \phi\Lambda$ reaction at nine fixed beam momenta. Experimental data are taken from Refs. [31–35]: filled circles, open circles, filled triangles, open squares, and filled squares. The curve notation is the same as in Fig. 5.

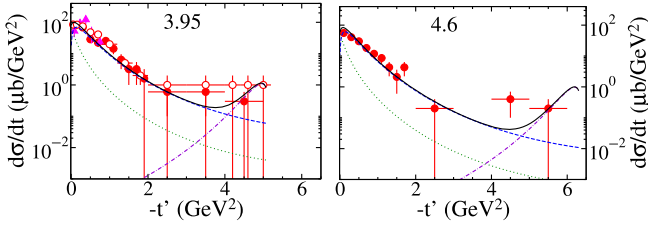


FIG. 7. t -dependent differential cross sections for the $K^-p \rightarrow \phi\Lambda$ reaction at $P_{\text{Lab}} = 3.95$ and 4.6 GeV, covering large momentum transfer. Experimental data are taken from Refs. [31–33]: filled circles, open circles, and filled triangles. The curve notation is the same as in Fig. 5. The violet dot-dashed curve denotes the u -channel nucleon contribution.

$-t'$ at Fig. 8(a) $P_{\text{Lab}} = 3.35$ GeV in both the helicity and GJ frames [30] and at Fig. 8(b) $P_{\text{Lab}} = 5$ GeV in the helicity frame [33]. In general, including the K -Reggeon exchange in addition to the K^* -Reggeon contribution improves the overall description of the SDMEs. In particular, the t -dependence of ρ_{00}^H and $\text{Re}\rho_{10}^H$ for the K -Reggeon exchange closely follows that of the corresponding experimental data. Similar features are observed in Fig. 9, where the SDMEs ρ_{00} , $\text{Re}\rho_{10}$, ρ_{11} , and ρ_{1-1} are shown at $P_{\text{Lab}} = 4.25$ GeV in both the Fig. 9(a) helicity and Fig. 9(b) GJ frames [31,32,34–36].

The SDME ρ_{00} is sensitive to the unnatural-parity-exchange contribution associated with the K -Reggeon exchange for the vector-meson helicity-zero state. Similarly, the combinations $\rho_{11} \pm \rho_{1-1}$ separate the natural- and unnatural-parity-exchange contributions, corresponding mainly to the K^* - and K -Reggeon exchanges, respectively, in the transverse vector-meson helicity sector $\lambda_V = \pm 1$. Overall, the results indicate that the reaction mechanism is dominated by natural-parity exchange, consistent with the behavior observed in the differential cross sections shown in Fig. 6. However, there remain some

discrepancies between the experimental data and our results, suggesting that additional unnatural-parity-exchange contributions, such as the axial-vector $K_1(1270)$ and $K_1(1400)$ mesons, may play a role. In contrast, natural-parity-exchange contributions, such as the scalar κ meson [$K_0^*(700)$], do not appear to be necessary. A detailed investigation of these effects is beyond the scope of this work.

B. Charm production: $K^-p \rightarrow D_s^{*-}\Lambda_c^+, J/\psi\Lambda$

We now turn to charm production described in Sec. II B and compare the results with those for the strangeness production in $K^-p \rightarrow \phi\Lambda$. Figure 10 shows the predicted total cross sections for the Fig. 10(a) $K^-p \rightarrow D_s^{*-}\Lambda_c^+$ and Fig. 10(b) $K^-p \rightarrow J/\psi\Lambda$ reactions as functions of P_{Lab} . In both cases, vector-Reggeon exchanges dominate, similar to the strangeness-production channel. Owing to the much larger mass gap in the final state of the $K^-p \rightarrow J/\psi\Lambda$ channel, its total cross section increases more gradually with P_{Lab} than those of the $K^-p \rightarrow \phi\Lambda$ and $K^-p \rightarrow D_s^{*-}\Lambda_c^+$ reactions. Our result for $K^-p \rightarrow J/\psi\Lambda$ is approximately 3 orders of magnitude smaller than that predicted in Ref. [67].

A direct comparison of the $\phi\Lambda$, $D_s^{*-}\Lambda_c^+$, and $J/\psi\Lambda$ production is shown in Fig. 11(a) as functions of s/s_{th} , where s_{th} is the threshold value of s for each reaction. The total cross section for the open-charm production in $K^-p \rightarrow D_s^{*-}\Lambda_c^+$ is suppressed by approximately 4–6 orders of magnitude compared with that for the reaction $K^-p \rightarrow \phi\Lambda$ depending on the kinematic region. This suppression can be attributed mainly to the larger energy-scale parameter in the charm sector, $s_{D(D^*)}^{KN:D_s^*\Lambda_c^+} > s_{K(K^*)}^{KN:\phi\Lambda}$, as well as to the smaller coupling strength, $g_{D_s^*D^*K} < g_{\phi K^*K}$. In the case of the hidden-charm production in $K^-p \rightarrow J/\psi\Lambda$, the total cross section is further suppressed by about 8–9 orders of

