

Extended inert doublet model with low scale seesaw mechanisms

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We have developed an extension of the inert doublet model in which the CP phases in the weak sector are generated from one-loop level corrections mediated by dark fields, while the strong CP phase remains vanishing at three loops. In this framework, the tiny masses of the active neutrinos are produced through a radiative inverse seesaw mechanism at a two-loop level, the masses of the first and second families of Standard Model (SM) charged fermions arise from a one-loop level radiative seesaw mechanism, and the third generation of SM charged fermion masses are generated at tree level. We have demonstrated that the proposed model successfully accounts for SM fermion masses and mixings. The radiative nature of the seesaw mechanisms is attributed to preserved discrete symmetries, which are required for ensuring the stability of fermionic and scalar dark matter candidates. The preserved discrete symmetries also allow for multicomponent dark matter, whose annihilation processes permit one to successfully reproduce the measured amount of dark matter relic abundance for an appropriate region of parameter space, which has been shown to be compatible with current dark matter direct detection limits. Besides that, we explore the model's ability to explain the 95 GeV diphoton excess observed by the CMS Collaboration, showing that it readily accommodates this anomaly. We also have shown that charged lepton flavor violating decays acquire rates within the current experimental sensitivity.

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I. INTRODUCTION

Despite the great success of the Standard Model (SM) as a theory of strong and electroweak interactions whose predictions have been experimentally verified with the highest degree of accuracy, there are several issues that the SM is unable to explain, such as the smallness of neutrino masses [1], the current amount of the dark matter

relic density [2], the tiny values of the electric dipole moments of the neutron and the elementary particles [3], and the SM charged fermion mass and mixing hierarchy. In the SM, the gauge invariance does not restrict the flavor structure of the Yukawa interactions. In particular, the Yukawa couplings of fermions to the Higgs field exhibit a wide range of values without any apparent underlying principle. While the discovery of the Higgs boson [4,5] confirms the existence of these interactions, current collider experiments are primarily sensitive to the Yukawa couplings of third-generation fermions [6–12]. The hypothesis that a more fundamental theory generates SM Yukawa couplings offers a promising avenue for understanding the flavor puzzle [13–32]. Such fundamental theory is expected to be highly symmetric at high energies; these Yukawa terms either vanish or converge to a common value. Theories focusing on explaining the origin of the SM fermion structure have been proposed and analyzed in detail in [33–54]. These models generate the top quark mass at the tree level. In contrast, the masses of the

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remaining particles, such as the bottom quark, tau lepton, and muon, arise at one-loop or higher-order levels, leading to a hierarchical mass spectrum.

The origin of quark mixing and the size of CP violation in this sector are also related issues. Nonperturbative effects in QCD can result in P and CP violations, indicated by a parameter θ , which is the sum of two terms: θ_{QCD} and θ_{QED} . The θ_{QED} appears due to the chiral rotation of the quark fields, which render the real and positive quark masses. The θ_{QCD} is the value of P - and CP -violating angles of QCD vacuum. The appearance of θ induces the neutron electric dipole moment (nEDM), which is constrained by the experiment [3] as $d_n \lesssim 3 \times 10^{-26} e$. It implies that $|\theta| \lesssim 10^{-10}$, which appears unnatural, and then corresponds to the strong CP problem. The popular solution to the strong CP problem in QCD is an assumption about the existence of global Peccei-Quinn (PQ) symmetry [55–57] or spontaneous CP symmetry breaking, which requires one to extend the SM particle content by adding a complex scalar field responsible for such CP breaking [58–61]. Recent studies performed in [62] have proposed a new solution to the strong CP problem based on the existence of a dark sector. Both the CP symmetry and Z_8 are spontaneously broken in such a way to give rise to a residual Z_2 discrete symmetry that allows the stability of the dark matter. The strong CP phase can arise via loop corrections mediated by dark fields. In the following work, we show that in our scenario, at the tree level, the active sector conserves the CP symmetry, then implying that the parameters θ_{QCD} and θ_{QED} vanish at tree level. The explicit CP violation appears in the dark sector. The CP violation can be transmitted to the SM quark sector via loop corrections mediated by the dark fields.

We achieve this in an extended inert doublet model (IDM) framework where the CP phases in the weak sector arise at one-loop level, whereas the strong CP phase remains vanishing at the three-loop level. In that theory, the tiny active neutrino masses are originated from an inverse seesaw mechanism at two-loop level; the first and second generation of SM charged fermion masses arise at one loop, whereas the third family of SM charged fermions obtains tree-level masses. It is worth mentioning that in most of the extended IDM, the tiny active neutrino masses are produced by a radiative seesaw mechanism at one loop [13,14,22–25,51,63–88], then requiring very small neutrino Yukawa couplings (of the order of the electron Yukawa coupling) or a tiny mass difference between the dark scalars and pseudoscalar seesaw messengers, to successfully accommodate the experimental values of the neutrino mass squared splittings. Two-loop neutrino mass models have been proposed and analyzed in the literature [89–97] in order to provide a more natural explanation for the tiny values of the active neutrino masses than those relying on radiative seesaw mechanisms at the one-loop level.

In this work, we consider an extension of the IDM where the inclusion of gauge singlet scalars augments the scalar sector, and the fermion sector is enlarged by charged vectorlike fermions and right-handed Majorana neutrinos. In that theory, the SM gauge symmetry is supplemented by including the spontaneously broken $U(1)_X$ global symmetry and the preserved Z_2 symmetry. The charged vectorlike fermions, dark scalars, and pseudoscalars mediate a one-loop level radiative seesaw mechanism that yields the masses of the first and second generation of SM charged fermions. Additionally, the charged vectorlike fermions and the dark scalars and pseudoscalars provide radiative corrections to the CP phases in the weak sector at one level and maintain the strong CP phase vanishing at the three-loop level. On the other hand, the tiny masses of the active neutrinos are produced by an inverse seesaw mechanism at the two-loop level where the lepton-number-violating Majorana mass terms arise at two loops. This is achieved thanks to the preserved Z_2 symmetry as well as to the remnant Z'_2 symmetry arising from the spontaneous breaking of the global $U(1)_X$ symmetry. These preserved discrete symmetries ensure the radiative nature of the above-mentioned seesaw mechanisms and the stability of the dark matter candidates. The model is compatible with the current pattern of SM fermion masses and mixings, with the constraints arising from charged lepton flavor violation, dark matter relic density, and direct detection. It also successfully accommodates the 95 GeV diphoton excess.

This paper is organized as follows. In Sec. II, we describe the model in detail. Its implications in SM fermion masses and mixings are discussed in Sec. III A. Section IV provides a detailed discussion of how the strong CP problem is addressed in the model under consideration. The consequences of the model in dark matter, charged lepton flavor violation, and the 95 GeV diphoton excess are discussed in Secs. VI, VII, and VIII, respectively. We state our conclusions in Sec. IX.

II. MODEL

A. Particle content

We start this section by explaining the reasoning that justifies the inclusion of extra scalar and fermions needed for the implementation of the radiative seesaw mechanisms that generates one-loop level masses for the first and second families of SM charged fermions, two-loop level tiny active neutrino masses, as well as two-loop level mixing mass terms between SM quarks and some heavy quarks. Besides that, the masses of the third generation of the SM charged fermions will be generated at tree level by the following operators:

$$\bar{q}_{iL}\tilde{\phi}u_{3R}, \quad \bar{q}_{iL}\phi d_{3R}, \quad \bar{l}_{iL}\phi l_{3R}, \quad i = 1, 2, 3, \quad (1)$$

where ϕ is the SM $SU(2)_L$ scalar doublet, which is expanded as

$$\phi = \begin{pmatrix} \phi^+ \\ \frac{v + \phi_R^0 + i\phi_I^0}{\sqrt{2}} \end{pmatrix}, \quad (2)$$

whereas q_{iL} and l_{iL} are the SM quark and lepton doublets, respectively. They are defined as follows:

$$q_{iL} = \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix}, \quad l_{iL} = \begin{pmatrix} \nu_{iL} \\ e_{iL} \end{pmatrix}, \quad i = 1, 2, 3. \quad (3)$$

To generate one-loop level masses for the first and second families of SM charged fermions, we need to forbid the following operators:

$$\bar{q}_{iL} \tilde{\phi} u_{nR}, \quad \bar{q}_{iL} \phi d_{nR}, \quad \bar{l}_{iL} \phi l_{nR}, \quad n = 1, 2 \quad (4)$$

at tree level and to allow other operators instead, which are described in the following. In order to successfully implement the radiative seesaw mechanism that yields one-loop masses for the first and second families of SM charged fermions, we need the following operators:

$$\begin{aligned} \bar{q}_{iL} \tilde{\eta} T_{kR}, \quad \bar{T}_{kL} \phi_2^* u_{nR}, \quad \bar{T}_{kL} \sigma^* T_{sR}, \quad k, s, n = 1, 2. \\ \bar{q}_{iL} \eta B_{kR}, \quad \bar{B}_{kL} \phi_2 d_{nR}, \quad \bar{B}_{kL} \sigma B_{nR}, \quad i = 1, 2, 3 \\ \bar{l}_{iL} \eta E_{kR}, \quad \bar{E}_{kL} \phi_2 l_{nR}, \quad \bar{E}_{kL} \sigma E_{nR}. \end{aligned} \quad (5)$$

This implies an extension of the SM gauge symmetry by the inclusion of extra $U(1)_X \times Z_2$ symmetry, with $U(1)_X$ assumed to be global. We further assume that the global $U(1)_X$ symmetry spontaneously breaks down to a preserved Z'_2 symmetry, which is needed to ensure the radiative nature of the seesaw mechanism that generates one-loop level masses for the first generation of SM charged fermions. The supplementary Z_2 symmetry is crucial for ensuring the two-loop level nature of the inverse seesaw mechanism that yields the active neutrino masses. The successful implementation of the above-mentioned radiative seesaw mechanism requires one to enlarge the SM scalar spectrum by including an inert $SU(2)_L$ scalar doublet η as well as the electrically neutral scalar singlets φ_1 , φ_2 , φ_3 , and σ . In addition, the fermion sector of the SM has to be augmented as well by including the following charged vector like fermions: two up-type quarks T_k , two down-type quarks B_k , and two exotic charged leptons E_k ($k = 1, 2$) in singlet representations of $SU(2)_L$. Besides that, implementing the two-loop level inverse seesaw mechanism to produce tiny masses of the light active neutrinos requires the following structure for the full neutrino mass matrix in the basis (ν_L, ν_R^C, N_R^C) :

$$M_\nu = \begin{pmatrix} 0_{3 \times 3} & m_{\nu D} & 0_{3 \times 3} \\ m_{\nu D}^T & 0_{2 \times 2} & M \\ 0_{3 \times 3} & M^T & \mu \end{pmatrix}, \quad (6)$$

where ν_{iL} ($i = 1, 2, 3$) corresponds to the active neutrinos, whereas ν_{iR} and N_{iR} ($i = 1, 2$) are the sterile neutrinos. Furthermore, the entries of the full neutrino mass matrix of Eq. (6) should obey the hierarchy $\mu_{nk} \ll (m_{\nu D})_{in} \ll M_{nk}$ ($i = 1, 2, 3, n, k = 1, 2$), where the submatrices $m_{\nu D}$ and M are generated at tree level, whereas the Majorana mass submatrix μ associated with the breaking of the lepton number in two units is radiatively generated at the two-loop level. Consequently in order to successfully implement the above-described inverse seesaw mechanism, we need the following operators:

$$\begin{aligned} \bar{l}_{iL} \tilde{\phi} \nu_{nR}, \quad \nu_{nR} \sigma^* \bar{N}_{kR}^C, \quad \bar{N}_{nR} \Psi_{kR}^C \varphi_1, \quad k, n = 1, 2. \\ \bar{\Psi}_{nR} \varphi_3 \Omega_{kR}^C, \quad \bar{\Omega}_{nR} \sigma \Omega_{kR}^C, \quad i = 1, 2, 3, \end{aligned} \quad (7)$$

where ν_{nR} , Ω_{nR} , Ψ_{nR} ($n = 1, 2$) are right-handed Majorana neutrinos which are required to be added to the fermionic spectrum of the SM.

The model under consideration corresponds to an extended inert doublet model. Including electrically neutral scalar singlets enlarges the scalar sector, and the fermion sector is augmented by adding charged vectors like fermions and right-handed Majorana neutrinos. The SM gauge symmetry is extended by the inclusion of the spontaneously broken global $U(1)_X$ symmetry and the preserved Z_2 discrete symmetry. In the model under consideration, the first and second families of SM charged fermions get one-loop level masses, thanks to the conserved Z'_2 symmetry arising from the spontaneous symmetry breaking of $U(1)_X$. In contrast, the third generation of SM charged fermions obtain their masses at tree level. The masses of the light-active neutrinos arise from a radiative inverse seesaw mechanism at a two-loop level. Furthermore, in the considered model, mixing mass terms between SM quarks and some heavy quarks are generated at the two-loop level and involve complex parameters arising from a CP -violating scalar potential, then allowing one to address the strong CP problem. The scalar, quark, and leptonic spectrum of the model, as well as their assignments under the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X \times Z_2$ symmetry are displayed in Tables I, II, and III, respectively.

The SM $SU(2)$ scalar doublet ϕ and the scalar singlet σ develop the following vacuum expectation values (VEVs):

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \quad \langle \sigma \rangle = \frac{v_\sigma}{\sqrt{2}}$$

TABLE I. Scalar assignments under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X \times Z_2$.

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_X$	Z_2
ϕ	1	2	$\frac{1}{2}$	0	0
η	1	2	$\frac{1}{2}$	-1	0
σ	1	1	0	-2	0
φ_1	1	1	0	1	1
φ_2	1	1	0	1	0
φ_3	1	1	0	-2	1

TABLE II. Quark assignments under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X \times Z_2$. Here $i = 1, 2, 3$ and $n = 1, 2$.

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_X$	Z_2
q_{iL}	3	2	$\frac{1}{6}$	0	0
u_{3R}	3	1	$\frac{2}{3}$	0	0
u_{nR}	3	1	$\frac{2}{3}$	2	0
d_{3R}	3	1	$-\frac{1}{3}$	0	0
d_{nR}	3	1	$-\frac{1}{3}$	-2	0
T_{nL}	3	1	$\frac{2}{3}$	1	0
T_{nR}	3	1	$\frac{2}{3}$	-1	0
B_{nL}	3	1	$-\frac{1}{3}$	-1	0
B_{nR}	3	1	$-\frac{1}{3}$	1	0

TABLE III. Lepton assignments under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X \times Z_2$. Here $i = 1, 2, 3$ and $n = 1, 2$.

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_X$	Z_2
l_{iL}	1	2	$-\frac{1}{2}$	-4	0
l_{nR}	1	1	-1	-2	0
l_{3R}	1	1	-1	-4	0
E_{nL}	1	1	-1	-1	0
E_{nR}	1	1	-1	-3	0
ν_{nR}	1	1	0	-4	0
N_{nR}	1	1	0	2	0
Ψ_{nR}	1	1	0	-1	1
Ω_{nR}	1	1	0	-1	0

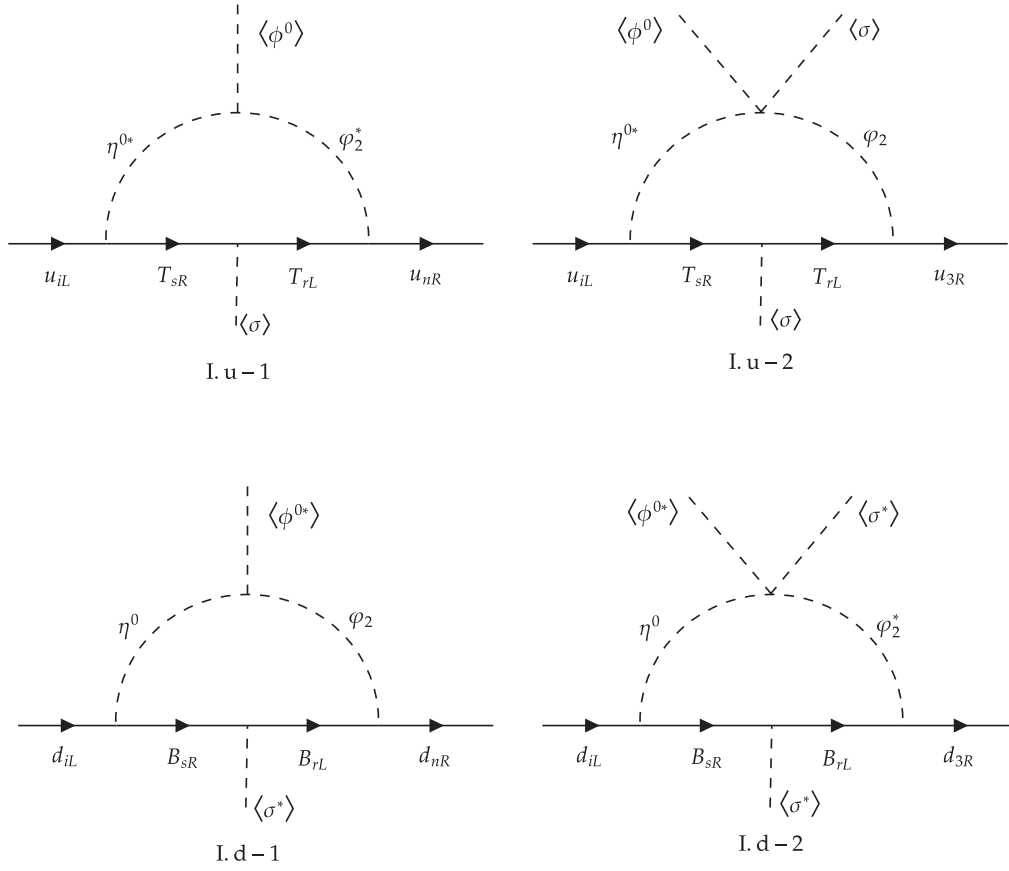
producing the following scheme of symmetry breaking:

$$\begin{aligned}
 &SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X \times Z_2 \\
 &\quad \downarrow v_\sigma \\
 &SU(3)_C \times SU(2)_L \times U(1)_Y \times Z'_2 \times Z_2
 \end{aligned} \tag{8}$$

$$\begin{aligned}
 &\quad \downarrow v \\
 &SU(3)_C \times U(1)_Q \times Z'_2 \times Z_2.
 \end{aligned} \tag{9}$$

