

String-based axial and helicity-flip GPDs: A comparison to lattice QCD

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We construct an analytic, string-based representation of the nucleon's axial and helicity-flip conformal moments of generalized parton distributions (GPDs) that holds for *any* skewness and for both the quark and gluon channels. The starting point is the Mellin-Barnes resummation of the conformal partial-wave expansion, where the moments are parametrized by open- (Reggeon) and closed-string (Pomeron) trajectories with slopes determined by experimental form factors and meson/glueball spectroscopy. The forward limits are fixed by the empirical unpolarized and polarized parton distributions. Polynomiality, crossing symmetry, and support are satisfied by construction. After next-to-leading order Dokshitzer-Gribov-Lipatov-Altarelli-Parisi/Efremov-Radyushkin-Brodsky-Lepage evolution to $\mu = 2$ GeV our analytic framework (i) reproduces some of the currently available lattice moments of $\mathbb{E}$ and $\mathbb{H}$ in the nonsinglet sector, (ii) predicts sea-quark and gluon polarized moments that will be testable by forthcoming simulations and experiments at Jefferson Lab and the future Electron-Ion Collider, and (iii) yields axial and helicity-flip GPDs in x -space in reasonable agreement with lattice QCD.

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I. INTRODUCTION

Generalized parton distributions (GPDs) bridge the gap between elastic form factors and ordinary parton distribution functions (PDFs), encoding how quarks and gluons with longitudinal momentum fraction x are distributed in the transverse plane at a given momentum transfer t [1–3]. Among other things, their Mellin moments give direct access to the energy-momentum tensor and hence to the decomposition of the proton's mass and spin, while the x – η – t -dependent kernels that enter deeply virtual Compton scattering (DVCS) and hard exclusive meson production are of paramount importance in current experiments carried out at Jefferson Lab and at the future Electron-Ion Collider (EIC) at Brookhaven National Laboratory.

During the past decade, lattice QCD has begun to deliver precise, nonperturbative information on low-order Mellin moments of all leading twist GPDs, both in the unpolarized and helicity channels [4–10]. These results provide stringent benchmarks for any phenomenological model and a unique opportunity to constrain the elusive gluon and sea-quark sectors. On the theory side, gauge/string duality suggests that high-energy amplitudes are dominated by t -channel exchange of open and closed strings, corresponding to Reggeon and Pomeron/odderon trajectories with approximately linear slopes [11,12]. Embedding this idea into the conformal partial-wave formalism of GPDs naturally leads to a compact, analytic parametrization that enforces all symmetry constraints [13,14].

The present work generalizes earlier work on the unpolarized sector [15,16] at arbitrary skewness η to *both* the axial/polarized and helicity-flip sectors. Starting from the conformal partial-wave expansion, we

- (1) model the conformal moments by universal open-/closed-string Regge exchanges whose intercepts are fixed by the quantum numbers of the corresponding quark and gluon operator and with their slopes fixed to form factors or hadron spectroscopy;

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- (2) fix the forward limit to the empirical unpolarized and polarized PDF fits, thereby eliminating additional free parameters and ensuring compatibility with experimental constraints;
- (3) include the exact skewness dependence through a hypergeometric kernel derived in cubic string field theory, ensuring polynomiality and crossing symmetry at all orders;
- (4) evolve the resulting moments to $\mu = 2$ GeV with next-to-leading order (NLO) Dokshitzer-Gribov-Lipatov-Altarelli-Parisi/Efremov-Radyushkin-Brodsky-Lepage (DGLAP-ERBL) kernels and compare them with the latest lattice data;
- (5) resum the conformal partial-wave expansion by means of a complex Mellin-Barnes integral.

The resulting parametrization (i) reproduces some of the available lattice data of nonsinglet axial/polarized and helicity-flip moments up to $|t| \simeq 3$ GeV² with no *post hoc* tuning, (ii) delivers quantitative predictions for singlet sea-quark and gluon polarized moments—quantities that so far remain unmeasured, and (iii) yields a consistent picture of the proton spin budget and its impact-parameter densities, in line with recent FLAG [17] averages and spin observables computed on the lattice [8,18]. Recently, the present formulation was used to suggest a novel interpretation of the spin and orbital momentum correlations in the proton at finite rapidity or skewness [19].

The paper is organized as follows: In Sec. II, we fix our conventions and review the necessary kinematical parameters required for our parametrization. Section III reviews the conformal partial-wave expansion and Mellin-Barnes resummation. Section IV constructs the string-based conformal moments. In Secs. IV F and V, we present our numerical results for the axial and helicity-flip GPD moments, and their x -space dependence, including comparisons to available lattice QCD simulations. In Sec. V, we include our updated results for the unpolarized sea-quark and gluon GPDs in the ERBL region as originally developed in [15,16]. In Sec. VI, we summarize our main findings and outline future applications. We have collected our conventions on the kinematics in Appendix A; parametrization of conformal moments and partial waves (PWs) used can be found in Appendix B.

II. QUARK AND GLUON GPDs

In this section we introduce our conventions used for the leading-twist quark and gluon GPDs. We work in a symmetric frame with average momentum $P = (p_1 + p_2)/2$ and momentum transfer $q = p_2 - p_1$. For additional details on the resulting kinematical variables, we refer the reader to Appendix A.

The flavor- q quark GPDs are defined through nonlocal, bilinear operators on the light cone

$$\langle p_2 | \bar{\psi}_q(-z^-)[- , +] \gamma^+ \psi_q(z^-) | p_1 \rangle = \int_{-1}^1 dx e^{-ixP^+z^-} (h^+ H_q(x, \eta, t; \mu) + e^+ E_q(x, \eta, t; \mu)), \quad (1a)$$

$$\langle p_2 | \bar{\psi}_q(-z^-)[- , +] \gamma^+ \gamma^5 \psi_q(z^-) | p_1 \rangle = \int_{-1}^1 dx e^{-ixP^+z^-} (\tilde{h}^+ \tilde{H}_q(x, \eta, t; \mu) + \tilde{e}^+ \tilde{E}_q(x, \eta, t; \mu)), \quad (1b)$$

$$\langle p_2 | F_a^{+\mu}(-z^-)[- , +] F_{a\mu}^+(z^-) | p_1 \rangle = \int_{-1}^1 dx e^{-ixP^+z^-} (h^+ H_g(x, \eta, t; \mu) + e^+ E_g(x, \eta, t; \mu)), \quad (1c)$$

$$\langle p_2 | F_a^{+\mu}(-z^-)[- , +] \tilde{F}_{\mu a}^+(z^-) | p_1 \rangle = \int_{-1}^1 dx e^{-ixP^+z^-} (\tilde{h}^+ \tilde{H}_g(x, \eta, t; \mu) + \tilde{e}^+ \tilde{E}_g(x, \eta, t; \mu)), \quad (1d)$$

where $[- , +]$ denotes a straight Wilson line enforcing gauge invariance, μ is the renormalization scale, and the nucleon spinors are normalized as $\bar{N}(p, \lambda) N(p, \lambda) = 2m_N$. Their Dirac structures factorize into spin-conserving and spin-flip projectors

$$\begin{aligned} h^+ &= \bar{N}(p_2) \gamma^+ N(p_1), \\ e^+ &= \frac{1}{2m_N} \bar{N}(p_2) i \sigma^{\nu+} q_\nu N(p_1), \end{aligned} \quad (2a)$$

$$\begin{aligned} \tilde{h}^+ &= \bar{N}(p_2) \gamma^+ \gamma^5 N(p_1), \\ \tilde{e}^+ &= -\frac{q^+}{2m_N} \bar{N}(p_2) \gamma^5 N(p_1). \end{aligned} \quad (2b)$$

For fixed (x, η, t) the quark GPDs are real and satisfy

$$H_q(x, \eta, t) = H_q(x, -\eta, t), \quad E_q(x, \eta, t) = E_q(x, -\eta, t), \quad (3)$$

while the polarized GPDs are odd in η , thanks to time-reversal parity (TP symmetry) and hermiticity. Support properties are $H_q(x, \eta, t) = 0$ for $|x| > 1$ and likewise for the remaining GPDs. Polynomiality of Mellin moments

$$\int_{-1}^1 dx x^{n-1} H_a(x, \eta, t) = \sum_{k=0}^{\lfloor n/2 \rfloor} A_{n,2k}^{(a)}(t) \eta^{2k} \quad (4)$$

encodes Lorentz-invariance constraints and is central to the conformal partial-wave formalism employed below.

