

Global quenches and correlator dynamics in de Sitter space

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We study nonequilibrium initial states of quantum fields in curved spacetime and develop a framework for describing global quenches as unitary perturbations of the initial density matrix. Using the Keldysh-Schwinger functional integral, we derive expressions for postquench correlators in arbitrary geometries and apply the method to both Minkowski and de Sitter backgrounds. In flat space, the approach reproduces the known relaxation behavior for massive fields and reveals qualitatively distinct dynamics in the massless case. In de Sitter space, we find that the late-time decay of quench-induced corrections depends sensitively on the mass and may influence the behavior of cosmological observables such as the power spectrum. The framework also extends straightforwardly to non-Gaussian initial states, providing a basis for future studies of loop effects and primordial non-Gaussianities.

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I. INTRODUCTION

Nonequilibrium processes in quantum systems attract significant attention both in condensed matter physics and in the context of quantum field theory and holography. One of the most universal and theoretically controlled ways to drive a system out of equilibrium is a quantum quench, usually defined as a sudden change of some Hamiltonian parameter, such as the mass or coupling constant. Studying the subsequent relaxation allows one to explore fundamental questions of thermalization, integrability, and the evolution of quantum correlations. In this context one usually distinguishes between the so-called local quench, where the system is perturbed in a spatially localized region, and the global quench, where the sudden change is uniform throughout space. Local quenches are actively studied in condensed matter physics [1–3], as well as in the investigation of chaotic and integrable features in different systems, including holographic models [4–11]. Global quenches are typically considered in the context of relaxation toward some equilibrium state, commonly referred to as “subsystem thermalization” [12–18]. They are also a standard tool for analyzing the dynamics of phase

transitions [16,19,20] and the evolution of quantum entanglement following a sudden perturbation [17,21].

In the simplest setting, a global quench is realized by a mass-dependent $m(t)$ quench protocol in the sudden limit [13,14], where the mass instantaneously changes from one value to another, $m \rightarrow M$. For flat spacetime, in free theories as well as certain interacting and conformal theories, the dynamics of such quenches is well understood when the initial state is either the vacuum of the pre-quench theory or specific squeezed states. In this situation, the postquench state can be written as a generalized Cardy-Calabrese (gCC) state, and the system relaxes toward a generalized Gibbs ensemble, with the relaxation rate depending on the number of spatial dimensions [13,14]. Namely, the gCC state can be written as a perturbation over the vacuum $|0_{\text{out}}\rangle$ after the quench as follows [14]:

$$| \text{gCC} \rangle \equiv e^{\frac{1}{2} \sum_k \kappa(k) \hat{a}^\dagger(k) \hat{a}(k)} | D \rangle, \\ | D \rangle \equiv e^{-\frac{1}{2} \sum_k \hat{a}^\dagger(k) \hat{a}^\dagger(-k)} | 0_{\text{out}} \rangle, \quad (1.1)$$

where $\kappa(k)$ is some given function, $|D\rangle$ is the Dirichlet state, and the annihilation operators $\hat{a}(k)$ also refer to the theory after the quench. Note that gCC state (1.1) is Gaussian and nonunitary construction, which nevertheless allow for analytic treatment of correlators in globally out-of-equilibrium systems.

In recent decades, it has been recognized that mechanisms analogous to global quenches may also play a role in the evolution of the early Universe. In this context, a global quantum quench can effectively describe processes experienced by fields after fast transient phenomena, such as

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phase transitions induced by a sharp change in temperature, or sudden features in the inflaton potential [22–28]. Most existing works focus on situations involving a change of phase [22–24], but it is also of interest to analyze non-equilibrium dynamics triggered by perturbations that do not induce a phase transition [25,28]. More generally, a quench may serve as a model for restructuring the initial quantum state during or shortly after inflation. For example, the mass-dependent quench protocol in the sudden limit for the inflaton potential was used in [26] to explain observable features in the scalar primordial power spectrum. In [25], a quench was analyzed in the $O(N)$ model in de Sitter spacetime, where it was shown that after a sudden mass shift, the effective mass dynamically returns to some asymptotic value, although at intermediate stages its square may become negative, corresponding to a temporary local spontaneous symmetry breaking. An interesting approach was suggested in [28], where a generalization of the Caldeira-Leggett model to curved spacetime produced a mass-dependent quench protocol as an effective description after integrating out environmental degrees of freedom. In this case, the sudden mass quench arises naturally, and both the vacuum state and Gaussian squeezed states were considered as initial states. It was shown that after relaxation the system approaches a gCC-like state in de Sitter space, and the analysis of two-point correlators allowed one to describe the corresponding modifications of the perturbation spectrum. A closely related setup was studied in [29], where flat spacetime is suddenly transformed into de Sitter spacetime. It was demonstrated that the evolution of local operators, such as the stress-energy

tensor, approaches the Bunch-Davies state at late times even if the initial state breaks de Sitter symmetry, allowing one to regard it as a universal attractor. This raises the question of how much memory of the initial state is retained in quantum dynamics in an expanding geometry, and what the nature of relaxation is in curved spacetime.

In this work, we propose another class of initial states suitable for describing postquench dynamics in quantum field theory in curved spacetime. We use the Keldysh functional integral formalism [30–33] to derive explicit expressions for two-point correlation functions after a global (in space) perturbation implemented as a unitary transformation of the equilibrium density matrix (see Appendix A). The idea of this method was introduced in [34], and in the case of a local quench in flat spacetime it reproduces known results [34,35]. We find that for Gaussian perturbations the resulting behavior qualitatively matches that of the sudden mass quench protocol for massive fields (see Appendix B). At the same time, for massless fields in flat spacetime we observe the absence of relaxation even in $D = 4$, which is the feature of the behavior known for $D = 2, 3$ in the $m(t)$ quench protocol.

More precisely, we consider a theory of a scalar field ϕ and perturb the density matrix of the pure vacuum state $\hat{\rho}_0$ as $\hat{\rho} = \hat{Q}\hat{\rho}_0\hat{Q}^\dagger$, where $\hat{Q} = \exp\{i\alpha \int d^{D-1}\mathbf{x} \sqrt{|g|}\hat{\phi}^2(\mathbf{x})\}$ with α an arbitrary mass-dimensional parameter $[\alpha] = [m]$. Then the evolution of the Keldysh two-point function G_K^Q after the quench can be expressed in terms of integrals over the equilibrium Keldysh function G_K and retarded Green's function G_R (Appendix A, [34]),

$$iG_K^Q(t, \mathbf{x}|t, \mathbf{y}) = iG_K(t, \mathbf{x}|t, \mathbf{y}) - 2\alpha \int d^{D-1}\xi \sqrt{|g(t_0, \xi)|} [iG_K(t, \mathbf{x}|t_0, \xi)G_R(t, \mathbf{y}|t_0, \xi) + iG_K(t, \mathbf{y}|t_0, \xi)G_R(t, \mathbf{x}|t_0, \xi)] \\ + 4\alpha^2 \int d^{D-1}\xi_1 d^{D-1}\xi_2 \sqrt{|g(t_0, \xi_1)|} \sqrt{|g(t_0, \xi_2)|} iG_K(t_0, \xi_1|t_0, \xi_2)G_R(t, \mathbf{x}|t_0, \xi_1)G_R(t, \mathbf{x}|t_0, \xi_2), \quad (1.2)$$

or in momentum space (assuming $g_{\mu\nu}$ depends only on time and spatial slices are homogeneous),

$$iG_K^Q(t, \mathbf{x}|t, \mathbf{y}) = \int \frac{d^{D-1}\mathbf{k}}{(2\pi)^{D-1}} e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})} \left\{ iG_K(\mathbf{k}|t, t) - 4\alpha \sqrt{|g(t_0)|} iG_K(\mathbf{k}|t, t_0)G_R(\mathbf{k}|t, t_0) \right. \\ \left. + 4\alpha^2 \sqrt{|g(t_0)|} \sqrt{|g(t_0)|} iG_K(\mathbf{k}|t_0, t_0)G_R(\mathbf{k}|t, t_0)G_R(\mathbf{k}|t, t_0) \right\} \\ = iG_K(t, \mathbf{x}|t, \mathbf{y}) - 4\alpha I_1(t, t_0, r) + 4\alpha^2 I_2(t, t_0, r), \quad (1.3)$$

where I_1 and I_2 denote the integrals giving the corresponding corrections to the two-point correlator, and we assume the equilibrium state to be homogeneous and isotropic, so the correlators depend only on $r = |\mathbf{x} - \mathbf{y}|$. In $D = 4$ Minkowski spacetime, for massive fields we find relaxation of the two-point correlator at late times $mt \gg 1$,

$$iG_K^Q(t, \mathbf{x}|t, \mathbf{y}) \simeq iG_K^{\text{postquench}}(r) + \frac{\alpha \sqrt{\alpha^2 + m^2}}{8\pi^{\frac{3}{2}}} \cdot \frac{1}{(mt)^{\frac{3}{2}}} \cos(2mt + \delta), \\ \delta = \frac{\pi}{4} - \arctan\left(\frac{\alpha}{m}\right), \quad (1.4)$$

where $iG_K^{\text{postquench}}(r)$ is the modified two-point correlator after the quench, and the relaxation rate qualitatively matches the result for the $m(t)$ quench protocol [13,14]. However, for massless $m = 0$ fields in Minkowski spacetime and $t \gg r$, the correlator does not relax even in four dimensions,

$$iG_K^0 \sim \alpha^2 \log \left[\frac{2t}{r} \right]. \quad (1.5)$$

For mass quenches this behavior appears only in lower dimensions, so a more detailed analysis of critical post-quench dynamics within our framework is left for future work.

