

Natural supersymmetry at a muon collider

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There is great interest within the particle physics community for building a $\mu^+\mu^-$ collider with center-of-mass (CoM) energies ranging from $\sqrt{s} \sim 1\text{--}14$ TeV. For beyond-the-Standard-Model (BSM) physics, natural supersymmetry seems perhaps the most motivated, plausible extension of the Standard Model. Here, we examine what can be accomplished by a muon collider with regards to natural SUSY at various muon collider CoM energies. In natural SUSY—especially in the guise that would emerge from the string landscape—one expects sparticles to be spread over two orders of magnitude in mass values. A muon collider with highly variable beam energies would be most useful for targeting 2-body reaction thresholds and Higgs boson resonances.

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I. INTRODUCTION

Perhaps the most important lesson learned from over 10 years of LHC running is that the Standard Model (SM) reigns supreme at energy scales up to a few hundred GeV. Nonetheless, if we follow Dirac's advice and follow where the (SM) equations lead us, then the gauge hierarchy problem, as exemplified by the Higgs mass quadratic divergences, still remains as a reminder that the SM is likely incomplete.

A. Status of weak scale supersymmetry

Weak scale supersymmetry (SUSY) emerged already in the early 1980s as perhaps an inevitable solution to the big hierarchy problem in that it is free of the destabilizing quadratic divergences that plague the Higgs mass [1,2]. The $N = 1$ spacetime SUSY naturally emerges in Calabi-Yau string compactifications on Ricci-flat manifolds of special holonomy [3]. Non-SUSY compactifications have been

argued to lead to instabilities such as Witten's bubble of nothing [4].

1. Naturalness and the Δ_{EW} measure

In the pre-LHC era, naturalness considerations seemed to imply that superpartner masses within the context of the minimal supersymmetric Standard Model (MSSM) should lie at or around the weak scale [5]. When supersymmetric matter failed to turn up in early LHC searches, greater scrutiny was placed on the naturalness question.

(i) The log-derivative measure of fine-tuning $\Delta_{p_i} \equiv \max_i |\partial \log m_Z^2 / \partial \log p_i|$ asked for small variation in m_Z^2 with respect to variation in high scale soft SUSY breaking parameters. However, neglecting correlations between the soft parameters—which in popular models such as mSUGRA/CMSSM are input as independent parameters of ignorance whereas in explicit constructs they all emerge as computable numbers in terms of the gravitino mass $m_{3/2}$ —leads to fine-tuning overestimates [6] in the range of $10 - 10^3$ [7].

(ii) Likewise, the measure $\Delta_{HS} \equiv \delta m_{H_u}^2 / m_{H_u}^2$ neglects the fact that the logarithmic evolution of $m_{H_u}^2$ is dependent on the value of the soft term $m_{H_u}^2$ itself, and furthermore this evolution, to be phenomenologically viable, must be large enough to result in a (radiative-driven) breakdown of electroweak symmetry. Use of Δ_{HS} also results in fine-tuning overestimates by factors of $\sim 10 - 10^3$ [7].

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A more conservative, model-independent measure of fine-tuning Δ_{EW} was instead invoked in Refs. [8,9]. Minimization of the scalar potential in the MSSM allows one to relate the measured value of m_Z to the MSSM Lagrangian parameters

$$m_Z^2/2 = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2$$

$$\simeq -m_{H_u}^2 - \mu^2 - \Sigma_u^u(\tilde{t}_{1,2}) \quad (1)$$

where $m_{H_u}^2$ and $m_{H_d}^2$ are soft SUSY breaking Higgs masses, μ is the (SUSY-conserving) Higgs/Higgsino superpotential mass term,¹ $\tan \beta \equiv v_u/v_d$ is the ratio of Higgs field vacuum expectation values and the Σ_u^u and Σ_d^d terms contain an assortment of radiative corrections, the most important of which are usually the $\Sigma_u^u(\tilde{t}_{1,2})$ [9]. The numerical measure Δ_{EW} is defined by

$$\Delta_{\text{EW}} \equiv \max_i |i^{\text{th}} \text{ term on RHS of Eq.(1)}| / (m_Z^2/2). \quad (2)$$

A value of $\Delta_{\text{EW}} \lesssim 30$ requires each independent contribution to m_Z to be within a factor of four of m_Z itself.

Several implications of $\Delta_{\text{EW}} \lesssim 30$ are the following.

- (i) Since the μ parameter is directly bounded, then the Higgsino-like electroweakinos of the MSSM should have mass values $m_{\tilde{\chi}_{1,2}^0}$ and $m_{\tilde{\chi}_1^\pm} \sim 100\text{--}350$ GeV. Thus, unlike most 20th century SUSY constructs, the Higgsinos are the lightest SUSY particles.
- (ii) The weak scale value of $m_{H_u}^2$ is radiatively driven to small negative values $\sim -(100\text{--}350 \text{ GeV})^2$. This usually requires nonuniversal Higgs masses, as is generically expected in actual supergravity constructs [11–13].
- (iii) The (usually) dominant radiative corrections $\Sigma_u^u(\tilde{t}_{1,2})$ are minimized by large, negative trilinear soft terms A_0 [8]. Large trilinears are also generic in SUGRA constructs. They also help to uplift the light Higgs mass $m_h \rightarrow 125$ GeV [14]. Under Δ_{EW} , top squarks may range up to $m_{\tilde{t}_1} \lesssim 3$ TeV and $m_{\tilde{t}_2} \lesssim 8$ TeV [15].
- (iv) The gluinos contribute to the $\Sigma_{u,d}^{u,d}$ terms at two-loop level and thus can range up to 6–9 TeV (model-dependent [15,16]).
- (v) First/second generation squark and slepton mass contributions to the weak scale are Yukawa-coupling suppressed. Thus, they can range up to 10–40 TeV. These latter values can lead to a mixed decoupling/quasidegeneracy solution to the SUSY flavor and CP problems [17] which are otherwise endemic to gravity mediation models.

