Probing of colliding wave spacetimes and a quantum Weyl conjecture

Ivo Sachs[1,*] and Marc Schneider[2,†]

[1]*Arnold-Sommerfeld Center for Theoretical Physics, Theresienstraße 37, 80333 München, Germany*
[2]*Department of Physics and EHU Quantum Center, University of the Basque Country UPV/EHU, Barrio Sarriena s/n, Leioa 48940, Spain*

(Received 6 March 2026; accepted 24 July 2026; published 21 August 2026)

We perform a quantum probing of colliding plane-wave spacetimes. In particular, we consider the Khan-Penrose and the Ferrari-Ibáñez solutions, which admit a strong and a weak singularity after the two waves collide. While we find that, like Schwarzschild, for the Khan-Penrose solution the singularity cannot be probed by quantum-field theory, the Ferrari-Ibáñez singularity can be traversed. Our results culminate in a quantum Weyl conjecture: The significant geometric property to classify spacetimes with respect to quantum probes is given by the Coulomb part of the Weyl tensor. We then use this conjecture to sketch a possible backreaction scenario for plane waves.

DOI: 10.1103/7wj9-tsyv

I. INTRODUCTION

Quantum probing revealed that the intuitive classical picture fails when quantized probes are considered. The concept contrasts the occurrence of a singularity with its physical significance. Colloquially speaking, addressing the question: Do singular systems lead to singular measurements? The answer is in the negative, and one of the showcase examples marks the hydrogen atom. Albeit incomplete with respect to classical electrodynamics, quantum mechanical evolution remains complete because of a probabilistic completion; i.e., there is a vanishing probability to find bound states, as well as scattering states of the electron to be found at the center, where the Coulomb field becomes singular. Hence, the electron is barred from probing the end point.

The concept of quantum mechanical completeness has been extended to quantum-field theory in curved spacetimes to capture dynamical scenarios in which absorption and emission processes occur [1]. One of the most intricate questions is the fate of quantum fields that encounter curvature singularities. The functional Schrödinger representation has proven itself to be extremely versatile to detect pathologies in a time-dependent framework. For the Schwarzschild black hole's interior, one observed a probabilistic suppression that led to a vanishing probability of finding any quanta on the singular hypersurface [2]. In this sense, Schwarzschild black holes comply with the idea of being gravitational analogs of the hydrogen atom; however, the discussion here focuses on scattering states, while bound states are not considered. In fact, our functional Schrödinger states describe the evolution of such field configurations and thus determine the probability density of a quantum field to be located on a particular hypersurface through the norm.

While the gravitational singularity that occurs inside Schwarzschild black holes seems to prevent the fields from reaching it [1], the focusing singularity in plane-wave spacetimes shows a different behavior. In fact, after passing through the wave, the quantum fields experience a focusing and will eventually reach the focal plane [3]. The different quality attributed to the singularity influences the behavior of the quantum fields directly. The obvious question arises: What exactly is this quality?

To answer this question, we consider colliding plane-wave spacetimes because they offer a rich geometric playground that combines plane waves and black hole interiors [4–9]. This class of spacetimes describes colliding plane waves which form a very diverse geometric landscape with four patches that are separated by the sandwich waves. Before the collision, one faces a flat region (denoted below as patch I), which resembles Minkowski spacetime except for the future boundaries which are the plane waves. After crossing one of the incoming waves, there lies a region with a focusing, null singularity (patches II and III). These regions correspond to pp-wave spacetimes [10]. The most intriguing region is patch IV, which forms after the waves have collided. There, the collision of the waves creates a scatter that propagates toward the future but is destined to collapse under its own gravitational pull, thus forming a spacelike singularity. There exist several solutions depending on the details of the system from which we choose to

*Contact author: ivo.sachs@lmu.de
†Contact author: marc.schneider@ehu.eus

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

work with the Khan-Penrose solution [4]. In this particular solution, the curved region features a gravitational collapse reminiscent of the formation of a black hole and, hence, amounts to a Schwarzschild interior [11,12]. However, there are milder solutions that admit weaker singularities after the collision, e.g., the Ferrari-Ibáñez spacetime [5], which we study for reference as well.

Since we are interested in the general properties that determine the evolution of quanta on curved backgrounds, (colliding) plane-wave geometries are ideal because every spacetime provides locally a plane-wave limit [13,14]. Therefore, probing all patches of this geometry allows us to link the folding singularities of the focal plane with the curvature singularity after the collision and extract the geometric properties that determine the fate of quantum probes. Regarding the classification of the geometric part, we will make use of the Penrose-Newman formalism [15] which allows us to decompose the different parts of the curvature tensor. These Penrose-Newman scalars provide a clear interpretation and allow us to decode the properties of gravity that determine the evolution of the quantum probes. In the underlying case, a strong connection with the Coulomb part of the Weyl tensor emerges, as such, substantiating the intuition of black holes being gravitational analogs of the hydrogen atom. We will formalize this observation into a quantum Weyl conjecture.

Our analysis will be organized as follows: In the next section, we discuss the geometric properties of colliding plane-wave spacetimes, that is, the different regions shaped by the waves. Our main focus will be the region after the collision. Then, in the third section, we discuss the geometric properties of the Khan-Penrose solution explicitly and perform a quantum probing of patch IV. This we contrast with the Ferrari-Ibáñez solution in Sec. IV. We collect our findings and weave them into the quantum Weyl conjecture in Sec. V, where we furthermore apply this conjecture to estimate a (quasiclassical) backreaction scenario. We close with a discussion of our results in the last section. Throughout the article, we use the mostly plus sign convention for the metric and natural units.

II. THE GEOMETRY OF COLLIDING PLANE WAVES

In this section, we describe the basic geometric facts about plane-wave spacetimes and their properties after collision. Plane-wave spacetimes have a wide range of applications and constitute even a particular limit to many spacetimes that seem initially not connected [16]. We start with the description of single plane waves and move toward the case where they collide.

A. Plane-wave spacetime

To describe a collision of two plane waves, we begin with a single, linearly polarized plane wave with profile $H(u,x,y)$ that travels in the v direction [10,17]. In Brinkmann coordinates,

$$g=-2\mathrm{d}u\otimes\mathrm{d}v-H(u,x,y)\mathrm{d}u\otimes\mathrm{d}u+\mathrm{d}x\otimes\mathrm{d}x+\mathrm{d}y\otimes\mathrm{d}y, \tag{1}$$

where u and v denote the two null directions. This patch describes how the wave propagates on an otherwise flat Minkowski spacetime. If the plane wave is sourced by matter, the latter enters through the stress-energy tensor

$$T_{uu}=-\frac{1}{8\pi}\mathrm{tr}_g(H(u,x,y)) \tag{2}$$

into the Einstein equations. Albeit intuitive from an overseeing perspective, the more instructive description of the dynamics provides the Einstein-Rosen patch. Through the coordinate transformations $u\to U$, $v\to V+\frac{X^2}{2}F(U)\frac{\mathrm{d}F}{\mathrm{d}U}(U)+\frac{Y^2}{2}G(U)\frac{\mathrm{d}G}{\mathrm{d}U}(U)$, $x\to F(U)X$, and $y\to G(U)Y$, the metric (1) becomes

$$g_{III}=-2\mathrm{d}U\otimes\mathrm{d}V+F^2(U)\mathrm{d}X\otimes\mathrm{d}X+G^2(U)\mathrm{d}Y\otimes\mathrm{d}Y. \tag{3}$$

The two functions $F(U)$ and $G(U)$ depend on the specifics of the wave profile $H(U,x,y)$ through the equations

$$\frac{\mathrm{d}^2F}{\mathrm{d}U^2}=H(U)F(U)\quad\text{and}\quad\frac{\mathrm{d}^2G}{\mathrm{d}U^2}=-H(U)G(U), \tag{4}$$

which for an impulsive wave of unit amplitude [that is, $H(U)\propto\delta(U)$] become $F(U)=1+U\Theta(U)$ and $G(U)=1-U\Theta(U)$ where $\Theta(U)$ denotes the Heaviside distribution.

From (4), we then infer, in particular, that null geodesics in the U direction originating at past null infinity experiences a flat spacetime unless it has crossed the wave where it is diverted and focused on a null singularity at $U=1$. The same type of spacetime can be constructed when the wave travels in the u direction instead. Then, the metric functions become V dependent

$$g_{II}=-2\mathrm{d}U\otimes\mathrm{d}V+F^2(V)\mathrm{d}X\otimes\mathrm{d}X+G^2(V)\mathrm{d}Y\otimes\mathrm{d}Y, \tag{5}$$

where the functions $F(V)$ and $G(V)$ can be distinct from the ones in (4) because they depend on the profile and strength of the wave in the U direction. This can analogously be transformed into the Brinkmann coordinates via choosing an appropriate profile $H(v,x,y)$ for the wave that propagates in the u direction.

B. Colliding plane waves

A collision of two impulsive waves combines the two previously discussed scenarios and therefore admits the spacetime $(\mathbb{K}, g)$, i.e., a manifold $\mathbb{K}$ and a metric g which in Einstein-Rosen coordinates reads (with parallel linear polarization) [11]

$$g = -2e^{-M(U,V)}\mathrm{d}U \otimes \mathrm{d}V + \gamma. \tag{6}$$

The spatial part is given through the 2×2 metric

$$\gamma = e^{-\beta(U,V)}(e^{\alpha(U,V)}\mathrm{d}X \otimes \mathrm{d}X + e^{-\alpha(U,V)}\mathrm{d}Y \otimes \mathrm{d}Y), \tag{7}$$

where α and β are functions of U and V only. This spacetime admits a planar symmetry in the $X - Y$ plane. There exists a canonical null tetrad which consists of the null vectors l and n that are tangent to the null geodesic congruences and perpendicular to the two Killing vectors ξ_X and ξ_Y [11,15]. These form the remaining basis vectors m and its complex conjugate $\bar{m}$. For parallel linear polarizations, they admit the fairly simple form

$$\begin{aligned} l &= 2e^{M}\frac{\partial}{\partial U}, \qquad n = \frac{\partial}{\partial V}, \\ m &= \frac{1+i}{2}e^{\frac{\beta-\alpha}{2}}\frac{\partial}{\partial X} + \frac{1-i}{2}e^{\frac{\beta+\alpha}{2}}\frac{\partial}{\partial Y}, \\ \bar{m} &= \frac{1-i}{2}e^{\frac{\beta-\alpha}{2}}\frac{\partial}{\partial X} + \frac{1+i}{2}e^{\frac{\beta+\alpha}{2}}\frac{\partial}{\partial Y}. \end{aligned} \tag{8}$$

These vectors play a central role in the construction of the curvature scalars that we exploit to understand the properties of the possible singularity.

