

Carrollian amplitudes and holographic correlators in $\text{AdS}_3/\text{CFT}_2$

Iustin Surubaru^{*} and Bin Zhu[†]

*School of Mathematics and Maxwell Institute for Mathematical Sciences,
University of Edinburgh, EH9 3FD Edinburgh, United Kingdom*



(Received 23 April 2025; accepted 5 July 2025; published 21 July 2025)

We study Carrollian amplitudes of massless scalars in $(1+2)$ Minkowski space. Using the prescription recently shown by Alday *et al.* [Carrollian amplitudes from holographic correlators, *J. High Energy Phys.* **03** (2025) 158] originally designed for the AdS_4 Witten diagrams, we show that AdS_3 Witten diagrams in position space in the flat space limit reduce to Carrollian amplitudes. The flat space limit in the bulk is implemented by the Carrollian limit at the boundary. Focusing on four-point correlators with contact and exchange diagrams, we show that the Carrollian limit makes the universality of the bulk point singularity manifest upon performing analytic continuation to the Lorentzian signature of the boundary correlators. Unlike four-point Carrollian amplitudes in $(1+3)$ dimensions, the $(1+2)$ dimensional ones are nondistributional, having analytic properties simpler than the AdS correlators. We also observe for the first time a double copy structure of Carrollian amplitudes.

DOI: [10.1103/PhysRevD.112.026023](https://doi.org/10.1103/PhysRevD.112.026023)

I. INTRODUCTION

Flat holography has gained a lot of interest in recent years. Two seemingly different proposals of flat holography have emerged: Celestial holography and Carrollian holography. The main conjecture of celestial holography is that the dual theory of quantum gravity in d -dimensional asymptotically flat space is a celestial conformal field theory (CFT) living on the $(d-2)$ -dimensional celestial sphere at null infinity.¹ In contrast, the main proposal of Carrollian holography is that the dual theory is a $(d-1)$ dimensional Carrollian CFT living at null infinity [2–6]. The Carrollian framework is reached in the limit in which the speed of light vanishes, i.e., $c \rightarrow 0$ [7–13]. In both proposals, the S matrix of the bulk theory is dually recovered through the correlators of the appropriate CFT living either on the whole of null infinity or the celestial sphere. It has been shown that from the perspective of scattering amplitudes, celestial and Carrollian holography are closely related by a change of basis of scattering amplitudes [2–6].

When expressed in position space at null infinity, flat space amplitudes are mapped to Carrollian amplitudes,

which behave as correlation functions in a Carrollian CFT. Most of the work on Carrollian amplitudes has focused on massless scattering in four dimensions, see, e.g., [2–4, 7, 14–29]. In this work, we initiate a study of Carrollian amplitudes associated with massless scalar scattering in three dimensions. This includes massless scalars with self-cubic and quartic interactions, and massless scalars coupled to Chern-Simons gauge fields and gravity.

The codimension-1 feature suggests that Carrollian holography would be more naturally related to the flat space limit of the AdS/CFT correspondence [30–32], which is by far the most successful realization of the holographic principle. Understanding the flat space limit of AdS/CFT has been a long-standing effort with different proposals [33–41]. See, e.g., [42] for a comprehensive discussion and references therein.

With the new notions of flat holography introduced in recent years, there has been increased interest in relating the new formalisms to the flat space limit of AdS/CFT [7, 18, 43–46]. In particular, the authors of [7] gave a prescription for how to reproduce Carrollian amplitudes of massless particles in four dimensions by taking the Carrollian limit of holographic correlators in $\text{AdS}_4/\text{CFT}_3$. The flat space limit in the bulk is realized by the Carrollian limit at the boundary. In the usual setup of taking the flat limit of AdS, the boundary operators are restricted to strips, a consequence of their dual bulk description. However, the advantage of the prescription in [7] is that the boundary insertions are not restricted *a priori*, which is more natural from an intrinsic boundary perspective. It turns out that the analytic continuation from Euclidean to Lorentzian signature is crucial for taking the Carrollian limit at the

^{*}Contact author: iustin.surubaru@ed.ac.uk

[†]Contact author: binzhu@ed.ac.uk

¹See, e.g., [1] for reviews.

boundary. The presence of a bulk point singularity arises as a consequence of the Carrollian limit. In this work, we apply their prescription to holographic correlators in $\text{AdS}_3/\text{CFT}_2$ and confirm that the matching also holds in this case. This provides strong evidence that Carrollian holography is a natural candidate related to the flat space limit of AdS/CFT .

In parallel, scattering amplitudes in gravity and Yang-Mills theory exhibit a remarkable double-copy structure [47,48], known as the Kawai-Lewellen-Tye (KLT) relations [47] and the Bern-Carrasco-Johansson double copy [48]. The double copy relations state that gravitational amplitudes can be obtained by a well-defined “squaring” of gauge theory amplitudes. The double copy relations have been well studied in a momentum eigenstate basis with a rich mathematical structure and a wide range of applications. See, e.g., [49,50] for reviews. As for other bases, e.g., boost eigenstate basis [51,52], the double-copy structure of celestial amplitudes has been studied in [52–56]. Surprisingly, in this work we find an interesting double copy structure of Carrollian amplitudes.

The paper is organized as follows. In Sec. II, we review some basics of Carrollian amplitudes and compute several examples of Carrollian amplitudes for external massless scalar particles in three dimensions. We mainly focus on four-point amplitudes with contact and exchange diagrams and show the double copy structure between the Carrollian four-point amplitudes of Chern-Simons theory coupled to scalar matter and scalars minimally coupled to gravity. In Sec. III, we review some basics of (planar) Bondi coordinates for flat space and AdS_3 and show how the bulk flat space limit is implemented as the Carrollian limit $c \rightarrow 0$ at the boundary. We show how in the flat space limit, the bulk-to-bulk and bulk-to-boundary propagators of Witten diagrams reduce to those in Feynman diagrams. In Sec. IV, using the prescription in [7], we show in detail how to reproduce the Carrollian amplitudes that we obtain in Sec. II by taking the Carrollian limit of holographic correlators in $\text{AdS}_3/\text{CFT}_2$. We also discuss the two-point correlator in AdS_3 shockwave backgrounds. In Sec. V, we discuss briefly some future directions.

II. CARROLLIAN AMPLITUDES FOR MASSLESS SCATTERINGS

In [4,19], it was shown that scattering amplitudes of massless particles in d -dimensional flat space can be recast as Carrollian correlators in $d - 1$ dimensions by Fourier transforms. The Carrollian correlators living at null infinities are called Carrollian amplitudes. We shall focus on three-dimensional flat space that we will parametrize using planar Bondi coordinates. The planar Bondi coordinates for four-dimensional flat space were recently used in [7,57,58]. One can find the Bondi coordinates in three dimensions in [59]. Here, we review some basics of them. The coordinates used are given by $u, r, z \in \mathbb{R}$. One can express the usual

flat-space Cartesian coordinates in terms of the planar Bondi coordinates,

$$\begin{aligned} X^\mu &= \frac{1}{4} u \partial_z^2 q^\mu + \frac{1}{2} r q^\mu \\ &= \left(\frac{1}{2} u + \frac{1}{2} r(1 + z^2), rz, -\frac{1}{2} u + \frac{1}{2} r(1 - z^2) \right), \end{aligned} \quad (2.1)$$

using the parametrization of a null vector

$$q^\mu(z) = (1 + z^2, 2z, 1 - z^2), \quad (2.2)$$

in terms of a point $z \in S^1$. The metric in the planar Bondi coordinates is [59]

$$ds_{\text{Flat}}^2 = -du dr + r^2 dz^2. \quad (2.3)$$

In these conventions, the spacetime interval between two points is written as

$$\xi_{12}^{\text{Flat}} = X_{12}^\mu (X_{12})_\mu = -u_{12} r_{12} + r_1 r_2 z_{12}^2. \quad (2.4)$$

The future null infinity corresponds to sending $r \rightarrow +\infty$, while the past null infinity corresponds to $r \rightarrow -\infty$. At null infinity $r \rightarrow \pm\infty$, the boundary metric becomes degenerate

$$q_{ab} = 0 du^2 + dz^2, \quad (2.5)$$

which corresponds to the Carrollian structure at null infinity $\mathcal{I} \cong \mathbb{R} \times S^1$, where we will treat z as the decompactified coordinate for the celestial circle.

The bulk Poincaré symmetries are generated by vector fields

$$\eta = (\mathcal{T} + u\alpha)\partial_u + \mathcal{Y}\partial_z, \quad \alpha = \partial_z \mathcal{Y}, \quad (2.6)$$

where $\mathcal{T}(z) = 1, z, z^2$ are the three translations, and $\mathcal{Y}(z) = 1, z, z^2$ are the three Lorentz transformations. At the boundary, these correspond to the global part of Carrollian symmetries. A Carrollian primary operator $\Phi_\Delta(u, z)$ is defined as an operator that transforms as

$$\delta_{(\mathcal{T}, \mathcal{Y})} \Phi_\Delta = \left[(\mathcal{T} + u\partial_z \mathcal{Y})\partial_u + \mathcal{Y}\partial_z + \frac{\Delta}{2} \partial_z \mathcal{Y} \right] \Phi_\Delta, \quad (2.7)$$

$\Phi_\Delta(u, z)$ is a Carrollian primary of conformal dimension Δ .

