

Comparing strange and nonstrange quark stars within resummed QCD at NLO

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We employ the renormalization group optimized perturbation theory (RGOPT) resummation method to evaluate the equation of state (EoS) for strange ($N_f = 2 + 1$) and nonstrange ($N_f = 2$) cold quark matter at next-to-leading order. This allows us to obtain the mass-radius relation for pure quark stars and compare the results with the predictions from perturbative QCD (pQCD) at next-to-next-to-leading order. Choosing the renormalization scale to generate maximum star masses of order $M = 2 - 2.6M_\odot$, we show that the RGOPT can produce mass-radius curves compatible with the masses and radii of some recently observed pulsars, regardless of their strangeness content. The scale values required to produce the desired maximum masses are higher in the strange scenario since the EoS is softer in this case. The possible reasons for such behavior are discussed. Our results also show that, as expected, the RGOPT predictions for the relevant observables are less sensitive to scale variations than those furnished by pQCD.

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I. INTRODUCTION

The recent observational data of nearly two solar mass pulsars [1–4] suggests that quark matter (QM) may be present inside compact stellar objects (CSO). The theoretical description of those objects requires the knowledge of the quantum chromodynamics (QCD) equation of state (EoS) which can be determined by means of effective models [5], Bayesian inference [6–14], as well as perturbative approximations to QCD, among other possibilities. The vast majority of those applications take the Bodmer-Witten hypothesis [15–17] into account so that both, quark and neutron stars, are generally described in terms of strange quark matter (SQM), see for instance [18]. However, although mostly ignored in the literature, the intriguing possibility that quark matter may not be strange,

so that two-flavor quark stars with a larger maximum mass can exist, has also been advocated [19]. Inspired by this argument, the authors of Ref. [20] have used an alternative self-consistent mean field approximation to study the EoS of cold dense matter within the framework of the two-flavor Nambu–Jona-Lasinio model (NJL) [21,22] (in recent years, more works exploring the properties of nonstrange quark matter (NSQM) have come out [23–32]). The results indicate that this approach can generate an EoS which allows for the existence of two-flavor quark stars whose maximum masses can be of order $2M_\odot$.

Nevertheless, it would be legitimate to argue that such a result could be an artifact of the model approximation adopted in Ref. [20]. After all, when compared to QCD, the (effective) NJL model has some limitations which include the absence of asymptotic freedom and nonrenormalizability. On the other hand, the determination of the QCD EoS for cold and compressed matter, from *ab initio* evaluations, presents further issues which have not yet been circumvented. For instance, within this regime lattice QCD (LQCD) simulations are still plagued by the infamous sign problem [33,34] while perturbative QCD (pQCD) applications are only reliable at extremely high baryon densities, of order $n_B \sim 40n_0$ [35–38] ($n_0 = 0.16 \text{ fm}^{-3}$), where asymptotic freedom allows for weak coupling expansions. Regarding pQCD it is important to mention

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that when the modified minimal subtraction ($\overline{\text{MS}}$) renormalization scale (Λ) is taken at the conventional “central” value, $2(\mu_u + \mu_d + \mu_s)/3$, with μ_f the quark flavor chemical potentials, the next-to-leading order (NLO) pQCD predicts quark star (QS) masses above $2M_\odot$ [39] and below $2M_\odot$ at next-to-next-to-leading order (NNLO) [35,40–43].

Although the cold dense pQCD NNLO pressure is formally perturbatively renormalization group (RG) invariant, the higher order residual scale dependence of pQCD results [35–37] is still rather sizeable, conventionally taken to lie between half and twice the central scale choice value. Quite recently, pQCD calculations reached next-to-next-to-next-to-leading order (NNNLO) [44–47] within the massless quark approximation, and all orders in the leading soft logarithms approximation [48], producing in such cases significant residual scale dependence reductions. However, accounting for the strange quark mass effects, sizeable at moderate and low μ values, is very involved and presently limited to NNLO, thus impinging on scale uncertainties reductions. In order to tackle the scale dependence problem, an alternative method, which better incorporates RG properties within (variationally) optimized perturbative evaluations, has been proposed [49]. This *renormalization group optimized perturbation theory* (RGOPT) technique has been successfully used, at vanishing temperatures and densities, to calculate quantities such as the QCD scale $\Lambda_{\overline{\text{MS}}}$ and strong coupling α_s [49], and the quark condensate up to four- and five-loop orders [50,51]. Later, the RGOPT was applied to scalar field theories at finite temperatures [52–55], indicating that the method greatly improves the scale dependence in thermal theories, when compared to *screened perturbation theory* (SPT) [56,57] and *hard thermal loop perturbation theory* (HTLpt) [57–59] based on the HTL effective theory [60]. The technique was also employed to describe cold and dense QCD [61], as well as the quark sector of QCD at finite temperatures and baryonic densities [62,63]. Once again, the RGOPT predictions display a residual scale dependence which is smaller than those furnished by pQCD and HTLpt. Recently, some of the present authors have used the RGOPT EoS for QCD in the chiral limit ($m_u = m_d \equiv 0$) to describe nonstrange quark stars (NSQS) obtaining encouraging results at NLO [32]. In particular, when the renormalization scale is appropriately chosen, the RGOPT resummation of the perturbative series is able to describe several observational data, specially from pulsars PSR J0030 + 0451 [64,65], PSR J0740 + 6620 [66–68], and the light compact object identified in the middle of a supernova remnant, HESS J1731-347 [69]. Note that recently a different approach was considered to constrain the scale X within a model, possessing a bag parameter constrained by LQCD, which describes an isospin-dense matter system [70].

Here, our major goal is to investigate how the inclusion of strangeness affects the $N_f = 2$ results obtained in Ref. [32]. In the present $N_f = 2 + 1$ case, the introduction

of a finite (strange) quark mass (m_s) modifies the RG operator introducing an extra scale dependence, through the running of $m_s(\Lambda)$, which needs to be taken into account by the resummation procedure. This is accordingly a nontrivial extension with respect to the determination of the RGOPT EoS for massless quarks at $N_f = 3$ [61–63] and $N_f = 2$ [32]. Following the usual procedure, here we will also choose the scale to be density dependent so that it can be written in terms of the baryonic chemical potential¹ (μ_B) as $\Lambda = X\mu_B/3$, where X represents a free parameter that can be adjusted in order to reproduce the desired maximum mass value for a given family of stars. Then, the QS properties determined from the novel $N_f = 2 + 1$ RGOPT EoS will be compared with those previously obtained with the same method when $N_f = 2$ [32] as well as with those furnished by $N_f = 2 + 1$ pQCD at NNLO [35,36], for the corresponding flavor species. We stress that our approach is not a fixed order perturbative calculation but actually resumming a certain class of higher order contributions, indeed in the massless quark approximation the NLO RGOPT pressure is definitely numerically closer to NNLO pQCD than to NLO pQCD [61], and the latter would be a poor approximation to consider compared to the state-of-the-art NNLO pQCD. Therefore, in the present case with $m_s \neq 0$ we also consider appropriate to compare NLO RGOPT with NNLO pQCD. Since α_s runs with $\Lambda = X\mu_B/3$ the value of X needed to produce a massive quark star will enable us to compare the coupling strength associated with each different number of flavors. This comparison will then allow us to draw further conclusions about the reliability of the distinct EoS provided by the RGOPT at NLO.

The work is organized as follows. In the next section we present the perturbative result for QCD at NLO considering the case of quarks with a generic finite mass parameter. The RGOPT resummation setup is presented in Sec. III. In Sec. IV we discuss the modifications required to assure thermodynamic consistency as well as charge neutrality and β equilibrium. The RGOPT and pQCD results for $N_f = 2$ and $N_f = 2 + 1$ are compared in Sec. V while Sec. VI contains our final conclusions.

