

Off forward non- s -channel helicity conserving contributions to exclusive vector quarkonium production from the spin dependent BFKL Pomeron

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A novel contribution to off forward, exclusive vector quarkonium production, $\gamma^{(*)} + p \rightarrow V + p$, at high energy is derived which corresponds to a t -channel exchange of a Balitsky-Fadin-Kuraev-Lipatov (BFKL) hard Pomeron, with a helicity flip of the proton. This “spin-dependent BFKL Pomeron” is required in a consistent expansion in powers of the momentum transfer $t \approx -\Delta_{\perp}^2$ beyond first order. The spin-dependent Pomeron violates s -channel helicity conservation at $\mathcal{O}(\Delta_{\perp}^2)$, and beyond. Expanding to leading twist only, it corresponds to generalized parton distributions $E_g(x, t)$ for vanishing skewness. We derive explicit expressions for the eikonal BFKL amplitudes, to all orders in dipole size times momentum transfer, for all helicity configurations of the particles in the initial and final states. We also provide numerical estimates of the helicity flip two gluon exchange amplitude at moderate x from a light cone quark model of the proton. The spin dependent BFKL Pomeron could, in principle, be discovered via double spin asymmetries in $e + p \rightarrow e + p + J/\psi$ with a transversely polarized proton and longitudinally polarized electron in the initial state.

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I. INTRODUCTION

The eikonal, forward Balitsky-Fadin-Kuraev-Lipatov (BFKL) Pomeron or unintegrated gluon distribution [1–4] corresponds to the matrix element of a two-gluon t -channel exchange operator between incoming and outgoing light cone proton states with the same helicity $\lambda_p = \pm 1/2$. Indeed, in the eikonal (high-energy) and $t \rightarrow 0$ limits a helicity flip of the proton is not possible; the same applies to the transition of the photon to the vector meson state. This leads to so-called s -channel helicity conservation (SCHC) in eikonal exclusive J/ψ or $\psi(2S)$ production when $t \rightarrow 0$ [5]. Specifically, in photoproduction or ultra-peripheral proton-proton or nucleus-proton collisions, the vector meson (VM) is transversely polarized, and in fact its polarization $\bar{\lambda}$ is equal to the polarization $\lambda = \pm 1$ of the incoming photon.

In off forward production, $-t > 0$, non-SCHC corrections appear. The first kind preserves still the helicity of the proton, and is thus associated with the usual spin independent nonforward BFKL Pomeron [5–7] involving orbital angular momentum transfer to the $c - \bar{c}$ pair which scatters off the proton. Thus, the polarization of the J/ψ may now be different from that of the photon, $\bar{\lambda} \neq \lambda$. In the limit of small $|t| \approx \Delta_{\perp}^2$, non-SCHC contributions to the cross section in deeply-inelastic scattering (photoproduction) begin at linear (quadratic) order in the VM transverse momentum $\Delta_{\perp}$.

Here, we are concerned mainly with a second kind of non-SCHC corrections associated with a helicity flip of the proton. These are related to the so-called “spin-dependent Pomeron” (SDP) [8–11]. Hard processes involve the partonic substructure of the proton, and orbital angular momentum may be transferred to the partons even in the eikonal limit, thereby allowing a helicity flip of the proton.¹ The SDP is parametrized in terms of two distinct dipole-proton scattering amplitudes, associated with $(\mathbf{r}_{\perp} \cdot \mathbf{\Delta}_{\perp})(\epsilon_{\perp}^{\Lambda} \cdot \mathbf{r}_{\perp})$ and $(\epsilon_{\perp}^{\Lambda} \cdot \mathbf{\Delta}_{\perp})$ correlations, where $\mathbf{r}_{\perp}$ is the dipole vector and $\epsilon_{\perp}^{\Lambda} = -(\Lambda, i)/\sqrt{2}$ the helicity vector; hereafter we con-

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¹The SDP is associated with the $n = 0$ BFKL eigenfunction with positive energy intercept.

ventionally normalize the proton helicity defining $\Lambda = 2\lambda_p = \pm 1$. Therefore, the SDPs contribute at leading non-SCHC power $\mathcal{O}(\Delta_\perp^2)$ to the cross section for exclusive vector quarkonium photoproduction (at small momentum transfer).

The ZEUS and H1 Collaborations at HERA have confirmed that the cross section ratio for longitudinal vs transverse J/ψ polarization in photoproduction on unpolarized protons is small [12,13]. Similarly, these experiments found that the amplitude for J/ψ helicity opposite to photon helicity is much smaller than the SCHC amplitude. However, these data do not constrain the contribution associated with a helicity flip of the proton.

A polarization measurement of coherently photoproduced J/ψ in ultraperipheral Pb-Pb collisions at the LHC has been reported by the ALICE Collaboration [14,15]. These data again are found to be consistent with SCHC. In a model where the photon scatters off a “Pomeron patch” in the nucleus [16] there could be a contribution though which involves a helicity flip within that patch.

The STAR Collaboration at RHIC has constrained the presence of a hadronic spin flip due to *soft* Pomeron exchange in polarized proton-proton elastic scattering at very low $|t| \leq 0.035 \text{ GeV}^2$ [17]. This process lacks a hard scale though, and so it does not relate in a computable way to the gluon (ladder) exchanges of perturbative QCD. Also, as the numerical estimates below will show, a significant SDP amplitude may require higher momentum transfer, $\Delta_\perp \sim 0.5 \text{ GeV}$ or greater.

In Sec. II we introduce the general BFKL eikonal amplitudes for exclusive VM production. Following, Sec. III is focused on their helicity decomposition, and in Sec. IV we consider the proton helicity flip case. In Sec. V we perform a model computation of the SDPs for moderately small x . Section VI concludes the paper with a summary and discussion.

II. THE $\gamma^{(*)} + p \rightarrow V + p$ AMPLITUDES

The general formula for the eikonal amplitude of exclusive VM production $\gamma(q, \lambda)p(P, \Lambda) \rightarrow V(\Delta, \bar{\lambda})p'(P', \Lambda')$ is

$$\mathcal{M}_{\lambda\Lambda;\bar{\lambda}\Lambda'} = 2N_c \int d^2\mathbf{r}_\perp \mathcal{P}_{\Lambda\Lambda'}(\mathbf{r}_\perp, \Delta_\perp) \mathcal{A}_{\lambda\bar{\lambda}}(\mathbf{r}_\perp, \Delta_\perp), \quad (1)$$

in terms of which the S -matrix reads as $(2\pi)\delta(q^- - \Delta^-)q^- \mathcal{M}_{\lambda\Lambda;\bar{\lambda}\Lambda'}$, after subtracting the no-scattering contribution. Our notation mostly follows Ref. [18], and our amplitudes below without proton helicity flip, $\Lambda' = \Lambda$, agree with Ref. [18] except for different signs of the $\gamma_L \rightarrow V_L$ and $\gamma_\lambda \rightarrow V_{-\lambda}$ amplitudes. In Eq. (1) the helicities of the incoming and outgoing proton, as well as of the photon and the VM are indicated explicitly. $\mathcal{A}_{\lambda\bar{\lambda}}(\mathbf{r}_\perp, \Delta_\perp)$ is the reduced amplitude representing the $\gamma - V$ wave function overlap. For a transverse photon with $\lambda = \pm 1$, the helicity dependence of the $\gamma - V$ overlaps reads as

$$\begin{aligned} \mathcal{A}_{\lambda=\pm 1, \bar{\lambda}=0}(\mathbf{r}_\perp, \Delta_\perp) &= eq_c \lambda e^{i\lambda\phi_r} \int_z e^{-i\delta_\perp \cdot \mathbf{r}_\perp} \mathcal{A}_{TL}(r_\perp, z), \\ \mathcal{A}_{\lambda=\pm 1, \bar{\lambda}=\pm 1}(\mathbf{r}_\perp, \Delta_\perp) &= eq_c \int_z e^{-i\delta_\perp \cdot \mathbf{r}_\perp} [\delta_{\lambda\bar{\lambda}} \mathcal{A}_{TT}^{nf}(r_\perp, z) \\ &\quad + \delta_{\lambda, -\bar{\lambda}} e^{2i\lambda\phi_r} \mathcal{A}_{TT}^f(r_\perp, z)]. \end{aligned} \quad (2)$$

Here, $\delta_\perp = (z - \bar{z})\Delta_\perp/2$ where z and $\bar{z} = 1 - z$ denote the momentum fractions of the quark and antiquark in the VM, respectively. We also have $\int_z \equiv \int_0^1 \frac{dz}{4\pi}$. In the nonrelativistic heavy-quark limit, the $q\bar{q}$ light-cone (LC) momentum imbalance is $z - \bar{z} \rightarrow 0$. The functions $\mathcal{A}_{TL}(r_\perp, z)$ and $\mathcal{A}_{TT}^{nf,f}(r_\perp, z)$ are collected in Appendix A, together with the $\gamma_L - V$ overlaps.