Since our starting point will be amplitudes of massless particles, we choose a convenient parametrization for generic null momenta,

$$p_i^\mu = \epsilon_i \omega_i q_i^\mu, \quad (2.8)$$

where $\epsilon_i = \pm 1$ corresponds to outgoing or incoming particles and ω_i is the light-cone energy of the associated momentum. Adapting the conventions in [43], the space-time signature we use in this section is $(-, +, +)$. The Lorentzian invariant inner product between two momenta is written as

$$p_i \cdot p_j = -2\epsilon_i \epsilon_j \omega_i \omega_j z_{ij}^2, \quad (2.9)$$

where $z_{ij} = z_i - z_j$. Under the Lorentz symmetry $SL(2, R)$,

$$z \rightarrow \frac{az + b}{cz + d} (ad - bc = 1), \quad (2.10)$$

the light-cone energy transforms as

$$\omega \rightarrow (cz + d)^2 \omega. \quad (2.11)$$

Starting from the scattering amplitudes $\mathcal{A}_n(\{\omega_1, z_1\}_{\epsilon_1}, \dots, \{\omega_n, z_n\}_{\epsilon_n})$ in the momentum basis, the Carrollian amplitudes depending on the coordinates u_i and z_i at null infinity are obtained by performing Fourier transforms with respect to the light-cone energies

$$\begin{aligned} C_n(\{u_1, z_1\}_{\epsilon_1}, \dots, \{u_n, z_n\}_{\epsilon_n}) \\ = \prod_{i=1}^n \left(\int_0^{+\infty} \frac{d\omega_i}{2\pi} e^{i\epsilon_i \omega_i u_i} \right) \mathcal{A}_n(\{\omega_1, z_1\}_{\epsilon_1}, \dots, \{\omega_n, z_n\}_{\epsilon_n}), \end{aligned} \quad (2.12)$$

where we neglected the labels of helicity as we mainly focus the cases where all the external particles are scalar.

These Carrollian amplitudes can be reinterpreted as correlators in a Carrollian 2D CFT at null infinity [4, 19],

$$\begin{aligned} C_n(\{u_1, z_1\}_{\epsilon_1}, \dots, \{u_n, z_n\}_{\epsilon_n}) \\ = \langle \Phi_{\Delta_1, \epsilon_1}(u_1, z_1) \dots \Phi_{\Delta_n, \epsilon_n}(u_n, z_n) \rangle, \end{aligned} \quad (2.13)$$

where $\Phi_{\Delta_i, \epsilon_i}(u_i, z_i)$ are conformal Carrollian primaries with conformal dimensions $\Delta_i = 1$.

Generically, the ∂_u descendants of conformal Carrollian primaries are also primaries. Each ∂_u derivative increases the conformal dimension of the operator by 1. These primaries will turn out to appear naturally when taking a flat space limit of AdS Witten diagrams, while also having better convergence properties. One can obtain correlators of ∂_u descendants simply by taking u derivatives of the correlator (2.13). Adopting the notations of [19], we have,

$$\begin{aligned} C_n^{m_1 \dots m_n}(\{u_1, z_1\}_{\epsilon_1}, \dots, \{u_n, z_n\}_{\epsilon_n}) &= \partial_{u_1}^{m_1} \dots \partial_{u_n}^{m_n} C_n(\{u_1, z_1\}_{\epsilon_1}, \dots, \{u_n, z_n\}_{\epsilon_n}) \\ &= \prod_{i=1}^n \left(\int_0^{+\infty} \frac{d\omega_i}{2\pi} (i\epsilon_i \omega_i)^{m_i} e^{i\epsilon_i \omega_i u_i} \right) \mathcal{A}_n(\{\omega_1, z_1\}_{\epsilon_1}, \dots, \{\omega_n, z_n\}_{\epsilon_n}). \end{aligned} \quad (2.14)$$

For later use, we introduce a shorthand notation for $C_n^{1 \dots 1}$,

$$\begin{aligned} \tilde{C}_n(\{u_1, z_1\}_{\epsilon_1}, \dots, \{u_n, z_n\}_{\epsilon_n}) \\ = C_n^{1 \dots 1}(\{u_1, z_1\}_{\epsilon_1}, \dots, \{u_n, z_n\}_{\epsilon_n}). \end{aligned} \quad (2.15)$$

In the next subsections we will introduce examples of four-point scalar amplitudes at tree level, which will be the main focus of this work. The two- and three-point cases are similar to those discussed in [7].

A. Contact diagram

The four-point celestial amplitudes involving massless scalars in three dimensions have been studied in [43]. Here,

we adapt a similar convention of kinematics variables. The scattering amplitudes $\mathcal{A}_4(\{\omega_1, z_1\}_{\epsilon_1}, \dots, \{\omega_n, z_n\}_{\epsilon_n})$ contain momentum-conserving δ functions

$$\mathcal{A}_4(\{\omega_1, z_1\}_{\epsilon_1}, \dots, \{\omega_4, z_4\}_{\epsilon_4}) = \mathcal{T}(s, t) \delta^{(3)} \left(\sum_{i=1}^4 \epsilon_i \omega_i q_i^\mu \right), \quad (2.16)$$

where s , t , and u are Mandelstam variables defined as $s = -(p_1 + p_2)^2$, $t = -(p_1 + p_3)^2$, $u = -(p_1 + p_4)^2$. Using the parametrization shown in (2.2), the δ functions can be written as

$$\delta^{(3)} \left(\sum_{i=1}^4 \epsilon_i \omega_i q_i^\mu(z_i) \right) = \frac{1}{4|z_{23}z_{24}z_{34}|} \delta \left(\omega_2 + \epsilon_1 \epsilon_2 \frac{z_{13}z_{14}}{z_{23}z_{24}} \omega_1 \right) \delta \left(\omega_3 - \epsilon_1 \epsilon_3 \frac{z_{12}z_{14}}{z_{23}z_{34}} \omega_1 \right) \delta \left(\omega_4 + \epsilon_1 \epsilon_4 \frac{z_{12}z_{13}}{z_{24}z_{34}} \omega_1 \right), \quad (2.17)$$

where $z_{ij} = z_i - z_j$. On the support of the δ functions, the Mandelstam variables are

$$s = -4 \frac{z_{12}^2 z_{13} z_{14}}{z_{23} z_{24}} \omega_1^2, \quad t = -\frac{1}{z} s, \quad u = \frac{1-z}{z} s, \quad (2.18)$$

where the conformal invariant cross ratio is

$$z = \frac{z_{12} z_{34}}{z_{13} z_{24}}. \quad (2.19)$$

For a 2-to-2 scattering process, one needs to specify the incoming and outgoing configurations, which correspond to the following constraints on the cross ratio,

$$\begin{aligned}
& \text{(a) } 12 \rightleftharpoons 34, \quad z > 1, \\
& \text{(b) } 13 \rightleftharpoons 24, \quad 0 < z < 1, \\
& \text{(c) } 14 \rightleftharpoons 23, \quad z < 0.
\end{aligned} \tag{2.20}$$

We shall focus on the case where particles 1 and 2 are incoming, while particles 3 and 4 are outgoing. The scattering angle θ is related to the cross ratio as $z^{-1} = \sin^2(\theta/2) < 1$. The other scattering channels can be analyzed in a similar

way, see, e.g., [43] for a discussion on the associated celestial amplitudes.

We begin with the simplest case, which is the contact diagram of four massless scalars,

$$\mathcal{T}(s, t)_{\text{contact}} = \kappa_4, \tag{2.21}$$

where κ_4 is the coupling constant for a ϕ^4 interaction. Using the definition in (2.12), the Carrollian amplitude is,

$$\begin{aligned}
C_{4,\text{contact}} &= \frac{\kappa_4}{(2\pi)^4} \frac{1}{4|z_{23}z_{24}z_{34}|} \int_0^\infty d\omega_1 \exp\left(-i\omega_1 \left[u_1 - u_2 \frac{z_{13}z_{14}}{z_{23}z_{24}} + u_3 \frac{z_{12}z_{14}}{z_{23}z_{34}} - u_4 \frac{z_{12}z_{13}}{z_{24}z_{34}} \right] \right) \\
&= \frac{-i\kappa_4 \text{sign}(z_{23}z_{24}z_{34})}{(2\pi)^4} \frac{1}{4z_{23}z_{24}z_{34}} \frac{1}{u_1 - u_2 \frac{z_{13}z_{14}}{z_{23}z_{24}} + u_3 \frac{z_{12}z_{14}}{z_{23}z_{34}} - u_4 \frac{z_{12}z_{13}}{z_{24}z_{34}}}.
\end{aligned} \tag{2.22}$$

The four-point Carrollian amplitude (2.22) is time dependent, corresponding to the electric branch of Carrollian CFT. In the literature of Carrollian theories, there are two distinct branches of solutions corresponding to two different types of Carroll invariant actions, which are referred to as the timelike (electric) and spacelike (magnetic) theories [11,12,60–62]. It has been shown that in $(1+3)$ dimensions, the Carrollian amplitudes associated with scattering configurations belong to the electric branch [4,19]. Our findings here confirm that this is also the case for $(1+2)$ dimensions.

One can check that the Carrollian amplitude (2.22) satisfies the Ward identities generated by $M_n = z^{n+1} \partial_u$ with $n = -1, 0, 1$. These corresponds to the global part of the BMS₃ supertranslation symmetry. Compared to the four-point Carrollian amplitudes in [7], (2.22) is nondistributional as the bulk translation symmetry is less constraining in three dimensions.

In Sec. IV, we will show that the Carrollian amplitude (2.22) can be reproduced by taking the Carrollian limit

$c \rightarrow 0$ of the corresponding boundary correlator in AdS₃/CFT₂. We shall see that the analytic continuation to the Lorentzian signature is essential in this matching as shown in AdS₄/CFT₃ in [7]. The $c \rightarrow 0$ limit extracts the universal leading singularity upon the analytic continuation.