II. PERTURBATIVE QCD PRESSURE FOR $N_f = 2 + 1$ AT NLO

Let us start by reviewing the complete order- α_s massive quark perturbative pressure, at vanishing temperatures and finite chemical potentials, in the case of symmetric matter ($\mu_d = \mu_u = \mu_s \equiv \mu$). By adding the massive vacuum ($\mu = 0$) results from Ref. [50] to the in-medium ($\mu \neq 0$) results from Refs. [39,71,72] one can write the *per flavor* pressure, at NLO, as [61]

¹In this situation the pressure needs to be modified to ensure thermodynamic consistency.

$$\begin{aligned}
P_f^{\text{PT}}(m_f, \mu) = & -N_c \frac{m_f^4}{8\pi^2} \left(\frac{3}{4} - L_m \right) + \Theta(\mu^2 - m_f^2) \frac{N_c}{12\pi^2} \left[\mu p_F \left(\mu^2 - \frac{5}{2} m_f^2 \right) + \frac{3}{2} m_f^4 \ln \left(\frac{\mu + p_F}{m_f} \right) \right] \\
& - \frac{g^2 d_A}{4(2\pi)^4} m_f^4 \left(3L_m^2 - 4L_m + \frac{9}{4} \right) - \Theta(\mu^2 - m_f^2) \frac{g^2 d_A}{4(2\pi)^4} \left\{ 3 \left[m_f^2 \ln \left(\frac{\mu + p_F}{m_f} \right) - \mu p_F \right]^2 - 2p_F^4 \right\} \\
& - \Theta(\mu^2 - m_f^2) \frac{g^2 d_A}{4(2\pi)^4} m_f^2 (4 - 6L_m) \left[\mu p_F - m_f^2 \ln \left(\frac{\mu + p_F}{m_f} \right) \right], \tag{2.1}
\end{aligned}$$

where $p_F = (\mu^2 - m_f^2)^{1/2}$ is the Fermi momentum, $L_m = \ln(m_f/\Lambda)$, $d_A = N_c^2 - 1$, Λ is the $\overline{\text{MS}}$ arbitrary renormalization scale, while Θ represents the Heaviside function, and $g^2 \equiv 4\pi\alpha_s$. Being independent of the medium, the pure vacuum contributions $\propto m^4$, although originally present in the basic calculation, are often simply discarded in the literature. However, since renormalization properties essentially depend on vacuum contributions, the latter actually play a crucial role in our construction based on RG properties, as will be clear below. It is often useful to further simplify Eq. (2.1) in order to take the relevant case $\mu > m$ into account. It then reads

$$\begin{aligned}
P_f^{\text{PT}}(m_f, \mu) = & \frac{N_c}{12\pi^2} \left[\mu p_F \left(\mu^2 - \frac{5}{2} m_f^2 \right) + \frac{3}{2} m_f^4 \left(L_\mu - \frac{3}{4} \right) \right] \\
& - \frac{g^2 d_A}{4(2\pi)^4} \left[m_f^4 \left(3L_\mu^2 - 4L_\mu + \frac{1}{4} \right) + \mu^2(\mu^2 + m_f^2) + m_f^2 \mu p_F (4 - 6L_\mu) \right], \tag{2.2}
\end{aligned}$$

where

$$L_\mu \equiv \ln[(\mu + p_F)/\Lambda]. \tag{2.3}$$

Note, however, that Eqs. (2.1), (2.2) are not perturbatively RG invariant, since the renormalization scale $\ln(\Lambda)$ enters the LO $\mathcal{O}(g^0)$ contribution. This is inherently related to the extra vacuum energy UV divergences of any massive theory, but perturbative RG invariance can be consistently recovered as specified next.

III. RGOPT RESUMMATION SETUP

Before applying the RGOPT prescription one must obtain a (perturbatively) RG invariant (RGI) pressure, which can be achieved by adding additional zero-point contributions to the pressure given by Eq. (2.1). The latter prescription can then be written as [50,61]

$$P_f^{\text{PT}}(m_f, \mu) \rightarrow P_f^{\text{RGI}}(m_f, \mu) = P_f^{\text{PT}}(m_f, \mu) - m_f^4 \sum_k s_k g^{2k-2}, \tag{3.1}$$

so that the resulting pressure becomes scale independent at a given perturbative order, up to neglected higher orders. The coefficients s_k are determined by applying the massive RG operator²

²Note that the s_i coefficients are also related to the so-called vacuum energy anomalous dimension coefficients [73,74], as discussed in more details, e.g., in Ref. [51].

$$\Lambda \frac{d}{d\Lambda} = \Lambda \frac{\partial}{\partial \Lambda} + \beta(g^2) \frac{\partial}{\partial g^2} - \gamma_m(g^2) m_f \frac{\partial}{\partial m_f}, \tag{3.2}$$

at successive perturbative orders. One then obtains [61]

$$s_0 = -N_c [(4\pi)^2 (b_0 - 2\gamma_0)]^{-1}, \tag{3.3}$$

and

$$s_1 = -\frac{N_c}{4} \left[\frac{b_1 - 2\gamma_1}{4(b_0 - 2\gamma_0)} - \frac{1}{12\pi^2} \right]. \tag{3.4}$$

In our notation, the coefficients of the β and γ_m RG functions are given by

$$\beta(g^2 \equiv 4\pi\alpha_s) = -2b_0 g^4 - 2b_1 g^6 + \mathcal{O}(g^8), \tag{3.5}$$

and

$$\gamma_m(g^2) = \gamma_0 g^2 + \gamma_1 g^4 + \mathcal{O}(g^6), \tag{3.6}$$

where

$$b_0 = \frac{1}{(4\pi)^2} \left(11 - \frac{2}{3} N_f \right), \tag{3.7}$$

$$b_1 = \frac{1}{(4\pi)^4} \left(102 - \frac{38}{3} N_f \right), \tag{3.8}$$

$$\gamma_0 = \frac{1}{2\pi^2}, \tag{3.9}$$

and

$$\gamma_1^{\overline{\text{MS}}} = \frac{1}{8(2\pi)^4} \left(\frac{202}{3} - \frac{20}{9} N_f \right). \quad (3.10)$$

Generally, the RG OPT pressure ($P^{\text{RG OPT}}$) is then obtained by implementing the following steps (see, e.g., Refs. [49,61] for a review):

- (1) First, one modifies the free and interaction terms in the (renormalized) Lagrangian density by performing the replacements

$$m_f \rightarrow m_f + \eta(1 - \delta)^a, \quad g^2 \rightarrow \delta g^2, \quad (3.11)$$

most conveniently within the renormalized RGI massive pressure, Eq. (3.1). In Eq. (3.11) δ is the new expansion parameter and the exponent a is specified below. In QCD applications, m_f represents the physical (current) quark mass, while η represents an *arbitrary* mass parameter, whose optimal value is subsequently fixed by RG properties and a variational stationary criterion, as will be specified below. This will generate a RG-dressed screening quark mass, $\eta(g^2, \mu)$, with a resummed coupling dependence due to RG resummation properties. Note that, for $a = 1$, Eq. (3.11) is equivalent to the more familiar “added and subtracted” mass term prescription typically adopted in SPT [57] or HTLpt [59,75].

- (2) Next, one expands the pressure resulting from Eq. (3.11) to the desired (modified) perturbative order in powers of δ , which is set to the unit value in order to recover the original theory at the end of calculations. However, part of the RG OPT procedure stems from the observation that Eq. (3.11) may generally spoil the perturbative RGI properties of the original pressure, Eq. (3.1) (even in the chiral limit, $m_f \rightarrow 0$). Therefore, the exponent a in Eq. (3.11) is instead (uniquely) fixed by the RG equation at LO in the massless case, $m_f = 0$, to the critical value

$$a = \frac{\gamma_0}{2b_0}, \quad (3.12)$$

so that the RG invariance is recovered (for $m_f \rightarrow 0$) at LO, as prescribed in previous RG OPT applications [49,61]. Remark that, when $m_f = 0$, the RG operator given by Eq. (3.2) reduces to

$$\Lambda \frac{d}{d\Lambda} = \Lambda \frac{\partial}{\partial \Lambda} + \beta(g^2) \frac{\partial}{\partial g^2}. \quad (3.13)$$

- (3) Obviously, the previous step leaves a remnant η dependence at any finite δ orders, similarly to the original OPT (or HTLpt [75]). This initially arbitrary

parameter may be fixed by requiring the pressure, obtained from the modified perturbative expansion, to satisfy a mass optimization prescription (MOP)

$$f_{\text{MOP}} \equiv \left. \frac{\partial P^{\text{RG OPT}}}{\partial \eta} \right|_{\bar{\eta}} = 0, \quad (3.14)$$

so that the optimal variational mass ($\bar{\eta}$) becomes a function of the *original* parameters (such as the couplings) as well as control parameters (like the chemical potential).

