

Analyzing the magnitude of two loop corrections to the muon magnetic dipole moment in the mass insertion approximation

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With the development of muon magnetic dipole moment (MDM) experiments, particularly the high-precision measurements at Fermilab National Accelerator Laboratory (FNAL), experimental data have become increasingly precise. Up to now, the deviation of muon MDM between the experimental data and the standard model prediction still exists, which may be caused by the new physics contribution. The two loop supersymmetric diagrams can produce important corrections to muon MDM. For many two loop diagrams, we analyze the order of the contribution to select the important two loop diagrams and neglect the tiny one, which is in favor of distinguishing the important two loop diagrams from all the two loop diagrams. In our analysis, we use the mass insertion approximation. There is no rotation matrix during the study and the resulting factors become more intuitive, so the mass insertion method is more suitable for this study of ours.

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I. INTRODUCTION

Muon magnetic dipole moment (MDM) has attracted the interests of physicists for many years. For muon MDM, the standard model (SM) contributions [1–4] can be divided into several parts: QED contribution is dominant [1,5]; Electroweak and hadronic contributions are also important [5,6]. The latest experiment data for muon MDM is reported by the Fermilab National Accelerator Laboratory (FNAL) [7]. Combined with the previous data, the world average value of the departure between experiment data and the SM prediction is [8]

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 38(63) \times 10^{-11}. \quad (1)$$

One explanation is that the deviation Δa_μ is caused by the new physics. A computation using supersymmetry (SUSY) for such small deviation is still interesting. Through the symmetry of fermions and bosons, SUSY can cancel out the divergences with each other, making the theoretical

calculations more accurate, which provides a natural solution to the hierarchy problem in the Higgs mass corrections. By the running of the renormalization group equation, the three gauge coupling constants intersect at one point at the very high energy scale around 10^{16} GeV. The lightest neutralino can be a dark matter candidate. SUSY has new particles beyond the SM, and these new particles can bring new corrections to muon $g-2$. Even the deviation for the muon MDM between the experimental data and the SM prediction is minimal, the SUSY may provide a more self-consistent explanation by introducing new particles, such as supersymmetry partner particles.

Up to now, SM has achieved great successes with the detection of 125 GeV Higgs boson [9]. However, SM cannot provide cold dark matter candidate, which is one shortcoming of SM. Therefore, physicists extend the SM to obtain new models. The minimal supersymmetric extension of the standard model (MSSM) [10] is one of the most popular models, which has been studied for many years. Based on the MSSM, there are also many extended models [11,12] with added gauge group and superfields especially right-handed neutrino and Higgs superfields. In these new models, there are many particles contributing to muon MDM at two loop level [13,14]. Therefore, so many two loop triangle diagrams for $\mu \rightarrow \mu\gamma$ must be faced. It is very important to distinguish the order of the each type two loop corrections for muon MDM [15].

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The two loop diagrams in the SM have been well studied to obtain the electroweak corrections to muon MDM [1]. In the new physics models such as 2HDM [16] and MSSM, some types of two loop diagrams are researched carefully by the authors [13,14]. For the two loop diagrams, the Barr-Zee type [17], the rainbow type [18], the diamond type [19,20] are all of interest and are calculated analytically. Furthermore, the decoupled results are also shown apparently with the supposition that all supersymmetric particles are heavy and of the same mass. When the internal fermion or sfermion are very heavy, the two loop diagrams with fermion/sfermion loops have large factors [21]. The exact results for the photonic supersymmetric two loop corrections can be found in Ref. [22].

In our previous work, we have researched the factors of the two loop contributions to muon MDM in mass eigenstate [15]. It benefits the selection of the two loop diagrams. However, the rotation matrixes of the mass mixing matrixes are not considered in the analysis, which may influence the factor obviously. For the one loop contributions from $(\chi^0, \tilde{L})$ and $(\chi^\pm, \tilde{\nu})$ in the mass eigenstate, there is an enhancement factor $\frac{m_\mu}{M_{NP}}$. However, this is not the truth, because the combined rotation matrixes suppress the results. Considering the effects of these factors as a whole, one can obtain the factor $\frac{m_\mu^2}{M_{NP}^2} \tan \beta$ in the end, which is analyzed by the authors in detail [23]. It is supposed that the SUSY particles are all heavy and with the same mass M_{NP} . The detailed analysis for the one loop contributions is shown here.

In the MSSM, we present the general results of the one loop diagrams in the following form [23,24]

$$\begin{aligned}
 a_\mu^{\text{MSSM,1L}} &= a_\mu^{\chi^0} + a_\mu^{\chi^\pm}, \\
 a_\mu^{\chi^0} &= \frac{1}{48\pi^2} \sum_{i,m} \frac{m_\mu^2}{m_{\mu_m}^2} \left\{ -\frac{|n_{im}^L|^2 + |n_{im}^R|^2}{4} F_1^N(x_{im}) \right. \\
 &\quad \left. + \frac{m_{\chi_i^0}}{m_\mu} \text{Re}[n_{im}^L n_{im}^R] F_2^N(x_{im}) \right\}, \\
 a_\mu^{\chi^\pm} &= \frac{1}{48\pi^2} \sum_k \frac{m_\mu^2}{m_{\tilde{\nu}_k}^2} \left\{ \frac{|c_k^L|^2 + |c_k^R|^2}{4} F_1^C(x_k) \right. \\
 &\quad \left. + 2 \frac{m_{\chi_k^\pm}}{m_\mu} \text{Re}[c_k^L c_k^R] F_2^C(x_k) \right\}. \quad (2)
 \end{aligned}$$

$i = 1 \dots 4$, $k = 1, 2$ and $m = 1, 2$ respectively denote the neutralino, chargino, and smuon indices. The mass ratios are defined as $x_{im} = m_{\chi_i^0}^2 / m_{\mu_m}^2$, $x_k = m_{\chi_k^\pm}^2 / m_{\tilde{\nu}_k}^2$. The concrete forms of the couplings are

$$\begin{aligned}
 n_{im}^L &= \frac{1}{\sqrt{2}} (g_1 N_{i1} + g_2 N_{i2}) U_{m1}^{\tilde{\mu}*} - y_\mu N_{i3} U_{m2}^{\tilde{\mu}*}, \\
 n_{im}^R &= \sqrt{2} g_1 N_{i1} U_{m2}^{\tilde{\mu}} + y_\mu N_{i3} U_{m1}^{\tilde{\mu}}, \\
 c_k^L &= -g_2 Z_+^{1k}, \quad c_k^R = y_\mu Z_-^{2k}. \quad (3)
 \end{aligned}$$

The smuon mass squared matrix is diagonalized by $U^{\tilde{\mu}}$. While, N_{i1} is the rotation matrix to diagonalize the neutralino mass matrix. To obtain the mass eigenvalues of chargino, two rotation matrixes Z_- and Z_+ are needed, whose concrete forms are in the Eqs. (37)–(41).

The one loop functions in Eq. (2) are shown as

$$\begin{aligned}
 F_1^N(x) &= \frac{2}{(1-x)^4} [1 - 6x + 3x^2 + 2x^3 - 6x^2 \log x], \\
 F_2^N(x) &= \frac{3}{(1-x)^3} [1 - x^2 + 2x \log x], \\
 F_1^C(x) &= \frac{2}{(1-x)^4} [2 + 3x - 6x^2 + x^3 + 6x \log x], \\
 F_2^C(x) &= \frac{3}{(1-x)^3} [-3 + 4x - x^2 - 2 \log x]. \quad (4)
 \end{aligned}$$

These functions are normalized as $F_i^j(1) = 1$.

In Eq. (2), on the surface there is the factor $\frac{m_{\chi^\pm}}{m_\mu} \left(\frac{m_{\chi^0}}{m_\mu}\right)$ in the second term for $a_\mu^{\chi^\pm}$ ($a_\mu^{\chi^0}$). In fact, the terms linear in $m_{\chi^{0,\pm}}$ are not enhanced by a factor $m_{\chi^{0,\pm}}/m_\mu$ compared to the other terms. Let us take $\frac{m_{\chi^\pm}}{m_\mu}$ as an example. From Eq. (2), we discuss the total effects of the following terms

$$\begin{aligned}
 &\sum_k \frac{m_\mu^2}{m_{\tilde{\nu}_k}^2} \frac{m_{\chi_k^\pm}}{m_\mu} \text{Re}[c_k^L c_k^R] F_2^C(x_k) \\
 &= \sum_k \frac{m_\mu^2}{m_{\tilde{\nu}_k}^2} \frac{m_{\chi_k^\pm}}{m_\mu} \text{Re}[-g_2 V_{k1} y_\mu U_{k2}] F_2^C(x_k) \\
 &\propto \frac{m_\mu^2}{M_{NP}^2} \tan \beta. \quad (5)
 \end{aligned}$$

Here, y_μ is the muon Yukawa coupling. When we take the concrete forms of the parameters in Eq. (5), the factor $\frac{m_\mu^2}{M_{NP}^2} \tan \beta$ is obtained. That is to say the terms linear in $m_{\chi^{0,\pm}}$ are enhanced merely by a factor $\tan \beta$ from the muon Yukawa coupling.

The two loop SUSY contributions also have similar features as the one loop condition [23]. Considering the combined rotation matrixes, the factor $\frac{m_\mu}{M_{NP}}$ on the surface in the two loop corrections is suppressed in the total analysis. To get the factors more clearly and thoroughly, we use the mass insertion approximation (MIA) [25,26] to analyse the two loop triangle diagrams contributing to muon MDM. With MIA method we use the electroweak interaction eigenstate and treat perturbatively the mass insertions. In the MIA approach it is not necessary to include rotation mass matrices. Therefore, the results are clear for the vertex couplings in MIA method. The energy scale of the new physics is supposed as Λ_{NP} , and we take $\Lambda_{NP} \sim M_{NP}$.

In the Sec. II, the used suppositions and analysis method are shown in detail. The detailed example and obtained

factors for the two loop diagrams are collected in Sec. III. The last section is devoted to the discussion and conclusion.

II. THE METHOD

There are many SUSY models, where MSSM is very representative. Though the analysis is employed in the MSSM, it is also adaptable to other SUSY models and even non-SUSY models. Instead of the mass eigenstate, we use MIA, which is calculated in the electroweak interaction eigenstate. This method has advantages in the study of lepton flavor violation to show important parameters apparently [26,27]. Similarly, the advantages can also be reflected in the muon MDM analysis [25]. There is not the rotation matrixes, and the obtained factors become more intuitive.