¹Twenty solutions to the SUSY μ problem are reviewed in Ref. [10].

2. Natural SUSY from the string landscape

While the above considerations do not preclude superparticles at or around the weak scale $\tilde{m} \sim 100$ GeV, LHC mass bounds and the Higgs mass $m_h \simeq 125$ GeV require much higher values. In addition, TeV-scale or higher values for soft SUSY breaking terms are preferred by the landscape of vacua which emerge in modern string theory. Restricting ourselves to hidden sector SUSY breaking via a single F -term, then Douglas' argument is that nothing in string theory prefers any single F -term value over any other [18]. The F terms should thus be distributed as random complex numbers in the landscape (and D -terms would be distributed as random real numbers). And since the hidden-sector SUSY breaking scale is determined as

$$m_{\text{hidden}}^4 = \sum_i F_i F_i^\dagger + \sum_\alpha D_\alpha D_\alpha \quad (3)$$

(including the possibility of additional F_i - and D_α -term breaking fields), then the soft terms, related as $m_{\text{soft}} \sim m_{\text{hidden}}^2/m_P$, should be statistically distributed as a power-law $f_{\text{SUSY}} \sim m_{\text{soft}}^{2n_F+n_D-1}$ in the landscape [18–20]. The textbook case of SUSY breaking via a single F term field would thus lead to a linear statistical selection of large soft terms. An upper bound on soft term selection comes from requiring the derived value of the weak scale in each viable pocket-universe (PU) within the landscape to lie within the ABDS window [21]: $m_Z^{OU}/2 \lesssim m_Z^{PU} \lesssim (2\text{--}5)m_Z^{OU}$ where $m_Z^{OU} = 91.2$ GeV in *our universe*. The ABDS window coincides with $\Delta_{\text{EW}} \lesssim 30$ for pocket universes with no fine-tuning (random selection of values). The gist of this is that the string landscape prefers large values of soft terms over small values so long as the derived value of the weak scale is not too large, i.e., $\Delta_{\text{EW}} \lesssim 30$. This then coincides with the nonappearance of sparticles at LHC, and with the measured value of the Higgs mass [22].

3. Expected sparticle and Higgs mass spectra

The expected natural SUSY sparticle mass spectra is thus shown in Fig. 1, and would be typically spread over two orders of magnitude in values. The vector bosons W and Z , Higgs boson h , and Higgsinos should all lie in the 100–300 GeV range. Binos, winos, and light top squarks $\tilde{t}_1$ could lie in the few-TeV range while $\tilde{t}_2$, $\tilde{b}_{1,2}$, gluinos, and heavy Higgs H , A and $H^\pm$ could lie within the 3–8 TeV range. First/second generation squarks and sleptons would likely lie in the 10–40 TeV range. Thus, the superparticle mass spectrum would likely be spread out over a range of two orders of magnitude. Given such a situation, how does this information affect plans for future accelerator projects throughout the world?

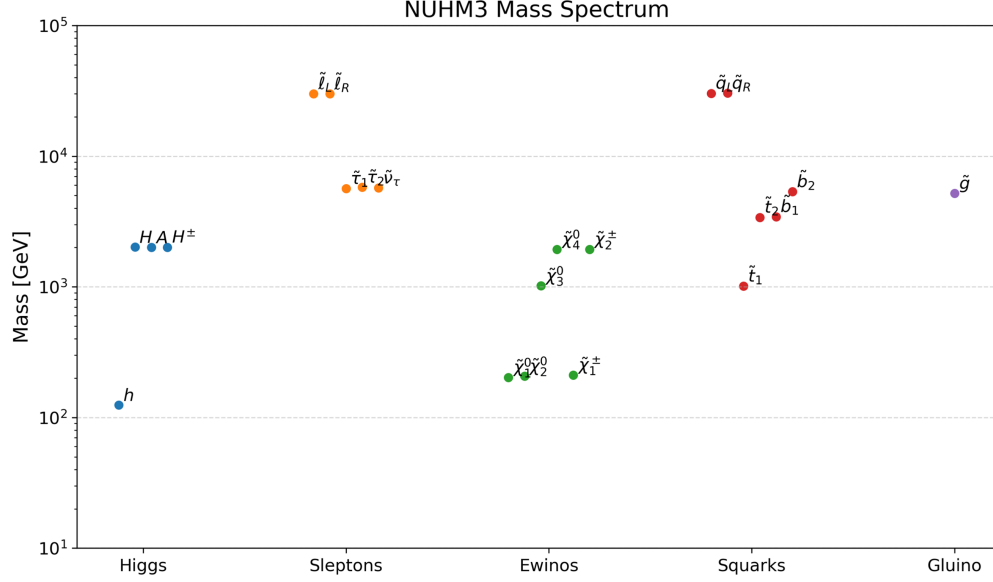


FIG. 1. Plot of typical natural SUSY mass spectra as expected from the string theory landscape.