We consider now the collision of two parallel linear polarized waves. Depending on the functions M, α, and β, the metric describes the following four portions:

(I) a flat region, before any wave has been crossed, that is, $M = \alpha = \beta \equiv 0$; thus, the metric becomes the Minkowski metric
(II) the focusing regions behind the wave in the U direction that focuses in $V = 1$ for which $M = 0$, $\alpha = \alpha(V)$, and $\beta = \beta(V)$, and the metric reads (5)
(III) the focusing regions behind a wave in the V-direction that focuses in $U = 1$ for which $M = 0$, $\alpha = \alpha(U)$, and $\beta = \beta(U)$, and the metric reads (4)
(IV) the curved region that forms after the plane waves have collided for which $M = M(U, V)$, as well as $\alpha = \alpha(U, V)$ and $\beta = \beta(U, V)$

To understand the asymptotic structures of all these patches, the Penrose-Newman scalars serve as a versatile tool. Since we are dealing with vacuum solutions, the relevant curvature stems from the five Weyl scalars defined via a contraction of the Weyl curvature tensor C with the canonical frame fields. Using the metric (6) and the frame fields (8), we find [15]

$$\begin{aligned} \Psi_0 &= -C(l, m, l, m) \\ &= 2ie^{2M}(\partial_U M \partial_U \alpha + \partial_U^2 \alpha - \partial_U \alpha \partial_U \beta), \\ \Psi_1 &= -C(l, m, l, n) = 0, \\ \Psi_2 &= -C(l, m, \bar{m}, n) = -e^{M}\partial_U \partial_V M, \\ \Psi_3 &= -C(l, n, \bar{m}, n) = 0, \\ \Psi_4 &= -C(\bar{m}, n, \bar{m}, n) \\ &= \frac{i}{2}(\partial_V \alpha \partial_V \beta - \partial_V M \partial_V \alpha - \partial_V^2 \alpha). \end{aligned} \tag{9}$$

The Ψ_0 and Ψ_4 scalars describe contributions of transverse waves propagating in the n or l directions, respectively, while Ψ_1 and Ψ_3 denote the respective longitudinal contributions; the scalar Ψ_2 is particular as a Coulomb-like contribution because it contains both derivatives with respect to U and V, while the other scalar only consists of one sort of derivative.

The Weyl scalars allow us to relate the patches of the plane-wave spacetime with the Petrov classification of spacetimes, so are the focusing regions of type N since all but either Ψ_0 or Ψ_4 vanish, while the region before any wave has been crossed is of type O. The curved patch itself does not fit the Petrov classification, but asymptotically, some solutions resemble a Petrov type D spacetime because all except Ψ_2 vanish (or grow slower than Ψ_2). Since plane waves generically lead to a focusing of geodesics, these space times will be geodesically incomplete, i.e., admit singularities [18]. We will see how this affects our results eventually.

In the following, we focus on two particular solutions with different strengths of singularities: the Khan-Penrose solution and the Ferrari-Ibáñez solution [11,19].

III. KHAN-PENROSE SPACETIME

In this section, we consider the collision of two impulsive null waves (δ-distributional profile) that is described by the Khan-Penrose spacetime [4]. We describe both its geometrical properties and the implication for the quantum probes.

A. Khan-Penrose spacetime: Geometry

The particular form of such a spacetime has been studied by Khan and Penrose in [4] and found to be of the form

$$\begin{aligned} g = &-2e^{-M(U,V)}\mathrm{d}U \otimes \mathrm{d}V \\ &+ F^2(U, V)\mathrm{d}X \otimes \mathrm{d}X + G^2(U, V)\mathrm{d}Y \otimes \mathrm{d}Y, \end{aligned} \tag{10}$$

where the metric coefficients are determined by the three functions

$$e^{-M(U,V)} = \frac{s(U, V)^{\frac{3}{2}}}{w(U)w(V)(p(U)q(V) + w(U)w(V))^2}, \tag{11}$$

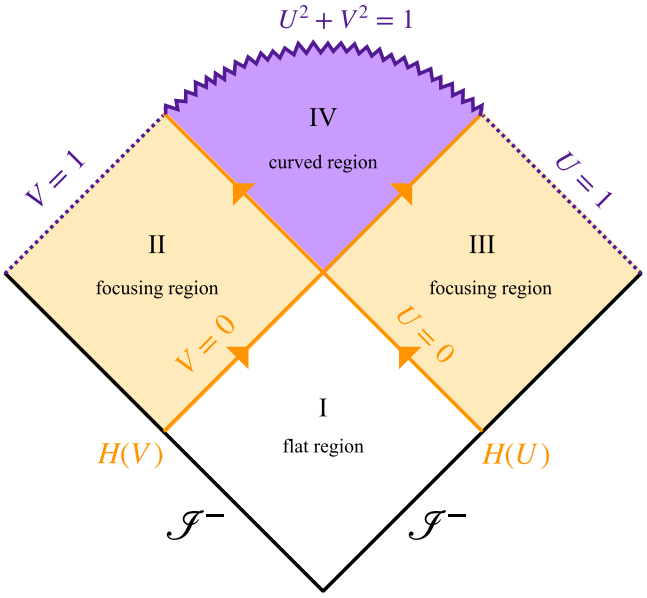


FIG. 1. Penrose diagram of the Khan-Penrose spacetime showing two colliding waves at $U = 0$ and $V = 0$ that cut the spacetime into four portions. In the orange shaded, focusing regions, null geodesics that have crossed the wave are focused at the focal plane $V = 1$ in II, or $U = 1$ in III. These mark the folding singularities. The purple shaded region is curved and admits a spacelike curvature singularity at $U^2 + V^2 = 1$.

$$F^2(U,V) = s(U,V)\frac{1 - p(U)w(V) - q(V)w(U)}{1 + p(U)w(V) + q(V)w(U)}, \tag{12}$$

$$G^2(U,V) = s(U,V)\frac{1 + p(U)w(V) + q(V)w(U)}{1 - p(U)w(V) - q(V)w(U)}. \tag{13}$$

For ease of notation, we have defined the three quantities $w(V) = \sqrt{1-q(V)^2}$, $w(U) = \sqrt{1-p(U)^2}$, and $s(U,V) = 1 - p(U)^2 - q(V)^2$.

By adopting the impulsive wave profile, we can derive the explicit forms of the functions $p(U) = U\Theta(U)$ and $q(V) = V\Theta(V)$. As such, the two waves propagate along $U = 0$ and the other along $V = 0$, thus cutting the manifold into four portions (cf. Fig. 1).

(i) Before crossing any wave: For $U < 0$ and $V < 0$, the three spacetime functions $e^{-M(U,V)} = F^2(U,V) = G^2(U,V) \equiv 1$, and (10) reduces to Minkowski spacetime in light-cone coordinates.

(ii) Crossing one wave: For either $U < 0$ and $V > 0$ or $U > 0$ and $V < 0$, the spacetime is flat again, but geodesics will be focused at the null singularity at either $V = 1$ or $U = 1$, respectively (or $w = 0$).

(iii) Region after the collision: For $U > 0$ and $V > 0$, the spacetime becomes curved, and any geodesic will end in a spacelike curvature singularity at $U^2 + V^2 = 1$ [or $s(U,V) = 0$].

As we can see, the singularity cannot be avoided by any geodesic, and therefore, the spacetime is g-incomplete [18] and not even globally hyperbolic due to the focusing singularities that completely enclose the future development [10].

Since we are mostly interested in the curved region, we will focus on patch IV,[1] except in Sec. V B we will consider backreaction. The curvature properties and in particular the singular asymptotics can be understood by the nonzero components of the Weyl tensor

$$\Psi_0 = \frac{\delta(V)}{w(U)} - \frac{3Uw(U)(w(U)w(V) + UV)}{w(V)^2 s(U,V)^2}, \tag{14}$$

$$\Psi_2 = \frac{(UV + w(U)w(V))^2}{w(U)w(V)s(U,V)^2} - \frac{\mathrm{UV}}{s(U,V)^2}, \tag{15}$$

$$\Psi_4 = \frac{\delta(U)}{w(V)} - \frac{3Vw(V)(w(U)w(V) + UV)}{w(U)^2 s(U,V)^2}, \tag{16}$$

which obviously diverge whenever $U^2 + V^2 = 1$. For the ease of notation, we have defined the quantities w and s as above. Note that the Kretschmann scalar can be composed out of the Weyl scalars via

$$K = R^{abcd}R_{abcd} = \mathrm{Re}(3\Psi_2^2 + \Psi_0\Psi_4), \tag{17}$$

where R_{abcd} denotes the Riemann curvature tensor. Since the Kretschmann scalar is a curvature invariant, the corresponding singularity is not removable. Similar to Schwarzschild spacetime, Ψ_2 diverges at least as fast as the squared reciprocal; such a singularity is considered to be strong in the sense of Tipler [20] and Królak and coworker [21]. From a physics perspective, this region can be understood as the scattered field that is produced by the collision which eventually suffers self-focusing. In this region, also Ψ_0 and Ψ_4 diverge in the same manner.

It should be mentioned that the folding singularity in patches II and III does not fulfill this criterion because, in both, $\Psi_2 \equiv 0$ and either $\Psi_4 = 0$ (in II) or $\Psi_0 = 0$ (in III) while the corresponding other scalar remains nonzero even diverges. Their main divergence can be read off from (14) and (16) to scale with the reciprocal only, that is, $(1 - V^2)^{-1}$ in II and $(1 - U^2)^{-1}$ in III. Since the condition for a strong singularity has not been met, the folding singularity is considered to be weak in the sense of Yurtsever [11].

Considered separately, region IV is a globally hyperbolic spacetime for a spatial slicing that mimics the Schwarzschild interior. For convenience, we define a temporal and a spatial coordinate perpendicular to the symmetry plane as follows [17]:

[1]It is clear that by replacing U with the function $p(U)$ and V with $q(V)$, we can chart the whole spacetime.

$$\xi = U + V \quad \text{and} \quad \eta = U - V, \tag{18}$$

while $X \to X$ and $Y \to Y$. Here, ξ defines a time, while η denotes a spatial coordinate in such a way that the region IV is foliated by Cauchy surfaces, and the past asymptotic boundaries are determined by the impulsive waves. Additionally, in the coordinate chart (ξ, η, X, Y), the metric assumes a diagonal form that facilitates the functional Schrödinger analysis.

B. Khan-Penrose spacetime: Quantum probing

To study the fate of quantum probes, we work in the functional Schrödinger representation of quantum-field theory which treats the evolution of entire field configurations rather than the individual fields. The functional Schrödinger approach is based on the canonical phase-space description which presumes a well-defined Cauchy evolution (or a null evolution) to be complete [22]. The Khan-Penrose spacetime naturally offers the possibility to construct the phase space either in the dual-null or temporal-spatial foliation [23] depending on the geometry of the individual region. For instance, while the singularity structure in regions II and III favors a dual-null foliation, region IV clearly suggests a timelike foliation of spacelike hypersurfaces.