There are a great many two loop SUSY triangle diagrams for $\mu \rightarrow \mu\gamma$. To save space in the text, we plot the two loop

self-energy diagrams, from which the two loop triangle diagrams can be obtained by attaching a photon on the internal charged lines in all possible ways. The one loop diagrams in electroweak state are much more than the one loop diagrams in mass eigenstate. Similarly, the two loop self-energy diagrams using MIA are much more than the two loop self-energy diagrams in the mass eigenstate. Therefore, we show some examples, and the other two loop diagrams are still denoted in the mass eigenstate.

In mass eigenstate, the two loop self-energy diagrams are plotted in Fig. 1. To simplify the discussion, we use the notations that S , V , and F represent virtual scalar boson, vector boson and Dirac (Majorana) particle, respectively. Some diagrams [Figs. 1(d), 1(f), 1(n), and 1(o)] have their Hermitian diagrams to produce the same contribution to muon MDM, and we do not show them here. Their Hermitian diagrams can be obtained by exchanging the vector boson and the corresponding scalar boson.

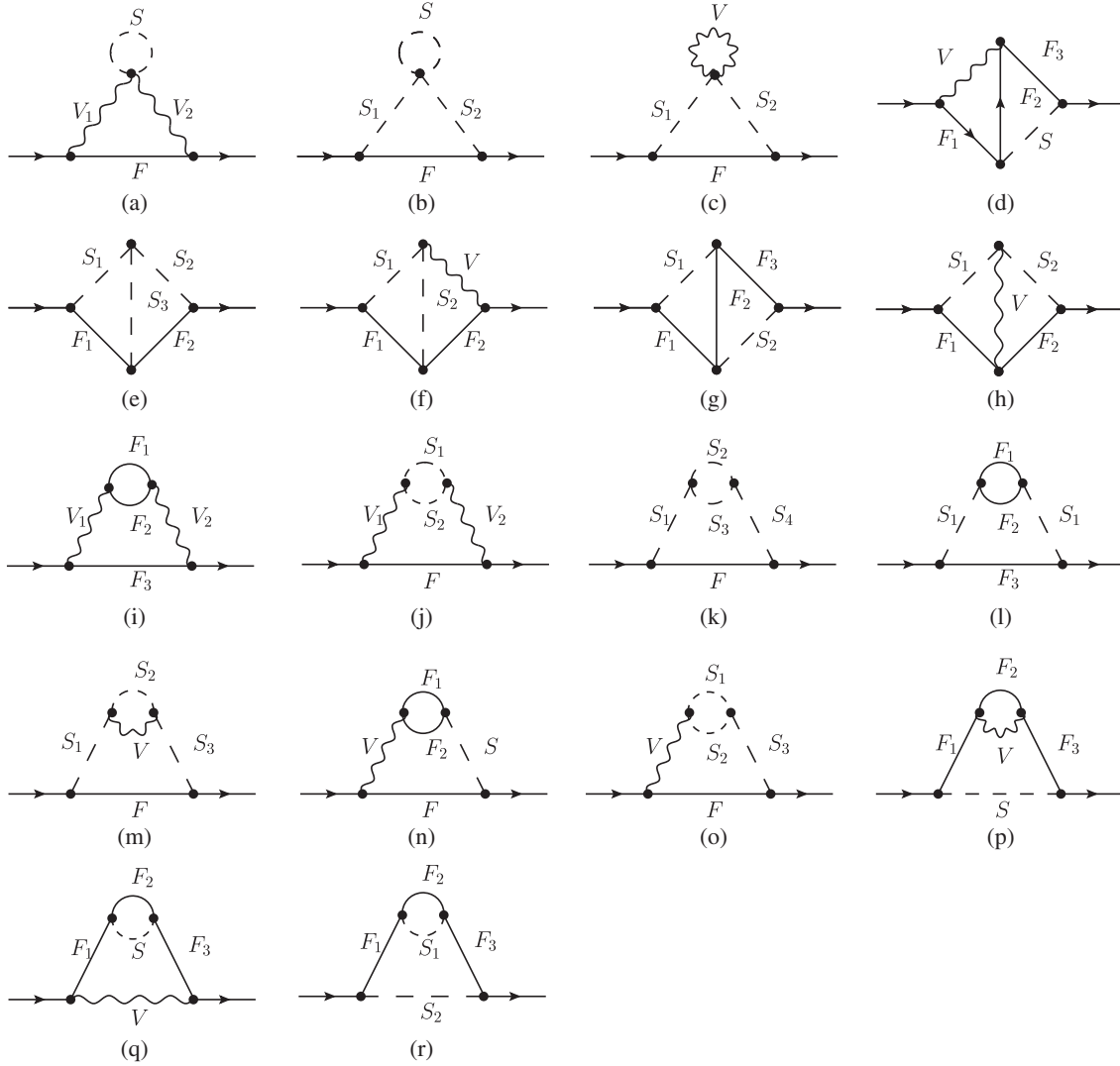


FIG. 1. The two loop self-energy diagrams for muon MDM in mass eigenstate.

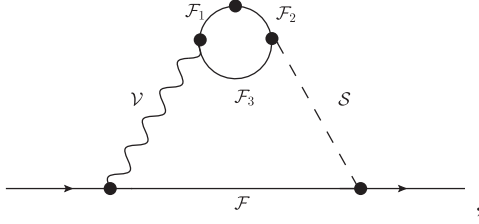


FIG. 2. The two loop self-energy diagram in electroweak eigenstate corresponding to Fig. 1(n).

In previous works, the authors show the decoupling results for dominant one-loop SUSY contributions to muon MDM, which all have the typical factor $\frac{m_\mu^2}{M_{NP}^2} \tan \beta$ with all SUSY particles masses being M_{NP} [23,25]. To simplify the discussion, we also use the supposition that all SUSY particles have the same mass M_{NP} in this work. Generally speaking, the masses of neutralino and sneutrino should be around 500 GeV, or even heavier. The slepton mass is no less than 700 GeV [28]. The lightest chargino mass is at the scale of 1100 GeV. The very heavy SUSY particles are colored particles including scalar quark and gluino, whose masses are not smaller than 1500 GeV. For the sake of simplicity, we suppose that the SUSY particles approximately have the same mass $M_{NP} \sim 1000$ GeV. From a numerical point of view, 1000 GeV is about the central value for the most SUSY particles' masses from 500 GeV to 1500 GeV, which is a reasonable approximation. This assumption greatly reduces the complexity of the analysis.

The sum of all the two loop triangle diagrams ($\mu \rightarrow \mu\gamma$) corresponding to each two loop self-energy diagram ($\mu \rightarrow \mu$) satisfies the Ward-identity [29,30]. We take two examples to explain the conversion between the two loop self-energy diagrams in mass eigenstate and the corresponding diagrams using MIA in electroweak eigenstate. For the Fig. 1(n), we take $V = \gamma$, $F = \mu$, $S = h_d^0$, $F_1 = F_2 = \chi^\pm$, whose corresponding diagram processed by MIA is shown in the Fig. 2 with $\mathcal{V} = \gamma$, $\mathcal{F} = \mu$, $S = h_d^0$, $\mathcal{F}_1 = \tilde{W}^\pm$, $\mathcal{F}_2 = \tilde{H}^\pm$, $\mathcal{F}_3 = \tilde{W}^\pm$. This is the Barr-Zee type two loop diagram with heavy fermion subloop, which contributes to

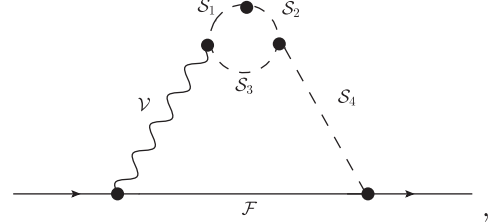


FIG. 3. The two loop self-energy diagram in electroweak eigenstate corresponding to Fig. 1(o).

muon MDM with the typical factor $\frac{m_\mu^2}{M_{NP}^2} \tan \beta$ [23]. Large $\tan \beta$ can improve this type of two loop diagram contribution.

The Barr-Zee type two loop diagram with scalar subloop in MIA is shown in Fig. 3. The corresponding particles are respectively $\mathcal{V} = \gamma$, $\mathcal{F} = \mu$, $S_1 = \tilde{t}_R$, $S_2 = \tilde{t}_L$, $S_3 = \tilde{t}_R$, $S_4 = h_d^0$. As discussed in Ref. [23], this diagram has the typical parameter $\frac{m_\mu^2}{M_{NP}^2} \tan \beta \frac{m_t^2}{m_w^2}$, which is analyzed in the next section.

III. THE ANALYSIS

In order to obtain representative analysis, we study the two loop diagrams in the MSSM [10]. Inserting MSSM particles into the two loop self-energy diagrams in generic form, a lot of two loop diagrams are produced from each type generic diagram. To save space in the text, we use the two loop diagrams in mass eigenstate to represent two loop diagrams in electroweak state. Each two loop diagram in mass eigenstate corresponds to several two loop diagrams in electroweak state, which are analyzed by MIA method in fact. Then, the results are obtained by MIA method, and the two loop diagrams are not so many to plot.