It is assumed that the global $U(1)_X$ symmetry is spontaneously broken down to the preserved Z'_2 symmetry by the vacuum expectation value of the scalar singlet σ . We note that the electric charge operator, $Q = T_3 + Y$, combines isospin and hypercharge, whereas Z'_2 is the residual symmetry resulting from the spontaneous breaking of $U(1)_X$, takes the form $Z'_2 = e^{i\pi X} = (-1)^X$, then implying that the Z'_2 field assignments of the model are defined as $(-1)^X$ being X the corresponding $U(1)_X$ charges. Hence, the particles with even X charge carry a zero Z'_2 charge, while the particles with odd X charge carry a unit Z'_2 charge. Due to the preserved $Z'_2 \times Z_2$ symmetry neither the extra $SU(2)$ scalar doublet η , nor the singlet scalar fields φ_1 , φ_2 , φ_3 acquire vacuum expectation values since they carry nontrivial charges under this preserved $Z'_2 \times Z_2$ symmetry. In what follows, we justify the particle content of the model. The scalar fields η and φ_2 are needed for the implementation of the one-loop level radiative seesaw mechanisms that yield the first and second generations of SM charged fermion masses. Such radiative seesaw mechanisms are also induced by the charged vectorlike fermions T_n , B_n , and E_n as indicated in the Feynman diagrams of Figs. 1 and 3. Besides that, the scalar singlets φ_1 and φ_3 , together with the neutral leptons Ω_{nR} and Ψ_{nR} ($n = 1, 2$) mediate the radiative seesaw mechanism at the two-loop level, as indicated in Fig. 4, that results in a dynamical generation of the lepton number violating $\mu_{nk} \bar{N}_{nR} \sigma N_{kR}^C$ Majorana mass terms, then yielding an inverse seesaw mechanism at two-loop level.

Notice that the preserved $Z'_2 \times Z_2$ symmetry is also crucial for avoiding the appearance of tree-level masses for active neutrinos as well as for the first and second families of SM charged fermions. It is worth mentioning that the preserved Z_2 and Z'_2 discrete symmetries are crucial for ensuring the radiative nature of the one-loop level radiative seesaw mechanisms that yield the masses of the first and second generation of SM charged fermions, as well as the inverse seesaw mechanism [45,46,48,98–111] that produces the tiny active neutrino masses. Furthermore, because of the conserved $Z'_2 \times Z_2$ symmetry, the model offers a natural stability mechanism for two-component dark matter in which a dark matter component carries an odd Z'_2 charge whereas the other dark matter component carries an odd- Z_2 charge. As a result, the particle spectrum in the model can be separated into two parts: the DM sector, which contains at least one of two odd- Z_2, Z'_2 charges, and the active sector, which carries both even- Z_2, Z'_2 charges. Although the explicit CP violation is in the dark sector, the preserved $Z_2 \times Z'_2$ symmetry prohibits the mixing between the active and dark sectors, then implying that the number of CP violations is not constrained by the SM Higgs couplings.

FIG. 1. One-loop corrections to the SM u -quark.

With the particle content and symmetries specified in Tables I, II, and III, the following quark and leptonic Yukawa terms arise:

$$\begin{aligned}
 -\mathcal{L}_Y^{(q)} = & \sum_{i=1}^3 y_i^{(u)} \bar{q}_{iL} \tilde{\phi} u_{3R} + \sum_{i=1}^3 \sum_{k=1}^2 x_{ik}^{(T)} \bar{q}_{iL} \tilde{\eta} T_{kR} + \sum_{k=1}^2 \sum_{n=1}^2 z_{kn}^{(u)} \bar{T}_{kL} \phi_2^* u_{nR} + \sum_{k=1}^2 \kappa_{k3}^{(u)} \bar{T}_{kL} \phi_2 u_{3R} \\
 & + \sum_{k=1}^2 \sum_{s=1}^2 y_{ks}^{(T)} \bar{T}_{kL} \sigma^* T_{sR} + \sum_{i=1}^3 y_i^{(d)} \bar{q}_{iL} \phi d_{3R} + \sum_{i=1}^2 \sum_{k=1}^2 x_{ik}^{(B)} \bar{q}_{iL} \eta B_{kR} \\
 & + \sum_{k=1}^2 \sum_{n=1}^2 z_{kn}^{(d)} \bar{B}_{kL} \phi_2 d_{nR} + \sum_{k=1}^2 \kappa_{k3}^{(d)} \bar{B}_{kL} \phi_2^* d_{3R} + \sum_{k=1}^2 \sum_{n=1}^2 y_{kn}^{(B)} \bar{B}_{kL} \sigma B_{nR}
 \end{aligned} \tag{10}$$

$$\begin{aligned}
 -\mathcal{L}_Y^{(l)} = & \sum_{i=1}^3 y_{i3}^{(l)} \bar{l}_{iL} \phi l_{3R} + \sum_{i=1}^3 \sum_{k=1}^2 x_{ik}^{(E)} \bar{l}_{iL} \eta E_{kR} + \sum_{n=1}^2 \sum_{k=1}^2 z_{nk}^{(l)} \bar{E}_{nL} \phi_2 l_{kR} + \sum_{n=1}^2 \sum_{k=1}^2 y_{nk}^{(E)} \bar{E}_{nL} \sigma^* E_{kR} \\
 & + \sum_{i=1}^3 \sum_{n=1}^2 (g_\nu)_{in} \bar{l}_{iL} \tilde{\phi} \nu_{nR} + \sum_{n=1}^2 \sum_{k=1}^2 (y_N)_{nk} \nu_{rR} \sigma^* \bar{N}_{kR}^C + \sum_{n=1}^2 \sum_{l=1}^2 (y_\Psi)_{nl} \bar{N}_{nR} \Psi_{lR}^C \phi_1 \\
 & + \sum_{n=1}^2 \sum_{k=1}^2 (x_\Omega)_{nk} \bar{\Psi}_{nR} \phi_3 \Omega_{kR}^C + \sum_{n=1}^2 \sum_{k=1}^2 (y_\Omega)_{nk} \bar{\Omega}_{nR} \sigma \Omega_{kR}^C + \text{H.c.}
 \end{aligned} \tag{11}$$

B. Scalar potential

With the scalar content and symmetries specified in Table I, the following scalar potential arises:

$$\mathcal{V} = V + V_{CPV} + V'_{CPV} \quad (12)$$

where

$$\begin{aligned} V = & -\mu_\phi^2 \phi^\dagger \phi - \mu_\eta^2 \eta^\dagger \eta - \mu_\sigma^2 \sigma^\dagger \sigma - \sum_{i=1}^3 \mu_{\varphi_i}^2 \varphi_i^\dagger \varphi_i + \frac{\lambda_\phi}{4} (\phi^\dagger \phi)^2 + \frac{\lambda_\eta}{4} (\eta^\dagger \eta)^2 + \frac{\lambda_\sigma}{4} (\sigma^\dagger \sigma)^2 + \sum_{i=1}^3 \frac{\lambda_{\varphi_i}}{4} (\varphi_i^\dagger \varphi_i)^2 \\ & + \lambda_{\eta\phi} (\phi^\dagger \phi) (\eta^\dagger \eta) + \lambda'_{\eta\phi} (\eta^\dagger \phi) (\phi^\dagger \eta) + \lambda_{\sigma\phi} (\sigma^\dagger \sigma) (\phi^\dagger \phi) + \lambda_{\sigma\eta} (\sigma^\dagger \sigma) (\eta^\dagger \eta) + \sum_{i=1}^3 \lambda_{\sigma\varphi_i} (\sigma^\dagger \sigma) (\varphi_i^\dagger \varphi_i) \\ & + \sum_{i=1}^3 \{ \lambda_{\phi\varphi_i} (\phi^\dagger \phi) (\varphi_i^\dagger \varphi_i) + \lambda_{\eta\varphi_i} (\eta^\dagger \eta) (\varphi_i^\dagger \varphi_i) \} + \sum_{i \neq j; i,j=1}^3 \lambda_{\varphi_i \varphi_j} (\varphi_i^\dagger \varphi_j) (\varphi_j^\dagger \varphi_i), \end{aligned} \quad (13)$$

and

$$\begin{aligned} V_{CPV} = & \left\{ \mathcal{F}_\phi \varphi_1 \varphi_2 \varphi_3 + \mathcal{G} \eta^\dagger \phi \varphi_2^\dagger + \mathcal{F}_{\sigma\varphi_1} \sigma \varphi_1 \varphi_1 + \mathcal{F}_{\sigma\varphi_2} \sigma \varphi_2 \varphi_2 + \text{H.c.} \right\}, \\ V'_{CPV} = & \left\{ f \eta^\dagger \phi \varphi_1 \varphi_3 + f' \eta^\dagger \phi \sigma \varphi_2 + f_\sigma \sigma^\dagger \varphi_3 \varphi_2^\dagger \varphi_1 + f'_\sigma \sigma^\dagger \varphi_3 \varphi_1^\dagger \varphi_2 + f_{\varphi_3 \sigma} \varphi_3^2 \sigma^{*2} + f_{\varphi_1 \varphi_3} \varphi_1^2 \varphi_3^{*2} + \text{H.c.} \right\}, \end{aligned} \quad (14)$$

All parameters in the potential V are real, whereas those in the CP -violating parts, V_{CPV} and V'_{CPV} , can be complex. However, by exploiting the freedom of rephasing the scalar fields, the complex phases can be absorbed into η , φ_1 , φ_2 , φ_3 , σ , and ϕ , making $\mathcal{F}_\phi$, $\mathcal{F}_{\sigma\varphi_1}$, $\mathcal{F}_{\sigma\varphi_2}$, f , f' , f_σ , and $f_{\sigma'}$ real. The remaining complex phases in $\mathcal{F}_\phi$, $\mathcal{G}$, $f_{\varphi_3 \sigma}$, and $f_{\varphi_1 \varphi_2}$ introduce CP violation in the scalar potential, thereby making the model explicitly CP -violating and generating complex corrections to the quark masses at the radiative level.

1. Theoretical constraints on the scalar potential

To ensure the validity of the effective description, we require the scalar sector to satisfy three sets of theoretical bounds: perturbativity, vacuum stability, and tree-level unitarity.

- (a) *Perturbativity.* All dimensionless couplings in the scalar potential are constrained to the perturbative regime.

$$|\lambda_i|, |f_j| < 4\pi, \quad (15)$$

where λ_i denote the quartic couplings and f_j represent the dimensionless coefficients of the CP -violating interactions. This condition guarantees the convergence of perturbative expansions.

- (b) *Vacuum stability.* The quartic part of the scalar potential, which dominates the behavior of the scalar potential in the region of very large values of the field

components, must be bounded from below in all field directions. Following the procedure used for analyzing the stability described in Refs. [112,113], we find that all self-couplings are positive,

$$\lambda_\phi, \lambda_\eta, \lambda_\sigma, \lambda_{\varphi_i} > 0, \quad (16)$$

together with copositivity constraints on the portal couplings,

$$\begin{aligned} \lambda_{\phi\eta} + \sqrt{\lambda_\phi \lambda_\eta} &> 0, & \lambda_{\phi\sigma} + \sqrt{\lambda_\phi \lambda_\sigma} &> 0, \\ \lambda_{\eta\sigma} + \sqrt{\lambda_\eta \lambda_\sigma} &> 0, \end{aligned} \quad (17)$$

$$\begin{aligned} \lambda_{\phi\varphi_i} + \sqrt{\lambda_\phi \lambda_{\varphi_i}} &> 0, & \lambda_{\eta\varphi_i} + \sqrt{\lambda_\eta \lambda_{\varphi_i}} &> 0, \\ \lambda_{\sigma\varphi_i} + \sqrt{\lambda_\sigma \lambda_{\varphi_i}} &> 0 \quad (i = 1, 2, 3), \end{aligned} \quad (18)$$

and analogous inequalities for the mixed couplings $\lambda_{\varphi_i \varphi_j}$. These constraints ensure that the potential does not develop runaway directions with $V \rightarrow -\infty$.

It is worth noting that the operator $\lambda'_{\eta\phi} (\eta^\dagger \phi) (\phi^\dagger \eta)$ does not in general reduce to $(\eta^\dagger \eta) (\phi^\dagger \phi)$. To account for this phase dependence, one may define an effective portal coupling

$$\lambda_{\eta\phi}^{\text{eff}} = \lambda_{\eta\phi} + \min(0, \lambda'_{\eta\phi}), \quad (19)$$

and use $\lambda_{\eta\phi} \rightarrow \lambda_{\eta\phi}^{\text{eff}}$ in the pairwise conditions above. This provides a conservative, sufficient criterion for vacuum stability.

- (c) *Tree-level unitarity*. The unitarity of $2 \rightarrow 2$ scalar scattering amplitudes imposes additional constraints on the quartic scalar couplings. In particular, the s -wave component of the amplitude must satisfy

$$|\text{Re } a_0| < \frac{1}{2}, \quad (20)$$

which translates into bounds on linear combinations of the couplings. Operationally, one constructs the scattering matrix $\mathcal{M}$ for all two-body scalar states, diagonalizes it, and imposes the condition

$$|\Lambda_i| < 8\pi, \quad (21)$$

where Λ_i are the eigenvalues of $\mathcal{M}$.

When combined with vacuum stability and perturbativity, the unitarity constraints significantly restrict the parameter space, ensuring the scalar potential remains consistent and theoretically controlled across the relevant energy scales.

2. Impact of CP -violating terms on theoretical bounds

The potential includes CP -violating contributions V_{CPV} and V'_{CPV} with cubic and quartic interactions. The theoretical constraints on the CP -violating scalar couplings are described below:

- (a) *Perturbativity*. All CP -violating couplings f_j must remain perturbative,

$$|f_j| < 4\pi. \quad (22)$$

- (b) *Vacuum stability*. Cubic terms are irrelevant at large field values, while quartic CP -violating couplings contribute to the vacuum stability conditions. The bounded from below conditions apply to their real parts, as follows:

$$\text{Re } f_{\varphi_3\sigma} > -\sqrt{\lambda_{\varphi_3}\lambda_\sigma}, \quad \text{Re } f_{\varphi_1\varphi_2} > -\sqrt{\lambda_{\varphi_1}\lambda_{\varphi_2}}. \quad (23)$$

In short, CP -violating terms extend the bounds: their real parts enter in the vacuum stability conditions, and their magnitudes are constrained by perturbativity.

