

GR from RG, 2D example: Jackiw-Teitelboim gravity induced from renormalization group flow

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We demonstrate how the two-dimensional gravity emerges within the “general relativity (GR) from renormalization group (RG)” program initiated in Refs. [1,2]. To achieve this, we consider a generic 2D conformal field theory (CFT) with a 3D holographic description, which we assume to be well described by pure Einstein-anti-de Sitter₃ gravity in the bulk. We study the holographic RG flow for the 2D CFT action and show that the RG corrected action at an arbitrary energy scale contains a 2D scalar-tensor gravity theory. In the simplest case, the flow induces Jackiw-Teitelboim (JT) gravity, where the bulk radial lapse function seeds the dynamical dilaton field of the JT gravity. We show that the standard $\overline{\text{T}\overline{\text{T}}}$ deformation of the 2D CFT is recovered as a special case in the Fefferman-Graham limit where the lapse is fixed. We further establish the robustness of the RG-induced gravity picture by verifying its consistency under holographic renormalization and by generalizing the result to a one-parameter family of boundary conditions. Our results provide a first-principles derivation of the JT gravity at a finite cutoff as an intrinsic manifestation of the holographic RG flow in a non-Fefferman-Graham gauge.

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I. INTRODUCTION

A. Motivation

The nature of gravity remains one of the most profound puzzles in theoretical physics. While gravity is traditionally viewed as a fundamental force, an alternative paradigm suggests that it is an *emergent* phenomenon—a coarse-grained, collective description of underlying nongravitational microscopic degrees of freedom. This viewpoint, pioneered by Sakharov [3] and echoed in black hole thermodynamics [4,5] and the fluid/gravity correspondence [6–8], has recently been revitalized through the “general relativity (GR) from renormalization group (RG)” program initiated in Refs. [1,2]. In this framework, the Einstein-Hilbert action is not postulated as fundamental but is instead *induced* by the RG flow of a quantum field theory (QFT). The central mechanism relies on the fact that every QFT possesses a universal coupling to its energy-momentum tensor [9]; under holographic RG flow, deformations

by operators quadratic in the stress tensor (such as $\overline{\text{T}\overline{\text{T}}}$) [10,11] naturally map to gravitational dynamics via bulk constraints [12–14].

In this work, we explore the emergence of two-dimensional (2D) gravity within this program. Studying 2D gravity is a worthwhile and highly instructive exercise for several reasons. First, 2D models such as Jackiw-Teitelboim (JT) gravity [15,16] serve as the essential laboratory for understanding the quantum structure of black hole horizons [17] and the holographic nature of nearly anti-de Sitter₂ (AdS₂) spacetimes [18,19]. Second, while 2D Einstein gravity is topologically trivial, scalar-tensor theories provide a rich dynamical landscape. Finally, the lower dimensionality of the AdS₃/CFT₂ correspondence [20,21] allows for a level of analytical precision—often yielding exact, closed-form solutions—that is difficult to achieve in higher dimensions. By demonstrating how JT gravity emerges from a generic 2D conformal field theory (CFT), we provide a robust lower-dimensional anchor for the GR from RG program.

B. Setting and analysis

We consider a $(1+2)$ -dimensional bulk spacetime described by pure Einstein gravity with a negative cosmological constant. According to the holographic dictionary [22,23], this bulk is dual to a 2D CFT residing on the asymptotic boundary. We employ a radial foliation of the

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bulk, where the radial coordinate r is identified with the RG energy scale of the boundary theory [24–31]. Crucially, we work in a coordinate system where the radial lapse function Φ is kept as an arbitrary degree of freedom, rather than being rigidly fixed as in the standard Fefferman-Graham (FG) gauge [32,33].

Our analysis proceeds in several key steps. We first derive the holographic RG flow equation for the boundary action at a finite radial cutoff, showing that it is driven by the 2D $\text{T}\bar{\text{T}}$ operator. By invoking the bulk Hamiltonian constraint, we replace this quadratic stress-tensor combination with boundary geometric quantities, effectively transforming the RG flow of the field theory into the radial evolution of a gravitational action.¹ We then perform an exact integration of this flow equation to find the effective boundary action at an arbitrary energy scale.

The primary result of this derivation is the emergence of a JT-like gravity theory on the cutoff surface, where the bulk radial lapse Φ is identified as the physical source for the induced dilaton field Ψ . Furthermore, we demonstrate that the “unfreezing” of the boundary metric—the transition of the background from a static structure to a dynamical field—is a direct consequence of the *RG flow of boundary conditions*: the rigid Dirichlet condition imposed at asymptotic infinity naturally evolves into a mixed Dirichlet-Neumann condition in the IR. We verify that this mechanism is consistent across both bare and holographically renormalized actions [37] and holds for a generalized family of boundary conditions.

C. Main results

The central result of this work is the exact, nonperturbative derivation of a two-dimensional gravity action from the holographic RG flow of a three-dimensional bulk. Specifically, we track the radial evolution of the boundary action from the UV conformal boundary to a finite-radius cutoff surface. We show that this evolution is governed by a deformation flow equation which, once integrated, yields an effective 2D scalar-tensor gravity theory on the cutoff surface. In the simplest case, this induced theory takes the form of Jackiw-Teitelboim gravity, as follows:

$$S_{\text{induced}} \propto \int d^2x \sqrt{-\gamma} \Psi (R[\gamma] - 2\Lambda(r)),$$

where the dynamical dilaton field Ψ is uniquely sourced by the bulk radial lapse function Φ .

We demonstrate that the standard $\text{T}\bar{\text{T}}$ deformation is a special case of this result, recovered precisely in the Fefferman-Graham limit where the lapse is held fixed. Furthermore, we prove that this mechanism is universal: for

¹Related studies investigating the connection between stress-tensor deformations and emergent gravitational dynamics via alternative frameworks include Refs. [34–36].

a generalized family of renormalized and nonrenormalized boundary conditions, the radial flow always induces JT-like dynamics, with the specific choice of boundary condition merely selecting a unique trajectory in the space of effective couplings. Finally, we provide a rigorous consistency check by showing that the integrability of this boundary RG flow is exactly equivalent to the transverse tensor components of the bulk Einstein equations.

D. Organization of the paper

In Sec. II, we establish the geometric foliation of the bulk and solve the AdS_3 Einstein equations perturbatively near the boundary. Section III derives the holographic RG flow equation and investigates the radial evolution of boundary conditions. In Sec. IV, we present our main result: the exact integration of the flow equation that yields the induced JT action. Section V repeats this derivation within the framework of holographic renormalization. We explore the universality of this induced gravity across a generalized family of boundary conditions in Sec. VI. Finally, we offer concluding remarks in Sec. VII. Technical identities are collected in Appendix.

II. GEOMETRIC SETUP AND ASYMPTOTIC DYNAMICS

In this section, we establish the geometric and dynamical framework required for our analysis. We begin by introducing a radial foliation of the spacetime, which naturally decomposes the metric into boundary and transverse components. We then define the bulk gravitational theory and systematically solve its equations of motion near the asymptotic boundary. This setup provides the foundational variables and their asymptotic behaviors, which are essential for driving the holographic renormalization group flow in the subsequent sections.

A. Geometric foliation

1. The spacetime arena

We consider a $(1+2)$ -dimensional spacetime $\mathcal{M}$ that is asymptotically AdS , equipped with coordinates x^μ and a Lorentzian metric $g_{\mu\nu}$. To track the holographic evolution from the UV to the IR, we introduce a radial foliation of $\mathcal{M}$ by a family of timelike hypersurfaces Σ_r . Each slice is parametrized by a radial coordinate $r \in (r_0, \infty)$, where $r_0 \geq 0$ represents the innermost extent of the foliation.

This foliation naturally induces a decomposition of the spacetime coordinates as $x^\mu = (x^a, r)$, where $a \in \{0, 1\}$. The coordinates x^a chart the intrinsic geometry of the hypersurfaces Σ_r , while r serves as the transverse coordinate. We identify the conformal boundary of the AdS spacetime with the limit $r \rightarrow \infty$, denoting this asymptotic boundary as $\Sigma := \Sigma_{r=\infty}$. To study the effective physics at an arbitrary energy scale, we consider a bulk subregion $\mathcal{M}_c$

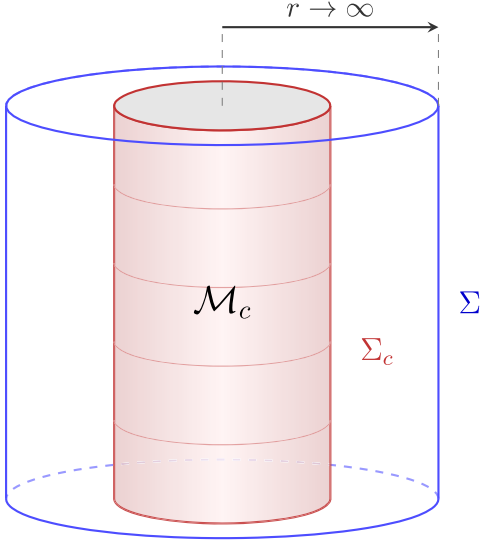


FIG. 1. Radial foliation of an asymptotically AdS_3 spacetime. The outer transparent cylinder represents the rigid conformal boundary Σ at $r \rightarrow \infty$. The inner shaded subregion $\mathcal{M}_c$ is bounded by a timelike cutoff hypersurface Σ_c at a finite radius $r = r_c$.

enclosed by a finite radial cutoff at $r = r_c$, bounded by the surface $\Sigma_c := \Sigma_{r_c}$ (see Fig. 1). More generally, we denote by $\mathcal{M}_r$ the bulk region capped at a generic fixed radius r , with Σ_r acting as its timelike boundary.

2. Metric decomposition and gauge choice

The radial foliation naturally lends itself to an Arnowitt-Deser-Misner (ADM)-like decomposition [38] of the space-time metric. The generic line element assumes the form

$$\begin{aligned} ds^2 &= \frac{\ell^2}{r^2} \Phi^2 dr^2 + h_{ab} (dx^a + U^a dr) (dx^b + U^b dr), \\ h_{ab} &:= \frac{r^2}{\ell^2} \gamma_{ab}, \end{aligned} \quad (2.1)$$

where Φ is the rescaled radial lapse function, U^a represents the shift vector tangential to Σ_r , and γ_{ab} is the conformal induced metric on each radial slice. To guarantee that the spacetime is asymptotically AdS, we impose the following standard falloff conditions in the large r limit:

$$\Phi \sim \mathcal{O}(1), \quad \gamma_{ab} \sim \mathcal{O}(1), \quad U^a \sim \mathcal{O}(r^{-2}). \quad (2.2)$$

It is important to note that the metric (2.1) generally involves six free functions, and we have not yet fixed the 3D diffeomorphisms. A standard choice in holographic literature is the FG gauge [32,33], which rigidly sets $\Phi = 1$ and $U^a = 0$, leaving only the 3 degrees of freedom encoded in the constant- r metric γ_{ab} .

However, for the purposes of our analysis, it is strictly crucial not to fix the radial lapse function Φ . As we will

demonstrate, retaining Φ as an arbitrary degree of freedom is the physical mechanism that eventually seeds the dilaton field of the induced 2D gravity theory. In contrast, the shift vector U^a does not play a dynamically generative role in this specific setup and may be conveniently set to zero later. Nonetheless, we keep U^a unfixed for the time being to maintain full generality in our geometric definitions.

3. Geometric variables

The unit normal one-form s to the hypersurfaces Σ_r , together with the corresponding projection tensor $h_{\mu\nu}$, are given by

$$s = s_\mu dx^\mu = \frac{\ell}{r} \Phi dr, \quad h_{\mu\nu} := g_{\mu\nu} - s_\mu s_\nu. \quad (2.3)$$

Here, $h_{\mu\nu}$ restricts the spacetime metric onto the tangent space of the hypersurface orthogonal to s^μ . We emphasize that henceforth, all lowercase Latin indices ($a, b, \dots$) on the boundary are exclusively raised and lowered using the conformal metric γ_{ab} .