B. Future accelerator facilities and a $\mu^+\mu^-$ collider

1. High luminosity LHC (HL-LHC)

The CERN high-luminosity LHC (HL-LHC) stands out as the forefront particle physics accelerator facility over the next decade. Recently, a review of HL-LHC capabilities for SUSY discovery in so-called plausible (natural) SUSY models was recently posted. In Ref. [23], it is found that the most lucrative avenues for SUSY discovery at HL-LHC are via Higgsino pair production (taking advantage of the relatively light Higgsinos with $\mu \sim 100\text{--}350$ GeV). In particular, the soft-opposite-sign dilepton plus jet(s) plus $\cancel{E}_T$ (SOSDJMET) signal arising from $pp \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0$ or $\tilde{\chi}_1^\pm \tilde{\chi}_2^\mp$ followed by $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 \ell^+ \ell^-$ seems most lucrative [24] (where an initial state jet radiation is used as a trigger) [25–27]. In fact, both ATLAS [28] and CMS [29] have some excess in this channel from Run 2 data, and it will be exciting to see what might be hiding in LHC Run 3 data. HL-LHC reach projections in this channel are shown in what we call the *Higgsino discovery plane* (HDP) in Ref. [30]. The HDP shows action in the $m_{\tilde{\chi}_{1,2}^0} \simeq \mu$ vs $\Delta m^0 \equiv m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$ plane. Natural SUSY models with their light Higgsinos end up having mass splittings in the $\Delta m^0 \sim 4\text{--}20$ GeV range for models with unified gaugino masses (highly preferred by theory). The mass splitting can be directly measured in LHC experiments since the soft dilepton invariant mass is bounded $m(\ell^+ \ell^-) < m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$. In order to achieve larger mass splittings, the gauginos must be quite light, and the gluino would drop below LHC search limits that $m_{\tilde{g}} \gtrsim 2.2$ TeV. In order to achieve lower mass splittings, the gauginos must move into the decoupling regime $\gtrsim 5$ TeV. Such heavy gluinos feed into top-squark soft term evolution and drive the

radiative corrections $\Sigma_u^u(\tilde{t}_{1,2})$ to large, unnatural values. (This means that LHC searches [31–33] for Higgsinos with GeV-or-below mass splittings which give rise to soft, displaced or disappearing tracks are not a lucrative search channel for the more plausible SUSY scenarios.) While HL-LHC can probe much of the natural parameter of the Higgsino discovery plane [34], some parameter space with higher $\mu \sim 250\text{--}350$ GeV seems out of reach.

Another promising search channel for HL-LHC is via top-squark pair production $pp \rightarrow \tilde{t}_1 \tilde{t}_1^*$. The current reach of LHC after Run 2 data requires that $m_{\tilde{t}_1} \gtrsim 1.1$ TeV based on several simplified model analyses with noncompressed stop decays as expected in natural SUSY with light Higgsinos [35,36]. In the future, the HL-LHC reach extends to about $m_{\tilde{t}_1} \sim 1.9$ TeV [37] while naturalness allows for $m_{\tilde{t}_1} \sim 1\text{--}3$ TeV.

LHC Run 2 search limits from gluino pair production require gluinos of mass $m_{\tilde{g}} \gtrsim 2.2$ TeV in noncompressed simplified models context [38,39]. In contrast, natural SUSY from the landscape [22] prefers $m_{\tilde{g}} \sim 2\text{--}6$ TeV, so that LHC in the $\tilde{g}\tilde{g}$ search channel is just beginning to probe the expected mass range. Gaugino (mainly wino) pair production at HL-LHC offers another promising natural SUSY discovery channel [40,41].

2. FCCee and FCChh

Beyond HL-LHC, CERN plans a future circular e^+e^- collider (FCCee) in a new 100 km tunnel which would operate as a Higgs (or Z) factory with $\sqrt{s} \sim 90\text{--}250$ GeV. While such a machine would have a Higgsino pair reach only to $m(\text{Higgsino}) \sim 125$ GeV, it could also search for light Higgsinos via deviations in electroweak precision observables (EWPO) like Δm_W and $\Delta \sin^2 \theta_{\text{eff}}$ [42,43].

If such deviations in EWPO show up, they would point to new physics, but the source of the new physics may not be clear. A higher energy linear e^+e^- collider with $\sqrt{s} > 2m(\text{Higgsino})$ is also a lucrative option for natural SUSY discovery [44–47]. The FCCee Higgs factory could be followed up at CERN by a new circular pp collider FCChh operating with $\sqrt{s} \sim 50\text{--}100$ TeV.

3. Muon collider

A more daring option is to pursue construction of a $\mu^+\mu^-$ collider. This old idea [48,49] has recently been pursued more strongly by the US P5 report [50] and the National Academy of Sciences (NAS) report² on future directions for high energy physics in the USA. Most proponents envision a $\mu^+\mu^-$ collider operating with center-of-mass (CoM) energy in the $\sqrt{s} \sim 1\text{--}10$ TeV range [51]. The technical challenge is to generate, cool and accelerate the μ^+ and μ^- beams and bring them together with a high enough luminosity, all before the beams decay. The beam decay remnants could also pose possible health hazards that would have to be overcome. The advantage of a muon collider of course is that one can accelerate the muons to TeV-scale energies with little energy loss from synchrotron radiation. Since the muons are fundamental, all their annihilation energy can be transferred into new particle production.

Our goal in this paper is to examine what a TeV-scale muon collider can do for the BSM case of natural supersymmetry. Since our goal is to present the overall big picture, we do not here engage in particular signal-to-background studies for specific production and decay processes. Previous studies of $\mu^+\mu^- \rightarrow \text{SUSY}$ have surprisingly been absent or else have examined rather implausible scenarios such as gluino pair production [51].³ In the more plausible natural SUSY scenarios, a plethora of reactions may be available to machines operating around $\sqrt{s} \sim 10$ TeV. In such a case, it may pay to have flexible beam energies which can allow the machine to scan over the various pair production thresholds which are expected to turn on. In addition, flexible beam energy could allow one to scan over heavy Higgs resonances, so the machine becomes a heavy Higgs factory as envisioned in Ref. [52]. To this end, we introduce a natural SUSY (natSUSY) benchmark point in Sec. II. The restrictions of naturalness under Δ_{EW} are such that almost all natural SUSY spectra are qualitatively similar, so a single well-chosen benchmark point should suffice to obtain the overall picture as to how natural SUSY would reveal itself at a muon collider. The natSUSY BM point has sparticle mass spectra spread over 2 decades of energy, with

sparticle masses ranging from 0.2–30 TeV. In Sec. III, we examine sparticle and Higgs pair production and resonance production processes at a $\mu^+\mu^-$ collider. We first focus on electroweakino pair production in Sec. III A, while in Sec. III B we examine squark pair production and in Sec. III C we examine Higgs pair production and resonance production reactions. We conclude in Sec. IV.