B. Exchange diagrams

In this section, we consider several examples of exchange diagrams. We begin with the simplest case of scalar exchange. For simplicity we only show the s -channel diagram

$$\mathcal{T}_{s,\text{scalar}} = \frac{\kappa_3^2}{s} = -\kappa_3^2 \frac{z_{23}z_{24}}{4z_{12}^2 z_{13}z_{14}\omega_1^2}, \tag{2.23}$$

where we expressed the s Mandelstam variable using (2.18) and κ_3 is the coupling constant for a ϕ^3 interaction. The Carrollian amplitude associated to scalar exchange is

$$C_{4,s,\text{scalar}} = -\frac{\kappa_3^2}{(4\pi)^4} \frac{1}{|z_{23}z_{24}z_{34}|} \frac{z_{23}z_{24}}{z_{12}^2 z_{13}z_{14}} \int_0^\infty \frac{d\omega_1}{\omega_1^2} \exp\left(-i\omega_1 \left[u_1 - u_2 \frac{z_{13}z_{14}}{z_{23}z_{24}} + u_3 \frac{z_{12}z_{14}}{z_{23}z_{34}} - u_4 \frac{z_{12}z_{13}}{z_{24}z_{34}} \right] \right), \tag{2.24}$$

which is IR divergent. One way to regularize it is to compute the ∂_u descendants of the Carrollian amplitude. We are interested in

$$\begin{aligned}
\tilde{C}_{4,s,\text{scalar}} &= \partial_{u_1} \partial_{u_2} \partial_{u_3} \partial_{u_4} C_{4,s,\text{scalar}} = -\frac{\kappa_3^2}{(4\pi)^4} \frac{\text{sign}(z_{23}z_{24}z_{34})}{z_{12}^2 z_{13}z_{14}z_{34}} \int_0^{+\infty} d\omega_1 \omega_1^2 \left(\frac{z_{13}z_{14}}{z_{23}z_{24}} \right) \left(\frac{z_{12}z_{14}}{z_{23}z_{34}} \right) \left(\frac{z_{12}z_{13}}{z_{24}z_{34}} \right) \\
&\quad \times \exp\left(-i\omega_1 \left[u_1 - u_2 \frac{z_{13}z_{14}}{z_{23}z_{24}} + u_3 \frac{z_{12}z_{14}}{z_{23}z_{34}} - u_4 \frac{z_{12}z_{13}}{z_{24}z_{34}} \right] \right) \\
&= -\frac{\kappa_3^2 \text{sign}(z_{23}z_{24}z_{34})}{(4\pi)^4} \frac{z_{13}z_{14}}{z_{23}^2 z_{24}^2 z_{34}^3} \frac{2i}{\left(u_1 - u_2 \frac{z_{13}z_{14}}{z_{23}z_{24}} + u_3 \frac{z_{12}z_{14}}{z_{23}z_{34}} - u_4 \frac{z_{12}z_{13}}{z_{24}z_{34}} \right)^3}.
\end{aligned} \tag{2.25}$$

As we mentioned before, one can check that the conformal dimensions for each operator in $\tilde{C}_4$ is $\Delta_i = 2$.

Scalar exchange is not the only option that can be studied. One can look at 3D Chern-Simons (CS) theory with scalar matter, where the exchanged field will be a gauge boson. The Feynman rules in Lorenz gauge can be found in [63]. The s channel gauge field exchange diagram contribution is

$$\begin{aligned} T_{s,CS} &= g^2 \frac{\epsilon^{\mu\nu\rho} (p_1 - p_2)_\mu (p_3 - p_4)_\nu (p_1 + p_2)_\rho}{2(p_1 + p_2)^2} \\ &= g^2 \frac{\epsilon^{\mu\nu\rho} p_{1,\mu} p_{2,\nu} p_{4,\rho}}{\langle 12 \rangle^2} \\ &= \frac{g^2 \langle 13 \rangle \langle 14 \rangle}{2 \langle 34 \rangle} = g^2 \omega_1 \frac{z_{13} z_{14}}{z_{34}}, \end{aligned} \quad (2.26)$$

where we used the following parametrization for the spinor-helicity variables in 3D $p_{a\beta} = \lambda_a \lambda_\beta$ with $\lambda_a = \sqrt{2\omega}(z, 1)$ and g is the coupling constant.

The Carrollian amplitude is,

$$\begin{aligned} C_{4,s,CS} &= -\frac{g^2 \text{sign}(z_{23} z_{24} z_{34})}{(2\pi)^4} \frac{z_{13} z_{14}}{4 z_{23} z_{24} z_{34}^2} \\ &\quad \times \frac{1}{\left(u_1 - u_2 \frac{z_{13} z_{14}}{z_{23} z_{24}} + u_3 \frac{z_{12} z_{14}}{z_{23} z_{34}} - u_4 \frac{z_{12} z_{13}}{z_{24} z_{34}}\right)^2}. \end{aligned} \quad (2.27)$$

One can also look at the exchange of a graviton minimally coupled to scalars. In three dimensions there is no dynamical degree of freedom for gravity. However, one can still compute the amplitude of external massless scalars with a nondynamical gravitational field being exchanged. One finds [37,64,65]

$$T_{s,GR} = \kappa^2 \frac{t^2 + st}{-s} = \kappa^2 \frac{\langle 13 \rangle^2 \langle 14 \rangle^2}{\langle 34 \rangle^2}, \quad (2.28)$$

where κ is the coupling constant. Interestingly, (2.28) is the double copy of (2.26). Using (2.18), it becomes

$$T_{s,GR} = 4\kappa^2 \omega_1^2 \frac{z_{13}^2 z_{14}^2}{z_{34}^2}. \quad (2.29)$$

Its Carrollian amplitude is then,

$$\begin{aligned} C_{4,s,GR} &= \frac{\kappa^2 \text{sign}(z_{23} z_{24} z_{34})}{(2\pi)^4} \frac{z_{13}^2 z_{14}^2}{z_{23} z_{24} z_{34}^3} \\ &\quad \times \frac{2i}{\left(u_1 - u_2 \frac{z_{13} z_{14}}{z_{23} z_{24}} + u_3 \frac{z_{12} z_{14}}{z_{23} z_{34}} - u_4 \frac{z_{12} z_{13}}{z_{24} z_{34}}\right)^3}. \end{aligned} \quad (2.30)$$

In Sec. IV, we will show that the above exchange amplitudes, (2.22), (2.25), (2.27), and (2.30), can be

reproduced from the Carrollian/flat space limit of their corresponding holographic correlators in $\text{AdS}_3/\text{CFT}_2$.

We also notice an interesting double copy relation among the gauge theory and gravity exchange Carrollian amplitudes. From (2.22), (2.27), and (2.30), we find

$$C_{4,s,GR} = C_{4,\text{contact}}^{-1} (C_{4,s,CS})^2, \quad (2.31)$$

provided the following identification of the coupling constants,

$$\kappa^2 = \frac{g^4}{8\kappa_4}. \quad (2.32)$$

The inverse of $C_{4,\text{contact}}$ serves as the KLT kernel.

III. FLAT LIMIT OF PROPAGATORS OF WITTEN DIAGRAM

In this section, we introduce the Bondi coordinates in AdS_3 and see how their flat space limit lands on the planar Bondi coordinates for $\mathbb{R}^{1,2}$. Geometrically, in these coordinates, the flat limit on AdS_3 corresponds to the Carrollian limit of the boundary. The building blocks of dynamical statements are the propagators appearing in Feynman or Witten diagrams. We shall see that after performing the flat space limit in Bondi coordinates, there is a natural identification between the two types of propagators.

A. Planar Bondi coordinates

The planar Bondi coordinates parametrizing flat space also exist in AdS . One can see them by expressing the AdS metric in embedding coordinates. For AdS_3 , they parametrize a hyperboloid in four-dimensional flat space $X^I = (X^+, X^-, X^0, X^1)$ with the metric

$$G_{IJ} dX^I dX^J = -dX^+ dX^- - (dX^0)^2 + (dX^1)^2. \quad (3.1)$$

AdS_3 is the hyperboloid defined by

$$X \cdot X = -X^+ X^- - (X^0)^2 + (X^1)^2 = -l^2. \quad (3.2)$$

In this section, we focus on *Lorentzian* AdS_3 , where there are two timelike directions in the embedding coordinates. The planar Bondi coordinates for AdS_3 are obtained as the specific parametrization,

$$X^I = r \left(1, \frac{u}{r} - \frac{u^2}{4l^2} + z^2, -\frac{l}{r} + \frac{u}{2l}, z \right). \quad (3.3)$$

One can check that the parametrization satisfies the definition for AdS_3 , $G_{IJ} X^I X^J = -l^2$ with l being the AdS radius.

Using (3.3), one finds the metric in the planar Bondi coordinates,

$$ds_{\text{AdS}_3}^2 = -\frac{r^2}{4l^2} du^2 - dudr + r^2 dz^2. \quad (3.4)$$

The conformal boundary is reached as $r \rightarrow \pm\infty$ with metric

$$ds_{\partial\text{AdS}_3}^2 = -\frac{1}{4l^2} du^2 + dz^2. \quad (3.5)$$

Both the bulk and boundary metrics reduce to those in the flat case as $l \rightarrow \infty$. In particular, one can identify the speed of light on the boundary c as $1/l$. The boundary metric becomes degenerate as $l \rightarrow \infty$, which corresponds to the Carrollian limit $c \rightarrow 0$ limit on the boundary.

The chordal distance defined by the embedding coordinates can be expressed in terms of the planar Bondi coordinates,

$$\begin{aligned} \xi_{12}^{\text{AdS}} &= (X_1 - X_2) \cdot (X_1 - X_2) \\ &= -\frac{r_1 r_2}{4l^2} u_{12}^2 - u_{12} r_{12} + r_1 r_2 z_{12}^2. \end{aligned} \quad (3.6)$$

In the flat space limit $l \rightarrow \infty$ it reduces to the desired invariant distance $\xi_{12}^{\text{AdS}} \rightarrow \xi_{12}^{\text{Flat}}$.

We can also consider Euclidean AdS_3 , which amounts to changing the sign in front of $(dX^0)^2$ in (3.1).

B. Propagators of Witten diagrams

The basic building blocks of boundary correlators in AdS are bulk-to-bulk and bulk-to-boundary propagators. Similarly, the building blocks of flat space amplitudes are Feynman propagators. One crucial first step in identifying the flat limit of AdS boundary correlators is matching the flat limit of bulk-to-bulk or bulk-to-boundary to Feynman propagators in position space. In this section we will show that this matching holds in AdS_3 .

