

Stealth singularities from self-gravitating fermions

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We present a new analytic solution to the Einstein-Dirac equations formulated by Finster *et al.* [*Phys. Rev. D* **59**, 104020 (1999)] to describe the stationary states of a pair of gravitationally interacting neutral fermions. The fermions' wave function in our analytic solution, as in their numerical ones, is both exponentially localized and normalizable. However, our solution differs from theirs in two key respects: it features a naked spacetime singularity at the origin, and the gravitational (Arnowitt-Deser-Misner) mass of the localized object is zero, making it gravitationally undetectable to an external observer. This is despite the arbitrarily large mass of the constituent fermions. This unexpected result may have significant implications for astronomy and cosmology, as it gives a mechanism by which mass could become “hidden” during the Universe's evolution.

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Introduction. The existence of so-called “naked” singularities (those without an accompanying horizon) in general relativity has been debated for many years [1]. It is clear that the Einstein equations admit solutions of this type, but the consensus is that naked singularities do not form as a result of gravitational collapse, at least not in an astrophysical context. This is now supported by the recent observational evidence of black holes [2] and black-hole mergers [3]. Furthermore, it is generally accepted that the matter sector associated with spacetimes containing naked singularities must exhibit a degree of unphysicality, such as in the negative-mass Schwarzschild solution [4], thus limiting their real-world relevance.

In the quantum regime, however, the matter sector is not necessarily subject to the same stringent physicality constraints [5], and therefore the existence of naked singularities at the quantum scale is still very much an open question. It is possible that such objects could have relevance in the context of the early Universe, for instance as alternatives to primordial black holes [6].

In this Letter, we demonstrate that spacetimes containing naked singularities can occur in a simple (semiclassical) quantum system: a spinor field minimally coupled to gravity

via the Einstein equations. Note that we are using the term “semiclassical” to indicate that the matter sector is treated quantum mechanically and the gravitational field purely classically, rather than to indicate any proximity to a classical behavior. The resulting spacetimes are asymptotically flat, but intriguingly we find that the total gravitational mass, measured, e.g., by the Arnowitt-Deser-Misner (ADM) mass, is identically zero, regardless of the mass of the spinor field, implying that these objects would be gravitationally undetectable by a test particle situated at a sufficient distance (beyond the order of a few Planck lengths). This raises the possibility of effectively concealing a clump of fermionic matter from external observation, assuming that the only interaction is via gravity.

Solitonlike objects with this property have previously been found in the context of gravitationally interacting skyrmions [7–9] and termed “stealth defects.” Given the similarities between the two cases, we describe the objects presented here as “stealth singularities.”

Compared to the Skyrme model, the system studied here is relatively simple, being composed of the minimally coupled Dirac and Einstein equations (together referred to as the Einstein-Dirac system):

$$(\not{D} - m)\Psi_a = 0; \quad G_{\mu\nu} = 8\pi GT_{\mu\nu}[\{\Psi_a\}], \quad (1)$$

where $\not{D}$ is the Dirac operator in curved spacetime, m is the fermion mass, Ψ_a is the spinor wave function for fermion a , $G_{\mu\nu}$ is the Einstein tensor, G is the Newton constant, and $T_{\mu\nu}$ is the stress-energy tensor.

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There are several senses in which this approach falls short of a complete theory of gravitationally interacting particles. First, the spacetime is treated completely classically. Second, the particles are treated in first quantization. Because of these two approximations, the stress-energy tensor $T_{\mu\nu}[\{\Psi_a\}]$ in the equation above has the same status as the probability density of a first-quantized wave function, i.e., no entanglement can develop between the particles and the spacetime. Furthermore, the entanglement between the particles is fixed to that provided by the assumed Hartree-Fock form of the N -particle wave function.

Reference [10] presents an alternative approach that treats the Dirac field in second quantization, i.e., as an operator. In that approach, the stress-energy tensor becomes an operator too. However, it would be inconsistent to put an operator-valued $T_{\mu\nu}$ into (1), so instead the expectation value of that operator is used. This again prevents the generation of any entanglement between the matter field and the spacetime, and indeed the same equations of motion result. Such approaches are often referred to in the literature as “semi-classical,” in the sense described above. The extent to which these approaches capture the properties of the fully quantum matter-and-spacetime configuration is not clear, though phase-space restrictions might make the two pictures similar for sufficiently low-energy states [11].