In de Sitter spacetime the situation is different; even equilibrium Green's functions of the free theory decay with time as a power of the conformal time $\eta = e^{-t}$ (taking the Hubble parameter $H = 1$). Therefore, regarding relaxation, one first needs to compare the decay of local operators in the nonequilibrium setup with the decay of the correlation function for the Bunch-Davies state. Second, given the secular growth of loop corrections even for the Bunch-Davies state [33,36–39], it would be interesting to analyze how controlled nonequilibrium perturbations affect long-time dynamics in the presence of growing loop contributions. Understanding the two-point correlator is central to all such questions, and the main goal of this work is to demonstrate the application of our method to these correlation functions in curved spacetime. In Sec. II, we analyze the behavior of the Keldysh propagator at equal times after the quench in the expanding Poincaré patch of de Sitter space. We find that for short spatial separations $r \lesssim \eta$, the qualitative behavior changes depending on whether the mass is above or below the critical value $m_{cr} = \sqrt{2}$ corresponding to the “conformally coupled” scalar. We expect that this may have nontrivial implications for local operators such as the stress-energy tensor, where one takes the coincidence limit $r \rightarrow 0$. However, the situation remains unclear and we leave the full analysis, including a proper regularization procedure for nonequilibrium correlators, for future work. In Sec. III, we discuss the application of our approach to non-Gaussian perturbations of equilibrium states. In Appendixes A and B, we describe our construction of the perturbed initial state and the corresponding evolution of correlators in curved spacetime, and we also present explicit computations in flat spacetime.

The proposed approach also enables systematic exploration of relaxation for non-Gaussian and anisotropic initial states in arbitrary geometries where Green's functions are known. In particular, the derivation presented in Appendix A applies with minimal modification if the parameter α is replaced by an arbitrary anisotropic spatial function $\alpha(\mathbf{x})$. Additionally, for future studies of loop corrections, we plan to generalize the approach to compute modified Green's functions at different time arguments

$t_1 \neq t_2$. We hope that this class of states will broaden the study of undergoing physics in the early Universe, and potentially find applications in the study of chaos, holographic models, and more generally nonequilibrium dynamics across different physical systems.

II. QUANTUM QUENCH IN THE EXPANDING POINCARÉ PATCH

In this section we study the behavior of the two-point correlator of a free minimally coupled scalar field after a Gaussian quench, mentioned above and described in details in Appendix A, in the Poincaré patch (EPP) of de Sitter space,

$$S[\phi] = \int d^D x \sqrt{|g|} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right], \quad (2.1)$$

$$ds^2 = \frac{1}{\eta^2} (d\eta^2 - d\mathbf{x}^2), \quad (2.2)$$

where $\eta = e^{-t}$ is conformal time, and we set the Hubble parameter to unity, $H = 1$. The canonical quantization of the field is as follows:

$$\begin{aligned} \hat{\phi}(\eta, \mathbf{x}) &= \int \frac{d^{D-1} \mathbf{k}}{(2\pi)^{D-1}} [\hat{a}_{\mathbf{k}} f_{\mathbf{k}}(\eta) e^{i\mathbf{k}\mathbf{x}} + \hat{a}_{\mathbf{k}}^\dagger f_{\mathbf{k}}^*(\eta) e^{-i\mathbf{k}\mathbf{x}}], \\ [\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{p}}^\dagger] &= (2\pi)^{D-1} \delta(\mathbf{k} - \mathbf{p}), \end{aligned} \quad (2.3)$$

where the harmonics $f_{\mathbf{k}}(\eta) e^{i\mathbf{k}\mathbf{x}}$ solve the free Klein-Gordon equation for the action (2.1). The solutions are given by Hankel functions, and we fix the modes so that the Bunch-Davies state $|\text{BD}\rangle$ corresponds to the Fock vacuum for the annihilation operators in (2.3),

$$\begin{aligned} f_{\mathbf{k}}(\eta) &= \eta^{\frac{D-1}{2}} h_\nu(k\eta) = \eta^{\frac{D-1}{2}} \frac{\sqrt{\pi}}{2} H_\nu^{(1)}(k\eta), \\ \nu &= \sqrt{\frac{(D-1)^2}{4} - m^2}, \quad \hat{a}_{\mathbf{k}} |\text{BD}\rangle = 0. \end{aligned} \quad (2.4)$$

In what follows we assume the most interesting case of light fields (complementary series) $m < \frac{D-1}{2}$, and we work only in $D = 4$ dimensions. It is convenient to keep in mind the following asymptotic expansions of the Hankel functions:

$$\begin{aligned}
h_\nu(k\eta) &\simeq \frac{1}{\sqrt{2k\eta}} e^{ik\eta} e^{-i\frac{\pi}{2}(\nu+\frac{1}{2})}, \quad \text{as } k\eta \gg |\nu|; \\
h_\nu(k\eta) &\simeq iA_- \frac{1}{(k\eta)^\nu} + iA_+(k\eta)^\nu + iB(k\eta)^{-\nu+2} + \dots, \quad \text{as } k\eta \ll |\nu|; \\
\text{where } A_- &= -\frac{\Gamma(\nu)}{2\sqrt{\pi}} 2^\nu, \\
A_+ &= \frac{\sqrt{\pi} e^{-i\pi\nu}}{2^{\nu+1} \Gamma(\nu+1) \sin(\pi\nu)}, \quad B = \frac{2^\nu}{8\sqrt{\pi}} \frac{\Gamma(\nu)}{2-\nu}.
\end{aligned} \tag{2.5}$$

Then, for the evolution of the two-point correlator after perturbing the state $|\text{BD}\rangle$ by the Gaussian operator (A10), (A15), we obtain in the EPP,

$$iG_K^Q(t, \mathbf{x}|t, \mathbf{y}) = iG_K(t, \mathbf{x}|t, \mathbf{y}) - 4\alpha I_1^{\text{EPP}}(t, t_0, r) + 4\alpha^2 I_2^{\text{EPP}}(t, t_0, r), \tag{2.6}$$

where $r = |\mathbf{x} - \mathbf{y}|$ and

$$\begin{aligned}
I_1^{\text{EPP}} &= - \int \frac{d^{D-1}\mathbf{k}}{(2\pi)^{D-1}} e^{i\mathbf{k}\mathbf{r}} \eta^{D-1} \text{Im}\{h_\nu^2(k\eta) h_\nu^{*2}(k\eta_0)\} \\
&= - \frac{1}{(2\pi)^{\frac{D-1}{2}} r^{\frac{D-3}{2}}} \int_0^\infty dk k^{\frac{D-1}{2}} J_{\frac{D-3}{2}}(kr) \eta^{D-1} \text{Im}\{h_\nu^2(k\eta) h_\nu^{*2}(k\eta_0)\} \\
&= - \frac{1}{2\pi^2} \frac{\eta}{r} \int_0^\infty d\xi \xi \cdot \sin\left(\xi \frac{r}{\eta}\right) \text{Im}\left\{h_\nu^2(\xi) \cdot h_\nu^{*2}\left(\xi \frac{\eta_0}{\eta}\right)\right\},
\end{aligned} \tag{2.7}$$

$$I_2^{\text{EPP}} = \frac{2}{\pi^2} \frac{\eta}{r} \int_0^\infty d\xi \xi \cdot \sin\left(\xi \frac{r}{\eta}\right) \left|h_\nu\left(\xi \frac{\eta_0}{\eta}\right)\right|^2 \text{Im}^2\left\{h_\nu(\xi) \cdot h_\nu^*\left(\xi \frac{\eta_0}{\eta}\right)\right\}. \tag{2.8}$$

