

Light scalar mesons in D^+, D_s^+ decays into three pseudoscalars

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In response to recent experimental findings from the BES III and LHCb Collaborations concerning the decay of open charm mesons into three pseudoscalars, we present evidence suggesting that the lightest scalar mesons can be characterized as diquark-antidiquark states mixed with the next-to-light, $q\bar{q}$, scalar mesons through six-fermion, instanton-induced interactions.

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I. INTRODUCTION

Recently, BES III [1,2] and LHCb [3,4] have provided detailed data on D_s^+ and D^+ , S -wave decays into three pseudoscalar mesons

$$(a) D_s^+ \rightarrow K^+ K^- \pi^+ \\ (b) D_s^+ \rightarrow \pi^+ \pi^- \pi^+ \quad (\text{Cabibbo allowed}) \quad (1.1)$$

$$(c) D^+ \rightarrow \pi^+ \pi^- \pi^+ \\ (d) D_s^+ \rightarrow K^+ \pi^- \pi^+ \quad (\text{Cabibbo suppressed}) \quad (1.2)$$

interpreted as decays into one pseudoscalar and one scalar meson, followed by the decay of the scalar into two pseudoscalar mesons.

The final states of these decays have been analyzed in terms of superpositions of pseudoscalar mesons and resonances, encompassing all meson resonances that undergo two-body decays. The estimated fraction of two mesons in the S -wave final state for (1.1)(a) is approximately 24%, while for (1.1)(b) and (1.2)(c), it is 85% and 62%, respectively. For (1.2)(d), the fraction is 46%.

Data provide valuable insights into the structure of the scalar mesons and may contribute to the resolution of the enigma surrounding the internal structure of the lightest scalar nonet.

In this paper, we employ the experimental data on decays (1.1) and (1.2) to assess the tetraquark hypothesis for the composition of the light scalar mesons σ , f_0 , and κ .

Furthermore, we investigate their mixing to the nonet of $q\bar{q}$ scalar mesons proposed in [5–8].

In Sec. II we review the arguments for describing the scalar meson multiplets as: (i) a heavier, P -wave $q\bar{q}$, nonet and (ii) a light diquark-antidiquark nonet, the two being mixed by interactions generated by QCD instantons [9,10].

In Sec. III, we consider the quark diagrams describing Cabibbo allowed decay, Eq. (1.1)(a). We introduce four phenomenological amplitudes, estimating three of them from the observed branching ratios of the $D_s^+ \rightarrow K^+ K^- \pi^+$ decay. In Sec. IV we give an upper bound to the missing fourth amplitude from the nonobservation of σ production in $D_s^+ \rightarrow \pi^+ \pi^- \pi^+$ decay, Eq. (1.1)(b), and predict the branching ratios of the other decay modes. In Secs. V and VI, we extend our considerations to the Cabibbo suppressed decays, Eq. (1.2). In this scheme we are able to reproduce the relative branching fractions of (1.1)(b) and (1.2)(c) decays.

For the last decay, (1.2)(d), the comparison of our predictions with the data from [2] is not so good. In part this may be due to rather large experimental errors. Also, unlike previous cases, BESIII fits the low energy $K^+ \pi^-$ data by a continuum distribution, rather than the Breit-Wigner corresponding to the $\kappa(700)$ resonance, as would be required by our model.

A more conclusive statement will require larger statistics and more precise data, fitted consistently with the two nonets of scalar resonances assumed in the present analysis.

II. SCALAR MESONS

A. The higher $q\bar{q}$ nonet

Positive-parity $q\bar{q}$ scalar mesons are expected to occur in P wave [11] and therefore be heavy. P -wave scalar mesons have indeed been observed above 1 GeV, see Table I, together with the other states required by the composition of orbital and spin angular momenta [12].

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TABLE I. Mass and widths of the heavier scalar mesons from PDG 2024 [12].