An interesting feature of the Einstein-Dirac system is the existence of “particlelike” solutions, where the gravitational self-interaction of the spinor wave function acts to create a spatially localized object, which is prevented from collapse by the effects of the uncertainty principle. Generally referred to as either “Dirac solitons” or “Dirac stars,” these objects were first studied by Finster *et al.* [12], henceforth FSY, who numerically constructed localized, spherically symmetric solutions containing two neutral fermions. This initial analysis has since been extended to solutions that include additional fields [13–15] and larger numbers of fermions [16,17] and has also been generalized to axisymmetric one-particle states [18].

In addition to these numerical studies, a number of analytic solutions to the (spherically symmetric) Einstein-Dirac system have also been discovered. These include a “power-law solution” or “lightlike singularity,” a Schwarzschild-like black-hole solution, and both massive and massless “worm-hole” solutions [11,19]. Although interesting in their own right, the physical relevance of these solutions is doubtful, since none of them contains a matter sector that is normalizable (due to divergences in the spinor fields either at spatial infinity or at a horizon).

In contrast, the analytic solution presented here has spinor fields that decay exponentially at large radii, and although the spacetime is singular at $r = 0$, the integral over the fermion density nonetheless converges, allowing the spinor fields to be consistently normalized. In this regard, our solution is similar in spirit to the particlelike solutions found numerically by FSY: it has a normalized, fermionic

quantum wave function that is spatially localized, with an associated spacetime that is asymptotically flat. There is one key difference, however: the presence of a central naked singularity, which significantly alters the properties of the resulting solitonlike objects.

Equations of motion. The equations of motion governing the two-fermion Einstein-Dirac system were first derived by FSY; we begin by outlining that derivation. The spacetime metric $g_{\mu\nu}$ and spinor wave functions $\Psi_{a \in \{1,2\}}$ take the following spherically symmetric forms:

$$g_{\mu\nu} = \text{diag}\left(-\frac{1}{T(r)^2}, \frac{1}{A(r)}, r^2, r^2 \sin^2 \theta\right); \quad (2)$$

$$\Psi_a(r, t) = \frac{\sqrt{T(r)}}{r} \begin{pmatrix} \alpha(r) e_a \\ i \sigma^r \beta(r) e_a \end{pmatrix} e^{-i\omega t}, \quad (3)$$

where $e_1 = (1, 0)^T$, $e_2 = (0, 1)^T$, σ^r is the radial spherical Pauli matrix, ω is the fermion energy (assumed common for each fermion), and (t, r, θ, ϕ) are the usual spherical coordinates. We use natural units $\hbar = c = 1$; the Newton constant G is retained explicitly. The two fields in (2), A and T , describe the radial profile of the spacetime metric; the two in (3), α and β , describe the fermion and antifermion components of the spinor wave function. Overall, the *Ansatz* (3) represents a pair of fermions of opposite spin arranged in a singlet state, where the total (two-particle) wave function can be constructed using the usual Hartree-Fock formalism: $\Psi = \Psi_1 \wedge \Psi_2$.

Using these *Ansätze*, one can derive expressions for the Dirac operator $\not{D}$, the energy-momentum tensor $T_{\mu\nu}$, the Ricci tensor $R_{\mu\nu}$, and the Ricci scalar $\mathcal{R}$. Substituting these into (1), one obtains the following set of four coupled differential equations:

$$\sqrt{A}\alpha' = +\frac{\alpha}{r} - (\omega T + m)\beta; \quad (4)$$

$$\sqrt{A}\beta' = -\frac{\beta}{r} + (\omega T - m)\alpha; \quad (5)$$

$$-1 + A + rA' = -16\pi G\omega T^2(\alpha^2 + \beta^2); \quad (6)$$

$$-1 + A - 2rA\frac{T'}{T} = 16\pi GT\sqrt{A}(\alpha\beta' - \alpha'\beta). \quad (7)$$

Here, $' \equiv d/dr$, and the fermion mass m is common for each fermion. Note that these are valid strictly for fermions with positive parity; negative parity solutions are discussed in Supplemental Material [20], along with the generalization to fermion number $N > 2$.