II. A PLAUSIBLE SUSY BENCHMARK MODEL

Using the naturalness measure Δ_{EW} , then it is found that many formerly popular SUSY models are now ruled out by naturalness when the light Higgs mass is restricted to be $m_h = 125.5 \pm 2.5$ GeV [53]. This is the case for the CMSSM model [54], the mGMSB model [55], mAMSB [56], and mirage mediation [57] (MM) for discrete choices of modular weights. For the cases of mGMSB and mAMSB, the small A_0 values make it difficult to accommodate $m_h \sim 125$ GeV unless 3rd generation matter scalars lie in the tens-of-TeV regime. This leads to large $\Sigma_u''(\tilde{t}_{1,2})$ contributions to the weak scale and consequently large electroweak fine-tuning. For the CMSSM model, low $\mu \sim 100\text{--}300$ GeV is only found in the focus point region [58], where also low A_0 is required. This region also has trouble generating $m_h \sim 125$ GeV unless 3rd generation scalar masses become huge.

Fortunately, a model which allows for low Δ_{EW} and $m_h \sim 125$ GeV is the original incarnation of the simplest possibility: gravity mediation (or supergravity, SUGRA). Under SUGRA, scalar mass *nonuniversality* is expected, unless some protective symmetry is acting, unlike the *ad hoc* universality which is input to the CMSSM. Thus, in SUGRA, we expect intra-generational soft term universality $m_{Q_i} = m_{U_i} = m_{D_i} = m_{L_i} = m_{E_i} \equiv m_0(i)$ since all elements of each generation fill out the 16-dimensional spinor rep of $SO(10)$. (Here we have in mind *local* GUTs where the GUT symmetry depends on the geography of field locations on the compact manifold [59].) This intragenerational universality also acts to reduce the fine-tuning contributions from a complete generation in the MSSM [60]. Thus, in SUGRA we expect $m_{H_u} \neq m_{H_d} \neq m_0(1) \neq m_0(2) \neq m_0(3)$ but where $m_{H_u} \sim (1.3\text{--}2)m_0(3)$ is typically required for $m_{H_u}^2$ to run to natural, barely negative values [7]. Furthermore, in the context of the string landscape, as mentioned previously there is a pull to large soft terms, but the pull on $m_0(1)$ and $m_0(2)$ has a generation independent upper bound arising from two-loop RG effects [61]. This leads to a decoupling/quasidegeneracy landscape solution to the SUSY flavor and CP problems which are otherwise endemic to SUGRA models [17]. Thus, we expect $m_0(1) \sim m_0(2) \sim 10\text{--}50$ TeV. In SUGRA, one also expects gaugino mass universality if each gaugino has the same functional dependence on hidden sector fields in the gauge kinetic function and also one expects generically large $A_0 \sim m_{3/2}$ soft terms which uplift $m_h \rightarrow 125$ GeV via mixing effects and also lowers fine-tuning via cancellations in the $\Sigma_u''(\tilde{t}_{1,2})$ terms [8].

²See *National Academy of Sciences report: Elementary Particle Physics: The Higgs and Beyond*, <https://nap.nationalacademies.org/resource/28839/EPP-one-pager.pdf>.

³In Ref. [51], Higgsino pair production and top-squark pair production are also discussed, albeit within the context of simplified models and using simplified naturalness calculations.

These sorts of considerations can be realized in the 4-extra parameter nonuniversal Higgs model NUHM4 with parameter choices

$$m_0(i), m_{H_u}, m_{H_d}, m_{1/2}, A_0, \tan\beta(\text{NUHM4}) \quad (4)$$

but where we will take $m_0(1) = m_0(2) = 30$ TeV for simplicity, and since these masses essentially decouple from our muon collider search considerations. In NUHMi ($i = 2-4$) models, it is convenient to trade m_{H_u} and m_{H_d} for the weak scale values m_A and μ via the scalar potential minimization conditions [62]. Thus, we will work with NUHM3 with parameter set

$$m_0(1, 2), m_0(3), m_{1/2}, A_0, \tan\beta, m_A, \mu \quad (\text{NUHM3}). \quad (5)$$

In Table I, we adopt the NUHM3 parameter set from Ref. [63] with $m_0(1, 2) = 30$ TeV, $m_0(3) = 6$ TeV, $m_{1/2} = 2.2$ TeV, $A_0 = -6$ TeV, $\tan\beta = 10$ with $\mu = 200$ GeV and $m_A = 2$ TeV. The spectra are generated using Isajet 7.91 [64] where the value of m_h typically agrees to be within ± 2 GeV with corresponding FeynHiggs [65] calculations [66].

