

Multicomponent dark matter from an $SU(2)_D \rightarrow Z_3^D \times Z_2^{\text{acc}}$ scenario

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A multicomponent weakly-interacting dark matter scenario is analyzed, where the candidate dark matter states arise from the $SU(2)_D \rightarrow Z_3^D \times Z_2^{\text{acc}}$ symmetry breaking triggered by a vacuum expectation value-developing scalar in the $J = 3/2$ representation. While the discrete symmetries guarantee at least two stable states, a closer look at the mass orderings of the contrived dark sector reveals the existence of a third dark matter candidate, whose stability effectively results from the tight phase space in the dark boson sector. Regions of masses and couplings where the three dark states furnish a comparable contribution to the observed relic abundance are determined numerically.

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I. INTRODUCTION

The identity of the long-ago observed dark matter in the skies persists as a puzzling mystery for the cosmology, astrophysics, and particle physics communities alike. Initially detected by gravitational means and later by its imprint on the cosmic microwave background spectrum, dark matter (DM) has been shown to typically permeate Galactic halos and constitute about 26% of the total energy budget of the Universe.

Attempts to provide a successful answer to this mystery have led these communities to entertain the possibility that dark matter is a yet undiscovered fundamental particle (or set of particles) with electroweak interactions. This entire line of thought, nicknamed the weakly interactive massive particle paradigm, has seen plenty of literature produced. The simplest cases have been considered in the past, ranging from dark matter candidates entirely neutral under the Standard Model (SM) gauge interactions (singlet DM [1–5]) to candidates that form composite states (bound-state DM [6–9]). Another direction has been brought by

scenarios where DM transforms under nontrivial multiplet representations of the SM electroweak gauge group [10–13]. This premise has been combined with the hypothesis that DM carries quantum numbers under discrete symmetries which apart from stabilizing it by preventing its decay, serve other purposes like the generation of tiny neutrino masses [14–17].

The work presented here belongs to the latter class, where DM is made of multiple species, which carry charges under symmetry groups that extend the SM one. The DM candidates arise from scalars in a higher representation of a dark-sector $SU(2)_D$ gauge symmetry, known as the quadruplet representation ($J = 3/2$ in the language of group theory), whose chosen vacuum expectation value (VEV) direction induces a remnant cyclic Z_3^D . This model was initially proposed by Borah, Ma, and Nanda [18], whose work took simplifying limits to ease the discussion of its phenomenology. Furthermore, the authors pointed out the presence of an additional symmetry, an accidental Z_2^{acc} , that enriches the setup.

In this paper we revisit that model and offer a classification of the possible DM scenarios according to the number of dark candidates that contribute significantly to the observed dark matter abundance. There is a strong dependence of these scenarios on the mass relations/kinematics between all states charged under $Z_3^D \times Z_2^{\text{acc}}$, and a nontrivial interplay between the two Higgs portals that offers the conditions necessary for three-component DM where each species contributes a comparable amount to the entire abundance. In addition, the direct detection

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prospects for the different mass orderings are discussed, and how these are modified by the multicomponent nature of DM. Likewise, we mention the impact of semiannihilation at the selected benchmark parameter choices. For other multispecies dark matter analyses, please refer to [17,19–30].

The description of the original model and notation is provided in Sec. II, followed by a discussion of the mass spectrum and the DM species mass orderings in Sec. III. Once equipped with the analytical mass dependence, the classification of the scalar DM scenarios and corresponding kinematic conditions is presented in detail in Sec. IV. This is followed by the numerical analysis of said scenarios with the help of automatized parameter space scans in Sec. V. Afterward, we offer our closing remarks.

II. DARK SYMMETRY BREAKING

The SM gauge group is extended by a local $SU(2)_D$ symmetry, under which only a new $J = 3/2$ multiplet $\Phi = (\phi_3, \phi_2, \phi_1, \phi_0)^T$ (quadruplet) transforms nontrivially. The full scalar potential $V(H, \Phi)$ includes the potential of the Higgs doublet $H = (H^+, H^0)$, with $H^0 = (v_h + \rho_h + i\sigma_h)/\sqrt{2}$,

$$V(H) = -\mu_H^2 H^\dagger H + \frac{\lambda_H}{2} (H^\dagger H)^2, \quad (1)$$

a *Higgs portal* operator parametrized by the quartic coupling $\lambda_{H\Phi}$,

$$V(H, \Phi) = V(\Phi) + V(H) + \lambda_{H\Phi} H^\dagger H \sum_k \phi_k^\dagger \phi_k, \quad (2)$$

and a rich quadruplet-only potential $V(\Phi)$, quoted in Appendix A. The breaking of $SU(2)_D$ is obtained through a nonzero VEV of the quadruplet Φ , with complex entries $\phi_i = v_i + (\rho_i + i\sigma_i)/\sqrt{2}$, which has been explored in Ref. [31]. When Φ acquires a VEV in the direction

$$\langle \Phi \rangle = \begin{pmatrix} v_3/\sqrt{2} \\ 0 \\ 0 \\ v_0/\sqrt{2} \end{pmatrix}, \quad (3)$$

it gives rise to a residual Z_3^D symmetry. Furthermore, Ref. [18] pointed out the existence of an accidental Z_2^{acc} symmetry upon $SU(2)_D$ breaking. Together, the full $Z_3^D \times Z_2^{\text{acc}}$ symmetry implies a richer setup able to embed a multicomponent DM sector. The physical fields after $SU(2)_2$ spontaneous symmetry breaking, but before electroweak symmetry breaking (EWSB), are

$$\tilde{\phi}_R = s_\theta \rho_0 - c_\theta \rho_3, \quad (4)$$

TABLE I. Bosonic field content and quantum numbers after $SU(2)_D \rightarrow Z_3^D \times Z_2^{\text{acc}}$ spontaneous symmetry breaking. Here ω is the unit cubic root, $\omega = e^{2\pi i/3}$. The first five rows correspond to spin zero fields with h representing the SM Higgs field. The last two rows contain the new gauge fields. See Sec. III for details.

Field	Z_3^D	Z_2^{acc}
η	ω	1
ϕ_I	1	-1
ϕ_R	1	-1
ζ	1	1
h	1	1
X'	ω	-1
X_3	1	1

$$\tilde{\zeta} = v_D + c_\theta \rho_0 + s_\theta \rho_3, \quad (5)$$

$$\phi_I = s_\theta \sigma_0 + c_\theta \sigma_3, \quad (6)$$

$$\eta = s_\theta \phi_1 + c_\theta \phi_2^*, \quad (7)$$

where θ is the quadruplet mixing angle. Upon EWSB, $\tilde{\phi}_R$ and $\tilde{\zeta}$ are subject to further mixing with ρ_h , resulting in three physical CP -even scalars, h, ϕ_R, ζ . The mixing between CP -even and CP -odd scalars is forbidden given that we are assuming real VEVs and parameters in the scalar potential, which implies CP conservation. The full, post-EWSB mixing matrices are listed in detail in Appendix B. The new fields, which only extend the SM gauge and scalar sectors, are portrayed together with the SM Higgs in Table I.

III. NEW STATES MASS SPECTRUM

The VEV direction in Eq. (3) results in three massive gauge bosons: a degenerate pair $X_{1,2}$ of mass $M_1 = M_2$, and a slightly heavier X_3 of mass M_3 ,

$$M_3^2 = \frac{9}{2} g_D^2 v_D^2, \quad M_1^2 = \frac{3}{2} g_D^2 v_D^2. \quad (8)$$

For convenience X_1 and X_2 will be combined into a complex vector pair X', X'^* with $X' = (X_1 - iX_2)/\sqrt{2}$ and $X'^* = (X_1 + iX_2)/\sqrt{2}$. The dark gauge bosons do not mix with the SM vectors after EWSB because Φ does not carry electroweak numbers, nor through kinetic mixing as is the case with dark $U(1)$'s. The mass relation

$$M_3 = \sqrt{3} M_1 \quad (9)$$

between the new gauge bosons will be relevant when discussing mass gaps in Sec. IV. The number of degrees of freedom in Φ results in the extended field content of Table I. There is a complex scalar η , a CP -odd scalar ϕ_I , and two CP -even scalars ϕ_R and ζ . The new states carry

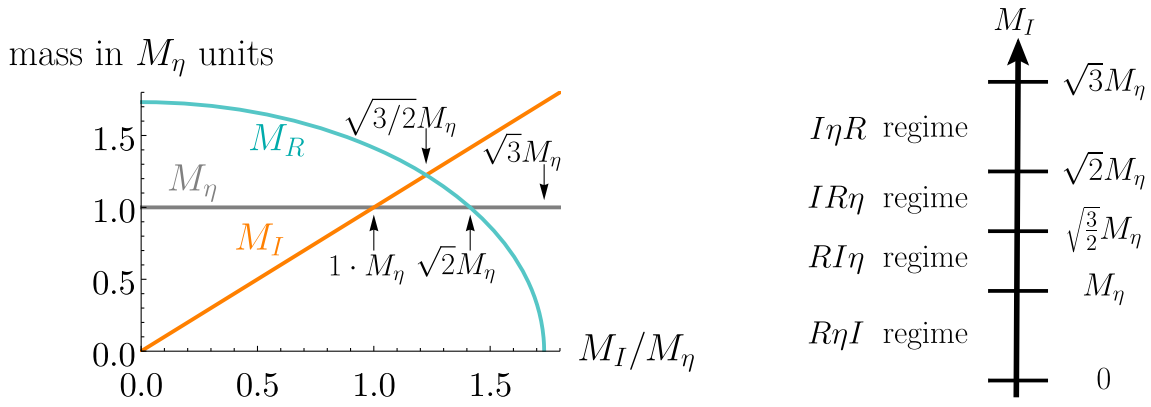


FIG. 1. Left: compressed ϕ_R , ϕ_I , and η mass spectrum from Eq. (14). Masses are expressed in units of M_η for illustration. Right: Possible mass orderings for ϕ_R , ϕ_I , and η as M_I varies at fixed M_η .

charges under Z_3^D or Z_2^{acc} , except for ζ , which mixes with the Higgs h due to the portal quartic $\lambda_{H\Phi}$. For reference, the mixing matrices and Goldstone bosons will be listed in Appendix B.