For localized solutions we require the spacetime to be asymptotically flat:

$$\lim_{r \rightarrow \infty} A(r) \rightarrow 1; \quad \lim_{r \rightarrow \infty} T(r) \rightarrow 1. \quad (8)$$

We also require each spinor wave function to be correctly normalized. This effectively quantizes the system by ensuring that each of the two spinor fields is occupied by precisely one fermion and implies that

$$\int_0^\infty \frac{4\pi r^2 dr}{\sqrt{A}} n_f = N = 2, \quad (9)$$

where $r^2 n_f = NT(\alpha^2 + \beta^2)$ is the (radial) fermion density.

The stealth solution. The conditions of asymptotic flatness and normalization, along with (4)–(7), fully define the two-fermion Einstein-Dirac system. For the special case of zero fermion energy ($\omega = 0$), together with a spatially flat metric ($A = 1$), we find that the equations of motion admit an exact solution that is both asymptotically flat and normalizable. Setting $\omega = 0$ and $A = 1$, and defining $x \equiv mr$, the equations of motion become

$$\alpha' = +\frac{\alpha}{x} - \beta; \quad (10)$$

$$\beta' = -\frac{\beta}{x} - \alpha; \quad (11)$$

$$-x \frac{T'}{T} = 8\pi GmT(\alpha\beta' - \alpha'\beta), \quad (12)$$

where now $' \equiv d/dx$, and (6) is automatically satisfied. The two Dirac equations can now be solved independent of the remaining Einstein equation:

$$\alpha(x) = c_1 e^{-x} + c_2 e^x; \quad (13)$$

$$\beta(x) = c_1 \left(1 + \frac{1}{x}\right) e^{-x} + c_2 \left(1 - \frac{1}{x}\right) e^x, \quad (14)$$

where c_1 and c_2 are constants with dimensions $(\text{length})^{-1/2}$. In order to obtain a localized solution, we are forced to set $c_2 = 0$. Then, substituting these forms into (12), we obtain

$$\frac{T'}{T^2} = \frac{h_m}{x^3} e^{-2x}, \quad (15)$$

where we have defined the dimensionless constant $h_m = 8\pi Gc_1^2 m$. This can be straightforwardly solved using integration by parts to give the following expression for T :

$$T(x) = \frac{2x^2}{2c_3 x^2 + h_m[(1-2x)e^{-2x} - 4x^2 \text{Ei}(-2x)]}, \quad (16)$$

where $\text{Ei}(x)$ is the exponential integral. Imposing asymptotic flatness fixes $c_3 = 1$, leaving only a single integration constant c_1 , which we shall relabel as c_m , since its value

will depend on the fermion mass. The normalization condition (9), setting $A = 1$, then becomes

$$2 = 4\pi \int_0^\infty x^2 n_f dx, \quad (17)$$

where the (radial) fermion density evaluates to

$$x^2 n_f = \frac{4c_m^2 m^{-1} (2x^2 + 2x + 1) e^{-2x}}{2x^2 + h_m[(1-2x)e^{-2x} - 4x^2 \text{Ei}(-2x)]}. \quad (18)$$

The normalization condition (17) is thus an implicit equation for c_m . To check whether the integral on the right-hand side converges, consider the asymptotic behavior of the integrand:

$$x^2 n_f \sim \begin{cases} \frac{1}{2\pi Gm^2} (1 + 4x + \mathcal{O}(x^2)) & x \ll 1; \\ \frac{4c_m^2}{m} \left(1 + \frac{1}{x} + \mathcal{O}\left(\frac{1}{x^2}\right)\right) e^{-2x} & x \gg 1. \end{cases} \quad (19)$$

This confirms that the integrand is well behaved at both $x = 0$ and $x = \infty$. In addition, $(1-2x)e^{-2x} - 4x^2 \text{Ei}(-2x)$ is strictly positive for all $x > 0$, and so the fermion density is guaranteed to be positive at all radii. Hence we conclude that the solution is normalizable for any $m > 0$.