The intrinsic dynamics of the foliation are governed by the extrinsic curvature of Σ_r , which encodes how each hypersurface is embedded within the bulk $\mathcal{M}$. It is defined via the Lie derivative along the normal vector [39,40], as follows:

$$K_{\alpha\beta} = \frac{1}{2} h^\mu{}_\alpha h^\nu{}_\beta \mathcal{L}_s h_{\mu\nu}. \quad (2.4)$$

To evaluate this efficiently, it is convenient to introduce the differential operator $\mathcal{D}_r := \partial_r - \mathcal{L}_U$, which incorporates both the strict radial derivative and the spatial shift contributions. Using this operator, the spatial components of the extrinsic curvature and its trace can be elegantly expressed in terms of the conformal metric γ_{ab} as

$$\begin{aligned} K_{ab} &= \frac{r^2}{\ell^3 \Phi} \gamma_{ab} + \frac{r^3}{2\ell^3 \Phi} \mathcal{D}_r \gamma_{ab}, \\ K &= \frac{2}{\ell \Phi} + \frac{r}{\ell \Phi} \mathcal{D}_r (\ln \sqrt{-\gamma}). \end{aligned} \quad (2.5)$$

B. AdS_3 gravity

1. Action and variational principle

The dynamics of the bulk geometry are governed by the three-dimensional Einstein-Hilbert action with a negative cosmological constant. When formulated to be compatible with a Dirichlet boundary condition on the radial slice Σ_r , the action takes the form

$$\begin{aligned} S_{\text{bulk}}^{\text{D}}[\mathcal{M}_r] &= \frac{1}{2\kappa_3} \int_{\mathcal{M}_r} d^3x \sqrt{-g} \left(\mathcal{R} + \frac{2}{\ell^2} \right) \\ &\quad + \frac{r^2}{\ell^2 \kappa_3} \int_{\Sigma_r} d^2x \sqrt{-\gamma} K, \end{aligned} \quad (2.6)$$

where $\kappa_3 = 8\pi G$ is the gravitational coupling, and $\mathcal{R}$ is the bulk Ricci scalar. The boundary integral in Eq. (2.6) is the standard Gibbons-Hawking-York term [41,42], which is necessary to render the Dirichlet variational principle well posed.

Varying the action (2.6) yields the bulk vacuum Einstein field equations:

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R}g_{\mu\nu} - \frac{1}{\ell^2}g_{\mu\nu} = 0, \quad \mathcal{R} = -\frac{6}{\ell^2}. \quad (2.7)$$

Simultaneously, tracking the on-shell boundary terms from the variation provides the Dirichlet symplectic potential evaluated on Σ_r [43–46], as follows:

$$\Theta_D(\Sigma_r) = -\frac{1}{2}\int_{\Sigma_r} \sqrt{-\gamma} T^{ab} \delta\gamma_{ab} + \text{corner terms}. \quad (2.8)$$

2. The Brown-York stress tensor

The momentum conjugate to the induced conformal metric γ_{ab} in the symplectic potential (2.8) is the Brown-York energy-momentum tensor (BY-EMT) [47]. It is defined entirely in terms of the boundary's extrinsic curvature as

$$T_{ab} = \frac{1}{\kappa_3} \left(K_{ab} - \frac{r^2}{\ell^2} K \gamma_{ab} \right), \quad T := \gamma^{ab} T_{ab} = -\frac{r^2}{\ell^2 \kappa_3} K. \quad (2.9)$$

3. Radial decomposition of field equations

To facilitate our holographic RG flow analysis, we project the 3D equations of motion (2.7) along and orthogonal to the foliation Σ_r . Using the metric decomposition (2.1), the Einstein equations split into a set of constraints and radial evolution equations, as follows:

$$R + \frac{2r^2}{\ell^4} + \kappa_3^2 \frac{\ell^2}{r^2} \text{TT} = 0, \quad (2.10a)$$

$$\nabla_b T_a^b = 0, \quad (2.10b)$$

$$\begin{aligned} & \frac{\kappa_3}{\Phi} \mathcal{D}_r T_{ab} - \frac{\ell^3 \kappa_3^2}{r^3} (T T_{ab} + 2 \text{TT} \gamma_{ab}) \\ & + \frac{\ell}{r} \left(\frac{\nabla_a \nabla_b \Phi}{\Phi} - \frac{\square \Phi}{\Phi} \gamma_{ab} \right) - \frac{\ell}{2r} R \gamma_{ab} = 0, \end{aligned} \quad (2.10c)$$

$$\mathcal{D}_r \gamma_{ab} + \frac{2}{r} \gamma_{ab} - \frac{2\ell^3 \kappa_3}{r^3} \Phi (T_{ab} - T \gamma_{ab}) = 0. \quad (2.10d)$$

Here, R denotes the intrinsic boundary Ricci scalar, and ∇_a is the covariant derivative built from the Levi-Civita connection of γ_{ab} . Note that Eq. (2.10d) is not a part of Einstein's equations; it is just the definition of BY-EMT

(2.9), written for completeness. We have also introduced the TT operator [9–11], defined as the characteristic quadratic combination of the stress tensor,

$$\text{TT} := T^{ab} T_{ab} - T^2. \quad (2.11)$$

Notice that in deriving the tensor evolution equation (2.10c), we utilized the fact that the boundary is two-dimensional. This allows us to apply the Cayley-Hamilton identity to T_{ab} , yielding the exact algebraic relation

$$T_a^c T_{cb} = T T_{ab} + \frac{1}{2} \text{TT} \gamma_{ab}. \quad (2.12)$$

The set of equations (2.10) establishes the dynamical backbone of our formulation. While similar decomposed equations were explored in Refs. [45,46], our expressions generalize those results by accommodating a completely arbitrary radial lapse function Φ , which is an essential ingredient of our analysis, as we will see in the following sections. As expected from the standard ADM formalism [38] (see Ref. [48] for a review), the Hamiltonian (2.10a) and momentum (2.10b) constraints are purely intrinsic to the hypersurface and remain entirely independent of the choices made for the lapse and shift functions.

C. Asymptotic solution space

1. Perturbative expansion near the boundary

To understand the UV behavior of the theory and prepare the necessary boundary data for our RG flow equations, we solve the equations of motion (2.10) perturbatively near the AdS boundary ($r \rightarrow \infty$) [29,37]. For this asymptotic analysis, we fix the spatial shift vector to $U^a = 0$, but critically, we leave the radial lapse Φ arbitrary. Compatible with the asymptotic AdS conditions (2.2), we introduce the following falloff ansatz for the metric components:

$$\begin{aligned} \Phi(r, x^a) &= 1 + \frac{\ell^2}{r^2} \Phi_1(x^a) + \frac{\ell^4}{r^4} \Phi_2(x^a) + \dots, \\ \gamma_{ab}(r, x^a) &= q_{ab}(x^a) + \frac{\ell^2}{r^2} C_{ab}(x^a) + \frac{\ell^4}{r^4} D_{ab}(x^a) + \dots. \end{aligned} \quad (2.13)$$

Here, $q_{ab}(x^a)$ is the arbitrary induced metric on the AdS conformal boundary. From this ansatz, it is straightforward to verify that the inverse conformal metric expands as

$$\gamma^{ab} = q^{ab} - \frac{\ell^2}{r^2} C^{ab} - \frac{\ell^4}{r^4} (D^{ab} - C^{ac} C_c^b) + \dots. \quad (2.14)$$

2. Solving the radial evolution

Substituting these expansions into the radial equations (2.10), we can determine the metric coefficients C_{ab} and D_{ab} perturbatively, order by order in $1/r$. The result is

$$\begin{aligned} C_{ab} &= \frac{12\pi\ell^2}{c} t_{ab} - \left(\Phi_1 + \frac{\ell^2}{2} R[q] \right) q_{ab}, \\ D_{ab} &= -\frac{3\pi\ell^4}{2c} R[q] t_{ab} - \frac{\ell^2}{4} D_a D_b \Phi_1 \\ &\quad + \frac{1}{2} \left(-\Phi_2 + \Phi_1^2 + \frac{\ell^4}{8} R[q]^2 + \frac{36\pi^2\ell^4}{c^2} \text{tt} \right) q_{ab}. \end{aligned} \quad (2.15)$$

In these expressions, t_{ab} acts as an integration constant from the perspective of radial integration; physically, it corresponds to the holographic energy-momentum tensor of the dual boundary UV theory [37,49]. The quantity c is the celebrated Brown-Henneaux central charge [21]:

$$c := \frac{12\pi\ell}{\kappa_3}. \quad (2.16)$$

Furthermore, the Hamiltonian and momentum constraints (2.10a) and (2.10b) evaluated at the boundary impose strict conservation and trace anomaly conditions on t_{ab} [49,50], as follows:

$$D_a t^{ab} = 0, \quad t := q^{ab} t_{ab} = \frac{c}{24\pi} R[q]. \quad (2.17)$$

Here, D_a denotes the covariant derivative compatible with the boundary metric q_{ab} , and tt is the $\text{T}\bar{\text{T}}$ operator at the AdS boundary, as follows:

$$\text{tt} := t_{ab} t^{ab} - t^2. \quad (2.18)$$

3. Unconstrained boundary data

At this stage, the asymptotic solution space is parametrized entirely by the boundary data q_{ab} , its conjugate momentum t^{ab} , and an infinite tower of arbitrary scalar fields Φ_i ($i = 1, 2, \dots$). It is crucial to emphasize that in writing these expansions, we have not yet imposed any specific boundary conditions. For instance, if one restricts to the well-known family of Bañados geometries [45,51], one fixes $\Phi_i = 0$ and restricts q_{ab} to a flat metric to strictly preserve standard Dirichlet boundary conditions, leading to $R[q] = 0$ and $t = 0$. In our setup, however, we keep the boundary fields and lapse coefficients completely general.

4. Semi-on-shell asymptotic quantities

To integrate the boundary action along the RG flow in later sections, we will need the asymptotic expansions of several composite geometric scalars and tensors. Substituting the solved metric coefficients back into our definitions, we obtain the following semi-on-shell expansions (meaning the bulk radial equations are satisfied, but the asymptotic constraints (2.17) are left explicit):

$$\begin{aligned} \sqrt{-\gamma} &= \sqrt{-q} \left[1 - \left(\frac{\ell}{r} \right)^2 \left(\Phi_1 + \frac{\ell^2}{4} R[q] \right) - \frac{1}{2} \left(\frac{\ell}{r} \right)^4 \left(\Phi_2 - \Phi_1^2 + \frac{\ell^2}{4} \square \Phi_1 + \frac{36\pi^2\ell^4}{c^2} \text{tt} \right) + \dots \right], \\ R[\gamma] &= R[q] + \frac{\ell^2}{r^2} \left(R[q] \Phi_1 + \frac{\ell^2}{4} R[q]^2 + \square \Phi_1 \right) + \dots, \\ T_{ab} &= -\frac{c}{12\pi\ell^2} \frac{r^2}{\ell^2} q_{ab} - 2t_{ab} + \frac{c}{12\pi\ell^2} \left(\Phi_1 + \frac{\ell^2}{2} R[q] \right) q_{ab} + \dots, \\ T &= -\frac{c}{6\pi\ell^2} \frac{r^2}{\ell^2} - \frac{cR[q]}{24\pi} - \frac{\ell^2}{r^2} \left(\frac{6\pi\ell^2}{c} t_{ab} t^{ab} + \frac{cR[q]}{24\pi} \Phi_1 + \frac{c}{24\pi} \square \Phi_1 \right) + \dots. \end{aligned} \quad (2.19)$$

These expansions, driven primarily by the boundary fields and the generic lapse coefficients Φ_1 and Φ_2 , constitute the necessary mathematical toolkit to evaluate the boundary action at spatial infinity.

III. HOLOGRAPHIC RG FLOW OF THE ACTION AND BOUNDARY CONDITIONS

With the geometric foliation and asymptotic solution space established, we are now equipped to study the radial evolution of the boundary theory. In the holographic

framework, moving the cutoff surface Σ_r from the asymptotic boundary into the bulk corresponds to integrating out high-energy degrees of freedom, realizing an exact RG flow in the dual theory [30,31]. In this section, we derive the explicit flow equation governing the boundary action, rigorously verify its consistency with the bulk dynamics, and demonstrate a profound physical consequence: a rigid Dirichlet boundary condition imposed in the UV inevitably evolves into a dynamically mixed boundary condition in the IR.

A. Dirichlet deformation flow equation

1. Radial evolution via diffeomorphisms

We first compute the radial deformation of the on-shell bulk action subject to Dirichlet boundary conditions. To do this, we employ the “freelance holography” machinery developed in Refs. [52,53] (see also Ref. [54] for a pedagogical review). A core insight of this approach is that the radial evolution of the on-shell action can be captured by evaluating its variation under a diffeomorphism generated by the radial vector field $\xi = \partial_r$.