In Eq. (1), $\mathcal{P}_{\Lambda\Lambda'}(\mathbf{r}_\perp, \Delta_\perp)$ represents the BFKL Pomeron exchange, defined as

$$\mathcal{P}_{\Lambda\Lambda'}(\mathbf{r}_\perp, \Delta_\perp) = \int d^2\mathbf{b}_\perp e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \left\{ 1 - \frac{1}{2N_c} \left[\frac{\langle P'\Lambda' | \text{tr}[V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp) + V(\mathbf{y}_\perp)V^\dagger(\mathbf{x}_\perp)] | P\Lambda \rangle}{\langle P\Lambda | P\Lambda \rangle} \right] \right\}, \quad (3)$$

with $\mathbf{x}_\perp = \mathbf{b}_\perp + \mathbf{r}_\perp/2$, $\mathbf{y}_\perp = \mathbf{b}_\perp - \mathbf{r}_\perp/2$. The eikonal Wilson lines $V(\mathbf{x}_\perp) = P \exp[-ig \int_{-\infty}^{\infty} dx^- A^+(x^-, \mathbf{x}_\perp)]$, with $A^+(x^-, \mathbf{x}_\perp) = t_a A_a^+(x^-, \mathbf{x}_\perp)$ the gluon field in covariant gauge, are used to construct the dipole amplitude $\text{tr}[V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp)]/N_c$. In (3) we pick up its C -even part, as dictated by the symmetries of the photon and the VM wave functions. The amplitude $\mathcal{P}_{\Lambda\Lambda'}(\mathbf{r}_\perp, \Delta_\perp)$ can be parametrized in the most general way as [8]

$$\begin{aligned} \mathcal{P}_{\Lambda\Lambda'}(\mathbf{r}_\perp, \Delta_\perp) &= \delta_{\Lambda, \Lambda'} \mathcal{P}(\mathbf{r}_\perp, \Delta_\perp) \\ &\quad + \cos(\phi_{r\Delta}) \Lambda e^{i\lambda\phi_r} \delta_{\Lambda, -\Lambda'} \mathcal{P}_S(\mathbf{r}_\perp, \Delta_\perp) \\ &\quad + \Lambda e^{i\lambda\phi_\Delta} \delta_{\Lambda, -\Lambda'} \mathcal{P}_S^\perp(\mathbf{r}_\perp, \Delta_\perp), \end{aligned} \quad (4)$$

where $\phi_{ab} = \phi_a - \phi_b$ and $\mathcal{P}(\mathbf{r}_\perp, \Delta_\perp)$, $\mathcal{P}_S(\mathbf{r}_\perp, \Delta_\perp)$ and $\mathcal{P}_S^\perp(\mathbf{r}_\perp, \Delta_\perp)$ are three in principle different nonperturbative real scalar functions. In Appendix C we briefly recall the connection to the generalized transverse momentum dependent distributions (GTMDs). $\mathcal{P}(\mathbf{r}_\perp, \Delta_\perp)$ is the “spin-independent Pomeron” (SIP) that is usually employed in calculations of J/ψ production in high-energy photon-proton or photon-nucleus scattering [5–7,18–23]. The SDPs $\mathcal{P}_S(\mathbf{r}_\perp, \Delta_\perp)$ and $\mathcal{P}_S^\perp(\mathbf{r}_\perp, \Delta_\perp)$, associated with a helicity flip of the proton, have so far not been identified in the context of eikonal, exclusive VM production. In the collinear limit, $\mathcal{P}$ is related to the gluon generalized parton distributions (GPDs) $H_g(x, t)$ at small x . Likewise, the combination $\mathcal{P}_S + 2\mathcal{P}_S^\perp$ is related to the GPD $E_g(x, t)$

[9,10]. The helicity-flip part should be linear in $\epsilon_\perp^\Lambda$ which explains the angular structure in (4) after taking into account that $\mathcal{P}_{\Lambda\Lambda'}(\mathbf{r}_\perp, \mathbf{\Delta}_\perp)$ is even in $\mathbf{r}_\perp \rightarrow -\mathbf{r}_\perp$. In this parametrization $\mathcal{P}_S \sim \Delta_\perp$ and $\mathcal{P}_S^\perp \sim \Delta_\perp$ for small $\Delta_\perp$. Each of the scalar functions depends on the variables $\mathbf{r}_\perp^2$, $\mathbf{\Delta}_\perp^2$, $\mathbf{r}_\perp \cdot \mathbf{\Delta}_\perp$ which can be revealed through a Fourier expansion, for example,

$$\mathcal{P}(\mathbf{r}_\perp, \mathbf{\Delta}_\perp) = \mathcal{P}_0(r_\perp, \Delta_\perp) + 2\mathcal{P}_e(r_\perp, \Delta_\perp) \cos(2\phi_{r\Delta}) + \dots, \quad (5)$$

and similarly for $\mathcal{P}_S(\mathbf{r}_\perp, \mathbf{\Delta}_\perp)$ and $\mathcal{P}_S^\perp(\mathbf{r}_\perp, \mathbf{\Delta}_\perp)$. The appearance of the “elliptic” Pomeron $\mathcal{P}_e(r_\perp, \Delta_\perp)$ in deeply virtual compton scattering at small x was first pointed out in Ref. [24] and in connection to the gluon GPD $E_{gT}(x, t)$. Its relevance to exclusive VM production was mentioned in Ref. [18]; we add below insight into the specific helicity channels and the power counting in $\Delta_\perp$. The azimuthal angular dependence of the dipole scattering amplitude on a proton, specifically, has been discussed in Ref. [25]. In the limit of small $\Delta_\perp$, $\mathcal{P}_0(r_\perp, \Delta_\perp) \sim \Delta_\perp^0$

while $\mathcal{P}_e(r_\perp, \Delta_\perp) \sim \Delta_\perp^2$; higher Fourier harmonics are proportional to higher powers of $\Delta_\perp$ and are expected to have very small amplitudes, so we will not consider them further.

III. SPIN-INDEPENDENT POMERON EXCHANGE

We first list the amplitudes for Pomeron exchange without helicity flip of the proton and discuss their scaling with $\Delta_\perp$ in the limit of small momentum transfer.² Even in the absence of a helicity flip of the proton we obtain new contributions to the cross section at $\mathcal{O}(\Delta_\perp^2)$ as compared to the classic work by Nikolaev *et al.* [5–7], which are due to interference of the SCHC $\gamma - V$ amplitude with the helicity flip $\gamma - V$ amplitude for the elliptic Pomeron, or gluon distribution.

We begin with the amplitudes for a transverse photon ($\lambda = \pm 1$) which survive in the $Q^2 \rightarrow 0$ photoproduction limit. The amplitude for a transverse VM can be decomposed into helicity nonflip ($\lambda = \bar{\lambda}$) and helicity flip ($\lambda = -\bar{\lambda}$) pieces. From the first term in the Fourier series (5), i.e., the isotropic Pomeron, the helicity nonflip SCHC amplitude is

$$\mathcal{M}_{\lambda\Lambda;\lambda\Lambda} = 4\pi N_c e q_c \int_z \int r_\perp d\mathbf{r}_\perp \mathcal{P}_0(r_\perp, \Delta_\perp) \mathcal{A}_{TT}^{nf}(r_\perp, z) J_0(r_\perp \delta_\perp) \sim \Delta_\perp^0. \quad (6)$$

Recall that $\delta_\perp = |z - \bar{z}| \Delta_\perp / 2$. For small momentum transfer this amplitude is independent of $\Delta_\perp$. For completeness we also list the nonflip amplitude for the elliptic Pomeron which starts out at order $\mathcal{O}(\Delta_\perp^4)$:

$$\mathcal{M}_{\lambda\Lambda;\lambda\Lambda}^e = -8\pi N_c e q_c \int_z \int r_\perp d\mathbf{r}_\perp \mathcal{P}_e(r_\perp, \Delta_\perp) \mathcal{A}_{TT}^{nf}(r_\perp, z) J_2(r_\perp \delta_\perp) \sim \Delta_\perp^4. \quad (7)$$

On the other hand, the $\gamma - V$ helicity flip amplitude is

$$\mathcal{M}_{\lambda\Lambda;-\lambda\Lambda} = 4\pi N_c e q_c e^{2i\lambda\phi_\Delta} \int_z \int r_\perp d\mathbf{r}_\perp \mathcal{P}_0(r_\perp, \Delta_\perp) \mathcal{A}_{TT}^f(r_\perp, z) J_2(r_\perp \delta_\perp) \sim \Delta_\perp^2. \quad (8)$$

This amplitude scales as $\Delta_\perp^2$ due to the transfer of 2 units of orbital angular momentum to compensate for the helicity flip. Each unit of orbital angular momentum comes with 1 power of $\Delta_\perp$, 1 power of the dipole size $r_\perp$, and finally 1 power of the LC momentum imbalance $|z - \bar{z}|$. Hence, we expect that the above non-SCHC contribution is smaller for Υ than for J/ψ but greater for $\psi(2S)$ than J/ψ mesons as the transverse size of the $\psi(2S)$ and the width of its light cone distribution amplitude [26] about $z = 1/2$ are greater than those of the J/ψ .