With these conventions, we now turn to the conformal partial-wave expansion that connects the local moments (4) to the full (x, η) dependence of GPDs.

III. CONFORMAL PARTIAL-WAVE EXPANSION

The leading-twist light-ray operators in Eq. (A1) form irreducible representations of the collinear conformal group $SL(2, \mathbb{R})$. Diagonalizing the corresponding evolution kernel in this basis yields an expansion of *any* quark or gluon GPD in terms of conformal PWs. In the central ERBL region ($|x| < \eta$) one has

$$H_a(x, \eta, t) = \sum_{n \in \mathcal{S}_a} (-1)^{n+1} P_n(x, \eta) \mathbb{H}_a(n, t, \eta), \quad (5)$$

where $\mathcal{S}_{q,g} = 2\mathbb{N} - 1$ for quark and helicity gluon GPDs, and $\mathcal{S}_g = 2\mathbb{N}$ for unpolarized gluon GPDs in accordance with C -parity. The basis polynomials

$$P_n(x, \eta) = \frac{(1 + x/\eta)^{n+1}}{(1 - x/\eta)^{n+2}} C_n^{\frac{3}{2}}\left(\frac{x}{\eta}\right) \quad (6)$$

are Gegenbauer polynomials dressed with the appropriate Jacobian factor to ensure proper normalization. They satisfy the orthogonality relation

$$\int_{-1}^1 dx (1 - x^2/\eta^2) P_n(x, \eta) P_m(x, \eta) = \frac{\delta_{nm}}{d_n},$$

with weight $d_n = \frac{\Gamma(n+3)}{2^4 \Gamma(n+\frac{3}{2})}$. The corresponding “conformal moments”

$$\mathbb{H}_a(n, t, \eta) = \int_{-1}^1 dx P_n(x, \eta) H_a(x, \eta, t) \quad (7)$$

play the role of “generalized form factors.” At $\eta = 0$ they reduce to ordinary Mellin moments and thus obey the polynomiality constraint

$$\int_{-1}^1 dx x^{n-1} H_a(x, \eta, t) = \sum_{k=0}^{\lfloor n/2 \rfloor} A_{n,2k}^{(a)}(t) \eta^{2k}. \quad (8)$$

A. Analytic continuation and Mellin-Barnes resummation

The series (5) converges only in the ERBL domain ($|x| < \eta$). To restore the support over the whole physical region ($|x| \leq 1$), we analytically continue $n \rightarrow j \in \mathbb{C}$ and resum the series by means of a Sommerfeld-Watson transform [20]

$$H_a(x, \eta, t) = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} \frac{dj}{\sin(\pi j)} P_j(x, \eta) \mathbb{H}_a(j, t, \eta). \quad (9)$$

The contour $\text{Re } j = c$ in the resulting Mellin-Barnes integral is chosen to the right of all singularities of $\mathbb{H}_a(j, t, \eta)$, while $P_j(x, \eta)$ denotes the unique analytic continuation of P_n satisfying $P_j(x, \eta) = P_j(x, -\eta) = P_{-j-1}(x, \eta)$. For $|x| > \eta$ one finds the leading asymptotics

$$P_j(x, \eta) \xrightarrow{|x| \gg \eta} (1 - x^2)^{-1} \left(\frac{x - \eta}{x + \eta} \right)^{\frac{1}{2}(j+2)},$$

such that the Mellin-Barnes representation interpolates smoothly, and uniquely, between the ERBL and DGLAP kinematics, once the conformal moments have been fixed.

B. Polynomiality, crossing, and the D -term

Equation (8) implies that $\sin(\pi j) \mathbb{H}_a(j, t, \eta)$ must be free of spurious poles at negative integer j . Together with the CP symmetry $\eta \rightarrow -\eta$ this condition fixes (9) up to an additive subtraction, the so-called D -term [3,21]. In channels where the D -term is non-negligible it can be included by adding a polynomial in η whose coefficients are independent functions of t . However, in the present work we retain it only in the flavor-singlet sector.

C. Practical remarks

The conformal moments $\mathbb{H}_a(j, t, \eta)$ in Eq. (5) provide the natural starting point for modeling as they diagonalize the one-loop evolution equations and map directly onto local matrix elements measured on the lattice. Once a model for $\mathbb{H}_a(j, t, \eta)$ is specified (Sec. IV), the inverse transform (9) can be evaluated numerically to recover the GPD. Convergence improves rapidly with increasing $|t|$ because the pole contribution of the secondary Regge trajectories is exponentially suppressed.

With the formalism established, we now construct a string-based ansatz for the conformal moments and confront it with lattice QCD.

IV. STRING-BASED CONFORMAL MOMENTS

Gauge/string duality suggests that high-energy amplitudes are dominated by t -channel exchanges of open (Reggeon-like) or closed (Pomeron/odderonlike) strings [11,12]. In the Mellin-Barnes framework of Eq. (9) this amounts to a simple Regge pole ansatz for the conformal moments $\mathbb{H}_a(j, t, \eta)$, separated into a skewness-independent part determined by the forward PDFs and a universal skewness-dependent prefactor. The following subsections give the explicit construction for all flavors and polarizations.

A. General structure

At an initial resolution of $\mu_0 = 1$ GeV we decompose

$$\mathbb{H}_a(j, \eta, t; \mu_0) = \mathcal{H}_a(j, t; \mu_0) + \mathcal{H}_{a\eta}(j, \eta, t; \mu_0), \quad (10)$$

where the first term captures the forward limit ($\eta \rightarrow 0$) and the second term restores skewness dependence. This separation is supported by explicit analyses of the gravitational form factors in gauge/string duality [15,22]. It is also expected from the kinematical structure of the light-cone expansion.

a. *Skewness-independent part.* For any quark flavor, or for gluons, we adopt the Reggeized Mellin transform

$$\mathcal{H}_a(j, t; \mu_0) = \int_0^1 dx \frac{f_a(x, \mu_0)}{x^{j_a - j + \alpha'_a t}}, \quad (11)$$

with $j_q = 1$ and $j_g = 2$. The physical intercepts are thus fixed by the leading piece in x of the polynomial parametrizing the PDF. Note that the gluon PDFs $f_g(x)$ in (11) contain an additional factor of x dictated by the corresponding field strength bilinears in (1). To parametrize the input PDFs $f_a(x, \mu_0)$ we utilize the global fits performed by the Asymmetry Analysis Collaboration (AAC) [23] (for polarized channels) and the Martin-Stirling-Thorne-Watt (MSTW) 2009 [24] (unpolarized channels).

b. *Skewness-dependent part.* Motivated by the computations in [22], we parametrize the skewness-dependent piece as

$$\mathcal{H}_{a\eta}(j, \eta, t; \mu_0) = [\hat{d}_j(\eta, t) - 1][\mathcal{H}_a(j, t; \mu_0)w - \mathcal{H}_a^S(j, t; \mu_0)], \quad (12)$$

where $\mathcal{H}_a^S$ is obtained from Eq. (11) by replacing $\alpha'_a \rightarrow \alpha_a^S$ (secondary trajectory). The universal kinematic kernel derived from the 2-to-2 open-/closed-string amplitude in cubic string field theory [13] reads

$$\hat{d}_j(\eta, t) = {}_2F_1\left(-\frac{j}{2} - \frac{j-1}{2}, \frac{1}{2} - j; -\frac{4m_N^2\eta^2}{t}\right). \quad (13)$$

This parametric dependence respects all physical constraints on the GPDs arising from unitarity and Lorentz invariance. Further, the functional form of (12) suggests that the skewness dependence arises from a degeneracy of mass spectra generating the corresponding Regge exchanges, as previously noted in [22].

B. Sea-quark singlet channel

For the flavor singlet $\sum_q^{N_f} \tilde{H}_q^{(+)}$ we use the same decomposition as in Eq. (10),

$$\sum_q^{N_f} \tilde{\mathbb{H}}_q^{(+)} = \sum_q^{N_f} \tilde{\mathcal{H}}_q^{(+)} + \sum_q^{N_f} \tilde{\mathcal{H}}_{q\eta}^{(+)}, \quad (14)$$

with odd conformal spin $j = 1, 3, \dots$. The forward piece is fixed by the empirical helicity PDFs

$$\sum_q^{N_f} \tilde{\mathcal{H}}_q^{(+)}(j, t; \mu_0) = \int_0^1 dx \frac{\sum_q \Delta q^{(+)}(x; \mu_0)}{x^{1-j+\tilde{\alpha}'_{u+d}t}}, \quad (15)$$

and the skewness dependence by Eq. (12) with $\tilde{\alpha}_{u+d}^S$. More specifically, the NLO PDFs fixed empirically by the AAC [23] are

$$\Delta f(x, \mu_0) = (\delta x^\nu - \kappa(x^\nu - x^\mu))f(x, \mu_0). \quad (16)$$

For the unpolarized PDFs $f(x, \mu_0)$ we will use the MSTW PDFs from [24] as noted earlier. Throughout, the initial resolution scale for both the unpolarized and polarized PDFs is $\mu_0 = 1$ GeV. Conveniently, this is also a scale at which holographic models are expected to work reliably.