The variation of the Einstein-Hilbert action, evaluated on shell and compatible with Dirichlet boundary conditions, under an arbitrary diffeomorphism ξ is given purely by a boundary term:

$$\delta_\xi S_{\text{bulk}}^{\text{D}}[\mathcal{M}_r]|_{\text{on-shell}} = -\frac{\ell^2}{2r^2} \int_{\Sigma_r} d^2x \sqrt{-\gamma} T^{ab} \delta_\xi \left(\frac{r^2}{\ell^2} \gamma_{ab} \right). \quad (3.1)$$

2. The holographic $\text{T}\bar{\text{T}}$ flow equation

By strictly choosing $\xi = \partial_r$, we extract the exact radial derivative of the on-shell action, as follows:

$$\begin{aligned} \frac{d}{dr} S_{\text{bulk}}^{\text{D}}[\mathcal{M}_r]|_{\text{on-shell}} &= -\frac{\ell^2}{2r^2} \int_{\Sigma_r} d^2x \sqrt{-\gamma} T^{ab} \partial_r \left(\frac{r^2}{\ell^2} \gamma_{ab} \right) \\ &= -\frac{\ell^2}{r^2} \int_{\Sigma_r} d^2x \sqrt{-\gamma} T^{ab} \left[\frac{\ell \Phi}{r} K_{ab} + \frac{r^2}{\ell^2} \nabla_a U_b \right] \\ &= -\frac{\ell^3 \kappa_3}{r^3} \int_{\Sigma_r} d^2x \sqrt{-\gamma} \Phi T^{ab} (T_{ab} - T \gamma_{ab}) \\ &= -\frac{\ell^3 \kappa_3}{r^3} \int_{\Sigma_r} d^2x \sqrt{-\gamma} \Phi \text{TT}. \end{aligned} \quad (3.2)$$

Let us unpack this sequence. The second line follows directly from substituting the definition of the extrinsic curvature (2.5). The third line is achieved by replacing K_{ab} with the Brown-York stress tensor via Eq. (2.9) and integrating the shift vector term by parts, which vanishes due to the strict conservation of the stress tensor (2.10b) ($\nabla_a T^{ab} = 0$). The final line elegantly compacts the expression using the standard definition of the $\text{T}\bar{\text{T}}$ operator (2.11).

Invoking the fundamental holographic dictionary at a finite radial cutoff, which equates the on-shell bulk action to the dual boundary action,

$$S_{\text{bulk}}^{\text{D}}[\mathcal{M}_r]|_{\text{on-shell}} = S_{\text{bdry}}^{\text{D}}[\Sigma_r], \quad (3.3)$$

we obtain the celebrated deformation flow equation:

$$\frac{d}{dr} S_{\text{bdry}}^{\text{D}}[\Sigma_r] = -\frac{\ell^3 \kappa_3}{r^3} \int_{\Sigma_r} d^2x \sqrt{-\gamma} \Phi \text{TT}. \quad (3.4)$$

This confirms that the radial evolution in pure Einstein-Hilbert gravity is intrinsically governed by a $\text{T}\bar{\text{T}}$ -like deformation [12–14].

3. Geometric formulation of the flow

While Eq. (3.4) frames the flow in terms of the boundary fluid’s stress tensor, it can be entirely recast into geometric variables. By substituting the Hamiltonian constraint (2.10a) to replace the TT operator with the intrinsic boundary curvature, we find an equivalent, purely gravitational representation of the flow, as follows:

$$\frac{d}{dr} S_{\text{bdry}}^{\text{D}}[\Sigma_r] = \frac{c}{12\pi r} \int_{\Sigma_r} d^2x \sqrt{-\gamma} \Phi \left(R[\gamma] + \frac{2r^2}{\ell^4} \right). \quad (3.5)$$

Let us pause to examine the profound physical implications of the rhs of Eq. (3.5). Remarkably, the radial evolution of the boundary QFT is driven by a term that is exactly proportional to the Jackiw-Teitelboim gravity Lagrangian [15,16]. In this expression, the bulk radial lapse function Φ naturally assumes the role of the 2D dynamical dilaton, coupling directly to the intrinsic boundary curvature $R[\gamma]$, while the r^2/ℓ^4 term provides the corresponding cosmological constant. This is a striking realization of the GR from RG program: the generator of the renormalization group flow is itself a gravitational action. This structural appearance strongly suggests that the effective boundary action at any finite energy scale must inherently possess a JT gravity sector. In Sec. IV, we will elevate this observation to a rigorous proof by introducing an exact JT-like ansatz for the finite-cutoff action and demonstrating that it nonperturbatively solves this very deformation flow equation.

B. A consistency check: Derivation of bulk equations from boundary

As a robust consistency check of our framework—and a demonstration of the profound link between RG flow and bulk dynamics [26,28,55]—we now show that the boundary deformation flow equation (3.5) precisely encodes the bulk spatial equations of motion. If our formalism is consistent, the variation of the radial flow of the action must commute with the radial flow of its variation. That is, computing $\frac{d}{dr} \delta S_{\text{bdry}}^{\text{D}}[\Sigma_r]$ in two distinct ways must yield identical results. As we will see, these two methods agree iff the tensor Einstein equation (2.10c) holds.

1. Method 1: Variation of the flow equation

First, we directly vary the geometric flow equation (3.5) with respect to the boundary metric. This trivially yields

$$\frac{d}{dr} \delta S_{\text{bdry}}^{\text{D}}[\Sigma_r] = \frac{\ell}{r \kappa_3} \delta \int_{\Sigma_r} d^2x \sqrt{-\gamma} \Phi \left(R[\gamma] + \frac{2r^2}{\ell^4} \right). \quad (3.6)$$

2. Method 2: Radial flow of the variational principle

Alternatively, we can start from the definition of the boundary action's on-shell variation, driven by the Brown-York stress tensor:

$$\delta S_{\text{bdry}}^{\text{D}}[\Sigma_r] = -\frac{1}{2} \int_{\Sigma_r} d^2x \sqrt{-\gamma} T^{ab} \delta \gamma_{ab}. \quad (3.7)$$

Taking the radial derivative of this expression requires applying the chain rule to the integrand, as follows:

$$-\frac{1}{2} \frac{d}{dr} \int_{\Sigma_r} d^2x \sqrt{-\gamma} T^{ab} \delta \gamma_{ab} = -\frac{1}{2} \int_{\Sigma_r} d^2x [\mathcal{D}_r(\sqrt{-\gamma} T^{ab}) \delta \gamma_{ab} + \sqrt{-\gamma} T^{ab} \mathcal{D}_r(\delta \gamma_{ab})]. \quad (3.8)$$

To evaluate the last term, we use the commutation relation between the variation and the radial differential operator, $\mathcal{D}_r \delta \gamma_{ab} = \delta \mathcal{D}_r \gamma_{ab} + \mathcal{L}_{\delta U} \gamma_{ab}$. When multiplied by $\sqrt{-\gamma} T^{ab}$, the Lie derivative term $\mathcal{L}_{\delta U} \gamma_{ab}$ integrates to a corner term due to the conservation of the stress tensor (2.10b) ($\nabla_a T^{ab} = 0$). Discarding this boundary term, we are left with evaluating $T^{ab} \delta \mathcal{D}_r \gamma_{ab}$.

Using the definition of the extrinsic curvature in terms of the stress tensor (2.10d) and applying the Cayley-Hamilton identity (2.12), we can rewrite this contraction as

$$\frac{d}{dr} \delta S_{\text{bdry}}^{\text{D}}[\Sigma_r] = -\frac{1}{2} \int_{\Sigma_r} d^2x \left\{ \left[\mathcal{D}_r(\sqrt{-\gamma} T^{ab}) - 2 \frac{\sqrt{-\gamma}}{r} T^{ab} \right] \delta \gamma_{ab} + \frac{\ell}{\kappa_3 r} \sqrt{-\gamma} \Phi \delta R - \frac{2\ell}{\kappa_3 r} \delta \left[\sqrt{-\gamma} \Phi \left(R + \frac{2r^2}{\ell^4} \right) \right] \right\}. \quad (3.12)$$

3. Synthesis and recovery of the equations of motion

We now demand that Method 1 (3.6) and Method 2 (3.12) agree. Notice that the last term in Eq. (3.12) is exactly equal to the expression of Method 1. Therefore, for the two approaches to be strictly consistent, the remainder of the integral in Eq. (3.12) must vanish identically, as follows:

$$\int_{\Sigma_r} d^2x \left\{ \left[\mathcal{D}_r(\sqrt{-\gamma} T^{ab}) - 2 \frac{\sqrt{-\gamma}}{r} T^{ab} \right] \delta \gamma_{ab} + \frac{\ell}{\kappa_3 r} \sqrt{-\gamma} \Phi \delta R \right\} = 0. \quad (3.13)$$

To extract the local equations of motion, we utilize the standard variation of the Ricci scalar,

$$\delta R = -\frac{1}{2} R \gamma^{ab} \delta \gamma_{ab} + \nabla^a \nabla^b \delta \gamma_{ab} - \gamma^{ab} \square \delta \gamma_{ab}, \quad (3.14)$$

and perform integration by parts on the derivative terms to isolate $\delta \gamma_{ab}$. Dropping the resulting total derivatives, Eq. (3.13) demands that

$$T^{ab} \delta \mathcal{D}_r \gamma_{ab} = \left(-\frac{2T^{ab}}{r} + \frac{\ell^3 \kappa_3}{r^3} \Phi \text{TT} \gamma^{ab} \right) \delta \gamma_{ab} + \frac{2\ell^3 \kappa_3}{r^3} \text{TT} \delta \Phi + \frac{\ell^3 \kappa_3}{r^3} \Phi \delta \text{TT}. \quad (3.9)$$

We can elegantly eliminate the δTT term by invoking the bulk Hamiltonian constraint (2.10a) and its variation, as follows:

$$\frac{\ell^3 \kappa_3}{r^3} \text{TT} = -\frac{\ell}{\kappa_3 r} \left(R + \frac{2r^2}{\ell^4} \right) \Rightarrow \frac{\ell^3 \kappa_3}{r^3} \delta \text{TT} = -\frac{\ell}{\kappa_3 r} \delta R. \quad (3.10)$$

Substituting this back, we can group the terms to explicitly reconstruct the variation of the JT-like action block:

$$T^{ab} \delta \mathcal{D}_r \gamma_{ab} = -\frac{\ell}{\kappa_3 r} \left\{ \frac{2\kappa_3}{\ell} T^{ab} \delta \gamma_{ab} - \Phi \delta R + \frac{2}{\sqrt{-\gamma}} \delta \left[\sqrt{-\gamma} \Phi \left(R + \frac{2r^2}{\ell^4} \right) \right] \right\}. \quad (3.11)$$

Inserting this result into Eq. (3.8), Method 2 yields

$$\mathcal{D}_r(\sqrt{-\gamma} T^{ab}) = \sqrt{-\gamma} \left[\frac{2T^{ab}}{r} + \frac{\ell R}{2r\kappa_3} \Phi \gamma^{ab} - \frac{\ell}{r\kappa_3} (\nabla^a \nabla^b \Phi - \gamma^{ab} \square \Phi) \right]. \quad (3.15)$$

Remarkably, upon expanding the lhs using the definition of T_{ab} (2.10d), this is nothing other than the transverse tensor Einstein equation (2.10c). We have thus successfully derived the spatial bulk dynamics strictly from the integrability of the boundary RG flow, relying solely on the Hamiltonian (2.10a) and momentum (2.10b) constraints.

C. RG flow of the Dirichlet boundary condition

It is a well-established property of Einstein gravity that it constitutes a well-defined initial-boundary value problem [56]. Once boundary data are specified at the asymptotic boundary Σ alongside a chosen boundary condition, the bulk equations of motion uniquely determine the induced data and the effective boundary conditions on any other constant radial slice Σ_r . We now investigate this exact radial flow.

1. The asymptotic Dirichlet condition

We begin by imposing a strict Dirichlet boundary condition at the asymptotic conformal boundary Σ , as follows:

$$\text{Dirichlet b.c. at } \Sigma: \quad \delta q_{ab} = 0. \quad (3.16)$$

While the induced metric q_{ab} is absolutely fixed on Σ , its conjugate momentum t_{ab} remains dynamical and undergoes arbitrary fluctuations. These fluctuations are only subject to the asymptotic Hamiltonian and momentum constraints (2.17), which explicitly demand that

$$\delta t = 0, \quad D_a \delta t^{ab} = 0. \quad (3.17)$$

Consequently, the UV dynamics of the dual theory are entirely governed by the conserved boundary stress tensor t^{ab} and the arbitrary lapse fluctuations $\delta\Phi_i$.

