

Erratum: Conservative binary dynamics from gravitational tail emission processes [Phys. Rev. D **108**, 124010 (2023)]

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A handful of mistakes and typos unfortunately slipped into some of the equations in the paper. Equation (17) should be

$$i(\mathcal{A}_{ij}^{(m-L-\text{ftail})})^{TT}(\omega, \omega \hat{\mathbf{n}}) = (-i)^r c_r^{(J)} G \Lambda_{ij,kl}^{TT} \frac{i\omega^3 L^q J^{pR(k)}(\omega)}{(r+1)(r+2)(r+3)(r+4)(r+5)} \\ \times \{ (r^4 + 10r^3 + 35r^2 + 50r + 48) \delta_{lp} [(1 - \delta_{0r}) \omega^2 \delta_{qi_1} k_{R-1} - k_q k_R] \\ + (r^4 + 12r^3 + 53r^2 + 102r + 84) \delta_{lq} k_p k_R \}. \quad (17)$$

This change propagates in the Ward identity equations, (19) and (25), by simply changing the sign of their right-hand sides:

$$k^\mu \mathcal{A}_{\mu l}^{(m-L-\text{tail})}(\omega, \omega \hat{\mathbf{n}}) \stackrel{d \rightarrow 3}{=} \delta_{r0} \frac{4}{15} G \omega^5 L^k J^{lk}(\omega), \quad (19)$$

$$\mathcal{A}_{00}^{(W,m-L-\text{ftail})}(\omega, \omega \hat{\mathbf{n}}) = \frac{4}{15} G \omega^3 k^l L^k J^{lk}(\omega), \\ \mathcal{A}_{0l}^{(W,m-L-\text{ftail})}(\omega, \omega \hat{\mathbf{n}}) = -\frac{4}{15} G \omega^4 L^k J^{lk}(\omega), \\ \mathcal{A}_{kl}^{(W,m-L-\text{ftail})}(\omega, \omega \hat{\mathbf{n}}) = 0. \quad (25)$$

Unrelated to the above, the signs of the right-hand sides of (33)–(35) should also be reversed:

$$i\mathcal{S}^{(e-L-\text{tot})} = -G^2 \frac{(12 + 50r + 35r^2 + 10r^3 + r^4)}{(r+1)^2(r+2)^2(r+3)!(2r+5)!!} e^{ikl} L^l \int \frac{d\omega}{2\pi} I^{ijR}(\omega) I^{kjR}(-\omega) \omega^{7+2r} \\ = -\frac{1}{2\Lambda^2} \int_{\mathbf{k}} \frac{d\omega}{2\pi} (\mathcal{A}_{ij}^{e-L-\text{tot}})^{TT}(\omega, \mathbf{k}) P[h_{ij}, h_{kl}] (\mathcal{A}_{kl}^{(e)})^{TT}(-\omega, -\mathbf{k}), \quad (33)$$

$$i\mathcal{S}^{(m-L-\text{tot})} = -G^2 \frac{4(36 + 50r + 35r^2 + 10r^3 + r^4)}{(r+1)^2(r+3)^2(r+3)!(2r+5)!!} e^{ikl} L^l \int \frac{d\omega}{2\pi} J^{ijR}(\omega) J^{kjR}(-\omega) \omega^{7+2r} \\ = -\frac{1}{2\Lambda^2} \int_{\mathbf{k}} \frac{d\omega}{2\pi} (\mathcal{A}_{ij}^{m-L-\text{tot}})^{TT}(\omega, \mathbf{k}) P[h_{ij}, h_{kl}] (\mathcal{A}_{kl}^{(m)})^{TT}(-\omega, -\mathbf{k}), \quad (34)$$

$$i\mathcal{S}^{(L^2)} \equiv i\mathcal{S}^{(e-L-\text{tot})}|_{r=0} = -\frac{1}{30} G^2 e^{ikl} L^l \int \frac{d\omega}{2\pi} I^{ij}(\omega) I^{jk}(-\omega) \omega^7. \quad (35)$$

The old version of equation (34) contained another typo, corrected above, in the numerator of the prefactor; that is the coefficient of r^2 is 35, not 32. In this way the prefactor of Eq. (34) becomes very similar, although not exactly equal, to its electric counterpart (33).