A. The techniques for the two loop function

Here, we show the techniques for the two loop diagram calculation. The used formulas to simplify the Feynman integrals are [20,31]

$$\begin{aligned}
 & \int \frac{d^D k_1 d^D k_2 (k_{1\mu} k_{1\nu}, k_{1\mu} k_{2\nu})}{(2\pi)^{2D} \mathcal{D}_0} \rightarrow \int \frac{d^D k_1 d^D k_2 (k^2, k_1 \cdot k_2) g_{\mu\nu}}{(2\pi)^{2D} D \mathcal{D}_0}, \\
 & \int \frac{d^D k_1 d^D k_2 (k_{1\mu} k_{1\nu} k_{1\rho} k_{1\sigma}, k_{1\mu} k_{1\nu} k_{1\rho} k_{2\sigma})}{(2\pi)^{2D} \mathcal{D}_0} \rightarrow \int \frac{d^D k_1 d^D k_2 (k_1^4, k_1^2 k_1 \cdot k_2) T_{\mu\nu\rho\sigma}}{(2\pi)^{2D} D(D+2) \mathcal{D}_0}, \\
 & \int \frac{d^D k_1 d^D k_2 k_{1\mu} k_{1\nu} k_{2\rho} k_{2\sigma}}{(2\pi)^{2D} \mathcal{D}_0} \rightarrow \int \frac{d^D k_1 d^D k_2}{(2\pi)^{2D} \mathcal{D}_0} \left(\frac{D(k_1 \cdot k_2)^2 - k_1^2 k_2^2}{D(D-1)(D+2)} T_{\mu\nu\rho\sigma} - \frac{(k_1 \cdot k_2)^2 - k_1^2 k_2^2}{D(D-1)} g_{\mu\nu} g_{\rho\sigma} \right), \\
 & \int \frac{d^D k_1 d^D k_2 k_{1\mu} k_{1\nu} k_{1\rho} k_{1\sigma} k_{2\alpha} k_{2\beta}}{(2\pi)^{2D} \mathcal{D}_0} \rightarrow \int \frac{d^D k_1 d^D k_2}{(2\pi)^{2D} \mathcal{D}_0} \left(\frac{D(k_1 \cdot k_2)^2 k_1^2 - k_1^4 k_2^2}{D(D+2)(D+4)(D-1)} T_{\alpha\beta\mu\nu\rho\sigma} - \frac{(k_1 \cdot k_2)^2 k_1^2 - k_1^4 k_2^2}{D(D+2)(D-1)} g_{\alpha\beta} T_{\mu\nu\rho\sigma} \right),
 \end{aligned}$$

$$\begin{aligned}
& \int \frac{d^D k_1 d^D k_2}{(2\pi)^{2D}} \frac{k_{1\mu} k_{1\nu} k_{1\rho} k_{2\sigma} k_{2\alpha} k_{2\beta}}{\mathcal{D}_0} \\
& \rightarrow \int \frac{d^D k_1 d^D k_2}{(2\pi)^{2D}} \frac{1}{\mathcal{D}_0} \left(\frac{(D+1)k_1^2 k_2^2 (k_1 \cdot k_2) - 2(k_1 \cdot k_2)^3}{D(D+2)(D+4)(D-1)} T_{\alpha\beta\mu\nu\rho\sigma} - \frac{k_1^2 k_2^2 (k_1 \cdot k_2) - (k_1 \cdot k_2)^3}{D(D+2)(D-1)} \right. \\
& \quad \left. \times [g_{\mu\sigma}(g_{\nu\alpha}g_{\rho\beta} + g_{\nu\beta}g_{\rho\alpha}) + g_{\mu\alpha}(g_{\nu\sigma}g_{\rho\beta} + g_{\nu\beta}g_{\rho\sigma}) + g_{\mu\beta}(g_{\nu\sigma}g_{\rho\alpha} + g_{\nu\alpha}g_{\rho\sigma})] \right), \tag{6}
\end{aligned}$$

with the time-space dimension $D = 4 - 2\epsilon$. $T_{\mu\nu\rho\sigma}$, $T_{\mu\nu\rho\sigma\alpha\beta}$, and $\mathcal{D}_0$ are shown as

$$\begin{aligned}
T_{\mu\nu\rho\sigma} &= g_{\mu\nu}g_{\rho\sigma} + g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho}, \\
T_{\mu\nu\rho\sigma\alpha\beta} &= g_{\mu\nu}T_{\rho\sigma\alpha\beta} + g_{\mu\rho}T_{\nu\sigma\alpha\beta} + g_{\mu\sigma}T_{\nu\rho\alpha\beta} + g_{\mu\alpha}T_{\nu\rho\sigma\beta} + g_{\mu\beta}T_{\nu\rho\sigma\alpha}, \\
\mathcal{D}_0 &= (k_1^2 - m_1^2)(k_2^2 - m_2^2)((k_1 - k_2)^2 - m_0^2). \tag{7}
\end{aligned}$$

The odd rank terms of k_1 and k_2 in the loop momenta are abandoned, because their integrals are zero. With the following decomposition formula, we reduce the complicated two-loop integrals to the simple two-loop vacuum integrals with one $1/\mathcal{D}_0$ and one-loop integrals

$$\begin{aligned}
\frac{1}{(k^2 - m_1^2)(k^2 - m_2^2)} &= \frac{1}{m_1^2 - m_2^2} \left(\frac{1}{k^2 - m_1^2} - \frac{1}{k^2 - m_2^2} \right), \\
\frac{1}{(k^2 - m^2)^n} &= \frac{1}{(n-1)!} \left(\frac{\partial}{\partial m^2} \right)^{n-1} \frac{1}{k^2 - m^2}. \tag{8}
\end{aligned}$$

The two-loop vacuum integral is obtained [31]

$$\begin{aligned}
& \Lambda_{\text{RE}}^{4\epsilon} \int \int \frac{d^D k_1 d^D k_2}{(2\pi)^{2D}} \frac{1}{(k_1^2 - m_1^2)(k_2^2 - m_2^2)((k_1 - k_2)^2 - m_0^2)} \\
& = \frac{\Lambda^2}{2(4\pi)^4} \frac{\Gamma^2(1+\epsilon)}{(1-\epsilon)^2} (4\pi x_R)^{2\epsilon} \left\{ -\frac{1}{\epsilon^2} (x_0 + x_1 + x_2) + \frac{1}{\epsilon} (-(x_0 + x_1 + x_2) \right. \\
& \quad + 2(x_0 \ln x_0 + x_1 \ln x_1 + x_2 \ln x_2)) + 2(x_0 \ln x_0 + x_1 \ln x_1 + x_2 \ln x_2) \\
& \quad \left. - 2(x_0 + x_1 + x_2) - x_0 \ln^2 x_0 - x_1 \ln^2 x_1 - x_2 \ln^2 x_2 - \Phi(x_0, x_1, x_2) \right\}, \tag{9}
\end{aligned}$$

with

$$\Phi(x, y, z) = (x + y - z) \ln x \ln y + (x - y + z) \ln x \ln z + (y + z - x) \ln y \ln z + \text{sign}(\lambda^2) \sqrt{|\lambda^2|} \Psi(x, y, z), \tag{10}$$

$$\lambda^2(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz. \tag{11}$$

The concrete form of $\Psi(x, y, z)$ reads as

(1) $\lambda^2 > 0$, $\sqrt{y} + \sqrt{z} < \sqrt{x}$:

$$\begin{aligned}
\Psi(x, y, z) &= 2 \ln \left(\frac{x + y - z - \lambda}{2x} \right) \ln \left(\frac{x - y + z - \lambda}{2x} \right) - \ln \frac{y}{x} \ln \frac{z}{x} \\
&\quad - 2L_{i_2} \left(\frac{x + y - z - \lambda}{2x} \right) - 2L_{i_2} \left(\frac{x - y + z - \lambda}{2x} \right) + \frac{\pi^2}{3}. \tag{12}
\end{aligned}$$

$L_{i_2}(x)$ is the Spence function

(2) $\lambda^2 > 0, \sqrt{x} + \sqrt{z} < \sqrt{y}$:

$$\Psi(x, y, z) = \text{Eq. (12)}(x \leftrightarrow y); \quad (13)$$

(3) $\lambda^2 > 0, \sqrt{x} + \sqrt{y} < \sqrt{z}$:

$$\Psi(x, y, z) = \text{Eq. (12)}(x \leftrightarrow z); \quad (14)$$

(4) $\lambda^2 < 0$:

$$\begin{aligned} \Psi(x, y, z) = 2 \Big\{ & Cl_2 \left(2 \arccos \left(\frac{-x + y + z}{2\sqrt{yz}} \right) \right) + Cl_2 \left(2 \arccos \left(\frac{x - y + z}{2\sqrt{xz}} \right) \right) \\ & + Cl_2 \left(2 \arccos \left(\frac{x + y - z}{2\sqrt{xy}} \right) \right) \Big\}. \end{aligned} \quad (15)$$

$Cl_2(x)$ denotes the Clausen function.

We show the used partial functions of $\Phi(x_1, x_2, x_3)$.

(1) The function $\frac{\partial \Phi}{\partial x_1}(x_1, x_2, x_3)$

$$\begin{aligned} \frac{\partial \Phi}{\partial x_1}(x_1, x_2, x_3) = & \frac{x_1 - x_2 - x_3}{\sqrt{|\lambda^2|}} \Psi(x_1, x_2, x_3) + \log(x_1) \log(x_2) \\ & + \log(x_1) \log(x_3) + 2 \log(x_1) - \log(x_2) \log(x_3). \end{aligned} \quad (16)$$

(2) The function $\frac{\partial^2 \Phi}{\partial x_1^2}(x_1, x_2, x_3)$

$$\begin{aligned} \frac{\partial^2 \Phi}{\partial x_1^2}(x_1, x_2, x_3) = & -\frac{4x_2x_3}{(|\lambda^2|)^{3/2}} \Psi(x_1, x_2, x_3) - \frac{2(-x_1 + x_2 + x_3) \log(x_1)}{\lambda^2} \\ & + \frac{\log(x_2) + \log(x_3) + 2}{x_1} - \frac{(x_3 - x_1 - x_2)(x_3 - x_1 + x_2) \log(x_2)}{x_1 \lambda^2} \\ & + \frac{(x_1 - x_2 + x_3)(-x_1 + x_2 + x_3) \log(x_3)}{x_1 \lambda^2}. \end{aligned} \quad (17)$$

(3) The function $\frac{\partial^2 \Phi}{\partial x_1 \partial x_2}(x_1, x_2, x_3)$

$$\begin{aligned} \frac{\partial^2 \Phi}{\partial x_1 \partial x_2}(x_1, x_2, x_3) = & \frac{1}{\lambda^2} (-2(x_1 - x_2 + x_3) \log(x_1) - 2(x_3 - x_1 + x_2) \log(x_2) \\ & + 4x_3 \log(x_3)) + \frac{2x_3(x_1 + x_2 - x_3)}{|\lambda^2|^{3/2}} \Psi(x_1, x_2, x_3). \end{aligned} \quad (18)$$