2. Flow of Dirichlet at finite distance

To see how this boundary condition evolves, we evaluate the variation of the induced metric at a finite radial distance using our asymptotic expansions (2.13). Tracking the variations yields

$$\begin{aligned} \delta\gamma_{ab}(r, x^a) = & \frac{\ell^2}{r^2} \left(\frac{12\pi\ell^2}{c} \delta t_{ab} - \delta\Phi_1 q_{ab} \right) \\ & + \frac{\ell^4}{r^4} \left(-\frac{3\pi\ell^4}{2c} R[q] \delta t_{ab} - \frac{\ell^2}{4} D_a D_b \delta\Phi_1 \right. \\ & \left. + \frac{1}{2} \left(-\delta\Phi_2 + \delta\Phi_1^2 + \frac{36\pi^2\ell^4}{c^2} \delta t t \right) q_{ab} \right) \\ & + \mathcal{O}((\ell/r)^6). \end{aligned} \quad (3.18)$$

Crucially, although we imposed strict Dirichlet boundary conditions at infinity ($\delta q_{ab} = 0$), this condition is visibly not preserved under radial evolution. Instead, the fluctuations of the boundary stress tensor (δt_{ab}) and the lapse function ($\delta\Phi_1$) directly act as sources, driving metric fluctuations $\delta\gamma_{ab} \neq 0$ at any finite cutoff surface.

3. Emergence of a mixed boundary condition

If the boundary condition is no longer Dirichlet, what is it? To answer this, we must similarly expand the variation of the Brown-York stress tensor at a finite distance using Eq. (2.19), as follows:

$$\delta T_{ab} = -2\delta t_{ab} + \frac{c}{12\pi\ell^2} \delta\Phi_1 q_{ab} + \mathcal{O}((\ell/r)^2). \quad (3.19)$$

Notice that the leading terms of $\delta\gamma_{ab}$ and δT_{ab} are sourced by the exact same UV fluctuations (δt_{ab} and $\delta\Phi_1$). By algebraically combining them to cancel these leading-order source terms, we uncover the true constraint operating at finite r , as follows:

$$\delta(e^{(\Phi-1)/2} \gamma_{ab}(r, x^a)) + \frac{6\pi\ell^4}{cr^2} \delta T_{ab}(r, x^a) = \mathcal{O}((\ell/r)^4). \quad (3.20)$$

This remarkable relation demonstrates that the boundary condition dynamically rotates along the RG flow. What begins as a purely Dirichlet condition at the asymptotic boundary evolves into a mixed (Robin-like) Neumann-Dirichlet condition on the finite-cutoff metric γ_{ab} [1,2,53,57,58].

Two features of this mixed condition stand out. First, the Dirichlet part is modified by the Weyl factor $e^{(\Phi-1)/2}$. The presence of this factor is a direct consequence of leaving the lapse function Φ arbitrary, differentiating our setup from standard Fefferman-Graham analyses. Second, the admixture with the Neumann condition (δT_{ab}) is explicitly suppressed by the central charge $1/c$ and by the radial running factor ℓ^2/r^2 . Therefore, as we evolve more into the IR (smaller r values), the Neumann admixture increases, and we move further from pure Dirichlet at large r .

D. Physical implications of the radial flow of boundary conditions

We now state the central argument regarding the radial evolution of the boundary action. The algebraic mixing of metric and stress-tensor fluctuations observed in the previous subsection has a direct and elegant interpretation at the level of the variational principle.

1. Variational principle along the flow

As established by the deformation flow equation (3.5), the effective action of the boundary theory at a finite cutoff Σ_r is related to the action at the asymptotic boundary Σ by integrating the RG flow. Formally, this integration yields an accumulated deformation term [45,53],

$$S_{\text{bdry}}^{\text{D}}[\Sigma_r] = S_{\text{bdry}}^{\text{D}}[\Sigma] + S_{\text{deform}}, \quad (3.21)$$

where S_{deform} represents the exact integral of the flow equation from infinity down to the radial slice r . While we will explicitly compute S_{deform} in Sec. IV and reveal that it contains the induced JT gravity action, its mere formal presence is sufficient to dictate the evolution of the boundary conditions. To understand these physical implications, consider the general variation of this relationship:

$$\delta S_{\text{bdry}}^{\text{D}}[\Sigma_r] = \delta S_{\text{bdry}}^{\text{D}}[\Sigma] + \delta S_{\text{deform}}. \quad (3.22)$$

This simple equation governs how boundary conditions are dynamically exchanged between the UV (asymptotic) and the IR (finite cutoff) regimes.

2. Flowing from the UV to the IR

If we impose a strict Dirichlet boundary condition at the asymptotic boundary Σ , we demand, by definition, that $\delta S_{\text{bdry}}^{\text{D}}[\Sigma] = 0$. Consequently, evaluating the variation at the finite cutoff surface Σ_r yields

$$0 \neq \delta S_{\text{bdry}}^{\text{D}}[\Sigma_r] = \delta S_{\text{deform}}. \quad (3.23)$$

This explicitly demonstrates that radial evolution, as governed by the bulk equations of motion, strictly breaks the Dirichlet boundary condition at finite distances. Instead, the well-posed variational principle at Σ_r dictates that

$$\delta(S_{\text{bdry}}^{\text{D}}[\Sigma_r] - S_{\text{deform}}) = 0. \quad (3.24)$$

This relation implies a dynamically mixed boundary condition on Σ_r . Both the induced metric and its conjugate stress tensor are free to fluctuate, but their variations are not independent—they are rigidly constrained by δS_{deform} . Physically, this describes a constrained “gravity coupled to matter” system residing at a finite radial distance, governed by the effective action $S_{\text{bdry}}^{\text{D}}[\Sigma_r]$.

3. Flowing from the IR to the UV

Conversely, one might attempt to work in the opposite direction by forcefully imposing a strict Dirichlet boundary condition at a finite distance, thereby requiring $\delta S_{\text{bdry}}^{\text{D}}[\Sigma_r] = 0$. In this scenario, the boundary condition at asymptotic infinity Σ can no longer be Dirichlet, as follows:

$$0 \neq \delta S_{\text{bdry}}^{\text{D}}[\Sigma] = -\delta S_{\text{deform}}, \quad (3.25)$$

which is equivalent to demanding that

$$\delta(S_{\text{bdry}}^{\text{D}}[\Sigma] + S_{\text{deform}}) = 0. \quad (3.26)$$

This generically leads to a mixed boundary condition at the asymptotic boundary Σ , describing an alternative effective gravity-matter system at infinity.

We close this part with the remark that the above discussions on the flow of boundary conditions and the analysis in Sec. III B about the consistency of the analysis are complementary to each other: the flow of boundary conditions is a result of the bulk equations of motion.

4. The physical origin of finite-cutoff instabilities

A crucial observation emerges from comparing the effective variational principles in Eqs. (3.24) and (3.26): there is a relative sign difference in front of the deformation term S_{deform} .

The term S_{deform} is directly responsible for activating the gravitational dynamics at the boundary. Thus, this sign reversal dictates that the induced gravitational action will carry opposite overall signs in the two scenarios.

Consequently, only one of these setups can yield a well-defined effective theory with healthy (positive-definite) kinetic terms for the boundary metric. This sign discrepancy provides a profound physical origin for the generic ill posedness [59–66]—such as the inevitable appearance of ghostlike instabilities—that plagues attempts to arbitrarily impose strict Dirichlet boundary conditions at a finite radial cutoff.

IV. EMERGENCE OF 2D GRAVITY: INTEGRATING THE FLOW EQUATION

Having established the differential RG flow equation and its profound implications for boundary conditions, we now turn to our primary objective: integrating this flow equation to uncover the exact effective action of the finite-cutoff theory. In this section, we show how the radial evolution inevitably induces a 2D scalar-tensor gravity theory—specifically, JT gravity—on the finite cutoff slice.

A. Recalling the statement of AdS/CFT

Before constructing an exact, local solution to the deformation flow equation (3.5), it is highly instructive to first recall the standard AdS/CFT correspondence in the saddle-point approximation. This will reveal the presence of JT gravity in the asymptotic expansion.

1. The holographic dictionary

According to the standard holographic dictionary [22,23] (see Refs. [52,53] and references therein), the effective action of the dual quantum field theory at a given cutoff surface is equivalent to the on-shell gravitational action evaluated in the enclosed bulk region. For a finite cutoff surface Σ_r and the true asymptotic boundary Σ , this implies that

$$\begin{aligned} S_{\text{bdry}}^{\text{D}}[\Sigma_r] &= S_{\text{bulk}}^{\text{D}}[\mathcal{M}_r]|_{\text{on-shell}}, \\ S_{\text{bdry}}^{\text{D}}[\Sigma] &= S_{\text{bulk}}^{\text{D}}[\mathcal{M}]|_{\text{on-shell}}, \end{aligned} \quad (4.1)$$

where the superscript D indicates that Dirichlet boundary conditions are imposed on the respective surfaces.

2. Localizing the bulk action

To evaluate these expressions, we substitute the bulk equations of motion into the action (2.6). Using the constant scalar curvature $\mathcal{R} = -6/\ell^2$, the volume element relation $\sqrt{-g} = \frac{r}{\ell} \Phi \sqrt{-\gamma}$, and the definition of the stress tensor trace (2.9), the on-shell bulk action reduces to

$$S_{\text{bulk}}^{\text{D}}[\mathcal{M}_r]|_{\text{on-shell}} = -\frac{2r}{\kappa_3 \ell^3} \int_{\mathcal{M}_r} d^3x \sqrt{-\gamma} \Phi - \int_{\Sigma_r} d^2x \sqrt{-\gamma} T. \quad (4.2)$$

To write the remaining bulk integral strictly as a boundary term, we can employ a mathematical trick by introducing an

auxiliary scalar field χ defined via the differential equation

$$\mathcal{D}_r(\sqrt{-\gamma}\chi) = \frac{\sqrt{-\gamma}\Phi}{\kappa_3} \frac{r}{\ell}. \quad (4.3)$$

By replacing the bulk integrand with this total radial derivative, the bulk on-shell action reduces entirely to a boundary integral on Σ_r , as follows:

$$S_{\text{bulk}}^{\text{D}}[\mathcal{M}_r]|_{\text{on-shell}} = - \int_{\Sigma_r} d^2x \sqrt{-\gamma} \left(T + \frac{2}{\ell^2} \chi \right). \quad (4.4)$$

3. Relating the UV and IR actions

We now seek the relationship between the field theory actions at the two respective boundaries, $S_{\text{bdry}}^{\text{D}}[\Sigma_r]$ and $S_{\text{bdry}}^{\text{D}}[\Sigma]$. Subtracting the two relations in Eq. (4.1) and utilizing Eq. (4.4), we obtain

$$S_{\text{bdry}}^{\text{D}}[\Sigma_r] = S_{\text{bdry}}^{\text{D}}[\Sigma] - \int_{\Sigma_r} d^2x \sqrt{-\gamma} \left(T + \frac{2}{\ell^2} \chi \right) + \int_{\Sigma} d^2x \sqrt{-\gamma} \left(T + \frac{2}{\ell^2} \chi \right). \quad (4.5)$$

One can readily verify that Eq. (4.5) is a valid mathematical solution to the differential flow equation (3.5), provided the scalar field χ satisfies Eq. (4.3).

4. Asymptotic expansion and induced JT gravity

While Eq. (4.5) is exact, its rhs explicitly mixes quantities evaluated at two different surfaces (Σ and Σ_r). For a coherent physical interpretation, we must express the action exclusively in terms of variables defined at a single surface. Using the asymptotic expansions derived in Sec. II C, we can evaluate the Σ integral. From the asymptotic properties (2.17), we have

$$\begin{aligned} \sqrt{-\gamma}|_{\Sigma} &= \sqrt{-q}, & T|_{\Sigma} &= -\frac{c}{6\pi\ell^2} \frac{r_{\infty}^2}{\ell^2} - \frac{cR[q]}{24\pi}, \\ \chi|_{\Sigma} &= \frac{cr_{\infty}^2}{24\pi\ell^2} - \frac{c\ell^2}{48\pi} R[q] \ln(r/\ell). \end{aligned} \quad (4.6)$$

Substituting these along with the finite- r expansions (2.19) into Eq. (4.5), dropping purely topological and total derivative terms, we are left with the following simplified expression:

$$\begin{aligned} S_{\text{bdry}}^{\text{D}}[\Sigma_r] &= S_{\text{bdry}}^{\text{D}}[\Sigma] + \int_{\Sigma} d^2x \sqrt{-q} \left[\frac{c(r^2 - r_{\infty}^2)}{12\pi\ell^4} \right. \\ &\quad \left. + \frac{\ell^2}{r^2} \left(\frac{3\pi\ell^2}{2c} \text{tt} + \frac{c(\Phi_1^2 - \Phi_2)}{24\pi\ell^2} - \frac{c\Phi_1}{48\pi} R[q] \right) \right] \\ &\quad + \mathcal{O}\left(\frac{\ell^4}{r^4}\right). \end{aligned} \quad (4.7)$$

This equation allows us to clearly interpret the physical mechanics of the flow. Suppose we impose the strict

Dirichlet boundary condition $\delta\gamma_{ab}|_{\Sigma_r} = 0$ at the finite cutoff Σ_r , which guarantees that a pure, nongravitational QFT resides there. Equation (4.7) dictates that doing so forces us to relax Dirichlet conditions at asymptotic infinity Σ . Consequently, the UV theory at infinity is no longer a pure QFT; metric fluctuations $\delta q_{ab} \neq 0$ are activated.