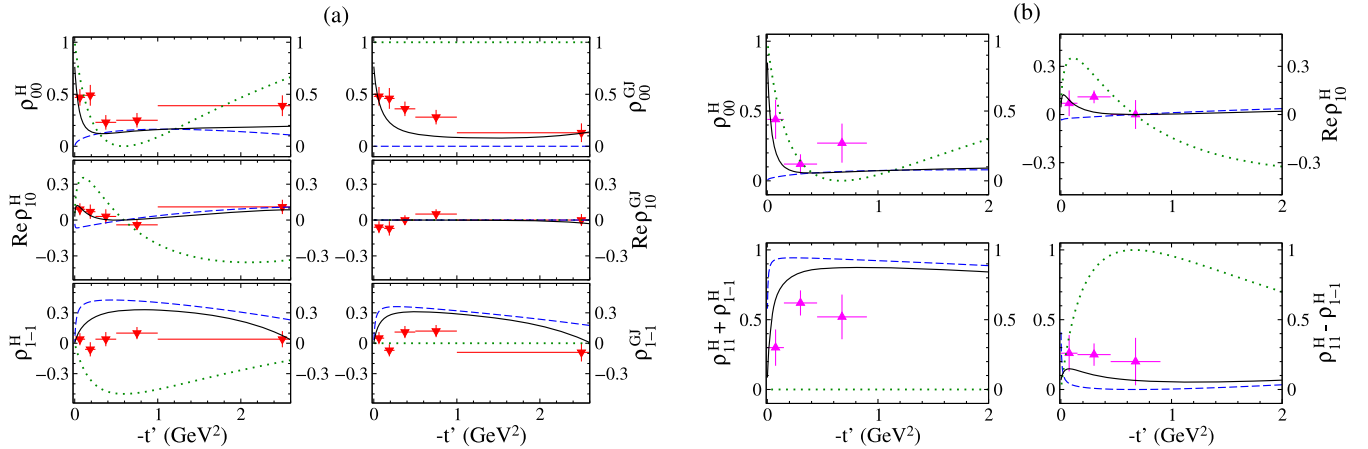


FIG. 8. SDMEs for the $K^-p \rightarrow \phi\Lambda$ reaction as functions of $-t'$. (a) ρ_{00} , $\text{Re}\rho_{10}$, and ρ_{1-1} in the helicity and GJ frames at $P_{\text{Lab}} = 3.35$ GeV. (b) ρ_{00} , $\text{Re}\rho_{10}$, and $\rho_{11} \pm \rho_{1-1}$ in the helicity frame at $P_{\text{Lab}} = 5$ GeV. Experimental data are taken from Refs. [30,33]: filled inverted triangles and filled triangles. The curve notation is the same as in Fig. 5.

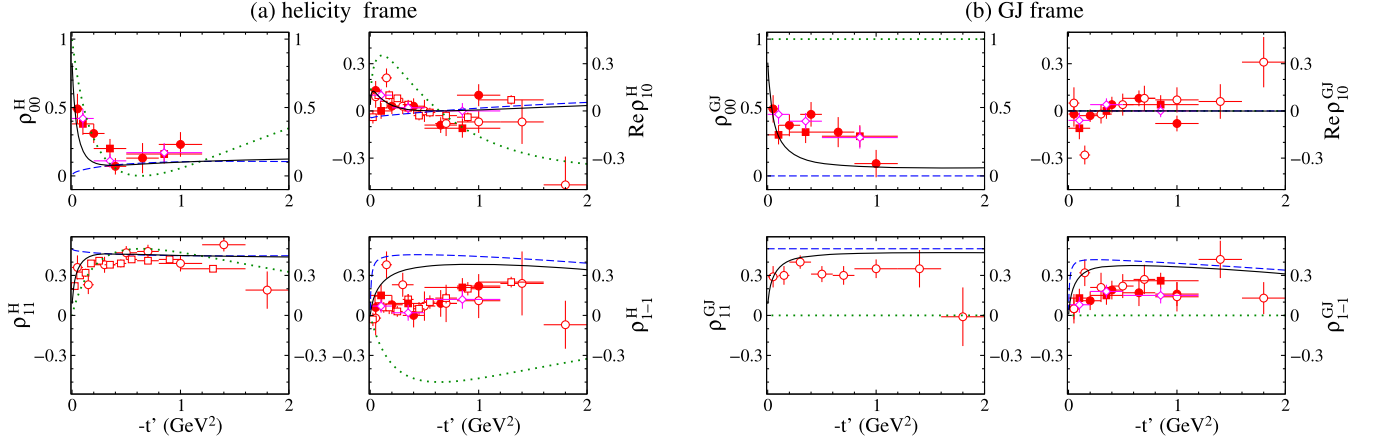


FIG. 9. SDMEs for the $K^- p \rightarrow \phi \Lambda$ reaction as functions of $-t'$. ρ_{00} , $\text{Re} \rho_{10}$, ρ_{11} , and ρ_{1-1} in the (a) helicity and (b) GJ frames at $P_{\text{Lab}} = 4.25$ GeV. Experimental data are taken from Refs. [31,32,34–36]: filled circles, open circles, open squares, and filled squares, and open diamonds. The beam momentum in Ref. [32] is $P_{\text{Lab}} = 3.95$ GeV. The curve notation is the same as in Fig. 5.

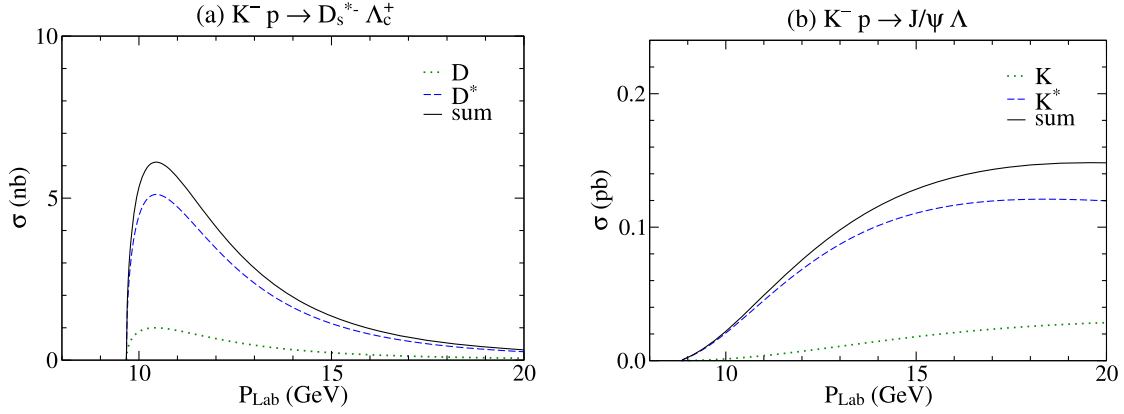


FIG. 10. Total cross sections for the (a) $K^- p \rightarrow D_s^{*-} \Lambda_c^+$ and (b) $K^- p \rightarrow J/\psi \Lambda$ reactions as functions of P_{Lab} . The green dotted and blue dashed curves denote the contributions from pseudoscalar- and vector-Reggeon exchanges, respectively, while the black solid curves represent their sum.