Properties of the solution. Overall, the solution can be written as

$$\alpha(x) = c_m e^{-x}; \quad (20)$$

$$\beta(x) = c_m \left(1 + \frac{1}{x}\right) e^{-x}; \quad (21)$$

$$T(x) = \left[1 + h_m \left(\frac{1-2x}{2x^2 e^{2x}} - 2\text{Ei}(-2x)\right)\right]^{-1}. \quad (22)$$

The metric field T has the following asymptotic behavior:

$$T \sim \begin{cases} \frac{2x^2}{h_m} (1 + 4x + \mathcal{O}(x^2)) & x \ll 1; \\ 1 - \frac{h_m}{2x^3 e^{2x}} \left(1 - \frac{3}{2x} + \mathcal{O}\left(\frac{1}{x^2}\right)\right) & x \gg 1. \end{cases} \quad (23)$$

The fermion spinor field Ψ_a decays exponentially at large x , but diverges at $x = 0$; however, the zero in T combines with this divergence in such a way as to maintain normalizability. At large radii, the metric tends toward an asymptotically flat spacetime, but does so via an exponential decay; thus the total ADM mass of the solution, given by the coefficient of a $1/x$ term at large radius, is identically zero.

This can be confirmed by considering the Komar mass [22], which here takes the following simple form:

$$M_K(x) = -\frac{x^2 \sqrt{AT'}}{GmT^2} = -\frac{h_m e^{-2x}}{Gmx}. \quad (24)$$

Since the ADM mass is defined as the large- x limit of the Komar mass, we conclude that the total gravitational mass is zero, and thus the solution is gravitationally undetectable to an external observer: hence the name “stealth” solution. The Komar mass is strictly negative for all radii, decreasing monotonically as $x \rightarrow 0$, with a divergence at $x = 0$. This indicates that the object is gravitationally repulsive, with the force increasing in strength the nearer one approaches, ultimately becoming infinite at the center. This strongly suggests that the spacetime may contain a singularity at $x = 0$, a conjecture that can be confirmed by evaluating the Ricci scalar:

$$\begin{aligned} \mathcal{R}(x) &= -8\pi G T_\mu^\mu = \frac{16\pi G m^3}{x^2} T(\alpha^2 - \beta^2) \\ &= \frac{-4m^2 h_m (2x + 1) e^{-2x}}{2x^4 - x^2 h_m [(2x - 1) e^{-2x} + 4x^2 \text{Ei}(-2x)]}. \end{aligned} \quad (25)$$

Expanding this at small x , we obtain

$$\mathcal{R}(x) \approx -\frac{4m^2}{x^2} \left(1 + 4x + \mathcal{O}(x^2 \log x) \right), \quad (26)$$

which clearly diverges at $x = 0$. This singularity is not accompanied, however, by an event horizon (see Supplemental Material [20] for confirmation of this via a null geodesic analysis); it is therefore a “naked” singularity, that in principle could be directly observed.

Further investigation of the solution’s properties requires a numerical approach, details of which are provided in Supplemental Material [20]. We find that, for each value of the fermion mass m , there exists a unique solution for the constant c_m . The radial profiles for a selection of these solutions, with m values ranging from 10^{-4} to 10^4 , are presented in Fig. 1. There is a distinct change in behavior as the fermion mass is increased. For small m , the radial fermion density initially plateaus at small x (until $x \approx 2Gm^2/\pi$), then decreases monotonically ($x^2 n_f \sim 1/x^2$) before ultimately decaying exponentially. As m increases, the central value of $x^2 n_f$ decreases, and the monotonically decreasing region is replaced by an increasing region ($x^2 n_f \sim x^3$), before exponential decay occurs as before. The density profile thus transitions from a distribution that is peaked at $x = 0$ (a “solid ball”) to one that has a peak at nonzero radius (a “hollow shell”).

The corresponding profiles for the metric field T are also shown in Fig. 1. At small m , we find that T initially exhibits a consistent increase ($T \sim x^2$) before asymptoting to $T = 1$ (a flat metric). For larger m this same increase still occurs, but is followed by a rapid exponential rise toward $T = 1$.