For low momentum transfer the second term in (5), i.e., the elliptic Pomeron, scales as $\sim \Delta_\perp^4$ for $\bar{\lambda} = \lambda$, but as $\sim \Delta_\perp^2$ for $\bar{\lambda} = -\lambda$,

$$\mathcal{M}_{\lambda\Lambda;-\lambda\Lambda}^e = 4\pi e q_c N_c e^{2i\lambda\phi_\Delta} \int_z \int r_\perp d\mathbf{r}_\perp \mathcal{P}_e(r_\perp, \Delta_\perp) \mathcal{A}_{TT}^f(r_\perp, z) [J_4(r_\perp \delta_\perp) + J_0(r_\perp \delta_\perp)] \sim \Delta_\perp^2, \quad (9)$$

without suppression by factors of $|z - \bar{z}|$.

Coming to the longitudinal VM, the first term in (5) leads to

$$\mathcal{M}_{\lambda\Lambda;0\Lambda} = -4\pi i e q_c N_c \lambda e^{i\lambda\phi_\Delta} \int_z \text{sign}(z - \bar{z}) \int r_\perp d\mathbf{r}_\perp \mathcal{P}_0(r_\perp, \Delta_\perp) \mathcal{A}_{TL}(r_\perp, z) J_1(r_\perp \delta_\perp) \sim \Delta_\perp^1. \quad (10)$$

²That is, the scaling with $\Delta_\perp$ of the leading twist contribution to the respective amplitude. However, in the BFKL approach one does not expand in powers of $r_\perp \Delta_\perp$, and high transverse momentum transfer is allowed.

This amplitude is proportional to a single power of $\Delta_\perp$ and a single power of $r_\perp$ (at small $\Delta_\perp$) but is quadratic in the LC momentum imbalance $|z - \bar{z}|$ since $\mathcal{A}_{TL}(r_\perp, z) \sim z - \bar{z}$; see Eq. (A7). The elliptic Pomeron contributes at order $\Delta_\perp^3$:

$$\mathcal{M}_{\lambda\Lambda;0\Lambda}^e = -4\pi i e q_c N_c \lambda e^{i\lambda\phi_\Delta} \int_z \text{sign}(z - \bar{z}) \int r_\perp dr_\perp \mathcal{P}_e(r_\perp, \Delta_\perp) \mathcal{A}_{TL}(r_\perp, z) [J_1(r_\perp \delta_\perp) - J_3(r_\perp \delta_\perp)] \sim \Delta_\perp^3. \quad (11)$$

Therefore, a test of SCHC violation via the cross section ratio for longitudinal vs transverse VM production σ_L/σ_T involves a suppression factor:

$$\frac{\sigma_L}{\sigma_T} \sim \langle (z - \bar{z})^4 r_\perp^2 \rangle_V \Delta_\perp^2. \quad (12)$$

On the other hand, the interference of the amplitude for a $\gamma - V$ helicity flip by 2 units, $\mathcal{M}_{\lambda\Lambda;-\lambda\Lambda}$ in (8), with the nonflip amplitude $\mathcal{M}_{\lambda\Lambda;\lambda\Lambda}$ in (6), is suppressed by 2 powers of $z - \bar{z}$:

$$\frac{\mathcal{M}_{\lambda\Lambda;-\lambda\Lambda} \mathcal{M}_{\lambda\Lambda;\lambda\Lambda}^* + \text{c.c.}}{|\mathcal{M}_{\lambda\Lambda;\lambda\Lambda}|^2} \sim \langle (z - \bar{z})^2 r_\perp^2 \rangle_V \Delta_\perp^2. \quad (13)$$

There is also an interference of $\mathcal{M}_{\lambda\Lambda;-\lambda\Lambda}^e$ and $\mathcal{M}_{\lambda\Lambda;\lambda\Lambda}$ which is not suppressed by powers of $z - \bar{z}$ at small $\Delta_\perp$. However, it involves the ratio of the elliptic to the isotropic BFKL unintegrated gluon distributions; see below.

We now list the amplitudes for a longitudinal virtual photon ($\lambda = 0$). These are

$$\begin{aligned} \mathcal{M}_{0\Lambda;0\Lambda} &= 4\pi e q_c N_c \int_z \int r_\perp dr_\perp \mathcal{P}_0(r_\perp, \Delta_\perp) \mathcal{A}_{LL}(r_\perp, z) J_0(r_\perp \delta_\perp) \sim \Delta_\perp^0, \\ \mathcal{M}_{0\Lambda;0\Lambda}^e &= -8\pi e q_c N_c \int_z \int r_\perp dr_\perp \mathcal{P}_e(r_\perp, \Delta_\perp) \mathcal{A}_{LL}(r_\perp, z) J_2(r_\perp \delta_\perp) \sim \Delta_\perp^4, \end{aligned} \quad (14)$$

where $\mathcal{A}_{LL}(r_\perp, z)$ is given in the first line of Eq. (A10). The first term, together with (6), represents the usual SCHC $\gamma - V$ amplitudes associated with the nonflip $T \rightarrow T$ and the $L \rightarrow L$ transitions. For the $L \rightarrow T$ transition we have

$$\begin{aligned} \mathcal{M}_{0\Lambda;\bar{\lambda}\Lambda} &= -4\pi i e q_c N_c \bar{\lambda} e^{-i\bar{\lambda}\phi_\Delta} \int_z \int r_\perp dr_\perp \mathcal{P}_0(r_\perp, \Delta_\perp) \mathcal{A}_{LT}(r_\perp, z) \text{sign}(z - \bar{z}) J_1(r_\perp \delta_\perp) \sim \Delta_\perp^1, \\ \mathcal{M}_{0\Lambda;\bar{\lambda}\Lambda}^e &= 4\pi i e q_c N_c e^{-i\bar{\lambda}\phi_\Delta} \int_z \int r_\perp dr_\perp \mathcal{P}_e(r_\perp, \Delta_\perp) \mathcal{A}_{LT}(r_\perp, z) (J_{2-\bar{\lambda}}(r_\perp \delta_\perp) - J_{2+\bar{\lambda}}(r_\perp \delta_\perp)) \sim \Delta_\perp^3, \end{aligned} \quad (15)$$

with $\mathcal{A}_{LT}(r_\perp, z)$ in the second line of Eq. (A10).

Let us summarize all contributions to the VM cross section due to the SIP exchange, organized by powers of $\Delta_\perp$. At leading power $\Delta_\perp^0$, we have the squares of the SCHC amplitudes $\mathcal{M}_{\lambda\Lambda;\lambda\Lambda}$ and $\mathcal{M}_{0\Lambda;0\Lambda}$, which are the only amplitudes that survive in the forward limit. At first power in momentum transfer $\Delta_\perp$, and for nonzero Q^2 , there is a contribution due to interference of $\mathcal{M}_{0\Lambda;\bar{\lambda}\Lambda}$ and $\mathcal{M}_{\lambda\Lambda;\lambda\Lambda}$ (for $\bar{\lambda} = \lambda$) as well as of $\mathcal{M}_{0\Lambda;0\Lambda}$ and $\mathcal{M}_{\lambda\Lambda;0\Lambda}$. At order $\Delta_\perp^2$, and in the $Q^2 \rightarrow 0$ photoproduction limit, we have (i) the interference of the SCHC amplitude $\mathcal{M}_{\lambda\Lambda;\lambda\Lambda}$ with the sum of the helicity flip amplitudes $\mathcal{M}_{\lambda\Lambda;-\lambda\Lambda} + \mathcal{M}_{\lambda\Lambda;-\lambda\Lambda}^e$, and (ii) the square of the $T \rightarrow L$ amplitude $\mathcal{M}_{\lambda\Lambda;0\Lambda}$ for the isotropic gluon distribution. For high momentum transfer of order of the mass m_c of the heavy quark one needs to sum all of the above amplitudes, of course. This is the regime we are focusing on.