We trade a subset of the quartic couplings for the scalar physical masses,

$$\lambda_1 = \frac{\sqrt{(M_h^2 - M_\zeta^2)^2 - \Delta}}{4v_D^2} + \frac{2M_h^2 + M_I^2 + 2M_\zeta^2 + 3M_\eta^2 - (M_I^2 - 3M_\eta^2)(1 - 2x)}{8v_D^2}, \quad (10)$$

$$\lambda_2 = \frac{3(M_\eta^2 - M_I^2 + (M_I^2 - 3M_\eta^2)(1 - 2x))}{8v_D^2}, \quad (11)$$

$$\lambda_3 = \frac{3M_\eta^2 - 3M_I^2 - (M_I^2 - 3M_\eta^2)(1 - 2x)}{4v_D^2}, \quad (12)$$

$$\lambda_h = \frac{M_h^2 + M_\zeta^2 - \sqrt{(M_h^2 - M_\zeta^2)^2 - \Delta}}{2v^2}, \quad (13)$$

$$M_R = \sqrt{3M_\eta^2 - M_I^2}, \quad (14)$$

where $\Delta \equiv 8\lambda_{H\Phi}^2 v_D^2 v^2$ and $x \equiv \sin^2(2\theta)$ parametrizes the quadruplet mixing.

The last of these equations is a mass relation that severely restricts the mass spectrum of the scalars with nontrivial $Z_3^D \times Z_2^{\text{acc}}$ charges. In fact, it is important to note, our relation (14) differs from the one obtained in Ref. [18], where a weaker condition on those scalar masses was obtained. This is most relevant to the results presented in this paper since our stringer constraining condition implies the existence of a third scalar DM candidate, stabilized by kinematics only. At fixed M_η , Eq. (14) not only forbids ϕ_R

and ϕ_I to take masses outside the $[0, \sqrt{3}M_\eta]$ range, but also clusters their masses around that of η . Such compressed spectrum is portrayed in the left panel of the diagram of Fig. 1.

As M_I varies, with M_η fixed, the ϕ_R , ϕ_I , and η change their relative mass sizes, giving rise to four possible orderings. The turnover points that define these orderings are M_η , $\sqrt{3/2}M_\eta \approx 1.22M_\eta$, and $\sqrt{2}M_\eta \approx 1.41M_\eta$ (see right panel of Fig. 1). As a means to aid the classification of the various mass orderings in Secs. IV B and IV C, we adopt the following labels:

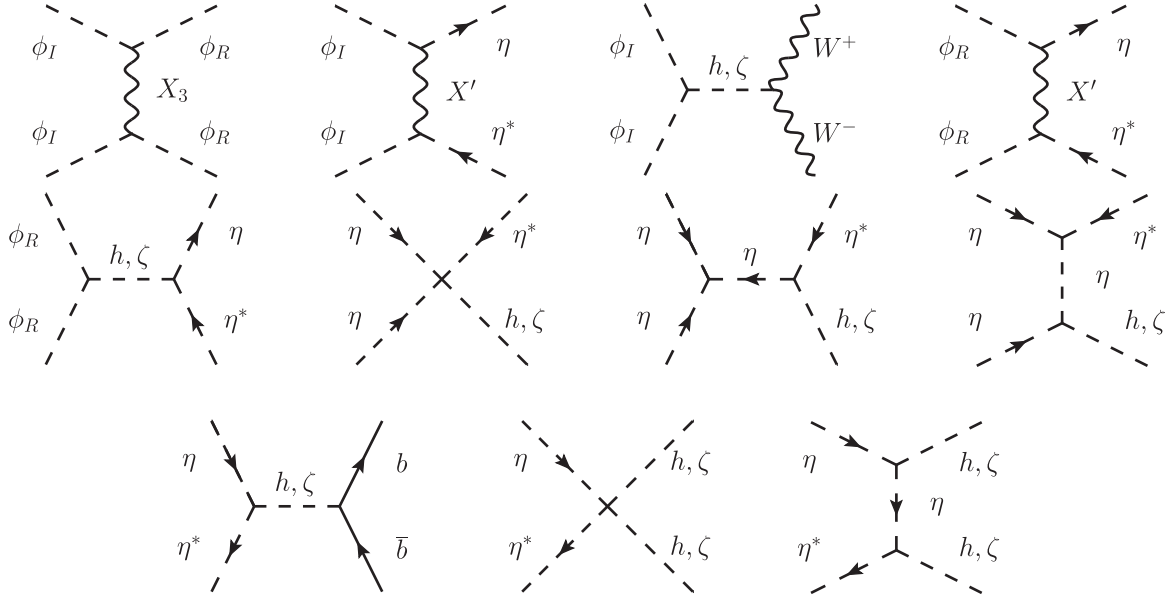
$$\begin{aligned} I\eta R \text{ regime } & M_I > M_\eta > M_R \text{ where } \sqrt{2}M_\eta < M_I < \sqrt{3}M_\eta, \\ IR\eta \text{ regime } & M_I > M_R > M_\eta \text{ where } \sqrt{3/2}M_\eta < M_I < \sqrt{2}M_\eta, \\ RI\eta \text{ regime } & M_R > M_I > M_\eta \text{ where } M_\eta < M_I < \sqrt{3/2}M_\eta, \\ R\eta I \text{ regime } & M_R > M_\eta > M_I \text{ where } 0 < M_I < M_\eta. \end{aligned} \quad (15)$$

These orderings, except for the $R\eta I$ one, result in narrow ranges for the M_η/M_I ratio.

IV. DARK SECTOR SCENARIOS

The Z_2^{acc} -carrying ϕ_R , ϕ_I states undergo self-annihilation into other dark states (DM interconversion) or into SM species, but only through the Higgs portal. Owing to this, the annihilation into the SM occurs preferentially in W bosons and b quarks. Meanwhile, the Z_3^D -carrying η participates in semiannihilation on top of its self-annihilation modes, provided the kinematics allows it. A subset of all possible two-to-two dark scalar annihilation topologies relevant to our benchmark choices in Sec. V is portrayed in Fig. 2.

Before reviewing the instability conditions in the dark sector, it is necessary to list the possible decay modes, which are dictated by the discrete group assignments in Table I. Owing to the small mass gaps between dark scalars, a reasonable approach to the instability conditions of all

FIG. 2. Several two-to-two self-annihilation and semiannihilation (for η only) amplitudes for the dark scalars.

new states consists in looking at their two-body decays at tree level. In setups where the gauge bosons are heavier than the scalar sector but still within 1 order of magnitude away from it, the (few) phase-space reduced three-body decays may as well be forbidden.

A. Decays

- (1) *Decays of η .* The complex scalar η carries a Z_3^D charge. For this reason it must include X' as a daughter if unstable. Since η is even under Z_2^{acc} , the other daughter must be Z_2^{acc} odd,

$$\eta \rightarrow X' \phi_R, \quad \eta \rightarrow X' \phi_I, \quad \eta \rightarrow X'^* X'^*. \quad (16)$$

The Z_2^{acc} symmetry excludes $X' \zeta$, $X' h$, and $X' X_3$ as decay modes.

- (2) *Decays of X' .* Owing to its Z_3^D charge, an unstable X' must have η among its decay products. Also, as a carrier of Z_2^{acc} , X' must also decay to the Z_2^{acc} -odd state,

$$X' \rightarrow \eta \phi_R, \quad X' \rightarrow \eta \phi_I. \quad (17)$$

The ζ , h , and X_3 cannot be final products since these are all even under Z_2^{acc} .

- (3) *Decays of ϕ_R .* Since this scalar only carries Z_2^{acc} charge, if unstable it must decay to ϕ_I or X' . When ϕ_I is a daughter, the other state must be neutral under both Abelian groups,

$$\phi_R \rightarrow \phi_I X_3, \quad \phi_R \rightarrow X' \eta^*. \quad (18)$$

- (4) *Decays of ϕ_I .* The decays of ϕ_I mirror those of ϕ_R because both carry the same charges,

$$\phi_I \rightarrow \phi_R X_3, \quad \phi_I \rightarrow X' \eta^*. \quad (19)$$

- (5) *Decays of ζ .* By mixing with the h , the ζ ends up with SM-like Higgs decay modes, and it is always unstable as long as $\lambda_{H\Phi} \neq 0$.
- (6) *Decays of X_3 .* As a singlet under $Z_3^D \times Z_2^{\text{acc}}$, the only possible decay modes of X_3 are

$$X_3 \rightarrow \phi_R \phi_I, \quad X_3 \rightarrow \eta \eta^*. \quad (20)$$

The decay $X_3 \rightarrow X' X'^*$, while allowed by the quantum numbers, is kinematically forbidden because the mass relation $M_3 = \sqrt{3} M_1$ guarantees $M_3 < 2 M_1$. In addition, as a gauge boson whose longitudinal mode is a linear combination of quadruplet pseudoscalars (see Appendix B), X_3 is forbidden to decay into the CP -even final states $\phi_R \phi_R$, $\phi_I \phi_I$, hh , $h\zeta$, or $\zeta\zeta$.

It can be concluded that all bosons with nontrivial $Z_3^D \times Z_2^{\text{acc}}$ quantum numbers decay into each other.