- (4) Although at LO and $m_f = 0$ the RG Eq. (3.13) is automatically fulfilled due to Eq. (3.12), at NLO and higher orders (or for $m_f \neq 0$) the resulting pressure no longer satisfies Eq. (3.13), due to reshuffled mass dependence. Thus, a possible alternative prescription to Eq. (3.14) is to (re)impose the RG Eq. (3.13) or, more precisely, Eq. (3.2) for $m_f \equiv m_s \neq 0$ ³

$$f_{\text{RG}} \equiv \Lambda \frac{\partial P_{\text{NLO}}^{\text{RG OPT}}}{\partial \Lambda} + \beta(g^2) \frac{\partial P_{\text{NLO}}^{\text{RG OPT}}}{\partial g^2} - \gamma_m(g^2) m_s \frac{\partial P_{\text{NLO}}^{\text{RG OPT}}}{\partial m_s} = 0. \quad (3.15)$$

At this point, some important additional remarks are in order: First, note that although these RG properties were originally obtained from vacuum ($\mu = T = 0$) contributions, they still hold when thermal and/or in-medium effects are considered [52–54,62,63]. Also, according to the original prescription, a is fixed once for all at LO, and the same value in Eq. (3.12) is used at successive perturbative orders of the (modified) perturbative δ expansion. Importantly, this guarantees [49] in particular that at higher orders there is always one solution $\bar{\eta}(g)$ compatible with asymptotic freedom behavior for $g \rightarrow 0$. Finally, note that even when treating massive theories as we do here, it is compelling to adopt this universal constant value for a in Eq. (3.12), in order to avoid otherwise a clumsy $a(g, m, \dots)$ dependent exponent, furthermore incompatible with standard renormalization. We emphasize that although other slightly different variational prescriptions could be thought of, the latter one provides a sensible way of comparing successive perturbative orders with the same prescription. This is further justified *a posteriori* since the strange quark mass (m_s) which is relevant here remains a rather moderate perturbation⁴ with respect to the dressed mass $\eta(g^2, \mu)$ determined from Eq. (3.14).

³Note in Eq. (3.15) the additional anomalous mass dimension consistently due to the physical mass m_s , on top of the massless RG operator.

⁴As we will see in the sequel, $m_s \ll \bar{\eta} \simeq \mathcal{O}(g\mu) \ll \mu$ roughly holds, at least as long as the QCD coupling, $g^2(\Lambda)$, remains moderate [i.e., for renormalization scales $\Lambda \sim \mathcal{O}(\mu)$ with μ not too small].

One can next apply the RGOPT replacements to the RGI pressure, Eq. (3.1), to similarly obtain the NLO pressure. Therefore, the pressure corresponding to the strange sector can be written as⁵

$$\begin{aligned}
 P_{\text{NLO},s}^{\text{RGOPT}}(m_s, \eta, \mu) = & P_{\text{LO},s}^{\text{RGOPT}}(m_s, \eta, \mu) - N_c \frac{\eta(\eta + m_s)^2}{(4\pi)^2 b_0 g^2} \left(\frac{\gamma_0}{b_0} \right) \left(\eta - \frac{\gamma_0 - 2b_0}{2(b_0 - 2\gamma_0)} m_s \right) \\
 & - N_c \frac{(\eta + m_s)^3}{4} \left[\eta \left(2 \frac{\gamma_0}{b_0} - 1 \right) + m_s \right] \left(\frac{b_1 - 2\gamma_1}{4(b_0 - 2\gamma_0)} - \frac{1}{12\pi^2} \right) \\
 & + N_c \frac{\eta(\eta + m_s)}{8\pi^2} \left(\frac{\gamma_0}{b_0} \right) [(\eta + m_s)^2 (1 - 2L_{\mu,s}) + 2\mu p_{F,s}] \\
 & - \frac{g^2 d_A}{4(2\pi)^4} \left[(\eta + m_s)^4 \left(\frac{1}{4} - 4L_{\mu,s} + 3L_{\mu,s}^2 \right) + \mu^2 (\mu^2 + (\eta + m_s)^2) \right. \\
 & \left. + \mu p_{F,s} (\eta + m_s)^2 (4 - 6L_{\mu,s}) \right], \tag{3.16}
 \end{aligned}$$

where the LO result reads

$$\begin{aligned}
 P_{\text{LO},s}^{\text{RGOPT}}(m_s, \eta, \mu) = & \frac{N_c}{12\pi^2} \left[\mu p_{F,s} \left(\mu^2 - \frac{5}{2} (\eta + m_s)^2 \right) + \frac{3}{2} (\eta + m_s)^4 \left(L_{\mu,s} - \frac{3}{4} \right) \right] \\
 & + N_c \frac{(\eta + m_s)^3}{(4\pi)^2 b_0 g^2} \left(\eta + \frac{b_0}{b_0 - 2\gamma_0} m_s \right), \tag{3.17}
 \end{aligned}$$

and $p_{F,s} = [\mu^2 - (\eta + m_s)^2]^{1/2}$, so that $L_{\mu,s} = \ln[(\mu + p_{F,s})/\Lambda]$ changes accordingly.

To consider the sector corresponding to the pair of light quarks one just needs to take $m_s \rightarrow m_u = m_d \equiv 0$ in Eq. (3.16). Then, upon considering Eq. (3.16) one can write the total RGOPT pressure at $N_f = 2 + 1$ as

$$P_{\text{NLO}}^{\text{RGOPT}}(m_s, \eta, \mu) = P_{\text{NLO},s}^{\text{RGOPT}}(m_s, \eta, \mu) + 2P_{\text{NLO},l}^{\text{RGOPT}}(0, \eta, \mu). \tag{3.18}$$

Finally, to make the expressions self-contained, let us also remark that when considering the approximation of N_f massless physical quarks, the result is just

$$P_{\text{NLO}}^{\text{RGOPT}}(\eta, \mu) = N_f P_{\text{NLO},l}^{\text{RGOPT}}(0, \eta, \mu), \tag{3.19}$$

in accordance with Refs. [32,61]. By comparing the results provided by Eqs. (3.18) and (3.19) we will be able to gauge how the presence of a realistic running quark mass, on top of the RG-dressed screening mass $\bar{\eta}(g^2, \mu)$, affects the RGOPT predictions.

⁵Note that Eqs. (3.16), (3.17) are not simply obtained from replacing $m_f \rightarrow m_s + \eta$ within the original NLO expression Eq. (2.2), due to the modified perturbative expansion from Eq. (3.11).

A. Optimization

As discussed above and in previous applications to QCD [61–63], concerning the NLO RGOPT pressure one may either consider the dressed mass $\bar{\eta}(g^2, \mu)$ obtained from solving the MOP Eq. (3.14) or, alternatively, the dressed mass obtained from the *reduced* RG relation, Eq. (3.15). However, our previous RGOPT application [61] to the $N_f = 3$ case (which is equivalent to the $m_s \rightarrow 0$ approximation within the $N_f = 2 + 1$ results considered here) indicates that at NLO, neither the MOP Eq. (3.14) nor the RG Eq. (3.13), when solved exactly, produce real-valued $\bar{\eta}(g^2, \mu)$ solutions over the full range of phenomenologically relevant values of the chemical potentials. This is due to a very nonlinear dependence in the (variational) mass.⁶ This issue can be remediated, however, upon performing a perturbative renormalization scheme change (RSC) such that a real solution is recovered [49]. In practice the RSC can be simply implemented at the relevant NLO by modifying the (variational) mass parameter which appears in the RGI pressure, Eq. (3.1) [that is, prior to the RGOPT prescription in Eq. (3.11), such that the RSC is perturbatively well defined]. Such modification can be carried out according to [49,61]

$$m_f \rightarrow m_f (1 + g^4 B_2). \tag{3.20}$$

⁶On the other hand, more perturbative (reexpanded) approximate real solutions could be considered, but at the price of losing most of the built-in higher order RG invariance properties of $\bar{\eta}(g^2, \mu)$.