Crucially, the deformation terms in Eq. (4.7) explicitly show that the UV theory is modified not only by matter-sector deformations (the standard tt term) but also by gravitational contributions. The most notable term is the $\Phi_1 R[q]$ coupling, which famously corresponds to JT gravity [15,16] living at the asymptotic boundary.

5. The Fefferman-Graham limit

It is worth noting what happens if we adopt the standard Fefferman-Graham gauge by fixing $\Phi_i = 0$. Under this restriction, the gravitational dilaton terms vanish, and the boundary action (4.7) dramatically simplifies to

$$S_{\text{bdry}}^{\text{D}}[\Sigma_r] = S_{\text{bdry}}^{\text{D}}[\Sigma] + \int_{\Sigma} d^2x \sqrt{-q} \left[\frac{c(r^2 - r_{\infty}^2)}{12\pi\ell^4} + \frac{3\pi\ell^4}{2cr^2} \text{tt} \right]. \quad (4.8)$$

In this gauge, the JT term drops out, leaving only the pure matter $\text{T}\bar{\text{T}}$ modification [10–12]. Note that while Eq. (4.7) is a perturbative result in the asymptotic expansion, the FG result (4.8) is exact.

6. Motivation for a local ansatz

While the preceding analysis elegantly illustrates how imposing a Dirichlet condition at a finite cutoff induces JT gravity in the UV, our primary goal is the exact reverse: we want to impose strict Dirichlet boundary conditions at asymptotic infinity and investigate the induced gravity on the boundary QFT at a finite distance. To achieve this cleanly, we need an exact, nonperturbative solution to the flow equation formulated entirely in terms of local geometric variables strictly residing on the cutoff surface Σ_r . We develop this targeted formulation in the subsequent subsection.

B. Induced JT gravity at finite cutoff

The asymptotic analysis in the previous subsection revealed the presence of JT gravity near the boundary Σ . We now seek an exact, nonperturbative solution to the deformation flow equation (3.5). Our goal is to construct a boundary action formulated entirely in terms of local quantities residing on the cutoff surface Σ_r .

1. The JT gravity ansatz

Motivated by the structure of the flow equation (3.5) and the form of the asymptotic result (4.7), we propose the following ansatz for the finite-cutoff boundary action:

$$S_{\text{bdry}}^{\text{D}}[\Sigma_r] = S_{\text{bdry}}^{\text{D}}[\Sigma] - \frac{1}{2} \int_{\Sigma_r} d^2x \sqrt{-\gamma} T + \frac{c}{12\pi} \int_{\Sigma_r} d^2x \sqrt{-\gamma} \Psi (R[\gamma] - 2\Lambda(r)). \quad (4.9)$$

In this expression, T is the trace of the stress tensor, $\Psi(r, x^a)$ is a dynamical scalar field playing the role of the JT dilaton, and $\Lambda(r)$ is an effective 2D cosmological constant. The second term on the rhs, as we will discuss

in Sec. IV C, acts as a generator for a Legendre transformation [52,57].

2. Localization of the flow equation

To verify if this ansatz satisfies the flow equation (3.5), we substitute Eq. (4.9) into the radial derivative. After discarding total derivative terms on the spatial slice Σ_r , we obtain the following integral constraint:

$$\int_{\Sigma_r} d^2x \sqrt{-\gamma} \left\{ \frac{c}{12\pi} (R[\gamma] - 2\Lambda) \partial_r \Psi + \frac{c\Psi}{6\pi r} (2\Lambda - r\Lambda') - \frac{c\Phi}{24\pi r} R[\gamma] + \frac{2\ell^4}{r^3} \Psi (T^{ab} \nabla_a \nabla_b \Phi + \Lambda \Phi T) \right\} = 0. \quad (4.10)$$

To extract a local physical theory, we demand that the integrand vanishes up to a total divergence. A natural choice is to set the integrand equal to $-\frac{c}{24\pi r} R[\gamma]$,² which yields the local dilaton evolution equation

$$\frac{c}{12\pi} (R[\gamma] - 2\Lambda) \partial_r \Psi + \frac{c\Psi}{6\pi r} (2\Lambda - r\Lambda') - \frac{c\Phi}{24\pi r} R[\gamma] + \frac{2\ell^4}{r^3} \Psi (T^{ab} \nabla_a \nabla_b \Phi + \Lambda \Phi T) = -\frac{c}{24\pi r} R[\gamma]. \quad (4.11)$$

3. Dilaton RG flow equation

Equation (4.11) can be recast as a first-order linear differential equation for the dilaton field Ψ , as follows:

$$\partial_r \Psi + \mathcal{X} \Psi = \mathcal{S}, \quad (4.12)$$

where the “drag” coefficient $\mathcal{X}$ and the “source” term $\mathcal{S}$ are given by

$$\mathcal{X} = \frac{\frac{4\Lambda}{r} (1 + \frac{6\pi\ell^4}{cr^2} \Phi T) - 2\Lambda' + \frac{24\pi\ell^4}{cr^3} T^{ab} \nabla_a \nabla_b \Phi}{R[\gamma] - 2\Lambda}, \quad \mathcal{S} = \frac{\Phi - 1}{2r} \frac{R[\gamma]}{R[\gamma] - 2\Lambda}. \quad (4.13)$$

The source term $\mathcal{S}$ is notably proportional to $(\Phi - 1)$, confirming that the nontrivial radial lapse Φ is the primary driver of the dilaton’s radial evolution. Using an integrating factor $\mu(r, x^a)$, the exact solution for the dilaton is

$$\Psi(r, x^a) = \frac{1}{\mu(r, x^a)} \left(\int^r dr' \mu(r', x^a) \mathcal{S}(r', x^a) + \Psi_0(x^a) \right), \quad \mu(r, x^a) = \exp \left(\int^r dr' \mathcal{X}(r', x^a) \right). \quad (4.14)$$

Here, $\Psi_0(x^a)$ is an integration constant independent of the radial coordinate.

4. Radial dependence and consistent expansions

The function $\Lambda(r)$ remains a free parameter of the integration. To match the expected UV behavior where gravity becomes subdominant, a careful analysis of the solution (4.14) reveals that we must assume the following expansions for the cosmological constant and the dilaton field:

$$\Lambda(r) = \left(\frac{r}{\ell} \right)^2 \Lambda_0 + \Lambda_1 + \left(\frac{\ell}{r} \right)^2 \Lambda_2 + \dots, \quad \Lambda_0 \neq 0, \quad \Psi(r, x^a) = \left(\frac{\ell}{r} \right)^2 \Psi_0(x^a) + \left(\frac{\ell}{r} \right)^4 \Psi_1(x^a) + \left(\frac{\ell}{r} \right)^6 \Psi_2(x^a) + \dots. \quad (4.15)$$

5. Iterative solution for dilaton coefficients

By substituting these expansions into the ordinary differential equation (4.12) and using the semi-on-shell expansions of the gravitational variables from Eq. (2.19), we solve for the dilaton coefficients iteratively. The leading-order coefficient Ψ_0 is fixed by the requirement that the boundary action successfully reduces to the undeformed action as $r \rightarrow \infty$, as follows:

²Throughout our analysis, we assume the Lorentzian cutoff surface Σ_r has a cylindrical topology ($\mathbb{R} \times S^1$), for which the Euler characteristic vanishes ($\chi = 0$). Within a fixed topological sector, shifting the finite-cutoff action by a term proportional to $R[\gamma]$ merely adds a topological constant which affects neither the local dynamics nor the variational principle. Furthermore, discarding spatial total derivatives on Σ_r corresponds to dropping codimension-2 corner terms (e.g., at $t \rightarrow \pm\infty$). These can be systematically managed either via corner conditions on $\partial\Sigma_r$ or by absorption into the Y -ambiguity/freedom of the covariant phase space formalism [43,44,67], allowing us to focus strictly on the local codimension-1 radial flow.

$$\Psi_0 = -\frac{\Phi_1}{2\ell^2\Lambda_0}. \quad (4.16)$$

Continuing the iteration, the first two subleading coefficients are determined as

$$\begin{aligned} \Psi_1 &= -\frac{\Phi_1}{4\ell^2\Lambda_0^2}(R[q] - 2\Lambda_1 + 2\Lambda_0\Phi_1), \\ \Psi_2 &= -\frac{1}{\Lambda_0} \left[\frac{\Phi_1^3}{4\ell^2} + \frac{\Phi_1\Phi_2}{4\ell^2} - \frac{\Phi_2}{16}R[q] + \frac{\Phi_1}{16}D^2\Phi_1 + \frac{9\pi^2\ell^2}{c^2}\Phi_1\text{tt} - \frac{\Lambda_1\Phi_1^2}{2\ell^2\Lambda_0} + \frac{\Lambda_1^2 - \Lambda_2\Lambda_0}{2\ell^2\Lambda_0^2}\Phi_1 \right. \\ &\quad \left. + \frac{3}{16} \left(1 + \frac{2}{\ell^2\Lambda_0} \right) \Phi_1^2 R[q] - \frac{\Lambda_1\Phi_1 R[q]}{2\ell^2\Lambda_0^2} + \frac{8 + \ell^2\Lambda_0(2 + \ell^2\Lambda_0)}{64\ell^2\Lambda_0^2} \Phi_1 R[q]^2 \right]. \end{aligned} \quad (4.17)$$

6. Verification via the boundary action expansion

With the iterative solutions in hand, we evaluate the asymptotic expansion of the full boundary action (4.9). To the first nontrivial order, the action becomes

$$S_{\text{bdry}}^{\text{D}}[\Sigma_r] = S_{\text{bdry}}^{\text{D}}[\Sigma] + \int_{\Sigma} d^2x \sqrt{-q} \left[\frac{cr^2}{12\pi\ell^4} + \frac{\ell^2}{r^2} \left(\frac{3\pi\ell^2}{2c} \text{tt} + \frac{c(\Phi_1^2 - \Phi_2)}{24\pi\ell^2} - \frac{c\Phi_1}{48\pi} R[q] \right) \right] + \mathcal{O}\left(\frac{\ell^4}{r^4}\right). \quad (4.18)$$

This expansion perfectly matches the result obtained from the direct holographic dictionary in Eq. (4.7). This agreement provides robust evidence that our local JT ansatz is correct, confirming that the bulk radial lapse function Φ is the unique physical source for the induced JT gravity on the finite-cutoff surface.

C. Induced JT gravity with Neumann boundary conditions

The presence of the term $-\frac{1}{2} \int_{\Sigma_r} d^2x \sqrt{-\gamma} T$ in our ansatz (4.9) warrants a deeper physical clarification. As we will demonstrate, this term is not merely a technical addition but serves as the generator of a canonical transformation between boundary conditions [68,69].

1. Legendre transformation and the Neumann action

In the context of the holographic variational principle, this term allows a transition from the Dirichlet boundary action, $S_{\text{bdry}}^{\text{D}}[\Sigma_r]$, to a new boundary action defined as [45,52,53]

$$S_{\text{bdry}}^{\text{N}}[\Sigma_r] = S_{\text{bdry}}^{\text{D}}[\Sigma_r] + \frac{1}{2} \int_{\Sigma_r} d^2x \sqrt{-\gamma} T. \quad (4.19)$$

The physical significance of $S_{\text{bdry}}^{\text{N}}$ becomes apparent when we evaluate its variation. Recalling that the variation of $S_{\text{bdry}}^{\text{D}}$ is driven by the stress tensor (3.7), the variation of the new action (4.19) takes the form

$$\delta S_{\text{bdry}}^{\text{N}}[\Sigma_r] = \frac{1}{2} \int_{\Sigma_r} d^2x \delta(\sqrt{-\gamma} T^{ab}) \gamma_{ab}. \quad (4.20)$$

This variation confirms that the action $S_{\text{bdry}}^{\text{N}}$ is compatible with Neumann boundary conditions, defined by $\delta(\sqrt{-\gamma} T^{ab}) = 0$. Under this condition, the variation of the action vanishes identically, rendering the variational principle well posed for a fixed stress tensor [57,70].