3. Scalar mass spectrum

The global minimum conditions for the scalar potential yield the following solutions for the mass dimensionful parameters μ_ϕ^2 and μ_σ^2 :

$$\mu_\phi^2 = \frac{1}{2}\lambda_{\sigma\phi}v_\sigma^2 + \frac{1}{4}\lambda_\phi v_\phi^2, \quad \mu_\sigma^2 = \frac{1}{2}\lambda_{\sigma\phi}v_\phi^2 + \frac{1}{4}\lambda_\sigma v_\sigma^2. \quad (24)$$

The charged scalar fields are found in the first components of the scalar doublets, denoted as ϕ and η . The first component of ϕ , referred to as $\phi^\pm$, is massless and is identified as the Goldstone bosons that are “eaten” by the longitudinal components of the $W^\pm$ gauge bosons. In contrast, the first component of η , called $\eta^\pm$, corresponds to physical fields that have mass

$$m_{\eta^\pm}^2 = -\mu_\eta^2 + \frac{1}{2}\lambda_{\eta\phi}v_\phi^2 + \frac{1}{2}\lambda_{\sigma\eta}v_\sigma^2. \quad (25)$$

Two CP -even neutral scalar fields, both carrying even Z_2 and Z'_2 charges, $\Re\phi$ and $\Re\sigma$, mix via a 2×2 matrix. We find the physical states for them as follows:

$$h = \cos\theta_h \Re\phi - \sin\theta_h \Re\sigma, \quad H = \sin\theta_h \Re\phi + \cos\theta_h \Re\sigma \quad (26)$$

with $\tan 2\theta_h = \frac{4\lambda_{\sigma\phi}v_\sigma v_\phi}{\lambda_\sigma v_\sigma^2 - \lambda_\phi v_\phi^2}$, and their masses are respectively given by

$$\begin{aligned} m_h^2 &= \frac{1}{4} \left(\lambda_\phi v_\phi^2 + \lambda_\sigma v_\sigma^2 - \sqrt{\lambda_\phi^2 v_\phi^4 + \lambda_\sigma^2 v_\sigma^4 - 2(\lambda_\phi \lambda_\sigma - 8\lambda_{\sigma\phi})v_\phi^2 v_\sigma^2} \right), \\ m_H^2 &= \frac{1}{4} \left(\lambda_\phi v_\phi^2 + \lambda_\sigma v_\sigma^2 + \sqrt{\lambda_\phi^2 v_\phi^4 + \lambda_\sigma^2 v_\sigma^4 - 2(\lambda_\phi \lambda_\sigma - 8\lambda_{\sigma\phi})v_\phi^2 v_\sigma^2} \right), \end{aligned} \quad (27)$$

where h is identified as the 126 GeV SM-like Higgs boson, and H is a new CP -even Higgs boson. In contrast, two CP -odd neutral scalar fields, both carrying even Z_2 and Z'_2 charges, $\Im\phi$ and $\Im\sigma$, are massless, and interpreted as the Goldstone boson eaten by the longitudinal component of the Z gauge boson and the Majoron, respectively. In the dark scalar sector of the model there is the field, φ_1 , which carries both odd Z_2 and Z'_2 charges. Due to the interaction terms between φ_1 , ϕ , and σ , $\Re\varphi_1$ and $\Im\varphi_1$ are physical dark CP -even and CP -odd scalars, respectively, with masses given by

$$\begin{aligned} m_{\Re\varphi_1}^2 &= -\mu_{\varphi_1}^2 + \frac{1}{2}\lambda_{\phi\varphi_1}v_\phi^2 + \frac{1}{2}\lambda_{\sigma\varphi_1}v_\sigma^2 + \sqrt{2}\mathcal{F}_{\sigma\varphi_1}v_\sigma, \\ m_{\Im\varphi_1}^2 &= -\mu_{\varphi_1}^2 + \frac{1}{2}\lambda_{\phi\varphi_1}v_\phi^2 + \frac{1}{2}\lambda_{\sigma\varphi_1}v_\sigma^2 - \sqrt{2}\mathcal{F}_{\sigma\varphi_1}v_\sigma. \end{aligned} \quad (28)$$

On the other hand, the dark singlet scalar field φ_3 has an odd Z_2 charge and is neutral under the preserved Z'_2 symmetry. After symmetry breaking, the model predicts a mixing matrix between the real and imaginary

components of φ_3 , which arises from the explicit CP -breaking term $f_{\varphi_3\sigma}\varphi_3^2\sigma^{*2} + \text{H.c.}$. The physical states are mixtures of the CP -odd and CP -even components, defined as

$$\begin{aligned} R_{\varphi_3} &= \cos \theta_{\varphi_3} \Re \varphi_3 - \sin \theta_{\varphi_3} \Im \varphi_3, \\ I_{\varphi_3} &= \sin \theta_{\varphi_3} \Re \varphi_3 + \cos \theta_{\varphi_3} \Im \varphi_3 \end{aligned} \quad (29)$$

where the mixing angle satisfies the relation $\tan \theta_{\varphi_3} = \frac{\Im f_{\varphi_3\sigma}}{\Re f_{\varphi_3\sigma}}$. The masses of these states are given by

where

$$\begin{aligned} M_{\Re\eta\varphi_2}^2 &= \begin{pmatrix} -\mu_\eta^2 + \frac{1}{2}(\lambda_{\eta\phi}v_\phi^2 + \lambda'_{\eta\phi}v_\phi^2 + \lambda_{\sigma\eta}v_\sigma^2) & \frac{1}{2}(\sqrt{2}\Re\mathcal{G} + \Re f'v_\sigma)v_\phi \\ \frac{1}{2}(\sqrt{2}\Re\mathcal{G} + \Re f'v_\sigma)v_\phi & -\mu_{\varphi_2}^2 + \frac{1}{2}(\lambda_{\phi\varphi_2}v_\phi^2 + \lambda_{\sigma\varphi_2}v_\sigma^2 + 2\sqrt{2}\mathcal{F}_{\sigma\varphi_2}v_\sigma) \end{pmatrix}, \\ M_{\Re\Im\eta\varphi_2}^2 &= \begin{pmatrix} 0 & -\frac{v_\phi}{2}(\sqrt{2}\Im\mathcal{G} - \Im f'v_\sigma) \\ -\frac{v_\phi}{2}(\sqrt{2}\Im\mathcal{G} - \Im f'v_\sigma) & 0 \end{pmatrix}, \\ M_{\Im\eta\varphi_2}^2 &= \begin{pmatrix} -\mu_\eta^2 + \frac{1}{2}(\lambda_{\eta\phi}v_\phi^2 + \lambda'_{\eta\phi}v_\phi^2 + \lambda_{\sigma\eta}v_\sigma^2) & \frac{1}{2}(\sqrt{2}\Re\mathcal{G} - \Re f'v_\sigma)v_\phi \\ \frac{1}{2}(\sqrt{2}\Re\mathcal{G} - \Re f'v_\sigma)v_\phi & -\mu_{\varphi_2}^2 + \frac{1}{2}(\lambda_{\phi\varphi_2}v_\phi^2 + \lambda_{\sigma\varphi_2}v_\sigma^2 - 2\sqrt{2}\mathcal{F}_{\sigma\varphi_2}v_\sigma) \end{pmatrix}. \end{aligned} \quad (32)$$

The mixing matrix presented in Eq. (31) yields four physical states, denoted as $\rho^T = (\rho_1, \rho_2, \rho_3, \rho_4)$. These states are related to the states contained in $RI_{\eta\varphi_2}^T = (\Re\eta^0, \Re\varphi_2, \Im\eta^0, \Im\varphi_2)$ through a 4×4 rotation matrix, referred to as R . The relationship can be expressed as $\rho^T = R^T RI_{\eta\varphi_2}^T$.

III. FERMION MASSES AND MIXINGS

A. Quark masses and mixings

In the model under consideration, the third quark family acquires its mass at the tree level, while the other SM quarks get their masses radiatively at one-loop level. To understand this in more detail, we will begin by analyzing the quark Yukawa interactions.

The up- and down-type quark mass terms can be written in the interaction basis as

$$\begin{aligned} \mathcal{L}_{\text{mass}}^q &= (\bar{u}_{1L} \bar{u}_{2L} \bar{u}_{3L})M_u \begin{pmatrix} u_{1R} \\ u_{2R} \\ u_{3R} \end{pmatrix} \\ &+ (\bar{d}_{1L} \bar{d}_{2L} \bar{d}_{3L})M_d \begin{pmatrix} d_{1R} \\ d_{2R} \\ d_{3R} \end{pmatrix} + \text{H.c.}, \end{aligned} \quad (33)$$

$$\begin{aligned} m_{R_{\varphi_3}}^2 &= -\mu_{\varphi_3}^2 + \frac{\lambda_{\phi\varphi_3}}{2}v_\phi^2 + \frac{\lambda_{\sigma\varphi_3}}{2}v_\sigma^2 + |f_{\varphi_3\sigma}|v_\sigma^2, \\ m_{I_{\varphi_3}}^2 &= -\mu_{\varphi_3}^2 + \frac{\lambda_{\phi\varphi_3}}{2}v_\phi^2 + \frac{\lambda_{\sigma\varphi_3}}{2}v_\sigma^2 - |f_{\varphi_3\sigma}|v_\sigma^2. \end{aligned} \quad (30)$$

The neutral scalar fields that carry an even Z_2 charge and an odd Z'_2 correspond to the second component of the scalar doublet η , denoted as η^0 , and to φ_2 . Due to the complex phases associated with $\mathcal{G}$ and f' , there is a mixing between the CP -even components ($\Re\eta, \Re\varphi_2$) and the CP -odd components ($\Im\eta, \Im\varphi_2$). The mixing matrix has a form

$$M_{\Re\eta\varphi_2}^2 = \begin{pmatrix} M_{\Re\eta\varphi_2}^2 & M_{\Re\Im\eta\varphi_2}^2 \\ M_{\Re\Im\eta\varphi_2}^2 & M_{\Im\eta\varphi_2}^2 \end{pmatrix} \quad (31)$$

where M_u and M_d are 3×3 mass matrices. These mass matrices can be expressed as a sum of contributions from different loop orders:

$$\begin{aligned} M_u &= M_u^{(0)} + M_u^{(1)} + M_u^{(2)} + \dots, \\ M_d &= M_d^{(0)} + M_d^{(1)} + M_d^{(2)} + \dots. \end{aligned}$$

At the tree level, only the terms in the third column are nonzero, which imposes a specific structure on the matrices:

$$M_u^{(0)} = \begin{pmatrix} 0 & 0 & \frac{y_1^{(u)}}{\sqrt{2}}v \\ 0 & 0 & \frac{y_2^{(u)}}{\sqrt{2}}v \\ 0 & 0 & \frac{y_3^{(u)}}{\sqrt{2}}v \end{pmatrix}, \quad M_d^{(0)} = \begin{pmatrix} 0 & 0 & \frac{y_1^{(d)}}{\sqrt{2}}v \\ 0 & 0 & \frac{y_2^{(d)}}{\sqrt{2}}v \\ 0 & 0 & \frac{y_3^{(d)}}{\sqrt{2}}v \end{pmatrix}. \quad (34)$$

When one-loop corrections are considered, they introduce additional nonzero elements. The one-loop diagrams contributing to the quark mass matrix are illustrated in Fig. 1.

We enforce an exact CP symmetry within the SM part of the Lagrangian, but allow for explicit CP breaking in the dark sector. This ensures that CP phases vanish at tree

level. We observe that the CP -violating phase ($e^{\pm i\alpha}$) in the dark matter sector, arising from the complex coupling $\mathcal{G}$ of the scalar interaction vertex $\mathcal{G}\eta^\dagger\phi\phi_2^\dagger + \text{H.c.}$, induces corrections to the first and second columns of the mass mixing matrix, while leaving the third column unaffected. The full mass matrices, up to one-loop order, are given by the sum of the tree-level and one-loop contributions, $M^{(0)} + M^{(1)}$. This results in the following structure:

$$\mathcal{M}_u = \begin{pmatrix} e^{i\alpha}(m_{11}^u)^{(1)} & e^{i\alpha}(m_{12}^u)^{(1)} & \frac{y_1^{(u)}}{\sqrt{2}}v + (m_{13}^u)^{(1)} \\ e^{i\alpha}(m_{21}^u)^{(1)} & e^{i\alpha}(m_{22}^u)^{(1)} & \frac{y_2^{(u)}}{\sqrt{2}}v + (m_{23}^u)^{(1)} \\ e^{i\alpha}(m_{31}^u)^{(1)} & e^{i\alpha}(m_{32}^u)^{(1)} & \frac{y_3^{(u)}}{\sqrt{2}}v + (m_{33}^u)^{(1)} \end{pmatrix}, \quad \mathcal{M}_d = \begin{pmatrix} e^{-i\alpha}(m_{11}^d)^{(1)} & e^{-i\alpha}(m_{12}^d)^{(1)} & \frac{y_1^{(d)}}{\sqrt{2}}v + (m_{13}^d)^{(1)} \\ e^{-i\alpha}(m_{21}^d)^{(1)} & e^{-i\alpha}(m_{22}^d)^{(1)} & \frac{y_2^{(d)}}{\sqrt{2}}v + (m_{23}^d)^{(1)} \\ e^{-i\alpha}(m_{31}^d)^{(1)} & e^{-i\alpha}(m_{32}^d)^{(1)} & \frac{y_3^{(d)}}{\sqrt{2}}v + (m_{33}^d)^{(1)} \end{pmatrix}. \quad (35)$$

Here, the entries $(m_{ij}^q)^{(1)}$ ($i, j = 1, 2, 3$) with $q = u, d$, which arise from one-loop corrections, have the following form:

$$e^{i\alpha}(m_{in}^u)^{(1)} = \sum_{r=1}^2 \frac{x_{ir}^{(T)} z_{rn}^{(u)} m_{T_r}}{16\pi^2} \left\{ \sum_{i=1}^4 R_{1i} R_{2i} f(m_{\rho_i}^2, m_{T_r}^2) - \sum_{i=1}^4 R_{3i} R_{4i} f(m_{\rho_i}^2, m_{T_r}^2) \right\} \\ - i \sum_{r=1}^2 \frac{x_{ir}^{(T)} z_{rn}^{(u)} m_{T_r}}{16\pi^2} \left\{ \sum_{i=1}^4 R_{1i} R_{4i} f(m_{\rho_i}^2, m_{T_r}^2) + \sum_{i=1}^4 R_{2i} R_{3i} f(m_{\rho_i}^2, m_{T_r}^2) \right\}, \quad (36)$$

$$e^{-i\alpha}(m_{in}^d)^{(1)} = \sum_{r=1}^2 \frac{x_{ir}^{(B)} z_{rn}^{(d)} m_{B_r}}{16\pi^2} \left\{ \sum_{i=1}^4 R_{1i} R_{2i} f(m_{\rho_i}^2, m_{B_r}^2) - \sum_{i=1}^4 R_{3i} R_{4i} f(m_{\rho_i}^2, m_{B_r}^2) \right\} \\ + i \sum_{r=1}^2 \frac{x_{ir}^{(B)} z_{rn}^{(d)} m_{B_r}}{16\pi^2} \left\{ \sum_{i=1}^4 R_{1i} R_{4i} f(m_{\rho_i}^2, m_{B_r}^2) + \sum_{i=1}^4 R_{2i} R_{3i} f(m_{\rho_i}^2, m_{B_r}^2) \right\}, \quad (37)$$

and

$$(m_{i3}^u)^{(1)} = \sum_{r=1}^2 \frac{x_{ir}^{(T)} \kappa_{r3}^{(u)} m_{T_r}}{16\pi^2} \left\{ \sum_{i=1}^4 R_{1i} R_{2i} f(m_{\rho_i}^2, m_{T_r}^2) - \sum_{i=1}^4 R_{3i} R_{4i} f(m_{\rho_i}^2, m_{T_r}^2) \right\} \\ - i \sum_{r=1}^2 \frac{x_{ir}^{(T)} \kappa_{r3}^{(u)} m_{T_r}}{16\pi^2} \left\{ \sum_{i=1}^4 R_{1i} R_{4i} f(m_{\rho_i}^2, m_{T_r}^2) + \sum_{i=1}^4 R_{2i} R_{3i} f(m_{\rho_i}^2, m_{T_r}^2) \right\}, \quad (38)$$

$$(m_{i3}^d)^{(1)} = \sum_{r=1}^2 \frac{x_{ir}^{(B)} \kappa_{r3}^{(d)} m_{B_r}}{16\pi^2} \left\{ \sum_{i=1}^4 R_{1i} R_{2i} f(m_{\rho_i}^2, m_{B_r}^2) - \sum_{i=1}^4 R_{3i} R_{4i} f(m_{\rho_i}^2, m_{B_r}^2) \right\} \\ + i \sum_{r=1}^2 \frac{x_{ir}^{(B)} \kappa_{r3}^{(d)} m_{B_r}}{16\pi^2} \left\{ \sum_{i=1}^4 R_{1i} R_{4i} f(m_{\rho_i}^2, m_{B_r}^2) + \sum_{i=1}^4 R_{2i} R_{3i} f(m_{\rho_i}^2, m_{B_r}^2) \right\}, \quad (39)$$

with $i = 1, 2, 3$ and $r, n = 1, 2$ and the loop function $f(m_1, m_2)$ is given by

$$f(m_1^2, m_2^2) = \frac{m_1^2}{m_1^2 - m_2^2} \ln \left(\frac{m_1^2}{m_2^2} \right). \quad (40)$$

The phase factor $e^{\pm i\alpha}$ acts as a crucial link between the dark and visible sectors. It encapsulates the CP -breaking effects originating in the dark sector and transmits them to the

quark mass matrices at the one-loop level. Through this mechanism, the model naturally accounts for the observed CP violation in the weak interactions, while the strong CP -phase remains parametrically suppressed. To determine the quantitative value of the weak phase, we shall conduct a systematic analysis.