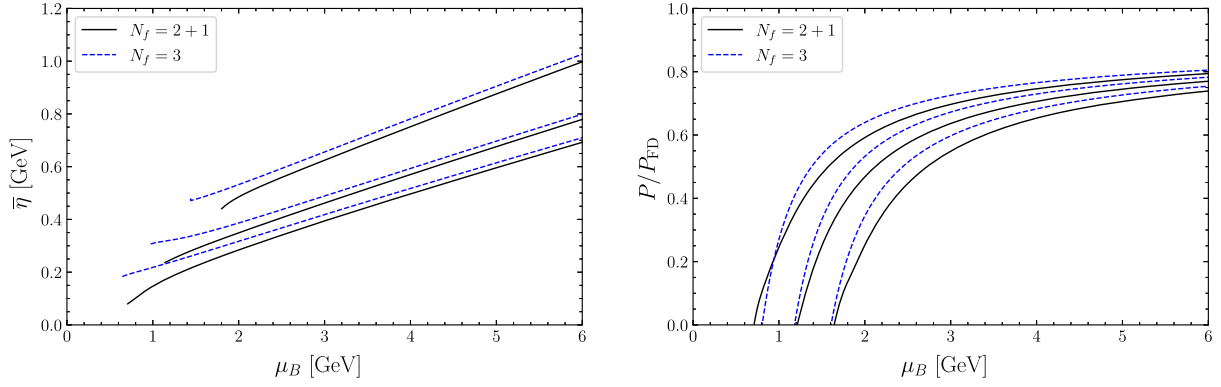


FIG. 1. Left: NLO optimized mass parameters $\bar{\eta}$ as functions of the baryon chemical potential for $N_f = 2 + 1$ ($m_s \neq 0$) (continuous line) and $N_f = 3$ ($m_s = 0$) (dashed lines). Upper lines correspond to $X = 1$, lower lines to $X = 4$, and central lines correspond to $X = 2$. Right: normalized pressure as functions of the baryon chemical potential for $N_f = 2 + 1$ and $N_f = 3$. The bottom lines correspond to $X = 1$, the upper lines to $X = 4$, while the central lines correspond to $X = 2$.

After applying this RSC to the original NLO pressure, one must reexpand perturbatively the latter to NLO, producing an extra term $-4g^2 m_f^4 s_0 B_2$ in the resulting pressure. As a by-product, after the RGOPT modifications, similar contributions with $m_f \rightarrow \eta + m_s$ and $m_f \rightarrow \eta$ are induced respectively in the first and second term on the right-hand side of Eq. (3.18). Although the dimensionless RSC parameter B_2 in Eq. (3.20) is arbitrary to start with, one seeks a sensible prescription to recover real solutions while maintaining B_2 reasonably close to $\overline{\text{MS}}$ results to justify *a posteriori* such a perturbative RSC. As Ref. [49] prescribes, B_2 is accordingly fixed (uniquely) upon requiring the nearest “contact” of the two curves parametrizing the MOP Eq. (3.14) and RG Eq. (3.13), considered as functions $f_{\text{MOP}}(\eta, g^2)$, $f_{\text{RG}}(\eta, g^2)$ respectively. Algebraically this corresponds to solving

$$f_{\text{RSC}} = \frac{\partial f_{\text{RG}}}{\partial g^2} \frac{\partial f_{\text{MOP}}}{\partial \eta} - \frac{\partial f_{\text{RG}}}{\partial \eta} \frac{\partial f_{\text{MOP}}}{\partial g^2} \equiv 0 \quad (3.21)$$

simultaneously with Eq. (3.14), $f_{\text{MOP}}(g^2, m) = 0$, in order to obtain the two (real) values $\bar{\eta}$, $\bar{B}_2$ (for the $N_f = 3$ case as well as for the $N_f = 2 + 1$ case at hand).

For the running coupling $g^2(\Lambda)$, we use the exact two-loop order, obtained from

$$\ln \frac{\Lambda}{\Lambda_{\overline{\text{MS}}}} = \frac{1}{2b_0 g^2} + \frac{b_1}{2b_0^2} \ln \left(\frac{b_0 g^2}{1 + \frac{b_1}{b_0} g^2} \right), \quad (3.22)$$

where $\Lambda_{\overline{\text{MS}}} \simeq 0.335$ GeV ($N_f = 3$) is fixed so that $\alpha_s(\Lambda = 1.5$ GeV) $\simeq 0.326$ [76], in rough consistency with the latest world-average values [77]. For the strange quark mass we consider the NLO running,

$$m_s = \hat{m}_s \left(\frac{g^2}{4\pi^2} \right)^{4/9} \left(1 + 0.895062 \frac{g^2}{4\pi^2} \right), \quad (3.23)$$

with $\hat{m}_s \simeq 0.253$ GeV corresponding to $m_s(2 \text{ GeV}) = 0.0935(8)$ GeV [77].

Figure 1 compares the optimized mass parameter, $\bar{\eta}$, as well as the normalized pressure as functions of the baryon chemical potential representing symmetric matter, $\mu_B = 3\mu$, for $N_f = 3$ and $N_f = 2 + 1$. Following the standard prescription, we have considered three different values of the renormalization scale, $\Lambda = X\mu_B/3$, with $X = 1, 2, 4$, to quantify the residual scale dependence. Regarding the variational masses, the figure suggests that they share a similar behavior, increasing linearly with μ_B roughly as $\bar{\eta} \sim \mathcal{O}(g\mu)$, although $\bar{\eta}$ actually entails an all-order g^2 dependence from RG resummation. Importantly, the fact that $\bar{\eta}$ accordingly behaves as a screening mass for moderate coupling g , also guarantees that the RSC Eq. (3.20) modifications induced to the pressure, $\propto -B_2 s_0 g^2 \bar{\eta}^4$, are of formally higher orders $\mathcal{O}(g^6)$, so that the physical pressure is only mildly dependent on the renormalization scheme choice. For the same μ_B value, and comparing with both $N_f = 2$ and $N_f = 3$ cases, the presence of a massive quark ($N_f = 2 + 1$) generates a lower pressure. This can be explained by first noting that in the basic NLO pressure Eq. (2.1) or (2.2), the $m_f \neq 0$ contributions are always negative as compared to the equivalent massless contributions (at least as long as $m_f \ll \mu$), and this feature remains true for the modified NLO expression Eq. (3.16). Then since $\bar{\eta} + m_s > \bar{\eta}$, the lowering of the pressure is obviously more pronounced for $m_s \neq 0$ in the RGOPT case.⁷ A lower pressure tends to enhance the remnant scale dependence, therefore slightly counteracting the scale dependence improvement obtained

⁷In contrast, the RSC modification $\sim -B_2 s_0 g^2 \bar{\eta}^4$ gives a positive extra contribution to the pressure, as $s_0 > 0$ in Eq. (3.3) and we always obtain $B_2 < 0$. But this contribution is subdominant compared to the “direct” $\bar{\eta}$ dependence entering other contributions.

from RGOPT, although the figure indicates that the observed scale dependence is not so much sensitive to the strange quark mass value.

IV. CHEMICAL EQUILIBRIUM AND THERMODYNAMIC CONSISTENCY

Before performing numerical evaluations to further analyze the relevant physical observables, let us redefine some thermodynamic quantities to account for chemical equilibrium and to ensure thermodynamic consistency. The latter requirement is important if one recalls that the renormalization scale (and hence the coupling) is taken to be density dependent. When considering the $N_f = 2 + 1$ case one can enforce chemical equilibrium and charge neutrality by requiring the following relations to be satisfied:

$$\mu_u = \mu_d - \mu_e \equiv \mu, \quad \mu_s = u_d, \quad (4.1)$$

$$\frac{2}{3}\rho_u - \frac{1}{3}\rho_d - \frac{1}{3}\rho_s - \rho_e = 0. \quad (4.2)$$

When $N_f = 2$, similar relations,

$$\mu_u = \mu_d - \mu_e \equiv \mu, \quad (4.3)$$

$$\frac{2}{3}\rho_u - \frac{1}{3}\rho_d - \rho_e = 0, \quad (4.4)$$

must be observed. The relevant baryon chemical potential describing each different number of flavors is defined as $\mu_B = \mu_u + 2\mu_d$ (recall that $\mu_d = \mu_s$). Seeking a fair comparison, the renormalization scale given by $\Lambda = X\mu_B/3$ is chosen in all cases.