C. Nonsinglet valence channels

For the isovector ($u-d$) and isoscalar ($u+d$) combinations of $\mathbb{E}_{u\pm d}^{(-)}(j, t; \mu_0)$ we use the valence input PDFs as

$$\mathbb{E}_{u\pm d}^{(-)}(j, t; \mu_0) = \int_0^1 dx \frac{u_v(x) \pm d_v(x)}{x^{1-j+\alpha'_{u\pm d}t}}, \quad (17)$$

with $q_v = q - \bar{q}$ and the additional assumption of $\bar{u} = \bar{d}$. Similarly, we use for the same combinations in the polarized sector

$$\tilde{\mathbb{H}}_{u\pm d}^{(-)}(j, t; \mu_0) = \int_0^1 dx \frac{\Delta u_v(x) \pm \Delta d_v(x)}{x^{1-j+\tilde{\alpha}'_{u\pm d}t}}, \quad (18)$$

where $\Delta q_v = \Delta q - \Delta \bar{q}$ and with $\Delta \bar{u} = \Delta \bar{d} = \Delta \bar{s}$, following the analysis in [23]. Because the corresponding D -term is generically small and flavor independent, we neglect the skewness-dependent piece in these channels. The slopes $\tilde{\alpha}'_{u-d}$ and $\tilde{\alpha}'_{u+d}$ are fixed by the dipole fit to the nucleon polarized form factor $G_A(t) = g_A(1 - t/m_A^2)^{-2}$, using the lattice results of Ref. [25] for m_A and Ref. [18] for g_A .

D. Gluon singlet channel

For the C-odd (pseudovector) gluon helicity GPD we write

$$\tilde{\mathbb{H}}_g^{(+)}(j, \eta, t; \mu_0) = \tilde{\mathcal{H}}_g^{(+)}(j, t; \mu_0) + \tilde{\mathcal{H}}_{g\eta}^{(+)}(j, \eta, t; \mu_0), \quad (19)$$

with

TABLE I. NLO Regge slopes used for the parametrization of conformal moments. All values are displayed in GeV^{-2} .

	α'_{u-d}	α'_{u+d}	$\alpha'^{(+)}_{u+d}$	α'_s	$\alpha'_{T/PV}$	$\alpha'_{S/PS}$
$\mathbb{H}$	0.6345	0.9492	0.5044	1.9657	0.5584	5.2081
$\mathbb{E}$	1.3929	1.1368	...	...	...	...
$\tilde{\mathbb{H}}$	0.3140	0.2975	0.7186	1.179	0.490	0.744

$$\tilde{\mathcal{H}}_g^{(+)}(j, t; \mu_0) = \int_0^1 dx \frac{x \Delta g(x, \mu_0)}{x^{2-j+\alpha'_g t}}, \quad (20)$$

with the skewness dependence determined by Eq. (12) with the pseudoscalar slope parameter $\tilde{\alpha}'_g$.

E. Choice of slopes and numerical values

For the parametrization of the unpolarized moments we refer the reader to [16]. However, for the quark singlet sector we relaxed the assumption of $\alpha'_{u+d} = \alpha'^{(+)}_{u+d}$ previously made in [16], which corresponds to vanishing $\tilde{u}$ and $\tilde{d}$ contributions. We fix the nonsinglet Regge slopes for the helicity moments $\tilde{\alpha}'_{u-d}$ and $\tilde{\alpha}'_{u+d}$ to the dipole form of [25] but the overall scale of the conformal moment is fixed to the values quoted in the PDG [26] (isovector) and FLAG [17] (isoscalar).

While the unpolarized singlet moments may either be fixed by experiment [27] or lattice QCD [28,29], fixing the slopes for the polarized singlet moments is less trivial. In the absence of both measurements and lattice data, we chose to fix the corresponding singlet slopes for $\tilde{\mathbb{H}}$ by the mass spectra of the η' meson trajectory quoted in the PDG [26] and the lattice glueball spectrum [30]. In the naive quark model E_s is zero by simple charge counting. In general, the helicity-flip gravitational form factors obtained from the E GPDs are expected to be small [31,32], which is indeed supported by lattice computations [28,29,33] and holography [34], where this statement even extends to the higher moments, or generalized form factors. Hence, the corresponding singlet moments will be set to zero. For the reader's convenience, we have collected the model parameters in Table I.

F. Results and comparison with lattice QCD

All conformal moments constructed in Secs. III and IV are evolved from the hadronic scale $\mu_0 = 1 \text{ GeV}$ to the conventional reference scale $\mu = 2 \text{ GeV}$ in the $\overline{\text{MS}}$ scheme using the NLO DGLAP-ERBL kernels of Refs. [35,36],¹ with the anomalous dimensions shifted by $j \rightarrow j - 1$ to match the spin of the corresponding Regge exchanges. In the following we compare the evolved moments with the most precise lattice determinations currently available. For the readers convenience, we have collected the lattice ensembles in Table II.

¹We encountered some typos in the normalization of the NLO anomalous dimensions listed in [35] which is why we chose to use those listed in [37–39] adapted to our conventions.

TABLE II. Collection of references and the corresponding lattice ensembles used for the comparison. All datasets use the $\overline{\text{MS}}$ scheme to evolve to the reference scale of $\mu = 2 \text{ GeV}$, except Ref. [4] which evolves to $\mu = 3 \text{ GeV}$.

Ref.	N_f	a (fm)	m_π (MeV)
[6,8,18]	2 + 1 + 1	0.093	260
[40]	2 (cA2.09.64)	0.080	130
[41]	2 + 1 (dataset 6)	0.124	352
[10]	2 + 1 + 1	0.090	130
[4]	2 + 1 + 1	0.042	310

1. Nonsinglet helicity-flip and polarized moments

Figures 1 and 2 show the nonsinglet helicity-flip ($\mathbb{E}$) and polarized ($\tilde{\mathbb{H}}$) moments for $j = 1-5$, respectively. The upper panels display the isovector combinations $\mathbb{E}_{u-d}(j, t)$ and $\tilde{\mathbb{H}}_{u-d}(j, t)$, while the corresponding isoscalar results $\mathbb{E}_{u+d}(j, t)$ and $\tilde{\mathbb{H}}_{u+d}(j, t)$ are shown below. The blue bands represent our string-based predictions evolved to $\mu = 2 \text{ GeV}$, and the lattice points are taken from the tabulated values in LHPC [41] and Refs. [8,18,40].² Except for the second moment of the E_{u-d} GPD, we find excellent agreement with the reported lattice QCD results for the first three moments. For the higher moments we notice some discrepancy but they are strongly suppressed and additionally more noisy on the lattice. The second moment of E_{u-d} is special as it is expected to be small by angular momentum conservation, a feature which is not shared by the corresponding string-based moment. Using linear Regge trajectories, such a behavior could in principle be generated by introducing a daughter trajectory with opposite sign of the coupling, though the slope would need to be rather small to explain the almost-flat behavior in the lattice data of Fig. 1, most likely hinting at different physics at play.

2. Singlet sea-quark and gluon polarized moments

Finally, Fig. 3 displays our novel predictions for the singlet sea-quark moments $\tilde{\mathbb{H}}_\Sigma$ (upper row) and the polarized gluon moments $\tilde{\mathbb{H}}_g$ (lower row) for $j = 1, 2, 3$. No lattice results exist to date. The curves shown here are therefore direct outputs of the model, providing concrete, testable predictions for forthcoming simulations and for DVCS measurements at Jefferson Lab and the future EIC.

V. AXIAL AND HELICITY-FLIP GPDS IN x -SPACE FROM CONFORMAL MOMENTS

In the previous sections we derived closed expressions for all polarized and helicity-flip conformal moments.

²We also note that our results are in agreement with those reported in Ref. [42].