We start from the scalar bulk-to-bulk propagator, which is a solution to the equation of motion,

$$(\square_1 + m^2) G_{BB}^{\text{AdS},\Delta}(X_1, X_2) = \frac{1}{\sqrt{-g}} \delta^{(4)}(X_{12}), \quad (3.7)$$

where $m^2 l^2 = \Delta(\Delta - 2)$ and the d'Alembertian in Bondi coordinates is,

$$\square = \left[\frac{1}{r^2} \partial_z^2 - 4\partial_u \partial_r - \frac{2}{r} \partial_u + \frac{r}{l^2} (3\partial_r + r\partial_r^2) \right]. \quad (3.8)$$

The bulk-to-bulk propagator will be a function of the chordal distance ξ_{12}^{AdS} and we perform a change of variables to the dimensionless variable,

$$\chi_{12} = -\frac{4l^2}{\xi_{12}^{\text{AdS}}}. \quad (3.9)$$

The solution to the wave equation then reads [38,66],

$$G_{BB}^{\text{AdS},\Delta}(X_1, X_2) = C(\Delta) (\chi_{12})^{\Delta} {}_2F_1\left(\Delta, \Delta - \frac{1}{2}, 2\Delta - 1; \chi_{12}\right), \quad (3.10)$$

with

$$C(\Delta) = -\frac{i(-1)^\Delta}{2\pi \times 4^\Delta l}, \quad (3.11)$$

where the $-i$ difference compared to [38,66] is due to Wick rotation from EAdS_3 and a different convention for the Green's function.

We are interested now in the flat space limit. This is obtained by taking $l \rightarrow \infty$ and thus $\chi_{12} \rightarrow \infty$. Note that for our purposes, Δ does not rescale with l as we take the flat space limit. Therefore, all scalars become massless regardless of their conformal dimension. The dominant term in the expansion of the hypergeometric function near infinity is given by,

$$\frac{1}{(\chi_{12})^{\Delta-\frac{1}{2}}} \frac{i(-1)^\Delta \sqrt{\pi} \Gamma(2\Delta - 1)}{\Gamma(\Delta) \Gamma\left(\Delta - \frac{1}{2}\right)}. \quad (3.12)$$

So in the $\chi_{12} \rightarrow \infty$ limit, the propagator (3.10) becomes,

$$G_{BB}^{\text{AdS},\Delta}(X_1, X_2) \xrightarrow{\chi_{12} \rightarrow \infty} \frac{1}{8\pi} \frac{\chi_{12}^{\frac{1}{2}}}{l} = \frac{1}{4\pi} \frac{1}{\sqrt{-\xi_{12}^{\text{AdS}} - i\epsilon}} = \frac{1}{4\pi} \frac{1}{\sqrt{\frac{r_1 r_2}{4l^2} u_{12}^2 + u_{12} r_{12} - r_1 r_2 z_{12}^2 - i\epsilon}}, \quad (3.13)$$

where the analytic continuation to the Lorentzian signature is implemented by $\xi_{12}^{\text{AdS}} \rightarrow \xi_{12}^{\text{AdS}} + i\epsilon$ (see, e.g., [67]). Taking $l \rightarrow \infty$ in the last line leads to

$$G_{BB}^{\text{AdS},\Delta}(X_1, X_2) \xrightarrow{l \rightarrow \infty, \text{Bondi}} \frac{1}{4\pi} \frac{1}{\sqrt{u_{12} r_{12} - r_1 r_2 z_{12}^2 - i\epsilon}}, \quad (3.14)$$

which agrees with the flat space Feynman propagator for massless scalar in position space [68].

We conclude that in Bondi coordinates, the AdS_3 massive scalar bulk-to-bulk propagator reduces to the massless Feynman propagator in the limit $l \rightarrow \infty$. In particular, the parameter Δ completely disappears. This property is the same as that in [7] for the AdS_4 case.

The bulk-to-boundary propagator in AdS can be obtained from the bulk-to-bulk propagator by sending one of the bulk points to the boundary. This means that $\xi_{12}^{\text{AdS}} \rightarrow \infty$ and $\chi_{12} \rightarrow 0$. Focusing on the first field being sent on the boundary, we see that the propagator (3.10) becomes,

$$G_{BB}^{\text{AdS},\Delta}(X_1, X_2)^{\chi_{12} \rightarrow 0} C(\Delta) (\chi_{12})^\Delta = C(\Delta) \left(\frac{l}{r_1} \right)^\Delta \left(\frac{-4l}{-\frac{r_2}{4l^2} u_{12}^2 - u_{12} + r_2 z_{12}^2 + i\epsilon} \right)^\Delta. \quad (3.15)$$

As we discussed in this section, the future null infinity (outgoing) corresponds to sending $r_1 \rightarrow +\infty$, while the past null infinity (incoming) corresponds to $r_1 \rightarrow -\infty$. The bulk-to-boundary propagator is taken for the $r_1 \rightarrow +\infty$ limit of the above,

$$G_{\partial B}^{\text{AdS},\Delta}(x, X) = \lim_{r_1 \rightarrow \infty} l^{1/2} \left(\frac{r_1}{l} \right)^\Delta G_{BB}^{\text{AdS},\Delta}(X_1, X_2) = l^{1/2} C(\Delta) \left(\frac{-4l}{-\frac{r_2}{4l^2} u_{xX}^2 - u_x - q_x \cdot X + i\epsilon} \right)^\Delta. \quad (3.16)$$

In the flat limit, this reduces to,

$$G_{\partial B}^{\text{AdS},\Delta}(x, X)^{l \rightarrow \infty, \text{Bondi}} \rightarrow \frac{-\alpha(\Delta)}{2\pi} \frac{\Gamma(\Delta)}{(-u - q \cdot X + i\epsilon)^\Delta}, \quad (3.17)$$

with

$$\alpha(\Delta) = -(-1)^{\Delta} 4^{\Delta} 2\pi \frac{C(\Delta)}{\Gamma(\Delta)} l^{\Delta+\frac{1}{2}} = \frac{i l^{\Delta-1/2}}{\Gamma(\Delta)}. \quad (3.18)$$

For the case $\Delta = 1/2$, the expression becomes the flat space Feynman propagator for massless scalars in position space [68], where one of the operators is sent to null infinity. In Secs. II and IV, we will mostly consider the case where $\Delta = 1$ for simplicity. The other integer values of Δ do not provide any new data on flat space as they can be obtained by ∂_u derivatives on the Carrollian primaries. This feature can also be found in [7] for the AdS4 case. The bulk-to-boundary propagator of incoming particles can be obtained in a similar way.

We have seen how the building blocks of Witten diagrams reduce to those of Feynman diagrams for the case of scalars. The spinning propagators can be found in [69]. One can perform a similar analysis to what we have in this section and reach the same conclusion.

A comment is in order. The flat limit of the bulk to boundary propagators has been considered before, see, e.g., [70]. In this section we performed a similar check using

Bondi coordinates, along the lines of [7]. One difference is that in Bondi coordinates, we do not need to perform an analytic continuation from Euclidean to Lorentzian in the propagator.

In the next section, we show how the Carrollian limit can be taken at the level of the holographic boundary correlators and its relation to the bulk point singularity.

IV. AdS₃ WITTEN DIAGRAMS, BULK POINT SINGULARITY, AND CARROLLIAN LIMIT

In the previous section, we saw how the various ingredients contributing to Witten diagrams from a bulk perspective behave under the flat space limit. From a boundary perspective, in [7], it was shown that the Carrollian limit of the AdS₄ boundary CFT correlators reproduces the associated Carrollian amplitudes, which is equivalent to saying that the boundary Carrollian limit corresponds to the bulk flat limit.² From a bulk perspective, the flat space amplitude can be recovered from the residue of the bulk-point singularity signaling the locality of the bulk interactions. To see the appearance of the bulk-point singularity on the boundary correlators, a key step in the derivation is performing an analytic continuation from the Euclidean signature of the usual Witten diagrams to the Lorentzian signature. The boundary Carrollian limit of the Lorentzian correlators focuses on and extracts the bulk point singularity contribution. In this section, we apply the same prescription to 2D holographic correlators in AdS₃/CFT₂, and show how to reproduce the four-point Carrollian amplitudes obtained in Sec. II. As we will see, there is no restriction on the boundary operator insertions *a priori*.

In this section, Euclidean correlators will be denoted by $\langle \dots \rangle_E$. Lorentzian correlators will not have any subscripts. We will use $x = (ct, z)$ as Euclidean coordinates with metric $ds_E^2 = c^2 dt^2 + dz^2$. The Euclidean time will be analytically continued to the Lorentzian time u .

A. Contact diagram

We start by considering a ϕ^4 interaction in AdS₃ with coupling κ_4 , the four-point boundary correlators are given by

$$\begin{aligned} & \langle O_{\Delta_1}(x_1) O_{\Delta_2}(x_2) O_{\Delta_3}(x_3) O_{\Delta_4}(x_4) \rangle_E^{\text{contact}} \\ &= \kappa_4 \int_{\text{AdS}_3} d^3 X \prod_{i=1}^4 G_{\partial B}^{\Delta_i}(x_i, X). \end{aligned} \quad (4.1)$$

²The main difference between the prescription in [7] and the traditional flat space limit of AdS (see, e.g., [37]) is that CFT correlators with fixed conformal dimensions are not localized on a strip in [7]. One might show that the prescription in [7] is not related to bulk excitations of finite energy. This would be natural from the Carrollian amplitudes perspective as we have seen in Sec. II, as all energies ω_i are being integrated.