From Table I, we see the gluino mass $m_{\tilde{g}} \sim 5.2$ TeV while first/second generation squarks and sleptons $m_{\tilde{q}, \tilde{\ell}} \sim 30$ TeV so the model is safe from LHC gluino/squark search limits. The lighter top-squark however is only $m_{\tilde{t}_1} \sim 1$ TeV, at the edge of present LHC exclusion limits using simplified models [38,39]. The several Higgsinos lie around ~ 200 GeV while heavy Higgs $m_{H_{A,H^\pm}} \sim 2$ TeV. Thus, the predicted spectra is spread over two orders of magnitude. In spite of this, the value of $\Delta_{\text{EW}} = 25$ so the model is EW natural. With such heavy second generation sleptons, one expects $(g-2)_\mu$ to be in accord with SM values [67], which is borne out by recent theory calculations [68].

Invoking naturalness in the QCD sector, we expect the presence of a PQ sector which would naturally be of DFSZ-form in SUSY (each requires two Higgs doublets). Thus, one might expect mixed SUSY DFSZ axion plus Higgsino-like WIMP dark matter [69] where the relic abundance is typically mainly axions [70]. Recent work solving the SUSY μ problem by first suppressing μ via discrete R -symmetries [71] (which naturally emerge from string compactifications [72]) and then regenerating via $H_u H_d$ coupling to PQ sector fields (Kim-Nilles [73]) can lead to small amounts of R -parity violation so that the WIMPs that are produced in the early universe may all decay away before the onset of big bang nucleosynthesis, leaving axion-only dark matter [74,75] (seemingly in accord with recent strong limits of relic WIMP dark matter from LZ [76]). Thus, we expect our spectra to be safe from recent LHC searches and from recent WIMP direct detection searches.

TABLE I. Input parameters and masses in GeV units for an NUHM3 model SUSY benchmark point with $m_t = 173.2$ GeV and generated from Isajet 7.91.

parameter	NUHM3
$m_0(1, 2)$	30000
$m_0(3)$	6000
$m_{1/2}$	2200
$\tan\beta$	10
A_0	-6000
μ	200
m_A	2000
$m_{\tilde{g}}$	5178.7
$m_{\tilde{u}_L}$	30256.0
$m_{\tilde{u}_R}$	30323.3
$m_{\tilde{e}_R}$	29961.9
$m_{\tilde{t}_1}$	1012.0
$m_{\tilde{t}_2}$	3405.3
$m_{\tilde{b}_1}$	3441.5
$m_{\tilde{b}_2}$	5336.2
$m_{\tilde{\tau}_1}$	5639.4
$m_{\tilde{\tau}_2}$	5772.0
$m_{\tilde{\nu}_\tau}$	5725.2
$m_{\tilde{\chi}_2^\pm}$	1936.1
$m_{\tilde{\chi}_1^\pm}$	211.3
$m_{\tilde{\chi}_4^0}$	1938.8
$m_{\tilde{\chi}_3^0}$	1018.7
$m_{\tilde{\chi}_2^0}$	207.1
$m_{\tilde{\chi}_1^0}$	202.2
m_h	124.5
$\Omega_{\tilde{\chi}_1^0}^{TP} h^2$	0.01
$BF(b \rightarrow s\gamma) \times 10^4$	3.0
$BF(B_s \rightarrow \mu^+ \mu^-) \times 10^9$	3.8
$\sigma^{SI}(\tilde{\chi}_1^0, p)$ (pb)	4.0×10^{-10}
$\sigma^{SD}(\tilde{\chi}_1^0, p)$ (pb)	7.5×10^{-6}
$\langle \sigma v \rangle _{v \rightarrow 0}$ (cm ³ /sec)	2.0×10^{-25}
Δ_{EW}	25

The spectra generated from our single benchmark point is rather generic from natural SUSY models that arise from the string landscape [9], with its pull to large soft terms [22]. Thus, our ensuing cross sectional plots for our benchmark point should also be quite similar to what is expected from many other natural SUSY models based on low Δ_{EW} but with $m_h \sim 125$ GeV.

III. NATURAL SUSY AT A MUON COLLIDER

Next, we investigate what can be accomplished by a TeV-scale muon collider in the case of natural SUSY. The full set of sparticle pair production reactions including all mixings were computed in Ref. [77] and presented in terms of polarized incoming leptons in Ref. [78]. These were embedded into Isajet [64] in 1994 which we use for our

total cross section calculations. The formulas are summarized in Appendix A of Ref. [79].

In this work we focus on the leading order tree-level new particle production processes. Sparticles can be created at muon colliders also via the higher order vector boson fusion (VBF) processes. These reaction can become dominant at collider energies typically beyond about $\sqrt{s} \sim 10$ TeV (depending on sparticle mass).[51,80] We leave it to future work to assess the relative advantages of the VBF processes.

A. Chargino and neutralino pair production

We first show total cross section rates $\mu^+\mu^- \rightarrow \text{SUSY}$ for the case of electroweakinos, since these include the vaunted light Higgsinos which are the hallmark of natSUSY. In Fig. 2(a), we show the chargino pair production rates $\sigma(\mu^+\mu^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^-, \tilde{\chi}_1^+\tilde{\chi}_2^-, \text{ and } \tilde{\chi}_2^+\tilde{\chi}_2^-)$. As a reference cross section, we also show the rate for $\mu^+\mu^- \rightarrow e^+e^-$ (dashed red curve). The threshold for charged Higgsino pair production begins around $\sqrt{s} = 2m(\text{Higgsino}) \sim 400$ GeV where $\tilde{\chi}_1^+\tilde{\chi}_1^-$ production followed by $\tilde{\chi}_1^- \rightarrow f\bar{f}'\tilde{\chi}_1^0$ decay (where f and f' are SM fermions). The $\tilde{\chi}_1^\pm$ decay products are very soft near threshold since most of the decaying particle rest energy goes into making $m_{\tilde{\chi}_1^0}$, and the $\tilde{\chi}_1^0$ LSP is expected to yield missing energy, even in the case of late-decaying $\tilde{\chi}_1^0$. The $\tilde{\chi}_1^+\tilde{\chi}_1^-$ cross section is nearly the same as for e^+e^- production since both processes are dominated by s -channel γ exchange. As the beam energy increases, the decay products will become more energetic and collimated due to the increased production energy of the $\tilde{\chi}_1^\pm$ s.