IV. SPIN-DEPENDENT POMERON EXCHANGE, PROTON HELICITY FLIP

The leading angular dependence of the SDPs $\mathcal{P}_S(\mathbf{r}_\perp, \Delta_\perp)$ and $\mathcal{P}_S^\perp(\mathbf{r}_\perp, \Delta_\perp)$ is given in (4). Retaining only the first Fourier harmonics, $\mathcal{P}_S(\mathbf{r}_\perp, \Delta_\perp) \approx \mathcal{P}_{S0}(\mathbf{r}_\perp, \Delta_\perp)$ and $\mathcal{P}_S^\perp(\mathbf{r}_\perp, \Delta_\perp) \approx \mathcal{P}_{S0}^\perp(\mathbf{r}_\perp, \Delta_\perp)$, the resulting amplitudes for a transverse photon, and their scaling with $\Delta_\perp$ for $\Delta_\perp \rightarrow 0$, are

$$\begin{aligned}
\mathcal{M}_{\lambda\Lambda;0,-\Lambda} &= 2\pi i N_c e q_c e^{i(\lambda+\Lambda)\phi_\Delta} \int_z \text{sgn}(z - \bar{z}) \int r_\perp dr_\perp \mathcal{A}_{TL}(r_\perp, z) \mathcal{P}_{S0}(r_\perp, \Delta_\perp) [J_{\lambda+\Lambda+1}(r_\perp \delta_\perp) - J_{\lambda+\Lambda-1}(r_\perp \delta_\perp)] \sim \Delta_\perp^2, \\
\mathcal{M}_{\lambda\Lambda;\lambda,-\Lambda} &= -2\pi N_c e q_c \Lambda e^{i\Lambda\phi_\Delta} \int_z \int r_\perp dr_\perp \mathcal{A}_{TT}^{nf}(r_\perp, z) \mathcal{P}_{S0}(r_\perp, \Delta_\perp) [J_2(r_\perp \delta_\perp) - J_0(r_\perp \delta_\perp)] \sim \Delta_\perp^1, \\
\mathcal{M}_{\lambda\Lambda;-\lambda,-\Lambda} &= 2\pi N_c e q_c e^{i(2\lambda+\Lambda)\phi_\Delta} \int_z \int r_\perp dr_\perp \mathcal{A}_{TT}^f(r_\perp, z) \mathcal{P}_{S0}(r_\perp, \Delta_\perp) [J_{2\lambda+\Lambda+1}(r_\perp \delta_\perp) - J_{2\lambda+\Lambda-1}(r_\perp \delta_\perp)] \sim \Delta_\perp^{2+\Lambda}, \quad (16)
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{M}_{\lambda\Lambda;0,-\Lambda} &= -4\pi i N_c e q_c \Lambda e^{i(\Lambda+\lambda)\phi_\Delta} \int_z \text{sgn}(z - \bar{z}) \int r_\perp dr_\perp \mathcal{A}_{TL}(r_\perp, z) \mathcal{P}_{S0}^\perp(r_\perp, \Delta_\perp) J_\lambda(r_\perp \delta_\perp) \sim \Delta_\perp^2, \\
\mathcal{M}_{\lambda\Lambda;\lambda,-\Lambda} &= 4\pi N_c e q_c \Lambda e^{i\Lambda\phi_\Delta} \int_z \int r_\perp dr_\perp \mathcal{A}_{TT}^{nf}(r_\perp, z) \mathcal{P}_{S0}^\perp(r_\perp, \Delta_\perp) J_0(r_\perp \delta_\perp) \sim \Delta_\perp^1, \\
\mathcal{M}_{\lambda\Lambda;-\lambda,-\Lambda} &= -4\pi N_c e q_c \Lambda e^{i(2\lambda+\Lambda)\phi_\Delta} \int_z \int r_\perp dr_\perp \mathcal{A}_{TT}^f(r_\perp, z) \mathcal{P}_{S0}^\perp(r_\perp, \Delta_\perp) J_2(r_\perp \delta_\perp) \sim \Delta_\perp^3. \quad (17)
\end{aligned}$$

Therefore, SDP exchanges associated with a helicity flip of the proton contribute to the VM photoproduction cross section at $\mathcal{O}(\Delta_\perp^2)$ through all the $\mathcal{O}(\Delta_\perp)$ amplitudes from above, including their interference. These are the two $\mathcal{M}_{\lambda\Lambda;\lambda,-\Lambda}(\Delta_\perp)$ without a helicity flip from photon to VM, as well as the double helicity flip amplitude $\mathcal{M}_{\lambda\Lambda;-\lambda,-\Lambda}(\Delta_\perp)$ with the first type of SDP exchange, and for opposite helicities of the incoming photon and proton. Note that at leading power of $r_\perp \delta_\perp$, i.e., at leading twist, each amplitude of Eq. (16) combines with the corresponding amplitude of (17) to $E_g \sim \mathcal{P}_{S0} + 2\mathcal{P}_{S0}^\perp$, in agreement with Ref. [27]. We also point out that, interestingly, none of these amplitudes involves factors of $z - \bar{z}$. Hence, for small momentum transfer $\Delta_\perp$, the ratio of proton helicity flip to nonflip cross sections scales as

$$\frac{\sigma_{\Lambda'=-\Lambda}}{\sigma_{\Lambda'=\Lambda}} \sim \langle r_\perp^2 \rangle_V \Delta_\perp^2. \quad (18)$$

Equation (18) lacks the large $(z - \bar{z})^4$ suppression factor of Eq. (12). On the other hand, the coefficient involves the squared ratio of the helicity flip Pomeron to the conventional BFKL Pomeron which at present is unknown. Below we present a first estimate of the eikonal, helicity flip two-gluon exchange amplitude for moderately small x from a nonperturbative LC quark model of the proton.

For a longitudinal virtual photon,

$$\begin{aligned}
\mathcal{M}_{0\Lambda;0,-\Lambda} &= -2\pi e q_c N_c \Lambda e^{i\Lambda\phi_\Delta} \int_z \int r_\perp dr_\perp \mathcal{P}_{S0}(r_\perp, \Delta_\perp) \mathcal{A}_{LL}(r_\perp, z) [J_2(r_\perp \delta_\perp) - J_0(r_\perp \delta_\perp)] \sim \Delta_\perp^1, \\
\mathcal{M}_{0\Lambda;\bar{\lambda},-\Lambda} &= -2\pi i N_c e q_c e^{i(-\bar{\lambda}+\Lambda)\phi_\Delta} \int_z \text{sgn}(z - \bar{z}) \int r_\perp dr_\perp \mathcal{A}_{LT}(r_\perp, z) \mathcal{P}_{S0}(r_\perp, \Delta_\perp) \\
&\quad \times [J_{-\bar{\lambda}+\Lambda+1}(r_\perp \delta_\perp) - J_{-\bar{\lambda}+\Lambda-1}(r_\perp \delta_\perp)] \sim \Delta_\perp^2, \quad (19)
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{M}_{0\Lambda;0,-\Lambda} &= 4\pi N_c e q_c \Lambda e^{i\Lambda\phi_\Delta} \int_z \int r_\perp dr_\perp \mathcal{P}_{S0}^\perp(r_\perp, \Delta_\perp) \mathcal{A}_{LL}(r_\perp, z) J_0(r_\perp \delta_\perp) \sim \Delta_\perp^1, \\
\mathcal{M}_{0\Lambda;\bar{\lambda},-\Lambda} &= -4\pi i N_c e q_c \Lambda e^{i(-\bar{\lambda}+\Lambda)\phi_\Delta} \int_z \text{sgn}(z - \bar{z}) \int r_\perp dr_\perp \mathcal{P}_{S0}^\perp(r_\perp, \Delta_\perp) \mathcal{A}_{LT}(r_\perp, z) J_{\bar{\lambda}}(r_\perp \delta_\perp) \sim \Delta_\perp^2. \quad (20)
\end{aligned}$$

Hence, at nonzero photon virtuality we obtain an additional contribution to the cross section at $\mathcal{O}(\Delta_\perp^2)$.

Isolating the contribution of the SDP requires access to the spin flip amplitude. We comment on the suggestion of Refs. [15,27] to use target single spin asymmetries via the cross section difference $\Delta\sigma(\mathbf{S}_\perp) = \sigma(\mathbf{S}_\perp) - \sigma(-\mathbf{S}_\perp)$, where the initial proton is transversely polarized³ with spin $\mathbf{S}_\perp$. Since the proton's spin enters the cross section alongside a factor of “i,” $\Delta\sigma(\mathbf{S}_\perp)$ ends up probing the *imaginary* part of the interference between the proton helicity flip and the nonflip

³An obvious alternative would be to detect the polarization of the recoil proton, but to our knowledge this is not part of the electron-ion collider (EIC) polarimetry program, though there are proposals for such measurements at JLab [28].

amplitudes

$$\Delta\sigma(\mathbf{S}_\perp) \sim \text{Im}[\mathcal{M}_{nf}(\gamma \rightarrow V)\mathcal{M}_f^*(\gamma \rightarrow V)]. \quad (21)$$

However, in the high energy limit employed here, the corresponding amplitudes $\mathcal{M}_{nf}$ and $\mathcal{M}_f$ are real, and so single spin asymmetries vanish. The real part of the interference can be captured through the *double* spin asymmetry $\Delta\sigma(\lambda_e = +1, \mathbf{S}_\perp) - \Delta\sigma(\lambda_e = -1, \mathbf{S}_\perp)$ where the incoming electron beam has longitudinal polarization λ_e . The presence of an additional spin supplies another factor of “i”, and so

$$\begin{aligned} &\Delta\sigma(\lambda_e = +1, \mathbf{S}_\perp) - \Delta\sigma(\lambda_e = -1, \mathbf{S}_\perp) \\ &\sim \text{Re}[\mathcal{M}_{nf}(\gamma_L \rightarrow V)\mathcal{M}_f^*(\gamma_T \rightarrow V) \\ &\quad - \mathcal{M}_{nf}(\gamma_T \rightarrow V)\mathcal{M}_f^*(\gamma_L \rightarrow V)]. \end{aligned} \quad (22)$$

A nonvanishing asymmetry is now realized thanks to the interference between longitudinal and transverse photons.