This change in behavior can be partially understood by performing an asymptotic analysis in m , full details of which can be found in Supplemental Material [20]. At small m , we find that the radial fermion density and metric

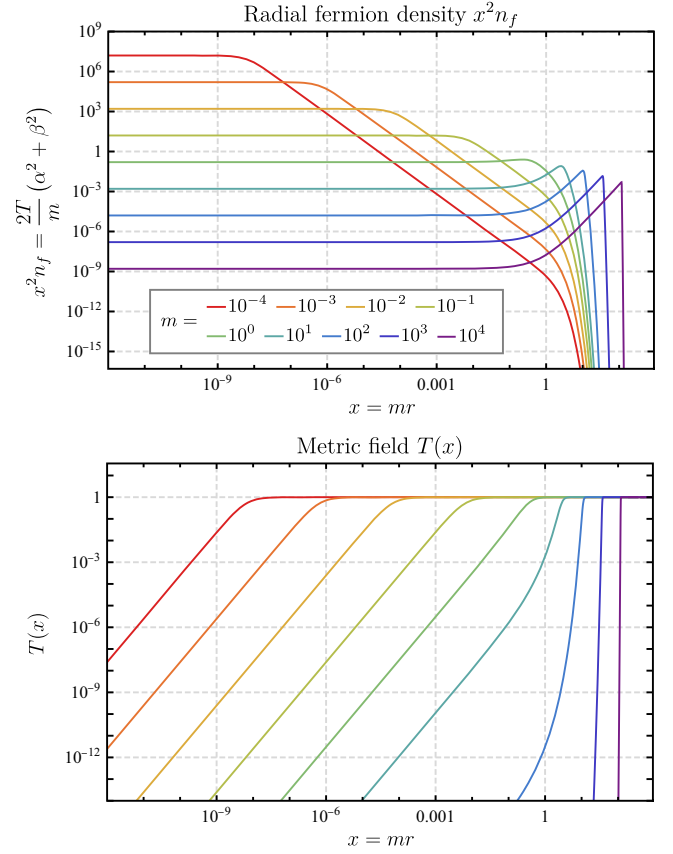


FIG. 1. Upper: the radial fermion density $x^2 n_f$ as a function of the normalized distance from the center of the object x for various values of the fermion mass m . Note the crossover from a solid ball to a hollow shell structure as the fermion mass increases. Lower: the metric field T as a function of the normalized distance from the center of the object x for the same values of the fermion mass m . Note that we have set $G = 1$, without loss of generality.

field T asymptote to the following forms:

$$x^2 n_f \sim \frac{4c_m^2 m^{-1}}{2x^2 + h_m}; \quad T \sim \left[1 + \frac{h_m}{2x^2} \right]^{-1}, \quad (27)$$

with $c_m \sim m^{3/2}$. In contrast, at large m we find

$$x^2 n_f \sim \frac{c_m^2 x^3 m^{-1}}{2x^3 e^{2x} + h_m}; \quad T \sim \left[1 + \frac{h_m}{2x^3 e^{2x}} \right]^{-1}, \quad (28)$$

with $c_m \sim (m^2/G)^{1/8} \exp[(2Gm^2)^{1/4}]$.

These expressions represent the extreme versions of the solid ball and hollow shell profiles shown in Fig. 1, to which the solution asymptotes at small and large m . Note that the localization mechanism is different in either case; for small m , the fermions are confined by a Newtonian-like $1/x^2$ term, whereas at large m they are confined by an exponential decay.

The explicit dependence of the radial extent R on the fermion mass is shown in Fig. 2, where we have defined

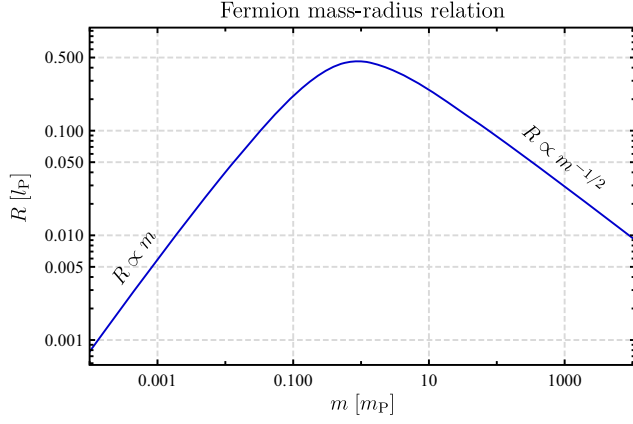


FIG. 2. The radial extent of the fermionic matter distribution R as a function of the fermion mass m . We have set $G = 1$, allowing quantities to be written in Planck units. On the solid ball side of the diagram ($m \lesssim m_P$) the object grows as the fermion mass increases; on the hollow shell side ($m \gtrsim m_P$) it shrinks. Note that R represents the physical radius of the solution, whereas the distributions in Fig. 1 are plotted as a function of the rescaled radial coordinate $x = mr$.