(4) The function $\frac{\partial^3 \Phi}{\partial x_1^3}(x_1, x_2, x_3)$

$$\begin{aligned} \frac{\partial^3 \Phi}{\partial x_1^3}(x_1, x_2, x_3) = & \frac{12x_2x_3(x_1 - x_2 - x_3)}{|\lambda^2|^{5/2}} \Psi(x_1, x_2, x_3) - \left(\frac{24x_2x_3}{\lambda^4} + \frac{2}{\lambda^2} \right) \log(x_1) \\ & + \left(\frac{(x_3 - x_1)^2 - x_2^2}{x_1^2 \lambda^2} + \frac{2x_2}{x_1 \lambda^2} - \frac{12x_2x_3(-x_1 - x_2 + x_3)}{x_1 \lambda^4} \right) \log(x_2) \\ & + \left(\frac{2x_3(2x_1 + x_2 - x_3)}{x_1^2 \lambda^2} + \frac{12x_2x_3(x_1 - x_2 + x_3)}{x_1 \lambda^4} + \frac{1}{x_1^2} \right) \log(x_3) \\ & - \frac{2[(x_3 - x_2)^2 - x_1(x_2 + x_3)]}{x_1^2 \lambda^2} - \frac{\log(x_2) + \log(x_3)}{x_1^2}. \end{aligned} \quad (19)$$

(5) The function $\frac{\partial^3 \Phi}{\partial x_1^2 \partial x_2}(x_1, x_2, x_3)$

$$\begin{aligned} \frac{\partial^3 \Phi}{\partial x_1^2 \partial x_2}(x_1, x_2, x_3) &= \frac{2(x_2 - x_1 - x_3)}{x_1 \lambda^2} + \left(\frac{2}{\lambda^2} + \frac{12x_3(x_1 + x_2 - x_3)}{\lambda^4} \right) \log(x_1) - \left(\frac{24x_2x_3}{\lambda^4} + \frac{2(x_1 + x_3)}{x_1 \lambda^2} \right) \log(x_2) \\ &+ \left(\frac{8x_3(x_2 - x_1 + x_3)}{\lambda^4} + \frac{2x_3(x_2 - x_3 + x_1)(x_2 - x_3 - x_1)}{x_1 \lambda^4} \right) \log(x_3) \\ &+ \frac{2x_3(\lambda^2 - 3(x_1 - x_3)^2 + 3x_2^2)}{|\lambda^2|^{5/2}} \Psi(x_1, x_2, x_3), \end{aligned} \quad (20)$$

B. The effective Lagrangian for muon MDM

Using the effective Lagrangian method, we adopt the dimension 6 operators for the process $l' \rightarrow l' + \gamma$,

$$\begin{aligned} \mathcal{O}_1^{L,R} &= \frac{1}{(4\pi)^2} \bar{l}(i\mathcal{D})^3 P_{L,R} l, & \mathcal{O}_2^{L,R} &= \frac{eQ_f}{(4\pi)^2} \overline{(i\mathcal{D}_\mu l)} \gamma^\mu F \cdot \sigma P_{L,R} l, \\ \mathcal{O}_3^{L,R} &= \frac{eQ_f}{(4\pi)^2} \bar{l} F \cdot \sigma \gamma^\mu P_{L,R} (i\mathcal{D}_\mu l), & \mathcal{O}_4^{L,R} &= \frac{eQ_f}{(4\pi)^2} \bar{l} (\partial^\mu F_{\mu\nu}) \gamma^\nu P_{L,R} l, \\ \mathcal{O}_5^{L,R} &= \frac{m_l}{(4\pi)^2} \bar{l}(i\mathcal{D})^2 P_{L,R} l, & \mathcal{O}_6^{L,R} &= \frac{eQ_f m_l}{(4\pi)^2} \bar{l} F \cdot \sigma P_{L,R} l. \end{aligned} \quad (21)$$

Here $\mathcal{D}_\mu = \partial_\mu + ieA_\mu$ and $P_{L,R} = \frac{1 \mp \gamma_5}{2}$. $F_{\mu\nu}$ represents the electromagnetic field strength. The operators $\mathcal{O}_{2,3,6}^{L,R}$ have relation with lepton MDM, which is the combination of the Wilson coefficients $C_{2,3,6}^{L,R}$. The Wilson coefficients of the operators $\mathcal{O}_{2,3,6}^{L,R}$ satisfy the following relations

$$C_2^{L,R} = (C_3^{L,R})^*, \quad C_6^{L,R} = (C_6^{L,R})^*. \quad (22)$$

With the on-shell conditions for the external leptons, transformation of the used terms in the effective Lagrangian are obtained as [18,31]

$$\begin{aligned} &C_2^{L,R} \mathcal{O}_2^{L,R} + (C_2^{L,R})^* (\mathcal{O}_2^{L,R})^* + C_6^{L,R} \mathcal{O}_6^{L,R} + (C_6^{L,R})^* (\mathcal{O}_6^{L,R})^* \\ &\Rightarrow (C_2^{L,R} + (C_2^{L,R})^* + C_6^{L,R}) \mathcal{O}_6^{L,R} + ((C_2^{L,R})^* + C_2^{L,R} + (C_6^{L,R})^*) (\mathcal{O}_6^{L,R})^* \\ &= \frac{eQ_l m_l}{(4\pi)^2} [\Re(C_2^{L,R} + (C_2^{L,R})^* + C_6^{L,R}) \bar{l} \sigma^{\mu\nu} l + i \Im(C_2^{L,R} + (C_2^{L,R})^* + C_6^{L,R}) \bar{l} \sigma^{\mu\nu} \gamma_5 l] F_{\mu\nu}. \end{aligned} \quad (23)$$

with $\Re(\dots)$ denoting to take the real part of a complex number, and $\Im(\dots)$ denoting to take the imaginary part of a complex number. The lepton MDM reads as

$$\mathcal{L}_{MDM} = \frac{e}{4m_l} a_l \bar{l} \sigma^{\mu\nu} l F_{\mu\nu}. \quad (24)$$

We finally get

$$a_l = \frac{4Q_l m_l^2}{(4\pi)^2} \Re(C_2^{L,R} + (C_2^{L,R})^* + C_6^{L,R}). \quad (25)$$

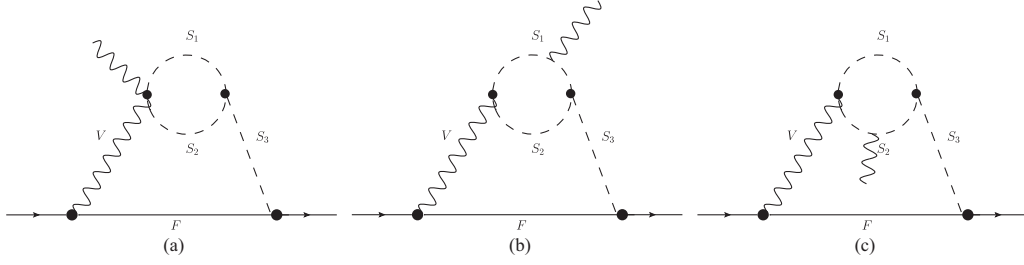
C. Example 1: Barr-Zee type diagrams with scalar subloop and neutral vector boson

For example, we analyze Barr-Zee type diagrams with charged scalar subloop, neutral vector boson and neutral Higgs. For simplicity, we use the triangle diagrams in the Fig. 4. The factor order of Fig. 5 in MIA can be obtained

from the operation $\Delta M_{XY,S}^2 \frac{\partial}{\partial m_S^2}$ on the results from the Fig. 4 with $X = R, L$ and $Y = R, L$. The charged scalar masses for left-handed and right-handed are treated as same in the end. The reason is the relation between up scalar propagator and the down propagator with a black spot denoting one mass insertion in MIA, and they are shown in the Fig. 6. In the Fig. 6, the Feynman rule for the up propagator is $\frac{i}{k^2 - m_S^2}$. While, the Feynman rule for the down propagator is $\frac{i}{k^2 - m_{S_1}^2} \Delta M_{XY,S}^2 \frac{i}{k^2 - m_{S_2}^2} \sim \frac{i}{k^2 - m_S^2} \Delta M_{XY,S}^2 \frac{i}{k^2 - m_S^2}$, because $m_{S_2} \sim m_{S_1} \rightarrow m_S$. We take the operation $\Delta M_{XY,S}^2 \frac{\partial}{\partial m_S^2}$ on $\frac{1}{k^2 - m_S^2}$ and obtain

$$\Delta M_{XY,S}^2 \frac{\partial}{\partial m_S^2} \frac{1}{k^2 - m_S^2} = \frac{1}{k^2 - m_S^2} \Delta M_{XY,S}^2 \frac{1}{k^2 - m_S^2}. \quad (26)$$

Using this relation, we can simplify the order analysis.

FIG. 4. The two loop triangle diagrams for muon MDM with neutral V and S_3 . S_1 and S_2 in the subloop are charged scalar particles.

The sum of Figs. 4(a)–4(c) satisfy the Ward-Identity. During C_2 , C_3 , and C_6 , and C_6 is the simplest one and its order is representative. Because C_6 is relatively simple and representative for the order analysis, we calculate C_6 for the triangle diagrams in the Fig. 4, and obtain the corresponding order of muon MDM.