At this stage we should specify how to best compare our RGOPT pressure results with pQCD, which for the full m_s dependence is known at NNLO [35,36,78]. As anticipated in the Introduction we consider more appropriate to compare NLO RGOPT with NNLO pQCD, since the NLO RGOPT expression given by Eq. (3.16) resums an all order RG dependence, and in the massless quark approximation the NLO RGOPT pressure is numerically closer [61] to NNLO pQCD than to NLO pQCD (we have also explicitly checked that a similar behavior holds for $m_s \neq 0$). To obtain the pQCD results at $N_f = 2$, we thus use the NNLO result for massless quarks given by [40,41,79],

$$\frac{P_{N_f=2}^{\text{pQCD}}}{P_{\text{FD}}} = 1 - 2\frac{g^2}{4\pi^2} - \frac{g^4}{(4\pi)^2} \left[10.3754 - 0.535832N_f + N_f \ln(N_f) + N_f \ln\left(\frac{g}{4\pi^2}\right) + \left(11 - \frac{2}{3}N_f\right) \ln\left(\frac{\Lambda}{\mu_f}\right) \right], \quad (4.5)$$

where the Fermi-Dirac pressure P_{FD} describing noninteracting massless quarks reads

$$P_{\text{FD}} = \frac{3N_f}{12\pi^2} \left(\frac{\mu_B}{3}\right)^4. \quad (4.6)$$

Concerning the pQCD results at $N_f = 2 + 1$, we consider the pocket formula given in Ref. [36], which reproduces well the NNLO results of Ref. [35]. This simple relation is given by

$$\frac{P_{N_f=2+1}^{\text{pQCD}}}{P_{\text{FD}}} = \left(c_1 - \frac{a(X)}{(\mu_B/\text{GeV}) - b(X)} \right), \quad (4.7)$$

$$a(X) = d_1 X^{-\nu_1}, \quad b(X) = d_2 X^{-\nu_2},$$

where $c_1 = 0.9008$, $d_1 = 0.5034$, $d_2 = 1.452$, $\nu_1 = 0.3553$, and $\nu_2 = 0.9101$.

As already emphasized, since the scale is chosen to be density dependent, the equations of state, Eqs (3.18), (3.19), and (4.5), need to be redefined to fulfill thermodynamic consistency. The pQCD EoS given by Eq. (4.7) is already thermodynamically consistent by construction [36]. To achieve thermodynamic consistency within $N_f = 2$ pQCD, and within the different N_f cases considered for

the RGOPT, we follow the same procedure adopted in our previous work⁸ [32], where extra terms b_f are added to the pressure, defined so that

$$\frac{db_f}{d\mu_f} = - \sum_i \frac{dP_f}{dw_i} \frac{dw_i}{d\mu_f}, \quad (4.8)$$

where w_i represent Lagrangian parameters that depend on the control parameter μ_f (see Refs. [32,80,81] for a deeper discussion).

V. NUMERICAL RESULTS

In this section, we first present the numerical results furnished by the RGOPT and pQCD for NSQM ($N_f = 2$)

⁸At this point, note a detail missed in Ref. [32]: the electron density obtained from charge neutrality, Eqs. (4.2) and (4.4), also depends on the parameters w_i . Consequently, the electron pressure is not thermodynamic consistent. Then, in principle, one also needs to add a term b_e to the leptonic pressure to completely guarantee thermodynamic consistency. For the sake of rigor, we have added this term in our present results. However, it turns out that the leptonic term, missed in Ref. [32], is small enough so that its contribution to the full pressure is almost negligible.

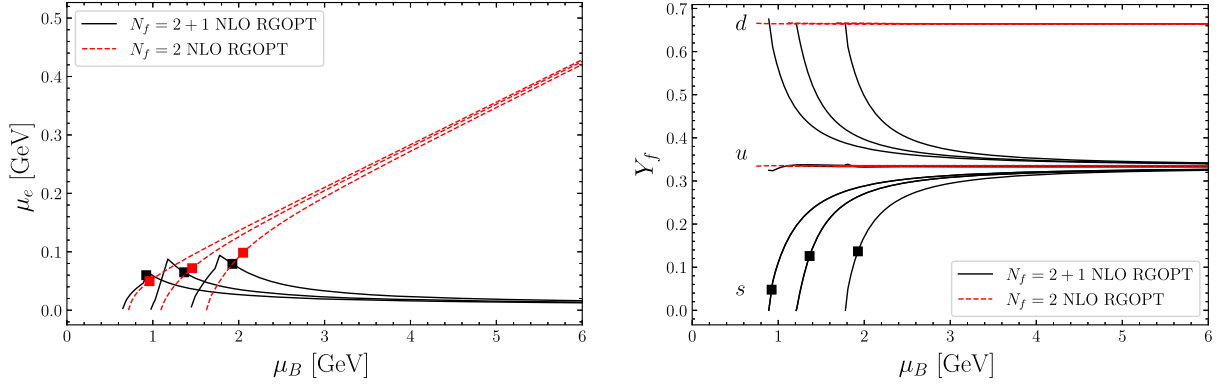


FIG. 2. Left: electron chemical potential as a function of μ_B , obtained from the charge neutrality condition. The case $N_f = 2 + 1$ is represented by the continuous line while the dashed line represents the case $N_f = 2$. The boundaries of each band are obtained by setting the renormalization scale coefficient to $X = 1$ (rightmost boundary) and $X = 4$ (leftmost boundary), while $X = 2$ corresponds to the central line. The kink, in the case $N_f = 2 + 1$, corresponds to the onset of the s quark. Right: quark fraction $Y_f = \rho_f/\rho_B$ as a function of μ_B . In both figures the squares represent the values at which $P = 0$, which defines the surface of the QS.

as well as SQM ($N_f = 2 + 1$) considering β equilibrium, Eqs. (4.1) and (4.3), and charge neutrality, Eqs. (4.4) and (4.2). Next, the EoS corresponding to each case is used to obtain the mass-radii relation describing nonstrange and strange quark stars.

A. NSQM and SQM for β -equilibrium matter

The electron chemical potential predicted by the NLO RGOPT is plotted in Fig. 2 (on the left panel) as a function of μ_B for $N_f = 2 + 1$ (solid lines) and $N_f = 2$ (dashed lines). In the case of SQM, the appearance of kinks in the curves signals the onset of strange quarks which induces a strong reduction of the number of electrons decreasing the electron chemical potential ($\mu_e \rightarrow 0$, at high- μ_B). Therefore, in this limit, the densities of the three quark flavors become similar. In other words, the system favors the replacement of the negative electric charge of electrons by one of the s -quarks. On the other hand, in the case of NSQM, as μ_B takes on higher values, the electron production increases to compensate for the increase in the u -quark number density, while maintaining charge neutrality. Note that the u -quark has a smaller Fermi momentum than the d -quark and, therefore, an increase of the number density is favored.

On the right panel of Fig. 2 the relative fraction of quarks with different flavors, $Y_f = \rho_f/\rho_B$ is shown. At low chemical potential the s -quark has still not set in and the fraction of d -quarks is approximately equal to the double of the u -quark fraction due to charge neutrality. After the onset of strangeness, s -quarks gradually replace d -quarks while the yield of u -quarks does not change. At large chemical potentials, when the Fermi momenta are much larger than the effective masses, all fractions become similar. This is what is frequently designated by strange matter. Note, however, that the QS surface is defined by the condition

$P = 0$, and, therefore, the s -quarks are already present on that layer of the QS.