As $\sqrt{s}$ increases, one passes the $\mu^+\mu^- \rightarrow \tilde{\chi}_1^\pm\tilde{\chi}_2^\mp$ threshold at just over $\sqrt{s} \sim 2$ TeV where charged Higgsino + wino production begins. This reaction takes place via s -channel Z and t -channel muon sneutrino exchange. The sneutrino exchange contribution is highly suppressed by the ~ 30 TeV sneutrinos while the Z -exchange diagram is suppressed because the Z couples to wino-wino and Higgsino-Higgsino via the Dirac kinetic term in the Lagrangian but to wino-Higgsino only via mixings: thus, the mixed $\tilde{\chi}_1^\pm\tilde{\chi}_2^\mp$ production is highly suppressed by about three orders of magnitude compared to $\tilde{\chi}_1^+\tilde{\chi}_1^-$ production.⁴ It may not even be visible depending on the integrated luminosity which can be achieved for a muon collider. In Ref. [51], integrated luminosity values of $0.2\text{--}10\text{ ab}^{-1}$ are entertained for $\sqrt{s}$ varying from $1\text{--}10$ TeV. Thus, we might expect just of order $1\text{--}10$ events of the mixed $\tilde{\chi}_1^\pm\tilde{\chi}_2^\mp$ chargino pair production reactions.

Near $\sqrt{s} \sim 4$ TeV, then $\tilde{\chi}_2^+\tilde{\chi}_2^-$ (charged wino pair production) turns on. These reaction rates can exceed even the

⁴The Z -boson couplings to charginos and neutralinos are derived on p. 172-173 of the text Ref. [79].

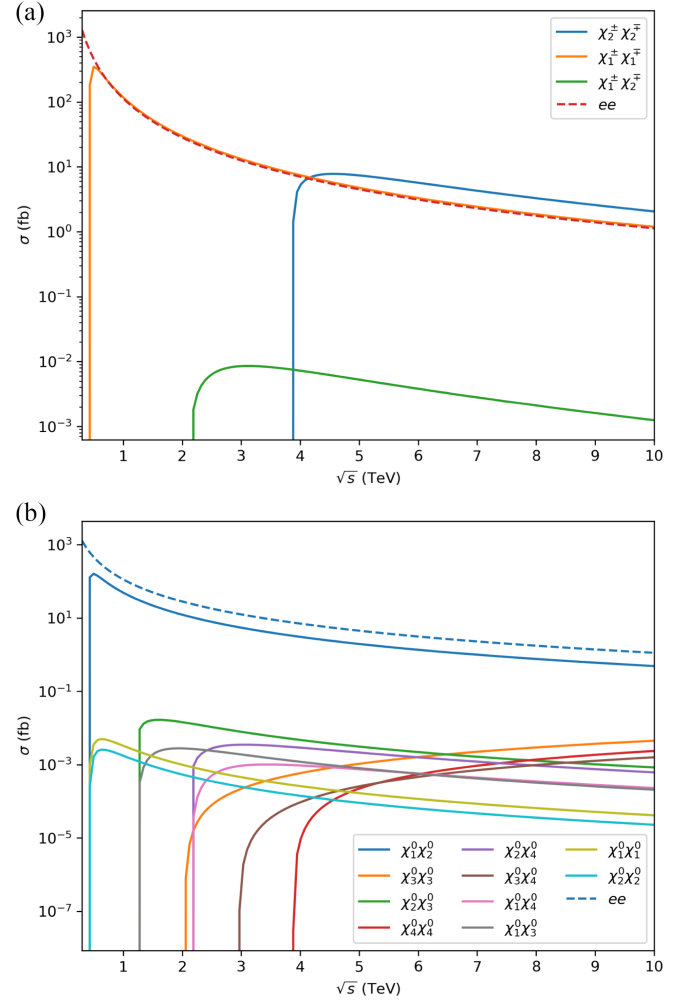


FIG. 2. Plot of (a) chargino pair production rates vs $\sqrt{s}$ and (b) neutralino pair production vs $\sqrt{s}$ at a $\mu^+\mu^-$ collider for the natural SUSY benchmark point listed in the text.

benchmark $\mu^+\mu^- \rightarrow e^+e^-$ process. The charged winos decay via $\tilde{\chi}_2^- \rightarrow W^-\tilde{\chi}_{1,2}^0$, $\tilde{\chi}_1^-Z^0$, and $\tilde{\chi}_1^-h$ each at around 25%. The WW , WZ , ZZ , Wh , and $Zh + E_T$ final states should be easily recognizable. If variable beam energy is possible, then threshold scans could determine $m(\tilde{\chi}_2^\pm)$ to good precision.

In Fig. 2(b), we show the rates for neutralino pair production $\tilde{\chi}_i^0\tilde{\chi}_j^0$ with $i, j = 1\text{--}4$ (ten reactions total). The reactions again turn on around $\sqrt{s} \sim 2\mu$ where neutral Higgsino pair production begins. Only the reaction $\tilde{\chi}_1^0\tilde{\chi}_2^0$ exceeds the fb level since production is only via s -channel Z exchange. It is comparable to but less than the benchmark e^+e^- production rate. The virtual Z only couples to the Higgsino components of the neutralinos so the neutral gaugino pair reactions are suppressed while cancellations in the $Z\tilde{\chi}_i^0\tilde{\chi}_i^0$ couplings suppress the $\tilde{\chi}_1^0\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0\tilde{\chi}_2^0$ production rates [79].