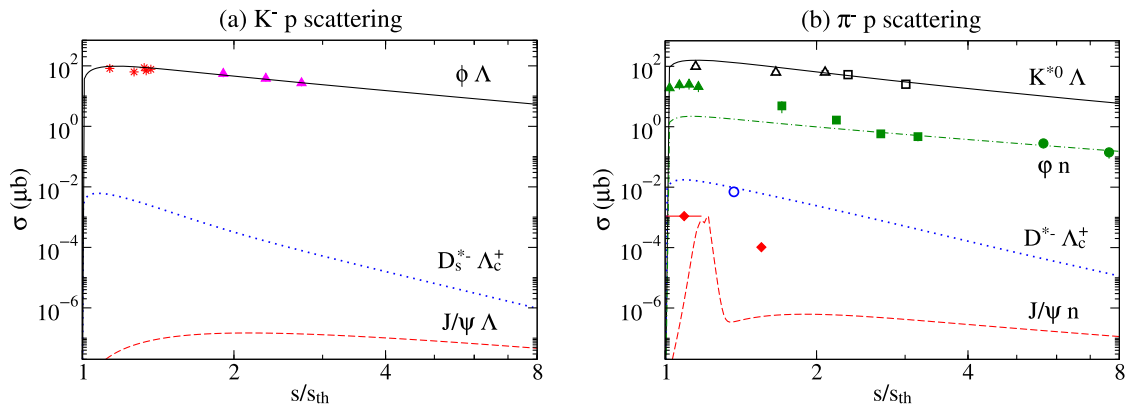


FIG. 11. (a) Total cross sections for the $K^- p \rightarrow (\phi \Lambda, D_s^{*-} \Lambda_c^+, J/\psi \Lambda)$ reactions as functions of s/s_{th} . Experimental data are taken from Refs. [29,33]: stars and filled triangles. (b) Total cross sections for the $\pi^- p \rightarrow (K^{*0} \Lambda, \phi n, D^{*-} \Lambda_c^+, J/\psi n)$ reactions. Experimental data are taken from Refs. [68] (open triangles), [66] (open squares), [69] (filled triangles), [33] (filled squares and circles), [70] (open circles), [71,72] (filled diamonds). Theoretical results are taken from Refs. [15,18].

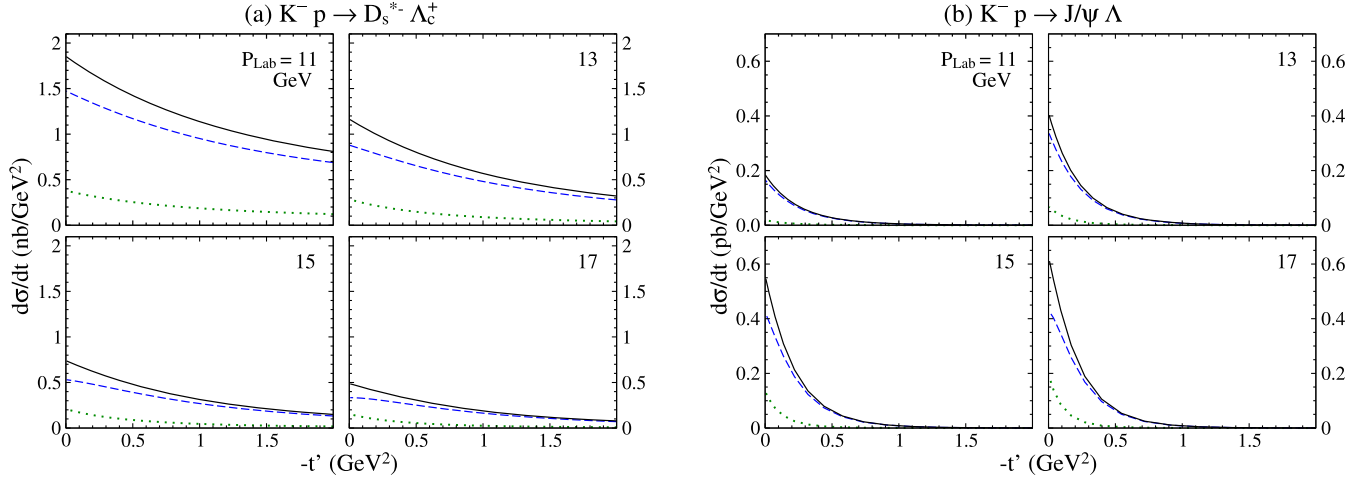


FIG. 12. t -dependent differential cross sections for the (a) $K^-p \rightarrow D_s^{*-}\Lambda_c^+$ and (b) $K^-p \rightarrow J/\psi\Lambda$ reactions at four fixed beam momenta. Curve notations are the same as in Fig. 10.

magnitude compared to that for the reaction $K^-p \rightarrow \phi\Lambda$, even though the same Regge parameters are employed. This stronger suppression mainly arises from the much smaller coupling strength in the charm sector, $g_{\phi K^*K} \simeq 3.5 \times 10^3 g_{J/\psi K^*K}$.

It is worthwhile to compare the results shown in Fig. 11 (a) with those of the relevant pion-induced reactions studied in Refs. [15,18] within a similar theoretical framework. In Fig. 11(b), we compare the total cross sections for the reactions $\pi^-p \rightarrow K^{*0}\Lambda$ [66,68], ϕn [33,69], $D^{*-}\Lambda_c^+$ [70], and $J/\psi n$ [71,72]. The hidden-strangeness and hidden-charm production channels (ϕn and $J/\psi n$) are more suppressed than the corresponding open-hadron production channels ($K^{*0}\Lambda$ and $D^{*-}\Lambda_c^+$) due to the Okubo-Zweig-Iizuka rule [73–75], similar to the kaon-induced reactions shown in Fig. 11(a). The predicted cross section for the $D^{*-}\Lambda_c^+$ channel is in good agreement with the available experimental upper limit [70]. The peak structure observed in the $\pi^-p \rightarrow J/\psi n$ reaction originates from the contributions of the hidden-charm pentaquark states $P_c(4380)$ and $P_c(4450)$ [76]. The resulting cross section near threshold is close to the experimental upper limit [71,72]. Note that the result for the $J/\psi n$ background production should be regarded as a lower bound on the cross section, since only the summed branching ratio $\mathcal{B}(\phi \rightarrow \rho\pi + \pi^+\pi^-\pi^0) = 15.32\%$ is experimentally available [53]. This leads to an ambiguity in determining the individual $\mathcal{B}(\phi \rightarrow \rho\pi)$ channel.

Figure 12 displays the predicted t -dependent differential cross sections for the $K^-p \rightarrow D_s^{*-}\Lambda_c^+$ and $K^-p \rightarrow J/\psi\Lambda$ reactions at four fixed beam momenta. As expected, forward peaks are observed in both cases, particularly in the latter case. The differential cross sections are expressed in units of nb/GeV^2 and pb/GeV^2 , respectively. Consequently, future high-intensity accelerator facilities will be required to measure such small cross sections, particularly for the $K^-p \rightarrow J/\psi\Lambda$ reaction.