2. The Neumann-JT formulation

Building on this insight, we can suggestively rewrite our exact solution (4.9) by absorbing the Legendre term into the lhs. This allows us to express the result as an effective Neumann action, as follows:

$$S_{\text{bdry}}^{\text{N}}[\Sigma_r] = S_{\text{bdry}}^{\text{D}}[\Sigma] + \frac{c}{12\pi} \int_{\Sigma_r} d^2x \sqrt{-\gamma} \Psi(R[\gamma] - 2\Lambda(r)). \quad (4.21)$$

3. Physical interpretation

Equation (4.21) is our main result so far, and we pause to discuss its physics. As discussed in Sec. III C, the radial flow generated by the $\text{T}\bar{\text{T}}$ boundary deformation naturally takes us from a Dirichlet boundary condition in the UV to a mixed boundary condition in the IR. Therefore, the radial flow itself does not directly yield a pure Neumann condition. Instead, Eq. (4.21) should be understood as describing a profound connection between the Dirichlet boundary condition in the UV (infinite r) and the Neumann boundary condition in the IR (finite r), with the induced JT-like deformation serving as a bridge between the two. Importantly, this JT-like deformation is *not* merely the integral of the generator of the radial evolution (which is given by the rhs of Eq. (3.5) and leads to the mixed condition). Rather, it inherently incorporates the Legendre

transformation necessary to shift the variational principle. In this picture, the Dirichlet action at infinity acts as the “initial condition,” and the JT bridge naturally yields a Neumann formulation at the finite-radius surface Σ_r . Note that the Neumann boundary condition means that variations of the 2D metric $\delta\gamma_{ab}$ are not restricted by any boundary condition (induced from the 3D bulk description). Having such an unrestricted $\delta\gamma_{ab}$ is precisely what is required by a standard 2D gravity description.

4. On the nature of the induced finite-cutoff action

Before proceeding to the renormalized theory, we pause to discuss the physical meaning of the induced boundary action and the corresponding variational principle. By the holographic dictionary (see, e.g., Refs. [52,53]), formulations such as Eqs. (3.5) or (4.21) represent the Hamilton-Jacobi functional of the bulk, rather than an autonomous off-shell 2D gravity theory. In our setup, due to the radial flow of the boundary conditions, the finite-cutoff variables $(\Phi, \gamma_{ab}, T^{ab})$ are allowed to fluctuate and have variations. We use the terminology of “dynamical fields” strictly in this sense. However, their fluctuations are not independent; they are related to each other through the dynamically generated mixed boundary condition.

Regarding the variational principle, one can study the general variation of the integrated action with respect to these dynamical fields. In fact, this parallels what we have done in Sec. III B, where taking the variation of the differential flow equation (3.5) yields the transverse tensor bulk equation of motion (2.10c). Equations such as Eq. (4.21) are exact radial integrations of the flow, thus their variation inherently yields the integrated version of the bulk equation of motion (3.15), relating the fields at the finite cutoff to their fixed asymptotic boundary values. This dovetails with the standard holographic Hamilton-Jacobi analyses [37,71]. However, there is a crucial subtlety in taking this variation: the dilaton Ψ is not an independent dynamical field; according to Eqs. (4.12) and (4.13), Ψ is a composite functional of the other dynamical fields, and its variation $\delta\Psi$ is entirely induced by their variations. This explains why one may not independently vary Ψ to extract an autonomous 2D gravity equation of motion such as $R[\gamma] - 2\Lambda = 0$.

V. RENORMALIZED HOLOGRAPHIC RG FLOW

In the previous sections, we analyzed the radial flow of the unrenormalized (bare) gravitational action. While this provided a clear local derivation of the induced JT dynamics, a complete holographic description requires handling the infinite volume divergences of AdS spacetime. In this section, we extend our framework to the renormalized holographic action. We demonstrate that the emergence of induced 2D gravity is a robust feature of the theory that survives the standard holographic renormalization

procedure [29,37,72], resulting in a finite and physically well-defined boundary effective action.

A. Renormalized Dirichlet deformation flow equation

1. Renormalized action and stress tensor

Boundary observables are rendered finite by supplementing the Einstein-Hilbert action with the standard holographic counterterm proportional to the boundary volume element [49,73]. The renormalized action for a bulk region $\mathcal{M}_r$ with Dirichlet boundary conditions is

$$\mathcal{S}_{\text{bulk}}^{\text{D}}[\mathcal{M}_r] = \frac{1}{2\kappa_3} \int_{\mathcal{M}_r} d^3x \sqrt{-g} \left(\mathcal{R} + \frac{2}{\ell^2} \right) + \frac{r^2}{\ell^2 \kappa_3} \int_{\Sigma_r} d^2x \sqrt{-\gamma} \left(K - \frac{1}{\ell} \right). \quad (5.1)$$

This inclusion does not alter the Dirichlet nature of the variational principle; rather, it shifts the momentum conjugate to the boundary metric. The resulting renormalized Brown-York energy-momentum tensor—often called the holographic stress tensor—is given by [29,37,49]

$$\mathcal{T}_{ab} = \frac{1}{\kappa_3} \left(K_{ab} - \frac{r^2}{\ell^2} K \gamma_{ab} + \frac{r^2}{\ell^3} \gamma_{ab} \right) = T_{ab} + \frac{c}{12\pi} \frac{r^2}{\ell^4} \gamma_{ab}. \quad (5.2)$$

While the bare tensor T_{ab} diverges as $r \rightarrow \infty$, the holographic tensor $\mathcal{T}_{ab}$ remains finite at the boundary. Specifically, its asymptotic limit is related to the dual QFT stress tensor as $\mathcal{T}_{ab} \rightarrow -t_{ab}$ (cf. (2.17), (2.19), which is compatible with the results in the FG gauge [37,49].

2. Renormalized bulk dynamics

Projecting the Einstein equations into the radial foliation using the renormalized variables (5.2) yields the following set of constrained evolution equations:

$$\mathcal{T} + \frac{c}{24\pi} R + \frac{6\pi\ell^4}{cr^2} \mathcal{T}\mathcal{T} = 0, \quad (5.3a)$$

$$\nabla_b \mathcal{T}_a^b = 0, \quad (5.3b)$$

$$\Phi^{-1} \mathcal{D}_r \mathcal{T}_{ab} - \frac{12\pi\ell^4}{cr^3} \left(\mathcal{T}\mathcal{T}_{ab} + \frac{3}{2} \mathcal{T}\mathcal{T} \gamma_{ab} \right) + \frac{c}{12\pi r} \left(\frac{\nabla_a \nabla_b \Phi}{\Phi} - \frac{\square \Phi}{\Phi} \gamma_{ab} \right) = 0, \quad (5.3c)$$

where $\mathcal{T}\mathcal{T} := \mathcal{T}^{ab} \mathcal{T}_{ab} - \mathcal{T}^2$. These equations demonstrate that the quadratic $\mathcal{T}\mathcal{T}$ structure persists in the renormalized theory, though with radial weights that differ from the bare case.

3. Derivation of the renormalized flow

Following the same logic as in Sec. III A, we evaluate the radial derivative of the renormalized on-shell action by considering its variation under the radial diffeomorphism $\xi = \partial_r$, as follows:

$$\begin{aligned} \frac{d}{dr} \mathcal{S}_{\text{bulk}}^{\text{D}}[\mathcal{M}_r]|_{\text{on-shell}} &= -\frac{\ell^2}{2r^2} \int_{\Sigma_r} \sqrt{-\gamma} \mathcal{T}^{ab} \partial_r \left(\frac{r^2}{\ell^2} \gamma_{ab} \right) \\ &= -\int_{\Sigma_r} \sqrt{-\gamma} \Phi \left(\frac{12\pi\ell^4}{cr^3} \mathcal{T}\mathcal{T} + \frac{\mathcal{T}}{r} \right). \end{aligned} \quad (5.4)$$

Invoking the holographic dictionary, we obtain the renormalized deformation flow equation was used. [13,45,53]

$$\frac{d}{dr} \mathcal{S}_{\text{bdry}}^{\text{D}}[\Sigma_r] = -\int_{\Sigma_r} \sqrt{-\gamma} \Phi \left(\frac{12\pi\ell^4}{cr^3} \mathcal{T}\mathcal{T} + \frac{\mathcal{T}}{r} \right). \quad (5.5)$$

Using the renormalized Hamiltonian constraint (5.3) to replace the $\mathcal{T}\mathcal{T}$ term with the boundary Ricci scalar, we find the equivalent geometric representation of the renormalized flow:

$$\frac{d}{dr} \mathcal{S}_{\text{bdry}}^{\text{D}}[\Sigma_r] = \frac{c}{12\pi r} \int_{\Sigma_r} \sqrt{-\gamma} \Phi \left(R + \frac{12\pi}{c} \mathcal{T} \right). \quad (5.6)$$

Equation (5.6) reveals that even after renormalization, the radial RG flow maintains a JT-like structure, with the lapse Φ sourcing a curvature term R , now accompanied by a trace contribution from the holographic stress tensor.

B. Integrating the renormalized action flow equation

With the renormalized flow equation (5.6) established, we now integrate it to find the exact local boundary action. As in the bare case, we seek a solution where the nontrivial radial lapse Φ seeds a gravitational dilaton field.

1. Renormalized JT ansatz

We consider the following ansatz for the renormalized boundary action at the cutoff surface Σ_r :

$$\mathcal{S}_{\text{bdry}}^{\text{D}}[\Sigma_r] = \mathcal{S}_{\text{bdry}}^{\text{D}}[\Sigma] + \frac{c}{12\pi} \int_{\Sigma_r} d^2x \sqrt{-\gamma} \Psi(R[\gamma] - 2\Lambda(r)). \quad (5.7)$$

Note that, unlike the unrenormalized ansatz (4.9), we do not include an explicit T term here; the counterterms already present in the renormalized bulk action (5.1) naturally account for the trace structure.

2. Dilaton RG flow equation

Substituting this ansatz into the renormalized flow equation (5.6) yields an integral constraint. Following the localization procedure—equating the integrand to the purely topological term $-\frac{c}{24\pi r} R[\gamma]$ —we obtain a first-order linear differential equation for the renormalized dilaton Ψ , as follows:

$$\partial_r \Psi + \mathcal{X} \Psi = \mathcal{S}. \quad (5.8)$$

For the renormalized flow, the drag and source coefficients are given by

$$\mathcal{X} = \frac{1}{r} \frac{4\Lambda[1 + (\frac{6\pi\ell^4}{cr^2} \mathcal{T} - 1)\Phi] + \frac{24\pi\ell^4}{cr^2} \mathcal{T}^{ab} \nabla_a \nabla_b \Phi - 2(\square\Phi + r\Lambda')}{R[\gamma] - 2\Lambda}, \quad (5.9)$$

$$\mathcal{S} = \frac{1}{2r} \frac{(2\Phi - 1)R[\gamma] + \frac{24\pi}{c} \Phi \mathcal{T}}{R[\gamma] - 2\Lambda}. \quad (5.10)$$

The source term $\mathcal{S}$ is now driven by both the curvature R and the holographic trace $\mathcal{T}$, weighted by the lapse function.