The $\tilde{\chi}_2^0 \rightarrow f\bar{f}\tilde{\chi}_1^0$ so the invariant mass $m(f\bar{f})$ is kinematically bounded by $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$ which presents a

distinctive signature. As the CoM energy $\sqrt{s}$ increases beyond threshold, the energy of the $f\bar{f}$ final state increases while its invariant mass remains kinematically fixed. A variety of other $\tilde{\chi}_i^0\tilde{\chi}_j^0$ reactions turn on starting at $\sqrt{s} \sim 2$ TeV where neutralino Higgsino + wino begins. These all present interesting signatures since $\tilde{\chi}_3^0 \rightarrow \tilde{\chi}_1^\pm W^\mp$ at $\sim 25\%$ each, along with $\tilde{\chi}_2^0 Z$ and $\tilde{\chi}_1^0 h$, while $\tilde{\chi}_4^0 \rightarrow \tilde{\chi}_1^\pm W^\mp$ at $\sim 25\%$, and $\tilde{\chi}_2^0 Z$ and $\tilde{\chi}_2^0 h$ at $\sim 20\%$. However, the individual reactions may be difficult to pick out since we expect only a handful of each to be produced at the projected integrated luminosities.

B. Squark pair production

While first/second generation sfermion pair production would not be accessible even to the highest energy muon colliders, the third generation squark pair production would be available. In Fig. 3, we show the cross sections for $\tilde{t}_1\tilde{t}_1^*$, $\tilde{t}_2\tilde{t}_2^*$, $\tilde{t}_1\tilde{t}_2^*$ + $\tilde{t}_2\tilde{t}_1^*$, $\tilde{b}_1\tilde{b}_1^*$, and $\tilde{b}_1\tilde{b}_2^*$ + $\tilde{b}_2\tilde{b}_1^*$ production vs $\sqrt{s}$. Naturalness requires the $\tilde{t}_1$ to be the lightest squarks for the NUHM3 model, and the $\mu^+\mu^- \rightarrow \tilde{t}_1\tilde{t}_1^*$ reaction threshold appears around $\sqrt{s} \sim 2$ TeV for our BM model. In this case, $\tilde{t}_1 \rightarrow b\tilde{\chi}_1^+$ at $\sim 50\%$ followed by $t\tilde{\chi}_{1,2}^0$ at $\sim 25\%$ each. Thus, from light stop pair production, we expect final states of $b\bar{b}$, $t\bar{t}$ and $t\bar{b}/\bar{t}b$ plus light Higgsino pairs, where the latter yield softer decay products along with large $\cancel{E}_T$. The $\tilde{t}_1\tilde{t}_1^*$ reactions occur at nearly the rate for e^+e^- production once production threshold is passed. As beam energy increases, the $\tilde{t}_1\tilde{t}_2$ threshold is passed, but these reactions only occur at the $\sim 10^{-3}$ fb level. Here, $\tilde{t}_2 \rightarrow tZ$ and $t h$ at $\sim 30\%$ and $\sim 40\%$, respectively. Around $\sqrt{s} \sim 7$ TeV, both $\tilde{t}_2\tilde{t}_2^*$ and $\tilde{b}_1\tilde{b}_1^*$ turn on and quickly approach the fb level cross sections. The $\tilde{b} \rightarrow \tilde{t}_1 W$ and $\tilde{\chi}_1^- t$ at 68% and 21%

respectively. The $\tilde{b}_1\tilde{b}_2^* + \tilde{b}_2\tilde{b}_1^*$ reactions turn on around $\sqrt{s} \sim 9$ TeV but at likely too low a rate to be observed. However, owing to the clean environment of a muon collider, these hadronic states stand out more distinctly than at hadronic colliders such as the LHC or FCC-hh. As the beam energy increases, the decay products of $\tilde{t}$ and $\tilde{b}$ become increasingly collimated, allowing clearer identification of the individual squark origins.

C. SUSY Higgs boson production

Heavy Higgs boson production is particularly exciting at a $\mu^+\mu^-$ collider [81]. From Eq. (1), the heavy Higgs contributions to Δ_{EW} are $(m_{H_d}/\tan\beta)^2/(m_Z^2/2)$ so for $\tan\beta \sim 10$ and $\Delta_{EW} \lesssim 30$ and with $m_{A/H} \sim m_{H_d}$ we expect $m_{H/A/H^\pm} \lesssim 3.5$ TeV [82].

The s -channel resonance production of heavy neutral Higgs bosons is given by

$$\sigma(\mu^+\mu^- \rightarrow H/A) = \frac{4\pi\Gamma_{H/A}^2 BF(H/A \rightarrow \mu^+\mu^-)}{(s - m_{H/A}^2)^2 + \Gamma_{H/A}^2 m_{H/A}^2} \quad (6)$$

and is plotted in Fig. 4 where the H and A resonances are clearly visible near $\sqrt{s} \sim 2$ TeV. The H/A cross sections at resonance exceed the baseline $\mu^+\mu^- \rightarrow Zh$ value. Having variable muon beam energy in the vicinity of H/A resonance production would allow good precision in mapping out the heavy neutral Higgs properties. And if sufficient precision on beam energy is attained, one might even separate out the two (overlapping) H and A resonances [52].