PDG name	M	Γ	Partial rates
$f_0(1370)$	1200–1500	50–500	2π dominant
$a_0(1450)$	1474 ± 19	265 ± 13	...
$K_0(1430)$	1425 ± 50	270 ± 80	...
$f_0(1500)$	1522 ± 25	108 ± 33	Glueball?
$f_0(1710)$	1733 ± 8	150 ± 11	$K\bar{K}$ dominant

$$J = s_q + s_{\bar{q}} + L, \quad L = 1, \quad J^{PC} = 0^{++}$$

$$(a_0(1450), \dots), 1^{++}(a_1(1260), \dots),$$

$$1^{+-}(b_1(1235), \dots), 2^{++}(a_2(1320), \dots). \quad (2.1)$$

Scalar, $q\bar{q}$, mesons can be classified in the $SU(3)_F$ representations $\mathbf{8} \oplus \mathbf{1}$. In particular, $f_0(1370)$ and $f_0(1710)$ are assumed to be associated to the $\bar{u}u + d\bar{d}$ and $s\bar{s}$ configurations, respectively. The additional singlet in Table I, $f_0(1500)$, has been proposed to be predominantly a glueball [13–16] and will not be considered in the following.

The $q\bar{q}$ nonet is represented by the matrix

$$S'(q\bar{q}) = \begin{pmatrix} \frac{f' + a^0}{\sqrt{2}} & a'^+ & \kappa'^+ \\ a'^- & \frac{f' - a^0}{\sqrt{2}} & \kappa'^0 \\ \kappa'^- & \bar{\kappa}'^0 & f'' \end{pmatrix} \quad (2.2)$$

with the shorthand notation $f' = f_0(1370)$, $f'' = f_0(1710)$; when f_0 is found, we mean $f_0 = f_0(980)$, the lightest of f_0, f', f'' .

B. The lower tetraquark nonet

Following Jaffe [5] and successive developments [6–8], we assume that the lightest scalar multiplet is made by spin zero diquark-antidiquark bound states forming the $SU(3)_F$ nonet. We will define the spin-0 diquark as

$$[qq]_{i\alpha} = \epsilon_{ijk} \epsilon_{\alpha\beta\gamma} (\bar{q}_c)^{j\beta} \gamma_5 q^{k\gamma}, \quad (2.3)$$

where Latin indices are for flavor, Greek for color and the subscript c is for charge conjugation.¹ This ensures Fermi statistics: the state is antisymmetric in color, spin and flavor. The spin-0, $\mathbf{3}_c$ diquark with flavors a and b is $[q^a q^b]_\alpha$. The connection between tetraquark states and $SU(3)_F$ matrices is given by the relations

$$S_j^i = \epsilon_{iab} \epsilon^{jcd} M_{cd}^{ab} \quad \text{where} \quad M_{cd}^{ab} = [q^a q^b]_\alpha [\bar{q}_c \bar{q}_d]^\alpha. \quad (2.4)$$

¹Where $\bar{q}_c \gamma_5 q = -iq^T(1 \otimes \sigma_2)q$.

For example, S_1^1 is the diquark combination $[ds][\bar{d}\bar{s}] = (f_0 - a_0)/\sqrt{2}$, $S_2^2 = [us][\bar{u}\bar{s}] = (f_0 + a_0)/\sqrt{2}$, and $S_3^3 = [ud][\bar{u}\bar{d}]$ with

$$S([qq][\bar{q}\bar{q}]) = \begin{pmatrix} \frac{f_0 - a^0}{\sqrt{2}} & a^+ & \kappa^+ \\ a^- & \frac{f_0 + a^0}{\sqrt{2}} & \kappa^0 \\ \kappa^- & \bar{\kappa}^0 & \sigma \end{pmatrix}. \quad (2.5)$$

The unmixed diagonal states are

$$\sigma = [ud][\bar{u}\bar{d}] \quad (2.6)$$

$$f_0, a_0 = \frac{1}{\sqrt{2}}([us][\bar{u}\bar{s}] \pm [ds][\bar{d}\bar{s}]). \quad (2.7)$$

When the mixing angle between the components $(S_1^1 + S_2^2)/\sqrt{2}$ and S_3^3 is very small we have the structure of an ideally mixed 4-quark nonet. If we had a mixing close to 90° the scalars would be better described in terms $q\bar{q}$ mesons: exchange $f_0 \leftrightarrow \sigma$ in (2.5). The latter case however is questionable because of the degeneracy $f_0(s\bar{s}) \sim a_0(q\bar{q})$, in addition to the fact that scalar mesons of the $q\bar{q}$ kind should be P -wave states.