The integral expression defines D functions (see, e.g., [71] for a review on Witten diagrams),

$$\begin{aligned} & \langle O_{\Delta_1}(x_1) O_{\Delta_2}(x_2) O_{\Delta_3}(x_3) O_{\Delta_4}(x_4) \rangle_E^{\text{contact}} \\ &= \kappa_4 D_{\Delta_1, \Delta_2, \Delta_3, \Delta_4}(x_i) \\ &= \kappa_4 \mathcal{N}_4 \frac{(x_{14}^2)^{\frac{1}{2}\Sigma_\Delta - \Delta_1 - \Delta_4} (x_{34}^2)^{\frac{1}{2}\Sigma_\Delta - \Delta_3 - \Delta_4}}{(x_{13}^2)^{\frac{1}{2}\Sigma_\Delta - \Delta_4} (x_{24}^2)^{\Delta_2}} \bar{D}_{\Delta_1, \Delta_2, \Delta_3, \Delta_4}(U, V), \end{aligned} \quad (4.2)$$

where

$$\mathcal{N}_4 = \frac{\pi}{2} \Gamma\left(\frac{1}{2}\Sigma_\Delta - 1\right) \prod_{i=1}^4 \frac{1}{\Gamma(\Delta_i)}, \quad \Sigma_\Delta = \sum_{i=1}^4 \Delta_i, \quad (4.3)$$

$$U = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \quad V = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2}, \quad (4.4)$$

where the normalization factors for bulk-to-boundary propagators are omitted [71].

We will consider the simplest, $\Delta_i = 1$ case. The D function can be expressed in terms of elementary functions [72,73]

$$\begin{aligned} & \langle O_1(x_1) O_1(x_2) O_1(x_3) O_1(x_4) \rangle_E^{\text{contact}} \\ &= \frac{\kappa_4 \pi}{2} \frac{1}{x_{13}^2 x_{24}^2} \bar{D}_{1,1,1,1}(U, V), \end{aligned} \quad (4.5)$$

with

$$\log(1 - Z) \rightarrow \log(1 - Z) + 2\pi i, \quad \text{Li}_2(Z) \rightarrow \text{Li}_2(Z) - 2\pi i \log(Z), \quad (4.9)$$

$$\log(\bar{Z}) \rightarrow \log(\bar{Z}) - 2\pi i, \quad \log(1 - \bar{Z}) \rightarrow \log(1 - \bar{Z}) - 2\pi i, \quad \text{Li}_2(\bar{Z}) \rightarrow \text{Li}_2(\bar{Z}) + 2\pi i \log(\bar{Z}) + 2\pi^2. \quad (4.10)$$

Therefore, after the analytic continuation (4.8) the elementary function $\bar{D}_{1,1,1,1}$ receives an additional contribution

$$\bar{D}_{1,1,1,1} \rightarrow \bar{D}_{1,1,1,1} + \frac{4\pi^2}{Z - \bar{Z}} + \frac{2\pi i}{Z - \bar{Z}} \log\left(\frac{1 - \bar{Z}}{1 - Z}\right). \quad (4.11)$$

The important feature of the new terms is that they are singular in the $Z \rightarrow \bar{Z}$ limit. This is relevant as we will see that on a kinematic basis, taking the boundary Carrollian limit is equivalent to the same $Z \rightarrow \bar{Z}$ limit. Without the singular terms, the Carrollian limit would be vanishing. We observe that the second term is subleading in the $Z \rightarrow \bar{Z}$

$$\begin{aligned} \bar{D}_{1,1,1,1}(U, V) &= \frac{1}{Z - \bar{Z}} \left[2\text{Li}_2(Z) - 2\text{Li}_2(\bar{Z}) \right. \\ &\quad \left. + \log(Z\bar{Z}) \log\left(\frac{1 - Z}{1 - \bar{Z}}\right) \right]. \end{aligned} \quad (4.6)$$

We alternatively expressed the U, V variables in terms of the usual boundary cross ratios

$$U = Z\bar{Z}, \quad V = (1 - Z)(1 - \bar{Z}). \quad (4.7)$$

As written, the CFT correlator does not display any bulk-point singularity as it has been derived in an Euclidean regime. In order to connect to have a well-behaved flat space limit, we will need to analytically continue to the Lorentzian regime. In the Euclidean signature, the cross ratios Z and $\bar{Z}$ are complex conjugated to each other. In the Lorentzian signature, they become independent. To perform the analytic continuation from Euclidean to Lorentzian, one starts from an Euclidean correlator and, depending on the time ordering between the four operators in the Lorentzian spacetime, one chooses a path for the continuation. Recall that the scattering configuration considered in Sec. II is $12 \rightarrow 34$, which means that we shall place operators 3 and 4 in the future of operators 1 and 2. This corresponds to the following analytic continuation [43],

$$12 \rightarrow 34: \quad Z - 1 \rightarrow e^{2\pi i}(Z - 1), \quad \frac{1}{\bar{Z}} \rightarrow e^{2\pi i} \frac{1}{\bar{Z}}. \quad (4.8)$$

Monodromies of the polylogarithms around all branch points are well known (see, e.g., [7,74]). One finds

limit, but the first generates the leading singularity of the contact diagram,

$$\hat{\Phi}_{1111}^{ls} = \frac{4\pi^2}{Z - \bar{Z}}. \quad (4.12)$$

The leading singularity corresponds to the bulk point singularity discussed in [37,75]. Notice that depending on the path of analytic continuation, the final result might be different, see, e.g., [7,37]. However, when the leading singularity exists, it is independent of the path.

To connect Witten diagrams to flat-space Carrollian correlators, we note that in the Carrollian limit,

$$(Z - \bar{Z})^2 = ((1 + U - V)^2 - 4U) \xrightarrow{c \rightarrow 0} \frac{4c^2}{z_{13}^4 z_{24}^4} [u_4 z_{12} z_{13} z_{23} - u_3 z_{12} z_{14} z_{24} + u_2 z_{13} z_{14} z_{34} - u_1 z_{23} z_{24} z_{34}]^2 + \mathcal{O}(c^4), \quad (4.13)$$

where we used $x_{ij}^2 = -c^2(u_i - u_j)^2 + (z_i - z_j)^2$. In contrast to the Carrollian limit of the cross-ratio difference in four dimensions [7], there is no constant term as $c \rightarrow 0$ and the $Z - \bar{Z}$ difference vanishes. Taking the Carrollian limit $c \rightarrow 0$ on the boundary leads to the limit $Z \rightarrow \bar{Z}$ in (4.13). As $c \rightarrow 0$, the contributing term will thus be the leading singularity discussed above. We then find that after analytic continuation to the Lorentzian signature, in the Carrollian limit $c \rightarrow 0$, the contact Witten diagram (4.5) becomes

$$\langle O_1(x_1) O_1(x_2) O_1(x_3) O_1(x_4) \rangle^{\text{contact}} = \kappa_4 \frac{\pi^3}{c |z_{23} z_{24} z_{34}|} \frac{1}{u_1 - u_2 \frac{z_{13} z_{14}}{z_{23} z_{24}} + u_3 \frac{z_{12} z_{14}}{z_{23} z_{34}} - u_4 \frac{z_{12} z_{13}}{z_{24} z_{34}}}, \quad (4.14)$$

which matches the Carrollian amplitude of the contact diagram in (2.22) up to a constant factor.

A comment is in order regarding the choice of the conformal dimensions of the primary operators. It was shown that from the irreducible representations of the Poincaré group at null infinity, the natural scaling dimension would be $\Delta = \frac{1}{2}$ for AdS_3 in the flat limit [16]. One can compute the Carrollian amplitude with $\Delta = \frac{1}{2}$ with its ∂_u descendants and match it to the corresponding Carrollian limit of the AdS_3 boundary correlator. The main difficulty is that there is no known expression for the corresponding AdS_3 boundary correlator in this case. However, the leading singularity of interest might be extracted following Appendix B in [7]. We leave a detailed analysis for future work.

Next, we shall follow the same prescription to compute several examples of the Carrollian limit of boundary correlators in $\text{AdS}_3/\text{CFT}_2$ with integer dimensions and we will always see the matching with the associated Carrollian amplitudes.

B. Exchange diagrams

The four-point correlator associated with a scalar exchange in the bulk is

$$\langle O_{\Delta_1}(x_1) O_{\Delta_2}(x_2) O_{\Delta_3}(x_3) O_{\Delta_4}(x_4) \rangle_E^{\text{scalar exchange}} = I_s + I_t + I_u, \quad (4.15)$$

where I_s corresponds to the s -channel exchange diagram with the exchange operator having conformal dimension Δ ,

$$I_s = \kappa_3^2 \int_{\text{AdS}_3} d^3 X d^3 Y \prod_{i=1}^2 G_{\partial B}^{\Delta_i}(x_i, X) \prod_{i=3}^4 G_{\partial B}^{\Delta_i}(x_i, Y) G_{BB}^{\Delta}(X, Y), \quad (4.16)$$

and κ_3 is the coupling for the ϕ^3 interaction. I_t and I_u can be obtained by the replacements $1 \leftrightarrow 3$ and $1 \leftrightarrow 4$, respectively. Exchange diagrams cannot be written as elementary functions for generic conformal dimensions. However, when $\Delta_3 + \Delta_4 - \Delta \in 2\mathbb{Z}^+$, they can be written as a finite sum of contact diagrams,

$$\begin{aligned} I_s &= \kappa_3^2 \sum_{k=k_{\min}}^{k_{\max}} a_k (x_{34}^2)^{k-\Delta_4} D_{\Delta_1, \Delta_2, \Delta_3-\Delta_4+k, k} \\ &= \frac{(x_{14}^2)^{\frac{1}{2}\Sigma_{\Delta}-\Delta_1-\Delta_4} (x_{34}^2)^{\frac{1}{2}\Sigma_{\Delta}-\Delta_3-\Delta_4}}{(x_{13}^2)^{\frac{1}{2}\Sigma_{\Delta}-\Delta_4} (x_{24}^2)^{\Delta_2}} \\ &\quad \times \sum_{k=k_{\min}}^{k_{\max}} a_k \mathcal{N}'_4 \bar{D}_{\Delta_1, \Delta_2, \Delta_3-\Delta_4+k, k}, \end{aligned} \quad (4.17)$$

where $k_{\min} = \frac{1}{2}(\Delta - \Delta_3 + \Delta_4)$, $k_{\max} = \Delta_4 - 1$. The coefficients a_k can be computed recursively. See [71] for explicit expressions for a_k and $\mathcal{N}'_4$.