In Fig. 3, we show the normalized pressures furnished by the NLO RGOPT and NNLO pQCD for both SQM and NSQM, thermodynamically consistent after applying Eq. (4.8). The first thing to note is that, within both approximations, the introduction of a finite strange quark mass decreases the scale dependence (this effect is much more noticeable in the case of pQCD). While those features result from involved combined effects from the running coupling, running mass, and extra thermodynamic consistency contributions, this behavior is explained partly from the fact that since $b_0(N_f = 3) < b_0(N_f = 2)$, the running coupling $g^2(\Lambda) \sim [2b_0 \ln(\Lambda/\Lambda_{\text{QCD}})]^{-1}$ has somewhat smaller scale variation for $N_f = 3$ than for $N_f = 2$. The latter effect is however partly compensated by the sizable massive contributions $\bar{\eta} \neq 0$, lowering the pressure (as explained in the last paragraph before Sec. III), which rather tends to increase the remnant scale dependence. As explained in Sec. III, the RGOPT procedure involves a resummed all order RG dependence, thus adding higher order Λ dependence that can further reduce the renormalization scale dependence, so that our results display a somewhat reduced scale dependence with respect to unresummed pQCD. However, the improvement that we obtain for the present cold quark matter case is much more moderate than its counterpart for hot QCD in [62,63]. In addition, the figure suggests that the NLO RGOPT produces EoSs that converge slower to the Fermi-Dirac limit (free massless quarks) than its NNLO pQCD counterpart. This happens because the former procedure produces a nontrivial medium-dressed mass $\bar{\eta}(g, \mu)$, so that the effective masses entering the quark propagators are $m_s + \bar{\eta}(g, \mu)$ (strange sector) and $\bar{\eta}(g, \mu)$ (nonstrange sector). Accordingly, at $N_f = 2$ the RGOPT quark propagator already involves a nontrivial effective quark mass while,

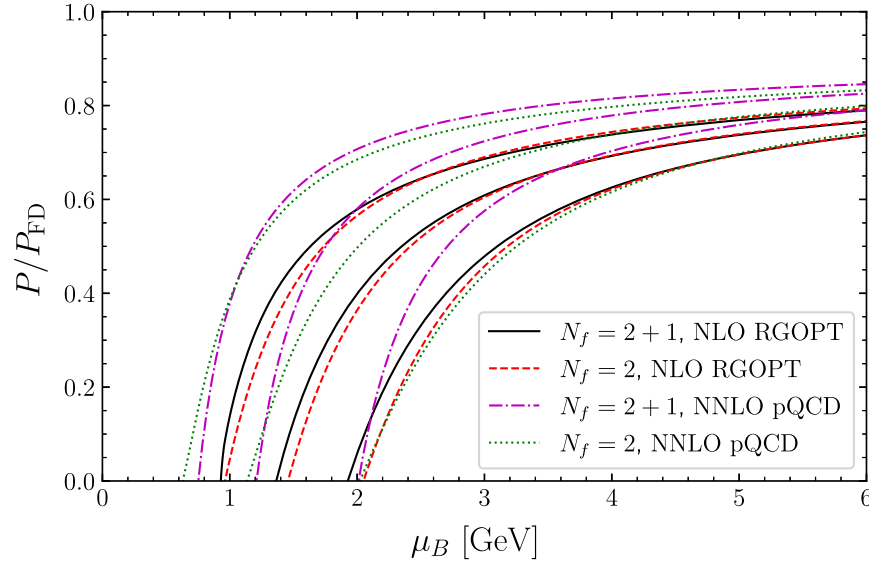


FIG. 3. Thermodynamic consistent NLO RGOPT pressure as function of μ_B for $N_f = 2 + 1$ (continuous lines) and $N_f = 2$ (dashed lines). For comparison the dot-dashed and dotted lines respectively represent the $N_f = 2 + 1$ and $N_f = 2$ NNLO pQCD pressure. For all curves the upper ones correspond to $X = 4$, the lower ones to $X = 1$, while the central scale, $X = 2$, is represented by the inner curves. All pressures have been normalized by the Fermi-Dirac value P_{FD} for free massless quarks.

at $N_f = 2 + 1$, the effect of including the s -quark physical (current) mass is more diluted. A similar behavior was initially noticed to occur within NLO pQCD [39], where the introduction of the strange quark mass was observed to push down the pressure at low- μ_B values ($\mu_B \lesssim 3$ GeV), softening the resulting EoS. However, later the authors of Ref. [35] found that the softening effect is less noticeable when finite masses are considered at NNLO.

B. Quark stars

Having discussed the EoS of β -equilibrium quark matter, for both SQM and NSQM, we next consider NS observations to constrain the scale X . We will apply our EoS to the description of quark stars (QS). We will not consider hybrid stars with a core of quark matter in order to avoid an extra uncertainty through the hadronic EoS.

In Fig. 4 we display the EoSs, pressure versus chemical potential, and pressure versus the energy density (top panels) and the respective quark fractions (bottom panel) for the values of X that reproduce $M_{\text{max}} = 2, 2.3$ and $2.6M_{\odot}$. The corresponding values⁹ for both approximations in the case of SQS and NSQS are given in Table I. In this table, one can see the NLO RGOPT may require X values slightly higher than those of NNLO pQCD, namely $X = 3.63\text{--}4.30$ for SQS and $X = 3.39\text{--}4.18$ for NSQS, implying that such stars may be composed by weakly interacting quarks [smaller $\alpha_s(X)$], even at low P -values. In contrast,

the NNLO pQCD produces the required maximum masses for values of X within the canonical range, namely $X = 2.95\text{--}3.56$ for SQS and $X = 2.17\text{--}2.53$ for NSQS. We also checked that using higher orders in the running coupling and mass instead of Eqs. (3.22) and (3.23) would have very moderate visible impact on our results, which is partly due to our use of exact two-loop running coupling, Eq. (3.22), less sensitive to higher order differences than the truncated power-expansion in $\ln^{-1}(\Lambda/\mu)$ more commonly used in the literature. Moreover, note that all our RGOPT results in Table I lie within a range of μ_B and X values such that the coupling α_s remains moderately perturbative, $0.35 \leq \alpha_s \leq 0.38$ roughly for the values of the scale at the surface of the stars with maximum masses.

We should also comment on the configurations shown in Table I by taking into account the stability constraints generally considered for quark matter: (i) For stable strange matter, the energy per baryon at the bulk should be below 930 MeV [15,17], while, for the same input parameters in a given model, ud -quark matter should be no more bound than iron, i.e., the energy per baryon should be above 930 MeV, see the discussion in Ref. [82]. (ii) For stable ud -quark matter, the energy per baryon should be below 930 MeV. Recently, other scenarios have been proposed within phenomenological quark models [19,29], and therefore the conditions of absolute stability are still under discussion.

Considering first the results obtained with pQCD given in Table I, they show stable NSQM only for $X \gtrsim 2.5$, corresponding to $M_{\text{max}} \gtrsim 2.56M_{\odot}$. Looking at the relevant pQCD X values for SQM, the corresponding energies per baryon are below 930 MeV in bulk for almost all values listed in Table I, i.e., for $X \gtrsim 3$. However, at the same X

⁹Notice that the X values of the RGOPT and pQCD for NSQM ($N_f = 2$) slightly differ from those of Ref. [32] since in that work the scale was chosen to be $\Lambda = X(\mu_u + \mu_d)/2$.