V. ESTIMATE OF THE SPIN DEPENDENT POMERON AT MODERATELY SMALL x

We now provide a numerical estimate of the magnitude of the spin dependent Pomeron at moderately small x from a light-front constituent quark model for the proton. Of course, such models lack a deeper theoretical justification and should be viewed as empirically motivated parametrizations of the structure of the proton at moderate and large x (and low resolution scales). In particular, some recent ideas about the fundamental, nonperturbative origin of the spin of the proton refer to the role of anomalous interactions

and topology [29–37], with the difference in the number of left- vs right-handed quarks given by the topological charge density in the proton. The light-front constituent quark model we employ assumes that the helicity wave function of the proton emerges simply by Melosh transformation of nonrelativistic Pauli spinors for massive quarks to the light front.

Empirically motivated light-front quark models have been used extensively in the literature to compute Dirac and Pauli form factors and anomalous magnetic moments [38–40] of proton and neutron, transverse momentum dependent parton distributions [41–43], GPDs [41], gravitational form factors [44–46], spin independent two-gluon (Pomeron) and three-gluon (Odderon) amplitudes [25,47–49], quark Wigner distributions [50], and polarized dipole scattering amplitudes for small- x helicity evolution [51]. Our numerical results for the helicity flip two-gluon exchange provide a first idea about its potential magnitude at moderate x , and they could be used in the future as initial conditions for QCD BFKL evolution to small x [10,11]. Furthermore, the expressions below demonstrate explicitly that the helicity flip two-gluon exchange is indeed eikonal, and they clarify its origin as due to orbital angular momentum transfer to the partons in the proton. The fact that parton orbital angular momentum modifies certain amplitudes even qualitatively is well known; for example, it changes the asymptotic behavior of the form factor ratio $Q^2 F_2(Q^2)/F_1(Q^2)$ from constant [52] to a logarithmic rise [53]. In the present context, it gives rise to a nonvanishing two-gluon exchange amplitude with proton helicity flip.

The eikonal dipole amplitude (3) in the two gluon exchange approximation, for charges in the fundamental representation, is given by [47]

$$\mathcal{P}_{\Lambda\Lambda'}(\mathbf{r}_\perp, \mathbf{\Delta}_\perp) = \frac{g^4 C_F}{2} \int \frac{d^2 \mathbf{q}_\perp}{(2\pi)^2} \frac{\cos(\frac{\mathbf{r}_\perp \cdot \mathbf{\Delta}_\perp}{2}) - \cos(\mathbf{r}_\perp \cdot (\mathbf{q}_\perp - \mathbf{\Delta}_\perp))}{\mathbf{q}_\perp^2 (\mathbf{q}_\perp - \mathbf{\Delta}_\perp)^2} G_{2,\Lambda\Lambda'}(\mathbf{q}_\perp, \mathbf{\Delta}_\perp - \mathbf{q}_\perp). \quad (23)$$

Here, $g^2/4\pi = \alpha_s$ is the QCD coupling; for the figures below we chose $\alpha_s = 0.35$. $G_{2,\Lambda\Lambda'}(\mathbf{q}_{1\perp}, \mathbf{q}_{2\perp})$ represents the matrix element of two eikonal color charge operators $J^{+a}(\mathbf{q}_\perp) = \int dx^- J^{+a}(x^-, \mathbf{q}_\perp)$ between proton states with helicities Λ and Λ' :

$$\langle K, \Lambda' | J^{+a}(\mathbf{q}_{1\perp}) J^{+b}(\mathbf{q}_{2\perp}) | P, \Lambda \rangle = \frac{1}{2} \delta^{ab} 16\pi^3 P^+ \delta(P^+ - K^+) \delta^{(2)}(\mathbf{P}_\perp - \mathbf{K}_\perp - \mathbf{q}_{1\perp} - \mathbf{q}_{2\perp}) G_{2,\Lambda\Lambda'}(\mathbf{q}_{1\perp}, \mathbf{q}_{2\perp}), \quad (24)$$

given by

$$\begin{aligned} G_{2,\Lambda\Lambda'}(\mathbf{q}_{1\perp}, \mathbf{q}_{2\perp}) &= \int [dx_i] \int [d^2 k_i] \sum_{\{\lambda_i\}} \sum_j [\Phi_{\Lambda'}^*(\lambda_i, x_i, \mathbf{k}_{i(j)\perp}) \Psi^*(x_i, \mathbf{k}_{i(j)\perp}) \\ &\quad - \frac{1}{2} \sum_{j' \neq j} \Phi_{\Lambda'}^*(\lambda_i, x_i, \mathbf{k}_{i(jj')\perp}) \Psi^*(x_i, \mathbf{k}_{i(jj')\perp})] \Phi_\Lambda(\lambda_i, x_i, \mathbf{k}_{i\perp}) \Psi(x_i, \mathbf{k}_{i\perp}), \end{aligned} \quad (25)$$

with $\mathbf{k}_{i(j)\perp} = \mathbf{k}_{i\perp} + (x_i - \delta_{ij})(\mathbf{q}_{1\perp} + \mathbf{q}_{2\perp})$, $\mathbf{k}_{i(jj')\perp} = \mathbf{k}_{i\perp} + x_i(\mathbf{q}_{1\perp} + \mathbf{q}_{2\perp}) - \mathbf{q}_{1\perp}\delta_{ij} - \mathbf{q}_{2\perp}\delta_{ij'}$. The explicit form of the integration measures over quark LC momentum fractions x_i and transverse momenta $\mathbf{k}_{i\perp}$ is given in Appendix B. For the spatial wave function $\Psi(x_i, \mathbf{k}_{i\perp})$ we employ a simple model due to Schlumpf [38,39]. The spin flip two-gluon exchange amplitude has already been studied long ago in a nonrelativistic quark-diquark model of the proton [54,55]. It was found that in the soft regime the spin-flip coupling is small and that it is quite sensitive to diquark correlations. Here, we restrict to hard gluon exchanges where $|\mathbf{q}_{1\perp}|, |\mathbf{q}_{2\perp}| \gg \Lambda_{\text{QCD}}$ due to the fact that the t -channel gluons also couple to a small dipole. In this regime diquark correlations in the proton may be less significant although such questions would deserve further scrutiny in the future.

The helicity wave functions $\Phi_\Lambda(\lambda_i, x_i, \mathbf{k}_{i\perp})$ for a proton with helicity Λ are obtained through a Melosh transformation of rest frame Pauli spinors to the light front [38,39,42,56]. Their explicit expressions for $\Lambda = +1$ and the $|uud\rangle$ flavor state are given in Ref. [42], for example. In the collinear limit, the only nonzero functions for

$\Lambda = +1$ are $\Phi_+(++-, x_i, \mathbf{k}_{i\perp} = 0) = 2/\sqrt{6}$, $\Phi_+(-+-, x_i, \mathbf{k}_{i\perp} = 0) = \Phi_+(-++-, x_i, \mathbf{k}_{i\perp} = 0) = -1/\sqrt{6}$, i.e., they reduce to the well-known nonrelativistic SU(2) spin wave functions, as the Melosh transformation for $\mathbf{k}_{i\perp} = 0$ is trivial. Note that in this case the product $\Phi_+^*(\lambda_i, x_i, \mathbf{k}_{i\perp} = 0)\Phi_-(\lambda_i, x_i, \mathbf{k}_{i\perp} = 0) = 0$ for any given set $\{\lambda_i\}$ of quark helicities: a helicity flip of the proton is not possible when the quarks are collinear and carry no orbital angular momentum. However, this is no longer the case when $\mathbf{k}_{i\perp} \neq 0$ due to the fact that the Melosh transformations depend on the quark transverse momenta. From the explicit form of the helicity wave functions $\Phi_\Lambda(\lambda_i, x_i, \mathbf{k}_{i\perp})$ one may also verify that $G_{2,\Lambda\Lambda'}(\mathbf{q}_{1\perp}, -\mathbf{q}_{1\perp}) \sim \delta_{\Lambda\Lambda'}$ i.e., the helicity flip amplitude vanishes in the forward limit.