In order to match the Carrollian correlator (2.25), we are interested in the case $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = \Delta = 2$ for which we have

$$I_s = \frac{\kappa_3^2}{(x_{13}^2)^2 (x_{24}^2)^2} \bar{D}_{2,2,1,1}, \quad (4.18)$$

where we ignored an overall normalization constant involving π . Following the same analytic continuation as we had in (4.8), the exchange diagram $\bar{D}_{2211}$ develops a leading singularity

$$\bar{D}_{2,2,1,1} \rightarrow \bar{D}_{2,2,1,1} + \hat{\Phi}_{2211}^{Is} + \dots, \quad (4.19)$$

where the leading singularity $\hat{\Phi}_{2211}^{Is}$ can be obtained from $\hat{\Phi}_{1111}^{Is}$ by taking a derivative [7],

$$\hat{\Phi}_{2211}^{Is} = -\frac{2(1-Z)}{Z-\bar{Z}} \partial_{\bar{Z}} \hat{\Phi}_{1111}^{Is} = -8\pi^2 \frac{(1-Z)}{(Z-\bar{Z})^3}. \quad (4.20)$$

In the Carrollian limit $c \rightarrow 0$, we have $(1-Z) \rightarrow \frac{z_{23} z_{14}}{z_{13} z_{24}}$, and $Z - \bar{Z}$ is given by (4.13). Hence, the resulting correlator is

$$\begin{aligned} \lim_{c \rightarrow 0} I_s &\sim \frac{\kappa_3^2}{z_{13}^4 z_{24}^4} \frac{z_{23} z_{14}}{z_{13} z_{24}} \frac{z_{13}^6 z_{24}^6}{c^3 (z_{23} z_{24} z_{34})^3} \left(u_1 - u_2 \frac{z_{13} z_{14}}{z_{23} z_{24}} + u_3 \frac{z_{12} z_{14}}{z_{23} z_{34}} - u_4 \frac{z_{12} z_{13}}{z_{24} z_{34}} \right)^3 \\ &= \frac{\kappa_3^2}{c^3} \frac{z_{13} z_{14}}{z_{23}^2 z_{24}^2 z_{34}^3} \frac{1}{\left(u_1 - u_2 \frac{z_{13} z_{14}}{z_{23} z_{24}} + u_3 \frac{z_{12} z_{14}}{z_{23} z_{34}} - u_4 \frac{z_{12} z_{13}}{z_{24} z_{34}} \right)^3}, \end{aligned} \quad (4.21)$$

which matches the associated Carrollian amplitude (2.25) up to an overall constant. For the t channel and u channel exchange diagram, one can simply perform permutations.

We consider next the Chern-Simons theory with scalar matter in AdS_3 . The massless Chern-Simons exchange diagram was computed in [76]. For $\Delta_i = 1$ the s channel contribution is

$$\begin{aligned} \langle O_1(x_1) O_1(x_2) O_1(x_3) O_1(x_4) \rangle_E^{\text{CS}} \\ = \frac{1}{x_{12}^2 x_{34}^2} ((Z - \bar{Z}) U \bar{D}_{2,2,1,1} + \log(V)). \end{aligned} \quad (4.22)$$

Upon analytic continuation to the Lorentzian signature, the leading singularity comes from $\bar{D}_{2,2,1,1}$ that can be found in (4.19) and (4.20). The Carrollian limit of this exchange diagram (4.22) gives us

$$I_{s,\text{CS}} \sim \frac{1}{c^2} \frac{z_{13} z_{14}}{z_{23} z_{24} z_{34}^2} \frac{1}{\left(u_1 - u_2 \frac{z_{13} z_{14}}{z_{23} z_{24}} + u_3 \frac{z_{12} z_{14}}{z_{23} z_{34}} - u_4 \frac{z_{12} z_{13}}{z_{24} z_{34}} \right)^2}, \quad (4.23)$$

which agrees with the corresponding Carrollian amplitude (2.27).

Finally, the exchange Witten diagram of gravitons in AdS_3 can be found in [76]. For $\Delta_i = 1$ in the s channel,

$$\begin{aligned} \langle O_1(x_1) O_1(x_2) O_1(x_3) O_1(x_4) \rangle_E^{\text{GR}} \\ = \frac{1}{x_{12}^2 x_{34}^2} (2 + U(U - V - 1) \bar{D}_{2,2,1,1}). \end{aligned} \quad (4.24)$$

Once again the leading singularity comes from $\bar{D}_{2,2,1,1}$. We obtain the following Carrollian limit of the Lorentzian correlator,

$$I_{s,\text{GR}} \sim \frac{1}{c^3} \frac{z_{13}^2 z_{14}^2}{z_{23} z_{24} z_{34}^3} \frac{1}{\left(u_1 - u_2 \frac{z_{13} z_{14}}{z_{23} z_{24}} + u_3 \frac{z_{12} z_{14}}{z_{23} z_{34}} - u_4 \frac{z_{12} z_{13}}{z_{24} z_{34}} \right)^3}, \quad (4.25)$$

which matches Carrollian amplitude (2.30).

The same procedure can be repeated for higher conformal dimension Δ_i and the results will match the ∂_u descendants of the associated Carrollian amplitude.

C. Two-point correlators in AdS shockwave backgrounds

In the previous sections we focused on tree level correlators on pure AdS_3 backgrounds. However, it is known that for certain kinematic regimes, quantum effects can be resummed to all orders in the relevant coupling constant. Initially studied in flat space, this is referred to as the eikonal regime, see, e.g., [77] for a comprehensive review. One of the famous results shown by 't Hooft is that the 2-to-2 eikonal amplitude can be produced by the two-point amplitude in a shockwave background [78] where one of the particles can be viewed as the source of the shockwave. Similarly, in AdS, properties of the eikonal amplitude can be derived by studying the two-point function of a scalar in a shockwave background [79]. In this section we will analyze the behavior of this background two-point scalar correlator under the Carrollian limit. The expression for the two-point correlator in the AdS_{d+1} shockwave backgrounds has the following integral representation,

$$\mathcal{E} = \int_0^\infty s^{2\Delta_1-1} ds \int_{H_{d-1}} \widetilde{dx} e^{2iq \cdot (sx) + (is)^{j-1} h(x)}, \quad (4.26)$$

where we omit the overall normalization constant that can be found in [79]. H_{d-1} is the transverse hyperbolic space determined by the shock, $h(x)$ is the metric deformation under the presence of the shock and it depends on Newton's constant G , and j is spin of the exchange particle. See [79] for full details.

The expansion of the two-point correlator in Newton's constant G takes the form

$$\mathcal{E} = \mathcal{E}_0 + \mathcal{E}_1 + \dots, \quad (4.27)$$

where $\mathcal{E}_1$ is the first nontrivial term that linearly depends on G . Focusing on AdS_3 , it was shown in [79], for the case of $\Delta_1 = \Delta_2 = 2$ and $j = 0$, that one can evaluate all the integrals over H_1 to obtain a closed form expression for $\mathcal{E}_1$. The result is

$$\begin{aligned} \mathcal{E}_1 \sim M(w) &= -\frac{4G}{3} w_2 F_1(1, 2, 4; 1-w) \\ &= -\frac{4Gw}{(1-w)^3} [1 + 2w \ln(w) - w^2], \end{aligned} \quad (4.28)$$

with w defined in terms of the cross-ratios Z and $\bar{Z}$: $w = Z/\bar{Z}$. We are interested in the Carrollian limit of the correlator. This is obtained by consider the bulk-point singular limit $Z \rightarrow \bar{Z}$. One finds that $Z \rightarrow \bar{Z}$ corresponds to $w \rightarrow 1$ in (4.28). In this limit, the two point is not singular

$$\mathcal{E}_1 \sim 4G, \quad (4.29)$$

and would not match the Carrollian limit of the corresponding scalar exchange diagram, which is an order of $1/c^3$.

This is consistent with the fact that in three-dimensional flat space there is no matching between four-point eikonal amplitudes and two-point amplitudes in a shockwave background. To see this, we remember that the regime of validity for the eikonal exponentiation is transparent in the impact parameter space,

$$\frac{\hbar}{\sqrt{s}} \ll G\sqrt{s}b^{4-D} \ll b, \quad (4.30)$$

where b is the impact parameter dual to the exchanged momentum q and D is the dimension of spacetime. It is straightforward to conclude that this condition cannot be satisfied in three dimensions $D = 3$. The eikonal exponentiation exists only in four or higher dimensions.

Another nontrivial background where the flat limit prescription can be applied could be the Bañados-Teitelboim-Zanelli (BTZ) black hole in three dimensions. Two-point functions of scalars have been computed in [80] and the flat limit of the BTZ geometry has been studied in [59]. As BTZ spacetimes are quotients of AdS_3 by a Killing vector field, they are usually expressed in a convenient set of coordinates where the quotient procedure is trivialized. It would be interesting to understand the geometry and Carrollian limit in the planar Bondi coordinates and to further relate this to the bulk two-point function in the flat limit of BTZ.

Notice that the planar Bondi coordinates used in this work are well adapted to both future and past null infinities in the large-radius limit. It is expected that this would not be true in generic asymptotically flat spacetimes. In order to study the backgrounds that we discussed above and see how the global structure of AdS and flat space is taken into account, one might need to develop perhaps new coordinate systems or improved prescriptions.

V. CONCLUDING REMARKS

There are many interesting and promising directions to explore in the near and far future. Here we list a few of them. It would be interesting to make a connection with the intrinsic Carrollian approach. See, e.g., [62,81–90]. In particular, [83] showed the existence of the infrared triangle in three dimensions. It would be interesting to show what the soft graviton theorem implies for Carrollian amplitudes of massless particles in three dimensions. To that end, one needs to consider spinning external particles. We hope to study it in the near future.