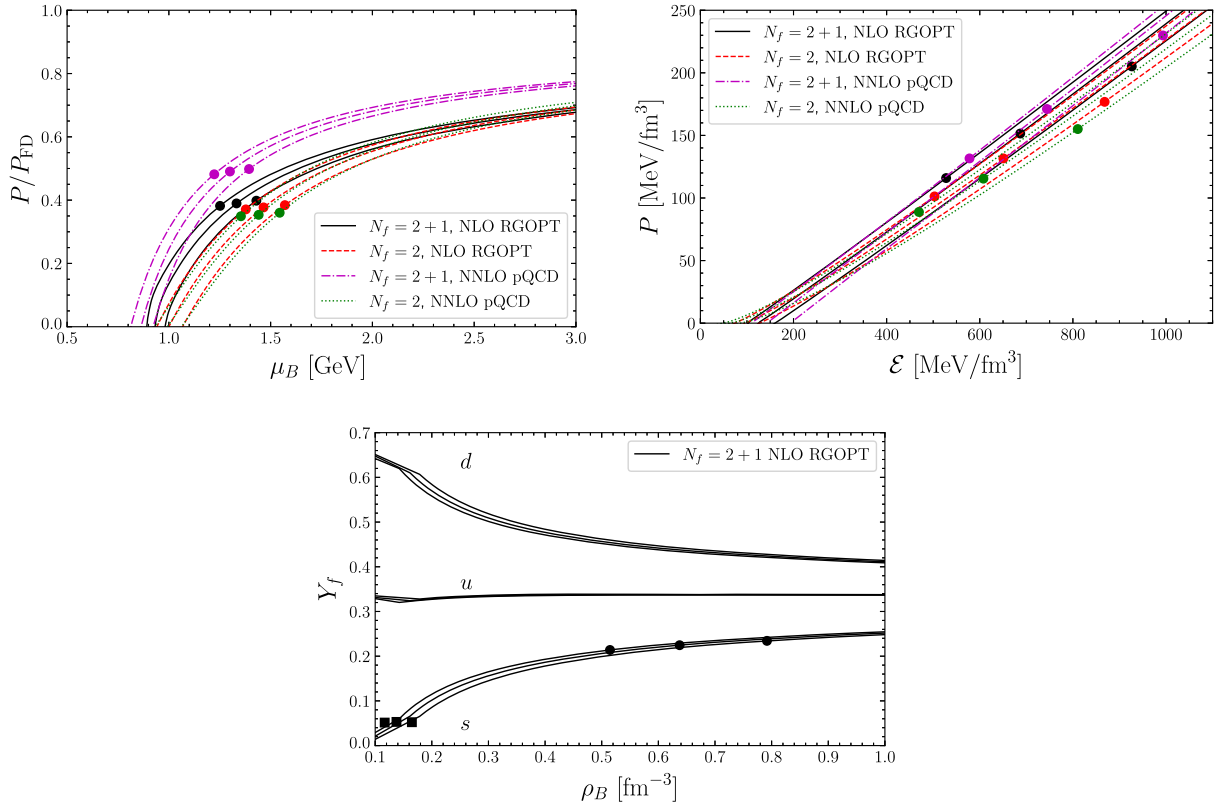


FIG. 4. Pressure versus baryon chemical potential (top left) and energy density (top right) of β -equilibrated matter for the $N_f = 2 + 1$ (continuous lines) and $N_f = 2$ (dashed lines) NLO RGOPT, and for $N_f = 2 + 1$ (dot-dashed lines) and $N_f = 2$ (dotted lines) NNLO pQCD. Bottom: the quark fractions as a function of the baryonic density for the same EoS. The curves were obtained with the scale values X that reproduce $M_{\max} = 2, 2.3$ and $2.6M_{\odot}$, given in Table I. The dots identify the values at the center of the maximum mass star configuration. The squares in the bottom panel identify the baryonic densities at which $P = 0$.

values, the NSQM energies per baryon are also below 930 MeV, but would give very large corresponding maximum masses $M_{\max} \gtrsim 2.7M_{\odot}$. In several recent studies [83,84] it is shown, using different agnostic descriptions

of the neutron star EoS and imposing pQCD constraints, that the maximum mass of the NS is below $\sim 2.5M_{\odot}$. Therefore, according to these studies models with $M_{\max} \geq 2.7M_{\odot}$ appear unfavored. However, some works propose

TABLE I. QS properties predicted by NLO RGOPT with $N_f = 2 + 1$ and $N_f = 2$, and by NNLO pQCD with $N_f = 2 + 1$ and $N_f = 2$ with the X scale values that reproduce $M_{\max} = 2, 2.3$ and $2.6M_{\odot}$. The considered properties are maximum mass $M_{\max}$ and corresponding radius $R_{\max}$, radius of the $1.4M_{\odot}$ and $0.77M_{\odot}$ stars, $R_{1.4}$ and $R_{0.77}$, central baryon density $\rho_B^{c,\max}$ and central baryon chemical potential μ_B^c , as well as surface baryonic density and chemical potential of the maximum mass configuration.

	X	$M_{\max} (M_{\odot})$	$R_{\max} \text{ (km)}$	$R_{1.4} \text{ (km)}$	$R_{0.77} \text{ (km)}$	$\rho_B^{c,\max} (\rho_0)$	$\mu_B^{c,\max} \text{ (GeV)}$	$\rho_B^{\text{surf},\max} (\rho_0)$	$\mu_B^{\text{surf},\max} \text{ GeV}$
$N_f = 2 + 1$ RGOPT	3.63	2.00	12.0	12.2	10.4	4.95	1.430	1.03	0.969
	3.97	2.30	13.7	13.5	11.4	3.98	1.337	0.86	0.930
	4.30	2.59	15.5	14.6	12.7	3.21	1.251	0.73	0.890
$N_f = 2 + 1$ pQCD	2.95	2.00	11.2	11.6	9.85	5.48	1.394	1.31	0.923
	3.26	2.30	13.3	12.9	11.0	4.40	1.300	1.04	0.863
	3.56	2.60	15.1	14.2	12.0	3.62	1.223	0.84	0.813
$N_f = 2$ RGOPT	3.39	2.00	12.4	12.9	11.1	4.17	1.570	0.74	1.067
	3.80	2.29	14.3	14.3	12.2	3.34	1.465	0.61	1.000
	4.18	2.60	16.3	15.9	13.4	2.74	1.378	0.49	0.941
$N_f = 2$ pQCD	2.17	2.00	13.1	13.9	12.2	3.92	1.544	0.50	1.053
	2.35	2.29	15.1	15.4	13.6	3.14	1.441	0.39	0.982
	2.53	2.61	17.2	17.5	15.1	2.58	1.354	0.31	0.920

the existence of quark stars with masses above $2.5M_\odot$: in Ref. [85] a scenario of uds QS with a maximum mass of $2.5 - 2.6M_\odot$ is associated to the binary neutron star merger GW190814, and in Ref. [28] the authors obtain within an interacting quark model $M_{\max} \leq 3.23M_\odot$, a value of the order proposed by Rhoades and Ruffini [86] as maximum NS mass determined just from Einstein's theory of relativity, the principle of causality, and Le Chatelier's principle. Moreover, note that pQCD constraints are treated differently in Ref. [70], where quark stars are discussed.

In the RGOPT case for SQM, the Bodmer-Witten hypothesis is fulfilled and NSQM is less bound than iron as far as $3.97 \lesssim X \lesssim 4.24$, corresponding to $2.30M_\odot \lesssim M_{\max} \lesssim 2.54M_\odot$, thus in a more constrained range than for pQCD, for which SQM matter is stable in the entire range of $2 - 2.5M_\odot$. The upper limit is obtained taking the same X range for NSQM: we obtain energies per baryon equal to 0.969(0.925) GeV, respectively, for $X = 3.97(4.30)$. More precisely, one has stable NSQM for $X \gtrsim 4.24$, corresponding to $M_{\max} \gtrsim 2.69M_\odot$, and therefore the condition $X < 4.24$ must be imposed.

The two scenarios obtained with both approaches (pQCD or RGOPT) for stable NSQM do not appear to be realistic according to several different studies that propose maximum masses of the order of $2.15 - 2.35M_\odot$ [83,87–89] and generally below $2.5M_\odot$ [83]. Therefore, within the possible limitations of our level of approximations, and considering realistic maximum masses between 2.3 and 2.5 solar masses, our descriptions seem to agree

with the Bodmer-Witten conjecture on SQM, with ud -quark matter being less bound than iron. This could suggest that pure quark stars would be made of uds matter, while ud -quark stars would carry an outer layer of hadronic matter or could be quark stars. These results indicate that the composition of dense matter is still not clear and further studies are required.

With respect to remnant scale dependence, it is important to notice that these values indicate that the RGOPT is moderately more stable to scale variations than pQCD. Indeed, to reproduce the maximum mass variation $\Delta M_{\max} = (2.6 - 2.0)M_\odot$ for SQS this method requires a relative scale variation $\Delta X = 0.67$, which is slightly higher than the one required by pQCD, $\Delta X = 0.61$. A similar situation occurs in the case of NSQS when the OPT requires $\Delta X = 0.79$ while pQCD only requires $\Delta X = 0.36$. On the different panels of Fig. 4, the dots identify the central chemical potential or baryonic density of the maximum mass configurations. At the same time, the squares on the bottom panel indicate the $P = 0$ condition that defines the QS surface. Note that on the surface s -quarks are already present.