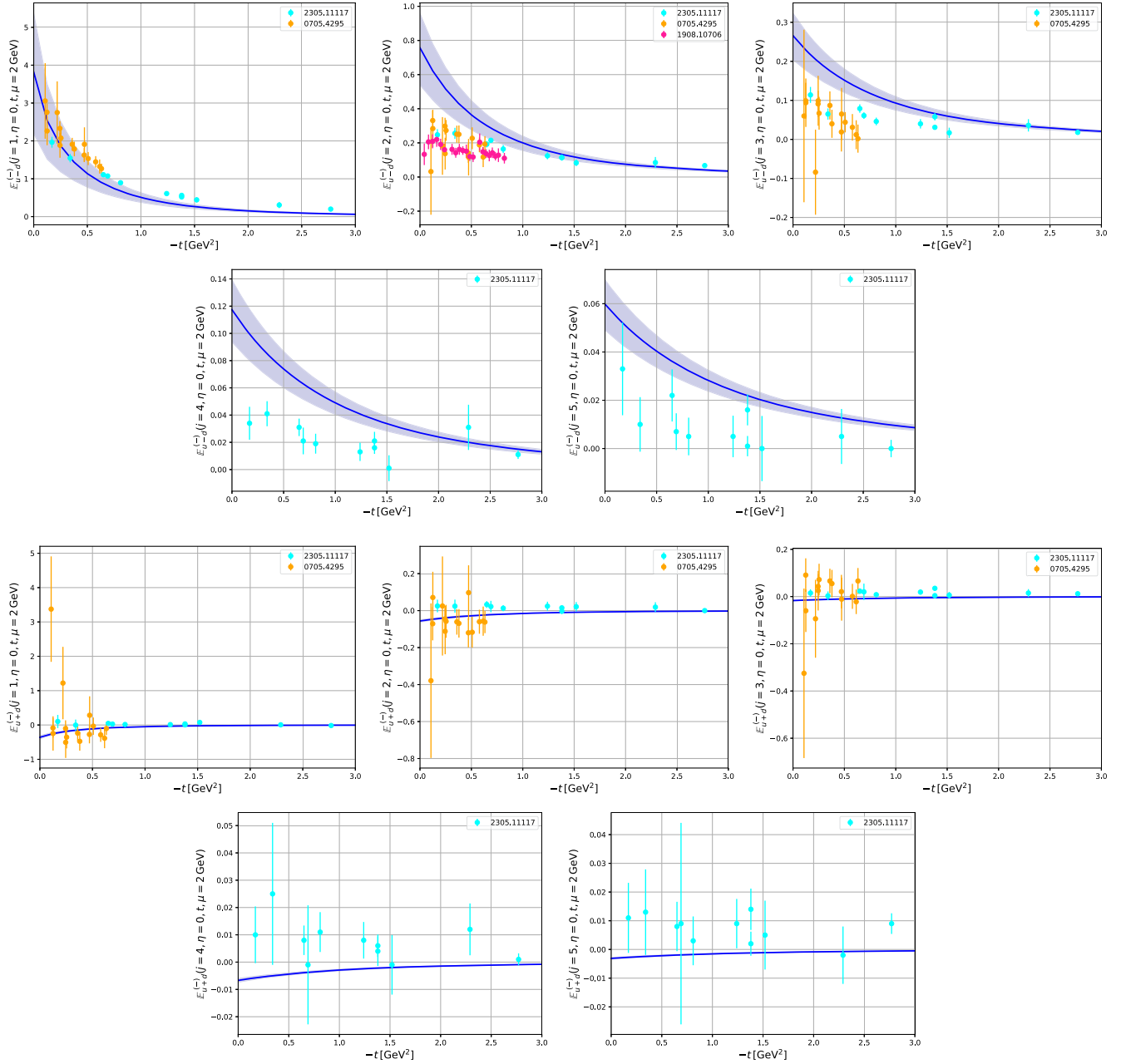


FIG. 1. Nonsinglet helicity-flip conformal moments $\mathbb{E}_{u-d}$ (top five blocks) and $\mathbb{E}_{u+d}$ (bottom five blocks) at $\mu = 2$ GeV for conformal spins $j = 1, \dots, 5$ (panels ordered left to right, top to bottom). Blue band, our string-based prediction. The lattice data are from LHPC [41] and Ref. [8].

To obtain the corresponding GPDs in x -space we used a Sommerfeld-Watson transform and evaluated the ensuing Mellin-Barnes integral numerically. The relevant

Mellin-Barnes representations are collected in Appendix B for the reader's convenience. For details of their derivation we refer the reader to Refs. [16,20].

$$\tilde{H}_q^{(+)}(x, \eta) = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} \frac{dj}{\sin(\pi j)} [p_j(x, \eta) + p_j(-x, \eta)] \tilde{\mathbb{H}}_q^{(+)}(j, \eta), \quad (21)$$

$$\tilde{H}_{u\pm d}^{(-)}(x, \eta, t) = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} \frac{dj}{\sin(\pi j)} p_j(x, \eta) \tilde{\mathbb{H}}_{u\pm d}^{(-)}(j, \eta, t), \quad (22)$$

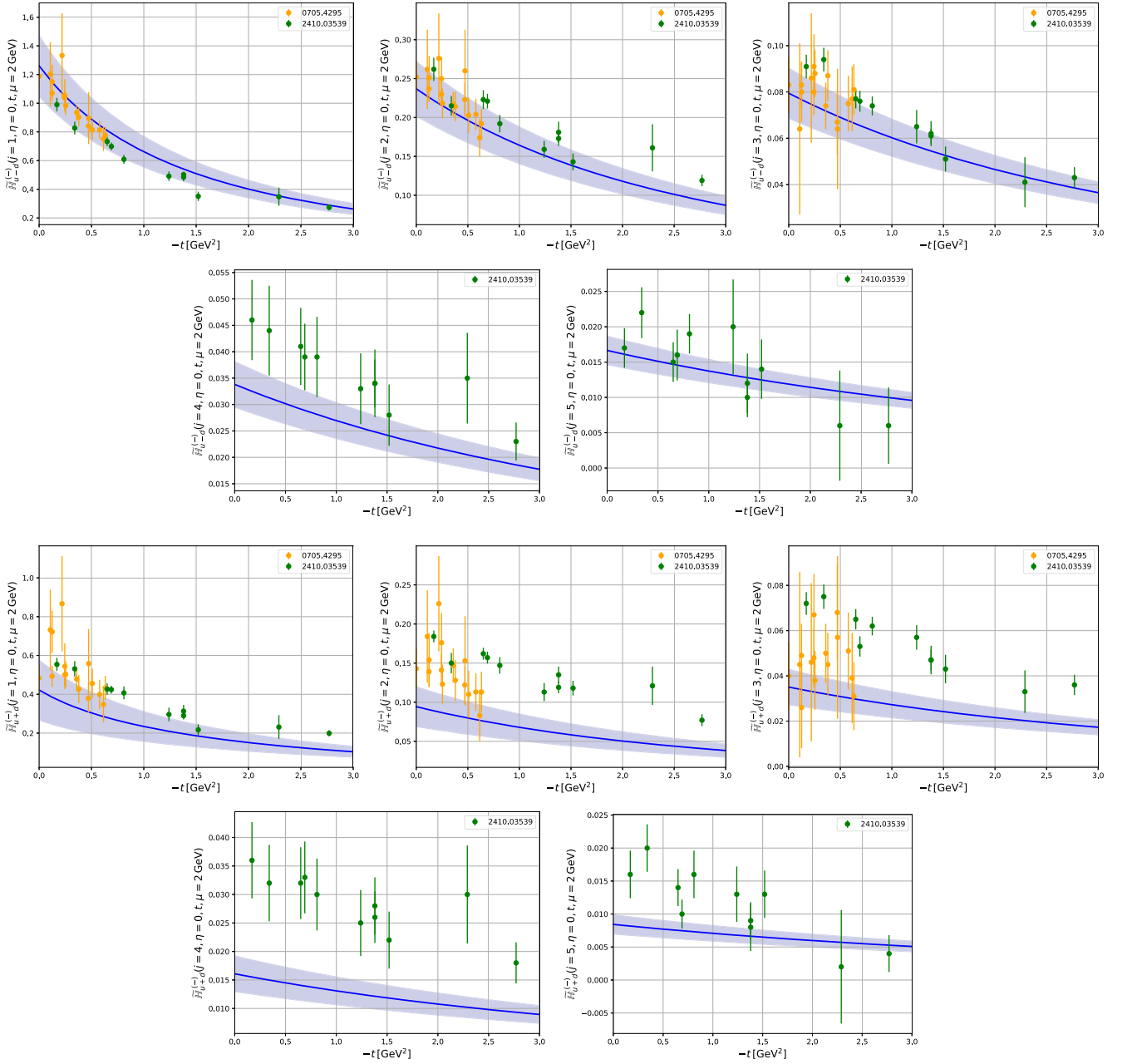


FIG. 2. Nonsinglet polarized conformal moments $\tilde{\mathbb{H}}_{u-d}$ (top block) and $\tilde{\mathbb{H}}_{u+d}$ (bottom block) evolved to $\mu = 2$ GeV. Symbols, lattice data of Ref. [18] and LHPC [41].