$$\begin{aligned}
 C_6 \propto & \frac{16\pi^2}{m_F} H_{V\bar{F}F} H_{VS_1S_2} H_{S_1S_2S_3} H_{S_3\bar{F}F} \int \int \frac{dk_1^D}{(2\pi)^D} \frac{dk_2^D}{(2\pi)^D} \frac{1}{(k_1^2 - m_{S_3}^2)(k_1^2 - m_F^2)(k_1^2 - m_V^2)} \\
 & \times \left[\left(\left(\frac{1}{D} + \frac{1}{2} \right) \frac{k_1 \cdot k_2}{(k_2^2 - m_{S_1}^2)} - \frac{2}{D} \frac{k_1 \cdot k_2 k_2^2}{(k_2^2 - m_{S_1}^2)^2} - \frac{2}{D-1} \frac{(k_1 \cdot k_2)^2}{(k_2^2 - m_{S_1}^2)(k_1^2 - m_{S_3}^2)} \right. \right. \\
 & - \frac{1}{4} \frac{k_1^2}{(k_2^2 - m_{S_1}^2)} + \frac{1}{D} \frac{k_1^2}{(k_1^2 - m_{S_3}^2)} + \frac{1}{D} \frac{k_1^2 k_2^2}{(k_2^2 - m_{S_1}^2)^2} + \frac{2}{D(D-1)} \frac{k_1^2 k_2^2}{(k_2^2 - m_{S_1}^2)(k_1^2 - m_{S_3}^2)} \\
 & + \left. \frac{1}{D} \frac{k_1^2 k_1 \cdot k_2}{(k_2^2 - m_{S_1}^2)(k_1^2 - m_{S_3}^2)} \right) \frac{1}{(k_2^2 - m_{S_1}^2)[(k_1 - k_2)^2 - m_{S_2}^2]} \\
 & + \left(\frac{1}{2} \frac{k_1 \cdot k_2}{(k_2^2 - m_{S_2}^2)} - \frac{2}{D} \frac{k_1 \cdot k_2 k_2^2}{(k_2^2 - m_{S_2}^2)^2} - \frac{2}{D-1} \frac{(k_1 \cdot k_2)^2}{(k_2^2 - m_{S_2}^2)(k_1^2 - m_{S_3}^2)} \right. \\
 & - \frac{1}{4} \frac{k_1^2}{(k_2^2 - m_{S_2}^2)} + \frac{1}{D} \frac{k_1^2 k_2^2}{(k_2^2 - m_{S_2}^2)^2} + \frac{2}{D(D-1)} \frac{k_1^2 k_2^2}{(k_2^2 - m_{S_2}^2)(k_1^2 - m_{S_3}^2)} \\
 & + \left. \frac{1}{D} \frac{k_1^2 k_1 \cdot k_2}{(k_2^2 - m_{S_2}^2)(k_1^2 - m_{S_3}^2)} \right) \frac{1}{(k_2^2 - m_{S_2}^2)[(k_1 - k_2)^2 - m_{S_1}^2]} \Big]. \quad (27)
 \end{aligned}$$

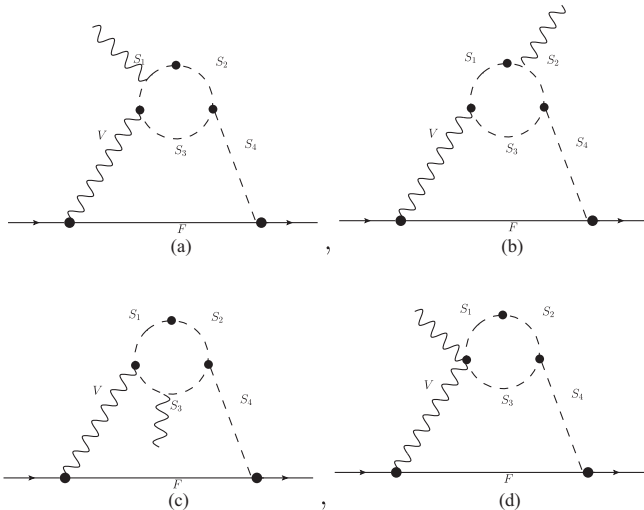


FIG. 5. The two loop triangle diagrams in MIA.

$H_{V\bar{F}F}$ ($H_{VS_1S_2}$) are the couplings of vector boson and fermion (vector boson and scalar), and they belong to electroweak couplings at the order of e ($\frac{e}{\sin \theta_W \cos \theta_W}$) with $V = \gamma(Z)$. $H_{V\bar{F}F}$ and $H_{VS_1S_2}$ do not affect the order analysis. So we neglect them in the following discussion.

To obtain the order of C_6^{MIA} for the corresponding mass insertion diagram, we use the operation on C_6 as $\Delta M_{XY,S}^2 \frac{\partial}{\partial m_{S_1}^2} C_6$ with $X = R, L$ and $Y = R, L$. In the end, the simplification $m_{S_2} \sim m_{S_1} \sim M_{NP} \gg m_V$ and m_F are taken. With the functions in Eqs. (9)–(20), we obtain the following results through tedious calculation



FIG. 6. The scalar propagators appearing in MIA.

$$\begin{aligned}
C_6^{\text{MIA}} &\propto \Delta M_{XY,S}^2 \frac{\partial C_6}{\partial m_{S_1}^2} \\
&\propto \frac{16\pi^2}{m_F} \Delta M_{XY,S}^2 H_{S_1 S_2 S_3} H_{S_3 \bar{F} F} \frac{1}{196608\pi^4 M_{NP}^4} (17 + \dots) \\
&\propto \frac{1}{12288\pi^2 m_F M_{NP}^4} \Delta M_{XY,S}^2 H_{S_1 S_2 S_3} H_{S_3 \bar{F} F} (17 + \dots).
\end{aligned} \quad (28)$$

The dots in Eq. (28) represent small terms comparing with the leading term. As $F \rightarrow \mu$, $V \rightarrow Z(\gamma)$, and $S_1 \rightarrow \tilde{t}_L$, $S_2 \rightarrow \tilde{t}_L$, $S_3 \rightarrow h_d^0$, then

$$\begin{aligned}
H_{S_3 \bar{F} F} &\rightarrow H_{h_d^0 \bar{\mu} \mu} \sim \frac{m_\mu}{v \cos \beta}, \\
\Delta M_{XY,S}^2 &\rightarrow \Delta M_{LR,\tilde{t}}^2 = m_t(A_t - \mu^* \cot \beta) \sim m_t A_t, \\
H_{S_1 S_2 S_3} &\rightarrow H_{\tilde{t}_L \tilde{t}_R h_d^0} \sim \mu m_t / v.
\end{aligned} \quad (29)$$

Here, we suppose $A_t \sim \mu$ and $\cot \beta$ is small.

Using the above replacements, C_6^{MIA} is transformed as

$$\begin{aligned}
C_6^{\text{MIA}} &\propto \frac{1}{12288\pi^2 m_\mu M_{NP}^4} m_t A_t \frac{\mu m_t}{v} \frac{m_\mu}{v \cos \beta} (17 + \dots) \\
&\propto \frac{1}{12288\pi^2 M_{NP}^4} \frac{A_t \mu m_t^2}{v^2 \cos \beta} (17 + \dots) \\
&\propto \frac{\alpha_e}{12288\pi \sin^2 \theta_W} \left(\frac{A_t \mu m_t^2}{m_W^2 M_{NP}^4 \cos \beta} \right) (17 + \dots). \quad (30)
\end{aligned}$$

In our discussion, $\tan \beta$ is large leading to $\sin \beta \sim 1$. Therefore, $\frac{1}{\cos \beta} \sim \tan \beta$. Then from Eq. (28), we obtain the core factor $\frac{A_t \mu m_t^2 \tan \beta}{m_W^2 M_{NP}^4}$ neglecting other terms in the big bracket, which is consistent with the result in Ref. [23]. In the end, the order factor of a_μ is obtained

$$a_\mu \propto m_\mu^2 \frac{A_t \mu m_t^2 \tan \beta}{m_W^2 M_{NP}^4} \propto \frac{m_\mu^2 \tan \beta}{M_{NP}^2}. \quad (31)$$

Here, we use the relation $A_t \sim \mu \sim M_{NP}$. So far, we have obtained that typical factor $\frac{m_\mu^2}{M_{NP}^2} \tan \beta$ for the Barr-Zee type two loop diagrams.

D. Example 2: Convert the Barr-Zee type two loop diagram with heavy fermion subloop and photon from mass eigenstate to MIA

For another example, the Barr-Zee type two loop diagram with heavy fermion subloop and photon are studied in mass eigenstate in Refs. [20,31]. Regardless of whether the results obtained by mass eigenstate method or MIA method, the order of magnitude is the same, which is a general rule. So, we transform the results of Barr-Zee

type two loop diagram with heavy fermion subloop and photon to MIA form, then the factor of the order is obvious. The two-loop corrections of $a_\mu^{2L, \gamma h^0}$ is obtained in mass eigenstate [20,31]

$$\begin{aligned}
a_\mu^{2L, \gamma h^0} &= \frac{e^2}{64\sqrt{2}\pi^4} H_{h^0 \bar{\mu} \mu} \sum_{F_1=F_2=\chi^\pm} \frac{x_\mu^{1/2}}{x_{F_1}^{1/2}} \Re(H_{h^0 \bar{F}_1 F_2}^L) \\
&\quad \times \mathcal{F}_1(x_h, x_{F_a}, x_{F_a}),
\end{aligned} \quad (32)$$

$$\begin{aligned}
\mathcal{F}_1(x, y, z) &= \frac{1}{x} \left[-4(2 + \ln y)(\ln x - 1) \right. \\
&\quad - \frac{\partial}{\partial z} \left(1 + 2 \frac{y-z}{x} \right) \Phi(x, y, z) \\
&\quad + \frac{\partial}{\partial z} \left(\left(1 + 2 \frac{y-z}{x} \right) \varphi_0(y, z) \right. \\
&\quad \left. \left. + 2(y-z)\varphi_1(y, z) \right) \right]. \quad (33)
\end{aligned}$$

The functions $\varphi_0(y, z)$ and $\varphi_1(y, z)$ read as

$$\begin{aligned}
\varphi_0(x, y) &= \begin{cases} (x+y) \ln x \ln y + (x-y)\Theta(x, y), & x > y \\ 2x \ln^2 x, & x = y \\ (x+y) \ln x \ln y + (y-x)\Theta(y, x), & x < y \end{cases} \\
\varphi_1(x, y) &= \begin{cases} -\ln x \ln y - \frac{x+y}{x-y} \Theta(x, y), & x > y \\ 4 - 2 \ln x - \ln^2 x, & x = y \\ -\ln x \ln y - \frac{x+y}{y-x} \Theta(y, x), & x < y \end{cases} \quad (34)
\end{aligned}$$

The function $\Theta(x, y)$ is

$$\Theta(x, y) = \ln x \ln \frac{y}{x} - 2 \ln(x-y) \ln \frac{y}{x} - 2 \text{Li}_2\left(\frac{y}{x}\right) + \frac{\pi^2}{3}. \quad (35)$$

with $L_{i_2}(x)$ denoting the Spence function.