The four-point Carrollian amplitudes we computed are nondistributional, which would make it a perfect playground to test some Carrollian bootstrap ideas. It would be interesting to decompose our four-point Carrollian amplitudes into the BMS blocks shown in [91–95] and see how the geodesic Witten diagrams associated with the conformal blocks in [96,97] behave in the Carrollian picture. Another interesting direction is finding differential equations satisfied by Carrollian amplitudes in the same spirit as [26]. We might find new solutions to the differential equations in comparison with the celestial version [98].

Perhaps the most exciting one, as mentioned in [7], is to see how far the prescription used in our work and [7] can be pushed beyond holography. Carrollian correlators were computed efficiently without knowing the Lagrangian of the Carrollian theory, hence bypassing the difficulty in quantizing a Carrollian theory. It would be interesting to apply it to other Carrollian theories without a Lagrangian description. For instance, it would be fascinating to compute four-point and higher-point correlators in the Carrollian/BMS Ising model in, e.g., [99,100].

ACKNOWLEDGMENTS

We would like to thank Tim Adamo, Romain Ruzziconi, and Stephan Stieberger for valuable comments on the draft. We would like to thank Tim Adamo, Wei Bu, Eduardo Casali, Riccardo Gonzo, Vijay Nimmeli, Benjamin Strittmatter, Tom Taylor, and Piotr Tourkine for useful discussions on related topics. I. S. is supported by an STFC studentship. B. Z. is supported by the Simons Collaboration on Celestial Holography.

DATA AVAILABILITY

No data were created or analyzed in this study.

[1] For recent reviews, see: A. M. Raclariu, Lectures on celestial holography, [arXiv:2107.02075](#); S. Pasterski, Lectures on celestial amplitudes, *Eur. Phys. J. C* **81**, 1062

(2021); A. Strominger, Lectures on the infrared structure of gravity and gauge theory, [arXiv:1703.05448](#); S. Pasterski, M. Pate, and A. M. Raclariu, Celestial holography,

- arXiv:2111.11392; L. Donnay, Celestial holography: An asymptotic symmetry perspective, *Phys. Rep.* **1073**, 1 (2024).
- [2] L. Donnay, A. Fiorucci, Y. Herfray, and R. Ruzziconi, Carrollian perspective on celestial holography, *Phys. Rev. Lett.* **129**, 071602 (2022).
- [3] A. Bagchi, S. Banerjee, R. Basu, and S. Dutta, Scattering amplitudes: Celestial and Carrollian, *Phys. Rev. Lett.* **128**, 241601 (2022).
- [4] L. Donnay, A. Fiorucci, Y. Herfray, and R. Ruzziconi, Bridging Carrollian and celestial holography, *Phys. Rev. D* **107**, 126027 (2023).
- [5] S. Banerjee, Null infinity and unitary representation of the Poincare group, *J. High Energy Phys.* **01** (2019) 205.
- [6] S. Banerjee, S. Ghosh, P. Pandey, and A. P. Saha, Modified celestial amplitude in Einstein gravity, *J. High Energy Phys.* **03** (2020) 125.
- [7] L. F. Alday, M. Nocchi, R. Ruzziconi, and A. Yellespur Srikant, Carrollian amplitudes from holographic correlators, *J. High Energy Phys.* **03** (2025) 158.
- [8] G. Barnich, A. Gomberoff, and H. A. González, Three-dimensional Bondi-Metzner-Sachs invariant two-dimensional field theories as the flat limit of Liouville theory, *Phys. Rev. D* **87**, 124032 (2013).
- [9] C. Duval, G. W. Gibbons, and P. A. Horvathy, Conformal Carroll groups and BMS symmetry, *Classical Quantum Gravity* **31**, 092001 (2014).
- [10] A. Bagchi, A. Mehra, and P. Nandi, Field theories with conformal Carrollian symmetry, *J. High Energy Phys.* **05** (2019) 108.
- [11] M. Henneaux and P. Salgado-Rebolledo, Carroll contractions of Lorentz-invariant theories, *J. High Energy Phys.* **11** (2021) 180.
- [12] J. de Boer, J. Hartong, N. A. Obers, W. Sybesma, and S. Vandoren, Carroll symmetry, dark energy and inflation, *Front. Phys.* **10**, 810405 (2022).
- [13] B. Chen, R. Liu, H. Sun, and Y. Zheng, Constructing Carrollian field theories from null reduction, *J. High Energy Phys.* **11** (2023) 170.
- [14] J. Salzer, An embedding space approach to Carrollian CFT correlators for flat space holography, *J. High Energy Phys.* **10** (2023) 084.
- [15] A. Saha, $w_{1+\infty}$ and Carrollian holography, *J. High Energy Phys.* **05** (2024) 145.
- [16] K. Nguyen and P. West, Carrollian conformal fields and flat holography, *Universe* **9**, 385 (2023).
- [17] K. Nguyen, Carrollian conformal correlators and massless scattering amplitudes, *J. High Energy Phys.* **01** (2024) 076.
- [18] A. Bagchi, P. Dhivakar, and S. Dutta, Holography in flat spacetimes: The case for Carroll, arXiv:2311.11246.
- [19] L. Mason, R. Ruzziconi, and A. Yellespur Srikant, Carrollian amplitudes and celestial symmetries, arXiv:2312.10138.
- [20] W. B. Liu, J. Long, and X. Q. Ye, Feynman rules and loop structure of Carrollian amplitudes, *J. High Energy Phys.* **05** (2024) 213.
- [21] E. Have, K. Nguyen, S. Prohazka, and J. Salzer, Massive Carrollian fields at timelike infinity, *J. High Energy Phys.* **07** (2024) 054.
- [22] S. Stieberger, T. R. Taylor, and B. Zhu, Carrollian amplitudes from strings, *J. High Energy Phys.* **04** (2024) 127.
- [23] T. Adamo, W. Bu, P. Tourkine, and B. Zhu, Eikonal amplitudes on the celestial sphere, *J. High Energy Phys.* **10** (2024) 192.
- [24] S. Banerjee, R. Basu, and S. Atul Bhatkar, Light transformation: A celestial and Carrollian perspective, *J. High Energy Phys.* **12** (2024) 122.
- [25] W. B. Liu, J. Long, H. Y. Xiao, and J. L. Yang, On the definition of Carrollian amplitudes in general dimensions, *J. High Energy Phys.* **11** (2024) 027.
- [26] R. Ruzziconi, S. Stieberger, T. R. Taylor, and B. Zhu, Differential equations for Carrollian amplitudes, *J. High Energy Phys.* **09** (2024) 149.
- [27] R. Ruzziconi and A. Saha, Holographic Carrollian currents for massless scattering, *J. High Energy Phys.* **01** (2025) 169.
- [28] S. Chakraborty, S. Hegde, and A. Maurya, Differential representation for Carrollian correlators, arXiv:2411.09641.
- [29] K. Nguyen and J. Salzer, Operator product expansion in Carrollian CFT, arXiv:2503.15607.
- [30] J. M. Maldacena, The large N limit of superconformal field theories and supergravity, *Adv. Theor. Math. Phys.* **2**, 231 (1998).
- [31] E. Witten, Anti-de Sitter space and holography, *Adv. Theor. Math. Phys.* **2**, 253 (1998).
- [32] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri, and Y. Oz, Large N field theories, string theory and gravity, *Phys. Rep.* **323**, 183 (2000).
- [33] L. Susskind, Holography in the flat space limit, *AIP Conf. Proc.* **493**, 98 (1999).
- [34] J. Polchinski, S matrices from AdS space-time, arXiv: hep-th/9901076.
- [35] S. B. Giddings, Flat space scattering and bulk locality in the AdS/CFT correspondence, *Phys. Rev. D* **61**, 106008 (2000).
- [36] R. Marotta, K. Skenderis, and M. Verma, Flat space spinning massive amplitudes from momentum space CFT, *J. High Energy Phys.* **08** (2024) 226.
- [37] M. Gary, S. B. Giddings, and J. Penedones, Local bulk S-matrix elements and CFT singularities, *Phys. Rev. D* **80**, 085005 (2009).
- [38] J. Penedones, Writing CFT correlation functions as AdS scattering amplitudes, *J. High Energy Phys.* **03** (2011) 025.
- [39] A. L. Fitzpatrick and J. Kaplan, Analyticity and the holographic S-matrix, *J. High Energy Phys.* **10** (2012) 127.
- [40] L. F. Alday and S. Caron-Huot, Gravitational S-matrix from CFT dispersion relations, *J. High Energy Phys.* **12** (2018) 017.
- [41] E. Hijano, Flat space physics from AdS/CFT, *J. High Energy Phys.* **07** (2019) 132.
- [42] Y. Z. Li, Notes on flat-space limit of AdS/CFT, *J. High Energy Phys.* **09** (2021) 027.
- [43] H. T. Lam and S. H. Shao, Conformal basis, optical theorem, and the bulk point singularity, *Phys. Rev. D* **98**, 025020 (2018).
- [44] L. P. de Gioia and A. M. Raclariu, Eikonal approximation in celestial CFT, *J. High Energy Phys.* **03** (2023) 030.