The quantum probing of the null singularity in regions II and III has been performed previously in [3] with the result that the folding singularity is reachable and the quantum field passes the focal plane, although in Sec. V B we will argue that backreaction becomes important in that region. However, in this section, our main focus is on the singularity in the curved region, since this region serves as a Penrose limit to a Schwarzschild black hole.

We start with a review of the temporal-spatial split that is adapted to the geometry of the collision region. We construct the Hamilton operator through a temporal-spatial split [24]: Consider the configuration space parametrized by the field ϕ pulled back to the spatial hypersurface Σ and $(\phi, \pi) \in T^*\mathcal{F}\Sigma$, where $\mathcal{F}\Sigma$ denotes the space of smooth functions on the spatial hypersurface Σ. The Lagrange density $\mathcal{L}$ defines a Lagrange transformation $\Lambda: T\mathcal{F}\Sigma \to T^*\mathcal{F}\Sigma$, with $\Lambda: (\phi, \dot{\phi}) \mapsto (\phi, \frac{\delta \mathcal{L}}{\delta \dot{\phi}}) = (\phi, \pi)$. Now, we construct the $(1+3)$ foliation by using the coordinate chart (ξ, η, X, V), where ξ and η are defined in (18). Then, for a massless, minimally coupled, scalar field, the Hamilton operator becomes

$$\mathcal{H} = \frac{\sqrt{-g_{\xi\xi}}}{2}\left(\frac{\pi^2}{\det(\sigma)} + \sigma(\nabla\phi, \nabla\phi)\right), \tag{19}$$

where we defined the metric σ as the induced metric on the spatial hypersurface Σ. Note, the operators ϕ are the configuration variable in the field-configuration space, while the momentum conjugate to ϕ is given by

$$\pi = \frac{\delta \mathcal{L}}{\delta \dot{\phi}} = \frac{\sqrt{\det(\sigma)}}{\sqrt{-g_{\xi\xi}}}\dot{\phi}, \tag{20}$$

where the overdot notation symbolizes the derivative with respect to ξ. The Hamilton operator rules the dynamics of the field configurations through a functional Schrödinger equation

$$i\partial_\xi \Psi_\xi[\phi] = H_\xi[\pi, \phi]\Psi_\xi[\phi], \tag{21}$$

and the wave functional $\Psi_\xi: \mathcal{F}\Sigma_\xi \to \mathbb{C}$ hence describes the evolution of the field configurations that are pulled back on Σ_ξ. By imposing the canonical quantization prescription $[\pi(x), \phi(x')] = i\delta(x, x')$, we can express $\pi \to -i\frac{\delta}{\delta\phi}$ as the functional derivative operator while ϕ becomes the multiplicative operator $\phi|\Psi\rangle = \langle\phi\rangle_{\mathrm{cl}}|\Psi\rangle$, which yields the classical expectation value $\langle\phi\rangle_{\mathrm{cl}} =: \phi$ as an eigenvalue.

The functional Schrödinger states live in the $\mathcal{L}^2(\mathcal{F}\Sigma_\xi, \mathcal{D}\phi)$ Hilbert space. Note, $\mathcal{D}\phi$ denotes a Lebesgue-like measure over all field configurations. In this formal measure space, the inner product is defined by integrating over all field configurations $\phi \in \mathcal{F}\Sigma_\xi$,

$$\|\Psi_\xi\|^2 = \int_{\mathcal{F}\Sigma_\xi} \mathcal{D}\phi |\Psi_\xi[\phi]|^2. \tag{22}$$

This functional integral can be interpreted as the probability to populate a certain hypersurface Σ_ξ [25]. This is the quantity we use to measure the probability for field quanta to be present on a spatial hypersurface of spacetime. In particular, we are interested in how this probability evolves as the hypersurface approaches the singularity. If this probability vanishes, the corresponding region is not accessible to a quantum field. On the other hand, if it diverges, the backreaction should be taken into account.

The geometries we consider below approach Minkowski space as $\xi \to -\infty$. Accordingly, the in-vacuum can be defined unambiguously [23]. As an initial condition, we then consider a plane wave over this state. The fact that we consider a free field theory, power counting arguments using (19) suggest a Gaussian state for the wave functional

$$\Psi_\xi[\phi] = N(\xi)\mathcal{G}_\xi[\phi] \;\text{ with }\; \mathcal{G}_\xi[\phi] = \exp\left(-\frac{1}{2}[\phi]\mathcal{K}(\xi)[\phi]\right). \tag{23}$$

We observe the two distinct contributions, a field-configuration-dependent exponential $\mathcal{G}_\xi[\phi]$ and a field-independent normalization $N(\xi)$. The functional kernel $\mathcal{K}_\xi: \mathcal{F}\Sigma_\xi \otimes \mathcal{F}\Sigma_\xi \to \mathbb{C}$,

$$[\phi]\mathcal{K}_\xi[\phi] = \iint_{\Sigma_\xi} \mathrm{d}^3\mu(x, x')\phi(x)K(x, x'; \xi)\phi(x') \tag{24}$$

inherits a time dependence from the metric. The measure $\mathrm{d}^3\mu(x,x') = \mathrm{d}^3x\mathrm{d}^3x'\sqrt{\det(\sigma)(x)}\sqrt{\det(\sigma)(x')}$. We will now see that the bilocal kernel function $K(x,x';\xi)$ completely determines the dynamics of the field configurations.

With the Gaussian ansatz (23), the norm factorizes $\|\Psi_\xi\|^2 = |N(\xi)|^2\|\mathcal{G}_\xi[\phi]\|^2$ into the modulus square of the time-dependent function $N(\xi)$ and the field-dependent functional integral over $\mathcal{G}_\xi[\phi]$. Through insertion of (23) into the functional Schrödinger equation (21), one finds the normalization to be

$$N(\xi) = N_0 \exp\left(-\frac{i}{2}\int_{\mathbb{K}} \sqrt{-\det(g)}K(x,x;\xi)\right), \tag{25}$$

where we defined the constant of integration N_0 which encodes the initial conditions. The planar symmetry in $\mathbb{K}$ allows us to perform a Fourier transformation in the spatial coordinates. Thus, the first contribution to the norm $|N(\xi)|^2$ is evaluated as follows:

$$\frac{|N(\xi)|^2}{|N_0|^2} = \exp\left(\int_{\mathbb{K}} \mathrm{d}^4x \int_k \frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^2}\sqrt{-\det(g)}\,\mathrm{Im}(K_{\boldsymbol{k}})(\xi)\right). \tag{26}$$

We observe that the normalization depends on the imaginary part of the kernel, while the norm of the field-dependent part contains its real part

$$\|\mathcal{G}_\xi[\phi]\|^2 = \int \mathcal{D}\phi \exp\left(-[\phi]\mathrm{Re}(\mathcal{K}_\xi)[\phi]\right). \tag{27}$$

When plugging (23) into (21), one obtains a Riccati equation for $K(x,x';\xi)$, which, in turn, can be transformed into the Klein-Gordon equation. As such, we can construct $K(x,x';\xi)$ from the mode solutions [1,26].

Without loss of generality [27], we may assume our probe theory to be a free, minimally coupled, massless scalar field $\phi(x)$ that obeys the Klein-Gordon equation $\Box\phi(x) = 0$. As we see in Fig. 2, there are essentially three distinct regions where the field propagates quite differently. Since the asymptotic regions are flat spacetime, there exists a natural vacuum such that the right-going modes (along U) are Minkowski solutions in I and II, while the left-going modes (along V) are Minkowski in I and III [23].

However, our main interest is in the behavior in the curved patch, and we will therefore construct the functional Schrödinger representation of our quantum-field theory only there. Next, let us solve the Klein-Gordon equation in region IV using the metric (10). For the general form of the metric, the Klein-Gordon equation in dual-null coordinates (U,V,X,Y) reads [23]

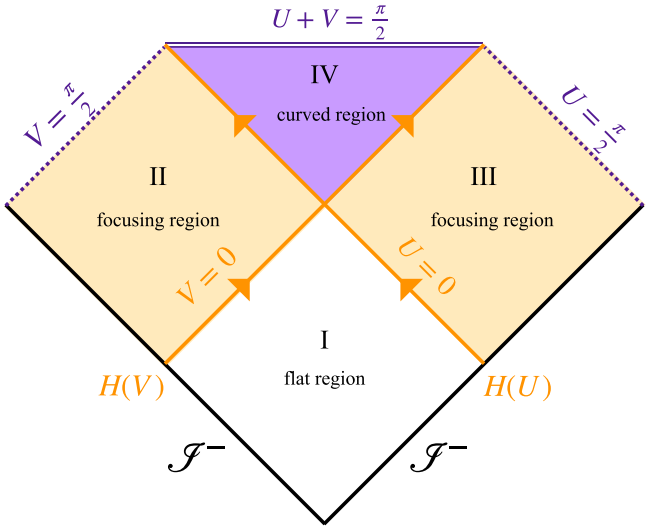


FIG. 2. Penrose diagram of the Ferrari-Ibáñez spacetime showing two colliding waves at $U=0$ and $V=0$ that cut the spacetime into four portions. In the orange shaded, focusing regions, null geodesics that have crossed the wave are focused at the focal plane $V=\frac{\pi}{2}$ in II, or $U=\frac{\pi}{2}$ in III. The purple shaded region is curved and admits a Cauchy horizon at $U+V=\frac{\pi}{2}$.

$$\Box\phi = -e^{M(U,V)}[2\partial_U\partial_V\phi + \partial_U\ln(FG)\partial_V\phi + \partial_V\ln(FG)\partial_U\phi] + \frac{\partial_X^2\phi}{F^2} + \frac{\partial_Y^2\phi}{G^2} = 0. \tag{28}$$

We are particularly interested in the behavior close to the singularity, that is, for $U^2+V^2\to 1$. The form of the metric components is rather complicated, and since our objective is to probe the singularity, we approximate the metric close to the singularity and find

$$g \simeq -\frac{(1-(U^2+V^2))^{\frac{3}{2}}}{U^2V^2}\mathrm{d}U\otimes\mathrm{d}V + \frac{(1-(U^2+V^2))^2}{2}\mathrm{d}X\otimes\mathrm{d}X + 2\mathrm{d}Y\otimes\mathrm{d}Y. \tag{29}$$

It is obvious that the null directions as well as the X direction vanish as we approach the singularity, while the Y direction assumes a finite-sized value. The planar symmetry is reflected in the Killing vector fields ∂_X and ∂_Y, which allows us to perform a Fourier decomposition

$$\phi(U,V,X,Y) = \iint \frac{\mathrm{d}^2\boldsymbol{k}}{4\pi^2}\chi_{\boldsymbol{k}}(U,V)e^{i\boldsymbol{k}\boldsymbol{x}}, \tag{30}$$

where $\boldsymbol{k} = (k_X,k_Y)$ and $\boldsymbol{x} = (X,Y)^T$. With the asymptotic metric (28) and the Fourier decomposition (30), the Klein-Gordon equation becomes, in the near singularity limit,

$$\frac{2U^2V^2}{\sqrt{1-(U^2+V^2)}}\partial_U\partial_V\chi + U\partial_V\chi + V\partial_U\chi + \left(\frac{k_X^2}{1-(U^2+V^2)} + (1-(U^2+V^2))k_Y^2\right)\chi = 0. \quad (31)$$

Note that we suppressed the dependences of χ for ease of notation.