Using similar assumption $m_{F_1} = m_{F_2} \gg m_{h^0}$, one can simplify the two-loop Barr-Zee type diagrams contributing to the muon MDM [20].

$$\begin{aligned}
a_\mu^{2L, \gamma h^0} &= \frac{e^2}{64\sqrt{2}\pi^4} H_{h^0 \bar{\mu} \mu} \sum_{F_1=F_2=\chi^\pm} \frac{x_\mu^{1/2}}{x_{F_1}^{1/2}} \Re(H_{h^0 \bar{F}_1 F_2}^L) \\
&\quad \times \left[1 + \ln \frac{x_{F_1}}{x_{h^0}} \right]. \quad (36)
\end{aligned}$$

In the MSSM, the mass matrix for charginos in the basis $(\tilde{W}^-, \tilde{H}_d^-)$, $(\tilde{W}^+, \tilde{H}_u^+)$ is [10]

$$M_{\chi^\pm} = \begin{pmatrix} M_2 & \frac{1}{\sqrt{2}} g_2 v_u \\ \frac{1}{\sqrt{2}} g_2 v_d & \mu \end{pmatrix}. \quad (37)$$

Considering that $\tan\beta = \frac{v_u}{v_d}$ is large and $\frac{1}{\sqrt{2}}g_2v_d \ll \frac{1}{\sqrt{2}}g_2v_u \ll M_2 \sim \mu$, the mass matrix of chargino in Eq. (37) is approximately

$$M_{\chi^\pm} \sim \begin{pmatrix} M_2 & \frac{1}{\sqrt{2}}g_2v_u \\ 0 & \mu \end{pmatrix}, \quad (38)$$

which is diagonalized by Z_+ and Z_-

$$Z_-^\dagger M_{\chi^\pm} Z_+^* = M_{\chi^\pm}^{\text{dia}}. \quad (39)$$

The approximate results of the rotation matrixes Z_- and Z_+ are deduced as

$$Z_- \sim \begin{pmatrix} 1 & \frac{-\mu g_2 v_u / \sqrt{2}}{M_2^2 - \mu^2} \\ \frac{\mu g_2 v_u / \sqrt{2}}{M_2^2 - \mu^2} & 1 \end{pmatrix}, \quad (40)$$

$$Z_+ \sim \begin{pmatrix} 1 & -\frac{M_2 g_2 v_u / \sqrt{2}}{M_2^2 - \mu^2} \\ \frac{M_2 g_2 v_u / \sqrt{2}}{M_2^2 - \mu^2} & 1 \end{pmatrix}. \quad (41)$$

The couplings $H_{h^0\bar{\mu}\mu}$ and $H_{h^0\bar{\chi}^\pm\chi^\pm}^L$ are in the following form

$$H_{h^0\bar{\mu}\mu} = \frac{m_l}{v \cos\beta} Z_R^{11},$$

$$H_{h^0\bar{\chi}_i^\pm\chi_i^\pm}^L = \frac{-e}{\sqrt{2}s_W} (Z_R^{11} Z_-^{2i} Z_+^{1i} + Z_R^{21} Z_-^{1i} Z_+^{2i}). \quad (42)$$

Here, Z_R is the rotation matrix to diagonalize the mass squared matrix of CP -even Higgs.

With the Eqs. (40)–(42), $a_\mu^{2L,\gamma h^0}$ in Eq. (36) turns to

$$\begin{aligned} a_\mu^{2L,\gamma h^0} &\propto \frac{e^2}{64\sqrt{2}\pi^4} \left(\frac{m_\mu}{v \cos\beta} Z_R^{11} \right) \frac{m_\mu}{m_\chi^\pm} \Re \left[\frac{-e}{\sqrt{2}s_W} (Z_R^{1k} Z_-^{21} Z_+^{11} + Z_R^{2k} Z_-^{11} Z_+^{21} + Z_R^{1k} Z_-^{22} Z_+^{12} + Z_R^{2k} Z_-^{12} Z_+^{22}) \right] \\ &\propto \frac{e^2}{64\sqrt{2}\pi^4} \left(\frac{m_\mu}{v \cos\beta} Z_R^{11} \right) \frac{m_\mu}{m_\chi^\pm} \Re \left[\frac{-e}{\sqrt{2}s_W} (Z_R^{1k} (Z_-^{21} + Z_+^{12}) + Z_R^{2k} (Z_+^{21} + Z_-^{12})) \right] \\ &\propto \frac{e^2}{64\sqrt{2}\pi^4} \frac{m_\mu}{v \cos\beta} Z_R^{11} \frac{m_\mu}{m_\chi^\pm} \frac{e}{\sqrt{2}s_W} \frac{g_2 v \sin\beta}{\sqrt{2}(M_2 + \mu)} (Z_R^{11} - Z_R^{21}) \\ &\propto \frac{1}{128\sqrt{2}\pi^4} \frac{e^4}{s_W^2} \left(\frac{m_\mu^2 \tan\beta}{m_\chi^\pm (M_2 + \mu)} \right) Z_R^{11} (Z_R^{11} - Z_R^{21}) \\ &\propto \frac{1}{256\sqrt{2}\pi^4} \frac{e^4}{s_W^2} Z_R^{11} (Z_R^{11} - Z_R^{21}) \times \left(\frac{m_\mu^2 \tan\beta}{M_{NP}^2} \right). \end{aligned} \quad (43)$$

Obviously, the typical order factor $\frac{m_\mu^2 \tan\beta}{M_{NP}^2}$ for $a_\mu^{2L,\gamma h^0}$ is obtained from the above deduction.

Because to calculate the two loop diagrams is very tedious and takes a lot of time, we give the above examples and do not show the calculation of other diagrams in detail. We extract the order factor of the two loop diagram to find the important two loop diagrams, which can reduce workload for the two loop research.

E. The order factors for the two loop diagrams

Similarly, we obtain the following results for Barr-Zee type two loop diagram

For $F_{\chi^\pm, \chi^\pm, \mu, Z, H^0}^{1,(n)}$ in Table I, the real photon can only be attached on the virtual lepton, which does not contribute to the lepton MDM and EDM in on-shell scheme [17].

For the Barr-Zee type two loop diagram with scalar subloop, we show it in the Fig. 3 with the particles $\mathcal{V} = \gamma, \mathcal{F} = \mu, \mathcal{S}_1 = \tilde{t}_R, \mathcal{S}_2 = \tilde{t}_L, \mathcal{S}_3 = \tilde{t}_R, \mathcal{S}_4 = h_d^0$. The factor analysis is [23]

$$\begin{aligned} \frac{m_\mu^2}{M_{NP}^2} \tan\beta \frac{\mu m_t}{m_W^2} \frac{m_t (A_t - \mu^* \cot\beta)}{M_{NP}^2} &\sim \frac{m_\mu^2}{M_{NP}^2} \tan\beta \frac{\mu m_t}{m_W^2} \frac{m_t A_t}{M_{NP}^2} \\ &\sim \frac{m_\mu^2}{M_{NP}^2} \tan\beta \frac{m_t^2}{m_W^2} \sim 4.6 \times \frac{m_\mu^2}{M_{NP}^2} \tan\beta. \end{aligned} \quad (44)$$

In the same way, the following results in Table II are obtained.

After comparison, the contribution factors from $F_{\tilde{L}, \tilde{L}, H^0, (\gamma, Z), \mu}^{1,(o)}$, $F_{\tilde{D}, \tilde{D}, H^0, (\gamma, Z), \mu}^{1,(o)}$, $F_{\tilde{\nu}, \tilde{\nu}, H^0, Z, \mu}^{1,(o)}$, and $F_{\tilde{L}, \tilde{\nu}, H^\pm, W, \nu}^{1,(o)}$ are not greater than that of $F_{\tilde{t}, \tilde{t}, H^0, (\gamma, Z), \mu}^{1,(o)}$. That is to say, for this type two loop diagram, the contribution from scalar

TABLE I. Barr-Zee type two loop diagrams with fermion subloop.

Factor	Two loop diagram
$\frac{m_\mu^2}{M_{NP}^2} \tan\beta$	$F_{\chi^\pm, \chi^\pm, \mu, Z, H^0}^{1,(n)}$, $F_{\chi^\pm, \chi^0, \nu, W, H^\pm}^{1,(n)}$

TABLE II. Barr-Zee type two loop diagrams with scalar subloop.

Factor	Two loop diagram
$4.6 \times \frac{m_\mu^2}{M_{NP}^2} \tan \beta$	$F_{\tilde{l}, \tilde{b}, H^0, W, \nu}^{1.(o)}, F_{\tilde{l}, \tilde{t}, H^0, Z, \mu}^{1.(o)}$

TABLE III. Barr-Zee type two loop diagrams with Higgs subloop.

Factor	Two loop diagram
$\frac{m_\mu^2}{M_{NP}^2} \tan \beta \frac{B_\mu}{M_{NP}^2} \sim \frac{m_\mu^2}{M_{NP}^2} \tan \beta$	$F_{H^\pm, H^\pm, H^0, (\gamma, Z), \mu}^{1.(o)}, F_{H^0, A^0, A^0, Z, \mu}^{1.(o)}, F_{H^\pm, (A^0, H^0), H^\pm, W, \nu}^{1.(o)}$

TABLE IV. The two loop diagrams with small factor.

Factor	Two loop diagram
$\frac{m_\mu^2}{M_{NP}^2}$	$F_{\gamma, S, (\gamma, Z), \mu}^{1.(a)}, F_{\chi^\pm, \chi^\pm, \mu, \gamma, (\gamma, Z)}^{1.(i)}, F_{S, S, \gamma, (\gamma, Z), \mu}^{1.(j)}$

TABLE V. The two loop diagrams have large factor $\frac{m_\mu^2}{M_V^2}$.

Factor	Two loop diagram
$\frac{m_\mu^2}{M_V^2}$	$F_{Z, S, Z, \mu}^{1.(a)}, F_{W, S, W, \nu}^{1.(a)}, F_{\chi^0, \chi^\pm, \nu, W, W}^{1.(i)}, F_{\chi^\pm, \chi^\pm, \mu, Z, Z}^{1.(i)}, F_{S, S, Z, Z, \mu}^{1.(j)}, F_{S1, S2, W, W, \nu}^{1.(j)}$

down quark, slepton and sneutrino are not larger than that from scalar top quark.

For this type diagram, when the particles in the scalar subloop are Higgs, the corresponding factors are shown in Table III as follows

Here B_μ is at the order of M_{NP}^2 . From the above analysis, one can find $\frac{m_\mu^2}{M_{NP}^2} \tan \beta$ is the typical factor.

For a concise display of results, we collect that the diagrams possess the same factor. To simplify the discussion, we adopt the supposition $M_Z \sim M_W \sim M_V$.