In Fig. 13, we present the individual predictions for the SDMEs as functions of $-t'$ at $P_{\text{Lab}} = 15 \text{ GeV}$ in both the helicity and GJ frames. In principle, for the vector-Reggeon exchange, the matrix elements $\rho_{\lambda\lambda'}$ with $|\lambda| = |\lambda'| = 1$ are strongly enhanced. This behavior originates from the spin structure $\epsilon^{\mu\nu\alpha\beta}\epsilon_\mu^*(\lambda_V)k_{2\alpha}k_{1\beta}$ of the amplitude in Eq. (9). In the vector meson rest frame, where $\mathbf{k}_2 = (M_V, 0, 0, 0)$ and $\mathbf{k}_1$ corresponds to the incoming kaon three-momentum $\mathbf{p}_K$, this factor is proportional to the vector product $\boldsymbol{\epsilon}^*(\lambda_V) \times \mathbf{p}_K$. In the helicity frame and at small momentum transfers, $\mathbf{p}_K$ has a large z component and a small transverse component, which yields $\boldsymbol{\epsilon}^*(\lambda_V) \times \mathbf{p}_K \simeq i\lambda_V \boldsymbol{\epsilon}^*(\lambda_V)|\mathbf{p}_K|$, thereby leading to a pronounced enhancement of $\rho_{|\lambda|=1,|\lambda'|=1}$. In the GJ frame, $\mathbf{p}_K$ is aligned with the quantization axis; consequently, $\rho_{\lambda\lambda'}$ vanishes whenever either $\lambda = 0$ or $\lambda' = 0$. For the pseudoscalar-Reggeon exchange, however, the situation is markedly different. In this case, the scattering amplitude is proportional to the scalar product $\boldsymbol{\epsilon}^*(\lambda_V) \cdot \mathbf{p}_K$ in Eq. (9), which strongly enhances ρ_{00} in the GJ frame, so that $\rho_{00} = 1$, while all other $\rho_{\lambda\lambda'}$ vanish [22].

C. Pentaquark contributions to $K^-p \rightarrow D_s^{*-}\Lambda_c^+, J/\psi\Lambda$

Having obtained reasonable estimates for the background contributions to the $K^-p \rightarrow D_s^{*-}\Lambda_c^+$ and $K^-p \rightarrow J/\psi\Lambda$ reactions in the previous subsection, we investigate the s -channel P_{cs} contributions illustrated in Fig. 4. Using the branching ratios $\mathcal{B}(P_{cs} \rightarrow \bar{D}_s^*\Lambda_c)$ and $\mathcal{B}(P_{cs} \rightarrow J/\psi\Lambda)$ determined in Eq. (33), we take the branching ratios for $P_{cs} \rightarrow \bar{K}N$ to be 10^{-8} and 10^{-7} , treating them as free parameters. The predicted total cross sections are shown in Fig. 14 as functions of the c.m. energy $W = \sqrt{s}$, where the results for $P_{cs}(4459)^0$ with $J^P = 1/2^-$ and $3/2^-$ are displayed in the left and right panels, respectively. The contribution from the P_{cs} state with $J^P = 3/2^-$ is larger than that with $J^P = 1/2^-$ for both reactions. Since the

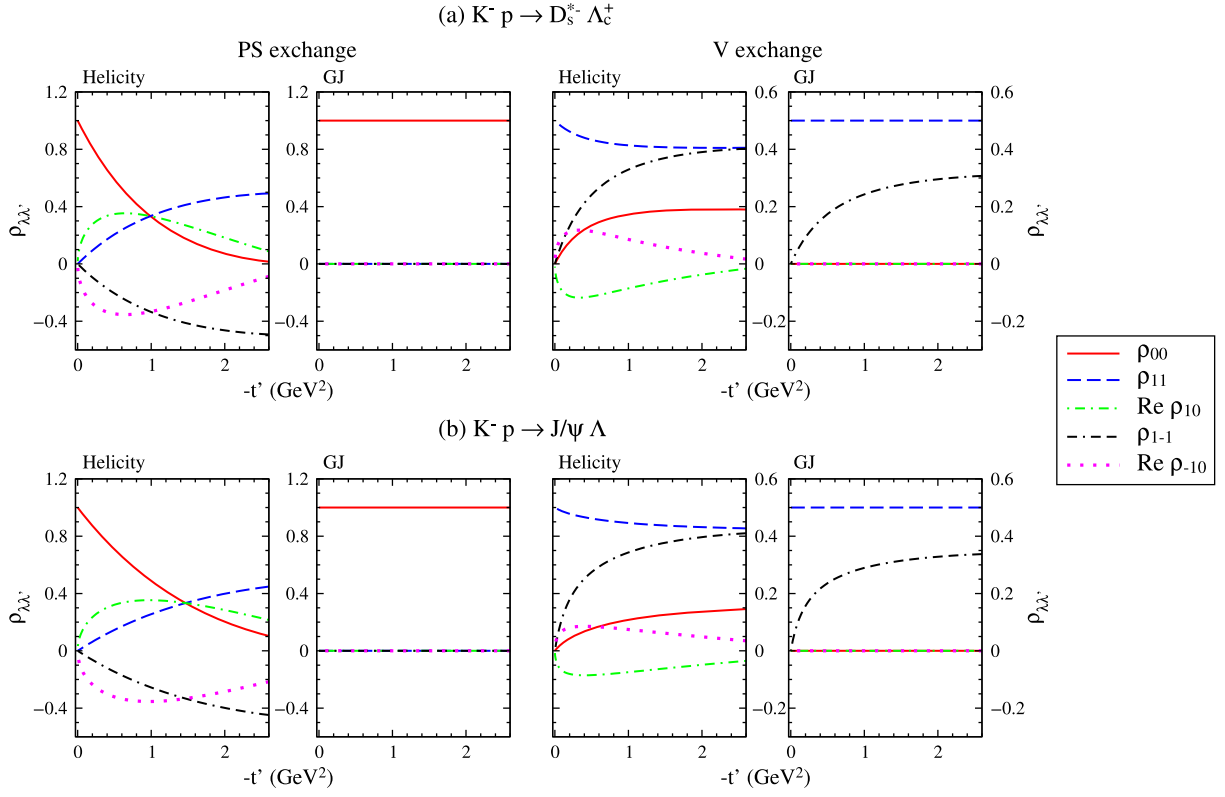


FIG. 13. SDMEs for the (a) $K^- p \rightarrow D_s^{*-} \Lambda_c^+$ and (b) $K^- p \rightarrow J/\psi \Lambda$ reactions as functions of $-t'$ at $P_{\text{Lab}} = 15$ GeV. The results for pseudoscalar- and vector-Reggeon exchanges are shown in the left and right panels, respectively, in both the helicity and GJ frames.