For simplicity, we consider a benchmark scenario where $m_{T_r} = m_{B_r}$ ($r = 1, 2$), which implies $\alpha_u = \alpha_d = \alpha$. Since the explicit breaking of the CP symmetry in the dark sector

is triggered only by a specific scalar interaction term $\mathcal{G}\eta^\dagger\phi\phi_2^\dagger + \text{H.c.}$, the parameters $m_{kn}^{u(d)}$ ($k = 1, 2, 3$, $n = 1, 2$) are taken to be real numbers. The mass matrices, $M_{u(d)}$, are diagonalized by the usual bi-unitary transformations $V_{L,R}^{u(d)}$, given by

$$\begin{aligned} V_L^{d\dagger} M_d V_R^d &= \text{Diag}(m_d, m_s, m_b), \\ V_L^{u\dagger} M_u V_R^u &= \text{Diag}(m_u, m_c, m_t), \end{aligned} \quad (41)$$

and the Cabbibo Kobayashi Maskawa (CKM) quark mixing matrix is determined as $V_{\text{CKM}} = (V_L^u)^\dagger V_L^d$. The bi-unitary matrices can be parametrized as

$$\begin{aligned} V_L^u &= \mathcal{O}_L^u \text{Diag}(e^{i\beta_1^u} e^{i\beta_2^u} e^{i\beta_3^u}), \\ V_R^u &= \mathcal{O}_R^u \text{Diag}(e^{i(\beta_1^u - \alpha)} e^{i(\beta_2^u - \alpha)} e^{i\beta_3^u}), \\ V_L^d &= \mathcal{O}_L^d \text{Diag}(e^{-i\beta_1^d} e^{-i\beta_2^d} e^{-i\beta_3^d}), \\ V_R^d &= \mathcal{O}_R^d \text{Diag}(e^{-i(\beta_1^d - \alpha)} e^{-i(\beta_2^d - \alpha)} e^{-i(\beta_3^d - \alpha)}), \end{aligned} \quad (42)$$

where $\mathcal{O}_{L,R}^{u(d)}$ are real orthogonal matrices which are approximately given by

$$\begin{aligned} \mathcal{O}_L^u &= \begin{pmatrix} (\mathcal{O}_L^u)_{11} & 0 & (\mathcal{O}_L^u)_{13} \\ 0 & (\mathcal{O}_L^u)_{22} & (\mathcal{O}_L^u)_{23} \\ (\mathcal{O}_L^u)_{13} & (\mathcal{O}_L^u)_{23} & (\mathcal{O}_L^u)_{33} \end{pmatrix}, \\ \mathcal{O}_R^u &= \begin{pmatrix} (\mathcal{O}_R^u)_{11} & (\mathcal{O}_R^u)_{12} & 0 \\ (\mathcal{O}_R^u)_{12} & (\mathcal{O}_R^u)_{22} & 0 \\ 0 & 0 & (\mathcal{O}_R^u)_{33} \end{pmatrix}, \\ \mathcal{O}_L^d &= \begin{pmatrix} (\mathcal{O}_L^d)_{11} & 0 & (\mathcal{O}_L^d)_{13} \\ 0 & (\mathcal{O}_L^d)_{22} & (\mathcal{O}_L^d)_{23} \\ (\mathcal{O}_L^d)_{13} & (\mathcal{O}_L^d)_{23} & (\mathcal{O}_L^d)_{33} \end{pmatrix}, \\ \mathcal{O}_R^d &= \begin{pmatrix} (\mathcal{O}_R^d)_{11} & (\mathcal{O}_R^d)_{12} & 0 \\ (\mathcal{O}_R^d)_{12} & (\mathcal{O}_R^d)_{22} & 0 \\ 0 & 0 & (\mathcal{O}_R^d)_{33} \end{pmatrix}. \end{aligned} \quad (43)$$

If we choose $\beta_2^u = \beta_3^u = 2\beta_1^u = 2\alpha$, $\beta_3^d = 2\beta_2^d = 2\beta_1^d = -2\alpha$, the CKM matrix is complex and contains only one phase which is α .

Using the up- and down-type quark mass matrices specified in Eqs. (35) and their matrix elements provided by Eq. (38), (39), we performed a fit of the CKM observables of the quark sector. The scalar rotation matrix R in Eq. (39) is taken to be an $SO(4)$ rotation matrix, with the Yukawa couplings on the order of $x \sim z \sim 10^{-1}$. Through this fitting process, we estimate the exotic quark masses m_{T_r} and the scalar masses m_{ρ_i} by selecting a

TABLE IV. The predicted values of the CKM observables.

Observable	Model value	Experimental value (Ref. [114])
$\sin \theta_{12}$	0.22518	0.22500 ± 0.00067
$\sin \theta_{23}$	0.04179	$0.04182^{+0.00085}_{-0.00074}$
$\sin \theta_{13}$	0.00370	0.00369 ± 0.00011
δ_{CP}	1.148	1.144 ± 0.027

suitable value for the loop function in Eq. (40), as follows:

$$m_{T_1} \approx 4.0 \text{ TeV}, \quad m_{T_2} \approx 5.4 \text{ TeV}, \quad m_{\rho_1} \approx 7.0 \text{ TeV}, \quad (45)$$

$$m_{\rho_2} \approx m_{\rho_3} \approx m_{\rho_4} \approx 6.8 \text{ TeV}. \quad (46)$$

The numerical values in Eqs. (45) and (46) correspond to representative benchmark points chosen from a scan over the parameter space, and are not meant to indicate a precise prediction; physical CP -violating observables are insensitive to small variations or rounding of these parameters.

The scalar masses m_{ρ_i} are consistently greater than the exotic quark masses m_{T_r} . This hierarchy can be explained using the one-loop function $f(m_1^2, m_2^2)$ as shown in Eq. (40). A lower value of the Yukawa couplings is needed for higher m_{ρ_i} to balance the entries in the quark mass matrices, which are detailed in Eq. (35). To fit the observables in the quark sector, we proceed to vary the quark sector parameters and find the best fit point that minimizes the following χ^2 function:

$$\chi^2 = \sum_i \left(\frac{\xi_{\text{obs},i} - \xi_i}{\xi_{\text{obs},i}} \right)^2 \quad (47)$$

where $\xi_{\text{obs},i}$ and ξ_i are the CKM observables of the quark sector and the model values of the observables respectively. The obtained values of the CKM observables are presented in Table IV.

Furthermore, with a variation of 5% around the best-fit values, δ_{CP} varies linearly over α as shown in Fig. 2.

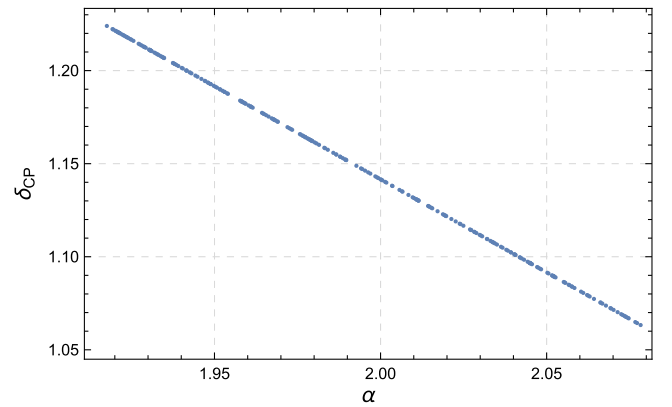


FIG. 2. δ_{CP} as a function over α value in CKM parametrization.

Our analysis allows a successful determination of the parameters of the CKM quark mixing matrix, with a specific focus on the dynamically generated CP phase originating from dark-sector interactions. The results demonstrate that this framework not only offers a robust description of CP violation but also establishes a direct link between this fundamental asymmetry and new physics beyond the Standard Model. The refined CKM parameters, which include the dark-sector contribution, show remarkable agreement with current experimental data, presenting a compelling alternative to traditional models.

B. Lepton masses and mixings

In general the SM charged-lepton mass matrix receives tree-level and radiative corrections at different loop order. The SM charged lepton mass matrix can be decomposed as

$$M^{(l)} = M^{(l)(0)} + M^{(l)(1)} + M^{(l)(2)} + \dots, \quad (48)$$

where $M^{(l)(0)}$ denotes the tree-level contribution and $M^{(l)(1)}$ the one-loop contribution, etc. At tree level the matrix has the rank-one “third column” texture

$$M^{(l)(0)} = \begin{pmatrix} 0 & 0 & \frac{y_1^{(l)}}{\sqrt{2}}v \\ 0 & 0 & \frac{y_2^{(l)}}{\sqrt{2}}v \\ 0 & 0 & \frac{y_3^{(l)}}{\sqrt{2}}v \end{pmatrix}. \quad (49)$$

One-loop diagrams (illustrated in Fig. 3) generate the first and second column entries as well as possible complex phases induced by the CP -violating dark-sector scalar interactions. Denoting the one-loop corrections by $m_{ij}^{(l)}$, the charged-lepton mass matrix up to one-loop order may be written as

$$\mathcal{M}^{(l)} = \begin{pmatrix} e^{-i\alpha}m_{11}^\ell & e^{-i\alpha}m_{12}^\ell & \frac{y_{13}^{(l)}}{\sqrt{2}}v \\ e^{-i\alpha}m_{21}^\ell & e^{-i\alpha}m_{22}^\ell & \frac{y_{23}^{(l)}}{\sqrt{2}}v \\ e^{-i\alpha}m_{31}^\ell & e^{-i\alpha}m_{32}^\ell & \frac{y_{33}^{(l)}}{\sqrt{2}}v \end{pmatrix}, \quad (50)$$

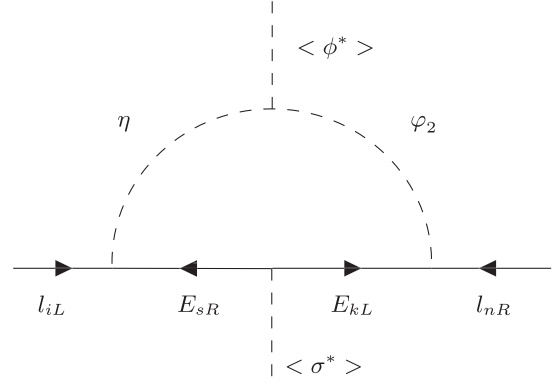


FIG. 3. One-loop corrections to the SM charged lepton mass matrices.

where the overall phase factor $e^{-i\alpha}$ indicates a CP phase communicated from the dark sector to the visible sector; the $m_{ij}^{(l)}$ denote one-loop-induced entries.

Because $M^{(l)(0)}$ is rank one, radiative corrections are necessary to generate nonzero masses for the electron and muon. The physical mass eigenvalues and the charged-lepton mixing matrices are obtained by the usual bi-unitary diagonalization:

$$V_L^{(l)\dagger} \mathcal{M}^{(l)} V_R^{(l)} = \text{diag}(m_e, m_\mu, m_\tau), \quad (51)$$

with $V_L^{(l)}$ and $V_R^{(l)}$ the left- and right-handed rotation matrices for the charged leptons.

1. Remarks

- (i) The left-handed rotation $V_L^{(l)}$ enters the leptonic mixing Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix, $U_{\text{PMNS}} = V_L^{(l)\dagger} V_L^{(\nu)}$, so the radiative structure of the charged-lepton mass matrix can affect the observed neutrino mixing angles and the leptonic Dirac CP -violating phase.
- (ii) The magnitude of the one-loop entries $m_{ij}^{(l)}$ is model dependent; a typical parametric estimate is

$$e^{-i\alpha}m_{kn}^\ell = \sum_{r=1}^2 \frac{x_{kr}^{(E)} z_{rn}^{(l)} m_{E_r}}{16\pi^2} \left\{ \sum_{i=1}^4 R_{1i} R_{2i} f(m_{\rho_i}^2, m_{E_r}^2) - \sum_{i=1}^4 R_{3i} R_{4i} f(m_{\rho_i}^2, m_{E_r}^2) \right\} \quad (52)$$

$$+ i \sum_{r=1}^2 \frac{x_{kr}^{(E)} z_{rn}^{(l)} m_{E_r}}{16\pi^2} \left\{ \sum_{i=1}^4 R_{1i} R_{4i} f(m_{\rho_i}^2, m_{E_r}^2) + \sum_{i=1}^4 R_{2i} R_{3i} f(m_{\rho_i}^2, m_{E_r}^2) \right\}. \quad (53)$$

Therefore, the loop-induced entries in $\mathcal{M}^{(l)}$ are not only responsible for generating the electron and muon masses but also serve as a new source of leptonic mixing and CP violation. This framework naturally accommodates the observed pattern of neutrino oscillations through the interplay between the dark-sector phases and the loop corrections in the charged lepton sector.

The full neutrino mass matrix in the basis (ν_L, ν_R^c, N_R^c) has the form

$$\mathcal{M}_\nu = \begin{pmatrix} 0_{3 \times 3} & M_\nu^D & 0_{3 \times 2} \\ (M_\nu^D)^T & 0_{2 \times 2} & M \\ 0_{2 \times 3} & M^T & \mu \end{pmatrix}, \quad (54)$$

where the submatrices, M_ν^D, M , are generated at tree level and are given by

$$(M_\nu)_\text{in} = (y_\nu)_\text{in} \frac{v}{\sqrt{2}}, \quad (M)_{nk} = (y_N)_{kn} \frac{v_\sigma}{\sqrt{2}}, \quad (55)$$

with $i = 1, 2, 3$ and $k, n = 1, 2$. The lepton-number-

$$e^{-i\beta} \mu_{sp} = \sum_{k=1}^2 \frac{(y_\Psi)_{sn} (x_\Omega^*)_{nk} (x_\Omega^\dagger)_{kr} (y_\Psi^T)_{rp} m_{\Omega_k}}{4(4\pi)^4} \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \frac{1}{\alpha(1-\alpha)} [I(m_{\Omega_k}^2, m_{RR}^2, m_{RI}^2) - I(m_{\Omega_k}^2, m_{IR}^2, m_{II}^2)],$$

$s, p, n, k, r = 1, 2$

with the loop integral given by [115]

$$I(m_1^2, m_2^2, m_3^2) = \frac{m_1^2 m_2^2 \log\left(\frac{m_3^2}{m_1^2}\right) + m_2^2 m_3^2 \log\left(\frac{m_1^2}{m_2^2}\right) + m_3^2 m_1^2 \log\left(\frac{m_2^2}{m_3^2}\right)}{(m_1^2 - m_2^2)(m_1^2 - m_3^2)},$$

$$m_{ab}^2 = \frac{\beta m_{(\varphi_1)_a}^2 + \alpha m_{(\varphi_3)_b}^2}{\alpha(1-\alpha)} (a, b = R: \text{or} : I). \quad (57)$$

The small masses of the active neutrinos arise from an inverse seesaw mechanism at the two-loop level and the mass matrices for the physical active and heavy neutrinos are respectively given by

$$\mathcal{M}_\text{active}^\nu = M_\nu^D (M^T)^{-1} \mu M^{-1} (M_\nu^D)^T, \quad (58)$$

$$\mathcal{M}_\text{sterile}^{\nu(-)} = -\frac{1}{2} (M + M^T) + \frac{1}{2} \mu, \quad (59)$$

$$\mathcal{M}_\text{sterile}^{\nu(+)} = \frac{1}{2} (M + M^T) + \frac{1}{2} \mu. \quad (60)$$

Owing to the smallness of the μ parameter, the two pairs of sterile neutrinos exhibit a tiny mass splitting and thus

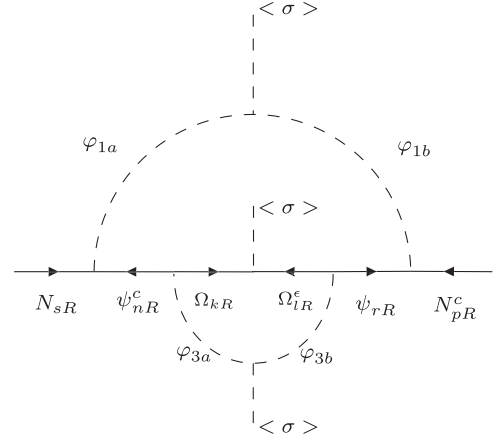


FIG. 4. Two-loop Feynman diagram contributing to the lepton-number-violating μ parameter.

violating Majorana mass term μ is not generated at tree level; instead, it originates from the two-loop diagram shown in Fig. 4. Its structure resembles the one obtained in Ref. [45] and can be expressed as

form pseudo-Dirac states. A detailed quantitative study of the neutrino sector will be presented together with the analysis of charged lepton flavor violation in the next section.

In general, the entries μ_{sp} are complex, since their loop-induced origin depends on the interaction couplings of the dark-sector scalar fields, such as $f_{\varphi_3 \sigma} \varphi_3^2 \sigma^{*2} + \text{H.c.}$ These terms not only generate the small Majorana mass contributions required by the inverse-seesaw mechanism but also act as an additional source of CP violation in the lepton sector.

Consequently, the model establishes a direct connection between the CP phases of the charged-lepton and neutrino sectors, linking them to a common dynamical origin in the dark sector. This interplay provides testable consequences in neutrino oscillation experiments, thereby offering a

direct probe of the CP -violating dynamics of the dark sector. While this topic contains many intricate details, we will not delve deeper within the scope of this work. A more detailed study will be presented in a subsequent publication.

C. Inverse seesaw realization and the Majoron

As discussed previously, light neutrino masses in this model are generated via an inverse seesaw mechanism involving the fields ν_L , ν_R^c , and N_R^c . The explicit structure of the neutrino mass matrix and its diagonalization have already been presented and will not be repeated here. General discussions of the inverse seesaw mechanism can be found in Refs. [116–118].

The global lepton-number symmetry of the model is spontaneously broken by the vacuum expectation value of the complex scalar σ . As a consequence, the imaginary component of σ remains as a physical, massless Goldstone boson,

$$\sigma = \frac{1}{\sqrt{2}}(v_\sigma + \rho + iJ), \quad (61)$$

which is identified as the Majoron. The appearance of a Majoron in inverse seesaw realizations with spontaneous lepton-number breaking is well known [119,120].

In the present model, the lepton-number-violating parameter μ is not generated at tree level but instead arises radiatively at the two-loop order. This radiative origin naturally accounts for the smallness of μ and constitutes a structural feature of the model.

Since the Majoron originates from the phase of σ , its couplings to neutrinos are induced by the same interactions responsible for the generation of μ . As a result, the effective Majoron-neutrino couplings scale as

$$g_{J\nu\nu} \sim \frac{\mu}{v_\sigma}, \quad (62)$$

and vanish in the lepton-number-conserving limit $\mu \rightarrow 0$. This behavior is characteristic of inverse seesaw realizations and differs from Majoron models in which the couplings are directly proportional to the light neutrino masses.

D. Phenomenological constraints on the Majoron

The phenomenology of Majorons coupled to Majorana neutrinos has been extensively studied in the literature. In general, such interactions can induce neutrino decays, contribute to stellar and supernova energy-loss processes, and affect cosmological observables such as the effective number of relativistic degrees of freedom N_{eff} [121–125].