Figure 5 shows the mass-radius relations obtained with the EoSs of Fig. 4 together with the predicted masses and radii of the pulsars PSR J0030 + 0451 [64,65] and PSR J0740 + 6620 [66,67] as well as the compact object HESS J1731-347 [69]. For reference, a band identifying the low mass compact object associated with GW190814 [90] is also shown. Comparing the results for NSQS with SQS,

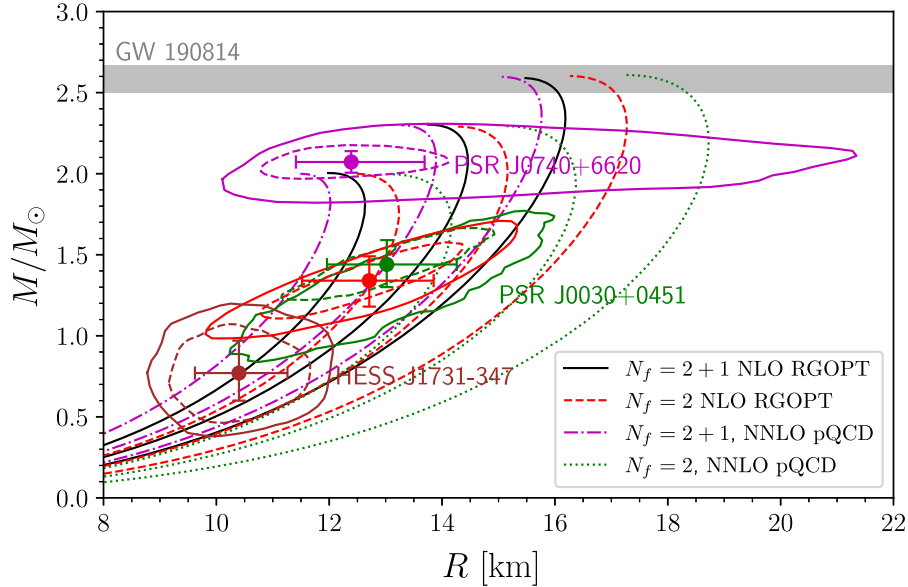


FIG. 5. Mass-radius relation given by the different approximations. Lines reproducing maximum QS masses of $2.6M_\odot$ correspond to the higher values of X of Table I, while the ones for $2M_\odot$ correspond to the lowest X values. Also included are the NICER data for pulsars PSR J0030 + 0451 [64,65] and PSR J0740 + 6620 [66,67], as well as the low mass compact star HESS J1731-347 [69]. In particular, the ellipses represent the 68% (dashed) and 95% (full) confidence interval of the 2-D distribution in the mass-radii domain while the error bars give the 1-D marginalized posterior distribution for the same data. A band identifying the mass of the low mass compact object associated with GW190814 [90] has also been included.

we find that the NLO RGOPT results show a slight change in the neutron star radii, which are lower for the SQS. This could be expected, since the onset of a new degree of freedom softens the EoS. As a consequence of the softening, in order to describe two solar mass stars or above, larger scale values X are required. In the case of SQM the X values are slightly larger than the ones obtained for pQCD. However, we note that for the maximum mass range considered $[2.2:2.6]M_\odot$, the NLO RGOPT predictions for both NSQS and SQS properties that are compatible with the present NS observations. More stringent constraints are necessary to distinguish the different scenarios. Considering the choice of scale adopted, the descriptions within both pQCD and RGOPT for $N_f = 2 + 1$ are compatible with each other, with NLO RGOPT predicting slightly larger radii. Compared with NNLO pQCD, NLO RGOPT gives a smaller reduction in the radii predicted by SQS (not more than 1 km) compared to NSQS. This is explained by the large difference between the pQCD expressions for $m \neq 0$ and $m = 0$, while RGOPT produces similar expressions in both cases, since even in the massless sector, the prescription introduces a resummed medium-dressed mass, $\bar{\eta}(g, \mu)$.

VI. CONCLUSIONS

We have considered the RGOPT resummation approach, at NLO, in order to compare strange and nonstrange pure QS. The latter case was originally investigated in Ref. [32], when massless up and down quarks ($N_f = 2$) were considered to represent the relevant degrees of freedom for the QCD EoS. Here, the $N_f = 2$ application was extended to the $N_f = 2 + 1$ case to include a new degree of freedom represented by the (massive) strange quark. Accordingly, the RG operator contains the anomalous mass dimension $\gamma_{m_s} \partial/\partial m_s$ that was absent in previous RGOPT applications to QCD where the chiral limit was considered.

Following the conventional practice, we have chosen the renormalization scale to be density dependent, $\Lambda = X\mu_B/3$. Consequently, parameters such as the strong coupling and the strange quark mass also run with μ_B implying that the pressure needs to be modified in order to produce thermodynamic consistent predictions.

After carrying out those modifications, in accordance with Ref. [32], we have determined the EoS representing the two scenarios, with and without the s -quark. In more academic applications, the value of the parameter characterizing the scale, is generally chosen conventionally to be $X \in [1, 4]$ with $X = 2$ representing the so-called central scale. In the present work, this arbitrary scale parameter was fixed in a more phenomenological way by selecting the values which give maximum QS masses of order $M_{\max} = 2, 2.3$ and $2.6M_\odot$ which represent some of the limits often considered in the literature. For the NLO RGOPT EoS the required values are, respectively,

$X = 3.63, 3.97$, and 4.30 when $N_f = 2 + 1$ and $X = 3.39, 3.80$, and 4.18 when $N_f = 2$ whereas for the NNLO pQCD EoS the values are $X = 2.95, 3.26$, and 3.56 when $N_f = 2 + 1$ and $X = 2.17, 2.35$, and 2.53 when $N_f = 2$. Accordingly, within the RGOPT framework, massive QS can only be produced at relatively larger scales (smaller couplings) compared with pQCD. The introduction of an extra degree of freedom, the s -quark, softens the EoS and to attain the same maximum masses the $N_f = 2 + 1$ scenario requires larger X scales in both descriptions. With these choices of the scale, it was shown that the EoS obtained are in accordance with properties of several recent neutron star mass-radius data, in particular, with data for pulsars PSR J0030 + 0451 [64,65] and PSR J0740 + 6620 [66,67], and for the low mass compact star HESS J1731-347. To understand these results, let us first recall that the introduction of finite quark masses tends to reduce the pressure predicted by both methods, as discussed earlier. This effect is amplified in the RGOPT approach, since the effective quark masses are $m_s + \bar{\eta}$ (in the strange heavy sector) and $\bar{\eta}$ (in the nonstrange light sector), implying that the corresponding EoS for the same scale X and baryonic chemical potential reach lower pressures than their pQCD counterparts. Therefore, the RGOPT predicts that QS with high masses can only be formed when the scale is rather high. Comparing the $N_f = 2 + 1$ and $N_f = 2$ RGOPT mass-radii curves one can see that, despite their similarities and good agreement with observations, the description of strange QS requires $X > 3.63$ implying that the coupling takes relatively lower numerical values. The $N_f = 2 + 1$ NNLO pQCD description of SQS also calls for somewhat higher X values which, however, lie within the more canonical range. Note also that the RGOPT is slightly more stable to scale variations than pQCD. In particular, to reproduce the maximum mass variation $\Delta M_{\max} = (2.6 - 2.0)M_\odot$ corresponding to possible different scenarios for SQS, NLO RGOPT requires a relative scale variation of $\Delta X = 0.67$, about 10% larger than the variation occurring in pQCD, $\Delta X = 0.61$. We have also studied more precisely the stability of SQM and NSQM scenarios upon examining the energy per baryon for our different QM approximations. For both pQCD and RGOPT, ud -quark matter appear less bound than iron, except for very large masses $M_{\max} \gtrsim 2.6M_\odot$. On the other hand, the NLO RGOPT predictions are in agreement with the SQM Bodmer-Witten conjecture within the mass range $2.30M_\odot \lesssim M_{\max} \lesssim 2.54M_\odot$. At the same time, they satisfy the condition that ud matter is less bound than iron. Note, that it is difficult to assert firm conclusions about the Bodmer-Witten conjecture without a truly nonperturbative contribution to the QCD vacuum energy, that neither pQCD nor RGOPT incorporate.

At this point, we must emphasize that a potentially more reliable RGOPT prediction will require a computation extended at NNLO, since this next perturbative order

contains sizeable massive contributions [35] from three-loop graphs with three-gluon vertices and correction to the quark-gluon vertex, which may turn out to affect the softness of the EoS with respect to the NLO case. While the RGOPT evaluation of the basic NNLO pressure with massive quarks was recently obtained [78], extending those results to consistently account for chemical equilibrium and thermodynamic consistency is somewhat involved and beyond the scope of the present work. Finally, we have observed that the method, once generalized for the inclusion of a genuine physical (current) mass $m_s(\Lambda)$, produces results which are slightly less sensitive to scale variations than those provided by pQCD, just like what happens when massless theories are considered [32,61–63].

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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