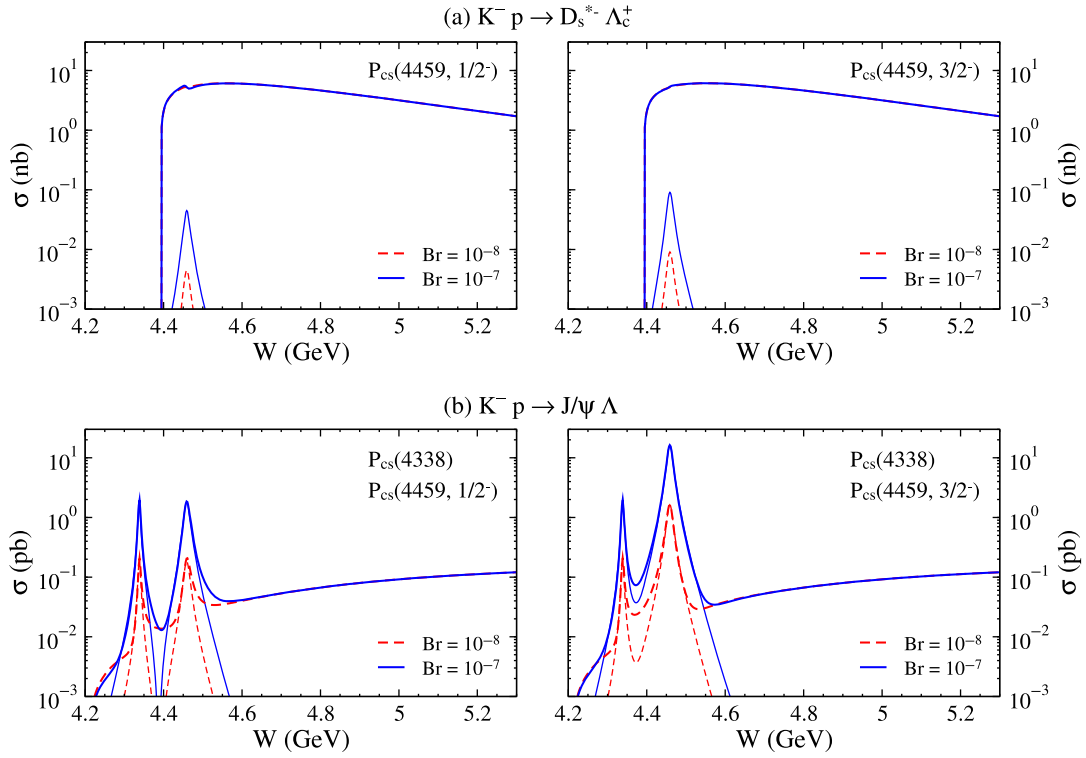


FIG. 14. Total cross sections for the (a) $K^- p \rightarrow D_s^{*-} \Lambda_c^+$ and (b) $K^- p \rightarrow J/\psi \Lambda$ reactions as functions of W , including contributions from the $P_{cs}(4338)^0$ and $P_{cs}(4459)^0$ states. Left and right panels correspond to $J^P = 1/2^-$ and $3/2^-$ for $P_{cs}(4459)^0$, respectively. The branching ratio $\mathcal{B}(P_{cs} \rightarrow \bar{K} N)$ is taken to be 10^{-8} and 10^{-7} .

background contribution to $K^-p \rightarrow D_s^{*-}\Lambda_c^+$ is much larger than that to $K^-p \rightarrow J/\psi\Lambda$, the $P_{cs}(4459)^0$ signal is difficult to observe in the $D_s^{*-}\Lambda_c^+$ channel. In contrast, both $P_{cs}(4338)^0$ and $P_{cs}(4459)^0$ give sizable contributions to $J/\psi\Lambda$ production. It is also seen that the $P_{cs}(4459)^0$ contribution exhibits a broader structure, reflecting its larger decay width.

IV. SUMMARY AND CONCLUSION

In this work, we investigated strangeness production in the $K^-p \rightarrow \phi\Lambda$ reaction within a hybrid Regge framework based on a QGSM-motivated prescription. The reaction dynamics are found to be governed predominantly by t -channel exchanges. In particular, the K^* -Reggeon exchange provides the leading contribution over the considered energy range, while the K -Reggeon exchange plays a non-negligible role, especially in reproducing the observed SDMEs. Contributions from the s -channel Λ and u -channel nucleon exchanges are small in the total cross section, although the u -channel nucleon exchange improves the description at backward angles.

Using the same hybrid Regge framework, we analyzed the open-charm $K^-p \rightarrow D_s^{*-}\Lambda_c^+$ and hidden-charm $K^-p \rightarrow J/\psi\Lambda$ reactions. The Regge trajectories $\alpha(t)$ and energy-scale parameters s_0 were determined consistently, thereby reducing model dependence and theoretical uncertainties. Our results indicate that the production rates for the open- and hidden-charm channels are strongly suppressed, by roughly 4–6 and 8–9 orders of magnitude, respectively, compared with the strangeness-production channel, depending on the kinematic region. This strong suppression originates mainly from the larger energy-scale parameters and much smaller effective coupling strengths in the charm sector.

We further explored possible s -channel contributions from the hidden-charm pentaquark states with strangeness, $P_{cs}(4338)^0$ and $P_{cs}(4459)^0$. While their effects are difficult to isolate in the open-charm channel due to the large nonresonant background, they may produce noticeable enhancements in the $J/\psi\Lambda$ channel, suggesting that

hidden-charm production could provide a more favorable environment for studying the P_{cs} states.

The SDME results allow us to disentangle the natural- and unnatural-parity-exchange contributions, which are largely dominated by K^* - and K -Reggeon exchanges, respectively, and further suggest that additional unnatural-parity-exchange contributions, such as those involving the axial-vector $K_1(1270)$ and $K_1(1400)$ mesons, may play a role. In addition, the tetraquark candidate $Z_{cs}(4000)^+$, observed in the $J/\psi K^+$ channel and having a possible $c\bar{c}u\bar{s}$ quark content [77], could contribute to the $K^-p \rightarrow J/\psi\Lambda$ reaction via t -channel exchange. Such effects were also not considered here and deserve further study.