Also, the global sign on the right-hand side of all appearances of the generic formula for the amplitude gluing should be a minus sign, i.e. [besides the already mentioned (33) and (34)], in Eq. (27) and in the second line of Eqs. (28), (29):

$$\begin{aligned}
i\tilde{\mathcal{S}}^{(r^2)} &= -\frac{1}{2\Lambda^2} \int_{\mathbf{k}} \frac{d\omega}{2\pi} \mathcal{A}_{\mu\nu}^{(e,r=0)}(\omega, \mathbf{k}) \mathcal{P}[h_{\mu\nu}, h_{\rho\sigma}] \mathcal{A}_{\rho\sigma}^{(e,r=0)}(-\omega, -\mathbf{k}) \\
&= -\frac{1}{2} \int_{\mathbf{k}} \frac{d\omega}{2\pi} (\mathcal{A}_{ij}^{(e,r=0)})^{TT}(\omega, \mathbf{k}) \mathcal{P}[h_{ij}, h_{kl}] (\mathcal{A}_{kl}^{(e,r=0)})^{TT}(-\omega, -\mathbf{k}),
\end{aligned} \tag{27}$$

$$\begin{aligned}
i\mathcal{S}^{(e-M\text{-tail})} &= -G^2 E \frac{2^{r+2}(r+3)(r+4)}{(r+1)(r+2)(2r+5)!} \times \int \frac{d\omega}{2\pi} (\omega^2)^{r+3} I^{ijR}(\omega) I^{ijR}(-\omega) \left[\frac{1}{\tilde{\epsilon}} - \gamma_r^{(e)} \right], \\
&= -\frac{1}{2\Lambda^2} \int_{\mathbf{k}} \frac{d\omega}{2\pi} (\mathcal{A}_{ij}^{e-M\text{-tail}})^{TT}(\omega, \mathbf{k}) P[h_{ij}, h_{kl}] (\mathcal{A}_{kl}^{(e)})^{TT}(-\omega, -\mathbf{k}),
\end{aligned} \tag{28}$$

$$\begin{aligned}
i\mathcal{S}^{(m-M\text{-tail})} &= -G^2 E \frac{2^{r+2}(r+2)^2(r+4)(r!)^2}{(2r+1)(2r+3)(2r+5)(2r)![(r+3)!]^2} \\
&\quad \times \int \frac{d\omega}{2\pi} J^{b|iRj}(\omega) J^{b'|kR'l}(-\omega) \omega^{2r+6} [\delta_{bb'} \delta_{ik} + (r+1) \delta_{ib'} \delta_{kb}] \left[\frac{1}{\tilde{\epsilon}} - \gamma_r^{(m)} \right], \\
&= -\frac{1}{2\Lambda^2} \int_{\mathbf{k}} \frac{d\omega}{2\pi} (\mathcal{A}_{ij}^{m-M\text{-tail}})^{TT}(\omega, \mathbf{k}) P[h_{ij}, h_{kl}] (\mathcal{A}_{kl}^{(m)})^{TT}(-\omega, -\mathbf{k}).
\end{aligned} \tag{29}$$

Finally, the signs of Eq. (D2) should be reversed: both the $\mp$'s should actually be $\pm$'s:

$$\int_{\mathbf{k}} \frac{k_{i_1} \dots k_{i_{2l}}}{\mathbf{k}^2 - (\omega \pm i\alpha)^2} = \left(\pm i \frac{\omega}{4\pi} \right) \delta_{i_1 \dots i_{2l}} \frac{\omega^{2l}}{(2l+1)!!} = \left(\pm i \frac{\omega}{16\pi^2} \right) \omega^{2l} \int d\Omega \hat{n}_{i_1} \dots \hat{n}_{i_{2l}}. \tag{D2}$$

The conclusions of the paper are unchanged.