$$\tilde{H}_g(x, \eta, t) = -\frac{1}{2i} \int_{c-i\infty}^{c+i\infty} \frac{dj}{\sin(\pi j)} {}^g p_j(x, \eta) \tilde{\mathbb{H}}_g(j, \eta, t), \quad (23)$$

$$\tilde{H}_g^{(+)}(x, \eta, t) = -\frac{1}{2i} \int_{c-i\infty}^{c+i\infty} \frac{dj}{\sin(\pi j)} [{}^g p_j(x, \eta) - {}^g p_j(-x, \eta)] \tilde{\mathbb{H}}_g^{(+)}(j, \eta, t). \quad (24)$$

Throughout, the integration contour is chosen such that $\text{Re } j = c$ is to the right of all singularities of $\tilde{\mathbb{H}}_{q,g}(j, \eta, t)$. The kernels $p_j, {}^g p_j$ are the standard analytic continuation of the

Gegenbauer polynomials that appear in the conformal partial-wave expansion [20]. Furthermore, the minus sign in Eqs. (23) and (24) reflects the gluon's Bose symmetry [20].

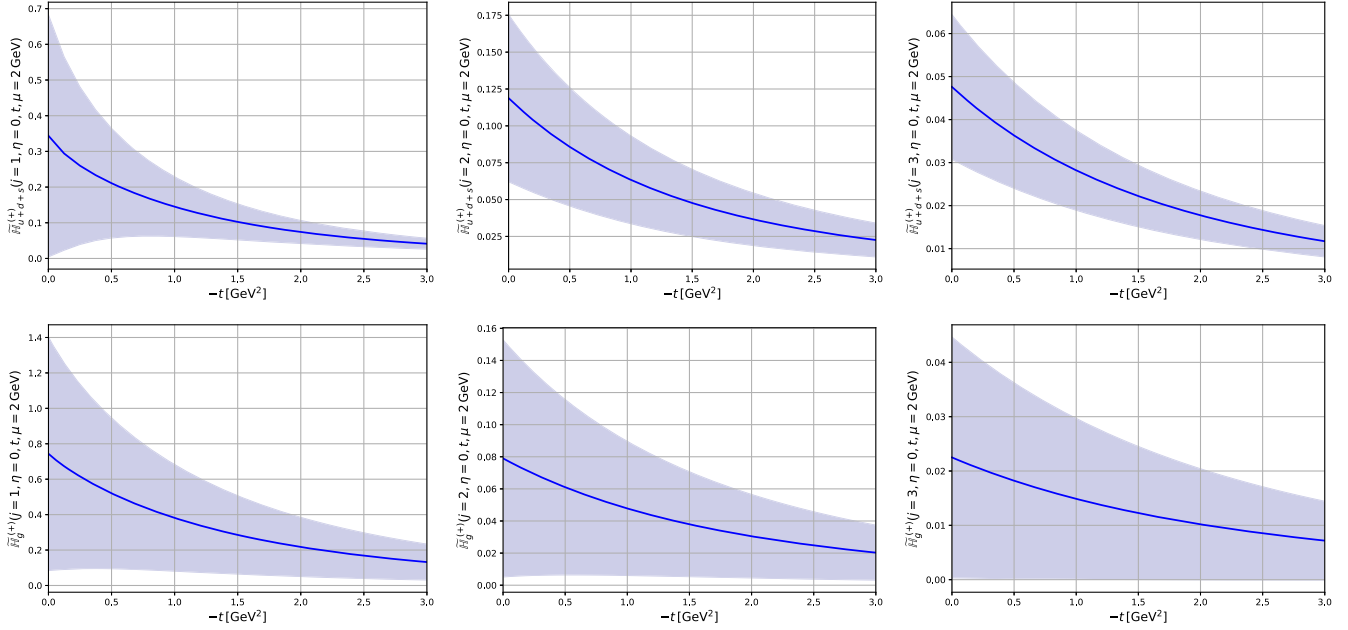


FIG. 3. Predictions for singlet polarized conformal moments at $\mu = 2$ GeV. Upper row: sea-quark singlet $\tilde{\mathbb{H}}_{\Sigma}(j, t)$. Lower row: pseudovector gluon $\tilde{\mathbb{H}}_g(j, t)$. Conformal spins $j = 2, 3, 4$ are ordered left to right. No lattice data are yet available.

A. Results and comparison with lattice QCD

All GPDs shown below are evolved from the input scale $\mu_0 = 1$ to $\mu = 2$ GeV at NLO. In every figure the dashed vertical lines mark the boundaries between the ERBL region ($|x| < \eta$) and the two DGLAP regions ($|x| > \eta$).

a. *Nonsinglet isovector channel.* Figure 4 compares our predictions with lattice data. Figure 4(a) shows $\tilde{H}_{u-d}$ for three representative kinematics, while Fig. 4(b) displays the helicity-flip distribution E_{u-d} for the same kinematics. The colored bands are the model uncertainties which we estimate by propagating the error of the input PDFs through

the evolution equations of the corresponding moments. The data points are extracted from Refs. [4,6,10]. Up to the scale, we reproduce the results well over the whole kinematic range, with stronger discrepancies arising for $x > \eta$ in the helicity case. We note, however, that the large- x behavior is dictated by the input PDFs rather than the t and η dependence, as can be seen in Fig. 5(a) by plotting our results over a wider kinematic range. Moreover, some lattice moments are known to have difficulty reproducing experimentally extracted PDF moments in the forward region. For the E^{u-d} GPD the mismatch in scale may be attributed to a contamination arising from the second

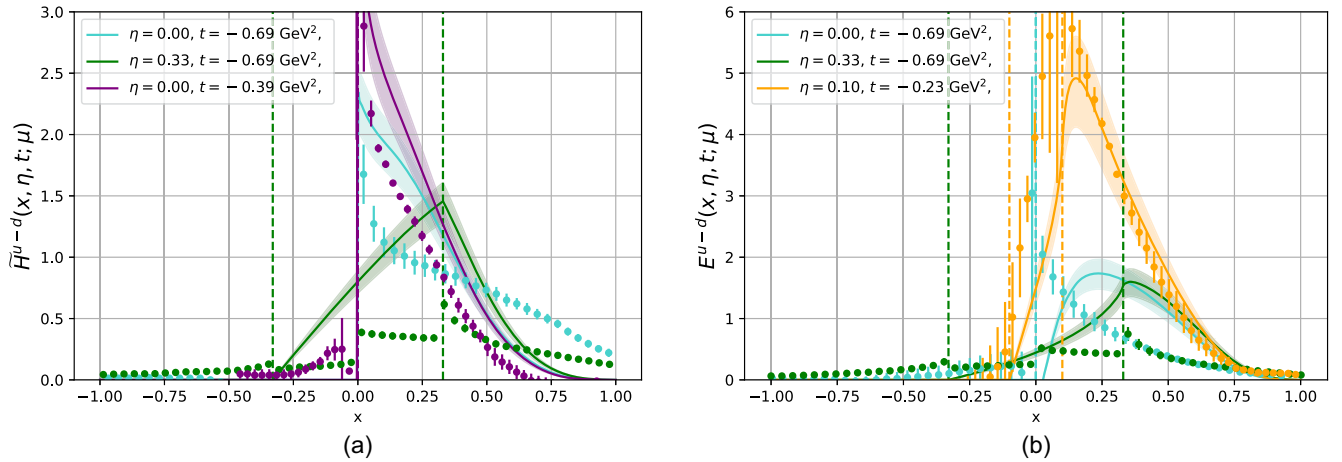


FIG. 4. (a) Evolved nonsinglet isovector axial GPD at a resolution of $\mu = 2$ GeV compared to [6] and (b) nonsinglet isovector GPD E compared to [6] (green, turquoise), [10] (orange) at $\mu = 2$ GeV, and [4] (purple) at $\mu = 3$ GeV. The dashed lines separate the ERBL region (inner) from the DGLAP region (outer).

singlet moment, though we are not able to test this hypothesis without breaking the tightly constrained Regge ansatz. It would be interesting to compare our results to models without Gaussian-like moments in the future.