Our predictions for charm-production observables, including total and differential cross sections as well as SDMEs, can serve as useful baselines for future measurements at J-PARC [2,27,28]. Within the same theoretical framework, the present study can be extended to other strangeness and charm production processes in kaon-induced reactions, such as $K^-p \rightarrow \phi\Sigma^0$, $D_s^{*-}\Sigma_c^+$, $J/\psi\Sigma^0$, as well as to pion-induced reactions like $\pi^-p \rightarrow K^*\Sigma$, $\bar{D}^*\Sigma_c$. The positive-parity states $D_{s0}^*(2317)$ and $D_{s1}(2460)$ lie about 40 MeV below the DK and D^*K thresholds, respectively, and are widely discussed in connection with DK and D^*K molecular interpretations [78,79]. It is therefore interesting to investigate their production mechanisms in the $K^-p \rightarrow D_{s0}^*(2317)\Lambda_c^+$ and $K^-p \rightarrow D_{s1}(2460)\Lambda_c^+$ reactions [80]. Such studies may provide further insight into their internal structure and underlying dynamics. Work along these lines is in progress.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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- [1] M. F. M. Lutz *et al.* ($\bar{\text{P}}\text{ANDA}$ Collaboration), Physics performance report for $\bar{\text{P}}\text{ANDA}$: Strong interaction studies with antiprotons, [arXiv:0903.3905](#).
 - [2] K. Aoki *et al.*, Extension of the J-PARC hadron experimental facility: Third white paper, [arXiv:2110.04462](#).

- [3] M. Andreotti *et al.*, Results of a search for the $h_c(^1P_1)$ state of charmonium in the $\eta_c\gamma$ and $J/\psi\pi^0$ decay modes, [Phys. Rev. D **72**, 032001 \(2005\)](#).
- [4] B. Singh *et al.* ($\bar{\text{P}}\text{ANDA}$ Collaboration), Feasibility study for the measurement of πN transition distribution

- amplitudes at PANDA in $\bar{p}p \rightarrow J/\psi\pi^0$, *Phys. Rev. D* **95**, 032003 (2017).
- [5] J. Haidenbauer and G. Krein, Production of charmed pseudoscalar mesons in antiproton-proton annihilation, *Phys. Rev. D* **89**, 114003 (2014).
- [6] J. Haidenbauer and G. Krein, Production of charmed baryons in $\bar{p}p$ collisions close to their thresholds, *Phys. Rev. D* **95**, 014017 (2017).
- [7] A. I. Titov and B. Kampfer, Exclusive charm production in $\bar{p}p$ collisions at $\sqrt{s} \lesssim 15$ GeV, *Phys. Rev. C* **78**, 025201 (2008).
- [8] R. Shyam and H. Lenske, Reaction $\bar{p}p \rightarrow \bar{\Lambda}_c^-\Lambda_c^+$ within an effective Lagrangian model, *Phys. Rev. D* **90**, 014017 (2014).
- [9] T. Sanghakhrit, S. I. Shim, Y. Yan, and A. Hosaka, Charmed baryon pair production in proton-antiproton collisions in effective Lagrangian and Regge approaches, *Eur. Phys. J. A* **58**, 32 (2022).
- [10] G. T. Bodwin, E. Braaten, and G. P. Lepage, Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium, *Phys. Rev. D* **51**, 1125 (1995); *Phys. Rev. D* **55**, 5853(E) (1997).
- [11] A. Lundborg, T. Barnes, and U. Wiedner, Charmonium production in $p\bar{p}$ annihilation: Estimating cross sections from decay widths, *Phys. Rev. D* **73**, 096003 (2006).
- [12] T. Barnes and X. Li, Associated charmonium production in low energy $p\bar{p}$ annihilation, *Phys. Rev. D* **75**, 054018 (2007).
- [13] H. Nouni *et al.*, J-PARC hadron experimental facility proposal P50, “Charmed baryon spectroscopy via the (π^-, D^{*-}) reaction”, available at https://j-parc.jp/researcher/Hadron/en/Proposal_e.html (2012).
- [14] S. H. Kim, A. Hosaka, H. C. Kim, H. Nouni, and K. Shirotori, Pion induced reactions for charmed baryons, *Prog. Theor. Exp. Phys.* **2014**, 103D01 (2014).
- [15] S. H. Kim, A. Hosaka, H. C. Kim, and H. Nouni, Production of strange and charmed baryons in pion induced reactions, *Phys. Rev. D* **92**, 094021 (2015).
- [16] S. I. Shim, A. Hosaka, and H. C. Kim, Heavy baryon production with an instanton interaction, *Prog. Theor. Exp. Phys.* **2020**, 053D01 (2020).
- [17] S. Y. Ryu *et al.*, J-PARC hadron experimental facility proposal P111, “ J/ψ production in π^-p reaction near threshold”, available at https://j-parc.jp/researcher/Hadron/en/Proposal_e.html (2025).