Associated with the chiral symmetry breaking $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$ we have an octet of pseudoscalar mesons Φ (pseudo-Goldstone bosons)²

$$\Phi = \begin{pmatrix} \frac{\eta_q + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_q - \pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_s \end{pmatrix}. \quad (2.8)$$

Scalar mesons interact with pseudoscalars with derivative interactions including $\partial_\mu \Phi \equiv \Phi_\mu$. The terms in the interaction Lagrangian involving the scalar and pseudoscalar mesons are

$$\mathcal{L} = A \text{Tr}(S \Phi_\mu \Phi^\mu) + B \text{Tr}(S) \text{Tr}(\Phi_\mu \Phi^\mu) + C \text{Tr}(S \Phi_\mu) \text{Tr}(\Phi^\mu) + D \text{Tr}(S) \text{Tr}(\Phi_\mu) \text{Tr}(\Phi^\mu). \quad (2.9)$$

In general the parameters A, B can be determined by the analysis of $\pi\pi$ or πK scattering whereas C, D can be determined in processes involving η and η' particles because of the explicit form of $\text{Tr}(\Phi_\mu)$. In the tetraquark picture we consider, for example, a transition like $[ds][\bar{d}\bar{s}] \rightarrow (s\bar{d})(d\bar{s})$, where the d and $\bar{d}$ quarks are being swapped, breaking the diquarks. Such a transition

²Where

$$\eta_q = \frac{\bar{u}\gamma_5 u + \bar{d}\gamma_5 d}{\sqrt{2}} \quad \text{and} \quad \eta_s = \bar{s}\gamma_5 s.$$

corresponds, in the notations given above, to $S_1^1 \rightarrow (\Phi_\mu)_3^2 (\Phi^\mu)_2^3$. The interaction term $S_1^1 (\Phi_\mu)_3^2 (\Phi^\mu)_2^3$ comes with $2B$ in the Lagrangian (2.9). With $2B$ we have also the $S_1^1 (\Phi_\mu)_3^1 (\Phi^\mu)_1^3$ interaction, which corresponds to the subleading transition $[ds][\bar{d}\bar{s}] \rightarrow (s\bar{u})(u\bar{s})$ where the $d\bar{d}$ pair converts into a $u\bar{u}$ pair. In the following we will include only the leading quark-swap amplitudes. In order to do this, we note that $S_1^1 (\Phi_\mu)_3^1 (\Phi^\mu)_1^3$ comes also from the term weighted by A in (2.9) and the combination $2B + A = 0$, which is satisfied for $B = 1/2$ and $A = -1$, removes this term. So the leading quark swapping interaction in the tetraquark model (subscript- f : flavor-swapping) corresponds to

$$\mathcal{L}_s = C_f \left(\frac{1}{2} \text{Tr}(S) \text{Tr}(\Phi_\mu \Phi^\mu) - \text{Tr}(S \Phi_\mu \Phi^\mu) \right) = C_f O_f. \quad (2.10)$$

Alternatively we can consider transitions in which $[ds][\bar{d}\bar{s}] \rightarrow (d\bar{d})(s\bar{s})$, i.e. involving the pseudoscalar singlet. This corresponds to $S_1^1 \rightarrow (\Phi_\mu)_2^2 (\Phi_\mu)_3^3$ which appears with coefficient $2D$ in (2.9). The operator weighted by D involves also terms like $S_1^1 \rightarrow (\Phi_\mu)_1^1 (\Phi_\mu)_3^3$ which are canceled by those deriving from the operator weighted by C , if we require $C = -1$ and $D = 1/2$. The transitions $[ds][\bar{d}\bar{s}] \rightarrow (s\bar{d})(d\bar{s})$ and $[ds][\bar{d}\bar{s}] \rightarrow (d\bar{d})(s\bar{s})$ represent two alternative quark swaps. In the first case, the d quarks are swapped with the $s\bar{s}$ pair being spectators, while in the second case, a d and $\bar{s}$ (or vice versa) are swapped. Under the assumption that there is no distinction between these two quark swappings, the corresponding operators can be grouped with the same effective coefficient

$$\begin{aligned} \mathcal{L} &= C_f O_f \\ &= C_f \left(-\text{Tr}(S \Phi_\mu \Phi^\mu) + \frac{1}{2} \text{Tr}(S) \text{Tr}(\Phi_\mu \Phi^\mu) \right. \\ &\quad \left. - \text{Tr}(S \Phi_\mu) \text{Tr}(\Phi^\mu) + \frac{1}{2} \text{Tr}(S) \text{Tr}(\Phi_\mu) \text{Tr}(\Phi^\mu) \right) \end{aligned} \quad (2.11)$$

where now O_f includes all the four terms in (2.9), with only one phenomenological parameter C_f in place of A, B, C, D .