Unfortunately, the coefficients in the differential equation prohibit a factorization due to the still complicated mixing of U and V. Therefore, we change to the coordinate chart (18) to naturally define the phase space. In this coordinate, the metric reads

$$g \simeq -\frac{|1-\frac{\xi^2+\eta^2}{2}|^{\frac{3}{2}}}{(\xi^2-\eta^2)^2}\mathrm{d}\xi\otimes\mathrm{d}\xi + \frac{|1-\frac{\xi^2+\eta^2}{2}|^{\frac{3}{2}}}{(\xi^2-\eta^2)^2}\mathrm{d}\eta\otimes\mathrm{d}\eta + \frac{|1-\frac{\xi^2+\eta^2}{2}|^2}{2}\mathrm{d}X\otimes\mathrm{d}X + 2\mathrm{d}Y\otimes\mathrm{d}Y. \quad (32)$$

From here, we can construct the d'Alembert operator accordingly in its explicit (asymptotical) form. Using again the planar symmetry to the Fourier transform in the X and Y coordinates, (31) then becomes

$$\frac{\xi^2-\eta^2}{\sqrt{\left|1-\frac{\xi^2+\eta^2}{2}\right|}}(\partial_\xi^2\chi - \partial_\eta^2\chi) + \left(\frac{1}{\xi+\eta}+\frac{1}{\xi-\eta}\right)\partial_\xi\chi + \left(\frac{1}{\xi+\eta}-\frac{1}{\xi-\eta}\right)\partial_\eta\chi + \left(\frac{k_X^2}{(\xi^2-\eta^2)\left|1-\frac{\xi^2+\eta^2}{2}\right|} + \left|1-\frac{\xi^2+\eta^2}{2}\right|k_Y^2\right)\chi = 0. \quad (33)$$

Although even in this chart, the differential operator will not be separable either, this system is more easily tractable because the singularity occurs for all X and Y whenever $\xi^2+\eta^2=2$ and the Klein-Gordon operator (33) retains a symmetry $\xi \leftrightarrow \eta$.

Without loss of generality, we can choose the spatial coordinate $\eta=\eta_o$ and constant and study only the limit in ξ close to the singularity. Different choices of η_o only affect the value for ξ at which the singularity occurs. Additionally, for a ξ close enough to the singularity, the spatial motion becomes *de facto* irrelevant. Hence, the choice $\eta_o=0$ simplifies the calculations without spoiling the general argument.

Thus, demanding $\eta_o=0$, we can modify our ansatz for $\chi_k(\xi,\eta)\approx\chi_k(\xi,0)$, which solves the asymptotic Klein-Gordon equation

$$\frac{\xi^2}{|1-\frac{\xi^2}{2}|^{\frac{1}{2}}}\frac{\mathrm{d}^2\chi}{\mathrm{d}\xi^2} + \frac{2}{\xi}\frac{\mathrm{d}\chi}{\mathrm{d}\xi^2} + \left(\frac{k_X^2\xi^{-2}}{|1-\frac{\xi^2}{2}|} + \left|1-\frac{\xi^2}{2}\right|k_Y^2\right)\chi = 0. \quad (34)$$

To construct the limit toward the singularity, we further introduce $\upsilon^2 := |1-\frac{\xi^2}{2}|$. The resulting Jacobi determinant yields $\mathrm{d}\xi = \frac{2\upsilon}{\sqrt{2-2\upsilon}}\mathrm{d}\upsilon$, which then leads to the equation

$$\frac{(2-2\upsilon^2)^{\frac{3}{2}}}{2\upsilon^2}\frac{\mathrm{d}}{\mathrm{d}\upsilon}\left(\frac{\sqrt{2-2\upsilon^2}}{2\upsilon}\frac{\mathrm{d}\chi}{\mathrm{d}\upsilon}\right) + \frac{1}{\upsilon}\frac{\mathrm{d}\chi}{\mathrm{d}\upsilon} + \left(\frac{k_X^2}{\upsilon^2(2-2\upsilon^2)} + \upsilon^2 k_Y^2\right)\chi = 0, \quad (35)$$

where the singularity is located at $\upsilon\to 0$. Hence, when performing the approximation $\upsilon \ll 1$, we can simplify the above equation tremendously. However, to appreciate the leading singularities, we introduce $\epsilon>0$ as a smallness parameter $\upsilon\to\epsilon\upsilon_o$ with υ_o a constant that carries the dimension. As such, $\partial_\upsilon \to \frac{1}{\epsilon}\partial_{\upsilon_o}$; hence, the simplified equation of motion reads after some arrangements

$$\frac{1}{\epsilon^2}\partial_{\upsilon_o}^2\chi + \left(\frac{1}{\epsilon^2\upsilon_o} + \epsilon\upsilon_o^2\right)\partial_{\upsilon_o}\chi + \left(\epsilon\frac{k_X^2\upsilon_o}{2} + \epsilon^5\upsilon_0^5 k_Y^2\right)\chi = 0. \quad (36)$$

Clearly, the dominant terms are the first two terms which are the only surviving ones for $\epsilon\to 0$. The k_Y^2 term can be safely neglected here. Up to $\mathcal{O}(\epsilon^2)$, the solution is given by a linear combination of the hypergeometric function and the Meijer G function

$$\chi_{k_X}(\xi,0) = c_{11}F_1\left(\frac{k_X}{3};1;-\frac{|1-\frac{\xi^2}{2}|^{\frac{3}{2}}}{3}\right) + c_2 \mathrm{G}_{1,2}^{2,0}\left(\begin{matrix}1-\frac{k_X}{3}\\0,0\end{matrix}\middle|\frac{|1-\frac{\xi^2}{2}|^{\frac{3}{2}}}{3}\right). \quad (37)$$

Considering only terms that are singular in the $\epsilon\to 0$ limit, we find a simple k_X-independent logarithmic solution

$$\chi(\xi) = c_1 + \frac{c_2}{2}\ln\left|1-\frac{\xi^2}{2}\right|. \quad (38)$$

This result is of particular interest since the logarithmic solution is also obtained for the modes in an interior Schwarzschild spacetime and a Kasner spacetime close to the singularity [1,25].

To perform the quantum probing, we need to explicitly construct the bilocal kernel function (24). The mode-sum decomposition with Huygens's principle and Abel's identity allows us to express the kernel in terms of the mode function $\chi_k(\xi,0)$ as follows [1,26]:

$$K_{\boldsymbol{k}}(\xi) = -\frac{i}{\sqrt{-\det(g)}}\partial_\xi \ln\left(\frac{\chi_{\boldsymbol{k}}(\xi,0)}{\chi_{\boldsymbol{k}}(\xi_o,0)}\right), \tag{39}$$

where ξ_o is an arbitrary initial value.[2] In our approximation of a fixed $\eta_o \ll \xi$, we find $\det(g) \sim -|1-\frac{\xi^2}{2}|^5/\xi^4$. Then, taking the solution (37), we can derive a near singularity approximation of (39)

$$K(\xi) \sim \frac{i6\sqrt{2}}{|1-\frac{\xi^2}{2}|^{7/2}(3\ln|1-\frac{\xi^2}{2}|-i4\pi)}. \tag{40}$$

We observe that, in contrast to the folding singularity of a single plane wave [3], the asymptotic kernel becomes independent of any momentum and any constant c_1 or c_2. This feature is reminiscent of the solution we find for Schwarzschild spacetime [1] and Kasner universes [25].[3]

In spite of the similarities to the Schwarzschild singularity, the approach toward the singularity and the strength are not exactly identical, which may have consequences for the wave functional. The first step for the quantum probing will be performed through evaluating the normalization (26) close to the singularity given by the integral of the imaginary part,

$$\ln\left(\frac{|N(\xi)|^2}{|N_0|^2}\right) \sim \mathcal{V}_{\rm PS}\int_\xi \frac{\sqrt{2}\mathrm{d}\xi}{|1-\frac{\xi^2}{2}|\ln|1-\frac{\xi^2}{2}|} \sim \mathcal{V}_{\rm PS}\ln\left(\ln\left|1-\frac{\xi^2}{2}\right|\right). \tag{41}$$

Here, $\mathcal{V}_{\rm PS} = \Xi_{\boldsymbol{k}}\mathrm{vol}(\Sigma_\xi)$ denotes the regularized phase-space volume, where Ξ_k reflects the regularized momentum contribution, and the integral over the spatial hypersurface is denoted by $\mathrm{vol}(\Sigma_\xi)$. Note, the value of $|N_0|^2$ is chosen such that the constant of integration cancels. When exponentiated, the normalization function reads

$$\frac{|N(\xi)|^2}{|N_0|^2} \sim \left|1-\frac{\xi^2}{2}\right|^{\mathcal{V}_{\rm PS}}. \tag{42}$$

The value of $N(\xi)$ is basically determined by the argument $1-\frac{\xi^2}{2}$ because the phase-space volume $\mathcal{V}_{\rm PS}$ is a semi-positive- definite object. Its value is defined through the hypersurface volume $\mathrm{vol}(\Sigma_\xi)$ and the momentum space volume $\Xi_{\boldsymbol{k}}$. Let us first analyze $\mathrm{vol}(\Sigma_\xi)$. To this aim, we consider the behavior of the induced metric essentially provided through a pullback of (32). As one can read off from the metric, the Y component stays finite while the remaining spatial components degenerate to zero. Hence, $\mathrm{vol}(\Sigma_\xi) \to 0$ for ξ approaching the singularity. In the limit where ξ approaches the singularity

$$\lim_{\xi\to\sqrt{2}}|N(\xi)|^2 = \lim_{\xi\to\sqrt{2}}\left|1-\frac{\xi^2}{2}\right|^{\mathcal{V}_{\rm PS}} = 1, \tag{43}$$

since $\lim_{\xi\to\sqrt{2}}\mathrm{vol}(\Sigma_\xi) = 0$. For the exponential term, we evaluate the functional integration over all $\phi \in \mathcal{F}\Sigma_\xi$. This quantity is proportional to the real part of the kernel

$$\|\exp(-[\phi]\mathrm{Re}(\mathcal{K}(\xi))[\phi])\| = \sqrt{\frac{C}{\mathrm{Det}[-\det(g)\mathrm{Re}(K_{\boldsymbol{k}}(\xi))]}} = \bar{C}\left|1-\frac{\xi^2}{2}\right|^{-\frac{3A}{2}}\ln^A\left|1-\frac{\xi^2}{2}\right|. \tag{44}$$

We have defined the constants C and $\bar{C}$ to absorb all numerical factors. Here, the quantity A denotes the number of momentum eigenstates within the regularized momentum space. This quantity can be regularized via a Riemann zeta function: Effectively, A stems from the k integral in the kernel; therefore, to regularize A, we need to regularize $\int \mathrm{d}k$ which, after discretization, turns our task into regularizing $\sum_{n=1}^{\infty} 1$. By analytic continuation, $A \to \zeta(0)$ which is $\zeta(0) = -\frac{1}{2}$. Since, the first term in (44) dominates for $\xi \to \sqrt{2}$, we find that norm

$$\lim_{\xi\to\sqrt{2}}\|\Psi_\xi[\phi]\| \propto \lim_{\xi\to\sqrt{2}}\left|1-\frac{\xi^2}{2}\right|^{\frac{3}{4}+\mathcal{V}_{\rm PS}} = 0. \tag{45}$$

In this sense, the quantum probes recover the result in Schwarzschild spacetime and the probability of finding quanta near the curvature singularity of the Khan-Penrose spacetime approaches zero.