Shortly beyond H/A resonance production, the $\mu^+\mu^- \rightarrow hA$ and ZH reactions turn on, albeit at likely unobservable levels. At $\sqrt{s} \gtrsim 4$ TeV, then the $\mu^+\mu^- \rightarrow HA$ and H^+H^-

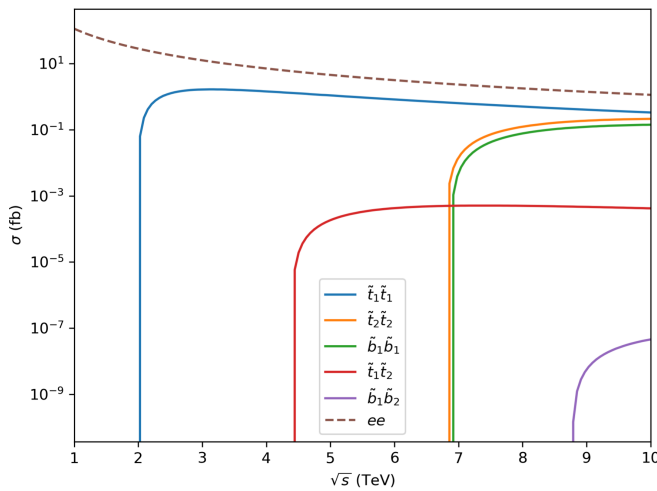


FIG. 3. Plot of (a) stop/sbottom pair production cross sections vs $\sqrt{s}$ at a $\mu^+\mu^-$ collider for the natural SUSY benchmark point listed in the text.

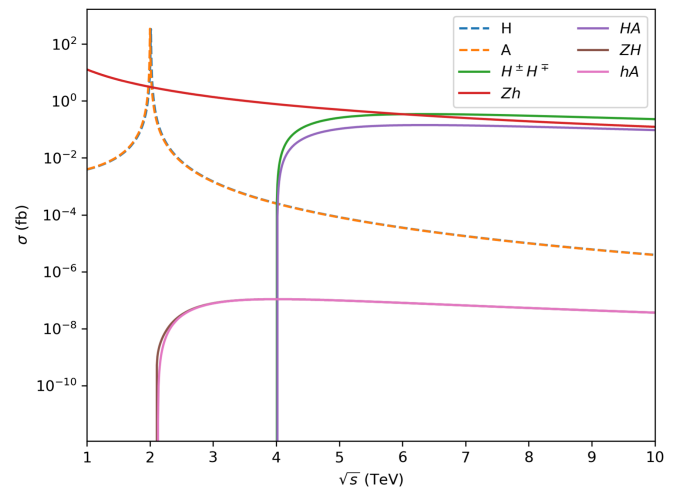


FIG. 4. Plot of Higgs boson production cross sections vs $\sqrt{s}$ at a $\mu^+\mu^-$ collider for the natural SUSY benchmark point listed in the text.

reactions turn on. The charged Higgs pair production reaction is quite difficult to see at hadron colliders [83], but occurs at a high rate at a TeV-scale $\mu^+\mu^-$ collider. In natural SUSY, since one expects $m_{H/A/H^\pm} \gg 2\mu$, then SUSY decay modes of the heavy Higgs states are always open. These decay modes are maximal for mixed Higgsino/gaugino decays and can dominate [84] the SM decay modes (which are usually assumed in most heavy Higgs analyses). For instance, for our BM scenario, the charged Higgs branching fraction $H^+ \rightarrow t\bar{b}$ is actually reduced to just 61% while the decay $H^+ \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_3^0$ is lifted to 29%.

IV. CONCLUSIONS

Natural supersymmetry with low Δ_{EW} is a highly plausible extension of the SM which (1) solves the gauge hierarchy problem; (2) explains why the weak scale $m_{W,Z,h} \sim 100$ GeV (because all contributions to the weak scale are comparable to or smaller than this value); and (3) is supported by an array of virtual quantum effects. NatSUSY in a landscape context supports and even predicts that $m_h \sim 125$ GeV with sparticles beyond present LHC limits. Previous work has examined in great detail how natSUSY might manifest itself at HL-LHC and linear e^+e^- colliders, but little or no work has been done for muon colliders, even though work toward realizing a muon collider has become a high priority item on future HEP facilities wish list.

Here we remedy this situation by examining what a muon collider with $\sqrt{s} = 1\text{--}10$ TeV can do in the natSUSY context. Using a very plausible natSUSY benchmark point which includes a solution to the SUSY flavor/ CP problems, we show that a variety of SUSY pair production reactions should be available to a muon collider, starting with Higgsino pair production for $\sqrt{s} \gtrsim 2|\mu|$. Mixed Higgsino/gaugino and gaugino pair production will allow access to the heavier electroweakino pair production reactions. Given control over the muon

collider beam energy, then precision scans at or around threshold production may allow for precision sparticle mass measurements which are otherwise rather difficult at hadron colliders due to the presence of E_T . Third generation squark pair production should also be accessible since top squark masses are bounded in natSUSY at the several TeV level. The capability of a $\mu^+\mu^-$ collider really shines for heavy Higgs production, where in natSUSY we expect heavy Higgs states with mass $\lesssim$ several TeV. With variable beam energy, the resonant production of the H/A states may be explored. At higher energies, HA and H^+H^- production should open up. Mapping out the threshold for charged Higgs pair production at a muon collider should allow precision measurement of $m_{H^\pm}$ which is otherwise extremely difficult at hadron collider (and likely beyond reach of linear e^+e^- colliders).

The main take-home lesson of this work is that, for natSUSY, around 25 reactions may be simultaneously available to a $\sqrt{s} \sim 10$ TeV muon collider. To make sense of these reactions, and to aid in a program of precision measurements of quantum numbers of the new states of matter, it would be beneficial to build a machine with *maximal flexibility in the CoM running energy*, so that the various reactions could be examined one-by-one, and precision threshold and resonance measurements could be made.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [64].

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