We are particularly interested in the behavior of the Keldysh propagator, since it is sensitive to the evolution of the state of the theory. In the last line of Eq. (2.7) we set $D = 4$. We will estimate the behavior of the two-point correlator at late times $\eta \ll 1$ in the regime where the quench happened sufficiently early, $\eta_0 \gg 1$, and we will also assume that the spatial separation is small, $r = |\mathbf{x} - \mathbf{y}| \ll \eta_0$. Let us note immediately that these expressions can be used in other regimes as well, in particular for studying loop corrections, where the integrals over internal vertices run over the whole spacetime. However, the main goal here is to demonstrate a controlled way to drive the system out of equilibrium, so we focus on the simplest case for estimates and leave a more advanced analysis for future work. The integrals (2.7)–(2.8) can then be estimated using the asymptotics (2.5) by splitting the ξ -integration into the ranges $[0, \nu \frac{\eta_0}{\eta}]$, $[\nu \frac{\eta_0}{\eta}, \nu]$, and (ν, ∞) . Replacing the harmonics by the corresponding asymptotics in these regions yields the estimates,

$$I_1^{\text{EPP}} \simeq a(r) \cdot \left(\frac{\eta}{\eta_0}\right)^{3-2\nu} + \text{subleading terms}, \tag{2.9}$$

$$\begin{aligned}
I_2^{\text{EPP}} &\simeq \left(\frac{\eta}{\eta_0}\right)^{3-2\nu} \cdot \left[b + \frac{A_-^2}{4\pi^2} \frac{\eta_0}{r} \int_\nu^{\nu \frac{\eta_0}{\eta}} \frac{d\ell}{\ell^{1+2\nu}} \sin\left(\ell \frac{r}{\eta_0}\right) \cdot (1 + \sin(2\ell - \pi\nu)) \right] \\
&\quad - \frac{1}{8\pi^2} \frac{\eta^2}{\eta_0^2} \text{Ci}\left(\nu \frac{r}{\eta}\right) + \text{subleading terms},
\end{aligned} \tag{2.10}$$

where Ci is the cosine integral and

$$a(r) \simeq \frac{A_-^3 \text{Im}\{A_+\}}{\pi^2} \frac{\nu^{3-2\nu}}{3-2\nu} + \frac{A_-^2}{4\pi^2} \frac{\eta_0}{r} \int_\nu^\infty \frac{d\ell}{\ell^{2\nu}} \sin\left(\ell \frac{r}{\eta_0}\right) \cos(2\ell - \pi\nu), \quad b = \frac{2A_-^4 \text{Im}^2\{A_+\}}{\pi^2} \frac{\nu^{3-2\nu}}{3-2\nu}. \tag{2.11}$$

Here the expression for the coefficient a is approximate because we extended the upper limit of the integral over ℓ to infinity instead of $\nu \frac{\eta_0}{\eta} \gg 1$, and the integral is always convergent due to oscillations. It is also important that the term with the cosine integral Ci in (2.10) is suppressed in the limit $\eta \ll r$ as $(\frac{\eta}{\eta_0})^3$. Nevertheless, we keep it because for $r \rightarrow 0$ it contains a

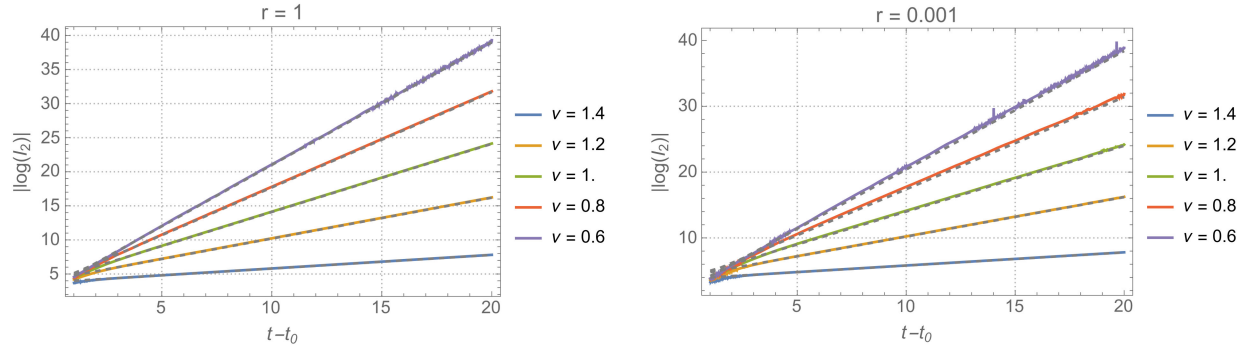


FIG. 1. The numerical plots of $|\log(I_2)|$ as a function of time $t - t_0$ for masses $m < \sqrt{2}$, $r = 1$ (left) and $r = 0.001$ (right). The dashed lines show the analytical estimate. We chose $-t_0 = \log(102) \simeq 4.6$.

subleading divergence, $\text{Ci}(\nu \frac{r}{\eta}) \sim \log(\frac{r}{\eta})$. While the tree-level two-point correlator diverges quadratically, $iG_K \sim \eta^2/r^2$, any further study of the evolution of expectation values of local operators, such as, e.g., the stress-energy tensor $\langle T_{\mu\nu} \rangle^Q \sim \hat{D}_{\mu\nu} iG_K^Q(t, \mathbf{x}|t, \mathbf{y})|_{\mathbf{x} \rightarrow \mathbf{y}}$ (with some differential operator $\hat{D}_{\mu\nu}$), will require careful ultraviolet regularization and subtraction of divergences.

An interesting feature of the modified (quenched) Keldysh function (2.6) is that for $m < m_{cr} = \sqrt{2}$ ($\nu > \nu_{cr} = \frac{1}{2}$) the decay proceeds with the power $3 - 2\nu$, whereas for $m > m_{cr}$ the leading decay power equals $2 < 3 - 2\nu$ for times $t \lesssim -\log(r)$ (or $\eta \gtrsim r$). This follows from (2.10), since in that case one can approximately replace $\sin(\ell \frac{r}{\eta_0})$ by its argument over the entire integration interval. For such $m > m_{cr}$ from the complementary series this may

be important for the evolution of local operators in the limit $r \rightarrow 0$ after the subtraction of UV divergences, because the decay of the correction is slower than that of the Bunch-Davies correlator, $iG_K \sim (\frac{\eta}{r})^{3-2\nu}$, which reaches this asymptotic behavior rather quickly even for $r < \eta$. This circumstance may spoil its “attractor” property in such a situation. Although fields with $m > \sqrt{2}$ are of limited practical relevance for inflationary physics, they remain interesting from a theoretical standpoint, and the full spectrum of primordial fields has yet to be determined. The above observation for the asymptotic behavior of the integral I_2^{EPP} can be checked numerically using the exact formula (2.8) for $m < m_{cr}$ and $m > m_{cr}$ (see Figs. 1 and 2).

In the case $m = \sqrt{2}$ we have $2 = 3 - 2\nu$, and the integrals for the nonequilibrium Keldysh propagator can be evaluated exactly,

$$iG_K^Q(t, \mathbf{x}|t, \mathbf{y}) = \frac{1}{4\pi^2} \frac{\eta^2}{r^2} + \frac{1}{2\pi^2} \cdot \left(\frac{\eta}{\eta_0} \right)^2 \cdot \left\{ -\alpha \frac{\eta_0}{2r} \log \left[\frac{2(\eta_0 - \eta) + r}{2(\eta_0 - \eta) - r} \right] + \alpha^2 \log \left[\frac{\sqrt{4(\eta_0 - \eta)^2 - r^2}}{r} \right] + \alpha^2 \frac{\eta_0 - \eta}{r} \log \left[\frac{2(\eta_0 - \eta) + r}{2(\eta_0 - \eta) - r} \right] \right\}. \quad (2.12)$$

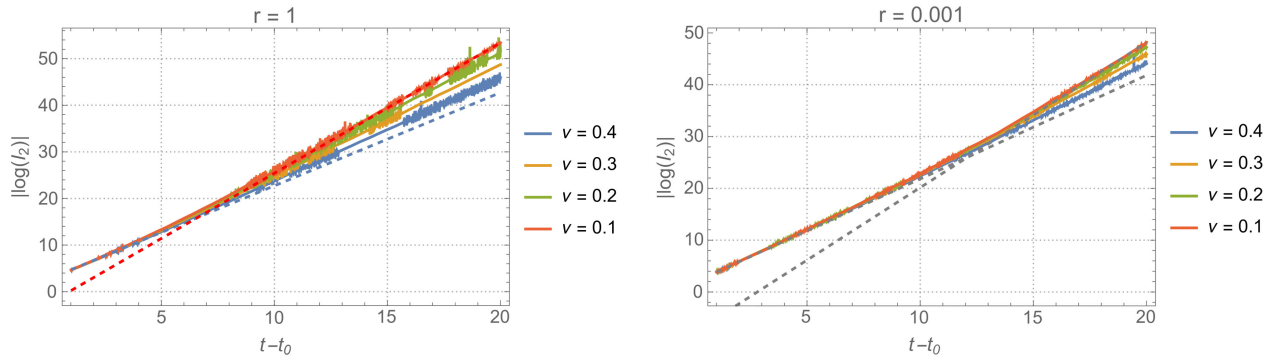


FIG. 2. The numerical plot of $|\log(I_2)|$ as a function of time $t - t_0$ for masses $m > \sqrt{2}$. Here we see that up to $t \lesssim -\log(r)$ the slope equals $2 < 3 - 2\nu$ (blue dashed line), independently of the mass, and then for $t > -\log(r)$ it matches the slope $3 - 2\nu$ (red dashed line for $\nu = 0.1$).