B. Kinematic (in)stability conditions

The kinematic instability conditions on the whole dark sector can be inferred from the decays listed in the previous section. Notice that only the minimal kinematic condition needs to be satisfied to render the parent particle unstable,

$$\phi_R: M_R > \text{Min}\{M_I + M_3, M_\eta + M_1\} \quad (21)$$

$$\phi_I: M_I > \text{Min}\{M_R + M_3, M_\eta + M_1\} \quad (22)$$

$$\eta: M_\eta > \text{Min}\{2M_1, M_R + M_1, M_I + M_1\} \quad (23)$$

$$X': M_1 > \text{Min}\{M_R + M_\eta, M_I + M_\eta\} \quad (24)$$

$$X_3: M_3 > \text{Min}\{M_R + M_I, 2M_\eta\}. \quad (25)$$

In order to avoid establishing a dark sector with too many stable states (whether from quantum numbers or from kinematic means), we will always seek the instability of X_3 . The minimal decay condition of X_3 is dependent on the ϕ_R , ϕ_I , and η mass ordering. In the notation of Eq. (25),

$$M_3 \xrightarrow{I\eta R} \begin{cases} 2M_\eta, & \text{if } M_I < \left(1 + \frac{\sqrt{2}}{2} M_\eta\right) \\ M_R + M_I, & \text{if } M_I > \left(1 + \frac{\sqrt{2}}{2} M_\eta\right). \end{cases} \quad (26)$$

$$M_3 \xrightarrow{IR\eta} 2M_\eta, \quad (27)$$

$$M_3 \xrightarrow{R\eta I} 2M_\eta, \quad (28)$$

$$M_3 \xrightarrow{R\eta I} \begin{cases} M_R + M_I, & \text{if } M_I < \left(1 - \frac{\sqrt{2}}{2} M_\eta\right) \\ 2M_\eta, & \text{if } M_I > \left(1 - \frac{\sqrt{2}}{2} M_\eta\right), \end{cases} \quad (29)$$

where $(1 - \sqrt{2})/2 M_\eta \approx 0.29M_\eta$ and $(1 + \sqrt{2})/2 M_\eta \approx 1.71M_\eta$. These conditions are obtained, in each mass ordering, by finding which is the smallest of $M_R + M_I$ and $2M_\eta$ in the allowed range of M_I/M_η . The interplay between sets of decay conditions is detailed in Appendix C, and in the next subsection we provide a summary of such interplay.

C. Viable multicomponent DM scenarios

The different mass orderings for the spectrum discussed earlier lead to different dark matter scenarios. These can be classified by the number of (effectively) stable DM candidates. Given the mass relations in Eqs. (9) and (14), it is expected that some scenarios cannot be realized consistently due to kinematic considerations, i.e., the compressed nature of the scalar and gauge mass spectra. We looked at the class of scenarios determined by the ϕ_R , ϕ_I , and η mass ordering (25), and within each, determined valid mass ranges for M_3 and M_1 within the hierarchy of scalar masses. One must note that ζ plays no role in classifying these mass orderings because, as commented in Sec. IV A, it does not appear as a daughter particle in any of the tree-level, two-body decays listed earlier. Naively, the presence of two discrete symmetries suggests two DM candidates, namely the lightest state charged under each of the discrete groups. However, in this section we show that in order to satisfy the kinematical constraints a consistent scenario actually leads to three candidates. Below we

follow the kinematical arguments that lead to this situation. First, the unviability of a two-component setup is explained in virtue of such conditions. Next, we describe how going to three component relaxes the previous constraint, and finally, argue that for one component the constraint is even stronger, making it unviable too.

1. Two-component DM

A priori, the $Z_3^D \times Z_2^{\text{acc}}$ symmetry seems to accommodate a multitude of possibilities for two-component DM in virtue of the two different quantum numbers involved. The specific instability conditions for each case are tabulated in Appendix C, where it is shown how they are mutually inconsistent in every ordering of M_R , M_I , and M_η . We found that all two-component scalar DM scenarios with X' (and hence X_3) heavier¹ than the two lightest dark scalars are inconsistent. The reason is that the minimal decay conditions often require the unstable, third heaviest scalar (for example, ϕ_R in the $R\eta I$ regime, or ϕ_I in $IR\eta$) to be heavier than the upper bound set by Eqs. (15) or by the mass relation in Eq. (14). Also, the fate of the scenarios with one scalar and one gauge boson DM candidate is similar: when X' is the lightest state and one of ϕ_R , ϕ_I , or η is the next to lightest, it is impossible to destabilize all other beyond-the-Standard Model (BSM) states in the absence of large gauge-scalar mass gaps. This is also true when one of ϕ_R , ϕ_I , or η is the lightest DM and X' is the next-to-lightest.

2. Three-component DM

Having proved the absence of consistent two-component DM scenarios, we now relax the minimal decay conditions on one more dark state with the purpose of identifying self-consistent setups. In these scenarios, in addition to the lightest Z_3^D and Z_2^{acc} states, one dark state is rendered stable by having its decays forbidden kinematically rather than by the discrete symmetry assignments. We find that a viable scenario can be obtained when all three DM candidates are scalars. One may ask if a gauge boson could be a DM candidate along with two scalars. However, in the case where the two lightest dark scalars and X' are the DM candidates, one must ensure that X_3 is unstable. Unfortunately, simultaneously satisfying the X_3 and third-heaviest scalar instability conditions turns impossible without invoking large mass gaps.

3. One-component DM

The gauge boson X' is the only state with nontrivial charge assignments under both discrete symmetries. Then, in principle, it can be the only stable DM candidate if lighter than all dark scalars. This scenario is in fact a more restricted version of the two-component DM setup where X' is the lightest state, because now the lightest dark scalar

¹Heavier yet within 1 order of magnitude from them.

TABLE II. Summary of the full mass orderings, range of $r = M_I/M_\eta$, and X' instability condition on g_D for the three-component scalar DM scenarios.

Regime	r range	Mass ordering	X' unstable if $g_D > g_D^*$
$I\eta R$	$\sqrt{2} < r < \sqrt{3}$	$M_3 > M_1 > M_I > M_\eta > M_R$ or $M_3 > M_I > M_1 > M_\eta > M_R$	$g_D^* = \sqrt{\frac{2}{3}} \frac{M_\eta}{v_D} \left(\sqrt{3 - r^2} + 1 \right)$
$IR\eta$	$\sqrt{\frac{3}{2}} < r < \sqrt{2}$	$M_3 > M_1 > M_I > M_R > M_\eta$	$g_D^* = \sqrt{\frac{2}{3}} \frac{M_\eta}{v_D} \left(\sqrt{3 - r^2} + 1 \right)$
$RI\eta$	$1 < r < \sqrt{3/2}$	$M_3 > M_1 > M_R > M_I > M_\eta$	$g_D^* = \sqrt{\frac{2}{3}} \frac{M_\eta}{v_D} (r + 1)$
$R\eta I$	$0 < r < 1$	$M_3 > M_R > M_1 > M_\eta > M_I$ or $M_3 > M_1 > M_R > M_\eta > M_I$	$g_D^* = \sqrt{\frac{2}{3}} \frac{M_\eta}{v_D} (r + 1)$

must satisfy a decay condition. Under our two-body decay treatment, this scenario is proved unviable, ruling out the one-component X' DM case.

Summarizing, we found that when X_3 and X' are the only unstable states, i.e., a three-scalar DM scenario, all instability conditions hold at the same time. Furthermore, this is true in each of the four orderings, $R\eta I$, $RI\eta$, $IR\eta$, and $I\eta R$. There are two reasons why this happens: first, the absence of contrived instability conditions on ϕ_R , ϕ_I , η , and second, due to a slightly increased flexibility in pushing X_3 and X' to heavier masses by increasing g_D alone. In order to ease the upcoming numerical analysis, we define the mass ratio $r = M_I/M_\eta$, and then the scalar masses look like

$$M_\eta, \quad M_I = rM_\eta, \quad M_R = M_\eta \sqrt{3 - r^2}, \quad (30)$$

where $0 < r < \sqrt{3}$. We also cast the instability condition on X' (either $M_1 > M_R + M_\eta$ or $M_1 > M_I + M_\eta$, depending on the dark scalar mass regime) as the condition $g_D > g_D^*$, where g_D^* is the smallest gauge coupling value at which X' is unstable. The complete dark sector mass orderings in the four main regimes, along with the r allowed range and the kinematic condition for the decay of X' , are collected in Table II.

It must be noted that in these scenarios, owing to an enhanced annihilation within the dark sector, the relative contributions to the observed DM abundance by the three DM species can differ greatly so that they resemble a setup with effectively one or two DM components. In what follows, we refer to these cases as pseudo-one and pseudo-two-component scenarios, bearing in mind that there is always a contribution from the three DM candidates. For this reason, rather than providing an exhaustive analysis of every case, we limit ourselves to revealing and describing regions of the parameter space where three-component DM is realized with the three abundances contributing comparably to the observed relic density.

V. NUMERICAL STUDY

In this section we select one of the dark scalar mass regimes shown in Table II and perform a numerical analysis on it. Such analysis is possible with the help of

Mathematica's model generator Sarahv4.15.1 [32], the spectrum calculator SPheno-v4.0.5 [33], and the C++-based dark matter analyzer MicrOMEGAsv6.0.5 [34], which computes the DM abundance and contributions by integrating the coupled system of Boltzmann equations of the DM candidates.

The two regimes where the dark scalar masses are the most compressed are the $RI\eta$ and $IR\eta$ regimes; then the value of any of the dark scalar masses acts as a ballpark estimate for the other two masses. Based on this observation, we choose the $RI\eta$ regime for the upcoming analysis. The starting point of our analysis of three-component DM is a look at the available free parameters, namely

$$g_D, v_D, r, x, M_\zeta, M_\eta, \quad \text{and} \quad \lambda_{H\Phi}. \quad (31)$$

In this set, we choose to fix g_D , v_D , r , x , and M_ζ , and scan over ranges of M_η and the Higgs portal coupling $\lambda_{H\Phi}$. The gauge coupling will be taken above the g_D^* value shown in the last column of Table II, which is M_η dependent. Meanwhile, the dark VEV will be fixed at 1 TeV. As indicated in the previous section, the value of $r = M_I/M_\eta$ within the $0 < r < \sqrt{3}$ range fixes the mass ordering between ϕ_R , ϕ_I , and η . The r ratio will be set to the value of the midpoint $\bar{r}$ of that r range, i.e., $\bar{r} = (1 + \sqrt{3/2})/2$ in the $RI\eta$ regime. For the quadruplet components, a small mixing parameter x is chosen, $x = 0.1$. Another of our assumptions is that the SM Higgs is the lightest of two $Z_3^D \times Z_2^{\text{acc}}$ singlets, $M_h < M_\zeta$. This is ensured, for example, with the arbitrary choice $M_\zeta^2 \approx 3 \times 10^4 \text{ GeV}^2$. The dark scalar masses will be parametrized in terms of M_η and scanned over the tens of GeV up to a few TeV.² Meanwhile, $\lambda_{H\Phi}$ will be varied from 10^{-5} to 10^{-2} to determine regions that barely escape direct detection searches. The scans performed below are possible through Sarah's extension SSPv1.2.5 [37].