In the present model, all Majoron-induced effects involving light neutrinos are controlled by the effective Majoron-neutrino coupling, which scales as

$$g_{J\nu\nu} \sim \frac{\mu}{v_\sigma}, \quad (63)$$

where the lepton-number-violating μ parameter is radiatively generated at the two-loop level as shown in the Feynman diagram of Fig. 4. This leads to a strong parametric suppression of Majoron interactions.

Astrophysical constraints from stellar cooling and supernova energy loss, including those derived from SN1987A, require the Majoron-neutrino coupling to satisfy

$$g_{J\nu\nu} \lesssim 10^{-5} - 10^{-6}, \quad (64)$$

in order to avoid excessive energy loss from supernova cores [125]. In the parameter region of interest, the couplings $g_{J\nu\nu} \sim \mu/v_\sigma$ lie well below these bounds. This can be seen considering that in the parameter space of the neutrino sector of the model, discussed in Secs. VIII and VII, we have $\text{Max}(\mu_{nk}) \sim \mathcal{O}(10)$ keV ($n, k = 1, 2$), $v_\sigma \approx 5$ TeV, which yields very tiny Majoron-neutrino coupling $g_{J\nu\nu} \sim \mathcal{O}(10^{-9})$, which is well below the cosmological bound provided by Eq. (64). As a result, Majoron emission does not significantly affect energy transport in stellar interiors or supernova environments.

Cosmological constraints are likewise satisfied. The suppressed Majoron-neutrino interaction prevents efficient thermalization of the Majoron in the early Universe for sufficiently large values of v_σ , ensuring consistency with bounds arising from big bang nucleosynthesis and cosmic microwave background observations on N_{eff} . We therefore conclude that the presence of a physical Majoron in the present model is compatible with existing laboratory, astrophysical, and cosmological constraints, while remaining directly linked to the origin of neutrino masses through the radiatively generated lepton-number-violating parameter μ at the two-loop level.

The Majoron also couples to the heavy neutral fermions present in the inverse seesaw sector, since these states acquire mass from Yukawa interactions involving the scalar field σ .

These couplings scale with the corresponding heavy fermion masses and are suppressed by the symmetry-breaking scale v_σ , but exhibit no direct suppression from the lepton-number-violating parameter μ .

These interactions do not impose new phenomenological constraints within the parameter range considered here. The heavy neutral fermions are too massive to be produced in stellar interiors or supernova cores; consequently, Majoron emission processes involving these heavy states do not contribute to astrophysical energy loss.

IV. STRONG CP PROBLEM

Having established a robust framework for weak CP violation, we now extend our analysis to consider the implications of the dark sector on the strong CP problem.

The existence of a nonzero θ term in quantum chromodynamics remains a profound puzzle, and our proposed mechanism offers a novel perspective. By exploring how the same dark-sector interactions that dynamically generate the weak CP phase might also influence the strong interaction, we can investigate potential solutions of the strong CP problem. We first parametrize the up- and down-quark mass matrices (35) as

$$M_u = (e^{i\alpha} C_u, r_u), \quad M_d = (e^{-i\alpha} C_d, r_d), \quad (65)$$

where $C_{u,d}$ are 3×2 submatrices containing the Yukawa entries affected by the dark-sector phase, while $r_{u,d}$ denote the third columns, which are taken real at the considered order. The effective strong CP phase is defined as

$$\theta_{\text{QCD}} = \text{Arg}[\text{Det}(M_u)\text{Det}(M_d)]. \quad (66)$$

Since the first two columns of M_u (M_d) carry a common factor $e^{i\alpha}$ ($e^{-i\alpha}$), one can factor out $e^{2i\alpha}$ ($e^{-2i\alpha}$) from the determinant:

$$\det(M_u) \propto e^{2i\alpha}, \quad \det(M_d) \propto e^{-2i\alpha}. \quad (67)$$

Multiplying them together gives

$$\det(M_u) \det(M_d) \propto e^{2i\alpha} e^{-2i\alpha} = 1, \quad (68)$$

and therefore

$$\theta_{\text{QCD}} = 0. \quad (69)$$

It means that the strong CP phase vanishes at the one-loop level. The next step is to examine whether this result remains stable once higher-order corrections are taken into account.

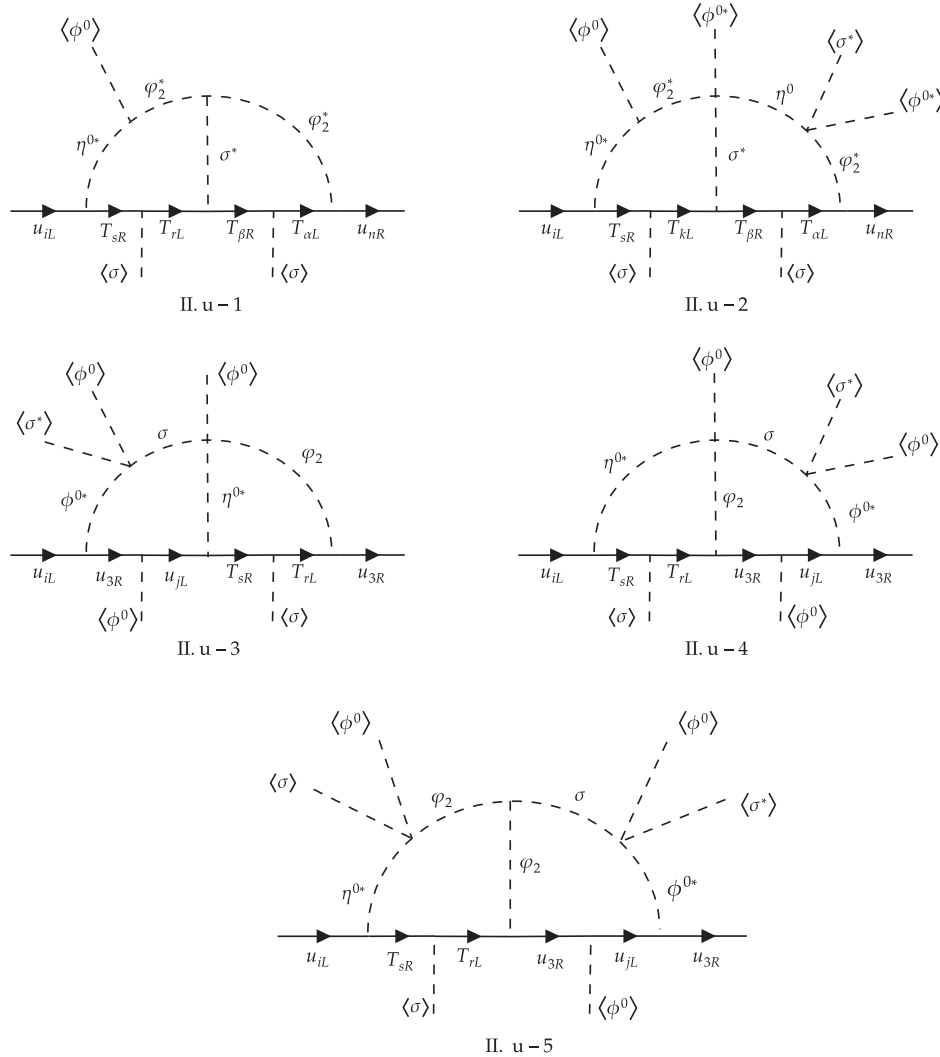


FIG. 5. Two-loop corrections to the SM u -quark.

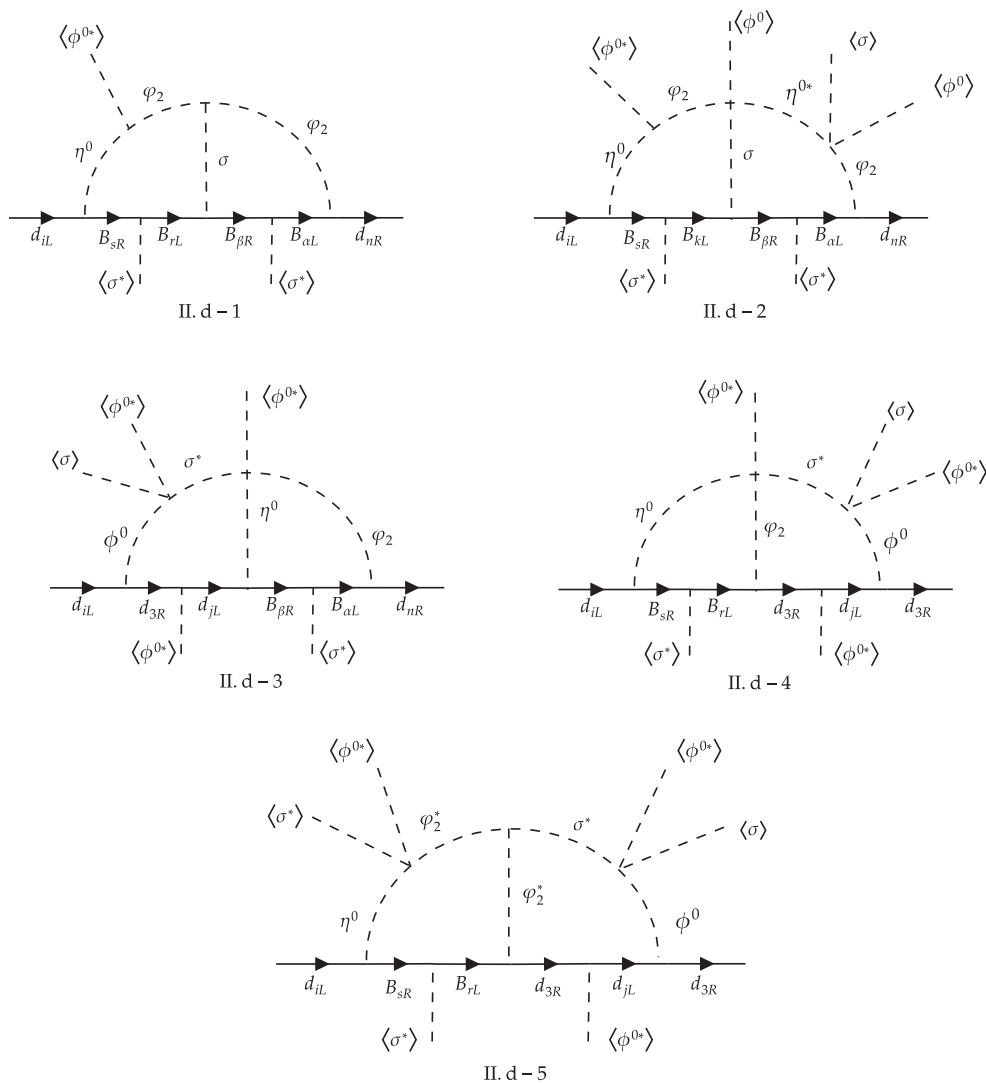


FIG. 6. Two-loop corrections to the SM d -quark.

As displayed in Figs. 5 and 6, the complete set of two-loop Feynman diagrams contributing to the up- and down-quark mass matrices exhibit a crucial structural feature: the appearance of complex phases is uniquely tied to the dark-sector interaction of the form

$$\mathcal{G}\eta^\dagger\phi\phi_2^\dagger + \text{H.c.} \quad (70)$$

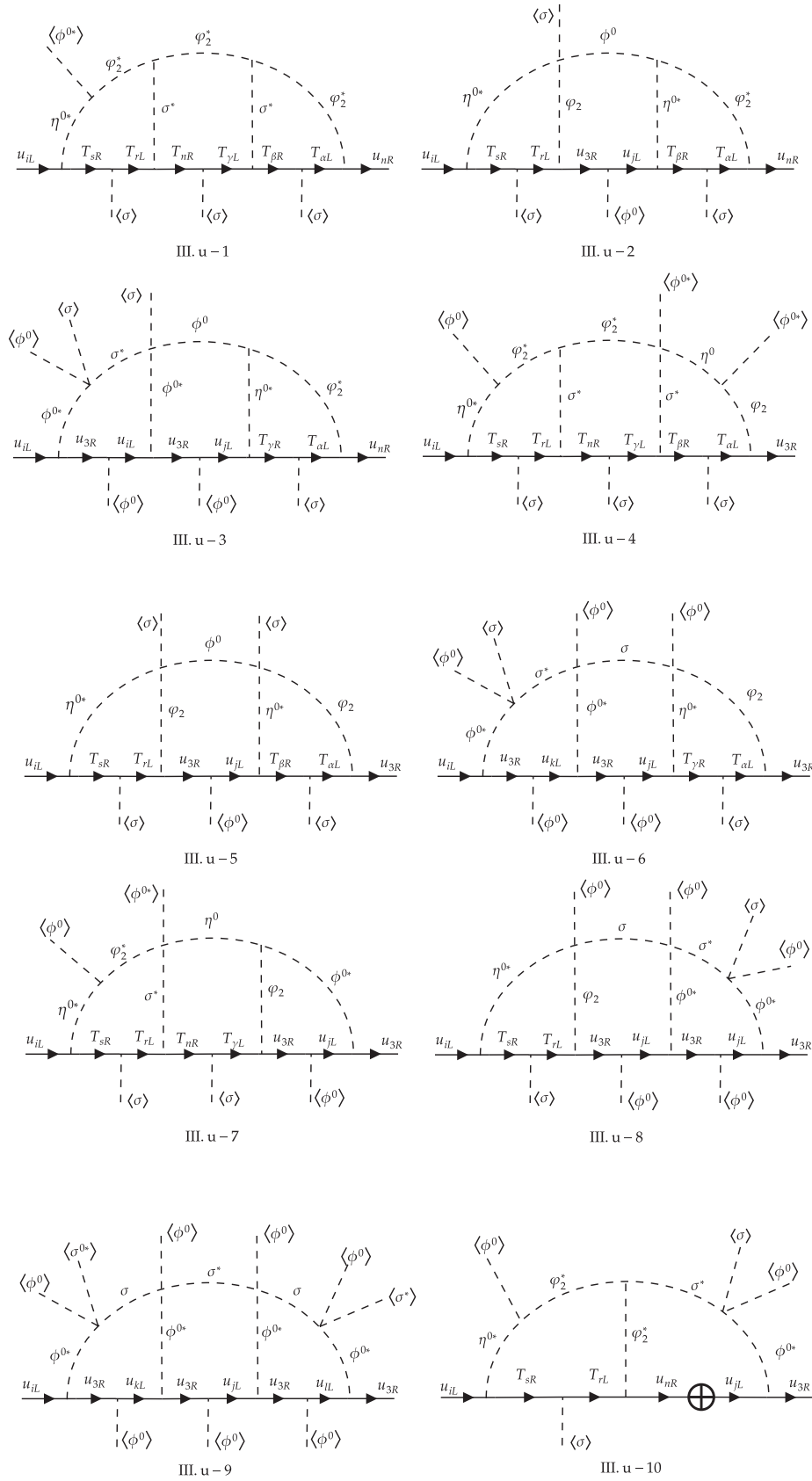
This term induces corrections solely in the first and second columns of the quark mass matrices, when written in the interaction basis (u, c, t) and (d, s, b) . As a result, the CP -violating phases are confined entirely to the light-quark sector, while the two-loop contributions to the third column remain purely real. This alignment ensures that the determinant phases of the mass matrices factorize, thereby protecting the strong CP parameter θ_{QCD} from acquiring a nonzero contribution up to two-loop order.

We now turn to the next order of radiative corrections. This step is crucial to determine whether higher-order

effects could destabilize the protection mechanism that prevents the emergence of a nonvanishing strong CP phase. In particular, we investigate whether three-loop diagrams such as those depicted in Figs. 7 and 8 can reintroduce complex phases into the quark mass matrices.

The diagrams III.q-1, III.q-2, and III.q-3 with $q = u, d$, shown in Figs. 7 and 8, contribute to the first two columns of the quark mass matrices. The complex phases that appear in these columns trace back to the same dark-sector portal responsible for the one- and two-loop corrections. As a consequence, the phase pattern of columns one and two at three loops is identical in structure to that found at lower orders: the entries in these columns carry a common overall phase (or its conjugate in the down sector) and therefore continue to factorize in the same manner as in the one- and two-loop analyses.

With the exception of the two representative topologies denoted III.u-10 and III.d-10, the three-loop diagrams (III.q- i with $i = 4, \dots, 9$) that contribute to the third column

FIG. 7. Three-loop corrections to the SM u -quark.

of the quark mass matrices exhibit a phase structure entirely analogous to the lower-order (one- and two-loop) contributions to the third column. In other words, for the bulk of three-loop topologies the complex-phase pattern entering $m_{i3}^{q(3)}$ follows the same selection rules and factorization properties that governed $m_{i3}^{q(1,2)}$.

The two exceptional diagrams, III.u-10 and III.d-10, at first glance appear to introduce a new uncompensated phase into the third column. This is because the candidate diagrams depicted in Fig. 7-III.u-10 and Fig. 8-III.d-10 seem to couple the dark vertex $\mathcal{G}\eta^\dagger\phi\varphi_2^\dagger + \text{H.c.}$ into a topology that dresses the third column operator. However, closing these three-loop diagrams requires the insertion of a mass subdiagram on an internal fermion line. Crucially, that mass insertion is itself generated at one loop and therefore carries the same dark-sector phase structure (but with the conjugate phase, once it appears as an insertion closing the fermion line).

Schematically, the three-loop contribution to the mass mixing term for a representative topology of this type factorizes as

$$(M_{i3}^q)^{(3)} \propto (e^{i\alpha}) \times ((M_{k3}^q)^{(1)})^* \times \mathcal{I}_3, \quad (71)$$

where $(M_{k3}^q)^{(1)}$ is the one-loop mass insertion (itself $\propto e^{i\alpha}$ in the basis where the dark phase appears on columns one and two), and $\mathcal{I}_3$ is the real-valued two-loop integral (after combining conjugate momentum regions and performing the usual subtractions).