In practice, we compute $\mathcal{P}_{\Lambda\Lambda'}(\mathbf{r}_\perp, \Delta_\perp)$ numerically from (23) and obtain the scalar functions $\mathcal{P}$, $\mathcal{P}_S$ and $\mathcal{P}_S^\perp$ via (4). For the SIP we extract $\mathcal{P}_0(\mathbf{r}_\perp, \Delta_\perp)$ and $\mathcal{P}_\epsilon(\mathbf{r}_\perp, \Delta_\perp)$ from $\mathcal{P}_{+,+1}(\mathbf{r}_\perp, \Delta_\perp)$. For $\mathcal{P}_S(\mathbf{r}_\perp, \Delta_\perp)$ and $\mathcal{P}_S^\perp(\mathbf{r}_\perp, \Delta_\perp)$ we determine the leading Fourier harmonics that appear in the amplitudes (16) and (17). A convenient way to extract them is through the following angular projections:

$$\begin{aligned} \mathcal{P}_{S0}(\mathbf{r}_\perp, \Delta_\perp) &= 8 \int_0^{2\pi} \frac{d\phi_\Delta}{2\pi} \int_0^{2\pi} \frac{d\phi_r}{2\pi} \cos(\phi_\Delta) \sin(2\phi_r) \text{Im} \mathcal{P}_{-1,+1}(\mathbf{r}_\perp, \Delta_\perp), \\ \mathcal{P}_{S0}^\perp(\mathbf{r}_\perp, \Delta_\perp) &= 2 \int_0^{2\pi} \frac{d\phi_\Delta}{2\pi} \int_0^{2\pi} \frac{d\phi_r}{2\pi} \cos(\phi_\Delta) [\text{Re} \mathcal{P}_{-1,+1}(\mathbf{r}_\perp, \Delta_\perp) - 2 \sin(2\phi_r) \text{Im} \mathcal{P}_{-1,+1}(\mathbf{r}_\perp, \Delta_\perp)]. \end{aligned} \quad (26)$$

Figure 1 shows $\mathcal{P}_0$ (left) and $\mathcal{P}_\epsilon$ (right) as functions of $\Delta_\perp$ for $r_\perp = 0.25$ fm which corresponds approximately to the size of a J/ψ or $\psi(2S)$. The Melosh rotation gives a percentage-level correction in the forward $\Delta_\perp \rightarrow 0$ limit for $\mathcal{P}_0$. Increasing $\Delta_\perp$ increases the phase space for the orbital motion of quarks, and so the effect of the Melosh rotation becomes more important, leading to corrections of several tens of percent at $\Delta_\perp \sim 1$ –2 GeV.

In Fig. 2 we show the SDPs $\mathcal{P}_{S0}$ and $\mathcal{P}_{S0}^\perp$ and their ratio to the isotropic Pomeron: $v_S = \mathcal{P}_{S0}/\mathcal{P}_0$ and $v_S^\perp = \mathcal{P}_{S0}^\perp/\mathcal{P}_0$. We also compare to the v_2 -coefficient defined from the elliptic Pomeron: $v_2 = \mathcal{P}_\epsilon/\mathcal{P}_0$. According to the model computation, both v_S and $v_S^\perp$ are negative and may reach magnitudes of a few percentages as $\Delta_\perp$ increases. The combination $v_S + 2v_S^\perp$ that enters the helicity flip amplitudes can reach up to $\sim 15\%$ for $\Delta_\perp \sim 1$ –2 GeV.

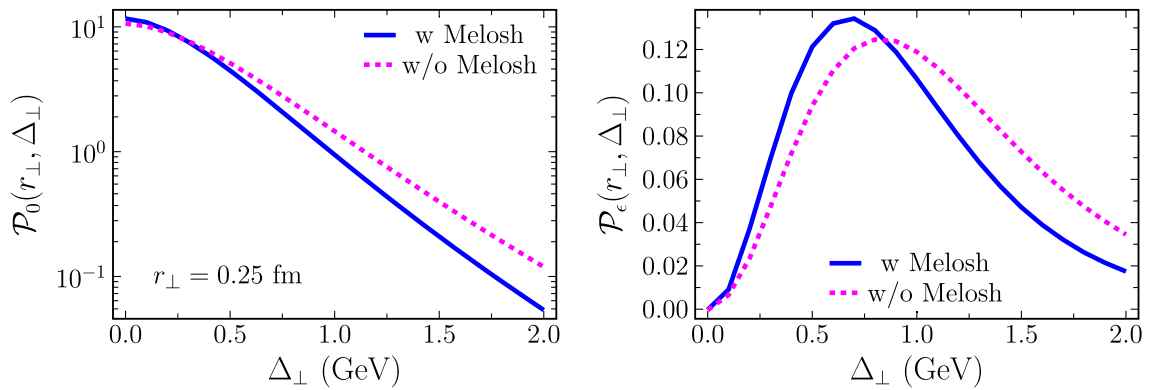


FIG. 1. Left: isotropic Pomeron $\mathcal{P}_0(\mathbf{r}_\perp, \Delta_\perp)$ with y-axis on a log scale. Right: elliptic Pomeron $\mathcal{P}_\epsilon(\mathbf{r}_\perp, \Delta_\perp)$.

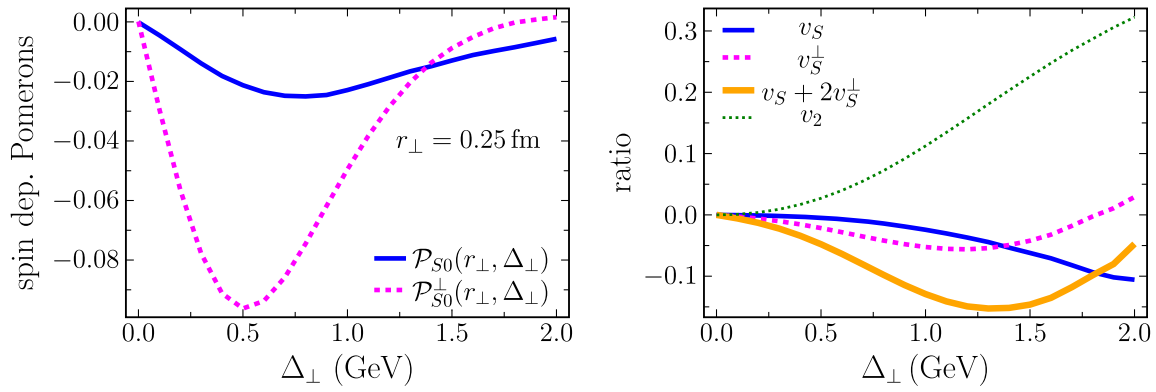


FIG. 2. Left: first harmonics of the spin-dependent Pomerons $\mathcal{P}_{S0}(r_\perp, \Delta_\perp)$ and $\mathcal{P}_{S0}^\perp(r_\perp, \Delta_\perp)$. Right: their ratios to the isotropic Pomeron $\mathcal{P}_0(r_\perp, \Delta_\perp)$. For comparison we also plot $v_2 = \mathcal{P}_e/\mathcal{P}_0$.

VI. SUMMARY AND DISCUSSION

In this paper we have computed the amplitudes for VM production due to eikonal BFKL (hard) Pomeron exchange, for all helicity configurations in the initial and final states, including helicity flip of the proton. These amplitudes are important for production at high energies and high momentum transfer.

SCHC in exclusive VM production at leading power in energy asserts that no helicity flip occurs in the transition of the photon to the J/ψ or $\psi(2S)$ meson, nor in the transition of the incoming to the scattered proton. This assertion holds in the forward limit of vanishing momentum transfer $\Delta_\perp$. Corrections to SCHC arise at nonzero $\Delta_\perp$; we have argued that these originate not only from higher Fourier harmonics of the usual nonflip BFKL Pomeron but also from the “spin dependent Pomeron” associated with a helicity flip of the proton.

The H1 and ZEUS experiments at HERA, and the ALICE experiment at the LHC, have tested helicity conservation in the photon to VM transition, and found it to be satisfied. However, we have argued that helicity nonconservation in this transition is suppressed not only by powers of the size of the VM times the momentum transfer but also by, either, powers of the (small) LC momentum imbalance $z - \bar{z}$ of quarks in the VM state, or by the (small) ratio of the “elliptic” to the isotropic BFKL unintegrated gluon distribution. On the other hand, helicity flips of the proton are not suppressed by powers of $z - \bar{z}$ but only by the ratio of the helicity flip vs nonflip unintegrated gluon distributions. At small $\Delta_\perp \ll m_c$ a collinear limit could be employed [57,58] to relate the “spin dependent Pomeron” to the GPD $E_g(x, t)$.

According to our numerical estimates the contribution to the $\gamma + p \rightarrow V + p$ cross section from proton helicity flip amplitudes is small when $\Delta_\perp R_V < 1$ but could reach a significant level for large $\Delta_\perp R_V \sim 1$, where R_V denotes the size of the VM. In this regime, the dipole (or $k_\perp$ -) factorization approach used here provides corrections to the collinear limit. A natural observable sensitive to proton

helicity flips from spin-dependent Pomerons would be spin asymmetries in exclusive J/ψ production [27,59], which could be measured in ultraperipheral $p^\dagger A$ collisions at RHIC [60] and/or the LHC [61,62]. However, as we have pointed out, single target spin asymmetries vanish in the eikonal limit, i.e., they are suppressed by powers of energy and may be difficult to access experimentally in high-energy collider experiments. Our proposal is to consider double spin asymmetries in ep collisions with transversely polarized proton and longitudinally polarized electron in the initial state, requiring also the detection of the recoil electron. We intend to provide numerical estimates for EIC kinematics in the future. If such measurement can be performed at the EIC then the expressions we derived will provide a basis for relating data for high momentum transfer to the dipole formalism.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: PHOTON-VECTOR MESON OVERLAPS