A representative two-parameter scan in the $RI\eta$ regime is that of Fig. 3, whose different panels show different ordering of the ϕ_R mass (the CP -even dark scalar) with

²For multicomponent DM in the sub-GeV regime, see Ref. [35,36].

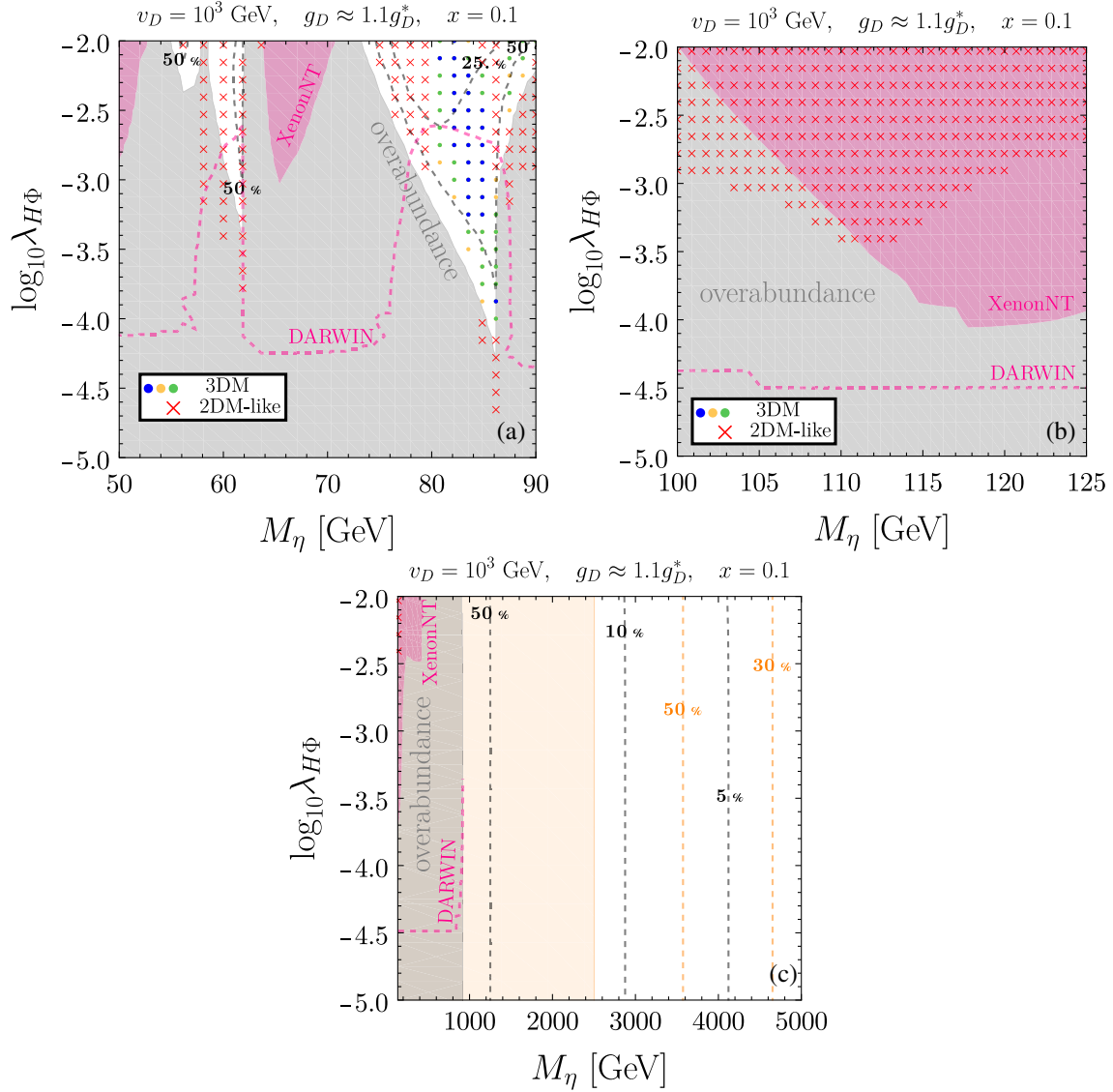


FIG. 3. Dark matter abundance and direct detection constraints on the $R\eta$ regime at fixed dark gauge coupling g_D , quadruplet VEV v_D , and quadruplet mixing x . The colored markers outline regions with three-component DM (blue, green, and yellow) or pseudo-two-component DM (red crosses). The rest of the regions are pseudo-one-component DM. Three CP -even scalar mass orderings are shown. (a) $M_R < M_h < M_\zeta$, where $g_D \sim 0.1$. (b) $M_h < M_R < M_\zeta$, with $g_D \sim 0.2$. (c) $M_h < M_R < M_\zeta$, for which $g_D \sim O(1)$.

respect to those of h and ζ .³ The dark gray solid line contains points where the total DM abundance amounts to 100% of the PLANCK measurement, $\Omega h^2 \approx 0.12$ [38], whereas dashed lines indicate contours of underabundance. Meanwhile, the gray shade covers regions where the total DM exceeds the observed abundance.

In the parameter window shown in panel (a) of Fig. 3, where $M_R < M_h < M_\zeta$, the most prominent features are the two dips where the overabundance exclusion relaxes. Funnels like these are typical of resonant annihilation

through s -channel Higgs exchange, which explains their occurrence at approximately $M_h/2$ and $M_\zeta/2$. Only one of the funnels, that of ζ , accommodates a narrow region where $\Omega_I \sim \Omega_\eta \sim \Omega_R$, namely, three-component DM.

In order to graphically indicate the ratios that differentiate between scenarios of distinct effective number of DM species, we first fix the (arbitrary) proportions that they must satisfy to fall in one or another category. Let us call the DM species abundances Ω_1 , Ω_2 , and Ω_3 , where $\Omega_1 < \Omega_2 < \Omega_3$ and Ω_i is any of Ω_R , Ω_I , or Ω_η . Pseudo-one-component DM, namely $\Omega_1 \lesssim \Omega_2 \ll \Omega_3$, satisfies $\Omega_2 < \frac{1}{10} \Omega_3$. For this case, the points will not be indicated in any plot, that is, all white regions are meant to be pseudo-one-component DM. For pseudo-two-component DM, namely $\Omega_1 \ll \Omega_2 \lesssim \Omega_3$

³Three panels are shown instead of a single one to avoid numerical artifacts caused by the crossing of CP -even mass eigenvalues in our code.

TABLE III. Chosen proportions for the three DM matter species abundances used to define the scenarios in our analysis.

DM scenarios	DM proportions
pseudo-one-component	$\Omega_1 \lesssim \Omega_2 \ll \Omega_3$ with $\Omega_2 < \frac{1}{10} \Omega_3$
pseudo-two-component	$\Omega_1 \ll \Omega_2 \lesssim \Omega_3$ with $\Omega_1 < \frac{1}{4} \Omega_2$ and $\Omega_2 > \frac{1}{10} \Omega_3$
Three component	$\Omega_1 \lesssim \Omega_2 \lesssim \Omega_3$

our chosen criteria are $\Omega_1 < \frac{1}{4} \Omega_2$ and $\Omega_2 > \frac{1}{10} \Omega_3$, which we identify as red crosses in each plot. Finally, three-component DM points are those which satisfy the condition $\Omega_1 \lesssim \Omega_2 \lesssim \Omega_3$. In this case we consider three different fractions⁴: $\Omega_1 > \frac{1}{2} \Omega_2$ and $\Omega_2 > \frac{1}{2} \Omega_3$ (blue dots), $\Omega_1 > \frac{1}{3} \Omega_2$ and $\Omega_2 > \frac{1}{3} \Omega_3$ (green dots), and $\Omega_1 > \frac{1}{4} \Omega_2$ and $\Omega_2 > \frac{1}{4} \Omega_3$ (yellow dots). This information is summarized in Table III.

Understanding why only one of the funnels in panel (a) of Fig. 3 harbors comparable partial abundances for the three scalars requires looking at the main contributions to annihilation there. First, we looked at the h funnel. Having M_η on the horizontal axis clearly indicates where η undergoes resonant s -channel annihilation through h , dominated by $\eta\eta^* \rightarrow b\bar{b}$. According to the mass relations in Eq. (30), when η encounters resonant annihilation near $M_h/2$ the heavier ϕ_I mass is barely past this value, but still annihilating mostly through $\phi_I\phi_I \rightarrow W^+W^-$ mediated by s -channel Higgs. Likewise, η sitting at the Higgs funnel implies that the ϕ_R mass is past $M_h/2$ too. But ϕ_R ends up a few GeV below $M_\zeta/2$, which enhances ϕ_R annihilation through s channel ζ . Yet, ϕ_R preferentially undergoes DM conversion into η through $\phi_R\phi_R \rightarrow \eta\eta^*$, which is possible kinematically as ϕ_R is heavier than both ϕ_I and η . At the h funnel, the dominant cross sections of ϕ_I and η have been checked to be comparable within 1 order, as opposed to the interconversion cross section of ϕ_R which is about one hundred times larger. Thus, Ω_R is relatively suppressed with respect to Ω_I and Ω_η around $M_h/2$, and an effectively pseudo-two-component DM scenario is rendered there (see red crosses). Next, we turn our attention to the ζ funnel. When η sits at around $M_\zeta/2$, its annihilation is resonant, with dominant channel $\eta\eta^* \rightarrow W^+W^-$. At that M_η value, the heavier ϕ_I is past its own ζ resonant annihilation but is still dominated by $\phi_I\phi_I \rightarrow W^+W^-$ with ζ exchange in the s channel. By the same reason, ϕ_R is also past its ζ resonant peak and undergoes conversion to η that, while dominated by $\phi_R\phi_R \rightarrow W^+W^-$, is closely followed by conversion to $\phi_R\phi_R \rightarrow \eta\eta^*$ through t -channel X' exchange. We have checked that the cross sections for these annihilation channels of η , ϕ_I , and ϕ_R are comparable within 1 order of magnitude when M_η is around $M_\zeta/2$. As a result, $\Omega_\eta \sim \Omega_I \sim \Omega_R$ in that region.