Because the mass insertion appears as a Hermitian term on the fermion propagator, it contributes with the complex conjugate of the one-loop entry, $(M_{k3}^q)^{(1)}$. Writing

$$(M_{k3}^q)^{(1)} \propto e^{i\alpha} (m_{k3}^q)^{(1)},$$

the product of phases in Eq. (71) becomes

$$e^{i\alpha} \times e^{-i\alpha} (m_{k3}^q)^{(1)} = (m_{k3}^q)^{(1)}.$$

Therefore, although the three-loop topologies III.u-10 and III.d-10 superficially connect the dark CP vertex to the third column operator, the necessary closure via a one-loop mass insertion enforces a conjugation of the phase and produces an overall real contribution. In other words, the would-be complex phase from the explicit dark vertex is compensated by the conjugate phase carried by the mass insertion, so that the net three-loop correction to m_{i3}^q is real. This mechanism explains why the column-wise factorization of the dark phase persists even after inclusion of the representative three-loop diagrams, and why no new source of strong CP violation is introduced at this order.

Taken together, these observations imply that the phase pattern established at one- and two-loop orders persists at three loops: columns one and two inherit the same dark-sector-induced phase structure, while the third column

remains real to the order considered. Accordingly, the factorization protection of the strong CP parameter is maintained at three-loop order unless a qualitatively different topology or coupling assignment (one that directly attaches an uncompensated dark phase to the third column operator) is introduced.

V. RADIATIVE CP STRUCTURE OF THE QUARK MASS MATRICES

We choose a rephasing basis in which all CP -even Yukawa couplings relevant for the radiative generation of quark masses are real. In this basis, the only interaction carrying a physical CP -violating phase and entering the quark mass diagrams considered below is

$$\mathcal{G}\eta^\dagger\Phi\varphi_2^\dagger + \text{H.c.} \quad (72)$$

All other scalar interactions required to close the loops are CP conserving.

Radiative contributions to the first two columns of the up-quark mass matrix,

$$(M_u)_{in}, \quad n = 1, 2, \quad (73)$$

must originate from one of the operators

$$\bar{q}_{iL}\tilde{\Phi}u_{3R}, \quad \bar{q}_{iL}\tilde{\eta}T_{kR}, \quad (74)$$

and terminate on the right-handed external field through

$$\bar{T}_{kL}\varphi_2^*u_{nR}. \quad (75)$$

The internal fermion lines consist of Dirac mass insertions of the vectorlike quark T , possible mixing mass insertions of the form

$$\bar{u}_{iL}u_{3R} + \text{H.c.}, \quad (76)$$

and the CP -even Yukawa interactions

$$\bar{q}_{iL}\tilde{\eta}T_{kR}, \quad \bar{T}_{kL}\varphi_2u_{3R}, \quad (77)$$

all of which are real in the chosen basis. As a result, the fermion subgraph does not introduce any complex phase, and any CP violation must originate from the scalar sector.

Since the external field u_{nR} ($n = 1, 2$) is reached only via φ_2^* , closing the scalar subgraph requires the presence of the scalar fields

$$\varphi_2^*, \varphi_2, \sigma^*, \eta^*, \Phi, \quad (78)$$

arranged such that all scalar lines can be consistently contracted. Under these assumptions, exactly one insertion of the CP -violating interaction

$$\mathcal{G}\eta^\dagger\Phi\varphi_2^\dagger \quad (79)$$

is required, while any additional loop insertions involve only CP -conserving vertices. Consequently, higher-loop corrections do not generate new independent phases and preserve the same column-wise phase structure as at lower orders. An analogous argument applies to the down-quark mass matrix.

For the third column, the radiative contributions again originate from

$$\bar{q}_{iL}\tilde{\Phi}u_{3R}, \quad \bar{q}_{iL}\tilde{\eta}T_{kR}, \quad (80)$$

but terminate on u_{3R} through

$$\bar{T}_{kL}\varphi_2u_{3R}. \quad (81)$$

In this case, the scalar subgraph can be closed entirely by CP -conserving scalar interactions, leading to a real contribution. If the CP -violating interaction $\eta^\dagger\Phi\varphi_2^\dagger$ is inserted, closure of the loop requires the simultaneous insertion of its Hermitian conjugate $\Phi^\dagger\eta\varphi_2$. Thus, CP -violating effects enter the third column amplitudes only through the CP -even combination $|\mathcal{G}|^2$, and no complex phase is generated.

The resulting structure of the quark mass matrices can therefore be written as

$$M_u = (e^{i\alpha}C_u, r_u), \quad M_d = (e^{-i\alpha}C_d, r_d), \quad (82)$$

where $C_{u,d}$ and $r_{u,d}$ are real in the chosen basis. As long as this structure is preserved, the rephasing-invariant combination

$$\bar{\theta} = \arg \det(M_u M_d) \quad (83)$$

vanishes identically within this class of radiative contributions.

VI. DARK MATTER PHENOMENOLOGY

Understanding the spectrum and stability of dark-sector states is essential to connect the model with cosmological observables and terrestrial searches. After spontaneous symmetry breaking, the residual discrete symmetry is $\mathbb{Z}_2 \times \mathbb{Z}'_2$. Such remnant discrete symmetry $\mathbb{Z}_2 \times \mathbb{Z}'_2$ is crucial for ensuring the stability of the scalar and/or fermionic dark matter candidates. Each dark matter candidate carries a pair of charges (q, q') with $q, q' \in \{0, 1\}$, which naturally classifies the dark-sector states into three distinct categories,

$$(q, q') \in \{(0, 1), (1, 0), (1, 1)\},$$

hereafter denoted as DM_1, DM_2, DM_3 (see Table V).

The stability pattern and mass ordering determine whether the model realizes a two- or three-component dark-matter scenario. In particular, if DM_1 and DM_2 are the lightest states and

$$m_{DM_3} > m_{DM_1} + m_{DM_2},$$

TABLE V. Classification of dark-sector fields according to their (Z_2, Z'_2) charges.

(Z_2, Z'_2)	Representative fields
$DM_1 \equiv (1, 0)$	$\eta, \varphi_2, \Omega_{nR}$
$DM_2 \equiv (0, 1)$	φ_3
$DM_3 \equiv (1, 1)$	φ_1, Ψ_{nR}

then DM_3 efficiently decays into a $DM_1 + DM_2$ pair, resulting in an effective two-component dark sector. Otherwise, all three species can contribute to the relic abundance.

In the following, we concentrate on the two-component scenario, which can be realized in two ways:

- (i) Scalar-scalar system: both dark components are scalars, namely φ_3 and a mixed state ρ_i arising from η and φ_2 .
- (ii) Scalar-fermion system: one dark component is the scalar φ_3 , while the other one is the fermion Ω_{nR} .

The relic abundances of these dark components are determined by the standard freeze-out mechanism. Rather than presenting the explicit coupled Boltzmann equations, we employ the public MICROMEGAS 6.2.3 package [126], which incorporates all annihilation, coannihilation, and conversion processes and provides numerical predictions for the thermal relic density. The code also computes elastic scattering cross sections relevant for direct detection, as well as annihilation rates relevant for indirect searches.

In order to extract quantitative predictions, we first identify representative benchmark points in the parameter space of the model. The choice is guided by following considerations:

- (1) Theoretical consistency: perturbativity of the couplings, vacuum stability of the scalar potential are imposed to ensure the validity of the effective description.
- (2) Collider limits: the masses of new gauge bosons and scalars are chosen above the current LHC bounds [127], while mixing angles between new states and the SM Higgs sector are kept within the limits set by Higgs precision data [127–129].
- (3) Dark-sector spectrum: The mass hierarchy in the dark sector is arranged such that DM_1 and DM_2 constitute the stable relics, providing a concrete realization of the two-component dark matter scenario discussed above.

Concretely, we scan over the following ranges of parameters, as summarized in Table VI:

- (i) Self-couplings. The SM Higgs quartic λ_ϕ is fixed by the observed 125 GeV Higgs mass, $m_h^2 \simeq \lambda_\phi v^2$, giving $\lambda_\phi \sim \frac{1}{2}$. For simplicity, we assume $\lambda_{\sigma, \eta, \varphi_i}$ are of the same order and sample them in the range $[0, 1]$.
- (ii) Mixed couplings. The portal couplings λ_{ij} ($i \neq j$) may be either comparable to the self-couplings ($\lambda_{ij} \sim \lambda_i$) or smaller ($\lambda_{ij} \sim 0.1\lambda_i$). The same

TABLE VI. Input parameters ranges for numerical study of DM.

Parameters	Ranges
$\lambda_{\phi,\eta,\sigma,\varphi_{1,2,3}}$	[0, 0.1]
$\lambda_{ij}(i \neq j)$	$[-0.1, 0.1]$ or $[-0.01, 0.01]$
v_σ	[100, 3000] GeV
m_{ϕ_3,ρ_1}	[200, 3000] GeV
$m_{\phi_3,\Omega_{1R}}$	[200, 3000] GeV

assumptions are applied to the couplings in the V_{CPV}, V'_{CPV} , the real couplings as well as the real and imaginary parts of $\mathcal{G}, f', f_\sigma, f'_\sigma$. In practice, we randomly sample these couplings in two ranges: $[-0.1, 0.1]$ and $[-0.01, 0.01]$.

- (iii) Dark matter masses. The masses of the two DM components, either (m_{ϕ_3}, m_{ρ_1}) or $(m_{\phi_3}, m_{\Omega_{nR}})$, are varied in the range [200, 3000] GeV. For the heavier dark state DM₃, we set $m_{\Psi_{nR}} = 5000$ GeV and $m_{\phi_1} = 5500$ GeV, such that $m_{DM_1} + m_{DM_2} < m_{DM_3}$.
- (iv) VEV. The vacuum expectation value v_σ is scanned within [100, 3000] GeV.

First, we assume that both DM components are scalar fields, φ_3 and ρ_1 , produced via the freeze-out mechanism. For the sake of simplicity, we consider a scenario where the scalar field φ_3 is mainly a weak singlet component whereas ρ_1 is mainly composed of the neutral CP -even component of a $SU(2)_L$ electroweak doublet. Hence, φ_3 annihilates to normal matter only via the Higgs portal, ρ_1 , whereas φ_3 annihilates to the SM fields via both the Higgs and gauge portals, as shown in Figs. 9 and 10.

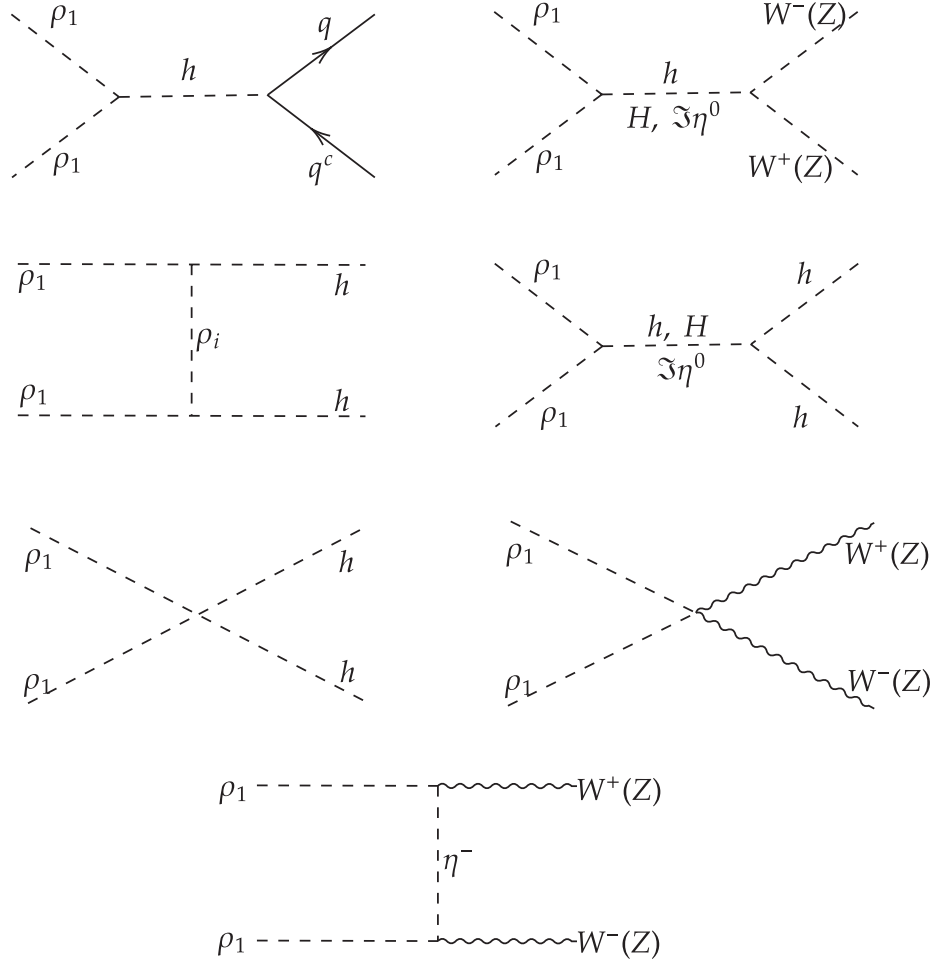
Furthermore, there is a conversion between two types of DM, where the heavier DM component can annihilate into the lighter one. We introduce an additional annihilation process,

$$\rho_1 \rho_1 \rightarrow \varphi_3 \varphi_3 \quad \text{if } m_{\rho_1} > m_{\varphi_3}, \quad (84)$$

or

$$\varphi_3 \varphi_3 \rightarrow \rho_1 \rho_1 \quad \text{if } m_{\varphi_3} > m_{\rho_1}. \quad (85)$$

The Feynman diagrams for conversion between two-component scalar DM are presented in Fig. 11. We establish

FIG. 9. Annihilation of the DM, ρ_1 , via a SM-like Higgs/new Higgs and gauge bosons portals.

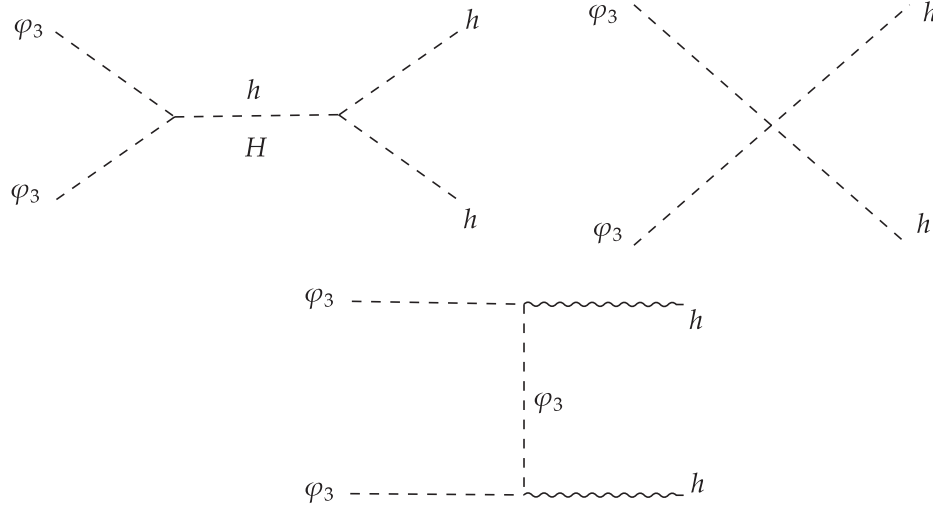
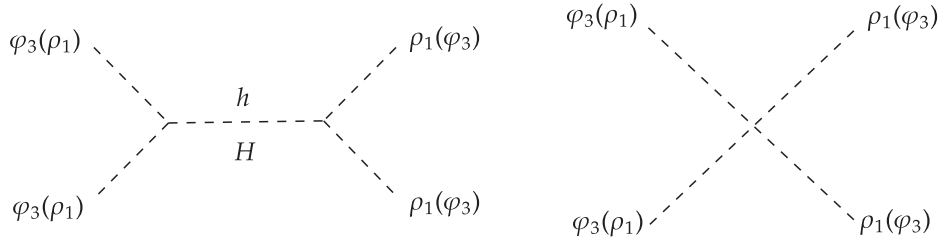
FIG. 10. Annihilation of the DM, φ_3 , via a SM-like Higgs/new Higgs portals.

FIG. 11. Conversion between two-component scalar DM.

the correlation between the masses of the two scalar DM components fulfilling DM relic abundances [130], as illustrated in Fig. 12. The contributions of both scalar DM candidates depend on their respective masses. The viable mass range for dark matter compatible with the relic density extends up to the TeV scale.

As a result, the steep dependence of the total dark matter density on the mixing parameter is primarily governed by the singlet contribution, while the doublet provides a more robust and less sensitive background. In particular, the constraint on the mass of the singlet scalar component φ_3 depends strongly on the range of the mixed couplings λ_{ij} . For small values, $\lambda_{ij} \in [-0.01, 0.01]$, the singlet mass is tightly bounded as $m_{\varphi_3} \lesssim 400$ GeV in order to reproduce the correct relic density. In contrast, for larger couplings, $\lambda_{ij} \in [-0.1, 0.1]$, the annihilation of φ_3 is sufficiently enhanced and its mass can reach the TeV scale without overclosing the Universe.