The light cone wave function $\gamma - V$ overlaps can be written in the form [18]

$$\mathcal{A}_{\lambda\bar{\lambda}}(\mathbf{r}_\perp, \mathbf{\Delta}_\perp) = eq_c \int_z e^{-i\delta_\perp \cdot \mathbf{r}_\perp} \int \frac{d^2 \mathbf{l}_\perp}{(2\pi)^2} \frac{e^{i\mathbf{l}_\perp \cdot \mathbf{r}_\perp}}{\mathbf{l}_\perp^2 + \varepsilon^2} \int \frac{d^2 \mathbf{l}'_\perp}{(2\pi)^2} e^{-i\mathbf{l}'_\perp \cdot \mathbf{r}_\perp} \phi(\mathbf{l}_{1\perp}, z) \frac{1}{z\bar{z}} A_{\lambda\bar{\lambda}}(\mathbf{l}_\perp, \mathbf{l}_{1\perp}, z). \quad (\text{A1})$$

Here, $q_c = 2/3$ is the fractional electric charge of the c -quark, $\phi(\mathbf{l}_{1\perp}, z)$ is the VM wave function, $\mathbf{l}_{1\perp} = \mathbf{l}'_\perp - z\mathbf{\Delta}_\perp$, and $\varepsilon^2 = m_c^2 + z\bar{z}Q^2$. The function $A_{\lambda\bar{\lambda}}(\mathbf{l}_\perp, \mathbf{l}_{1\perp}, z)$ can be expressed as a Dirac trace,

$$A_{\lambda\bar{\lambda}}(\mathbf{l}_\perp, \mathbf{l}_{1\perp}, z) = \frac{1}{(2q^-)^2} \text{tr}[(\not{l} + m_c) \not{\epsilon}(\lambda, q) (\not{l} - \not{q} + m_c) \gamma^- (\not{l}' - \not{\Delta} + m_c) \not{E}(\bar{\lambda}, \mathbf{\Delta}_\perp) (\not{l}' + m_c) \gamma^-], \quad (\text{A2})$$

with $\epsilon^\mu(\lambda, q)$ and $E^\mu(\bar{\lambda}, \mathbf{\Delta})$ the polarization of the photon and the VM, respectively. We have $\epsilon^\mu(\lambda = 0, q) = (Q/q^-, 0, \mathbf{0}_\perp)$ and $\epsilon^\mu(\lambda = \pm 1, q) = (0, 0, \epsilon_\perp^\lambda)$, where $\epsilon_\perp^\lambda = (-\lambda, -i)/\sqrt{2}$. The VM polarization is

$$E^\mu(\bar{\lambda} = 0, \mathbf{\Delta}) = \frac{1}{M_V} \Delta^\mu - \frac{M_V}{\Delta^-} n^\mu, \quad E^\mu(\bar{\lambda} = \pm 1, \mathbf{\Delta}) = \left(\frac{\epsilon_\perp^{\bar{\lambda}} \cdot \mathbf{\Delta}_\perp}{\Delta^-}, 0, \epsilon_\perp^{\bar{\lambda}} \right), \quad (\text{A3})$$

where M_V is the VM mass. l and l' represent on shell momenta with $l^- = l'^- = zq^-$. The traces $A_{\lambda\bar{\lambda}}(\mathbf{l}_\perp, \mathbf{l}_{1\perp}, z)$ evaluate to

$$\begin{aligned} A_{\lambda=\pm 1, \bar{\lambda}=0}(\mathbf{l}_\perp, \mathbf{l}_{1\perp}, z) &= -4M_V z\bar{z}(z - \bar{z})(\epsilon_\perp^\lambda \cdot \mathbf{l}_\perp), \\ A_{\lambda=\pm 1, \bar{\lambda}=\pm 1}(\mathbf{l}_\perp, \mathbf{l}_{1\perp}, z) &= 2[(\epsilon_\perp^\lambda \cdot \epsilon_\perp^{\bar{\lambda}*})(\mathbf{l}_\perp \cdot \mathbf{l}_{1\perp} + m^2) + (z - \bar{z})^2 (\epsilon_\perp^\lambda \cdot \mathbf{l}_\perp)(\epsilon_\perp^{\bar{\lambda}*} \cdot \mathbf{l}_{1\perp}) - (\epsilon_\perp^\lambda \cdot \mathbf{l}_{1\perp})(\epsilon_\perp^{\bar{\lambda}*} \cdot \mathbf{l}_\perp)]. \end{aligned} \quad (\text{A4})$$

From these we can work out the overlaps $\mathcal{A}_{\lambda\bar{\lambda}}(\mathbf{r}_\perp, \mathbf{\Delta}_\perp)$ to uncover the decomposition in (2) and the associated functions $\mathcal{A}_{TL}(r_\perp, z)$ and $\mathcal{A}_{TT}^{nf,f}(r_\perp, z)$. For TT polarizations, for example, after performing the integrals over $\mathbf{l}_\perp$ and $\mathbf{l}'_\perp$ we obtain

$$\begin{aligned} \mathcal{A}_{\lambda=\pm 1, \bar{\lambda}=\pm 1}(\mathbf{r}_\perp, \mathbf{\Delta}_\perp) &= \frac{eq_c}{\pi} \int_z e^{-i\delta_\perp \cdot \mathbf{r}_\perp} \frac{1}{z\bar{z}} \left[(\epsilon_\perp^\lambda \cdot \epsilon_\perp^{\bar{\lambda}*}) \left(-\varepsilon K_1(\varepsilon r_\perp) \frac{\partial \phi_T}{\partial r_\perp} + m_c^2 K_0(\varepsilon r_\perp) \phi_T(r_\perp, z) \right) \right. \\ &\quad \left. + (\epsilon_\perp^\lambda \cdot \hat{\mathbf{r}}_\perp)(\epsilon_\perp^{\bar{\lambda}*} \cdot \hat{\mathbf{r}}_\perp) 4z\bar{z}\varepsilon K_1(\varepsilon r_\perp) \frac{\partial \phi_T}{\partial r_\perp} \right]. \end{aligned} \quad (\text{A5})$$

Plugging in the explicit expressions for the transverse polarization vectors we finally arrive at

$$\mathcal{A}_{TT}^{nf}(r_\perp, z) = \frac{1}{\pi} \frac{1}{z\bar{z}} \left[-(z^2 + \bar{z}^2) \varepsilon K_1(\varepsilon r_\perp) \frac{\partial \phi_T}{\partial r_\perp} + m_c^2 K_0(\varepsilon r_\perp) \phi_T(r_\perp, z) \right], \quad \mathcal{A}_{TT}^f(r_\perp, z) = \frac{2}{\pi} \varepsilon K_1(\varepsilon r_\perp) \frac{\partial \phi_T}{\partial r_\perp}, \quad (\text{A6})$$

that were introduced in the second line of (2). A similar calculation leads to $A_{\lambda 0}(\mathbf{l}_\perp, \mathbf{l}_{1\perp}, z) = -4M_V z\bar{z}(z - \bar{z})(\epsilon_\perp^\lambda \cdot \mathbf{l}_\perp)$ leading to the first line of (2) and $\mathcal{A}_{TL}(r_\perp, z)$ given as

$$\mathcal{A}_{TL}(r_\perp, z) = \frac{\sqrt{2}i}{\pi} M_V (z - \bar{z}) \varepsilon K_1(\varepsilon r_\perp) \phi_L(r_\perp, z). \quad (\text{A7})$$

The functions $\phi_{T,L}(r_\perp, z)$ correspond to the two nonperturbative wave functions of the transversely or longitudinally polarized VM; see, e.g., Ref. [19].

For the longitudinal photon with virtuality Q^2 we find

$$A_{\lambda=0, \bar{\lambda}=0}(\mathbf{l}_\perp, \mathbf{l}_{1\perp}, z) = -8QM_V z^2 \bar{z}^2, \quad A_{\lambda=0, \bar{\lambda}=\pm 1}(\mathbf{l}_\perp, \mathbf{l}_{1\perp}, z) = 4Qz\bar{z}(z - \bar{z})(\epsilon_\perp^{\bar{\lambda}*} \cdot \mathbf{l}_{1\perp}). \quad (\text{A8})$$

The $\gamma_L^* - V$ wave function overlaps are

$$\mathcal{A}_{\lambda=0, \bar{\lambda}=0}(\mathbf{r}_\perp, \mathbf{\Delta}_\perp) = eq_c \int_z e^{-i\delta_\perp \cdot \mathbf{r}_\perp} \mathcal{A}_{LL}(r_\perp, z), \quad \mathcal{A}_{\lambda=0, \bar{\lambda}=\pm 1}(\mathbf{r}_\perp, \mathbf{\Delta}_\perp) = eq_c \bar{\lambda} e^{-i\bar{\lambda} \phi_r} \int_z e^{-i\delta_\perp \cdot \mathbf{r}_\perp} \mathcal{A}_{LT}(r_\perp, z), \quad (\text{A9})$$

where

$$\mathcal{A}_{LL}(r_\perp, z) = -\frac{4}{\pi} QM_V z\bar{z} K_0(\varepsilon r_\perp) \phi_L(r_\perp, z), \quad \mathcal{A}_{LT}(r_\perp, z) = -\frac{\sqrt{2}i}{\pi} Q(z - \bar{z}) K_0(\varepsilon r_\perp) \frac{\partial \phi_T}{\partial r_\perp}. \quad (\text{A10})$$

APPENDIX B: POMERON AMPLITUDE AT MODERATELY SMALL x FROM A LIGHT-FRONT CONSTITUENT QUARK MODEL

We now provide explicit expressions for the Pomeron exchange amplitudes at moderately small x off a proton described by a light-front quark model. Our expressions generalize similar expressions from Refs. [47,49] where, however, a helicity flip of the proton had not been considered.