Let us make here a remark concerning the dynamical reasons why in de Sitter space there occurs a change in the characteristic behavior of the relaxation of the two-point correlator, while the correlator (2.12) is similar in its form to the expression in flat space for a massless field (B9). The point is that a global quench can be viewed as a perturbation in the form of a set of wave packets or “quasiparticles” [12,34], distributed over some equilibrium state at the initial time t_0 . These wave packets then begin to spread, and the observed relaxation of correlators can be explained as a consequence of the destructive interference of these wave packets. In the case of flat space, such a picture qualitatively explains the absence of relaxation in the massless case; for a massive field the group velocity $v_{\text{gr}}(k) = \frac{k}{\sqrt{k^2 + m^2}}$ differs for different modes k of the wave packets, which leads to their spreading and destructive interference, whereas for massless fields the group velocity $v_{\text{gr}} = 1$ is the same for all modes, and therefore destructive interference between different modes does not occur and the correlators are not required to relax, which is what we observe in (B9). In the case of massive fields in flat space, relaxation proceeds to some stationary distribution over the original vacuum, which is described by the first term (nonoscillating in time) in the brackets of the integral $I_{2,4D}^{\text{flat}}$ (B6). In contrast, in de Sitter space even the vacuum two-point functions decay with time due to the volume expansion of space. Thus, in de Sitter space there is a combination of two effects; damping due to the volume expansion of space and, additionally, due to the superposition of wave packets. For fields from the complementary series ($m < \frac{D-1}{2}$) the “Compton wavelength” is larger than the curvature scale of the space, and the modes with small momenta “freeze”, and the degree of decay of the correlators decreases (the exponent $3 - 2\nu$ appears). At short distances the mass

does not play a role, and the degree of decay of the correlators is determined only by the volume expansion factor and is equal to 2, as in the first term of formula (2.12). In the case when $m = m_{cr}$, these exponents become equal, and the correlators behave as for a conformally coupled scalar, that is, at the level of correlators the situation with $m = \sqrt{2}$ is equivalent to the conformal coupling of the scalar field with the ξ -term: $m = 0, \xi = \frac{1}{6}$. For this reason (2.12) is similar in form to the result for a massless field in flat space (B9), but nevertheless differs and decays in time due to the volume expansion factor and the different causal structure of the space. It further turns out that in de Sitter space with $D = 4$ (and higher) there is a “gap” in mass between the “conformal scalar” and the principal series, namely the interval $\sqrt{2} < m < \frac{3}{2}$. In this case $2 < 3 - 2\nu$, the field becomes sufficiently heavy, and the contribution to the correlator with decay exponent 2, responsible for the damping of some energy distribution over the vacuum due to the volume expansion, becomes dominant over a certain time interval (as in Fig. 2). Finally, for large field masses $m > \frac{3}{2}$ in the principal series, the “Compton wavelength” becomes smaller than the curvature scale of the space, and one can expect an oscillatory behavior of the correlator analogous to that shown in Fig. 4, but with additional decay due to the volume expansion of space.

Another application of nonequilibrium dynamics on the de Sitter background that is relevant for the early Universe is the study of the scalar power spectrum [25–28]. If, for example, our scalar field describes fluctuations of the inflaton, then for the power spectrum $\mathcal{P}_s(k)$ and the spectral index $n_s(k)$ we have

$$\mathcal{P}_s(k) = \frac{k^3}{2\pi^2} iG_K^Q(k|t, t), \quad n_s(k) - 1 = \frac{d \log \mathcal{P}_s}{d \log k}, \quad (2.13)$$

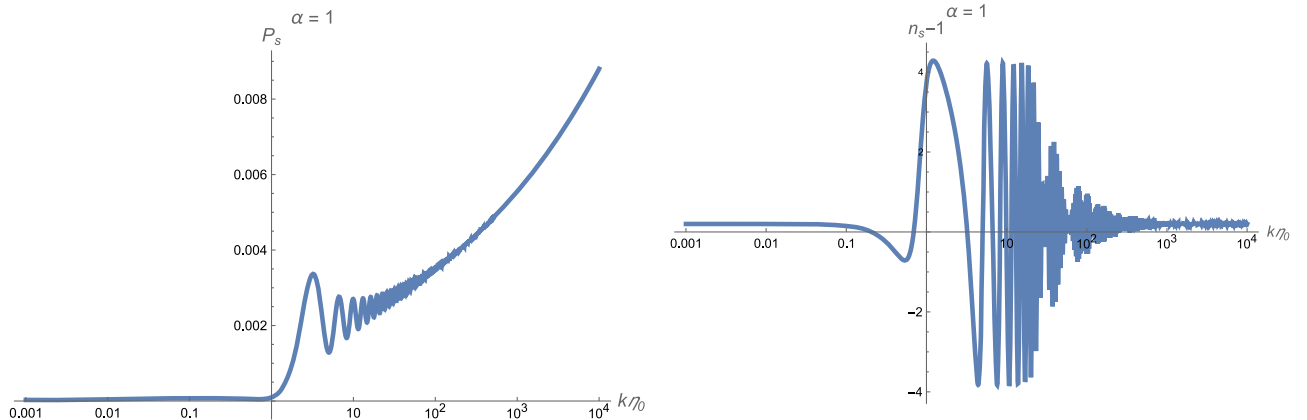


FIG. 3. Power spectrum $\mathcal{P}_s$ and spectral index $n_s - 1$ as the functions of the scale $k\eta_0$ for $\alpha = 1$. We chose $\eta_0 = 1$ and $\eta = 10^{-6}$ so that all depicted scales are superhorizon.

where $iG_K^Q(\mathbf{k}|t, t)$ is simply the Fourier transformation of the two-point correlator (2.6). In our Gaussian-quench setup, the plots for $\alpha = 1$ and a small mass ($\nu = 1.4$) look as in Fig. 3.

As seen from this figure, the spectral index features a sharp and comparatively broad first peak, followed by more frequent oscillations with decreasing amplitude. A similar pattern appears for the $m_{\text{eff}}(t)$ protocol for an inflaton potential [26,27] and for the $O(N)$ model in de Sitter [25]. We will return to cosmological observables for non-Gaussian initial-state perturbations, as well as to systems with a large number N of fields, in future work.

III. COMMENT ON THE NON-GAUSSIAN QUENCH DYNAMICS

One of the key distinctions of our approach to describe relaxation in a nonequilibrium initial state is that analytical calculations are not restricted only to Gaussian distributions in the initial density matrix, but non-Gaussianities can also be incorporated. In this section, we demonstrate how to correctly define such an initial state and apply the method in the two-dimensional case $D = 2$ for the two-point correlator in flat spacetime.

Let us consider, for example, the case with the insertion of a cubic operator $\hat{V}$ (with coupling “constant” λ) and the corresponding functional operator Q , defined in (A13),

$$\begin{aligned}\hat{V}[\hat{\phi}] &= \int d^{D-1}\mathbf{x} \sqrt{|g(t_0, \mathbf{x})|} : \hat{\phi}^3 : (\mathbf{x}), \\ Q\left[\Phi, -\frac{\delta}{\delta\Pi}\right] &= \exp\left\{\lambda \int d^{D-1}\mathbf{x} \sqrt{|g(t_0, \mathbf{x})|} \left[3 : \Phi^2 : (\mathbf{x}) \frac{\delta}{\delta\Pi(\mathbf{x})} - \frac{1}{4} \frac{\delta^3}{\delta\Pi^3(\mathbf{x})}\right]\right\}.\end{aligned}\quad (3.1)$$

Here, normal ordering¹ refers to the subtraction of the counterterm proportional to $\Delta(x, x) = iG_K(x, x)$, the two-point correlator evaluated at coincident points in some ultraviolet regularization scheme,

$$\begin{aligned}: \phi^2 : (x) &= \phi(x)^2 - \Delta(x, x), \\ : \phi^3 : (x) &= \phi(x)^3 - 3\Delta(x, x)\phi(x).\end{aligned}\quad (3.2)$$