IV. WEAKLY SINGULAR COLLIDING WAVE SOLUTIONS

The Khan-Penrose spacetime is one representation of colliding plane-wave spacetimes; there exists actually a plethora of colliding plane-wave spacetimes, e.g., Chandrasekhar-Xanthopoulos [7], Bell-Székeres [6], and the Ferrari-Ibáñez spacetime [5,11]. The latter is of particular interest, since it provides a rich family of different geometries. We will now consider an unusual case, namely, the degenerate Ferrari-Ibáñez solution which features a weak singularity in patch IV. The Penrose diagram can be found in Fig. 2.

[2]This kernel is related to the Wightman function as being twice its inverse [26].

[3]This apparently universal feature of k independence while approaching the singularity is reminiscent of the Belinskii-Khalatnikov-Lifshitz conjecture [28,29] because it suggests that the spatial contribution reduces only to a contact term while the temporal one diverges [25].

A. Ferrari-Ibáñez spacetime: Geometry

The degenerate Ferrari-Ibáñez spacetime is found by multiplying F^2 by $1/(1-(U^2+V^2))$ and G^2 by $1-(U^2+V^2)$ in (10). This modification changes its nature completely. Considering the Weyl scalars

$$\Psi_0 = \frac{\delta(V)}{(1+U)^2 w(U)} - \frac{3}{(1+Uw(V)+Vw(U))^3},$$
$$\Psi_2 = \frac{1}{(1+Uw(V)+Vw(U))^3},$$
$$\Psi_4 = \frac{\delta(U)}{(1+V)^2 w(V)} - \frac{3}{(1+Uw(V)+Vw(U))^3}, \tag{46}$$

and comparing them with the Khan-Penrose solution, we immediately see that they admit no longer a singularity at $U^2+V^2=1$, although this divergence is still present in the metric. In fact, all Weyl scalars remain finite over the whole manifold (except for the path where the impulsive waves travel). This suggests that the former curvature singularity has been replaced by a coordinate singularity. In fact, the region $U^2+V^2=1$ describes now a Cauchy horizon.[4] We will see if and in which sense our previous result persists.

For the sake of the calculations, we will switch to a slightly different parametrization that expresses the metric coefficients through trigonometric functions for which the metric becomes more compact

$$e^{-M(U,V)} = 2(1+\sin(p(U)+q(V))),$$
$$F^2(U,V) = \frac{1-\sin(p(U)+q(V))}{1+\sin(p(U)+q(V))}, \tag{47}$$
$$G^2(U,V) = (1+\sin(p(U)+q(V)))^2\cos^2(p(U)-q(V)),$$

where $p(U)$ and $q(V)$ are defined as before. The geometry distinguishes again four patches (cf. Fig. 2 for the Penrose diagram), while I–III resemble essentially those in the Khan-Penrose spacetime. Region IV can still be understood as a curved region; however, now its future is limited by a Cauchy horizon instead of a singularity.

The above representation of the metric is best suited to analyze the curved spacetime region because IV remains globally hyperbolic. We adapt our description to this via the coordinate transformation (18) such that the curved patch admits the metric

$$g = -(1+\sin(\xi))[\mathrm{d}\xi\otimes\mathrm{d}\xi - \mathrm{d}\eta\otimes\mathrm{d}\eta] + \frac{1-\sin(\xi)}{1+\sin(\xi)}\mathrm{d}X\otimes\mathrm{d}X + (1+\sin(\xi))^2\cos^2(\eta)\mathrm{d}Y\otimes\mathrm{d}Y. \tag{48}$$

As before, the coordinates induce a natural $(1+3)$ decomposition where now $0\le\xi<\frac{\pi}{2}$ and $-\frac{\pi}{2}\le\eta\le\frac{\pi}{2}$, while X and Y remain the same. In this parametrization, the geodesic boundary is located at $\xi=\frac{\pi}{2}$ if we set $\eta=\eta_o=0$ as before. However, this boundary is not realized as a curvature singularity but a Cauchy horizon. This is obvious because none of the Weyl scalars diverge with the consequence that the Kretschmann scalar (17) remains finite as well.

B. Ferrari-Ibáñez spacetime: Quantum probing

The quantum probing of the degenerate Ferrari-Ibáñez solution involves steps very similar to the previous section. Therefore, we will report only the important steps and refer to the relevant equations if necessary. To solve the Klein-Gordon equation in the curved patch, we work in the coordinate chart (48), for which the d'Alembert operator becomes separable such that

$$\phi(\xi,\eta,X,Y) = \int\frac{\mathrm{d}^2\boldsymbol{k}}{4\pi^2}\sum_\alpha \psi_\alpha(\xi;\boldsymbol{k})\varrho_\alpha(\eta;\boldsymbol{k})e^{i\boldsymbol{k}\boldsymbol{x}}, \tag{49}$$

where we defined the separation constant α, plus the two-dimensional vectors $\boldsymbol{k}=(k_X,k_Y)^T$ and $\boldsymbol{x}=(X,Y)^T$. This feature simplifies the calculations tremendously compared to the nonseparable Khan-Penrose d'Alembert operator. The two ordinary differential equations for $\psi_\alpha(\xi;\boldsymbol{k})$ and $\varrho_\alpha(\eta;\boldsymbol{k})$ read [23]

$$\frac{\mathrm{d}^2\psi_\alpha}{\mathrm{d}\xi^2}(\xi;\boldsymbol{k}) - \tan(\xi)\frac{\mathrm{d}\psi_\alpha}{\mathrm{d}\xi}(\xi;\boldsymbol{k}) + \left(\alpha + \frac{k_X^2(1+\sin(\xi))^4}{\cos^2(\xi)}\right)\psi_\alpha(\xi;\boldsymbol{k}) = 0, \tag{50}$$

$$\frac{\mathrm{d}^2\varrho_\alpha}{\mathrm{d}\eta^2}(\eta;\boldsymbol{k}) - \tan(\eta)\frac{\mathrm{d}\varrho_\alpha}{\mathrm{d}\eta}(\eta;\boldsymbol{k}) + \left(\alpha - \frac{k_Y^2}{\cos^2(\eta)}\right)\varrho_\alpha(\eta;\boldsymbol{k}) = 0. \tag{51}$$

Since we are particularly interested in the vicinity of the Cauchy horizon, i.e., around $\xi\approx\frac{\pi}{2}$, we can expand the trigonometric functions in (50) to $\mathcal{O}(\xi-\frac{\pi}{2})$ such that we end up with the asymptotically expanded differential equation

$$\frac{\mathrm{d}^2\psi_\alpha}{\mathrm{d}\xi^2}(\xi;\boldsymbol{k}) + \frac{1}{\xi-\frac{\pi}{2}}\frac{\mathrm{d}\psi_\alpha}{\mathrm{d}\xi}(\xi;\boldsymbol{k}) + \left(\frac{16k_X^2}{(\xi-\frac{\pi}{2})^2} + \alpha - \frac{32k_X^2}{3}\right)\psi_\alpha(\xi;\boldsymbol{k}) = 0. \tag{52}$$

Clearly, the above equation is a Bessel differential equation, and therefore we find as a solution the Bessel function of the first and second kinds

[4] For a Cauchy surface $\mathcal{S}$, a future Cauchy horizon is defined via $\overline{D^+(\mathcal{S})} - I^+(\mathcal{S})$, that is, the future boundary of the future domain of dependence $D^+(\mathcal{S})$ [18]. We defined the closed set $\overline{D^+(\mathcal{S})} \coloneqq D^+(\mathcal{S}) \cup \partial D^+(\mathcal{S})$, where $\partial D^+(\mathcal{S})$ denoted the boundary.

$$\psi_\alpha(\xi;\boldsymbol{k}) = c_1 \mathrm{J}_{i4k_X}\left(-\frac{i}{3}\sqrt{96k_X^2-9\alpha}\left(\xi-\frac{\pi}{2}\right)\right) + c_2 \mathrm{Y}_{i4k_X}\left(-\frac{i}{3}\sqrt{96k_X^2-9\alpha}\left(\xi-\frac{\pi}{2}\right)\right). \quad (53)$$

To get a better understanding of their behavior at the singularity, we consider a small argument expansion, or alternatively perform an asymptotic expansion and only evaluate (52) for singular coefficients; both cases yield

$$\psi_\alpha(\xi;\boldsymbol{k}) \sim c_1 e^{i4k_X \ln(\frac{\pi}{2}-\xi)} + c_2 e^{-i4k_X \ln(\frac{\pi}{2}-\xi)}. \quad (54)$$

This solution is familiar from the Schwarzschild horizon where, due to the coordinate singularity of the Schwarzschild coordinates, the wave gets infinitely blueshifted. Since the Ferrari-Ibáñez singularity manifests itself as a Cauchy horizon, we observe a similar blueshifting in the exponent. On the other hand, this suggests that we can continue this solution across the Cauchy horizon by analytically continuing the logarithmic term $i4k_X \ln(\frac{\pi}{2}-\xi) \to i4k_X \ln|\xi-\frac{\pi}{2}| \pm 4\pi k_X$, which is analogous to Unruh's prescription [30]. To close this analysis of the Klein-Gordon equation, let us present the solution to Eq. (51) for $\varrho_\alpha(\eta;\boldsymbol{k})$, that is,

$$\varrho_\alpha(\eta;\boldsymbol{k}) = \frac{z_1 {}_2F_1(\frac{B_-^1}{4},\frac{B_+^1}{4}(1-2k_Y+\beta);1-l;\cos^2(\eta))}{i^{k_Y}\cos^{k_Y}(\eta)} + i^{k_Y} z_2 \cos^{k_Y}(\eta) {}_2F_1\left(\frac{B_-^2}{4},\frac{B_+^2}{4};l+1;\cos^2(\eta)\right), \quad (55)$$

where we introduced $\beta=\sqrt{4\alpha+1}$, $B_\pm^1 = 1-2k_Y \pm \beta$, and $B_\pm^2 = 1+2k_Y \pm \beta$ for the ease of notation. It has been shown that when properly normalized, these modes form a complete orthonormal basis [23]

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathrm{d}\eta \cos(\eta)\varrho_\alpha(\eta;k_Y)\varrho^*_{\alpha'}(\eta;k'_Y) = \delta_{\alpha\alpha'}\delta(k_Y-k'_Y). \quad (56)$$

Since there is no particularly interesting limit associated with the η coordinate, we close the discussion here.