3. Asymptotic matching and the UV dilaton

We solve Eq. (5.8) using the same asymptotic expansions for Ψ and Λ as in Eq. (4.15). Near the boundary, the induced action behaves as

$$\mathcal{S}_{\text{bdry}}^{\text{D}}[\Sigma_r] = \mathcal{S}_{\text{bdry}}^{\text{D}}[\Sigma] - \frac{c\Lambda_0}{6\pi} \int_{\Sigma_r} d^2x \sqrt{-q} \Psi_0. \quad (5.11)$$

To ensure that the cutoff action successfully reduces to the undeformed boundary action as $r \rightarrow \infty$, we must set the integration constant to zero: $\Psi_0 = 0$. This is a key difference from the unrenormalized case, reflecting the fact that holographic renormalization has already “cleaned” the UV data. The subleading dilaton coefficients can then be determined iteratively, as follows:

$$\Psi_1 = \frac{1}{8\Lambda_0} \left[\Phi_1 R[q] - \left(\frac{12\pi\ell}{c} \right)^2 \text{tt} \right], \quad (5.12)$$

$$\begin{aligned} \Psi_2 &= \left(\frac{1}{8\Lambda_0} \right)^2 [-4\Lambda_1 + 4\Lambda_0 \Phi_1 + (2 + \ell^2 \Lambda_0) R[q]] \\ &\quad \times \left[\Phi_1 R[q] - \left(\frac{12\pi\ell}{c} \right)^2 \text{tt} \right]. \end{aligned} \quad (5.13)$$

4. Resulting effective action

The asymptotic expansion of the boundary action to the first nontrivial order is given by

$$\mathcal{S}_{\text{bdry}}^{\text{p}}[\Sigma_r] = \mathcal{S}_{\text{bdry}}^{\text{p}}[\Sigma] + \frac{3\pi\ell^4}{cr^2} \int_{\Sigma} d^2x \sqrt{-q} \times \left[\text{tt} - \left(\frac{c}{12\pi\ell} \right)^2 \Phi_1 R[q] \right] + \mathcal{O}(r^{-4}). \quad (5.14)$$

This result is highly significant. In the Fefferman-Graham gauge ($\Phi_1 = 0$), the expansion terminates and becomes exact, successfully recovering the seminal T $\bar{\text{T}}$ deformation as the unique finite-cutoff modification [12,74]. However, in our general setup with a nontrivial lapse, we see that the boundary QFT is modified not only by the T $\bar{\text{T}}$ operator but also by a dynamical coupling between the lapse fluctuation Φ_1 and the boundary curvature—precisely the signature of induced JT gravity. It is also instructive to compare the unrenormalized (4.18) and renormalized (5.14) results. The renormalized case, as expected, does not involve the r^2 term, and it does not have the 2D cosmological constant. Note also that the relative coefficients of tt and $\Phi_1 R[q]$ differ by a factor of 2. As a final remark, in order to have a 2D JT gravity with standard conventions, one could have defined Φ_1 with a negative sign.

VI. BEYOND DIRICHLET: A GENERALIZED FAMILY OF BOUNDARY CONDITIONS

Our analysis so far has focused on the holographic RG flow originating from strict Dirichlet boundary conditions at the AdS boundary. However, the GR from RG framework is not restricted to this specific choice. To demonstrate the robustness of the RG-induced gravity mechanism, we generalize our framework by introducing a one-parameter family of boundary conditions. We show that for this entire family, the radial evolution is universally governed by an induced JT-like dynamics, albeit with shifted effective couplings.

A. Nonrenormalized boundary conditions

1. A generalized variational principle

We consider a family of bulk theories parametrized by a real number w . This family is defined by modifying the weight of the Gibbons-Hawking-York term in the bulk action, as follows:

$$\mathcal{S}_{\text{bulk}}^w[\mathcal{M}_r] = \frac{1}{2\kappa_3} \int_{\mathcal{M}_r} d^3x \sqrt{-g} \left(\mathcal{R} + \frac{2}{\ell^2} \right) + \frac{wr^2}{\kappa_3 \ell^2} \int_{\Sigma_r} d^2x \sqrt{-\gamma} K. \quad (6.1)$$

The parameter w characterizes the specific boundary

condition imposed on the cutoff surface. Setting $w = 1$ recovers the standard Dirichlet action (2.6), while other values of w correspond to different mixtures of the metric and its conjugate momentum in the variational principle [45,46,52,57,65,75].

2. Generalized deformation flow

Following the diffeomorphism-based derivation used in Sec. III A, we evaluate the radial evolution of the boundary action $\mathcal{S}_{\text{bdry}}^w$. The resulting deformation flow equation takes the form

$$\frac{d}{dr} \mathcal{S}_{\text{bdry}}^w[\Sigma_r] = -\frac{12\pi\ell}{c} \left(\frac{\ell}{r} \right)^3 \int_{\Sigma_r} d^2x \sqrt{-\gamma} \Phi \times \left[w\text{TT} + (1-w) \left(\frac{c}{12\pi\ell} \right)^2 \frac{2r^4}{\ell^6} \right]. \quad (6.2)$$

As expected, for $w = 1$, this reduces to the Dirichlet T $\bar{\text{T}}$ flow equation (3.4). Utilizing the bulk Hamiltonian constraint, we can recast this into its equivalent gravitational formulation:

$$\frac{d}{dr} \mathcal{S}_{\text{bdry}}^w[\Sigma_r] = \frac{cw}{12\pi r} \int_{\Sigma_r} d^2x \sqrt{-\gamma} \Phi \left[R[\gamma] + \left(2 - \frac{1}{w} \right) \frac{2r^2}{\ell^4} \right]. \quad (6.3)$$

Remarkably, the flow maintains the structure of the JT gravity action for all $w \neq 0$. The effect of modifying the boundary conditions is entirely captured by a rescaling of the effective Newton's constant ($G_{\text{eff}} \sim 1/w$) and a shift in the effective cosmological constant of the boundary theory.

3. Integration and the shifted dilaton

To integrate this generalized flow, we propose a modified ansatz for the boundary action:

$$\mathcal{S}_{\text{bdry}}^w[\Sigma_r] = \mathcal{S}_{\text{bdry}}^w[\Sigma] - \frac{(2w-1)}{2} \int_{\Sigma_r} d^2x \sqrt{-\gamma} T + \frac{c}{12\pi} \int_{\Sigma_r} d^2x \sqrt{-\gamma} \Psi (R[\gamma] - 2\Lambda(r)). \quad (6.4)$$

Substituting this into the flow equation (6.3) yields a differential equation for Ψ identical in form to Eq. (4.12), with the same drag and source coefficients (4.13). The primary distinction lies in the matching conditions at the UV boundary. To ensure the correct asymptotic behavior and maintain consistency with the undeformed theory at infinity, the leading-order dilaton coefficient must be

$$\Psi_0 = -\frac{(2w-1)\Phi_1}{2\ell^2\Lambda_0}. \quad (6.5)$$

Since all subleading dilaton coefficients $\Psi_{i \geq 1}$ are determined iteratively from Ψ_0 , they are modified by the boundary condition parameter w . This demonstrates that the induced 2D gravity is a universal feature of the AdS_3 radial foliation, with the choice of boundary condition merely selecting a specific trajectory within the space of scalar-tensor effective theories.

4. The critical case $w=1/2$ and conformal boundary conditions

An illuminating special case arises when we set $w = 1/2$, a choice that famously corresponds to imposing conformal boundary conditions [61,64–66,76–81] (which, uniquely in 3D gravity, also coincides with the Neumann boundary condition [45,46]). At this precise value, the effective boundary cosmological constant term proportional to $(2 - 1/w)$ in the flow equation (6.3) exactly vanishes. The deformation flow thus elegantly reduces to a purely topological-like curvature coupling,

$$\frac{d}{dr} \mathcal{S}_{\text{bdry}}^{w=1/2}[\Sigma_r] = \frac{c}{24\pi r} \int_{\Sigma_r} d^2x \sqrt{-\gamma} \Phi R[\gamma], \quad (6.6)$$

and concurrently, the leading-order dilaton coefficient trivializes, $\Psi_0 = 0$.

Furthermore, the value $w = 1/2$ acts as a critical transition point in the parameter space of boundary conditions. As w crosses this threshold, the dilaton seed coefficient Ψ_0 (6.5) undergoes a sign reversal. In the framework of JT gravity, the sign of the dilaton dictates the overall sign of the effective gravitational action; thus, crossing $w = 1/2$ formally flips the sign of the boundary metric's kinetic terms. Generally, an uncompensated sign change in the effective gravitational action signals the onset of thermodynamic instabilities, highlighting the profound role that the choice of boundary condition plays in securing a well-posed initial-boundary value problem. However, in our setup, this does not represent a fatal pathology. As seen in Eq. (6.5), the overall sign of Ψ_0 depends not only on $(2w - 1)$ but also on the integration constant Λ_0 , which represents the leading-order 2D cosmological constant. One can systematically exploit this freedom to absorb the sign flip: by choosing $\Lambda_0 > 0$ for $w < 1/2$ and $\Lambda_0 < 0$ for $w > 1/2$, one maintains a strictly consistent overall sign for the induced boundary theory across the threshold. A more exhaustive analysis of how specific boundary conditions govern the thermodynamic stability of the finite-cutoff theory is postponed to future work.

B. Renormalized boundary conditions

Finally, we extend our analysis to a generalized family of *renormalized* boundary conditions [52,54]. This family allows us to explore the RG flow of theories that are not purely Dirichlet but involve a specific mixture of the metric and the holographic stress tensor.

1. Generalized renormalized action

We introduce a one-parameter family of bulk theories, parametrized by w , defined by the following bulk action:

$$\mathcal{S}_{\text{bulk}}^w[\mathcal{M}_r] = \frac{1}{2\kappa_3} \int_{\mathcal{M}_r} d^3x \sqrt{-g} \left(\mathcal{R} + \frac{2}{\ell^2} \right) + \frac{r^2}{\ell^2 \kappa_3} \int_{\Sigma_r} d^2x \sqrt{-\gamma} \left(wK - \frac{(2w-1)}{\ell} \right). \quad (6.7)$$

Crucially, this family possesses a well-defined variational principle for all w . Furthermore, unlike the unrenormalized cases discussed in Sec. VI A, this construction ensures that the on-shell action and all derived physical quantities remain finite as the regulator is removed ($r \rightarrow \infty$). This systematic subtraction of infinite volume divergences is what justifies the designation of this family as renormalized [45,52,54].

Invoking the framework of renormalized holography at a finite radial cutoff, the dual boundary action $\mathcal{S}_{\text{bdry}}^w[\Sigma_r]$ satisfies a generalized radial deformation flow equation:

$$\frac{d}{dr} \mathcal{S}_{\text{bdry}}^w[\Sigma_r] = - \int_{\Sigma_r} d^2x \sqrt{-\gamma} \Phi \left(\frac{12\pi\ell^4}{cr^3} w\mathcal{T}\mathcal{T} + \frac{\mathcal{T}}{r} \right). \quad (6.8)$$

By employing the renormalized Hamiltonian constraint (5.3), we recast this flow into its equivalent gravitational formulation:

$$\frac{d}{dr} \mathcal{S}_{\text{bdry}}^w[\Sigma_r] = \int_{\Sigma_r} d^2x \sqrt{-\gamma} \Phi \left[\frac{cw}{12\pi r} R[\gamma] + (2w-1) \frac{\mathcal{T}}{r} \right]. \quad (6.9)$$

2. Exact integration and UV limit

To integrate the flow (6.9), we propose an ansatz that includes a holographic trace term to account for the w -dependent weighting, as follows:

$$\mathcal{S}_{\text{bdry}}^w[\Sigma_r] = \mathcal{S}_{\text{bdry}}^w[\Sigma] + (1-w) \int_{\Sigma_r} d^2x \sqrt{-\gamma} \mathcal{T} + \frac{c}{12\pi} \int_{\Sigma_r} d^2x \sqrt{-\gamma} \Psi(R[\gamma] - 2\Lambda(r)). \quad (6.10)$$

Substituting this into the flow equation leads back to the same dilaton evolution equation (5.8) derived for the Dirichlet case. Consequently, the integration constant must again be $\Psi_0 = 0$ to match the undeformed dual theory at infinity. The first nontrivial subleading order of the

effective action is then

$$\begin{aligned} \mathcal{S}_{\text{bdry}}^w[\Sigma_r] &= \mathcal{S}_{\text{bdry}}^w[\Sigma] + \frac{3\pi\ell^4}{cr^2} \int_{\Sigma} d^2x \sqrt{-q} \\ &\times \left[(2w-1)\text{tt} - \left(\frac{c}{12\pi\ell} \right)^2 \Phi_1 R[q] \right] + \mathcal{O}(r^{-4}). \end{aligned} \quad (6.11)$$

3. The nontrivial flow of renormalized conformal boundary conditions

The specific choice $w = 1/2$ corresponds to imposing the renormalized conformal boundary condition, which coincides with the renormalized Neumann boundary condition in 3D gravity [45,46]. Setting $w = 1/2$ yields profound simplifications. In the exact flow equation (6.9), the explicit dependence on the holographic trace $\mathcal{T}$ perfectly cancels out, leaving a radial evolution driven solely by the curvature term.

Consequently, in the expansion (6.11), the matter-sector tt deformation completely drops out. If one were to enforce the standard Fefferman-Graham gauge by rigidly fixing the lapse ($\Phi_i = 0$), the expansion formally terminates and the action remains entirely unchanged, yielding exactly $\mathcal{S}_{\text{bdry}}^{w=1/2}[\Sigma_r] = \mathcal{S}_{\text{bdry}}^{w=1/2}[\Sigma]$. This perfectly recovers the observation made in Refs. [45,46] that renormalized conformal/Neumann boundary conditions do not run under the standard holographic RG flow.