Following the procedure outlined in Appendix A, we obtain the expression for the two-point correlator after such a non-Gaussian disturbance,

$$\begin{aligned}iG_K^Q(t, \mathbf{x}|t, \mathbf{y}) &= \langle \hat{\phi}(\mathbf{x}) \hat{\phi}(\mathbf{y}) \rangle_t^Q = iG_K(t, \mathbf{x}|t, \mathbf{y}) \\ &\quad - 9\lambda^2 \int d^{D-1}\xi_1 d^{D-1}\xi_2 \sqrt{|g(t_0, \xi_1)|} \sqrt{|g(t_0, \xi_2)|} \cdot G_R(t, \mathbf{x}|t_0, \xi_1) G_R(t, \mathbf{y}|t_0, \xi_2) G_K^2(t_0, \xi_1|t_0, \xi_2),\end{aligned}\quad (3.3)$$

or, in momentum space, for the correction $i\Delta G_K$ to the tree-level Keldysh propagator iG_K ,

$$\begin{aligned}i\Delta G_K(t, \mathbf{x}|t, \mathbf{y}) &= -18\lambda^2 \int \frac{d^{D-1}\mathbf{k}}{(2\pi)^{D-1}} \int \frac{d^{D-1}\mathbf{q}}{(2\pi)^{D-1}} \sqrt{|g(t_0)|} \sqrt{|g(t_0)|} e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})} \\ &\quad \times G_R(\mathbf{k}|t, t_0) G_R(-\mathbf{k}|t, t_0) G_K\left(\frac{\mathbf{k}+\mathbf{q}}{2}\middle|t, t_0\right) G_K\left(\frac{\mathbf{k}-\mathbf{q}}{2}\middle|t, t_0\right).\end{aligned}\quad (3.4)$$

It is easy to see that this expression is essentially a loop integral over the momentum $\mathbf{q}$ and, in particular, contains ultraviolet divergences associated with such loops. Therefore, in the general case, instantaneous interaction counterterms at the initial Cauchy surface, $t = t_0$, with renormalized couplings must be included. For now, we

postpone this issue and instead examine the two-dimensional flat space, $D = 2$, where the integral in (3.4) is finite. Then,

$$\begin{aligned}i\Delta G_K(t, \mathbf{x}|t, \mathbf{y}) &= \frac{18\lambda^2}{m^2} \int \frac{d\xi}{2\pi} \int \frac{d\zeta}{2\pi} e^{i\xi m(\mathbf{x}-\mathbf{y})} \\ &\quad \times \frac{1 - \cos\left(2\sqrt{\xi^2 + 1}mt\right)}{\xi^2 + 1} \\ &\quad \times \frac{1}{\sqrt{(\xi^2 - \zeta^2)^2 + 8(\xi + \zeta) + 16}}.\end{aligned}\quad (3.5)$$

¹In general, all additional terms in the Hamiltonian should be normal ordered $\delta\hat{H} \sim \lambda : \hat{V}[\hat{\phi}] :$. In the Gaussian quench case the result does not depend on this, which is why we did not impose it earlier.

As in the Gaussian quench case in flat space, this integral contains a time-independent contribution and a time-dependent “tail” represented by the cosine term in (3.5). The ζ -integral can be performed exactly, and the time dependence of the tail may be estimated via the stationary phase method for $mt \gg 1$, yielding,

$$i\Delta G_K(t, \mathbf{x}|t, \mathbf{y}) \approx I - \frac{9\lambda^2 \sqrt{\pi}}{2m^{5/2} \sqrt{t}} \cos\left(2mt + \frac{\pi}{4}\right),$$

$$I = \frac{36\lambda^2}{m^2} \int_{-\infty}^{+\infty} \frac{d\xi}{2\pi} \frac{e^{i\xi m(x-y)}}{|\xi| + 2} \frac{1}{\xi^2 + 1} K\left(\frac{4\sqrt{|\xi|}}{|\xi| + 2}\right), \quad (3.6)$$

where $K(z)$ is the complete elliptic integral of the first kind. As we see, the relaxation rate at late times for the massive theory is the same as in the Gaussian case (see also Appendix B), and appears to be insensitive to the particular quench type, at least for the two-point function in a massive theory.

We emphasize once again that in the case $D > 2$ the integrals determining the corrections to the two-point function after the non-Gaussian quench contain loop ultraviolet divergences. This means that a complete and exhaustive analysis of a non-Gaussian quench in this situation includes the consideration of theories with the corresponding interaction in the Lagrangian and the renormalization of this theory. Then operators with the corresponding renormalized coupling constants can be consistently included in the operator perturbing the density matrix as in (3.1), and for superrenormalizable theories it will be necessary to remove only a finite number of divergences. At present we have developed the formalism only for a theory that is Gaussian in the quantum field; in future work we will attempt to generalize this method to interacting theories. However, due to the restructuring of the initial state during the evolution, when considering interacting theories one will have either to deal with secular contributions in the perturbative expansion in the coupling constants, or to resort to a numerical analysis based on the quasiclassical expansion in the Keldysh-Schwinger functional integral [40].

Nevertheless, from (3.1) it is clear that such an initial state modifies higher-point correlators in a nontrivial way. In particular, one may study the evolution and relaxation of systems with nontrivial initial correlations encoded in the density matrix. Of special interest is the question, still not completely settled, of how initial correlations can imprint themselves during the inflationary period of the Universe, which can be understood by studying multipoint correlators of primordial fluctuations. We will return to the behavior of QFT in $D > 2$ under non-Gaussian quenches in future work.

IV. CONCLUSION

In this work, we examined the dynamics of quantum fields in a specific class of nonequilibrium initial states on curved spacetime backgrounds. Developing the approach of [34], we expressed the postquench two-point correlation function in terms of the Wigner functional of the initial density matrix, with the quench implemented as a unitary transformation of the initial state: $\hat{\rho}_Q = \hat{Q}\hat{\rho}_0\hat{Q}^\dagger$. This formulation, based on the Keldysh-Schwinger functional integral, allows one to clearly separate the modification of the initial state from the subsequent unitary evolution and is applicable to an arbitrary curved spacetime geometry. As a result, the method is particularly useful in situations where the standard description via sudden Hamiltonian changes or explicit mode evolution becomes technically cumbersome or inefficient.

As a first example, we considered in detail the Gaussian (quadratic) quench, corresponding to an initial-state perturbation of the form (A15). Such a quench modifies the initial statistical distribution rather than the Hamiltonian itself, and the postquench dynamics can be expressed entirely through equilibrium Green’s functions. In this case, the Keldysh function after the quench G_K^Q takes the compact form (1.2) and (1.3), so that the effect of a global quench reduces to a finite number of correction terms added to the standard correlators.

We then illustrated the method in flat spacetime, see Appendix B. For massive fields, we obtained relaxation of the correlator to a modified postquench value with an oscillatory power-law tail, decaying at large times as shown in (B7). This behavior matches the well-known results for mass-dependent quench protocols and therefore provides a nontrivial consistency check of the method. However, for massless fields in $D = 4$ we found qualitatively different dynamics; the correlator does not relax even at late times, see (B9). This indicates that the class of nonequilibrium states considered here cannot in general be reduced to the standard mass-quench scenario. A more detailed analysis of this “critical” postquench dynamics will be pursued in future work.

Our primary interest concerns the dynamics in de Sitter space, see Sec. II. For the Gaussian quench, we analyzed the late-time asymptotics of the two-point function. We found that the decay of the quench-induced corrections depends sensitively on the mass of the field: for light fields ($m < \sqrt{2}$, or equivalently $\nu > 1/2$) the corrections decay as $(\eta/\eta_0)^{3-2\nu}$, whereas heavier fields exhibit a slower decay regime in the limit of close spatial points. This suggests that postquench dynamics of local observables such as the stress-energy tensor may depend nontrivially on the mass spectrum, which is relevant for discussions of the stability of the Bunch-Davies state and for understanding secular (time-growing) quantum corrections in de Sitter space. In addition, we analyzed the possible implications of this

type of postquench description for cosmological observables in the expanding Poincaré patch. In particular, we computed the power spectrum $\mathcal{P}_s(k)$ and the spectral index $n_s(k)$ according to (2.13). The resulting spectra exhibit a pronounced initial peak followed by decaying oscillations (see Fig. 3). Such features are qualitatively similar to those encountered in models with time-dependent effective mass $m_{\text{eff}}(t)$ [26,27] and in $O(N)$ models in de Sitter space [25]. This should not be regarded as an alternative to these approaches. Instead, our point is that nontrivial initial state distributions should also be incorporated in a full and consistent treatment of the problem.

Finally, in Sec. III we showed that the method naturally generalizes to non-Gaussian initial states generated by higher-order operators, using the cubic perturbation $\hat{V} \sim \int : \phi^3 :$ as an example. In the simplest case of $D = 2$ Minkowski spacetime, we found contributions that also decay at late times with a power-law behavior. This demonstrates the applicability of the method to systematic studies of non-Gaussianity and higher-order correlation functions, including in cosmological settings where primordial non-Gaussianities may play a role.