In the following, we construct the bilocal kernel function $K_{\boldsymbol{k}}(\alpha;\xi)$. As before, we express the kernel via (39) in terms of the mode function $\psi_\alpha(\xi;\boldsymbol{k})$. Since we will work in the vicinity of the singularity, we use the asymptotic mode solution in (52) and find for the bilocal kernel function

$$K_{\boldsymbol{k}}(\alpha;\xi) = \frac{k_X}{(\xi-\frac{\pi}{2})^2\cos(\eta)}\left(\frac{2c_2}{c_2+c_1(\xi-\frac{\pi}{2})^{i8k_X}}-1\right). \quad (57)$$

As we see, $K_{\boldsymbol{k}}(\alpha;\xi)$ diverges quadratically at the Cauchy horizon and remains k_X dependent. Note, to get (57), we expanded the metric determinant close to the value $\xi=\frac{\pi}{2}$, which yields

$$\sqrt{-\det(g)} = (1+\sin(\xi))^2\cos(\xi)\cos(\eta) \sim 4\left(\frac{\pi}{2}-\xi\right)\cos(\eta). \quad (58)$$

Let us now study how this inflicts the dynamics of the field configurations in the wave functional. In particular, we ask whether the field configurations can migrate onto or even across the $\xi=\frac{\pi}{2}$ hypersurface.

We start with the derivation of the normalization function $|N(\xi)|^2$. As follows from (26), this quantity involves an integral of the imaginary part of the kernel. Evaluated in the vicinity of the Cauchy horizon, the integral assumes the following form:

$$\int \mathrm{d}^4x \int_k \frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^2}\sqrt{-\det(g)}\,\mathrm{Im}(K_{\boldsymbol{k}}(\alpha;\xi)) = \frac{\bar{C}_2\mathcal{V}_{\mathrm{PS}}}{4\pi^2}\int_\xi \frac{\mathrm{d}\xi}{\xi-\frac{\pi}{2}} = \frac{\bar{C}_2\mathcal{V}_{\mathrm{PS}}}{4\pi^2}\ln\left|\xi-\frac{\pi}{2}\right|. \quad (59)$$

In combination with the measure, (59) amounts to a logarithmic dependence, thus, rendering $|N_\xi|^2 \propto |\xi-\frac{\pi}{2}|^{\bar{C}_3\mathcal{V}_{\mathrm{PS}}}$. Its behavior is almost fully determined by the quantities in the phase-space volume, that is, the momentum space parameter Ξ_k and the volume of the hypersurface $\mathrm{vol}(\Sigma_\xi)$. For simplicity, we choose $\eta=0$ without loss of generality.

Let us first discuss the volume of the hypersurface. For this quantity, we evaluate its limit toward $\xi=\frac{\pi}{2}$ by considering a unit volume on Σ_ξ. For $\xi\to\frac{\pi}{2}$, the η as well as the Y direction assume a finite value, and the X direction shrinks to zero; thus, the unit volume is not conserved and becomes eventually zero, i.e., $\lim_{\xi\to\frac{\pi}{2}}\mathrm{vol}(\Sigma_\xi)=0$, which is similar for plane-wave space times and thus for the portions II and III. Since $\mathrm{vol}(\Sigma_\xi)\to 0$, we may conclude $|N(\xi)|^2\to 1$ at the Cauchy horizon.

To fully sustain this verdict, we need to analyze the momentum space quantity Ξ_k. Since there is a choice for the constants of integration, they can in principle admit a $\boldsymbol{k}$ dependence. Assuming this was not the case, Ξ_k contains the integral over k_X. Since k_X is an odd function integrated over a symmetric interval, we can conclude that Ξ_k vanishes. This is consistent with the popular choice of vacuum in this spacetime which sets $c_1\equiv 0$ and $c_2\in\mathbb{R}$ in (54) [23,31]. On the other hand, the momentum space volume remains independent of any coordinate value; thus, its value is independent of the hypersurface and may therefore be ignored. As a consequence, the kernel function admits no imaginary part; thus, the integral yields zero and $N(\xi)\to 1$. Therefore, the norm for any field configuration to reach the singularity at $\xi=\frac{\pi}{2}$ becomes 1.

Moving to the second contribution, we perform the functional over the field configurations $\phi \in \mathcal{F}\Sigma_\xi$, which yields for the asymptotic kernel

$$\begin{aligned}&\| \exp\left(-[\phi]\mathrm{Re}(\mathcal{K}(\xi))[\phi]\right)\| \\ &= \left|\frac{C}{\mathrm{Det}[-\det(g)\mathrm{Re}(K(\xi))]}\right|^{\frac{1}{2}} \propto \mathrm{Det}^{-\frac{1}{2}}(\bar{C}_1 k_X),\end{aligned} \tag{60}$$

with C and all (upcoming) $\bar{C}$ collecting the appearing constants. As we can immediately observe, the ξ dependence in the determinant (58) cancels the one in (57) exactly, leaving an effectively constant functional determinant. The functional determinant is evaluated via the ζ-function method which for any quantity $\mathcal{Q}$ with eigenvalues λ_n [32,33]

$$\mathrm{Det}(\mathcal{Q}) = \prod_n \lambda_n = \exp\left(-\frac{\mathrm{d}}{\mathrm{d}s}\zeta(s;\mathcal{Q})\right)\Big|_{s=0}, \tag{61}$$

with $\zeta(s) \coloneqq \sum_n \frac{1}{\lambda_n^s}$. Notice, $\mathrm{Re}(s) > 1$ for the sum to converge. Combining the two formulas shows that the functional determinant can be understood as a product of the eigenvalues $\bar{C}_1 k_X$. From the explicit definition of the ζ function in (61), it follows that

$$\begin{aligned}\zeta(s) &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\mathrm{d}k_X}{(\bar{C}_1 k_X)^s} \\ &= \lim_{k_X\to\infty} -\frac{k_X((\bar{C}_1 k_X)^s + (-\bar{C}_1 k_X)^s)}{(1-s)(-\bar{C}_1 k_X)^{2s}} = 0.\end{aligned} \tag{62}$$

The last equation only holds under the condition $\mathrm{Re}(s) > 1$ which is consistent with the condition for the ζ function itself. Since the ζ function is zero, so will be the derivative. With (61), it follows then immediately that the functional determinant becomes 1 in the vicinity of the Cauchy horizon. Hence,

$$\lim_{\xi\to\frac{\pi}{2}}\|\Psi_\xi[\phi]\| = 1 \tag{63}$$

for the degenerate Ferrari-Ibáñez solution. As a consequence, the quantum field will reach the Cauchy horizon and may be able to pass through (when backreactions are neglected).

V. CLASSIFICATION AND CONSEQUENCES

Collecting the results from different patches of the colliding wave spacetime and previous analyses of the Schwarzschild [1] and Kasner spacetimes [25], we observe an interesting pattern when quantum probes approach a singularity. All three spacetimes—patch IV in Khan-Penrose spacetime, Schwarzschild spacetime, and Kasner spacetime—admit a complete evolution such that no field configuration can reach the geodesic border. In contrast, the folding singularities in patches II and III as well as the spacelike singularity in patch IV of the Ferrari-Ibáñez spacetime can be reached.

A. The Ψ_2 conjecture

Although all the complete spacetimes differ in symmetry, one important similarity marks the divergence of the Coulomb part of the Weyl tensor Ψ_2. While for the Khan-Penrose patch, Ψ_0, Ψ_2, as well as Ψ_4 diverge at the singularity, Ψ_2 is the only nonvanishing contribution in the Schwarzschild interior. Since, there, Ψ_2 diverges as well, this suggests a pattern. The analyses of the folding singularities in patches II and III agree with the result for the single plane wave in [3]; i.e., the field configurations will be able to reach the focal plane. For both patches $\Psi_2 \equiv 0$, while only one of the transversal components, either Ψ_0 or Ψ_4, diverge while the respective other vanishes.

This shows that the transversal components Ψ_0 and Ψ_4 albeit divergent, do not inflict a depletion of a wave functional's norm but a divergent Ψ_2 will. This is substantiated by the result from the Ferrari-Ibáñez spacetime where none of the Weyl scalars diverges in patch IV, and the wave functional is able to propagate the field configurations onto the geodesic boundary. At this point, we would like to formulate our conjecture that determines the evolution of quantum field in singular space times (with nonvanishing Weyl curvature).

Conjecture (quantum Weyl conjecture). The evolution of quantum probes is shielded from the curvature singularity if the singularity manifests as a divergence of the Penrose-Newman Weyl scalar Ψ_2.

Since the Ψ_2 Penrose-Newman scalar is commonly referred to as the Coulomb part of the curvature, this interpretation enforces the idea of a gravitational hydrogen atom for a spacelike singularity. In both cases, the Coulomb part of the potential inflicts a probabilistic obstacle shielding the quantum degree of freedom from the singularity. Note, due to considering only matter-free spacetimes, there may be the caveat that also Ricci Penrose-Newman scalars[5] contribute, but at least for the Khan-Penrose spacetime, the important contributions

$$\begin{aligned}\Lambda &= \frac{R}{24} = \frac{e^M}{24}(\partial_U\Omega\partial_V\Omega - 2\partial_U\partial_V M - 2\partial_U\partial_V\Omega), \\ \Phi_{11} &= \frac{1}{4}(Ric(l,n) + Ric(m,\bar{m})) \\ &= \frac{1}{4}\partial_U\partial_V M + \frac{3}{16}\partial_U\partial_V\Omega - \frac{3}{32}\partial_U\Omega\partial_V\Omega,\end{aligned} \tag{64}$$

[5]Cosmological Friedmann-Lemaître-Robertson-Walker spacetimes have not yet been analyzed within the functional Schrödinger approach, but judging from the Heisenberg picture, the singularity can be passed [34,35] even for arbitrary sectional curvature [36].

with both $\Lambda \supset \Psi_2$ and $\Phi_{11} \supset \Psi_2$; hence, they contribute similarly. The other Penrose-Newman scalars related to the Ricci tensor contain only one type of derivative; hence, they are reminiscent of Ψ_0 or Ψ_4.