In panel (b) of Fig. 3, the dark scalar ϕ_R is found between the two Higgses, $M_h < M_R < M_\zeta$. At these M_η values the h and ζ funnels have degraded and annihilation is no longer resonant. While η dominates the DM abundance, it is closely followed by ϕ_I (as depicted by the red crosses) provided that the portal coupling is large enough. Annihilation of η and ϕ_I is still determined by Higgs portal interactions, leading to $\eta\eta^* \rightarrow VV^*$ and $\phi_I\phi_I \rightarrow VV^*$ with $V = W, Z$. These are mediated by h and ζ in the s channel and have comparable size. Yet, the absence of resonant annihilation rules out the entire window by overabundance considerations.

Panel (c) of Fig. 3 corresponds to a ϕ_R heavier than the two Higgses, $M_h < M_\zeta < M_R$. In this region η comprises most of the relic, effectively a pseudo-one-component DM scenario, and the PLANCK measurement is fulfilled at a near-TeV M_η . The (nonresonant) self-annihilation of η is driven by $\eta\eta^* \rightarrow \zeta\zeta$ through a four-point vertex or a t channel η , (see the last two diagrams in Fig. 2), followed by dark-to-dark conversion $\eta\eta^* \rightarrow \phi_I\phi_I$ via X' exchange in the t channel (reverse of the second). Since the η annihilation stops being determined by the Higgs portal coupling, the relic contours become nearly vertical.

To continue our analysis, we discuss the role of dark matter semiannihilation in the scans above. The presence of dark states φ carrying a Z_3^D quantum charge introduces annihilations of the type $\varphi\varphi \rightarrow \varphi^*\chi$ where χ is Z_3^D neutral (dark or visible). In this model φ is either η or X' , but it is enough to consider η as it is the lighter of the two. In the region near the h and ζ funnels, annihilation of the dark scalars (η included) is overwhelmingly driven by resonant s -channel self-annihilation through h and ζ , as discussed earlier. While these $\eta\eta^* \rightarrow \text{SM SM}$ channels decrease in size away from the funnels, they still dominate annihilation as M_η varies in the hundred-GeV ballpark. This is the case, for instance, in the region of panel (b) of Fig. 3, which is nevertheless excluded due to DM relic overabundance. Further away from the funnels, at even heavier M_η , DM becomes only pseudo one component, dominantly η . This is indeed the case of the region shown in panel (c) of Fig. 3. There, at the selected parameters, semiannihilation remains subdominant due a ratio of thermal cross sections $\langle v\sigma \rangle_{\text{self}} / \langle v\sigma \rangle_{\text{semi}} \approx 4$ at the decoupling temperature, as checked with the help of MicrOMEGAs. To depict the predominance of self-annihilation in that region, the same overabundance shade and underabundance contours are

⁴These fractions are not mutually exclusive.

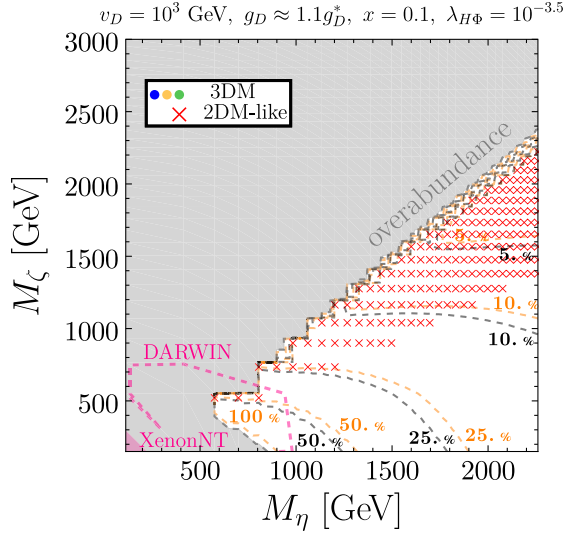


FIG. 4. The M_η and M_ζ interplay in the hundred-GeV to few-TeV region, corresponding to panel (c) of Fig. 3 but now at the fixed Higgs portal quartic. The dashed contours indicate underabundance values with all channels active (gray) and without $\eta\eta \rightarrow \eta^*\zeta$ semiannihilation (orange). The regions below the pink contours are excluded by direct detection considerations.

repeated in orange but now with η self-annihilation turned off.⁵ The effect of switching off self-annihilation is manifested as a considerable shift toward larger M_η in said contours.

In order to grasp the extent to which $\eta\eta \rightarrow \eta^*\zeta$ can be rendered comparable to $\eta\eta^* \rightarrow \zeta\zeta$ in our regions of study, we fixed the $\lambda_{H\Phi}$ quartic and let M_ζ vary while keeping the rest of the parameters at the same values as before. In Fig. 4, M_ζ is varied from hundreds of GeV to a few TeV, which includes regions where the ratio M_ζ/M_η approaches 1. Underabundant DM relic dashed contours are shown in the case when all annihilation channels open (gray) and with semiannihilation turned off (orange). We have checked that η semiannihilation only becomes comparable to self-annihilation at $M_\zeta/M_\eta > 1$ ratios, which for this window lies within the overabundant exclusion. As for the underabundant region, the effect of semiannihilation is subdominant. Regardless, here, three regions are discerned: in the region where ζ is much lighter than η , at fixed M_η we notice a soft variation of the underabundance contours as M_ζ is varied. In fact, the contours become more vertical toward the smallest M_ζ value shown, and start being insensitive to it. A second region is that where contours are almost horizontal, where M_ζ becomes closer to M_η . Despite being marked as a pseudo-two-component region, we have checked that η is the

most abundant of the two species. Here, although η is already past its self-annihilation resonant peak at $M_\zeta/2$, higher M_ζ brings this peak slightly closer to M_η , causing an increase of self-annihilation and a decrease in the η relic. A third region is that where M_ζ/M_η is very close to 1. At fixed M_η , heavier M_ζ starts limiting the phase space available for $\eta\eta^* \rightarrow \zeta\zeta$, which lowers this cross section and leaves higher DM abundance.

In the spirit of complementarity, the sensitivity of the three-component DM regions to the choices of benchmark parameters in the windows above is shown for different choices of the main dark parameters, namely the dark gauge coupling and dark VEV, in the panels of Fig. 5. In panel (a) of Fig. 5 one sees that when v_D is larger than that in panel (a) of Fig. 3, the h and ζ funnels erode. This is so because as v_D grows while keeping the scalar masses at the same ballpark as before, the interconversion amplitudes between the ϕ_R , η , and ϕ_I decrease in size due to X' and X_3 becoming heavier in the propagators. This causes the region of comparable abundances to be buried by the overabundance shade. The opposite effect, i.e., deeper funnels, is achieved with a v_D smaller than the one chosen earlier, as shown in panel (b) of Fig. 5. The tradeoff is, however, the preponderance of pseudo-one-component points with acceptable DM abundance. Finally, for a g_D choice much larger than the one used before, the funnels deteriorate once again as shown in panel (c) of Fig. 5. This is due to enhanced DM interconversion via X' and X_3 exchange, which relies on gauge-scalar vertices.

In regard to indirect detection prospects, a dedicated scan was not pursued due to two reasons. First, the dark sector and the SM communicate through a Higgs portal quartic that is small, and second, the $Z_3^D \times Z_2^{\text{acc}}$ symmetry results in our model sharing dark-to-visible annihilation channels with other works based on stabilizing $Z_{N>2}$ symmetries. It is reasonable to state that points with acceptable relic abundance could display similar thermally averaged annihilation cross section sizes within 1 order of magnitude to those found in such constructions, which are typically safe from indirect detection bounds [17]. For reference, around the ζ funnel of panel (a) in Fig. 3 there are $\phi_I\phi_I \rightarrow WW$ and $\eta\eta^* \rightarrow WW$ self-annihilations in the range $10^{-27} - 10^{-30}$ cm³/s, whereas around a mass of 1 TeV the abundance-dominating η undergoes $\eta\eta^* \rightarrow \zeta\zeta$ and $\eta\eta \rightarrow \eta^*\zeta$ annihilations around 10^{-29} cm³/s.

To conclude this section, a description of dark matter signals from nuclear-recoil based scattering, i.e., direct detection, is presented. The panels of Fig. 3 illustrate the comparison between the spin-independent (SI) cross section of the model at the chosen parameters against the XenonNT [39] limit (magenta shade) and the future sensitivity of the DARWIN Collaboration [40] (magenta dashed line). Owing to the presence of three DM species, the constraint imposed by these experiments on the i -th species is not on its SI cross section σ_i^{SI} but on its SI cross section rescaled by its relative abundance with respect to

⁵This depiction is preferred over showing the contours with semiannihilation switched off because the effect is barely discernible.