- (1) The diagrams have the factor $\frac{m_\mu^2}{M_{NP}^2}$ are in Table IV [15,18] are with S denoting the charged scalar particles ($\tilde{L}, \tilde{U}, \tilde{D}, H^\pm$). The factor $\frac{m_\mu^2}{M_{NP}^2}$ does not have the large improvement term $\tan \beta$, so these diagrams can be neglected safely.
- (2) The factor $\frac{m_\mu^2}{M_V^2}$ is large, which is bigger than the factor $\frac{m_\mu^2}{M_{NP}^2} \tan \beta$ with $M_{NP} \sim 1000$ GeV. Supposing $M_V \sim 90$ GeV and $\tan \beta \sim 50$, the ratio of the first factor to the second factor is

$$\frac{\frac{m_\mu^2}{M_V^2}}{\frac{m_\mu^2}{M_{NP}^2} \tan \beta} = \frac{M_{NP}^2}{M_V^2 \tan \beta} = \frac{1000^2}{90^2 \times 50} \sim 2.47. \quad (45)$$

So the diagrams possessing the factor $\frac{m_\mu^2}{M_V^2}$ are important, and they are collected in Table V.

with $S = \tilde{L}, H^\pm, \tilde{U}, \tilde{D}, H^0(A^0), \tilde{\nu}$ and $(S1, S2) = (\tilde{\nu}, \tilde{L}); (\tilde{U}, \tilde{D}); (H^0, H^\pm)$. In the on-shell scheme, the contribution from $F_{\chi^0, \chi^0, \mu, Z, Z}^{1.(i)}$ does not have large factor $\frac{m_\mu^2}{M_V^2}$, and it is classified as negligible [31].

- (3) There are many two loop diagrams including several types that have the typical factor $\frac{m_\mu^2}{M_{NP}^2} \tan \beta$. These two loop self-energy diagrams are collected in the Table VI, and they account for more than half of all two loop diagrams.

TABLE VI. The two loop diagrams have the typical factor $\frac{m_\mu^2}{M_{NP}^2} \tan \beta$.

Factor	Two loop diagram
$\frac{m_\mu^2}{M_{NP}^2} \tan \beta$	$F_{\tilde{L}, S, \tilde{L}, \chi^\pm}^{1.(b)}, F_{\tilde{\nu}, S, \tilde{\nu}, \chi^\pm}^{1.(b)}, S = \tilde{\nu}, \tilde{L}, H^0, H^\pm, F_{\tilde{L}, \tilde{L}, (\gamma, Z, W), \chi^0}^{1.(c)}, F_{\tilde{\nu}, \tilde{\nu}, (Z, W), \chi^\pm}^{1.(c)}$ $F_{\chi^0, \mu, \chi^0, \tilde{L}, \tilde{L}}^{1.(g)}, F_{\chi^\pm, \mu, \chi^\pm, \tilde{\nu}, \tilde{\nu}}^{1.(g)}, F_{\chi^0, \nu, \chi^\pm, \tilde{L}, \tilde{\nu}}^{1.(g)}, F_{\chi^0, \chi^\pm, \tilde{L}, \tilde{\nu}, W}^{1.(h)}, F_{\chi^\pm, \chi^\pm, \tilde{\nu}, \tilde{\nu}, Z}^{1.(h)}, F_{\chi^0, \chi^0, \tilde{L}, \tilde{L}, Z}^{1.(h)}$ $F_{\mu, \chi^\pm, \chi^\pm, \tilde{\nu}, \tilde{\nu}}^{1.(l)}, F_{\mu, \chi^0, \chi^0, \tilde{L}, \tilde{L}}^{1.(l)}, F_{\nu, \chi^\pm, \chi^0, \tilde{L}, \tilde{L}}^{1.(l)}, F_{\nu, \chi^\pm, \chi^\pm, \tilde{\nu}, \tilde{\nu}}^{1.(l)}, F_{\tilde{L}, \tilde{L}, \tilde{L}, (\gamma, Z), \chi^0}^{1.(m)}$ $F_{\tilde{\nu}, \tilde{\nu}, \tilde{\nu}, Z, \chi^\pm}^{1.(m)}, F_{\tilde{\nu}, \tilde{L}, \tilde{\nu}, W, \chi^\pm}^{1.(m)}, F_{\tilde{L}, \tilde{\nu}, \tilde{L}, W, \chi^0}^{1.(n)}, F_{\chi^\pm, \chi^\pm, \mu, Z, H^0}^{1.(n)}, F_{\chi^\pm, \chi^0, \nu, W, H^\pm}^{1.(n)}$ $F_{\tilde{l}, \tilde{b}, H^0, W, \nu}^{1.(o)}, F_{\tilde{l}, \tilde{t}, H^0, Z, \mu}^{1.(o)}, F_{\tilde{L}, \tilde{L}, H^0, (\gamma, Z), \mu}^{1.(o)}, F_{\tilde{D}, \tilde{D}, H^0, (\gamma, Z), \mu}^{1.(o)}, F_{\tilde{\nu}, \tilde{\nu}, H^0, Z, \mu}^{1.(o)}$ $F_{\tilde{L}, \tilde{\nu}, H^\pm, W, \nu}^{1.(o)}, F_{H^\pm, H^\pm, H^0, (\gamma, Z), \mu}^{1.(o)}, F_{H^0, A^0, A^0, Z, \mu}^{1.(o)}, F_{H^\pm, (A^0, H^0), H^\pm, W, \nu}^{1.(o)}$ $F_{\chi^0, \chi^\pm, \chi^0, W, \tilde{L}}^{1.(p)}, F_{\chi^0, \chi^0, \chi^0, Z, \tilde{L}}^{1.(p)}, F_{\chi^\pm, \chi^\pm, \chi^\pm, (\gamma, Z), \tilde{\nu}}^{1.(p)}, F_{\chi^\pm, \chi^0, \chi^\pm, W, \tilde{\nu}}^{1.(p)}$ $F_{\chi^\pm, \chi^\pm, \chi^\pm, H^0, \tilde{\nu}}^{1.(r)}, F_{\chi^\pm, \chi^0, \chi^\pm, H^\pm, \tilde{\nu}}^{1.(r)}, F_{\chi^0, \chi^0, \chi^0, H^0, \tilde{L}}^{1.(r)}, F_{\chi^0, \chi^\pm, \chi^0, H^\pm, \tilde{L}}^{1.(r)}$ $F_{\chi^0, F, \chi^0, \tilde{S}, \tilde{L}}^{1.(r)}, (F, \tilde{S}) = (\nu, \tilde{\nu}), (l, \tilde{L}), F_{\chi^\pm, F, \chi^\pm, \tilde{S}, \tilde{\nu}}^{1.(r)}, (F, \tilde{S}) = (\nu, \tilde{L}), (l, \tilde{\nu})$

TABLE VII. The two loop diagrams have the large factor $\frac{m_\mu^2}{M_V^2} \tan \beta$.

Factor	Two loop diagram
$\frac{m_\mu^2}{M_V^2} \tan \beta$	$F_{\nu, \chi^0, \chi^\pm, W, \tilde{\nu}}^{1.(d)}, F_{\nu, \chi^\pm, \chi^0, W, \tilde{L}}^{1.(d)}, F_{\mu, \chi^0, \chi^0, Z, \tilde{L}}^{1.(d)}, F_{\mu, \chi^\pm, \chi^\pm, Z, \tilde{\nu}}^{1.(d)}$ $F_{\chi^\pm, \mu, \tilde{\nu}, \tilde{Z}}^{1.(f)}, F_{\chi^0, \mu, \tilde{L}, \tilde{Z}}^{1.(f)}, F_{\chi^0, \nu, \tilde{L}, \tilde{\nu}, W}^{1.(f)}, F_{\chi^\pm, \nu, \tilde{\nu}, \tilde{L}, W}^{1.(f)}$ $F_{\nu, \chi^\pm, \nu, \tilde{L}, W}^{1.(g)}, F_{\nu, \chi^0, \nu, \tilde{\nu}, W}^{1.(g)}, F_{\mu, \chi^0, \mu, \tilde{L}, Z}^{1.(g)}, F_{\mu, \chi^\pm, \mu, \tilde{\nu}, Z}^{1.(g)}$

TABLE VIII. The two loop diagrams have the middle factor $\frac{m_\mu^2}{M_{NP}^2} \tan \beta \frac{\lambda_{HSS}}{M_{NP}}$.

Factor	Two loop diagram
$\frac{m_\mu^2}{M_{NP}^2} \tan \beta \frac{\lambda_{HSS}}{M_{NP}}$	$F_{\chi^\pm, \chi^\pm, \tilde{\nu}, \tilde{\nu}, H^0}^{1.(e)}, F_{\chi^0, \chi^0, \tilde{L}, \tilde{L}, H^0}^{1.(e)}, F_{\chi^0, \chi^\pm, \tilde{L}, \tilde{\nu}, H^\pm}^{1.(e)}$

- (4) The following diagrams in Table VII have the very large factor $\frac{m_\mu^2}{M_V^2} \tan \beta$. So these diagrams are most important to study muon MDM.
- (5) This type diagrams in Table VIII have the vertex $S-H-S$ possessing mass dimension, which is supposed as λ_{HSS} . Their contributions have the factor $\frac{m_\mu^2}{M_{NP}^2} \tan \beta \times \frac{\lambda_{HSS}}{M_{NP}}$, which is not larger than $\frac{m_\mu^2}{M_{NP}^2} \tan \beta$.
- (6) Similar as the above condition, this type diagram in Table IX has two vertexes $H-S-S$ and the couplings λ_{HSS}^2 . From analysis, their typical factor is $\frac{m_\mu^2}{M_{NP}^2} \tan \beta \times \frac{\lambda_{HSS}^2}{M_{NP}^2} \leq \frac{m_\mu^2}{M_{NP}^2} \tan \beta$. Because λ_{HSS} is not larger than M_{NP} in general.
- (7) This type diagram $F_{F1, F2, F3, S1, S2}^{1.(r)}$ has been researched by the authors [21]. In our supposition, this type diagram also has the typical factor $\frac{m_\mu^2}{M_{NP}^2} \tan \beta$. If the internal sfermions are very heavy, they can produce nondecoupling and logarithmically enhanced contributions to muon MDM. Supposing that the mass of heavy squark is M_{SH} and $M_{SH} \gg M_{NP}$, the logarithmically enhanced factor $\log \frac{M_{SH}^2}{M_{NP}^2}$ appears leading to the factor $\frac{m_\mu^2}{M_{NP}^2} \tan \beta \log \frac{M_{SH}^2}{M_{NP}^2}$ [21]. They are shown in Table X.
- (8) The diagrams in Table XI all have the lepton-Higgs-lepton vertexes, which lead to additional suppression factor $\frac{m_\mu}{V_{ew}} \tan \beta \lesssim 0.02$ [10]. V_{ew} denotes the vacuum expectation value of the Higgs field with $V_{ew} \sim 250$ GeV. Their total factors are shown as $(\frac{m_\mu^2}{M_H^2} \tan \beta, \frac{m_\mu^2}{M_{NP}^2} \tan \beta) \times \frac{m_\mu}{V_{ew}} \tan \beta$, which can be neglected safely. This type two loop diagrams can be neglected.