One may wonder why Ψ_2 is more significant than Ψ_0 and Ψ_4. The answer lies in the invariance of these objects. While Ψ_2 is Lorentz invariant, Ψ_0 and Ψ_4 have boost weight -2 and $+2$. Hence, a divergence in one of the two can be suppressed by a sufficiently strong boost. A divergence of the Coulomb part given by Ψ_2 cannot be mitigated through a Lorentz transformation, and to our knowledge, we could not find a spacetime that features a divergent Ψ_0 or Ψ_4 while keeping a finite Ψ_2.

B. Backreaction

Since colliding plane waves provide a (sort of) idealization of high energy scattering on a single plane wave (which in turn can be thought of as an idealization of a trans-Planckian $2 \to 2$ scattering), it would be interesting to analyze whether taking backreaction into account, one can interpolate between the folding singularity of a single plane wave and the spacelike singularity of the Khan-Penrose spacetime.

A convenient approach is to exploit the semiclassical Einstein equation $G = 8\pi\langle T\rangle_{\text{ren}}$ that relates the Einstein tensor with the stress-energy tensor. Already, this system imposes severe technical difficulties; hence, we rely on a more pragmatic approach using our conjecture. The following approach should be understood as a classical (or quasiclassical) estimate of the backreactions. A full determination would require specifying the quantum state and performing a full stress-energy tensor renormalization procedure. However, we show how the classical stress-energy tensor provides a guideline given a natural choice for the vacuum state in a pp-wave spacetime. We start with the explicit form for the stress-energy tensor of a free, massless scalar field

$$T = \nabla\phi \otimes \nabla\phi - \frac{g}{2} g(\nabla\phi, \nabla\phi) \tag{65}$$

in a given background. Based on our previous findings, we are interested which contributions of the stress-energy tensor become relevant to understand the need for backreaction when approaching the singularity. It should be understood that the expectation value $\langle T_{ab}\rangle_{\text{ren}}$ of the renormalized stress-energy tensor differs in general from (65) due to normal ordering that involves a choice of vacuum.

In order to probe the stability of the folding singularity, we send a field $\varphi(x)$ representing an in-state with respect to vacuum, that is not *per se* a test field, from region I through the plane wave $H(V)$ (cf. Fig. 3). As an initial metric, we use the plane-wave metric that describes patches I and II in Fig. 2,

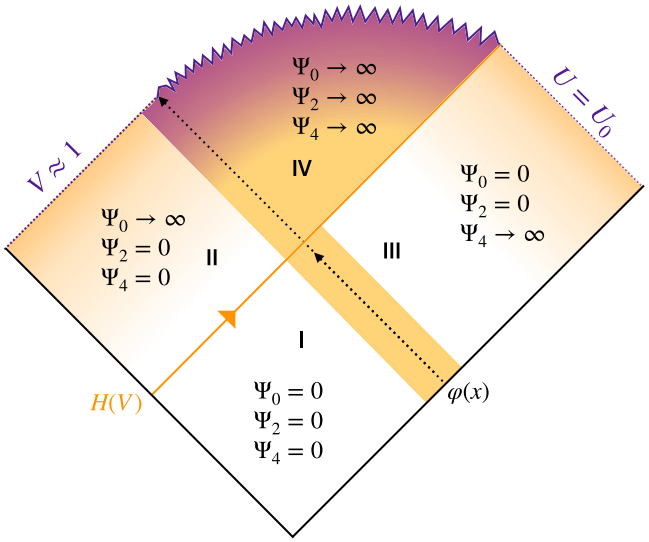


FIG. 3. The null-field wave packet $\varphi(x)$ (depicted as the orange bar) is radiated across an impulsive plane wave $H(V)$ into region II. After crossing the plane wave, the field scatters and creates a nonvanishing, actually diverging, Ψ_2 Weyl scalar through backreaction. The folding singularity in II transforms into a spacelike singularity and therefore into a Khan-Penrose spacetime. The focusing that occurs through the wave packet itself may lead to a folding singularity at some $U_0 \gg 1$.

$$g_{\text{pw}} = -2\mathrm{d}U \otimes \mathrm{d}V + \gamma_{\text{pw}}(V). \tag{66}$$

Since the pp-wave propagates along $V = 0$, the geometry focuses in the X direction along the V coordinate.

Then, sending a $\varphi(x)$ experiences a spacetime of the form of patch II after crossing $H(V)$. We can obtain a physical intuition of the backreaction recalling that T_{XX} represents the (anisotropic) stress component in the X direction. Since this direction collapses due to focusing, T_{XX} diverges, thus triggering a backreaction. Alternatively, one could also interpret the scalar profile as an incoming wave itself, with profile $H(U)$. Hence, for the backreacted metric, we make the particular ansatz [12,37]

$$\tilde{g}_{\text{BR}} = -2e^{-M(U,V)}\mathrm{d}U \otimes \mathrm{d}V + e^{\Omega(U,V)}\mathrm{d}X \otimes \mathrm{d}X + \mathrm{d}Y \otimes \mathrm{d}Y. \tag{67}$$

We could also choose a more general form of the spacetime to describe an inhomogeneous profile of the incoming scalar, but the above form serves best for illustrational purposes. Derivation of the curvature tensors yields the following nonvanishing components of the Einstein tensor:

$$G_{UU} = 2\partial_U M \partial_U \Omega - 4(\partial_U \Omega)^2 + 2\partial_U^2 \Omega, \tag{68}$$

$$G_{VV} = 2\partial_V M \partial_V \Omega - 4(\partial_V \Omega)^2 + 2\partial_V^2 \Omega, \tag{69}$$

$$G_{\text{UV}} = 4\partial_V \Omega \partial_U \Omega - 2\partial_V \partial_U \Omega, \tag{70}$$

$$G_{XX} = e^{M-4\Omega}\partial_U\partial_V M, \tag{71}$$

$$G_{YY} = -e^{M}(8\partial_U\Omega\partial_V\Omega - \partial_U\partial_V M - 4\partial_U\partial_V\Omega). \tag{72}$$

The backreacted metric features a U dependence taking into account the influence of the field itself, but how consistent is such an assumption? The answer depends not only on local curvature invariants, but also on global properties, like the particular vacuum state. Gibbons [38] argued that there exists a natural choice for vacuum in such spacetimes, that is an ingoing vacuum state, with respect to a scalar field that propagates along a Killing direction. For such vacuum, the expectation value of the renormalized stress-energy tensor does not contribute [38–40]. Albeit particular, this setup seems natural to understand how an ingoing null field backreacts in a pp-wave spacetime.

To understand the contribution from the incoming scalar, let us assume a (weak) U dependence for the field $\varphi(x)$.[6] Then, a plane wave with U dependence

$$\varphi(x) = \int_{\vec{k}} \frac{\mathrm{d}^3k}{(2\pi)^3} f_{\vec{k}}(V) e^{-ik_U U} e^{i\boldsymbol{kx}} \tag{73}$$

will imply a nonzero T_{UU} component $T_{UU} \propto k_U^2$ by (65). The Einstein equation then implies that a nonvanishing T_{UU} sources a nonvanishing G_{UU}. Since G_{UU} contains only terms proportional to $\partial_U\Omega$ and $\partial_U M$, the backreacted metric must then depend explicitly, but weakly, on U. In other words, even the slightest U dependence of $\varphi(x)$ will bring the system closer to a Khan-Penrose-type scenario. So far, we have only shown that $\Omega(U,V)$ carries an explicit U dependence.

Recalling the form of the solution to the Klein-Gordon equation on a plane-wave background [17,41]

$$\varphi(x) = \frac{2}{\sqrt[4]{\det(\gamma)}} e^{-ik_U U} e^{i\boldsymbol{kx}}, \tag{74}$$

where we have defined the spatial metric γ to be of the form

$$\gamma = \begin{pmatrix} e^{4\Omega(U,V)} & 0 \\ 0 & 1 \end{pmatrix}. \tag{75}$$

In the above equation, we used the fact that the metric function Ω is both U and V dependent. This implies an interesting consequence, because then, the T_{XX} and T_{YY} components are nonvanishing:

$$T_{XX} = e^{M-2\Omega}\partial_U\Omega\partial_V\Omega, \tag{76}$$

$$T_{YY} = e^{M+2\Omega}\partial_U\Omega\partial_V\Omega, \tag{77}$$

which in turn requires G_{XX} and G_{YY} to be nonvanishing. Analyzing the form of G_{XX}, we see that it scales like $e^{-2\Omega}\Psi_2$. The focusing singularity of the initial metric has been chosen to occur in the XX component; hence, $\Omega \to -\infty$ at the folding singularity V_0. Here, V_0 is close to 1 with a small deviation implied by the weak U dependence of Ω. Since $T_{YY} = e^{4\Omega}T_{XX}$, this contribution is suppressed by an exponential factor of Ω; therefore, we may assume that G_{YY} remains subdominant.

Knowing that T_{XX} diverges then implies that the G_{XX} component of the Einstein tensor diverges. As a consequence, the metric function M is actually a function of U and V. Furthermore,

$$\begin{aligned}\Psi_2 &= e^{2\Omega}G_{XX} = -8\pi e^{2\Omega}T_{XX} \\ &= -8\pi e^{M(U,V)-2\Omega(U,V)}\partial_U\Omega(U,V)\partial_V\Omega(U,V)\end{aligned} \tag{78}$$

diverges since $\partial_V\Omega$ diverges at the folding singularity. Hence, a slight U dependence of a scalar field in an impulsive pp-wave background creates a strong singularity with divergent Ψ_2, which in turn, by our conjecture, shields the field from populating this singularity, thus, implying again a quantum-field theoretically complete behavior. In this sense, once quantum fields are focused in a null singularity, they will deform the singularity immediately such that the interplay with gravity protects quantum-field theory from reaching the singularity.

In summary, backreaction can be effectively understood from the point of view of the Weyl quantum conjecture because Ψ_2 is the relevant object for estimating the occurrence and strength of backreaction effects.

VI. DISCUSSION

Plane-wave spacetimes are of significant interest since every spacetime provides a plane-wave limit [13] when sufficiently enlarged. In this sense, they are extraordinarily versatile to estimate statements that can be attributed to local processes. A formidable application of this limit is the link between the singular structure of the colliding wave region and the Kasner singularities, which in a limiting case mimic the Schwarzschild singularity. Geometrically, they admit folding, null singularities which turn into spacelike singularities after the waves collide. Hence, there are two distinct regions—those before the collapse and the one after. Hence, colliding plane-wave spacetimes offer an ideal system to retrieve information about what determines how quantum probes approach a singularity.

In this article, we focused on the curved spacetime region that is created due to the scattered fields after the collision. The two models we considered were the Khan-Penrose spacetime and a slight modification known as the degenerate Ferrari-Ibáñez spacetime. Both spacetimes were picked to illustrate the different behaviors quantum probes may admit.

[6] This is a generalization of the null reduction performed in [3].