Additionally, in Figs. 5(a) and 5(b), we compare our results to the unpublished results based on the works in Refs. [7,9]. The helicity sector is, up to a scale, reasonably well reproduced again with larger discrepancies for larger values of x . One may attribute this discrepancy to the different values of the axial charge extracted from Ref. [18] and the values reported by FLAG [17], to which we fit the scale. For the E^{u-d} GPD we again note the contamination from the second moment, which seems to enforce $E^{u-d}(x, \eta = 0, t) \simeq 0$, though this region is again very sensitive to the PDF parametrization in our case.

b. *Nonsinglet isoscalar channel.* The axial (a) and helicity-flip (b) GPDs for the isoscalar $u + d$ combination are plotted in Fig. 6. While lattice results are not yet available, the shapes of $\tilde{H}_{u+d}$ and E_{u+d} already offer stringent and testable benchmarks for future simulations and experiments.

c. *Singlet quark and gluon channels.* Finally, Fig. 7 presents our parameter-free predictions for the sea-quark singlet distribution $\tilde{H}_\Sigma$ [Fig. 7(a)] and for the polarized gluon distribution $\tilde{H}_g$ [Fig. 7(b)]. These results, particularly their pronounced η dependence in the ERBL domain, will be directly accessible to deeply virtual Compton scattering at Jefferson Lab 12 GeV and to the future Electron-Ion Collider.

Finally, we follow up on previous results presented by some of us in [15,16] in Fig. 8, where we present the NLO evolved unpolarized singlet GPDs. In the ERBL region, the

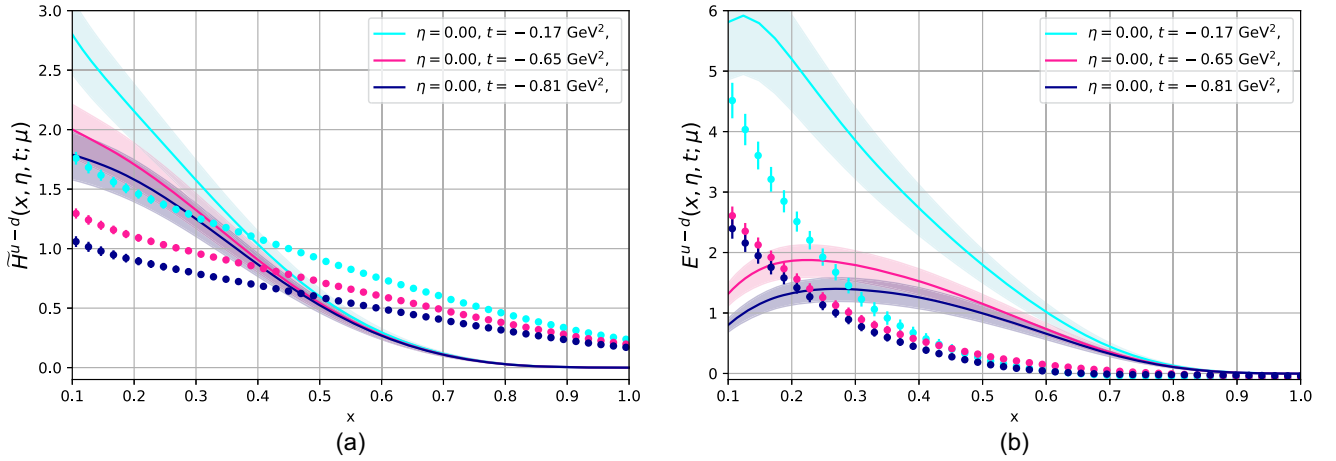


FIG. 5. (a) Evolved nonsinglet isovector axial GPD at a resolution of $\mu = 2$ GeV compared to [6] and (b) nonsinglet isovector GPD E compared to [6] (green, turquoise), [10] (orange) at $\mu = 2$ GeV, and [4] (purple) at $\mu = 3$ GeV. The dashed lines separate the ERBL region (inner) from the DGLAP region (outer).

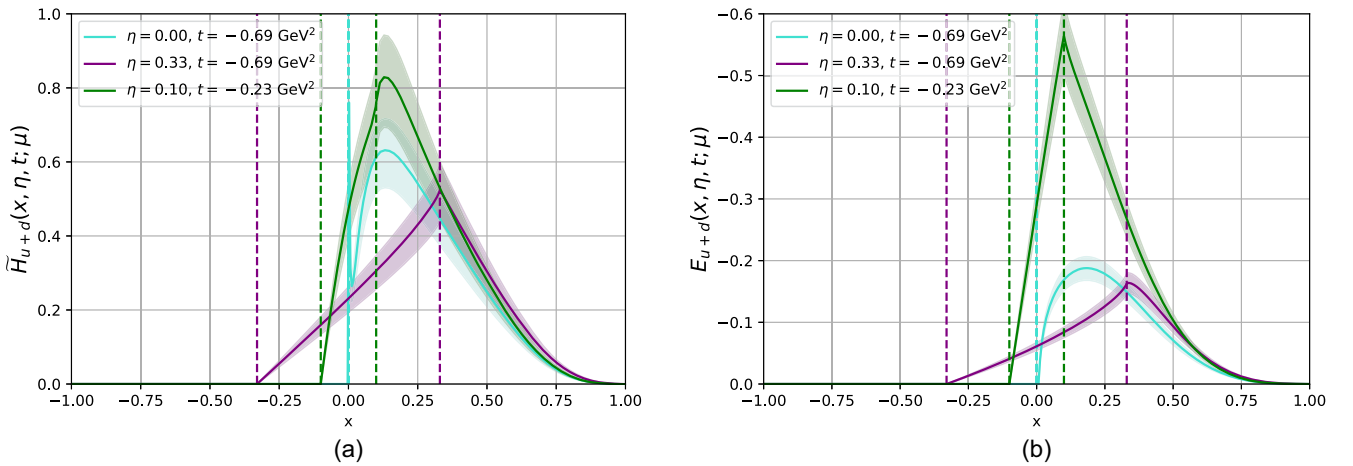


FIG. 6. (a) Evolved nonsinglet axial isoscalar $\tilde{H}$ GPD at a resolution of $\mu = 2$ GeV. (b) Evolved nonsinglet isoscalar E GPD at a resolution of $\mu = 2$ GeV. The dashed lines separate the ERBL region (inner) from the DGLAP region (outer).

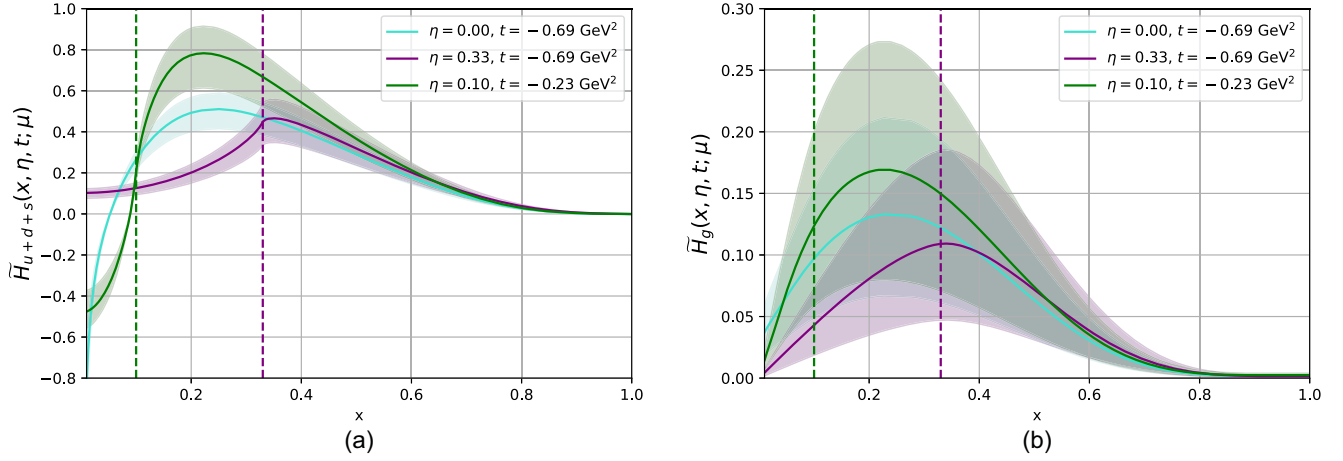


FIG. 7. Evolved singlet axial sea-quark (a) and gluon (b) GPDs at a resolution of $\mu = 2$ GeV. The dashed lines separate the ERBL region (inner) from the DGLAP region (outer).