Future directions include analyzing the impact of such nonequilibrium initial states on secular loop corrections in de Sitter space [41], as well as computing the behavior of the energy-momentum tensor and other local observables after proper regularization in the coincident limit. Moreover, it is of interest to consider such a class of nonequilibrium states not only in an expanding spacetime, but also to study, using such states, the response of fields, for example, on backgrounds with horizons, such as black holes. As we already noted in the Introduction, a local quench can be used to test

the chaotic behavior of the theory, which has already been partially done, for example, in [8] for the BTZ black hole. Our approach makes it possible to consider a more general class of nonequilibrium states in this context, and we will also attempt to pursue this in the near future. The approach developed here provides a convenient and analytically controlled framework for these investigations.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [42].

APPENDIX A: QUANTUM QUENCH IN CURVED SPACETIME

Consider a free massive minimally coupled scalar field ϕ in curved spacetime,

$$S[\phi] = \frac{1}{2} \int_{t_0}^{\infty} dt \int d^{D-1} \mathbf{x} \sqrt{|g(t, \mathbf{x})|} [\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2]. \quad (\text{A1})$$

Then the time evolution of the expectation value of a composite operator $\hat{O}[\hat{\phi}(\mathbf{x})]$ is given by

$$\langle \hat{O}[\hat{\phi}(\mathbf{x})] \rangle_t \equiv \text{tr} \{ \hat{\rho}(t) \hat{O} \} = \text{tr} \{ \hat{\rho} \hat{U}^\dagger(t, t_0) \hat{O} \hat{U}(t, t_0) \} \equiv \text{tr} \{ \hat{\rho} \cdot \hat{U}^\dagger(t, t_0) \hat{O} \hat{U}^\dagger(+\infty, t) \cdot \hat{U}(+\infty, t_0) \}, \quad (\text{A2})$$

where $\hat{\rho}$ is the initial density matrix and $\hat{U}(t, t_0)$ is the evolution operator. Expanding the trace as an integral over the basis of field eigenstates $|\xi_1\rangle$, and inserting the corresponding resolutions of the identity at the marked points in (A2), as well as using the standard transition from the evolution operator to the path integral, we obtain the expression

$$\langle \hat{O}[\hat{\phi}(\mathbf{x})] \rangle_t = \int \mathcal{D}\chi(\mathbf{x}) \mathcal{D}\xi_1(\mathbf{x}) \mathcal{D}\xi_2(\mathbf{x}) \langle \xi_1 | \hat{\rho} | \xi_2 \rangle \int_{\phi_+(t_0)=\xi_1}^{\phi_+(+\infty)=\chi} \mathcal{D}\phi_+ \int_{\phi_-(t_0)=\xi_2}^{\phi_-(+\infty)=\chi} \mathcal{D}\phi_- e^{iS[\phi_+] - iS[\phi_-]} \cdot O[\phi_-(t, \mathbf{x})]. \quad (\text{A3})$$

In the Schwinger-Keldysh functional integral formalism it is convenient to introduce the variables

$$\phi_{cl} = \frac{1}{2}(\phi_+ + \phi_-), \quad \phi_q = \phi_+ - \phi_-.$$

Then the action on the Keldysh time contour takes the form,

$$\begin{aligned}
S_K[\phi_{cl}, \phi_q] &= S[\phi_+] - S[\phi_-] \\
&= \int_{t_0}^{\infty} dt \int d^{D-1}\mathbf{x} \sqrt{|g|} [g^{\mu\nu} \partial_\mu \phi_q \partial_\nu \phi_{cl} + m^2 \phi_q \phi_{cl}] \\
&= - \int d^{D-1}\mathbf{x} \sqrt{|g|} \cdot (\xi_1(\mathbf{x}) - \xi_2(\mathbf{x})) \cdot g^{\mu 0} \partial_\mu \phi_{cl}(t_0, \mathbf{x}) - \int d^D x \sqrt{|g|} \phi_q \left[\frac{1}{\sqrt{|g|}} \partial_\mu \sqrt{|g|} g^{\mu\nu} \partial_\nu + m^2 \right] \phi_{cl} \\
&= - \int d^{D-1}\mathbf{x} \cdot (\xi_1(\mathbf{x}) - \xi_2(\mathbf{x})) \cdot \pi(t_0, \mathbf{x}) - \int d^D x \sqrt{|g|} \phi_q \cdot \text{EOM}[\phi_{cl}], \tag{A4}
\end{aligned}$$

where we have introduced the canonical momentum $\pi(t, \mathbf{x}) = \sqrt{|g|} g^{00} \partial_0 \phi(t, \mathbf{x})$, and denoted the operator in the square brackets in the second line acting on ϕ_{cl} as $\text{EOM}[\phi_{cl}]$. We also assumed that the mixed components g^{i0} vanish for spatial index i . Using the further convenient notations $\beta(\mathbf{x}) = \xi_1(\mathbf{x}) - \xi_2(\mathbf{x})$ and $\Phi(\mathbf{x}) = \frac{1}{2}(\xi_1(\mathbf{x}) + \xi_2(\mathbf{x}))$, we can rewrite (A3) in the form,

$$\begin{aligned}
\langle \hat{O}[\hat{\phi}(\mathbf{x})] \rangle_t &= \int \mathcal{D}\chi(\mathbf{x}) \mathcal{D}\Phi(\mathbf{x}) \mathcal{D}\beta(\mathbf{x}) \left\langle \Phi + \frac{\beta}{2} \middle| \hat{\rho} \middle| \Phi - \frac{\beta}{2} \right\rangle \\
&\quad \times \int_{\phi_{cl}(t_0)=\Phi}^{\phi_{cl}=\chi} \mathcal{D}\phi_{cl} \int_{\phi_q(t_0)=\beta}^{\phi_q=0} \mathcal{D}\phi_q \exp \left\{ -i \int d^{D-1}\mathbf{x} \beta(\mathbf{x}) \pi(t_0, \mathbf{x}) \right\} \\
&\quad \times \exp \left\{ i \int d^D x \sqrt{|g|} \phi_q \cdot \text{EOM}[\phi_{cl}] \right\} O \left[\phi_{cl} - \frac{\phi_q}{2} \right] \\
&= \int \mathcal{D}\Phi(\mathbf{x}) \mathcal{D}\Pi(\mathbf{x}) \mathcal{D}\beta(\mathbf{x}) \int_{\phi_{cl}=\Phi} \mathcal{D}\phi_{cl} \exp \left\{ -i \int d^{D-1}\mathbf{x} \beta(\mathbf{x}) \Pi(\mathbf{x}) \right\} \\
&\quad \times \delta(\Pi(\mathbf{x}) - \pi(t_0, \mathbf{x})) \cdot \delta(\text{EOM}[\phi_{cl}]) \cdot O[\phi_{cl}] = \int \mathcal{D}\Phi(\mathbf{x}) \mathcal{D}\Pi(\mathbf{x}) W[\Phi, \Pi] \cdot O[\phi|_{\text{on-shell}}], \tag{A5}
\end{aligned}$$

where in the fourth line we have omitted ϕ_q inside the composite operator O , since the contraction $\overline{\phi_q} \phi_q = 0$ in the Schwinger-Keldysh technique and the mixed contractions $\overline{\phi_q} \phi_{cl}$ vanish at coincident times due to causality (see also [43]). Also, in the fourth line, we integrated over ϕ_q , producing the delta-function, which enforces $\text{EOM}[\phi_{cl}] = 0$, and introduced the field $\Pi(\mathbf{x})$ through the additional delta function. In the last line, we introduced the Wigner functional and carried out the integration over ϕ_{cl} using the delta-constraints, which project ϕ_{cl} onto the solutions of the equation of motion with the specified initial conditions,

$$W[\Phi, \Pi] = \int \mathcal{D}\beta(\mathbf{x}) \exp \left\{ -i \int d^{D-1}\mathbf{x} \beta(\mathbf{x}) \pi(\mathbf{x}) \right\} \left\langle \Phi + \frac{\beta}{2} \middle| \hat{\rho} \middle| \Phi - \frac{\beta}{2} \right\rangle, \tag{A6}$$

$$\text{EOM}[\phi|_{\text{on-shell}}] = 0, \quad \phi(t_0, \mathbf{x})|_{\text{on-shell}} = \Phi(\mathbf{x}), \quad \partial_t \phi(t_0, \mathbf{x})|_{\text{on-shell}} = \frac{1}{\sqrt{|g|} g^{00}} \Pi(\mathbf{x}). \tag{A7}$$