R as the average value of the (physical) coordinate radius r , weighted by the radial fermion density:

$$R = 4\pi \int_0^\infty r T(\alpha^2 + \beta^2) dr. \quad (29)$$

The radial extent of all solutions is within one Planck radius and shrinks to zero at both $m = 0$ (as $R \propto m$) and $m = \infty$ (as $R \propto m^{-1/2}$), in agreement with an analytical asymptotic analysis (see Supplemental Material [20] for details). The transition in behavior from solid ball to hollow sphere occurs at the order of one Planck mass.

These results demonstrate that the stealth solution remains highly localized, irrespective of the value of the fermion mass. This is counterintuitive: one might have expected the weak gravitational attraction that exists between low-mass fermions to result in a bound state that extends over a large volume (as seen, indeed, in the original nonsingular FSU solutions). For the stealth solution, however, the self-consistently determined properties of the naked singularity alter the situation significantly, preventing the system from ever reaching a classical limit.

Discussion. We have demonstrated that the Einstein-Dirac equation system admits a “stealth” solution with the following properties: a fermionic wave function that is exponentially localized and normalizable; a spacetime containing a naked singularity at the origin; and zero ADM mass. How should we interpret the existence of such a solution, and how physical is it?

Perhaps the most surprising feature of the solution is its vanishing ADM mass, combined with the observation—see

Fig. 2—that the radial extent of the object tends to zero both as $m \rightarrow 0$ and as $m \rightarrow \infty$. This implies that arbitrarily large masses could be “hidden” in arbitrarily small spatial volumes. This behavior is at odds with our usual interpretation of how matter and gravity interact, suggesting caution when applying intuition from classical gravity to the quantum regime.

Although the fermion mass m can take any positive value, the fermion energy ω is identically zero, i.e., the stealth solution “lives” precisely midway between the particle continuum ($\omega \geq m$) and the antiparticle continuum ($\omega \leq -m$). The ADM mass and the fermion energy are thus completely decoupled from the fermion mass. Curiously, $\omega = 0$ here does *not* imply that the particle and antiparticle components of the wave function, α and β , have the same functional form; indeed, one of these diverges at the origin while the other does not. This is in contrast to the case of Majorana zero modes, which are “pinned” to the center of the spectral gap precisely because they are their own antiparticles [23].

The existence of a solution with such properties is likely related to the presence of the naked singularity. In a classical context, naked singularities are associated with unphysicality of the matter sector, and indeed we find that the stealth solution violates the weak, strong, and dominant energy conditions (see Supplemental Material [20] for details). It is not clear, however, whether one would expect such conditions to hold in the context of quantum-mechanical matter [24]. Indeed, since our fermion wave function is a valid solution to the Dirac equation, and moreover is normalizable, it would appear to be a well-defined quantum state. Exploring this question further, especially in the context of these stealth solutions, would be a useful subject for future research. We note in passing that a similar result concerning normalizability and naked singularities has been reported for Dirac fields in a Reissner-Nordström background [25].

Although the stealth solution exists only at $\omega = 0$, at least for finite fermion mass, in the limit $m \rightarrow 0$ the family of stealth solutions does join onto a branch of $\omega \neq 0$ solutions discovered numerically by one of us [26]. These “case [G]” solutions also exhibit naked spacetime singularities, but have a nonzero ADM mass.

Finally, we should again emphasize that the Einstein-Dirac system studied here is not a full theory of quantum gravity, but only a semiclassical approximation. It is worth noting, however, that the crossover from solid ball to hollow shell solutions occurs at $m \sim m_P$, even though our theoretical approach neglects much of the expected physics of quantum gravity (e.g., radiative corrections due to the graviton field, or entanglement between the matter and the spacetime).

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Data availability. There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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