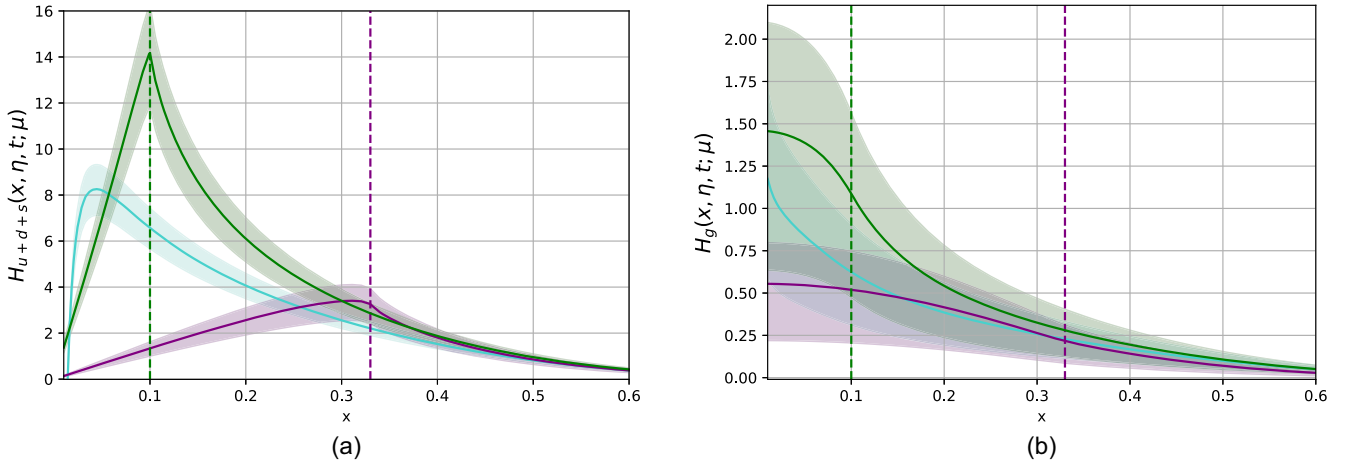


FIG. 8. Evolved unpolarized singlet quark (a) and gluon (b) GPDs at a resolution of $\mu = 2$ GeV. The dashed lines separate the ERBL region (inner) from the DGLAP region (outer). The kinematics correspond to those in Fig. 7.

sea-quark and gluon GPDs differ significantly due to the different values of c used in the evaluation of the Mellin-Barnes integral in [15,16].

B. Discussion

Across all channels the model reproduces the available lattice data with reasonable accuracies and delivers concrete predictions where no data exist, with the exception of $\mathbb{E}^{u-d}(j=2,0,t)$. The good agreement observed in the isovector sector (Fig. 4) lends credibility to the string-based parametrization using NLO PDF fits. The isoscalar and singlet results indicate regions in (x, η, t) space where future lattice simulations and experimental measurements can provide the strongest constraints. We look forward to comparing to more data at finite skewness in the future.

VI. CONCLUSIONS AND OUTLOOK

This work introduces a compact, string-based Regge parametrization of axial and helicity-flip leading-twist quark and gluon conformal moments. By construction, the formulation respects polynomiality, crossing symmetry, and support constraints exactly, while relying on a minimal set of free parameters: Regge slopes drawn from hadron and glueball spectroscopy, as well as empirical form factors, and forward limits fixed by the empirical MSTW unpolarized and AAC polarized parton distributions. After NLO DGLAP-ERBL evolution to the reference scale $\mu = 2$ GeV, the results reproduce most currently available lattice data of nonsinglet axial and helicity-flip moments up to $|t| \simeq 3$ GeV², without any tuning beyond those inputs. Equally important, it delivers the first

quantitative predictions for the sea-quark and gluon polarized moments and for the corresponding x -space GPDs, supplying clear benchmarks for forthcoming lattice calculations, deeply virtual Compton scattering at Jefferson Lab, and the future Electron-Ion Collider.

The analytic structure of the model makes it well suited for global GPD fits. Its explicit Mellin-Barnes representation allows efficient numerical evaluation, while the Regge form ties the small- x and large- t behaviors to well-understood physical trajectories. In practical terms, this means the parameter set can be embedded directly into Monte Carlo analyses of exclusive processes without introducing additional scheme-dependent ambiguities.

Several extensions are natural. On the theory side, the same strategy can be applied to transversity, higher-twist distributions, and modern small- x resummation schemes or generalized to off-forward transverse momentum dependent distributions. On the phenomenological side, a combined analysis of DVCS, exclusive meson production, and lattice moments should sharpen the determination of the sea-quark and gluon GPDs and test the Regge-string picture over a wider kinematic range. The forthcoming Electron-Ion Collider will provide precisely the leverage in skewness and momentum transfer that is needed to carry out these tests.

In summary, the present study demonstrates that a string-based Regge framework is not only compatible with the rigorous constraints of QCD and lattice data, but also offers predictive power in channels that remain experimentally unexplored. Its success in the nonsinglet sector and its concrete projections for singlet distributions underscore its potential as a standard tool in future global analyses of the nucleon structure.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [43,44].

APPENDIX A: KINEMATIC VARIABLES

We follow the symmetric notation of Refs. [14,35] and introduce

$$\tilde{q} \equiv \frac{q+q'}{2}, \quad p \equiv \frac{p_1+p_2}{2}, \quad \Delta \equiv p_1-p_2 = q'-q. \quad (\text{A1})$$

The Lorentz invariants constructed from these vectors read

$$\tilde{q}^2 = -\tilde{Q}^2, \quad \xi = \frac{\tilde{Q}^2}{2p \cdot \tilde{q}}, \quad \eta = \frac{\Delta \cdot \tilde{q}}{2p \cdot \tilde{q}}, \quad (\text{A2a})$$

$$Q^2 = -q^2, \quad Q'^2 = -q'^2, \quad x_B = \frac{Q^2}{2p_1 \cdot q}. \quad (\text{A2b})$$

A convenient “average resolution” scale is

$$\tilde{Q}^2 = \frac{1}{2} \left(Q^2 + Q'^2 + \frac{\Delta^2}{2} \right). \quad (\text{A3})$$

Using Eqs. (A2) one derives the exact relations

$$\xi = \frac{Q^2 + Q'^2 + \Delta^2/2}{2Q^2/x_B - Q^2 + Q'^2 + \Delta^2},$$

$$\eta = \frac{Q^2 - Q'^2}{2Q^2/x_B - Q^2 + Q'^2 + \Delta^2}, \quad (\text{A4})$$

as well as the inverse transformations

$$Q^2 = \left(1 + \frac{\eta}{\xi} \right) \tilde{Q}^2 - \frac{\Delta^2}{4}, \quad (\text{A5a})$$

$$Q'^2 = \left(1 - \frac{\eta}{\xi} \right) \tilde{Q}^2 - \frac{\Delta^2}{4}, \quad (\text{A5b})$$

$$x_B = \frac{(\xi + \eta)\tilde{Q}^2 - \xi\Delta^2/4}{(1 + \eta)\tilde{Q}^2 - \xi\Delta^2/2}. \quad (\text{A5c})$$

Choosing lightlike reference vectors

$$n^\mu = (1, 0, 0, -1)/\sqrt{2}, \quad n^{*\mu} = (1, 0, 0, 1)/\sqrt{2}, \quad (\text{A6})$$

with $n \cdot n^* = 1$, and $n^2 = n^{*2} = 0$, any four-vector decomposes as $a^\mu = a^+ n^{*\mu} + a^- n^\mu + a_\perp^\mu$, where $a^\pm = a \cdot n^{(*)}$ and $a_\perp \cdot n^{(*)} = 0$. In this basis the active parton carries a plus-momentum fraction

$$x = \frac{k^+}{P^+}, \quad P^+ = p^+ \equiv \frac{p_1^+ + p_2^+}{2}. \quad (\text{A7})$$

The skewness defined in Eq. (A4) reduces in the purely spacelike DVCS limit ($Q'^2 = 0$) to the familiar light-cone expression

$$\eta = \frac{\Delta^+}{2P^+} = \frac{p_1^+ - p_2^+}{p_2^+ + p_1^+}, \quad |\eta| \leq 1. \quad (\text{A8})$$