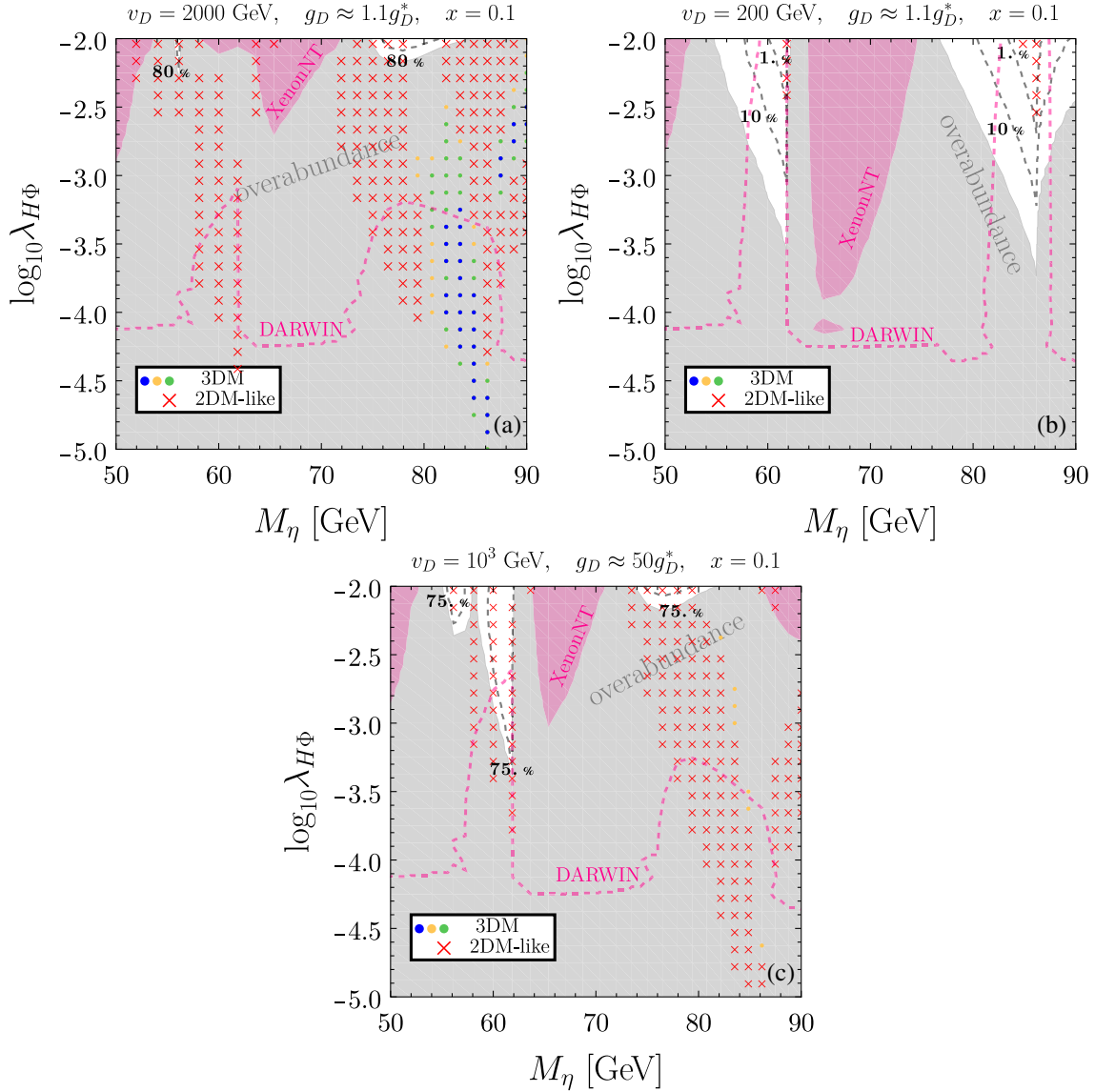


FIG. 5. Scans analog to panel (a) of Fig. 3, now with different v_D [panels (a) and (b)] and g_D [panel (c)]. In panels (a) and (b), $g_D \sim 0.5$, while $g_D \sim O(1)$ in panel (c).

the observed value,⁶ $\sigma_i^{\text{SI}} \times \Omega_i / \Omega_{\text{PLANCK}}$. For panel (a) of Fig. 3, which is the only one with a region where all three abundances are comparable, we opted to compare the XenonNT and DARWIN limits with the weighted sum

$$\sum_{i=R,I,\eta} \frac{\Omega_i}{\Omega_{\text{PLANCK}}} \sigma_i^{\text{SI}}. \quad (32)$$

Away from the ζ funnel the three abundances start differing and the comparison with the sum above can still be done, if taken as a conservative bound. As observed in that figure, the DARWIN projected sensitivity rules out parts of the

⁶This is so because experimental limits assume the PLANCK abundance value in the recoil rate calculation.

Higgs and ζ funnels, but still leaves points that are 3DM while hitting the observed DM abundance. For panels (b) and (c) in Fig. 3, the direct detection limit is applied to $\sigma_\eta^{\text{SI}} \times \Omega_\eta / \Omega_{\text{PLANCK}}$ directly because η is by far the dominant species there.

A common challenge for multicomponent DM realizations is the task of distinguishing its signals from those in single-component DM. Merely on direct detection grounds, this could be achieved (in principle) by registering signals at different mass values. This poses a difficulty for multicomponent DM mass spectra where the species are nearly degenerate, in which case the signal significance becomes too sensitive to the mass splitting(s) (see for example Ref. [41]). The $RI\eta$ mass ordering chosen at the start of this section has, at our benchmark, $(M_I - M_\eta)/M_\eta \sim 0.1$ and $(M_R - M_I)/M_I \sim 0.2$ which is near degenerate and

therefore subject to such experimental disadvantage (the same can be said about the $IR\eta$ mass ordering). The other two mass orderings offer slightly larger mass splittings, although that could come at the expense of uneven contributions to the relic density. For this reason, a signal in direct detection must be complemented with collider searches whose missing energy distributions take advantage of the different charge and parity quantum numbers of ϕ_R , ϕ_I , and η .

VI. CONCLUSIONS

An analysis has been presented of a BSM model in order to determine its viability as a multicomponent dark matter scenario. The model involves an additional (to the SM gauge symmetry) $SU(2)_D$ local symmetry spontaneously broken to a Z_3^D discrete subgroup through the nonzero VEV, a quadruplet scalar. Furthermore, there is an accidental Z_2^{acc} in the broken phase of the model such that, effectively, one has a $SU(2)_D \rightarrow Z_3^D \times Z_3^{\text{acc}}$ scenario.

The mass spectrum of the new $SU(2)_D$, i.e., gauge bosons and scalar components of the quadruplet, render interesting kinematic constraints which are found to restrict the possibilities for a viable set of DM candidates. We find that together with the presence of the two discrete symmetries involved, the kinematic constraints only allow for a viable three-component dark matter scenario.

A thorough numerical study has been performed that shows the regions of parameter space where the model provides a DM abundance consistent with current measurements. The individual contributions from the three DM candidates to the total abundance is shown to vary throughout the parameter in a way that, effectively, one can have regions of parameter space where all three candidates contribute significantly, hence called the three-component DM scenario, as well as regions in which only one or two of the three contribute do so, hence called pseudo-one- and pseudo-two-component scenarios.

We also conclude that next generation direct detection searches, depending on specific parameter choices that take advantage of resonant DM annihilation, have the potential to find members of a multicomponent dark sector. Discerning between the species, though, may require complementary search strategies in colliders. This represents a departure from the familiar expectation to find a unique DM particle candidate.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data supporting this study's findings are available within the article.

APPENDIX A: THE QUADRUPLER POTENTIAL

A $J = 3/2$ quadruplet Φ can be represented either in terms of the J_z eigenvalues or as a fully symmetric three-index tensor with components ϕ^{ijk} . The correspondence between entries in each representation follows from

$$\Phi = \begin{pmatrix} \phi_3 \\ \phi_2 \\ \phi_1 \\ \phi_0 \end{pmatrix} = \begin{pmatrix} \phi^{111} \\ \sqrt{3}\phi^{112} \\ \sqrt{3}\phi^{122} \\ \phi^{222} \end{pmatrix}. \quad (\text{A1})$$

As shown in Ref. [18], the scalar potential of Φ appearing in Eq. (2) has the structure

$$\begin{aligned} V(\Phi) = & -\mu_\Phi^2 \sum_k \phi_k^\dagger \phi_k + \frac{\lambda_1}{2} \left(\sum_k \phi_k^\dagger \phi_k \right)^2 \\ & - \frac{10\lambda_2}{9} [(\Phi\tilde{\Phi})_3(\Phi\tilde{\Phi})_3]_1 - \frac{5\lambda_3}{18} [(\Phi\Phi)_3(\Phi\Phi)_3 \\ & + \text{H.c.}]_1 - \frac{5\lambda_4}{9} [(\Phi\Phi)_3(\Phi\tilde{\Phi})_3 + \text{H.c.}]_1, \end{aligned} \quad (\text{A2})$$

where the components of $\tilde{\Phi}$ are defined through $\tilde{\Phi}^{ijk} = \epsilon^{ii'} \epsilon^{jj'} \epsilon^{kk'} \Phi_{i'j'k'}^*$ (indices refer to the tensorial notation). The objects $(\Phi\Phi)_3$ and $(\Phi\tilde{\Phi})_3$ are $J = 1$ objects whose $J_z = +1, 0, -1$ components are determined by the Clebsch-Gordan coefficients in a $3/2 \otimes 3/2$ product, and so on. Other contractions of Φ with higher or lower J are either vanishing, or equivalent to the $J = 1$ products included above.

APPENDIX B: GAUGE AND SCALAR MIXING

We list in this Appendix the algebra of gauge and scalar mixing. In the gauge sector, mass mixing with the Z is absent due to the lack of scalars simultaneously charged under both dark and SM symmetries. Likewise, kinetic mixing is absent from the non-Abelian nature of $SU(2)_D$. The mass matrices of the scalars display the full mixing after the spontaneous breaking of the $SU(2)_D$ and electro-weak symmetries.