- [18] S. H. Kim, H. Ch. Kim, and A. Hosaka, Heavy pentaquark states $P_c(4380)$ and $P_c(4450)$ in the J/ψ production induced by pion beams off the nucleon, *Phys. Lett. B* **763**, 358 (2016).
- [19] A. Sibirtsev and K. Tsushima, J/ψ production in πN collisions, *arXiv:nucl-th/9810029*.
- [20] J. J. Wu and T. S. H. Lee, Production of J/ψ on the nucleon and on deuteron targets, *Phys. Rev. C* **88**, 015205 (2013).
- [21] S. H. Kim, H. Ch. Kim, and A. Hosaka, $K^0\Lambda$ and $D^-\Lambda_c^+$ production induced by pion beams off the nucleon, *Phys. Rev. D* **94**, 094025 (2016).
- [22] S. H. Kim, Y. Oh, and A. I. Titov, Decay angular distributions of K^* and D^* vector mesons in pion-nucleon scattering, *Phys. Rev. C* **95**, 055206 (2017).
- [23] A. B. Kaidalov, Hadronic mass relations from topological expansion and string model, *Z. Phys. C* **12**, 63 (1982).
- [24] K. G. Borekov and A. B. Kaidalov, Production of charmed baryons in hadron-hadron collisions, *Yad. Fiz.* **37**, 174 (1983); [*Sov. J. Nucl. Phys.* **37**, 100 (1983)].
- [25] A. B. Kaidalov and O. I. Piskunova, Production of charmed particles in the quark-gluon string model, *Yad. Fiz.* **43**, 1545 (1986); [*Sov. J. Nucl. Phys.* **43**, 994 (1986)].
- [26] A. B. Kaidalov and P. E. Volkovitsky, Binary reactions in $\bar{p}p$ collisions at intermediate energies, *Z. Phys. C* **63**, 517 (1994).
- [27] T. Takahashi *et al.*, Beam and SKS spectrometers at the K1.8 beam line, *Prog. Theor. Exp. Phys.* **2012**, 02B010 (2012).
- [28] K. Agari *et al.*, The K1.8BR spectrometer system at J-PARC, *Prog. Theor. Exp. Phys.* **2012**, 02B011 (2012).
- [29] J. S. Lindsey and G. A. Smith, Production properties and decay modes of the ϕ meson, *Phys. Rev.* **147**, 913 (1966).
- [30] L. Lyons, A. M. Cooper-Sarkar, and A. G. Clark, $K^-p \rightarrow \Lambda^0$ or Σ^0 + vector meson at 3.1–3.6 GeV/c, *Nucl. Phys. B* **125**, 207 (1977).
- [31] M. Aguilar-Benitez, S. U. Chung, R. L. Eisner, and N. P. Samios, Study of nonstrange mesons produced in K^-p interactions at 3.9 and 4.6 GeV/c, *Phys. Rev. D* **6**, 29 (1972).
- [32] A. Rouge, H. Videau, I. Videau, J. P. De Brion, L. Moscoso, and R. Rader, The reactions $K^- + p \rightarrow \Lambda(\Sigma^0) +$ vector meson at 3.95 GeV/c, *Nucl. Phys. B* **44**, 365 (1972).
- [33] D. S. Ayres, R. Diebold, A. F. Greene, S. L. Kramer, J. S. Levine, A. J. Pawlicki, and A. B. Wicklund, ϕ -meson production in π^-p and K^-p interactions from 3 to 6 GeV/c, *Phys. Rev. Lett.* **32**, 1463 (1974).
- [34] M. J. Losty *et al.*, Nonstrange vector meson production in K^-p interactions at 4.2 GeV/c, *Nucl. Phys. B* **133**, 38 (1978).
- [35] A. J. De Groot, W. Hodgland, B. Jongejans, J. C. Kluyver, G. G. G. Massaro, W. J. Metzger, H. G. J. M. Tiecke, and J. Timmermans, A study of the hypercharge exchange reactions $K^-p \rightarrow 1^- \frac{1}{2}^+$ at an incoming momentum of 4.25 GeV/c, *Nucl. Phys. B* **74**, 77 (1974).
- [36] R. D. Field, R. L. Eisner, S. U. Chung, and M. Aguilar-Benitez, Transversity-amplitude analysis of the reactions $K^-p \rightarrow (\omega, \phi)\Lambda$, *Phys. Rev. D* **7**, 2036 (1973).
- [37] R. Aaij *et al.* (LHCb Collaboration), Evidence of a $J/\psi\Lambda$ structure and observation of excited Ξ^- states in the $\Xi_b^- \rightarrow J/\psi\Lambda K^-$ decay, *Sci. Bull.* **66**, 1278 (2021).
- [38] R. Aaij *et al.* (LHCb Collaboration), Observation of a $J/\psi\Lambda$ resonance consistent with a strange pentaquark candidate in $B^- \rightarrow J/\psi\Lambda\bar{p}$ decays, *Phys. Rev. Lett.* **131**, 031901 (2023).
- [39] B. Wang, L. Meng, and S. L. Zhu, Spectrum of the strange hidden charm molecular pentaquarks in chiral effective field theory, *Phys. Rev. D* **101**, 034018 (2020).
- [40] F. Z. Peng, M. J. Yan, M. Sánchez Sánchez, and M. P. Valderrama, The $P_{cs}(4459)$ pentaquark from a combined effective field theory and phenomenological perspective, *Eur. Phys. J. C* **81**, 666 (2021).
- [41] R. Chen, Can the newly reported $P_{cs}(4459)$ be a strange hidden-charm $\Xi_c\bar{D}^*$ molecular pentaquark?, *Phys. Rev. D* **103**, 054007 (2021).