On the other hand, the relic density of the doublet Higgs component ρ_1 is mainly governed by gauge-mediated annihilation channels into electroweak gauge bosons such as W^+W^- and ZZ . These processes remain efficient even for small Higgs mixing, so the allowed mass region of ρ_1 is only mildly affected by the choice of λ_{ij} and can consistently extend to the TeV scale in both coupling scenarios.

This distinction between singlet and doublet behavior illustrates the different interaction structures: while the singlet φ_3 is dominantly probed through Higgs-portal direct detection, the doublet ρ_1 is more naturally constrained by gauge-mediated annihilation channels and can be effectively tested in indirect and collider searches.

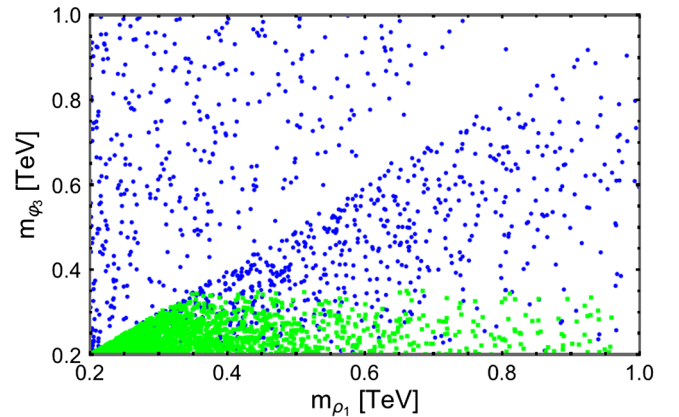


FIG. 12. Relation between the masses of two scalar dark matter components m_{ρ_1} and m_{φ_3} satisfying 3σ experimental constraint of DM relic density $\Omega_{\text{DM}} h^2 = 0.11933 \pm 0.00091$ [130]. The blue and green points are plotted for two cases of mixed couplings $\lambda_{ij} \in [-0.1, 0.1]$ and $[-0.01, 0.01]$, respectively.

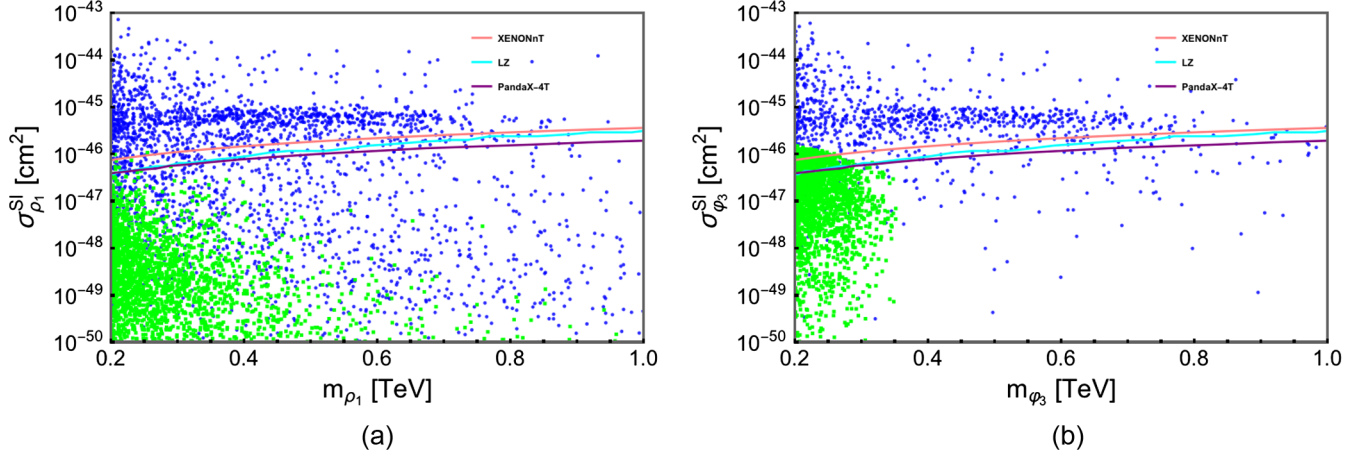


FIG. 13. The left and right panels demonstrate the dependence of the spin-independent scattering cross section of each DM component ρ_1 and φ_3 with nucleons on the DM masses m_{ρ_1} and m_{φ_3} , respectively. The blue and green points correspond to two cases of mixed couplings $\lambda_{ij} \in [-0.1, 0.1]$ and $[-0.01, 0.01]$, respectively. The pink, cyan, and purple solid lines represent the current upper experimental limits reported by XENONnT [131], LZ [132], and PandaX-4T [133], respectively.

Having clarified the distinct relic density dependence of the singlet and doublet components, we now turn to the implications for direct detection. Imposing the 3σ constraint on the relic density [130], we obtain the left and right panels of Fig. 13, which display the spin-independent (SI) scattering cross section of ρ_1 and φ_3 with nucleons as a function of their masses m_{ρ_1} and m_{φ_3} . Two representative scenarios of mixed couplings are considered: $\lambda_{ij} \in [-0.1, 0.1]$ (blue points) and $\lambda_{ij} \in [-0.01, 0.01]$ (green points).

We observe that for relatively large mixing, $\lambda_{ij} \in [-0.1, 0.1]$, both ρ_1 and φ_3 exhibit sizable SI cross sections, with the majority of φ_3 points lying above the current exclusion limits. In contrast, for smaller mixing values, $\lambda_{ij} \in [-0.01, 0.01]$, most parameter points fall below the experimental upper bounds, thus remaining viable. This trend directly reflects the quadratic dependence of the SI scattering rate on the mixed couplings, $\sigma_{\text{DM}}^{\text{SI}} \propto \lambda_{ij}^2$, so that suppressing λ_{ij} naturally reduces the cross section.

These results reinforce the complementarity between relic density and direct detection constraints: while the singlet scalar φ_3 is strongly probed through Higgs-portal interactions and subject to stringent direct detection limits, the doublet ρ_1 benefits from gauge-mediated annihilation channels and is correspondingly less constrained, even in the large-mixing regime.

Next, we analyze a scenario involving a fermion and a scalar DM candidate. In this case, we consider Ω_{1R} and φ_3

as two-component DM fields. The annihilation process of the fermionic field, Ω_{1R} , into SM fields is illustrated by the diagrams in Fig. 14.

The conversion between fermion and scalar DM components is presented in Fig. 15, which assumes that $m_{\Omega_{1R}} > m_{\varphi_3}$.

The relation between the masses of two DM components $m_{\Omega_{1R}}$ and m_{φ_3} satisfying 3σ experimental constraint for DM relic abundance [130] is shown in Fig. 16. The left and right panels in Fig. 17 show the dependence of spin-independent scattering cross section of each DM component Ω_{1R} , φ_3 with nucleon on the DM masses $m_{\Omega_{1R}}$ and m_{φ_3} , within two scenarios of mixed couplings $\lambda_{ij} \in [-0.1, 0.1]$ (blue points) and $\lambda_{ij} \in [-0.01, 0.01]$ (green points). We see that in the left panel, the DM component Ω_{1R} shows the satisfied SI-nucleon cross section in both cases of mixed couplings λ_{ij} . This result can be explained because the fermionic DM candidate Ω_{1R} interacts with the SM-like Higgs boson due to mixing h with H . The relevant coupling strength is suppressed by $\sin^2 \theta_h \simeq \frac{v^2}{v_g^2} \ll 1$. Hence, the SM-like Higgs field portal negligibly contributes to the annihilation cross section. Moreover, the Ω_{1R} scatters with nucleons via only the SM-like Higgs portal due to their mixing with the new Higgs, H , not interacting with SM quarks and gluons. Therefore, Ω_{1R} can escape the current detection of DM, which results in the quite low predicted σ^{SI} . Turning to

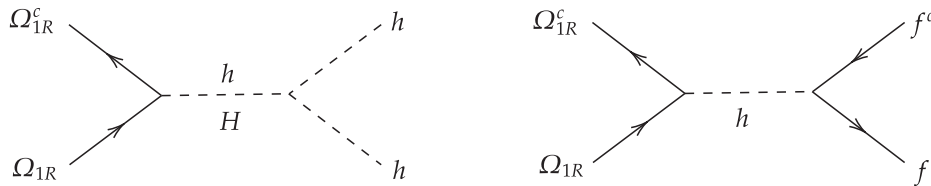


FIG. 14. Annihilation of fermion DM Ω_{1R} .

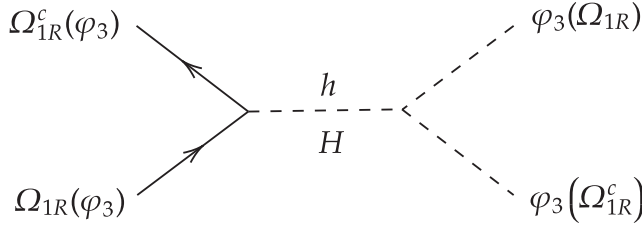


FIG. 15. Conversion between fermion DM Ω_{1R} and scalar DM φ_3 .

the right panel, the predicted values of the DM component φ_3 $\sigma_{\varphi_3}^{\text{SI}}$ are shown to be lower and satisfy measurement limits if $\lambda_{ij} \in [-0.01, 0.01]$, compared to the case $\lambda_{ij} \in [-0.1, 0.1]$. This behavior of φ_3 is quite consistent as in the right panel in Fig. 13.

VII. 95 GEV DIPHOTON EXCESS

In this section, we examine the exciting implications of our model for the 95 GeV diphoton excess recently reported by the CMS Collaboration. We suggest that this excess in the diphoton final state, observed around 95 GeV, could be attributed to the real component σ_R of the scalar singlet σ , which we assume has a mass of 95 GeV.

The electroweak scalar singlet σ_R is primarily produced via a gluon fusion mechanism involving heavy exotic

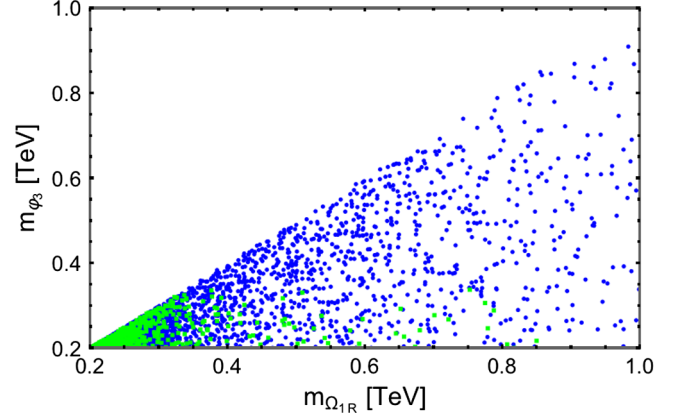


FIG. 16. The figure demonstrates the relation between the masses of two DM components $m_{\Omega_{1R}}$ and m_{φ_3} satisfying 3σ experimental constraint $\Omega_{\text{DM}} h^2 = 0.11933 \pm 0.00091$ [130]. The blue and green points are plotted for two cases, mixed couplings $\lambda_{ij} \in [-0.1, 0.1]$ and $[-0.01, 0.01]$, respectively.

quarks T_n and B_n (with $n = 1, 2$) in a triangular loop. Its diphoton decay is mediated by triangular loops featuring the virtual exchange of vectorlike quarks, charged vectorlike leptons E_n (for $n = 1, 2$), and electrically charged scalars. As a result, the cross section for the production of the diphoton scalar resonance at the LHC can be represented as follows:

$$\sigma_{\text{total}}(pp \rightarrow \sigma_R \rightarrow \gamma\gamma) = \frac{\pi^2}{8} \frac{1}{m_{\sigma_R} \Gamma_{\sigma_R}} \Gamma(\sigma_R \rightarrow \gamma\gamma) \frac{1}{s} \int_{\frac{m_{\sigma_R}^2}{s}}^1 \frac{dx}{x} f_g(x) f_g\left(\frac{m_{\sigma_R}^2}{sx}\right) \Gamma(\sigma_R \rightarrow gg), \quad (86)$$

where $m_{\sigma_R} \simeq 95$ GeV represents the mass of the resonance, and Γ_{σ_R} denotes its total decay width. The function $f_g(x)$ is the gluon distribution, and $\sqrt{s} = 13$ TeV is the LHC center-of-mass energy.

The diphoton excess observed at 95 GeV can be interpreted as a scalar resonance with a signal strength described by [134,135]

$$\mu_{\gamma\gamma}^{(\text{exp})} = \frac{\sigma_{\text{exp}}(pp \rightarrow \sigma_R \rightarrow \gamma\gamma)}{\sigma_{\text{SM}}(pp \rightarrow h \rightarrow \gamma\gamma)} = 0.35 \pm 0.12, \quad (87)$$

where σ_{SM} corresponds to the total cross section for a hypothetical SM Higgs boson at the same mass.

The corresponding decay widths of the resonance into photon and gluon pairs are respectively given by

$$\Gamma(\sigma_R \rightarrow gg) = \frac{K^{gg} \alpha_s^2 m_{\sigma_R}^3}{256\pi^3} \left| \sum_{n=1}^2 \frac{y_{T_n}}{m_{T_n}} F(x_{T_n}) + \sum_{n=1}^2 \frac{y_{B_n}}{m_{B_n}} F(x_{B_n}) \right|^2, \quad (88)$$

$$\begin{aligned} \Gamma(\sigma_R \rightarrow \gamma\gamma) = & \frac{\alpha^2 m_{\sigma_R}^3}{512\pi^3} \left| \sum_{n=1}^2 \frac{N_c Q_{T_n}^2 y_{T_n}}{m_{T_n}} F_{1/2}(\zeta_{T_n}) + \sum_{n=1}^2 \frac{N_c Q_{B_n}^2 y_{B_n}}{m_{B_n}} F_{1/2}(\zeta_{B_n}) \right. \\ & \left. + \sum_{n=1}^2 \frac{Q_{E_n}^2 y_{E_n}}{m_{E_n}} F_{1/2}(\zeta_{E_n}) + \frac{C_{\sigma H^\pm H^\mp}}{\sqrt{2} m_{H^\pm}^2} F_0(\zeta_{H^\pm}) \right|^2, \end{aligned} \quad (89)$$

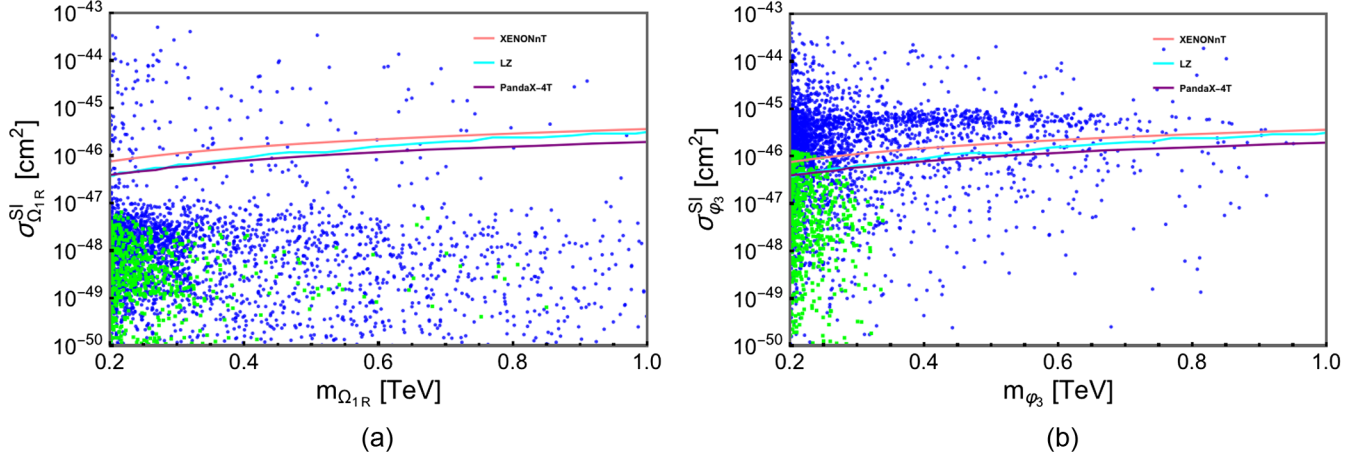


FIG. 17. The left and right panels demonstrate the dependence of the spin-independent scattering cross section of each DM component Ω_{1R} , ϕ_3 with nucleons on the DM masses $m_{\Omega_{1R}}$ and m_{ϕ_3} , respectively. The blue and green points are for two cases of mixed couplings $\lambda_{ij} \in [-0.1, 0.1]$ and $[-0.01, 0.01]$, respectively. The pink, cyan, and purple solid lines present the current upper experimental limits reported by XENONnT [131], LZ [132], and PandaX-4T [133] experiments, respectively.

where $K^{gg} \sim 1.5$ is a QCD loop enhancement factor that accounts for the higher order QCD corrections, $\zeta_i = 4M_i^2/m_\sigma^2$, with $M_i = m_{T_n}, m_{B_n}, m_{E_n}, m_{H^\pm}^2$ ($n = 1, 2$) and the loop functions $F_{1/2}(\zeta)$ and $F_0(\zeta)$ are given by

$$\begin{aligned} F_{1/2}(\zeta) &= -2\zeta(1 + (1 - \zeta)f(\zeta)), \\ F_0(\zeta) &= (1 - \zeta f(\zeta))\zeta, \end{aligned} \quad (90)$$

where

$$f(\zeta) = \begin{cases} |\arcsin \sqrt{1/\zeta}|^2 & \text{for } \zeta \geq 1 \\ -\frac{1}{4} \left(\ln \left(\frac{1+\sqrt{1-\zeta}}{1-\sqrt{1-\zeta}} \right) - i\pi \right)^2 & \text{for } \zeta < 1. \end{cases} \quad (91)$$

Figure 18 shows the plot of the ratio $\mu_{\gamma\gamma}$ as a function of the scalar mass $m_{H^\pm}$. In this plot, the masses of new exotic quarks were estimated using Eq. (38), which gives $m_{T_1} = m_{B_1} \simeq 4460$ GeV and $m_{T_2} = m_{B_2} \simeq 3858$ GeV. To evaluate the ratio $\mu_{\gamma\gamma}$, we consider a benchmark scenario where $v_\sigma \simeq 5$ TeV, the charged exotic lepton masses are set to be equal to $m_{E_1} = m_{E_2} \simeq 200$ GeV, and the coupling strength $C_{\sigma H^\pm H^\mp}$ of the trilinear scalar interaction $H^\pm H^\mp \mp \sigma$ takes the values of 2 to 20 TeV. These values fit the ratio $\mu_{\gamma\gamma}$ at the 1σ level for the charged scalar mass in the range $850 \leq m_{H^\pm} \leq 2500$ GeV. As indicated in Fig. 18, our model is consistent with the 95 GeV diphoton excess. The plot shows that the $\mu_{\gamma\gamma}$ ratio is sensitive to a variation of the trilinear scalar coupling $C_{\sigma H^\pm H^\mp}$, then implying, as indicated by Eq. (89), that the contribution of virtual exchange of electrically charged scalars in the triangular loop of the σ decay is relevant for $C_{\sigma H^\pm H^\mp} \sim 5$ to 10 TeV in

a wide range of charged scalar masses $m_{H^\pm}$. In addition, increasing the value of the trilinear scalar coupling $C_{\sigma H^\pm H^\mp}$ yields an increase that the lower and upper bounds on charged scalar masses $m_{H^\pm}$ are compatible with the 95 GeV diphoton excess.