The dark scalar-gauge interaction is encoded in the gauge covariant derivative of the quadruplet,

$$D_\mu \Phi = \partial_\mu \Phi - ig_D X_\mu^a T^a \Phi, \quad (\text{B1})$$

where $a = 1, 2$, and 3 (sum of repeated indices is implicit). The T^a are $SU(2)_D$ generators in the (four-dimensional) $J = 3/2$ representation,

$$J_1 = \frac{1}{2} \begin{pmatrix} 0 & \sqrt{3/2} & 0 & 0 \\ \sqrt{3/2} & 0 & \sqrt{3/2} & 0 \\ 0 & \sqrt{3/2} & 0 & \sqrt{3/2} \\ 0 & 0 & \sqrt{3/2} & 0 \end{pmatrix}, \quad J_2 = \frac{1}{2} \begin{pmatrix} 0 & \sqrt{3/2} & 0 & 0 \\ -\sqrt{3/2} & 0 & \sqrt{3/2} & 0 \\ 0 & -\sqrt{3/2} & 0 & \sqrt{3/2} \\ 0 & 0 & -\sqrt{3/2} & 0 \end{pmatrix}$$

$$J_3 = \begin{pmatrix} 3/2 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & -1/2 & 0 \\ 0 & 0 & 0 & -3/2 \end{pmatrix}.$$

The $SU(2)_D$ breaking pattern $\langle \Phi \rangle = (1/\sqrt{2})(v_3, 0, 0, v_0)$ induces the mixing matrix for the dark gauge bosons

$$\mathcal{M}_X^2 = \begin{pmatrix} \frac{3}{2}g_D^2 & 0 & 0 \\ 0 & \frac{3}{2}g_D^2 & 0 \\ 0 & 0 & \frac{9}{2}g_D^2 \end{pmatrix} v_D^2, \quad (\text{B2})$$

in the $\{X_1, X_2, X_3\}$ basis. The X_1 and X_2 turn out degenerate and can be combined into a complex vector $X' = (X_1 - iX_2)/\sqrt{2}$ and its charge-conjugate $X'^* = (X_1 + iX_2)/\sqrt{2}$.

In the basis $\{\sigma_h, \sigma_0, \sigma_3\}$ of imaginary parts of h , ϕ_0 , and ϕ_3 , the pseudoscalar mixing matrix postminimization reads as

$$\mathcal{M}_{\text{CP-odd}}^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2}(-2\lambda_2 - 3\lambda_3 + (-2\lambda_2 + \lambda_3)c_{4\theta}^{-1})s_\theta^2 & \frac{1}{4}(-2\lambda_2 - 3\lambda_3 + (-2\lambda_2 + \lambda_3)c_{4\theta}^{-1})s_{2\theta} \\ 0 & \frac{1}{4}(-2\lambda_2 - 3\lambda_3 + (-2\lambda_2 + \lambda_3)c_{4\theta}^{-1})s_{2\theta} & \frac{1}{2}(-2\lambda_2 - 3\lambda_3 + (-2\lambda_2 + \lambda_3)c_{4\theta}^{-1})c_\theta^2 \end{pmatrix} v_D^2, \quad (\text{B3})$$

where c_θ and s_θ are the sine and cosine of θ , respectively, while $c_{4\theta}^{-1}$ stands for the secant of 4θ .

Next, the (symmetric) mixing matrix of CP -even states in the basis $\{\rho_h, \rho_0, \rho_3\}$ of real parts of h , ϕ_0 , and ϕ_3 displays the following entries after minimization:

$$(\mathcal{M}_{\text{CP-even}}^2)_{1,1} = 2\lambda_h v^2, \quad (\text{B4})$$

$$(\mathcal{M}_{\text{CP-even}}^2)_{2,2} = \frac{v_D^2}{4} (2(4\lambda_1 + 2\lambda_2 + \lambda_3)c_\theta^2 + (2\lambda_2 - \lambda_3)(-1 + 3c_{2\theta})c_{4\theta}^{-1}), \quad (\text{B5})$$

$$(\mathcal{M}_{\text{CP-even}}^2)_{3,3} = \frac{v_D^2}{4} (4\lambda_1 + 2\lambda_2 + \lambda_3 + 3(2\lambda_2 - \lambda_3)c_{4\theta}^{-1})s_{2\theta}, \quad (\text{B6})$$

$$(\mathcal{M}_{\text{CP-even}}^2)_{1,2} = 2\lambda_{H\Phi} v v_D c_\theta, \quad (\text{B7})$$

$$(\mathcal{M}_{\text{CP-even}}^2)_{1,3} = 2\lambda_{H\Phi} v v_D s_\theta, \quad (\text{B8})$$

$$(\mathcal{M}_{\text{CP-even}}^2)_{2,3} = \frac{v_D^2}{4} (-2(4\lambda_1 + 2\lambda_2 + \lambda_3)s_\theta^2 - (2\lambda_2 - \lambda_3)(1 + 3c_{2\theta})c_{4\theta}^{-1}). \quad (\text{B9})$$

Finally, the 2×2 complex scalar mixing matrix written in the basis $\{\phi_1, \phi_2\}$ reads as

$$\mathcal{M}_{\text{complex}}^2 = -\frac{1}{12} v_D^2 (2\lambda_2 + 3\lambda_3 + 3(2\lambda_2 - \lambda_3))c_{4\theta}^{-1} \begin{pmatrix} 2s_\theta^2 & s_{2\theta} \\ s_{2\theta} & 2c_\theta^2 \end{pmatrix} v_D^2. \quad (\text{B10})$$

The massless modes of the CP -odd and complex scalar mixing matrices in Eqs. (B3) and (B10) that become the longitudinal components of Z , X_3 , and X' are, respectively, $G_Z = \sigma_h$, $G_{X_3} = -c_\theta \sigma_0 + s_\theta \sigma_3$, and $G_{X'} = -c_\theta \phi_1 + s_\theta \phi_2$.

APPENDIX C: NEW STATE INSTABILITY CONDITIONS

The conditions that ensure unstable dark vector bosons are listed below.

- (1) **$I\eta R$** ordering. For this regime, $\sqrt{2}M_\eta < M_I < \sqrt{3}M_\eta$. The decay conditions for X' and X_3 are

$$M_1 > \text{Min}\{M_R + M_\eta, M_I + M_\eta\},$$

$$M_3 > \begin{cases} 2M_\eta & \text{if } M_I < \left(1 + \frac{\sqrt{2}}{2}\right)M_\eta \\ M_R + M_I & \text{if } M_I > \left(1 + \frac{\sqrt{2}}{2}\right)M_\eta. \end{cases}$$

For X' the minimal condition is clearly

$$M_1 > M_R + M_\eta. \quad (\text{C1})$$

The two decay modes of X_3 provide weaker conditions than this one. The minimal condition for X' is enough to guarantee the instability of both X' and X_3 . Notice that, no matter what, X_3 is heavier than ϕ_I because either $M_3 > 2M_\eta > M_I^{\text{max}} = \sqrt{3}M_\eta$ or $M_3 > M_R + M_I > M_I$. The condition (C1) is not strong enough to place X' heavier than ϕ_I ; then $M_1 > M_I$ and $M_I > M_1$ are acceptable. Since (C1) implies that X' is separately heavier than ϕ_R and η ,

$$M_3 > M_I > M_1 > M_\eta > M_R \quad \text{or}$$

$$M_3 > M_1 > M_I > M_\eta > M_R,$$

leaving ϕ_I as either the third or fourth heaviest odd Z_2^{acc} particle. More specifically, the reason two orderings exist is because $M_R + M_\eta$ can take values in $[M_\eta, 2M_\eta]$ but M_I lives in $[\sqrt{2}M_\eta, \sqrt{3}M_\eta]$, which is fully contained in the $[M_\eta, 2M_\eta]$ range. This regime is self-consistent, with ϕ_I rendered kinematically stable.

- (2) **$IR\eta$** ordering. This is defined by $\sqrt{3/2}M_\eta < M_I < \sqrt{2}M_\eta$. The decay conditions for X' and X_3 are

$$M_1 > \text{Min}\{M_R + M_\eta, M_I + M_\eta\},$$

$$M_3 > \overset{IR\eta}{2M_\eta}.$$

For X' , the $IR\eta$ regime tells us that $M_I + M_\eta > M_R + M_\eta$; then the minimal condition is

$$M_1 > M_R + M_\eta. \quad (\text{C2})$$

For X_3 ,

$$M_3 = \sqrt{3}M_1 > M_\eta + M_\eta \Rightarrow M_1 > \frac{1}{\sqrt{3}}(M_\eta + M_\eta),$$

which is weaker than what (C2) implies:

$$M_1 > M_R + M_\eta > M_\eta + M_\eta.$$

It is sufficient to adopt the minimal condition for X' . Is X' heavier than the heaviest scalar, ϕ_I ? The answer is yes: in the $IR\eta$ regime $M_I^{\text{max}} = \sqrt{2}M_\eta$ and the inequality right above tells us $M_1 > 2M_\eta > M_I^{\text{max}}$; then

$$M_3 > M_1 > M_I > M_R > M_\eta,$$

meaning ϕ_I is the third heaviest Z_2^{acc} particle. This regime is self-consistent, with ϕ_I rendered kinematically stable by phase space.

- (3) **$RI\eta$** ordering. This is defined by $M_\eta < M_I < \sqrt{3/2}M_\eta$. The decay conditions for X' and X_3 are

$$M_1 > \text{Min}\{M_R + M_\eta, M_I + M_\eta\},$$

$$M_3 > \overset{RI\eta}{2M_\eta}.$$

For X' , in the $RI\eta$ regime $M_R + M_\eta > M_I + M_\eta$; then

$$M_1 > M_I + M_\eta. \quad (\text{C3})$$

As for X_3 ,

$$M_3 = \sqrt{3}M_1 > M_\eta + M_\eta \Rightarrow M_1 > \frac{1}{\sqrt{3}}(M_\eta + M_\eta),$$

but this is weaker than the boxed condition above, which implies

$$M_1 > M_I + M_\eta > M_\eta + M_\eta.$$

Therefore, it is enough to adopt the minimal condition for X' to simultaneously guarantee the X' and X_3 decays. Still, we can fix the entire mass ordering: $M_1 > M_I + M_\eta > 2M_\eta > M_R^{\text{max}}$ in this regime (which has a value of $M_R^{\text{max}} = \sqrt{2}M_\eta$); then the mass ordering is fixed:

$$M_3 > M_1 > M_R > M_I > M_\eta.$$

We conclude that this regime is self-consistent, with ϕ_R rendered kinematically stable by phase space.