The invariant four-momentum transfer and its transverse component are thus given by

$$t = \Delta^2 = -\frac{\Delta_\perp^2}{1-\eta^2} - \frac{4\eta^2 m_N^2}{1-\eta^2}, \quad \Delta_\perp \equiv \mathbf{q}_\perp. \quad (\text{A9})$$

The kinematics of DVCS allow an interpretation of the skewness η in terms of rapidity Δy . This can be seen as follows. Let p_1^μ and p_2^μ be the initial and final nucleon momenta. Their average is $P^\mu = \frac{1}{2}(p_1^\mu + p_2^\mu)$ and the momentum transfer is $\Delta^\mu = p_1^\mu - p_2^\mu$. Throughout we keep $\mathbf{P}_\perp = 0$ so that all transverse motion resides in $\Delta_\perp$. The dimensionless longitudinal asymmetry implies $p_1^+ = P^+(1+\eta)$ and $p_2^+ = P^+(1-\eta)$. For any on-shell four-vector with $\mathbf{p}_\perp = 0$ the (longitudinal) rapidity is

$$y = \frac{1}{2} \ln \left(\frac{p^+}{p^-} \right) = \ln \left(\frac{p^+}{m_N} \right), \quad (\text{A10})$$

using $p^+ p^- = m_N^2$ in the last step. The difference between the outgoing and incoming nucleon rapidities is

$$\Delta y = y_2 - y_1 = \ln \left(\frac{p_2^+}{p_1^+} \right) = \ln \left(\frac{1+\eta}{1-\eta} \right). \quad (\text{A11})$$

Equation (A11) can be inverted to a one-to-one mapping

$$\eta = \tanh \left(\frac{\Delta y}{2} \right), \quad \Delta y = 2 \operatorname{artanh}(\eta). \quad (\text{A12})$$

APPENDIX B: MOMENTS AND GPDS

The GPD moments are parametrized using Reggeized versions of the MSTW [24] and AAC [23] NLO PDF sets

$$\begin{aligned} \mathbb{E}_{u\pm d}^{(-)}(j, \eta, t; \mu) &= \int dx \frac{u_v(x) \pm d_v(x)}{x^{j-1+\alpha' t}}, \\ \tilde{\mathbb{H}}_{u\pm d}^{(-)}(j, \eta, t; \mu) &= \int dx \frac{\Delta u_v(x) \pm \Delta d_v(x)}{x^{j-1+\alpha' t}}, \\ \tilde{\mathbb{H}}_q^{(+)}(j, \eta, t; \mu) &= \int dx \frac{\Delta u_v(x) + \Delta d_v(x) + \Delta S(x)}{x^{j-1+\alpha' t}}, \\ \tilde{\mathbb{H}}_g^{(+)}(j, \eta, t; \mu) &= \int dx \frac{x \Delta g(x)}{x^{j-2+\alpha' t}}, \end{aligned} \quad (\text{B1})$$

where $\Delta S = 2(\Delta \bar{u} + \Delta \bar{d}) + \Delta s + \Delta \bar{s}$ and assuming isospin symmetry of the sea. The normalizations in the nonsinglet sector are enforced to satisfy

$$\begin{aligned} \mathbb{E}_{u\pm d}^{(-)}(j=1, 0, 0; \mu) &= \frac{\mu_{u\pm d}}{2m_N}, \\ \tilde{\mathbb{H}}_{u\pm d}^{(+)}(j=1, 0, 0; \mu) &= g_A^{u\pm d}, \end{aligned} \quad (\text{B2})$$

with $\mu_{u\pm d} = \mu_u \pm \mu_d$, $g_A^{u\pm d} = g_A^u \pm g_A^d$ the flavor separated nonsinglet contributions to the anomalous magnetic moment and axial charge of the proton, respectively [17,26]. The singlet moments have the additional skewness dependence fixed by (12). All conformal moments are evolved from the hadronic scale $\mu_0 = 1$ GeV to the conventional reference scale $\mu = 2$ GeV using the NLO DGLAP-ERBL kernels of Refs. [35,36] with the NLO anomalous dimensions of [37–39]. Note that the analytic continuation to complex spin- j at NLO requires an additional Sommerfeld-Watson transform to carry out the resulting fractional finite sums, as has been previously studied in [45] and more recently in [46] with respect to convergence properties, though the non-diagonal part of the evolution equation only accounts for a few percent of the resummed GPD. The GPDs in the momentum fraction x -space are determined from the conformal moments (B1) by the Mellin-Barnes integrals

$$\begin{aligned} E_{u\pm d}^{(-)}(x, \eta, t; \mu) &= \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} \frac{dj}{\sin(\pi j)} p_j(x, \eta) \mathbb{E}_{u\pm d}^{(-)}(j, \eta, t; \mu), \\ \tilde{H}_{u\pm d}^{(-)}(x, \eta, t; \mu) &= \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} \frac{dj}{\sin(\pi j)} p_j(x, \eta) \tilde{\mathbb{H}}_{u\pm d}^{(-)}(j, \eta, t; \mu), \\ \tilde{H}_{u+d+s}^{(+)}(x, \eta, t; \mu) &= \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} \frac{dj}{\sin(\pi j)} (p_j(x, \eta) + p_j(x, \eta)) \tilde{\mathbb{H}}_q^{(+)}(j, \eta, t; \mu), \\ \tilde{H}_g^{(+)}(x, \eta, t; \mu) &= \frac{-1}{2i} \int_{c-i\infty}^{c+i\infty} \frac{dj}{\sin(\pi j)} (p_j^g(x, \eta) - p_j^g(x, \eta)) \tilde{\mathbb{H}}_g^{(+)}(j, \eta, t; \mu), \end{aligned} \quad (\text{B3})$$

where the quark and gluon PWs are given in Refs. [16,20] both in the ERBL and DGLAP regimes. We have included them here for convenience. The quark PWs are

$$p_j(x, \eta) = \theta(\eta - |x|) \frac{1}{\eta^j} \mathcal{P}_j \left(\frac{x}{\eta} \right) + \theta(x - \eta) \frac{1}{x^j} \mathcal{Q}_j \left(\frac{x}{\eta} \right) \quad (\text{B4})$$

with

$$\begin{aligned}\mathcal{P}_j\left(\frac{x}{\eta}\right) &= \frac{2^j \Gamma(3/2 + j)}{\Gamma(1/2) \Gamma(j)} \left(1 + \frac{x}{\eta}\right) {}_2F_1\left(-j, j+1, 2 \middle| \frac{1}{2} \left(1 + \frac{x}{\eta}\right)\right), \\ \mathcal{Q}_j\left(\frac{x}{\eta}\right) &= \frac{\sin(\pi j)}{\pi} {}_2F_1\left(\frac{j}{2}, \frac{j+1}{2}; \frac{3}{2} + j \middle| \frac{\eta^2}{x^2}\right),\end{aligned}\tag{B5}$$

and the gluon PWs are

$$\begin{aligned}g\mathcal{P}_j(x, \eta) &= \theta(\eta - |x|) \frac{1}{\eta^{j-1}} g\mathcal{P}_j\left(\frac{x}{\eta}\right) + \theta(x - \eta) \frac{1}{x^{j-1}} g\mathcal{Q}_j\left(\frac{x}{\eta}\right), \\ g\mathcal{P}_j\left(\frac{x}{\eta}\right) &= \frac{2^{j-1} \Gamma(3/2 + j)}{\Gamma(1/2) \Gamma(j-1)} \left(1 + \frac{x}{\eta}\right)^2 {}_2F_1\left(-j, j+1, 3 \middle| \frac{1}{2} \left(1 + \frac{x}{\eta}\right)\right), \\ g\mathcal{Q}_j\left(\frac{x}{\eta}\right) &= -\frac{\sin(\pi j)}{\pi} {}_2F_1\left(\frac{j-1}{2}, \frac{j}{2}; \frac{3}{2} + j; \frac{\eta^2}{x^2}\right).\end{aligned}\tag{B6}$$

For the numerical evaluation of the Mellin-Barnes integrals, we chose $c = 0.95$ for the spin-1 exchanges and $c = 1.95$ for the spin-2 exchanges, respectively, to recover the correct sums in the conformal partial-wave expansion with spin- j Regge exchanges. All the above formulas are numerically implemented in the `core.py` module of the `stringy-gpds` PYTHON package.

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