TABLE IX. The two loop diagrams have the large factor $\frac{m_\mu^2}{M_{NP}^2} \tan \beta \frac{\lambda_{HSS}^2}{M_{NP}^2}$.

Factor	Two loop diagram
$\frac{m_\mu^2}{M_{NP}^2} \tan \beta \frac{\lambda_{HSS}^2}{M_{NP}^2}$	$F_{\tilde{\nu}, H^0, \tilde{\nu}, \tilde{\nu}, \chi^\pm}^{1.(k)}, F_{\tilde{\nu}, H^\pm, \tilde{L}, \tilde{\nu}, \chi^\pm}^{1.(k)}, F_{\tilde{L}, H^0, \tilde{L}, \tilde{L}, \chi^0}^{1.(k)}, F_{\tilde{L}, H^\pm, \tilde{\nu}, \tilde{L}, \chi^0}^{1.(k)}$

TABLE X. The two loop diagrams have the large factor $\frac{m_\mu^2}{M_{NP}^2} \tan \beta \log \frac{M_{SH}^2}{M_{NP}^2}$.

Factor	Two loop diagram
$\frac{m_\mu^2}{M_{NP}^2} \tan \beta \log \frac{M_{SH}^2}{M_{NP}^2}$	$F_{\chi^0, F, \chi^0, \tilde{S}, \tilde{L}}^{1.(r)}, (F, \tilde{S}) = (u_i, \tilde{U}), (d_i, \tilde{D})$ $F_{\chi^\pm, F, \chi^\pm, \tilde{S}, \tilde{\nu}}^{1.(r)}, (F, \tilde{S}) = (u_i, \tilde{D}), (d_i, \tilde{U})$

TABLE XI. The two loop diagrams have the small factor suppressed by $\frac{m_\mu}{V_{ew}}$.

Factor	Two loop diagram
$(\frac{m_\mu^3}{M_H^2} \tan^2 \beta, \frac{m_\mu^3}{M_{NP}^2} \tan^2 \beta)$	$F_{\mu, \chi^\pm, \chi^\pm, H^0, \tilde{\nu}}^{1.(g)}, F_{\mu, \chi^0, \chi^0, H^0, \tilde{L}}^{1.(g)}, F_{\nu, \chi^0, \chi^\pm, H^\pm, \tilde{\nu}}^{1.(g)}$ $F_{\nu, \chi^\pm, \chi^0, H^\pm, \tilde{L}}^{1.(g)}$

TABLE XII. The two loop diagrams have the large factor $\frac{m_\mu^2}{M_{NP}^2} \tan \beta \log x_\mu$.

Factor	Two loop diagrams
$\frac{m_\mu^2}{M_{NP}^2} \tan \beta \log x_\mu$	$F_{\mu, \chi^\pm, \chi^\pm, \gamma, \tilde{\nu}}^{1.(d)}, F_{\chi^0, \mu, \tilde{L}, \tilde{L}, \gamma}^{1.(f)}, F_{\mu, \chi^\pm, \mu, \tilde{\nu}, \gamma}^{1.(q)}, F_{\mu, \chi^0, \mu, \tilde{L}, \gamma}^{1.(q)}$

- (9) The factor $\frac{m_\mu^2}{M_{NP}^2} \tan \beta \log x_\mu$ shown in Table XII is larger than the factor $\frac{m_\mu^2}{M_{NP}^2} \tan \beta$, because $\log \frac{m_\mu}{M_{NP}} \sim -9$ with $M_{NP} = 1000$ GeV. This factor is negative, and it is enhanced by one order of magnitude by the large logarithmic function. This condition has been discussed by the authors [22] for the two loop diagrams in which the internal photon couples to at least one muon. If the internal photon is deleted, these two loop diagrams turn to the one loop SUSY diagrams of $\mu \rightarrow \mu$, where the internal particles are slepton-neutralino and sneutrino-chargino.

IV. DISCUSSION AND CONCLUSION

As it is well known that the two loop diagram contributions to muon MDM are important. However, there are so many two loop diagrams and the calculation of each two loop diagram is much more difficult than that of one loop diagram. So, it becomes important to identify the

TABLE XIII. The size of the factor.

Factor size	The factors
Small factor	$\frac{m_\mu^2}{M_{NP}^2}, \frac{m_\mu^3 \tan^2 \beta}{M_H^2 V_{ew}}, \frac{m_\mu^3 \tan^2 \beta}{M_{NP}^2 V_{ew}}$
Considerable factor	$\frac{m_\mu^2 \tan \beta}{M_{NP}^2}, \frac{m_\mu^2}{M_V^2}, \frac{m_\mu^2 \lambda_{HSS} \tan \beta}{M_{NP}^3}, \frac{m_\mu^2 \lambda_{HSS}^2 \tan \beta}{M_{NP}^4}$
Large factor	$\frac{m_\mu^2 \tan \beta}{M_V^2}, \frac{m_\mu^2 \tan \beta}{M_{NP}^2} \log \frac{m_\mu}{M_{NP}}, \frac{m_\mu^2}{M_{NP}^2} \tan \beta \log \frac{M_{SH}}{M_{NP}^2}$

magnitude of the two loop contributions. Based on the mass eigenstate, we have analyzed the factors of the two loop triangle diagrams for $\mu \rightarrow \mu\gamma$ in the work [15]. The effects from the rotation matrixes of the mass mixing matrixes are not taken into account, and the obtained results are not desirable. In order to solve this shortcoming, we use mass insertion approximation here and get overall clear factors of the two loop diagrams for $\mu \rightarrow \mu\gamma$.

To simplify the analysis, we suppose that all supersymmetric particles have the same mass M_{NP} in this work. This approximation greatly simplifies the analysis. In electroweak eigenstate state, there is not the rotation matrix for supersymmetric particles. For the two loop diagrams in the Fig. 1, their factors of the contributions to muon MDM are categorized by size in Table XIII.

From these factors, one can find that $\frac{m_\mu^2}{M_{NP}^2}, \frac{m_\mu^3 \tan^2 \beta}{M_H^2 V_{ew}}, \frac{m_\mu^3 \tan^2 \beta}{M_{NP}^2 V_{ew}}$ are small, and can be ignored safely. The considerable factors are $\frac{m_\mu^2 \tan \beta}{M_{NP}^2}, \frac{m_\mu^2}{M_V^2}, \frac{m_\mu^2 \lambda_{HSS} \tan \beta}{M_{NP}^3}, \frac{m_\mu^2 \lambda_{HSS}^2 \tan \beta}{M_{NP}^4}$. Most two loop diagrams possess the typical factor $\frac{m_\mu^2 \tan \beta}{M_{NP}^2}$. The rest are big ones including $\frac{m_\mu^2 \tan \beta}{M_V^2}$ and $\frac{m_\mu^2 \tan \beta}{M_{NP}^2} \log \frac{m_\mu}{M_{NP}}$. The large logarithm $\log \frac{m_\mu}{M_{NP}}$ gives negative corrections. As discussed in Ref. [21], if squarks are very heavy, the corrections from Fig. 1(r) type diagram are nondecoupling and logarithmically enhanced by $\frac{m_\mu^2 \tan \beta}{M_{NP}^2} \log \frac{M_{SH}}{M_{NP}^2}$. In this condition, our results are consistent with the conclusion of the authors [21]. In the end, the most important two-loop

TABLE XIV. The most important two loop diagrams.

The most important two loop diagrams
$F_{\mu, \chi^\pm, \chi^\pm, \gamma, \tilde{\nu}}^{1.(d)}, F_{\chi^0, \mu, \tilde{L}, \tilde{L}, \gamma}^{1.(f)}, F_{\mu, \chi^\pm, \mu, \tilde{\nu}, \gamma}^{1.(q)}, F_{\mu, \chi^0, \mu, \tilde{L}, \gamma}^{1.(q)}$ $F_{\chi^0, F, \chi^0, \tilde{S}, \tilde{L}}^{1.(r)}, (F, \tilde{S}) = (u_i, \tilde{U}), (d_i, \tilde{D})$ $F_{\chi^\pm, F, \chi^\pm, \tilde{S}, \tilde{\nu}}^{1.(r)}, (F, \tilde{S}) = (u_i, \tilde{D}), (d_i, \tilde{U})$ $F_{\nu, \chi^0, \chi^\pm, W, \tilde{\nu}}^{1.(d)}, F_{\nu, \chi^\pm, \chi^0, W, \tilde{L}}^{1.(d)}, F_{\mu, \chi^0, \chi^0, Z, \tilde{L}}^{1.(d)}, F_{\mu, \chi^\pm, \chi^\pm, Z, \tilde{\nu}}^{1.(d)}$ $F_{\chi^\pm, \mu, \tilde{\nu}, \tilde{\nu}, Z}^{1.(f)}, F_{\chi^0, \mu, \tilde{L}, \tilde{L}, Z}^{1.(f)}, F_{\chi^0, \nu, \tilde{L}, \tilde{\nu}, W}^{1.(f)}, F_{\chi^\pm, \nu, \tilde{\nu}, \tilde{L}, W}^{1.(f)}$ $F_{\nu, \chi^\pm, \nu, \tilde{L}, W}^{1.(q)}, F_{\nu, \chi^0, \nu, \tilde{\nu}, W}^{1.(q)}, F_{\mu, \chi^0, \mu, \tilde{L}, Z}^{1.(q)}, F_{\mu, \chi^\pm, \mu, \tilde{\nu}, Z}^{1.(q)}$

diagrams are collected in the following Table XIV. Though numerical values depend on the used models, the order analysis for the two loop diagrams benefits to select important diagrams from all the two loop diagrams. Ignoring the two loop diagrams with small factor can reduce the effort of the two loop calculations.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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