- (4) **$R\eta I$** ordering. This is defined by $0 < M_I < M_\eta$. Only X' and X_3 are required to be unstable. The decay conditions for X' and X_3 are

$$M_1 > \text{Min}\{M_R + M_\eta, M_I + M_\eta\}$$

$$M_3 > \begin{cases} M_R + M_I & \text{if } M_I < \left(1 - \frac{\sqrt{2}}{2} M_\eta\right) \\ 2M_\eta & \text{if } M_I > \left(1 - \frac{\sqrt{2}}{2} M_\eta\right). \end{cases}$$

For X' , in the $R\eta I$ regime clearly $M_R + M_\eta > M_I + M_\eta$, and then

$$M_1 > M_I + M_\eta. \quad (\text{C4})$$

For X_3 , its two decays give weaker conditions than (C4). Notice that no matter what, X_3 is heavier than ϕ_R ,

$$M_3 > 2M_\eta > M_R^{\max} \quad \text{or} \quad M_3 > M_R + M_I > M_R,$$

where $M_R^{\max} > \sqrt{3}M_\eta \approx 1.73M_\eta$. Yet, X' is not necessarily heavier than ϕ_R . Now, the X' decay condition simultaneously implies $M_1 > M_I$ and $M_1 > M_\eta$, so together with the regime ordering, two orderings arise:

$$M_3 > M_R > M_1 > M_\eta > M_I \quad \text{or}$$

$$M_3 > M_1 > M_R > M_\eta > M_I,$$

which tells us that ϕ_R , being stable by phase space, is not necessarily the third heaviest particle with odd Z_2^{acc} parity, as it can also be the fourth heaviest.⁷ We conclude that this regime is self-consistent, with ϕ_R rendered kinematically stable by phase space.

⁷ X' is not DM, even when $M_R > M_1$, because it is unstable.

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- [1] Vanda Silveira and A. Zee, Scalar phantoms, *Phys. Lett.* **161B**, 136 (1985).
 - [2] C. P. Burgess, Maxim Pospelov, and Tonnies ter Veldhuis, The minimal model of nonbaryonic dark matter: A singlet scalar, *Nucl. Phys.* **B619**, 709 (2001).
 - [3] John McDonald, Gauge singlet scalars as cold dark matter, *Phys. Rev. D* **50**, 3637 (1994).
 - [4] James M. Cline, P. Scott, K. Kainulainen, and C. Weniger, Update on scalar singlet dark matter, *Phys. Rev. D* **88**, 055025 (2013); **92**, 039906(E) (2015).
 - [5] Ekata Singh, Hrishabh Bharadwaj, and Devisharan, Exploring new physics via effective interactions, *Int. J. Mod. Phys. A* **39**, 2450038 (2024).
 - [6] Daniele S. M. Alves, S. R. Behbahani, P. Schuster, and J. G. Wacker, Composite inelastic dark matter, *Phys. Lett. B* **692**, 323 (2010).
 - [7] Pouya Asadi, Matthew Baumgart, Patrick J. Fitzpatrick, Emmett Krupczak, and Tracy R. Slatyer, Capture and decay of electroweak WIMponium, *J. Cosmol. Astropart. Phys.* **02** (2017) 005.
 - [8] Benedict von Harling and Kalliopi Petraki, Bound-state formation for thermal relic dark matter and unitarity, *J. Cosmol. Astropart. Phys.* **12** (2014) 033.
 - [9] Alfredo Aranda, Luis Barajas, and Jose A. R. Cembranos, Magnetic dipole moments for composite dark matter, *J. Cosmol. Astropart. Phys.* **03** (2016) 034.
 - [10] Laura Lopez Honorez, Emmanuel Nezri, Josep F Oliver, and Michel H G Tytgat, The inert doublet model: An archetype for dark matter, *J. Cosmol. Astropart. Phys.* **02** (2007) 028.
 - [11] Ivania M. Ávila, Valentina De Romeri, Laura Duarte, and José W. F. Valle, Phenomenology of scotogenic scalar dark matter, *Eur. Phys. J. C* **80**, 908 (2020).
 - [12] Yu-Pan Zeng, Chengfeng Cai, Dan-Yang Liu, Zhao-Huan Yu, and Hong-Hao Zhang, Probing quadruplet scalar dark matter at current and future pp colliders, *Phys. Rev. D* **101**, 115033 (2020).
 - [13] Bryan Ostdiek, Constraining the minimal dark matter fiveplet with LHC searches, *Phys. Rev. D* **92**, 055008 (2015).
 - [14] Genevieve Belanger, Kristjan Kannike, Alexander Pukhov, and Martti Raidal, Z_3 scalar singlet dark matter, *J. Cosmol. Astropart. Phys.* **01** (2013) 022.
 - [15] Cesar Bonilla, Ernest Ma, Eduardo Peinado, and Jose W.F. Valle, Two-loop Dirac neutrino mass and WIMP dark matter, *Phys. Lett. B* **762**, 214 (2016).
 - [16] Arnab Dasgupta, Sin Kyu Kang, and Oleg Popov, Radiative Dirac neutrino mass, neutrinoless quadruple beta decay, and dark matter in B-L extension of the standard model, *Phys. Rev. D* **100**, 075030 (2019).
 - [17] Geneviève Bélanger, Alexander Pukhov, Carlos E. Yaguna, and Óscar Zapata, The Z_5 model of two-component dark matter, *J. High Energy Phys.* **09** (2020) 030.
 - [18] Debasish Borah, Ernest Ma, and Dibyendu Nanda, Dark $SU(2) \rightarrow Z_3 \times Z_2$ gauge symmetry, *Phys. Lett. B* **842**, 137981 (2023).
 - [19] C. Boehm, Pierre Fayet, and J. Silk, Light and heavy dark matter particles, *Phys. Rev. D* **69**, 101302 (2004).
 - [20] Ernest Ma, Supersymmetric model of radiative seesaw majorana neutrino masses, *Ann. Fond. Louis de Broglie* **31**, 285 (2006).
 - [21] Taeil Hur, Hye-Sung Lee, and Salah Nasri, A Supersymmetric $U(1)$ -prime model with multiple dark matters, *Phys. Rev. D* **77**, 015008 (2008).
 - [22] Kathryn M. Zurek, Multi-component dark matter, *Phys. Rev. D* **79**, 115002 (2009).
 - [23] Brian Batell, Maxim Pospelov, and Adam Ritz, Direct detection of multi-component secluded WIMPs, *Phys. Rev. D* **79**, 115019 (2009).

- [24] Francesco D’Eramo and Jesse Thaler, Semi-annihilation of dark matter, *J. High Energy Phys.* **06** (2010) 109.
- [25] Daniel Feldman, Zuowei Liu, Pran Nath, and Gregory Peim, Multicomponent dark matter in supersymmetric hidden sector extensions, *Phys. Rev. D* **81**, 095017 (2010).
- [26] Keith R. Dienes and Brooks Thomas, Dynamical dark matter: I. Theoretical overview, *Phys. Rev. D* **85**, 083523 (2012).
- [27] Christian Gross, Oleg Lebedev, and Yann Mambrini, Non-Abelian gauge fields as dark matter, *J. High Energy Phys.* **08** (2015) 158.
- [28] Fatemeh Elahi and Sara Khatibi, Multi-component dark matter in a non-Abelian dark sector, *Phys. Rev. D* **100**, 015019 (2019).
- [29] Geneviève Bélanger, Alexander Pukhov, Carlos E. Yaguna, and Óscar Zapata, The Z_7 model of three-component scalar dark matter, *J. High Energy Phys.* **03** (2023) 100.
- [30] Subhaditya Bhattacharya, Lipika Kolay, and Dipankar Pradhan, Multiparticle scalar dark matter with $\mathbb{Z}_N$ symmetry, [arXiv:2410.16275](https://arxiv.org/abs/2410.16275).
- [31] A. Adulpravitchai, A. Blum, and M. Lindner, Non-Abelian discrete groups from the breaking of continuous flavor symmetries, *J. High Energy Phys.* **09** (2009) 018.
- [32] Florian Staub, SARAH 4: A tool for (not only SUSY) model builders, *Comput. Phys. Commun.* **185**, 1773 (2014).
- [33] W. Porod and F. Staub, SPheno 3.1: Extensions including flavour, CP -phases and models beyond the MSSM, *Comput. Phys. Commun.* **183**, 2458 (2012).
- [34] G. Alguero, G. Bélanger, F. Boudjema, S. Chakraborti, A. Goudelis, S. Kraml, A. Mjallal, and A. Pukhov, micrOMEGAs 6.0: N-component dark matter, *Comput. Phys. Commun.* **299**, 109133 (2024).
- [35] Amalia Betancur, Andrés Castillo, Guillermo Palacio, and Juan Suarez, Multicomponent scalar dark matter at high-intensity proton beam experiments, *J. Phys. G* **49**, 075003 (2022).
- [36] Subhaditya Bhattacharya and Dipankar Pradhan, Direct search signal of two-component dark matter, [arXiv:2501.17862](https://arxiv.org/abs/2501.17862).
- [37] Florian Staub, Thorsten Ohl, Werner Porod, and Christian Speckner, A tool box for implementing supersymmetric models, *Comput. Phys. Commun.* **183**, 2165 (2012).
- [38] N. Aghanim *et al.*, Planck 2018 results. VI. Cosmological parameters, *Astron. Astrophys.* **641**, A6 (2020); **652**, C4(E) (2021).
- [39] E. Aprile *et al.*, First dark matter search with nuclear recoils from the XENONnT experiment, *Phys. Rev. Lett.* **131**, 041003 (2023).
- [40] J. Aalbers *et al.*, DARWIN: Towards the ultimate dark matter detector, *J. Cosmol. Astropart. Phys.* **11** (2016) 017.
- [41] Juan Herrero-Garcia, Andre Scaffidi, Martin White, and Anthony G. Williams, On the direct detection of multi-component dark matter: sensitivity studies and parameter estimation, *J. Cosmol. Astropart. Phys.* **11** (2017) 021.