- [42] F. L. Wang and X. Liu, Emergence of molecular-type characteristic spectrum of hidden-charm pentaquark with strangeness embodied in the $P_{\psi s}^{\Lambda}(4338)$ and $P_{cs}(4459)$, *Phys. Lett. B* **835**, 137583 (2022).
- [43] M. J. Yan, F. Z. Peng, M. Sánchez Sánchez, and M. P. Valderrama, $P_{\psi s}^{\Lambda}(4338)$ pentaquark and its partners in the molecular picture, *Phys. Rev. D* **107**, 074025 (2023).
- [44] F. L. Wang, S. Q. Luo, H. Y. Zhou, Z. W. Liu, and X. Liu, Exploring the electromagnetic properties of the $\Xi_c^{(*)}\bar{D}_s^{(*)}$ and $\Omega_c^{(*)}\bar{D}_s^{(*)}$ molecular states, *Phys. Rev. D* **108**, 034006 (2023).
- [45] J. T. Zhu, S. Y. Kong, and J. He, $P_{\psi s}^{\Lambda}(4459)$ and $P_{\psi s}^{\Lambda}(4338)$ as molecular states in $J/\psi\Lambda$ invariant mass spectra, *Phys. Rev. D* **107**, 034029 (2023).
- [46] A. Feijoo, W. F. Wang, C. W. Xiao, J. J. Wu, E. Oset, J. Nieves, and B. S. Zou, A new look at the P_{cs} states from a molecular perspective, *Phys. Lett. B* **839**, 137760 (2023).
- [47] F. L. Wang and X. Liu, Surveying the mass spectra and the electromagnetic properties of the $\Xi_c^{(*)}D^{(*)}$ molecular pentaquarks, *Phys. Rev. D* **109**, 014043 (2024).
- [48] B. Wang, K. Chen, L. Meng, and S. L. Zhu, Spectrum of the molecular pentaquarks, *Phys. Rev. D* **109**, 074035 (2024).
- [49] L. Roca, J. Song, and E. Oset, Molecular pentaquarks with hidden charm and double strangeness, *Phys. Rev. D* **109**, 094005 (2024).
- [50] Z. Y. Yang, F. Z. Peng, M. J. Yan, M. Sánchez Sánchez, and M. P. Valderrama, Molecular P_{ψ} pentaquarks from light-meson exchange saturation, *Phys. Rev. D* **111**, 014012 (2025).
- [51] R. Chen, Strong decays of the newly $P_{cs}(4459)$ as a strange hidden-charm $\Xi_c\bar{D}^*$ molecule, *Eur. Phys. J. C* **81**, 122 (2021).
- [52] C. W. Xiao, J. J. Wu, and B. S. Zou, Molecular nature of $P_{cs}(4459)$ and its heavy quark spin partners, *Phys. Rev. D* **103**, 054016 (2021).
- [53] S. Navas *et al.*, Particle data group, *Phys. Rev. D* **110**, 030001 (2024).
- [54] Y. Oh, K. Nakayama, and T. S. H. Lee, Pentaquark $\theta^+(1540)$ production in $\gamma N \rightarrow K\bar{K}N$, *Phys. Rep.* **423**, 49 (2006).
- [55] S. H. Kim, J. K. Ahn, Sh. H. Kim, S. i. Nam, and M. K. Cheoun, Double-strangeness exchange reactions in a hybrid Regge-plus-resonance approach, *Phys. Rev. C* **107**, 065202 (2023).
- [56] T. A. Rijken, V. G. J. Stoks, and Y. Yamamoto, Soft-core hyperon-nucleon potentials, *Phys. Rev. C* **59**, 21 (1999).
- [57] V. G. J. Stoks and T. A. Rijken, Soft-core baryon-baryon potentials for the complete baryon octet, *Phys. Rev. C* **59**, 3009 (1999).
- [58] M. M. Brisudova, L. Burakovsky, and J. T. Goldman, Effective functional form of Regge trajectories, *Phys. Rev. D* **61**, 054013 (2000).
- [59] P. Colangelo, F. De Fazio, and T. N. Pham, Nonfactorizable contributions in B decays to charmonium: The case of $B^- \rightarrow K^- h_c$, *Phys. Rev. D* **69**, 054023 (2004).
- [60] F. K. Guo, C. Hanhart, G. Li, U. G. Meissner, and Q. Zhao, Effect of charmed meson loops on charmonium transitions, *Phys. Rev. D* **83**, 034013 (2011).
- [61] J. X. Lu, M. Z. Liu, R. X. Shi, and L. S. Geng, Understanding $P_{cs}(4459)$ as a hadronic molecule in the $\Xi_b^- \rightarrow J/\psi\Lambda K^-$ decay, *Phys. Rev. D* **104**, 034022 (2021).
- [62] X. W. Wang and Z. G. Wang, Strong decays of $P_{cs}(4338)$ and its high isospin cousin via QCD sum rules, *Phys. Rev. D* **110**, 014008 (2024).
- [63] S. H. Kim, Y. Oh, S. Son, S. Sakinah, and M. K. Cheoun, Effective Lagrangian for strong and electromagnetic interactions of high-spin resonances, *Phys. Rev. D* **111**, 054031 (2025).
- [64] C. Fernández-Ramírez, A. Pilloni, M. Albaladejo, A. Jackura, V. Mathieu, M. Mikhasenko, J. A. Silva-Castro, and A. P. Szczepaniak (JPAC Collaboration), Interpretation of the LHCb $P_c(4312)^+$ signal, *Phys. Rev. Lett.* **123**, 092001 (2019).
- [65] K. Schilling, P. Seyboth, and G. E. Wolf, On the analysis of vector meson production by polarized photons, *Nucl. Phys. B* **15**, 397; *Nucl. Phys.* **18**, 332(E) (1970).
- [66] D. J. Crennell, H. A. Gordon, K. W. Lai, and J. M. Scarr, Two-body strange-particle final states in $\pi^- p$ interactions at 4.5 and 6 GeV/c, *Phys. Rev. D* **6**, 1220 (1972).
- [67] S. Clymton, H. J. Kim, and H. C. Kim, Production of hidden-charm strange pentaquarks P_{cs} from the $K^- p \rightarrow J/\psi\Lambda$ reaction, *Phys. Rev. D* **104**, 014023 (2021).
- [68] O. I. Dahl, L. M. Hardy, R. I. Hess, J. Kirz, and D. H. Miller, Strange-particle production in $\pi^- p$ interactions from 1.5 to 4.2 BeV/c. 1. Three-and-more-body final states, *Phys. Rev.* **163**, 1377 (1967).
- [69] H. Courant, Y. I. Makdisi, M. L. Marshak, E. A. Peterson, K. Ruddick, and J. Smith-Kintner, ϕ production in $\pi^- p$ collisions near threshold, *Phys. Rev. D* **16**, 1 (1977).
- [70] J. H. Christenson, E. Hummel, G. A. Kreiter, J. Sculli, and P. Yamin, Limits on charm production in hadronic interactions near threshold, *Phys. Rev. Lett.* **55**, 154 (1985).
- [71] K. Jenkins *et al.*, A search for the reaction $\pi^- p \rightarrow \psi n$ near threshold, *Phys. Rev. D* **17**, 52 (1978).
- [72] I. H. Chiang *et al.*, Search for exclusive J/ψ production, *Phys. Rev. D* **34**, 1619 (1986).
- [73] S. Okubo, ϕ -meson and unitary symmetry model, *Phys. Lett.* **5**, 165 (1963).
- [74] G. Zweig, CERN Reports, CERN-TH-401 and CERN-TH-412, 1964.
- [75] J. Iizuka, Systematics and phenomenology of meson family, *Prog. Theor. Phys. Suppl.* **37**, 21 (1966).
- [76] R. Aaij *et al.* (LHCb Collaboration), Observation of $J/\psi p$ resonances consistent with pentaquark states in $\Lambda_b^0 \rightarrow J/\psi K^- p$ decays, *Phys. Rev. Lett.* **115**, 072001 (2015).
- [77] R. Aaij *et al.* (LHCb Collaboration), Observation of new resonances decaying to $J/\psi K^+$ and $J/\psi\phi$, *Phys. Rev. Lett.* **127**, 082001 (2021).
- [78] A. Faessler, T. Gutsche, V. E. Lyubovitskij, and Y. L. Ma, D^*K molecular structure of the $D_{s1}(2460)$ meson, *Phys. Rev. D* **76**, 114008 (2007).
- [79] L. Liu, K. Orginos, F. K. Guo, C. Hanhart, and U. G. Meissner, Interactions of charmed mesons with light pseudoscalar mesons from lattice QCD and implications on the nature of the $D_{s0}^*(2317)$, *Phys. Rev. D* **87**, 014508 (2013).
- [80] H. Zhu and Y. Huang, Production of the $D_{s0}(2317)$ and $D_{s1}(2460)$ by kaon-induced reactions on a proton target, *Phys. Rev. D* **100**, 054031 (2019).