If one can find the retarded Green function for the given gravitational background,

$$\left[\frac{1}{\sqrt{|g|}} \partial_\mu \sqrt{|g|} g^{\mu\nu} \partial_\nu + m^2 \right] G_R(t, \mathbf{x} | t', \mathbf{y}) = - \frac{\delta(t - t') \delta^{(3)}(\mathbf{x} - \mathbf{y})}{\sqrt{|g|}}, \tag{A8}$$

then the on shell field is expressed explicitly as

$$\phi(t, \mathbf{x})|_{\text{on-shell}} = - \int d^{D-1}\mathbf{y} [\sqrt{|g(t_0, \mathbf{y})|} g^{00}(t_0, \mathbf{y}) \partial_t G_R(t, \mathbf{x} | t_0, \mathbf{y}) \cdot \Phi(\mathbf{y}) + G_R(t, \mathbf{x} | t_0, \mathbf{y}) \cdot \Pi(\mathbf{y})]. \tag{A9}$$

Now we want to study the dynamics of correlation functions of local operators after the system is driven out of an equilibrium state described by some initial density matrix $\hat{\rho}_0$. If, at time t_0 , the Hamiltonian $\hat{H}$ of the system undergoes a perturbation $\delta \hat{H} = -\alpha \delta(t - t_0) \hat{V}[\hat{\phi}(\mathbf{x})]$, then the density matrix describing the subsequent evolution becomes

$$\hat{\rho}_Q(t_0) = \hat{Q}\hat{\rho}_0\hat{Q}^\dagger, \quad \hat{Q} = e^{i\alpha\hat{V}[\hat{\phi}(\mathbf{x})]}. \quad (\text{A10})$$

More generally, we can simply consider perturbed initial states of the form (A10) without referring to the Hamiltonian quench interpretation. Using the identity

$$i\frac{\delta}{\delta\Pi(\mathbf{y})}\exp\left\{-i\int d^{D-1}\mathbf{x}\beta(\mathbf{x})\Pi(\mathbf{x})\right\} = \beta(\mathbf{y})\exp\left\{-i\int d^{D-1}\mathbf{x}\beta(\mathbf{x})\Pi(\mathbf{x})\right\}, \quad (\text{A11})$$

the modified Wigner functional (A6) after the quench takes the form

$$\begin{aligned} W_Q[\Phi, \Pi] &= \int \mathcal{D}\beta(\mathbf{x}) \exp\left\{-i\int d^{D-1}\mathbf{x}\beta(\mathbf{x})\pi(\mathbf{x})\right\} \left\langle \Phi + \frac{\beta}{2} \left| \hat{Q}\hat{\rho}_0\hat{Q}^\dagger \right| \Phi - \frac{\beta}{2} \right\rangle \\ &= Q\left[\Phi, \frac{\delta}{\delta\Pi}\right] W_0[\Phi, \Pi], \end{aligned} \quad (\text{A12})$$

where $W_0[\Phi, \Pi]$ corresponds to the unperturbed density matrix $\hat{\rho}_0$, and we introduced the functional operator

$$Q\left[\Phi, \frac{\delta}{\delta\Pi}\right] = \exp\left\{-i\alpha\left(V\left[\Phi - \frac{i}{2}\frac{\delta}{\delta\Pi}\right] - V\left[\Phi + \frac{i}{2}\frac{\delta}{\delta\Pi}\right]\right)\right\}. \quad (\text{A13})$$

Substituting (A12) into (A5) and integrating by parts over $\Pi(\mathbf{x})$, we obtain the general postquench evolution,

$$\langle \hat{O}[\hat{\phi}(\mathbf{x})] \rangle_t^Q = \int \mathcal{D}\Phi(\mathbf{x}) \mathcal{D}\Pi(\mathbf{x}) W_0[\Phi, \Pi] Q\left[\Phi, -\frac{\delta}{\delta\Pi}\right] O[\phi|_{\text{on-shell}}]. \quad (\text{A14})$$

Thus, the problem of postquench evolution reduces to computing expectation values with the equilibrium density matrix, but with the observable modified by the functional operator Q .

As a simple example of a global quench, consider a Gaussian initial state, which can be interpreted as an instantaneous perturbation of the mass term,

$$\begin{aligned} \hat{V}[\hat{\phi}] &= \int d^{D-1}\mathbf{x} \sqrt{|g(t_0, \mathbf{x})|} \hat{\phi}^2(\mathbf{x}), \\ Q\left[\Phi, -\frac{\delta}{\delta\Pi}\right] &= \exp\left\{2\alpha \int d^{D-1}\mathbf{x} \Phi(\mathbf{x}) \frac{\delta}{\delta\Pi(\mathbf{x})}\right\}. \end{aligned} \quad (\text{A15})$$

Finally, using (A9) and the representation of the Keldysh propagator in terms of the Wigner functional,

$$iG_K(t, \mathbf{x}|t', \mathbf{y}) = \frac{1}{2} \text{tr}(\hat{\rho} \cdot \{\hat{\phi}(t, \mathbf{x}), \hat{\phi}(t', \mathbf{y})\}) = \int \mathcal{D}\Phi(\mathbf{x}) \mathcal{D}\Pi(\mathbf{x}) W[\Phi, \Pi] \phi(t, \mathbf{x})|_{\text{on-shell}} \phi(t', \mathbf{y})|_{\text{on-shell}}, \quad (\text{A16})$$

we obtain the evolution of the equal-time two-point correlator,

$$\begin{aligned} iG_K^Q(t, \mathbf{x}|t, \mathbf{y}) &= \langle \hat{\phi}(\mathbf{x}) \hat{\phi}(\mathbf{y}) \rangle_t^Q \\ &= \int \mathcal{D}\Phi(\mathbf{x}) \mathcal{D}\Pi(\mathbf{x}) W_0[\Phi, \Pi] \exp\left\{2\alpha \int d^{D-1}\boldsymbol{\xi} \sqrt{|g(t_0, \boldsymbol{\xi})|} \Phi(\boldsymbol{\xi}) \frac{\delta}{\delta\Pi(\boldsymbol{\xi})}\right\} \phi(t, \mathbf{x})|_{\text{on-shell}} \phi(t, \mathbf{y})|_{\text{on-shell}} \\ &= iG_K(t, \mathbf{x}|t, \mathbf{y}) - 2\alpha \int d^{D-1}\boldsymbol{\xi} \sqrt{|g(t_0, \boldsymbol{\xi})|} [iG_K(t, \mathbf{x}|t_0, \boldsymbol{\xi}) G_R(t, \mathbf{y}|t_0, \boldsymbol{\xi}) + iG_K(t, \mathbf{y}|t_0, \boldsymbol{\xi}) G_R(t, \mathbf{x}|t_0, \boldsymbol{\xi})] \\ &\quad + 4\alpha^2 \int d^{D-1}\boldsymbol{\xi}_1 d^{D-1}\boldsymbol{\xi}_2 \sqrt{|g(t_0, \boldsymbol{\xi}_1)|} \sqrt{|g(t_0, \boldsymbol{\xi}_2)|} iG_K(t_0, \boldsymbol{\xi}_1|t_0, \boldsymbol{\xi}_2) G_R(t, \mathbf{x}|t_0, \boldsymbol{\xi}_1) G_R(t, \mathbf{x}|t_0, \boldsymbol{\xi}_2). \end{aligned} \quad (\text{A17})$$

It is often convenient to evaluate these integrals in momentum space. If the background metric is homogeneous and isotropic, depending only on time, and the equilibrium state $\hat{\rho}_0$ shares these symmetries, then the result takes the form

$$iG_K^Q(t, \mathbf{x}|t, \mathbf{y}) = \int \frac{d^{D-1}\mathbf{k}}{(2\pi)^{D-1}} e^{ik(\mathbf{x}-\mathbf{y})} \left\{ iG_K(\mathbf{k}|t, t) - 4\alpha\sqrt{|g(t_0)|} iG_K(\mathbf{k}|t, t_0) G_R(\mathbf{k}|t, t_0) \right. \\ \left. + 4\alpha^2 \sqrt{|g(t_0)|} \sqrt{|g(t_0)|} G_K(\mathbf{k}|t_0, t_0) G_R(\mathbf{k}|t, t_0) G_R(\mathbf{k}|t, t_0) \right\}. \quad (\text{A18})$$

Note that for expectation values of polynomial operators we always obtain only a finite number of terms modifying the result relative to the unperturbed initial state. Moreover, expressions of the type (A17) can be derived for higher-order correlation functions, as well as for non-Gaussian (nonquadratic) initial perturbations or spatially inhomogeneous quenches. Indeed, all arguments above remain valid if one replaces the constant α by a spatial function $\alpha(\mathbf{x})$, which simply appears inside the corresponding integrals.