For the Khan-Penrose spacetime, we found that the curvature singularity is reminiscent of the Schwarzschild singularity and prohibits the quantum fields to reach the singular hypersurface.[7] In contrast, the Ferrari-Ibáñez solution yields that any field configuration is able to reach the singular hypersurface. The key to understanding this difference was found by analyzing the singular structure, which in this spacetime was given by a weak singularity as in the regions before the collision but after crossing one wave. Quantum probes exhibit similar behavior in patches II and III due to the absence of significant obstructions to field configurations. The normalizability of the wave functional remains intact, allowing fields to propagate through the focusing planes. This behavior extends to the curved region of the Ferrari-Ibáñez spacetime. Geometrically, the divergence at the Cauchy horizon resembles the one at the Schwarzschild horizon. Knowing how the spacetime continues beyond the Cauchy horizon ensures that fields will propagate into this region as well.

Let us understand the generality of our analysis of a massless scalar field theory by discussing different setups: A mass term will always be subdominant close to the singularity due to a metric determinant as a prefactor which vanishes toward the singularity. This makes sense, since the mass is an infrared quantity, while kinetic terms dominate the behavior at the singularity. For spinor fields, we would expect behavior similar to that for scalar field because the Khan-Penrose spacetime resembles Schwarzschild black holes; thus, the differences due to spin and charge will not be present. For higher spin gauge fields, the analysis would feature additional constraints from the gauge condition; e.g., the Gupta-Bleuler condition $\nabla E = 0$ applied to $\Psi[a]$ becomes $\nabla \frac{\delta}{\delta a}\Psi[a] = 0$ with a the physical degree of freedom [44]. However, it has been shown that spin does not avert singularities [45]; thus, we regard the scalar analysis to be sufficient.

Embedding our findings into the landscape of quantum completeness shows that the analogy of the gravitational hydrogen atom might be more than a mere intuitive picture. In fact, considering the results for the Schwarzschild interior and comparing them with the colliding plane-wave spacetimes unveils a pattern. In both spacetimes, the curved region is qualified by a diverging Penrose-Newman scalar Ψ_2 that describes the Coulomb part of the Weyl tensor. Taking the analogy seriously, quantum completeness seems to be inferred by a diverging Ψ_2 rather than any other Weyl Penrose-Newman scalar. Comparison with the plane-wave patches II and III shows that none of the other nonzero, and even diverging, scalars Ψ_0 or Ψ_4 lead to a depletion of the norm. The Ferrari-Ibáñez solution also substantiates that a divergence in Ψ_2 is necessary rather than any finite contribution.

Through our conjecture, we were able to construct a versatile path of considering backreactions in a specific but natural scenario in which the effect can be described as quasiclassical. In fact, we identified the dominant contribution that triggers backreaction. Such field configurations mimic a second plane-wave profile that crosses the classical wave, such that the folding, null singularity "bends" into a spacelike singularity. From the standpoint of quantum-field theory, this feature is beneficial: The singularity is not operationally accessible, and consistency is maintained through backreaction on the geometry, which induces a Coulomb-like Weyl curvature component. We would like to stress that if we had chosen a generic in-vacuum, the vacuum fluctuations would contribute to the backreaction through the semiclassical Einstein equation as well. Such a scenario has been studied, e.g., in [23] which required one to compute the expectation value of the stress-energy tensor.

ACKNOWLEDGMENTS

We would like to thank Santiago Londoño Castillo and Simon Kuhn for early collaboration. M. S. wants to thank the LMU Munich and the Arnold-Sommerfeld Center for Theoretical Physics for the hospitality during the finalization of the project. Thank you to the anonymous referee for their constructive and detailed comments. This work is supported by the Excellence Cluster Origins of the DFG under Germany's Excellence Strategy EXC-2094 390783311 as well as EXC 2094/2: ORIGINS 2 and by the Basque Government Grant No. IT1628-22 as well as by Grant No. PID2021-123226NB-I00 (funded by MCIN/AEI/10.13039/501100011033 and by "ERDF A way of making Europe").

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

[7] A geometrically inspired proposal that uses a changing vacuum can be found in [42,43].

[1] S. Hofmann and M. Schneider, Classical versus quantum completeness, Phys. Rev. D **91**, 125028 (2015).

[2] L. Eglseer, S. Hofmann, and M. Schneider, Quantum populations near black-hole singularities, Phys. Rev. D **104**, 105010 (2021).

[3] I. Sachs, M. Schneider, and M. Urban, Quantum probing of null singularities, Phys. Rev. D **104**, 125020 (2021).

[4] K. Khan and R. Penrose, Scattering of two impulsive gravitational plane waves, Nature (London) **229**, 185 (1971).

[5] V. Ferrari and J. Ibañez, A new exact solution for colliding gravitational plane waves, Gen. Relativ. Gravit. **19**, 383 (1987).

[6] P. Szekeres, Colliding gravitational waves, Nature (London) **228**, 1183 (1970).

[7] S. Chandrasekhar and B. C. Xanthopoulos, On colliding waves in the Einstein-Maxwell theory, Proc. R. Soc. A **398**, 223 (1985).

[8] U. Yurtsever, New family of exact solutions for colliding plane gravitational waves, Phys. Rev. D **37**, 2790 (1988).

[9] S. Hayward, Colliding waves and black holes, Classical Quantum Gravity **6**, 1021 (1989).

[10] R. Penrose, A remarkable property of plane waves in general relativity, Rev. Mod. Phys. **37**, 215 (1965).

[11] U. Yurtsever, Structure of the singularities produced by colliding plane waves, Phys. Rev. D **38**, 1706 (1988).

[12] U. Yurtsever, Colliding almost-plane gravitational waves: Colliding plane waves and general properties of almost-plane-wave spacetimes, Phys. Rev. D **37**, 2803 (1988).

[13] M. Blau, M. Borunda, M. O'Loughlin, and G. Papadopoulos, Penrose limits and spacetime singularities, Classical Quantum Gravity **21**, L43 (2004).

[14] A. Feinstein, Penrose limits, the colliding plane-wave problem and the classical string backgrounds, Classical Quantum Gravity **19**, 5353 (2002).

[15] R. Penrose, W. Rindler, and J. N. Goldberg, Spinors and space-time, volume I: Two-spinor calculus and relativistic fields (1986).

[16] R. Penrose, Any space-time has a plane wave as a limit, in *Differential Geometry and Relativity* (Springer, New York, 1976), pp. 271–275.

[17] J. Garriga and E. Verdaguer, Scattering of quantum particles by gravitational plane waves, Phys. Rev. D **43**, 391 (1991).

[18] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge University Press, Cambridge, England, 1973).

[19] U. Yurtsever, Singularities and horizons in the collisions of gravitational waves, Phys. Rev. D **40**, 329 (1989).

[20] F. J. Tipler, Energy conditions and spacetime singularities, Phys. Rev. D **17**, 2521 (1978).

[21] C. J. Clarke and A. Królak, Conditions for the occurence of strong curvature singularities, J. Geom. Phys. **2**, 127 (1985).

[22] K. H. Choi, S. Hofmann, and M. Schneider, Local diagnostic program for unitary evolution in general spacetimes, Phys. Rev. D **111**, 025019 (2025).

[23] U. Yurtsever, Quantum field theory in a colliding plane-wave background, Phys. Rev. D **40**, 360 (1989).

[24] S. A. Hayward, Dual-null dynamics, Ann. l'IHP Phys. Théor. **59**, 399 (1993), https://www.numdam.org/item/AIHPA_1993__59_4_399_0/.

[25] S. Hofmann, M. Schneider, and M. Urban, Quantum complete prelude to inflation, Phys. Rev. D **99**, 065012 (2019).

[26] D. Long and G. M. Shore, The schrödinger wave functional and vacuum states in curved spacetime, Nucl. Phys. **B530**, 247 (1998).

[27] S. Hofmann and M. Schneider, Non-Gaussian ground-state deformations near a black-hole singularity, Phys. Rev. D **95**, 065033 (2017).

[28] V. A. Belinski, E. Lifshitz, and I. Khalatnikov, Oscillatory approach to the singular point in relativistic cosmology, Sov. Phys. **13**, 745 (1971).

[29] V. A. Belinski, I. Khalatnikov, and E. Lifshitz, A general solution of the Einstein equations with a time singularity, Adv. Phys. **31**, 639 (1982).

[30] W. G. Unruh, Notes on black-hole evaporation, Phys. Rev. D **14**, 870 (1976).

[31] A. Ashtekar and A. Magnon, Quantum fields in curved space-times, Proc. R. Soc. A **346**, 375 (1975).

[32] I. Sachs and A. Wipf, Generalized Thirring models, Ann. Phys. (N.Y.) **249**, 380 (1996).

[33] G. Cognola, E. Elizalde, and S. Zerbini, Fluctuations of quantum fields via zeta function regularization, Phys. Rev. D **65**, 085031 (2002).

[34] A. Ashtekar, T. De Lorenzo, and M. Schneider, Probing the big bang with quantum fields, Adv. Theor. Math. Phys. **25**, 1651 (2021).

[35] A. Ashtekar, A. del Río, and M. Schneider, Space-like singularities of general relativity: A phantom menace?, Gen. Relativ. Gravit. **54**, 45 (2022).

[36] A. Ashtekar and A. del Río, Probing cosmological singularities with quantum fields: Open and closed FLRW universes, Phys. Rev. D **106**, 085003 (2022).

[37] L. Hadjiivanov and D. T. Stoyanov, On the free scalar massless field in two-dimensional space-time, Rep. Math. Phys. **15**, 361 (1979).

[38] G. Gibbons, Quantized fields propagating in plane-wave spacetimes, Commun. Math. Phys. **45**, 191 (1975).

[39] E. Zauderer, On a modification of Hadamard's method for obtaining fundamental solutions for hyperbolic and parabolic equations, IMA J. Appl. Math. **8**, 8 (1971).

[40] F. G. Friedlander, *The Wave Equation on a Curved Space-Time* (Cambridge University Press, Cambridge, England, 1975), Vol. 2.

[41] I. Sachs and T. Tran, On non-perturbative unitarity in gravitational scattering, Eur. Phys. J. C **79**, 914 (2019).

[42] J. L. Alonso, C. Bouthelier-Madre, J. Clemente-Gallardo, and D. Martínez-Crespo, Geometric flavors of quantum field theory on a cauchy hypersurface. Part I: Gaussian analysis and other mathematical aspects, J. Geom. Phys. **203**, 105264 (2024).

[43] J. L. Alonso, C. Bouthelier-Madre, J. Clemente-Gallardo, and D. Martínez-Crespo, Geometric flavours of quantum field theory on a cauchy hypersurface. Part II: Methods of quantization and evolution, J. Geom. Phys. **203**, 105265 (2024).

[44] M. M. Schneider, Black holes are quantum complete, Ph.D. thesis, Munich U., 2018.

[45] J. Stewart and P. Hájíček, Can spin avert singularities?, Nat. Phys. Sci. **244**, 96 (1973).