- [45] L. P. de Gioia and A. M. Raclariu, Celestial amplitudes from conformal correlators with bulk-point kinematics, [arXiv:2405.07972](#).
- [46] A. Bagchi, P. Dhivakar, and S. Dutta, AdS Witten diagrams to Carrollian correlators, *J. High Energy Phys.* **04** (2023) 135.
- [47] H. Kawai, D. C. Lewellen, and S. H. H. Tye, A relation between tree amplitudes of closed and open strings, *Nucl. Phys.* **B69**, 1 (1986).
- [48] Z. Bern, J. J. M. Carrasco, and H. Johansson, New relations for gauge-theory amplitudes, *Phys. Rev. D* **78**, 085011 (2008).
- [49] Z. Bern, J. J. M. Carrasco, M. Chiodaroli, H. Johansson, and R. Roiban, The duality between color and kinematics and its applications, *J. Phys. A* **57**, 333002 (2024).
- [50] T. Adamo, J. J. M. Carrasco, M. Carrillo-González, M. Chiodaroli, H. Elvang, H. Johansson, D. O’Connell, R. Roiban, and O. Schlotterer, Snowmass white paper: The double copy and its applications, [arXiv:2204.06547](#).
- [51] N. Arkani-Hamed, M. Pate, A. M. Raclariu, and A. Strominger, Celestial amplitudes from UV to IR, *J. High Energy Phys.* **08** (2021) 062.
- [52] S. Stieberger and T. R. Taylor, Symmetries of celestial amplitudes, *Phys. Lett. B* **793**, 141 (2019).
- [53] W. Fan, A. Fotopoulos, S. Stieberger, and T. R. Taylor, On Sugawara construction on celestial sphere, *J. High Energy Phys.* **09** (2020) 139.
- [54] E. Casali and A. Puhm, Double copy for celestial amplitudes, *Phys. Rev. Lett.* **126**, 101602 (2021).
- [55] E. Casali and A. Sharma, Celestial double copy from the worldsheet, *J. High Energy Phys.* **05** (2021) 157.
- [56] I. Surubaru and B. Zhu, Conformal blocks from celestial graviton amplitudes, *J. High Energy Phys.* **06** (2025) 174.
- [57] T. He and P. Mitra, Asymptotic symmetries and Weinberg’s soft photon theorem in Mink_{d+2} , *J. High Energy Phys.* **10** (2019) 213.
- [58] R. Gonzo and A. Pokraka, Light-ray operators, detectors and gravitational event shapes, *J. High Energy Phys.* **05** (2021) 015.
- [59] G. Barnich, A. Gomberoff, and H. A. Gonzalez, The Flat limit of three dimensional asymptotically anti-de Sitter spacetimes, *Phys. Rev. D* **86**, 024020 (2012).
- [60] C. Duval, G. W. Gibbons, P. A. Horvathy, and P. M. Zhang, Carroll versus Newton and Galilei: Two dual non-Einsteinian concepts of time, *Classical Quantum Gravity* **31**, 085016 (2014).
- [61] D. Hansen, N. A. Obers, G. Oling, and B. T. Sogaard, Carroll expansion of general relativity, *SciPost Phys.* **13**, 055 (2022).
- [62] S. Baiguera, G. Oling, W. Sybesma, and B. T. Sogaard, Conformal Carroll scalars with boosts, *SciPost Phys.* **14**, 086 (2023).
- [63] M. Ben-Shahar and H. Johansson, Off-shell color-kinematics duality for Chern-Simons, *J. High Energy Phys.* **08** (2022) 035.
- [64] B. M. Barker, S. N. Gupta, and R. D. Haracz, One-graviton exchange interaction of elementary particles, *Phys. Rev.* **149**, 1027 (1966).
- [65] S. R. Huggins and D. J. Toms, One graviton exchange interaction of nonminimally coupled scalar fields, *Classical Quantum Gravity* **4**, 1509 (1987).
- [66] E. D’Hoker, D. Z. Freedman, and L. Rastelli, AdS/CFT four point functions: How to succeed at z integrals without really trying, *Nucl. Phys.* **B62**, 395 (1999).
- [67] D. Simmons-Duffin, TASI lectures on conformal field theory in Lorentzian signature (2019), https://www.desy.de/~bargheer/string-journal-club/presentations/2023-05-16_Sebastian-Harris_Simmons-Duffin:_TASI-Lorentzian-CFT.pdf.
- [68] H. H. Zhang, K. X. Feng, S. W. Qiu, A. Zhao, and X. S. Li, On analytic formulas of Feynman propagators in position space, *Chin. Phys. C* **34**, 1576 (2010).
- [69] E. D’Hoker, D. Z. Freedman, S. D. Mathur, A. Matusis, and L. Rastelli, Graviton and gauge boson propagators in $\text{AdS}(d+1)$, *Nucl. Phys.* **B62**, 330 (1999).
- [70] S. Komatsu, M. F. Paulos, B. C. Van Rees, and X. Zhao, Landau diagrams in AdS and S-matrices from conformal correlators, *J. High Energy Phys.* **11** (2020) 046.
- [71] A. Bissi, A. Sinha, and X. Zhou, Selected topics in analytic conformal bootstrap: A guided journey, *Phys. Rep.* **991**, 1 (2022).
- [72] A. Denner, U. Nierste, and R. Scharf, A compact expression for the scalar one loop four point function, *Nucl. Phys.* **B67**, 637 (1991).
- [73] N. I. Usyukina and A. I. Davydychev, An approach to the evaluation of three and four point ladder diagrams, *Phys. Lett. B* **298**, 363 (1993).
- [74] J. L. Bourjaily, H. Hannesdottir, A. J. McLeod, M. D. Schwartz, and C. Vergu, Sequential discontinuities of Feynman integrals and the Monodromy Group, *J. High Energy Phys.* **01** (2021) 205.
- [75] J. Maldacena, D. Simmons-Duffin, and A. Zhiboedov, Looking for a bulk point, *J. High Energy Phys.* **01** (2017) 013.
- [76] L. Rastelli, K. Roumpedakis, and X. Zhou, $\text{AdS}_3 \times S^3$ tree-level correlators: Hidden six-dimensional conformal symmetry, *J. High Energy Phys.* **10** (2019) 140.
- [77] P. Di Vecchia, C. Heissenberg, R. Russo, and G. Veneziano, The gravitational eikonal: From particle, string and brane collisions to black-hole encounters, *Phys. Rep.* **1083**, 1 (2024).
- [78] G. ’t Hooft, Graviton dominance in ultrahigh-energy scattering, *Phys. Lett. B* **198**, 61 (1987).
- [79] L. Cornalba, M. S. Costa, J. Penedones, and R. Schiappa, Eikonal approximation in AdS/CFT: From shock waves to four-point functions, *J. High Energy Phys.* **08** (2007) 019.
- [80] E. Keski-Vakkuri, Bulk and boundary dynamics in BTZ black holes, *Phys. Rev. D* **59**, 104001 (1999).
- [81] J. de Boer, J. Hartong, N. A. Obers, W. Sybesma, and S. Vandoren, Carroll stories, *J. High Energy Phys.* **09** (2023) 148.
- [82] B. Chen, H. Sun, and Y. Zheng, Quantization of Carrollian conformal scalar theories, *Phys. Rev. D* **110**, 125010 (2024).
- [83] J. Cotler, K. Jensen, S. Prohazka, A. Raz, M. Riegler, and J. Salzer, Quantizing Carrollian field theories, *J. High Energy Phys.* **10** (2024) 049.
- [84] P. Kraus and R. M. Myers, Carrollian partition functions and the flat limit of AdS, *J. High Energy Phys.* **01** (2025) 183.

- [85] P. Kraus and R. M. Myers, Carrollian partition function for bulk Yang-Mills theory, [arXiv:2503.00916](#).
- [86] J. Figueroa-O'Farrill, A. Pérez, and S. Prohazka, Quantum Carroll/fracton particles, *J. High Energy Phys.* **10** (2023) 041.
- [87] L. Ciambelli, C. Marteau, A. C. Petkou, P. M. Petropoulos, and K. Siampas, Flat holography and Carrollian fluids, *J. High Energy Phys.* **07** (2018) 165.
- [88] L. Ciambelli, R. G. Leigh, C. Marteau, and P. M. Petropoulos, Carroll structures, null geometry and conformal isometries, *Phys. Rev. D* **100**, 046010 (2019).
- [89] J. Cotler, K. Jensen, S. Prohazka, M. Riegler, and J. Salzer, Soft gravitons in three dimensions, *J. High Energy Phys.* **07** (2025) 002.
- [90] G. Poulas and S. Vandoren, On Carroll partition functions and flat space holography, *J. High Energy Phys.* **06** (2025) 232.
- [91] A. Bagchi, M. Gary, and Zodinmawia, Bondi-Metzner-Sachs bootstrap, *Phys. Rev. D* **96**, 025007 (2017).
- [92] A. Bagchi, M. Gary, and Zodinmawia, The nuts and bolts of the BMS bootstrap, *Classical Quantum Gravity* **34**, 174002 (2017).
- [93] B. Chen, P. X. Hao, R. Liu, and Z. F. Yu, On Galilean conformal bootstrap, *J. High Energy Phys.* **06** (2021) 112.
- [94] B. Chen and R. Liu, The shadow formalism of Galilean CFT₂, *J. High Energy Phys.* **05** (2023) 224.
- [95] B. Chen, P. x. Hao, R. Liu, and Z. f. Yu, On Galilean conformal bootstrap. Part II. $\xi = 0$ sector, *J. High Energy Phys.* **12** (2022) 019.
- [96] E. Hijano, P. Kraus, E. Perlmutter, and R. Snively, Witten diagrams revisited: The AdS geometry of conformal blocks, *J. High Energy Phys.* **01** (2016) 146.
- [97] E. Hijano, P. Kraus, E. Perlmutter, and R. Snively, Semi-classical Virasoro blocks from AdS₃ gravity, *J. High Energy Phys.* **12** (2015) 077.
- [98] W. Fan, A. Fotopoulos, S. Stieberger, T. R. Taylor, and B. Zhu, Elements of celestial conformal field theory, *J. High Energy Phys.* **08** (2022) 213.
- [99] Z. f. Yu and B. Chen, Free field realization of the BMS Ising model, *J. High Energy Phys.* **08** (2023) 116.
- [100] P. X. Hao, W. Song, Z. Xiao, and X. Xie, BMS-invariant free fermion models, *Phys. Rev. D* **109**, 025002 (2024).