APPENDIX B: QUENCH DYNAMIC IN FLAT SPACETIME

Let us write the quantization of the field in flat spacetime in the standard way,

$$\hat{\phi}(t, \mathbf{x}) = \int \frac{d^{D-1}\mathbf{k}}{(2\pi)^{D-1}} [\hat{a}_k f_k(t) + \hat{a}_k^\dagger f_k^*(t)], \quad (\text{B1})$$

where

$$[\hat{a}_p, \hat{a}_k^\dagger] = (2\pi)^{D-1} \delta(\mathbf{k} - \mathbf{p}), \\ f_k(t) = \frac{1}{\sqrt{2\sqrt{k^2 + m^2}}} e^{-i\sqrt{k^2 + m^2}t}. \quad (\text{B2})$$

The corresponding propagators for the vacuum initial state are

$$iG_K(\mathbf{k}|t, t_0) = \text{Re}\{f_k(t)f_k^*(t_0)\} = \frac{1}{2\sqrt{k^2 + m^2}} \cos\left(\sqrt{k^2 + m^2}(t - t_0)\right), \\ G_R(\mathbf{k}|t, t_0) = -2\theta(t - t_0) \text{Im}\{f_k(t)f_k^*(t_0)\} = -\frac{1}{\sqrt{k^2 + m^2}} \sin\left(\sqrt{k^2 + m^2}(t - t_0)\right) \theta(t - t_0). \quad (\text{B3})$$

In what follows we denote $r = |\mathbf{x} - \mathbf{y}|$ and assume $t > t_0$. Then, in flat spacetime, we can rewrite (A18) as

$$iG_K^Q(t, \mathbf{x}|t, \mathbf{y}) = iG_K(t, \mathbf{x}|t, \mathbf{y}) - 4\alpha I_1^{\text{flat}}(t, t_0, r) + 4\alpha^2 I_2^{\text{flat}}(t, t_0, r), \\ I_1^{\text{flat}}(t, t_0, r) = -\frac{1}{4} \int \frac{d^{D-1}\mathbf{k}}{(2\pi)^{D-1}} e^{ik(\mathbf{x}-\mathbf{y})} \frac{1}{k^2 + m^2} \sin\left(2\sqrt{k^2 + m^2}(t - t_0)\right), \\ I_2^{\text{flat}}(t, t_0, r) = \frac{1}{2} \int \frac{d^{D-1}\mathbf{k}}{(2\pi)^{D-1}} e^{ik(\mathbf{x}-\mathbf{y})} \frac{1}{(k^2 + m^2)^{\frac{3}{2}}} \sin^2\left(\sqrt{k^2 + m^2}(t - t_0)\right). \quad (\text{B4})$$

Using the Fourier-Bessel representation,

$$\int \frac{d^{D-1}\mathbf{p}}{(2\pi)^{D-1}} e^{ipr} f(p) = \frac{1}{(2\pi)^{\frac{D-1}{2}} r^{\frac{D-3}{2}}} \int_0^{+\infty} dp p^{\frac{D-1}{2}} J_{\frac{D-3}{2}}(pr) f(p), \quad (\text{B5})$$

for $D = 4$, and setting $t_0 = 0$ due to time-translation symmetry, the integrals in (B4) become

$$I_{1,4D}^{\text{flat}}(t, t_0, r) = -\frac{1}{8\pi^2 r} \int_0^\infty d\xi \frac{\xi}{\xi^2 + 1} \sin(mr \cdot \xi) \sin\left(2mt \cdot \sqrt{\xi^2 + 1}\right), \\ I_{2,4D}^{\text{flat}} = \frac{1}{8\pi^2} \frac{1}{mr} \int_0^\infty d\xi \frac{\xi}{(\xi^2 + 1)^{\frac{3}{2}}} \sin(mr \cdot \xi) \left[1 - \cos\left(2mt \cdot \sqrt{\xi^2 + 1}\right)\right]. \quad (\text{B6})$$

We are interested in the late-time regime $t - t_0 \gg m^{-1}$ and distances $r \ll t - t_0$. In this limit, the contributions from I_1 and the oscillatory part of I_2 are suppressed and can be estimated via the method of steepest descent. The nonoscillatory part of

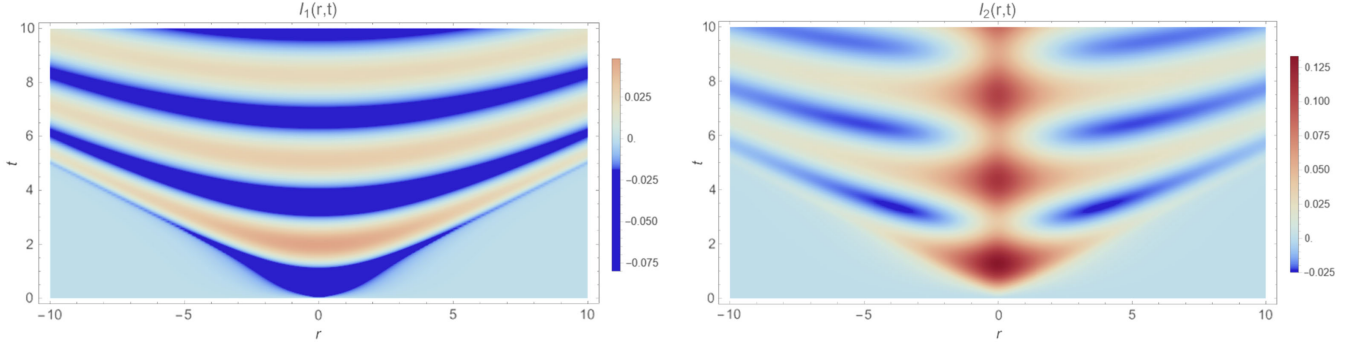


FIG. 4. Numerical values of the integrals $I_1(t, r)$ and $I_2(t, r)$ in the case $D = 2$. The “horizon effect” at $|r| < 2t$ is clearly visible.

I_2 survives and yields the postquench correction to the Keldysh function,

$$\begin{aligned}
 iG_K^Q(t, \mathbf{x}|t, \mathbf{y}) &= iG_K^{\text{postquench}}(t, \mathbf{x}|t, \mathbf{y}) + \text{tail}(t, r), \\
 iG_K^{\text{postquench}}(t, \mathbf{x}|t, \mathbf{y}) &= \frac{1}{4\pi^2} \frac{m}{r} K_1(mr) + \frac{\alpha^2}{2\pi^2} K_0(mr), \\
 \text{tail}(t, r) &\simeq \frac{\alpha m}{8\pi^{3/2}} \frac{1}{(mt)^{3/2}} \cos\left(2mt + \frac{\pi}{4}\right) + \frac{\alpha^2}{8\pi^{3/2}} \frac{1}{(mt)^{3/2}} \sin\left(2mt + \frac{\pi}{4}\right) \sim \frac{1}{(mt)^{3/2}} \cos(2mt + \delta), \quad (\text{B7})
 \end{aligned}$$

with some phase δ , defined in (1.4). The relaxation rate coincides with the well-studied mass-quench dynamics in flat space [13,14].

The same can be done in two dimensions ($D = 2$),

$$\begin{aligned}
 I_{1,2D}^{\text{flat}}(t, t_0, r) &= -\frac{1}{4\pi m} \int_0^\infty d\xi \frac{1}{\xi^2 + 1} \cos(mr \cdot \xi) \sin(2mt \cdot \sqrt{\xi^2 + 1}), \\
 I_{2,2D}^{\text{flat}} &= \frac{1}{4\pi m^2} \int_0^\infty d\xi \frac{1}{(\xi^2 + 1)^{3/2}} \cos(mr \cdot \xi) [1 - \cos(2mt \cdot \sqrt{\xi^2 + 1})], \quad r = x - y. \quad (\text{B8})
 \end{aligned}$$

In this case, the relaxation tail behaves as $\text{tail}(t) \sim \frac{1}{\sqrt{mt}} \cos(2mt + \delta_{2d})$, consistent with the general expectations in even spatial dimensions. In addition, no subleading UV divergences appear, and although the integrals cannot be obtained in closed form, they can be computed numerically (see Fig. 4). As expected, the result for massive fields is quantitatively identical to the standard $m(t)$ -quench protocol, including the “horizon effect” at $|r| > 2t$ [13].

The situation changes drastically in the massless case $m = 0$, where late-time suppression does not occur. However, in this case the postquench dynamics (B4) can be computed exactly for $D = 4$,

$$\begin{aligned}
 iG_K^Q(t, \mathbf{x}|t, \mathbf{y}) &= \frac{1}{4\pi^2 r^2} + \frac{\alpha}{4\pi^2 r} \log \left[\frac{2t+r}{2t-r} \right] \\
 &\quad + \frac{\alpha^2}{2\pi^2} \left\{ \log \left[\frac{\sqrt{4t^2 - r^2}}{r} \right] + \frac{t}{r} \log \left[\frac{2t+r}{2t-r} \right] \right\}, \quad (\text{B9})
 \end{aligned}$$

which does not relax at $t \rightarrow \infty$ due to the first term in the braces. This type of “nonrelaxation” behavior is also known in $D = 2$ and $D = 3$ [13,14], but does not occur in higher dimensions for a mass-quench. The origin and implications of this feature for our initial state will be discussed elsewhere.

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