In our setup, σ_R is predominantly a singletlike state and its tree-level couplings to SM fermions and gauge bosons arise only through scalar mixing. In the parameter region relevant for Fig. 18, we have $\tan \theta_h \sim 10^{-1} - 10^{-2}$. Consequently, both the production rates and the tree-level partial widths into SM final states scale approximately as

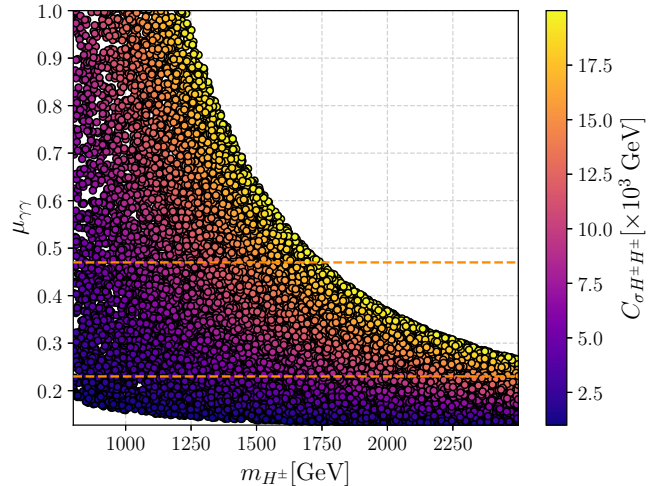


FIG. 18. Signal strength for a 95 GeV diphoton excess as a function of the charged scalar mass $m_{H^\pm}$. The orange horizontal lines correspond to the 1σ upper and lower experimental bounds, respectively.

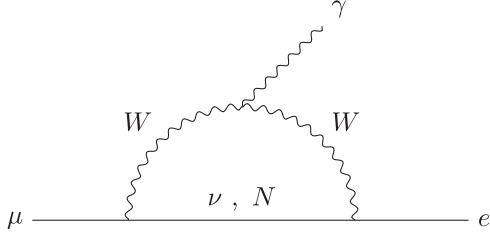


FIG. 19. One-loop contribution to $\mu \rightarrow e\gamma$. Here, ν is for light active neutrino and N is for heavy sterile neutrino both in physical basis.

$\tan^2 \theta_h$. As a consequence, signal strengths in channels such as $b\bar{b}$ and $\tau^+\tau^-$ are generically suppressed to the level $\mu_{b\bar{b}}, \mu_{\tau\tau} \sim \tan^2 \theta_h \lesssim 10^{-2} - 10^{-4}$ and are not expected to yield observable excesses. By contrast, the $\gamma\gamma$ mode can be enhanced by the loop contributions of vectorlike charged fermions (and charged scalars), while the total width is simultaneously reduced due to the suppressed tree-level decays. This makes the diphoton final state, and potentially $Z\gamma$, the most promising channel for probing σ_R near 95 GeV within this model.

VIII. CHARGED LEPTON FLAVOR VIOLATION

The most stringent limits on charged lepton flavor violation (cLFV) are obtained from the radiative decay $\mu \rightarrow e\gamma$. In our framework, this process arises at the one-loop level and receives contributions both from light neutrino mixing and from the exchange of heavy sterile states. The branching ratio can be written as [136–138]

$$\text{Br}(l_i \rightarrow l_j \gamma) = \frac{\alpha_W^3 s_W^2 m_{l_i}^5}{256\pi^2 m_W^4 \Gamma_i} |G_{ij}|^2, \quad (92a)$$

$$G_{ij} \simeq \sum_{k=1}^3 [(1 - RR^\dagger)U_\nu]^*_{ik} ((1 - RR^\dagger)U_\nu)_{jk} G_\gamma \left(\frac{m_{\nu_k}^2}{m_W^2} \right) + 2 \sum_{l=1}^2 (R^*)_{il} (R)_{jl} G_\gamma \left(\frac{m_{N_{Rl}}^2}{m_W^2} \right), \quad (92b)$$

$$G_\gamma(x) = \frac{10 - 43x + 78x^2 - 49x^3 + 18x^3 \ln x + 4x^4}{12(1-x)^4}. \quad (92c)$$

Here $\Gamma_\mu = 3 \times 10^{-19}$ GeV is the total muon decay width, and U_ν denotes the PMNS matrix, since we are working in the basis where the SM charged-lepton mass matrix is diagonal. The matrix R encodes the mixing between active and heavy states and is defined as

$$R = \frac{1}{\sqrt{2}} m_D^* M^{-1}, \quad (93)$$

with M and m_D the Majorana and Dirac neutrino mass matrices, respectively.

The first term in Eq. (92) corresponds to the modified contribution of light neutrinos, while the second term arises from loops with heavy sterile states as it shows in Fig. 19. Current experimental limits, most notably from the MEG II experiment [139], $\text{Br}(\mu \rightarrow e\gamma) < 1.5 \times 10^{-13}$ at 90% C.L., place strong bounds on the allowed active-sterile mixing parameters contained in R .

To address the concerns regarding quantitative fits, we provide explicit benchmark points in Table VII for

TABLE VII. Five benchmark points with M_D , M , μ branching ratio $\text{Br}(\mu \rightarrow e\gamma)$ and neutrino mass-squared differences.

Benchmark	M_D (GeV)	M (GeV)	μ (eV)	Results
1	$\begin{pmatrix} 1.511 & 4.247 \\ 4.686 & 1.701 \\ 1.630 & 4.452 \end{pmatrix}$	$\begin{pmatrix} 3742.6 & 0 \\ 0 & 1729.1 \end{pmatrix}$	$\begin{pmatrix} 455.2 & 3770.7 \\ 7435.5 & 222.8 \end{pmatrix}$	$\text{Br}(\mu \rightarrow e\gamma) = 5.9 \times 10^{-14}$ $\Delta m_{21}^2 = 7.05 \times 10^{-5}$ $\Delta m_{31}^2 = 2.48 \times 10^{-3}$
2	$\begin{pmatrix} 1.790 & 4.785 \\ 4.659 & 1.563 \\ 4.364 & 1.542 \end{pmatrix}$	$\begin{pmatrix} 2402.1 & 0 \\ 0 & 2920.0 \end{pmatrix}$	$\begin{pmatrix} 418.1 & 3781.0 \\ 7568.3 & 215.2 \end{pmatrix}$	$\text{Br}(\mu \rightarrow e\gamma) = 3.5 \times 10^{-14}$ $\Delta m_{21}^2 = 7.08 \times 10^{-5}$ $\Delta m_{31}^2 = 2.38 \times 10^{-3}$
3	$\begin{pmatrix} 2.265 & 3.246 \\ 1.531 & 4.782 \\ 4.900 & 1.643 \end{pmatrix}$	$\begin{pmatrix} 2656.8 & 0 \\ 0 & 2341.2 \end{pmatrix}$	$\begin{pmatrix} 330.1 & 3713.0 \\ 7004.9 & 219.1 \end{pmatrix}$	$\text{Br}(\mu \rightarrow e\gamma) = 7.5 \times 10^{-14}$ $\Delta m_{21}^2 = 6.83 \times 10^{-5}$ $\Delta m_{31}^2 = 2.53 \times 10^{-3}$
4	$\begin{pmatrix} 1.608 & 4.759 \\ 4.748 & 1.555 \\ 3.722 & 1.667 \end{pmatrix}$	$\begin{pmatrix} 1989.5 & 0 \\ 0 & 3209.7 \end{pmatrix}$	$\begin{pmatrix} 403.3 & 3805.5 \\ 7046.1 & 252.5 \end{pmatrix}$	$\text{BRBr}(\mu \rightarrow e\gamma) = 4.7 \times 10^{-14}$ $\Delta m_{21}^2 = 7.14 \times 10^{-5}$ $\Delta m_{31}^2 = 2.42 \times 10^{-3}$
5	$\begin{pmatrix} 3.880 & 1.707 \\ 4.293 & 1.501 \\ 1.579 & 4.664 \end{pmatrix}$	$\begin{pmatrix} 3113.9 & 0 \\ 0 & 2003.5 \end{pmatrix}$	$\begin{pmatrix} 421.1 & 3807.1 \\ 7719.3 & 239.9 \end{pmatrix}$	$\text{Br}(\mu \rightarrow e\gamma) = 3.6 \times 10^{-14}$ $\Delta m_{21}^2 = 7.06 \times 10^{-5}$ $\Delta m_{31}^2 = 2.43 \times 10^{-3}$

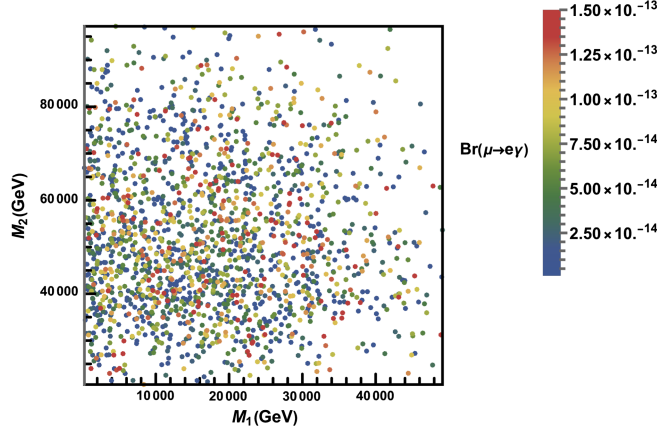


FIG. 20. Allowed region in the M_1 - M_2 plane consistent with charged lepton flavor violating constraints.

the two-loop inverse seesaw model. For each benchmark, the Dirac mass matrix M_D , the Majorana mass M , and the lepton-number-violating parameter μ are chosen such that the resulting neutrino mass eigenvalues reproduce the observed mass-squared differences Δm_{21}^2 and $|\Delta m_{31}^2|$ within the current 3σ experimental ranges [140]. We stress that the physical PMNS angles $\theta_{12,13,23}$ emerge from the interplay of the neutrino and charged-lepton sectors. Given the significant parametric freedom in this setup, their values cannot be uniquely determined. Nevertheless, the corresponding branching ratios for $\mu \rightarrow e\gamma$ remain consistent with the latest MEG II bound, demonstrating that our framework can quantitatively accommodate neutrino oscillation data while satisfying charged lepton flavor violation constraints. The allowed region in the M_1 - M_2 plane consistent with charged lepton flavor violating constraints is displayed in Fig. 20. This indicates that the model remains a viable framework for simultaneously explaining neutrino oscillations and charged lepton flavor violation. In addition, Fig. 21, shows the plot of $\text{Br}(\mu \rightarrow e\gamma)$ as a function of $\text{Tr}[RR^\dagger]$ from Eq. (93) and the lightest value of heavy Majorana neutrinos m_N . All the points of the plot

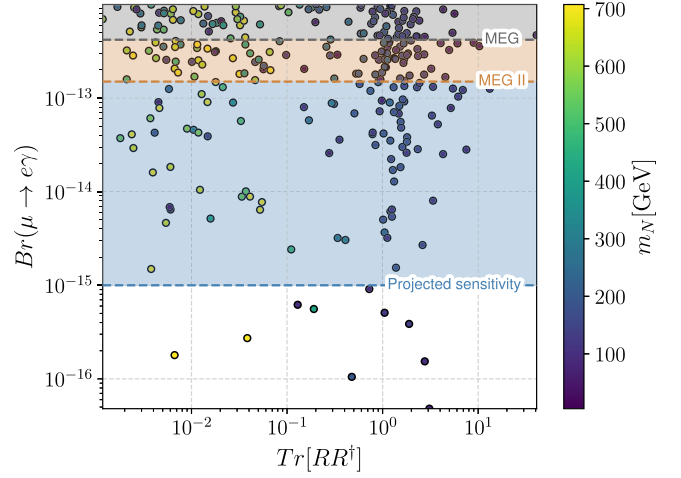


FIG. 21. Branching ratios of $\mu \rightarrow e\gamma$. The projected sensitivity dotted line is for hypothetically sensitivity of 10^{-15} for future experiments.

successfully comply with neutrino oscillation experimental data and are consistent with the assumption of the diagonal M matrix. The branching ratios of $\mu \rightarrow e\gamma$ are in the reach of actual MEG II sensitivity and can reach the projected sensitivity of 10^{-15} for future experiments (Ref. [141] and Ref. [142]).

To conclude this section, we discuss the implications of our model for electron-muon conversion. In the low-momentum regime, the dominant contributions to lepton flavor violation come from dipole operators, as described by the effective Lagrangian approach of Ref. [143]. This framework establishes relations between the branching ratios of various LFV processes:

$$\begin{aligned} \text{Br}(\mu \rightarrow 3e) &\simeq \frac{1}{160} \text{Br}(\mu \rightarrow e\gamma), \\ \text{CR}(\mu\text{Ti} \rightarrow e\text{Ti}) &\simeq \frac{1}{200} \text{Br}(\mu \rightarrow e\gamma), \\ \text{CR}(\mu\text{Al} \rightarrow e\text{Al}) &\simeq \frac{1}{350} \text{Br}(\mu \rightarrow e\gamma) \end{aligned} \quad (94)$$

The μ - e conversion ratio, $\text{CR}(\mu-e)$, is defined as [144]

$$\text{CR}(\mu-e) = \frac{\Gamma(\mu^- + \text{nucleus}(A, Z) \rightarrow e^- + \text{nucleus}(A, Z))}{\Gamma(\mu^- + \text{nucleus}(A, Z) \rightarrow \nu_\mu + \text{nucleus}(A, Z-1))}. \quad (95)$$

Given these relations and our model's prediction for $\mu \rightarrow e\gamma$, the resulting rates for $\mu \rightarrow 3e$ and $\mu-e$ conversion in titanium and aluminum are expected to be $\sim 10^{-15}$. This places them two orders of magnitude below our $\mu \rightarrow e\gamma$ rate and well beneath the current experimental bounds of approximately 10^{-12} .

IX. CONCLUSIONS

In summary, we have proposed an extension of the inert doublet model that successfully accommodates some of the unaddressed SM issues. This model is based on the SM gauge symmetry, supplemented by a spontaneously broken

global $U(1)_X$ symmetry and a preserved Z_2 discrete symmetry.

In our model, the first and second families of SM charged fermions acquire their masses through a one-loop level radiative seesaw mechanism, while the third generation of SM charged fermions obtains their masses at tree level. The masses of the light-active neutrinos arise from a radiative inverse seesaw mechanism at the two-loop level.

We successfully address the strong CP problem by preserving the CP symmetry in the SM sector's Lagrangian at the tree level. In this approach, explicit CP violation occurs in the dark scalar sector. The CP -violating phases are then transferred to the SM quark sector at the loop level, which is crucial for providing a viable solution to the strong CP problem. We recover the CP phase in the weak sector through one-loop level corrections mediated by dark fields, while the strong CP phase remains vanishing at the three-loop level.

The radiative nature of the seesaw mechanisms is attributed to preserved discrete symmetries, which are required for ensuring the stability of fermionic and scalar dark matter candidates. The preserved discrete symmetries also allow for multicomponent dark matter, whose annihilation processes will enable us to successfully reproduce the measured amount of dark matter relic abundance while satisfying the current dark matter direct detection limits.

Additionally, the model complies with the constraints related to charged lepton flavor-violating processes, with

the predicted rates for these decays falling within the current experimental sensitivity.

Finally, we interpret the 95 GeV diphoton excess reported by the CMS Collaboration as a scalar resonance with a mass of 95 GeV, which aligns with the latest experimental data.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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