We write the proton state in the light-front (LF) quark model in the form

$$|P, \Lambda\rangle = \int [dx_i] \int [d^2\mathbf{k}_{i\perp}] \sum_{j_1, j_2, j_3} \frac{\epsilon^{j_1 j_2 j_3}}{\sqrt{N_c!}} \sum_{\{\lambda_i\}} \Phi_\Lambda(\lambda_i, \mathbf{k}_{i\perp}) \Psi(x_i, \mathbf{k}_{i\perp}) |\{x_i P^+, \mathbf{k}_{i\perp}, \lambda_i, j_i\}\rangle. \quad (\text{B1})$$

Here, x_i denotes the LC momentum fraction of the i th quark where $i = 1 \dots N_c = 3$; $\mathbf{k}_{i\perp}$ is its transverse momentum relative to the center-of-mass transverse momentum $\mathbf{P}_\perp$ of the proton; j_i refers to its color; λ_i to its helicity. The integrations over x_i and $\mathbf{k}_{i\perp}$ are given by

$$[dx_i] = \prod_{i=1 \dots 3} \frac{dx_i}{2x_i} \delta\left(1 - \sum_i x_i\right),$$

$$[d^2\mathbf{k}_{i\perp}] = \prod_{i=1 \dots 3} \frac{d^2\mathbf{k}_{i\perp}}{(2\pi)^3} (2\pi)^3 \delta^{(2)}\left(\sum_i \mathbf{k}_{i\perp}\right). \quad (\text{B2})$$

The spatial wave function $\Psi(x_i, \mathbf{k}_{i\perp})$ is symmetric under the exchange of any two quarks and invariant under simultaneous rotations or reflections of all transverse momenta $\mathbf{k}_{i\perp}$. For numerical estimates we employ a simple model due to Schlumpf [38,39],

$$\Psi(x_i, \mathbf{k}_i) \sim \sqrt{x_1 x_2 x_3} e^{-\mathcal{M}^2/2\beta^2}, \quad \mathcal{M}^2 = \sum_i \frac{\mathbf{k}_{i\perp}^2 + m_q^2}{x_i}. \quad (\text{B3})$$

The quark mass $m_q = 0.26$ GeV and the parameter $\beta = 0.55$ GeV have been tuned to reproduce the electromagnetic “radius” and the anomalous magnetic moments of proton and neutron⁴; they also lead to reasonably good agreement of the Dirac and Pauli form factors of the proton with data. The normalization of the spatial wave function follows from the requirement that $\langle K, \Lambda' | P, \Lambda \rangle = 16\pi^3 P^+ \delta(P^+ - K^+) \delta^{(2)}(\mathbf{P}_\perp - \mathbf{K}_\perp) \delta_{\Lambda\Lambda'}$.

The helicity wave functions $\Phi_\Lambda(\lambda_i, \mathbf{k}_{i\perp})$ in (B1) for a proton with helicity Λ are obtained through a Melosh transformation of rest frame Pauli spinors to the LF. Their explicit expressions for $\Lambda = +1$ are given in Ref. [42]; here we add only that the functions for $\Lambda = -1$ are obtained by a sign flip of Φ , and the exchange $k_{iL} \leftrightarrow -k_{iR}$, where $k_{R/L} = k_\perp^1 \pm i k_\perp^2 = k_\perp e^{\pm i\phi_k}$, and $\lambda_i \rightarrow -\lambda_i$.

The eikonal dipole amplitude at order $(gA^+)^2$ (two gluon exchange) has been expressed in Eq. (23) in terms of the color charge correlator $G_{2,\Lambda\Lambda'}(\mathbf{q}_{1\perp}, \mathbf{q}_{2\perp})$ of Eq. (25). This color charge correlator vanishes, for any Λ, Λ' when either $\mathbf{q}_{1\perp}$ or $\mathbf{q}_{2\perp}$ go to zero as gluons with wavelength greater than the size of the proton decouple. Furthermore, in the forward $t \rightarrow 0$ limit, $G_{2,\Lambda\Lambda'}(\mathbf{q}_\perp, -\mathbf{q}_\perp) \sim \delta_{\Lambda\Lambda'}$, i.e., a helicity flip of the proton is then not possible.⁵ Equation (25) provides an explicit illustration for the origin of the proton helicity flip in the eikonal limit: it is due to the Melosh transformation from Pauli to LC helicity spinors which depends on the quark transverse momenta [42,56]. A transverse momentum transfer can then lead to nonzero overlap of the helicity wave functions for $\Lambda \neq \Lambda'$ unlike in a nonrelativistic quark model of the proton.

APPENDIX C: GTMDS OF THE EIKONAL DIPOLE AND THE SPIN DEPENDENT POMERONS

The connection of the Pomeron $\mathcal{P}_{\Lambda\Lambda'}(\mathbf{r}_\perp, \mathbf{\Delta}_\perp)$ to the GTMDS has been established in Refs. [8,9]. From Eqs. (14) and (15) in [9] we have

$$\int d^2\mathbf{r}_\perp e^{-i\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \mathcal{P}_{\Lambda\Lambda'}(\mathbf{r}_\perp, \mathbf{\Delta}_\perp) = -\frac{g^2(2\pi)^3}{4N_c M(\mathbf{k}_\perp^2 - \mathbf{\Delta}_\perp^2/4)} \left[M\delta_{\Lambda\Lambda'} f_{1,1}(\mathbf{k}_\perp, \mathbf{\Delta}_\perp) + \Delta_\perp \frac{\mathbf{k}_\perp^2}{M^2} \delta_{\Lambda, -\Lambda'} \Lambda e^{i\Lambda\phi_k} f_{1,2}(\mathbf{k}_\perp, \mathbf{\Delta}_\perp) \right. \\ \left. + \Delta_\perp \Lambda \delta_{\Lambda, -\Lambda'} e^{i\Lambda\phi_\Delta} \left(f_{1,3}(\mathbf{k}_\perp, \mathbf{\Delta}_\perp) - \frac{1}{2} f_{1,1}(\mathbf{k}_\perp, \mathbf{\Delta}_\perp) \right) \right], \quad (\text{C1})$$

⁴We have omitted the flavor wave function in (B1) as this does not play a role in our analysis.

⁵This statement refers specifically to two gluon exchange where in the $t \rightarrow 0$ limit $\mathbf{q}_{1\perp} = -\mathbf{q}_{2\perp}$ are anticollinear. For a C -odd three gluon exchange a helicity flip of the proton is possible even in the forward limit where it is related to the gluon Sivvers function [8,63–65].

where the $f_{1,i}(\mathbf{k}_\perp, \mathbf{\Delta}_\perp)$ are the real parts of the GTMDs, and M is the proton mass. The imaginary parts of the GTMDs are related to the Odderons which are not relevant in the context of VM production. The Pomeron amplitudes are related to the $f_{1,i}$ as follows [9]:

$$\begin{aligned}
 \mathcal{P}(\mathbf{r}_\perp, \mathbf{\Delta}_\perp) &= \frac{g^2(2\pi)^3}{4N_c} \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} e^{i\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \frac{f_{1,1}(\mathbf{k}_\perp, \mathbf{\Delta}_\perp)}{\mathbf{k}_\perp^2 - \mathbf{\Delta}_\perp^2/4}, \\
 \mathcal{P}_S(\mathbf{r}_\perp, \mathbf{\Delta}_\perp) &= \frac{g^2(2\pi)^3}{4N_c} \frac{\Delta_\perp}{M} \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} e^{i\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \left[2 \frac{(\mathbf{r}_\perp \cdot \mathbf{k}_\perp)^2}{r_\perp^2 M^2} - \frac{\mathbf{k}_\perp^2}{M^2} \right] \frac{f_{1,2}(\mathbf{k}_\perp, \mathbf{\Delta}_\perp)}{\mathbf{k}_\perp^2 - \mathbf{\Delta}_\perp^2/4}, \\
 \mathcal{P}_S^\perp(\mathbf{r}_\perp, \mathbf{\Delta}_\perp) &= \frac{g^2(2\pi)^3}{4N_c} \frac{\Delta_\perp}{M} \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} e^{i\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \left[-\frac{1}{2} f_{1,1}(\mathbf{k}_\perp, \mathbf{\Delta}_\perp) + \left(-\frac{(\mathbf{r}_\perp \cdot \mathbf{k}_\perp)^2}{r_\perp^2 M^2} + \frac{\mathbf{k}_\perp^2}{M^2} \right) f_{1,2}(\mathbf{k}_\perp, \mathbf{\Delta}_\perp) \right. \\
 &\quad \left. + f_{1,3}(\mathbf{k}_\perp, \mathbf{\Delta}_\perp) \right] \frac{1}{\mathbf{k}_\perp^2 - \mathbf{\Delta}_\perp^2/4}. \tag{C2}
 \end{aligned}$$

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