However, our generalized framework reveals a striking new perspective: moving beyond the FG gauge by unfixing the radial lapse demonstrates that this boundary condition *does* in fact flow. Remarkably, this flow is no longer driven by the standard fluidic tt operator, but is strictly mediated by the emergent JT-gravity geometric deformation $\Phi_1 R[q]$. This encapsulates the essence of the GR from RG mechanism: the radial lapse activates a purely gravitational RG flow even for boundary conditions that are otherwise completely stable against matter-sector deformations. As we will discuss in Sec. VII, this purely geometric flow has a beautiful physical interpretation as the macroscopic manifestation of boundary fluctuations, whereas the effects of the boundary condition and w appear in the tt operator and in how it gravitates.

4. Universality via Legendre transformations

An insightful perspective on the ansatz (6.10) emerges through the lens of canonical transformations [52,53,55,68]. The second term on the rhs acts as the generator of a Legendre transformation connecting the Dirichlet and w -type boundary conditions at the finite cutoff surface Σ_r , as follows:

$$\mathcal{S}_{\text{bdry}}^w[\Sigma_r] = \mathcal{S}_{\text{bdry}}^D[\Sigma_r] + (1-w) \int_{\Sigma_r} d^2x \sqrt{-\gamma} \mathcal{T}. \quad (6.12)$$

Taking the asymptotic limit $r \rightarrow \infty$, the Hamiltonian constraint ensures that the integral of $\mathcal{T}$ becomes purely topological (proportional to $\int \sqrt{-q} R[q]$). Thus, we establish the equivalence of the on-shell boundary actions at spatial infinity, as follows:

$$\mathcal{S}_{\text{bdry}}^w[\Sigma] = \mathcal{S}_{\text{bdry}}^D[\Sigma]. \quad (6.13)$$

This establishes a profound result: the induced JT gravity action we have derived is a universal feature of the AdS_3 radial flow. Any theory in the w -family can be systematically reconstructed starting from the Dirichlet result and applying the appropriate Legendre transformation. This underscores the fact that the emergence of 2D gravity is an intrinsic geometric property of the radial foliation, independent of the specific boundary condition chosen to anchor the flow.

VII. DISCUSSION AND OUTLOOK

In this work, we have provided a concrete and analytical realization of the GR from RG program [1,2] in the context of 2D gravity. By tracking the holographic radial evolution of a generic 2D CFT action, we demonstrated that JT gravity is not merely a phenomenological construct but an inevitable low-energy effective description induced by the renormalization group flow.

A. Beyond the Fefferman-Graham gauge

A distinctive feature of our 2D derivation, compared to the higher-dimensional case [1], is the absolute necessity of going beyond the standard FG gauge. In four dimensions, the holographic RG flow induces an Einstein-Hilbert term that is inherently dynamical. In two dimensions, however, the Einstein-Hilbert action is purely topological. If one strictly adheres to the FG gauge ($\Phi = 1$), the induced action at the cutoff surface remains a topological invariant, failing to capture the rich gravitational dynamics characteristic of the IR. Our results show that the “activation” of 2D gravity is explicitly tied to the radial lapse function Φ . By retaining Φ as a generic degree of freedom, the topological term $\int R$ is upgraded to the dynamical JT term $\int \Psi R$, where the lapse effectively seeds the dilaton.

B. Induced gravity as boundary fluctuations

This leads to a deep geometric and physical question: since the lapse function Φ can be fixed to unity by a bulk coordinate transformation, what is the physical origin of this induced gravity? To see this clearly, consider our starting point: a coordinate system (r, x^a) where the radial lapse $\Phi(r, x^a) \neq 1$ and the physical cutoff Σ_r is defined as a surface of constant radius r .

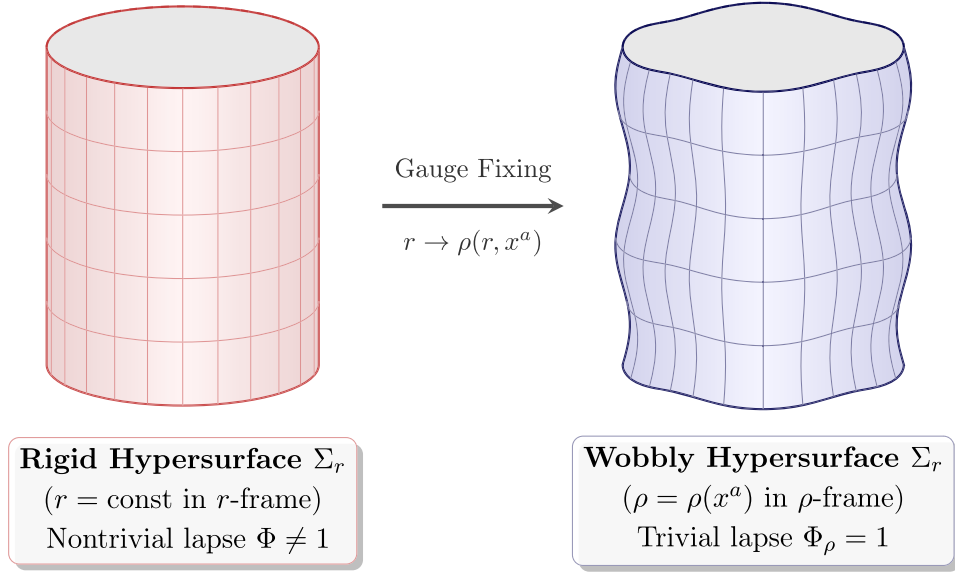


FIG. 2. Geometric interpretation of the induced gravity. Left: In the coordinate system (r, x^a) , the physical cutoff Σ_r is a constant-radius hypersurface. Here, the nontrivial radial lapse $\Phi \neq 1$ manifests as a dynamical dilaton field. Right: After a radial coordinate transformation to the Fefferman-Graham frame (ρ, x^a) , the lapse is fixed to unity ($\Phi_\rho = 1$). However, the physical surface Σ_r is no longer at a constant coordinate; it acquires a wobbly profile $\rho(x^a)$, encoding the gravitational reparametrization modes of the boundary.

One can always perform a radial coordinate transformation $r \rightarrow \rho(r, x^a)$ to reach a new frame (ρ, x^a) where the lapse is set to unity ($\Phi_\rho = 1$), corresponding to the standard FG gauge. However, this transformation redefines the meaning of the radial scale. Crucially, the original physical cutoff Σ_r (which was a uniform, constant-coordinate hypersurface in the r -frame) is no longer a surface of constant coordinate in the new ρ -frame. Instead, as illustrated in Fig. 2, the physical surface Σ_r is mapped to a nontrivial, wobbly profile $\rho = f(x^a)$.

This provides a beautiful and unified picture of emergent gravity in 2D: *induced gravity is a manifestation of the fluctuations of the holographic cutoff surface*. In the rigid (r, x^a) frame, the boundary is straight, and the gravitational degrees of freedom appear as a dynamical dilaton field Ψ sourcing the curvature. In the FG (ρ, x^a) frame, the lapse is trivial, but the gravitational dynamics are now encoded in the reparametrization modes (the “wobble”) of the physical cutoff surface relative to the FG scale [18,19,82]. Our derivation confirms that the induced JT gravity is not an auxiliary structure, but an intrinsic description of the boundary’s capacity to fluctuate—a feature that is made manifest by the bulk radial lapse under holographic RG flow.

C. Unfreezing the metric

A persistent conceptual hurdle in the emergent gravity paradigm is the question of how a fixed background metric in the UV can become a dynamical, fluctuating field in the IR. We have addressed this through the *RG flow of boundary conditions* [1,2]. We showed that a rigid Dirichlet boundary condition imposed at asymptotic

infinity ($\delta\gamma_{ab} = 0$) does not remain Dirichlet at finite scales. Instead, it systematically evolves into a mixed Dirichlet-Neumann condition, where the metric fluctuations are functionally tied to the energy-momentum tensor. This “unfreezing” mechanism suggests that the transition from a nongravitating QFT to a gravitating theory happens as a result of integrating out UV degrees of freedom, and that the irrelevant $\text{T}\bar{\text{T}}$ deformation can be recast as boundary gravity.

D. Dirichlet-to-mixed flow and the Neumann bridge

An intriguing feature of this construction is the natural transition from a fixed Dirichlet boundary condition in the UV to a dynamically mixed condition in the IR. As explored in Sec. IV C, the exact integration of the deformation flow inherently generates the trace term needed to perform a Legendre transformation, allowing us to elegantly repack the finite-cutoff theory into a Neumann polarization. This equivalent formulation plays a functional role in understanding the emergence of gravity: in the Neumann description, the variational principle does not tether the induced metric to any fixed background structure. Instead, the boundary metric is treated as a completely free, dynamically fluctuating field, which is a fundamental requirement for any standard theory of gravity. In this sense, the holographic RG flow does more than just induce the gravitational action; it establishes the appropriate variational framework for the metric to function as a dynamical entity.

In conclusion, the GR from RG program suggests that the search for a quantum theory of gravity by quantizing the

metric may be fundamentally misplaced. If the metric and its dynamics are collective IR descriptions of QFT degrees of freedom, the graviton is a collective excitation akin to a phonon in a fluid. Our derivation of JT gravity from the 2D CFT lapse provides a robust, lower-dimensional laboratory for this emergent viewpoint, suggesting that gravity is not a fundamental force, but rather the hydrodynamics of quantum field theory under coarse graining.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX: USEFUL RELATIONS

In this Appendix, we collect several identities and radial evolution equations for the geometric and boundary variables. These relations are essential for verifying the consistency of the holographic RG flow and for performing the exact integration of the boundary action.

1. Evolution of metric components

Using the radial foliation and the definition of the extrinsic curvature (2.5), the radial evolution of the determinant and the inverse induced conformal metric is given by

$$\begin{aligned}\mathcal{D}_r\sqrt{-\gamma} &= -\frac{2}{r}\sqrt{-\gamma} - \frac{\kappa_3\ell^3}{r^3}\sqrt{-\gamma}\Phi T, \\ \mathcal{D}_r\gamma^{ab} &= \frac{2}{r}\gamma^{ab} - \frac{2\kappa_3\ell^3}{r^3}\Phi(T^{ab} - T\gamma^{ab}).\end{aligned}\quad (\text{A1})$$

2. Stress tensor and curvature dynamics

Combining the bulk Einstein equations with the identities above, we derive the radial evolution for the boundary-weighted stress tensor and curvature scalars, as follows:

$$\begin{aligned}\mathcal{D}_r(\sqrt{-\gamma}T^{ab}) &= \sqrt{-\gamma}\left[\frac{2}{r}T^{ab} + \frac{\ell\Phi}{2r\kappa_3}R\gamma^{ab} \right. \\ &\quad \left. - \frac{\ell}{r\kappa_3}(\nabla^a\nabla^b\Phi - \gamma^{ab}\square\Phi)\right],\end{aligned}\quad (\text{A2a})$$

$$\mathcal{D}_r(\sqrt{-\gamma}T) = -\sqrt{-\gamma}\frac{\ell\Phi}{r\kappa_3}\left(R + \frac{4r^2}{\ell^4} - \frac{\square\Phi}{\Phi}\right),\quad (\text{A2b})$$

$$\mathcal{D}_rT = \frac{2T}{r} - \frac{\ell\Phi}{r\kappa_3}\left[R + \frac{4r^2}{\ell^4} - \left(\frac{\ell\kappa_3T}{r}\right)^2 - \frac{\square\Phi}{\Phi}\right],\quad (\text{A2c})$$

$$\begin{aligned}\mathcal{D}_r(\sqrt{-\gamma}R) &= \frac{2\ell^3\kappa_3}{r^3}\sqrt{-\gamma}T^{ab}\nabla_a\nabla_b\Phi \\ &= \frac{2\ell^3\kappa_3}{r^3}\nabla_a(\sqrt{-\gamma}T^{ab}\nabla_b\Phi).\end{aligned}\quad (\text{A2d})$$

3. Evolution of the JT block

The specific combination of terms that constitutes the core of the induced JT gravity action evolves as

$$\mathcal{D}_r\left[\sqrt{-\gamma}\left(R + \frac{2r^2}{\ell^4}\right)\right] = 2\kappa_3\sqrt{-\gamma}\left(-\frac{\Phi T}{\ell r} + \frac{\ell^3}{r^3}T^{ab}\nabla_a\nabla_b\Phi\right).\quad (\text{A3})$$

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