There is an additional interaction of the $\text{Tr}(S \Phi_\mu \Phi^\mu)$ type to be considered though. It is induced by QCD instantons producing an effective interaction reducing the $U(N_f)_L \times U(N_f)_R$ global symmetry of the quark model in the chiral limit to $SU(N_f)_L \times SU(N_f)_R$ times baryon number. The effect can be described by the 't Hooft 6-fermion interaction [9,10]

$$\mathcal{L}_I \propto \det[(q_L)_{i\alpha} (q_R)_{j\alpha}^i] \quad (2.12)$$

where i and j denote flavor indices. With three light quark flavors, $\mathcal{L}_I$ is proportional to the product of three quark and three antiquark fields, antisymmetrized in flavor and color, and it includes a term of the type

$$\text{Tr}(J^{[4q]} J^{[2q]}) \quad (2.13)$$

where

$$(J^{[4q]})_j^i = [\bar{q} \bar{q}]^{i\alpha} [q q]_{j\alpha} \quad (J^{[2q]})_j^i = \bar{q}_{j\alpha} q^{i\alpha}. \quad (2.14)$$

The two components of the effective Lagrangian (2.13) can annihilate a tetraquark and create a $q\bar{q}$ and viceversa. Therefore the interaction Lagrangian (2.11) can be extended to include the instanton induced interaction as in

$$\mathcal{L} = C_f O_f + C_I O_I. \quad (2.15)$$

The second term induces transitions $4q \leftrightarrow 2q$; notice that the O_I operator is itself of the form $\text{Tr}(S \Phi_\mu \Phi^\mu)$. This follows from the chiral realization of the currents in (2.13) and

$$\langle PP | \text{Tr}(J^{[4q]} J^{[2q]}) | S \rangle \propto \text{Tr}(S \Phi_\mu \Phi^\mu) + \dots \quad (2.16)$$

where, as commented in [7] [Eq. (16)], the dots denote higher-order terms in Φ and in the chiral expansion. At low energy, instanton effects are expected to be sizeable.

The instanton interaction induces the mixing of the two nonet fields of the form

$$\mathcal{L}_{S'S} = \gamma \text{Tr}(S'S) \quad (2.17)$$

where S' is the conventional scalar $q\bar{q}$ nonet.

The mixing provided by the effective QCD action, Eq. (2.17), among other effects, is responsible for the otherwise forbidden decay $f_0(980) \rightarrow \pi^+ \pi^-$ of the f_0 tetraquark [7]. Indeed the Zweig rule allowed decays are $f_0 \rightarrow K\bar{K}$ and $f' \rightarrow \pi\pi$, but not $f_0 \rightarrow \pi\pi$.

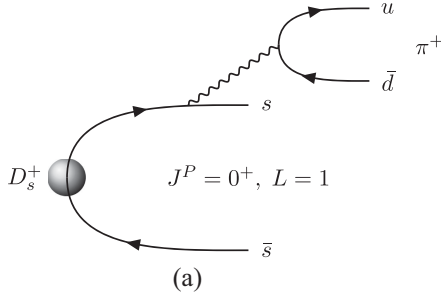
Mixing can go either way ($\lambda < 1$)

$$f_0(980) \equiv f_0 = f_{0,4q} + \lambda f'_{2q} \quad (2.18)$$

$$f_0(1370) \equiv f' = -\lambda f_{0,4q} + f'_{2q} \quad (2.19)$$

where we mean that the mass eigenstate f_0 is essentially the 4-quark eigenstate $f_{0,4q}$ mixed with a 2-quark component f'_{2q} (the prime makes reference to S'). Similarly the mass eigenstate f' is essentially f'_{2q} . Thanks to the mixing, it acquires a tetraquark component and it displays the corresponding decay modes, e.g. $f'' \rightarrow \pi^+ \pi^-$ via its mixing with σ , and can be created by 4-quark operators, as we shall discuss in Sec. III.

The ordering $M[a_0(1450)] > M[K_0(1430)]$ can also be attributed to the mixing with the corresponding light scalar

FIG. 1. D_s decay, Cabibbo allowed, tree-level diagram.

mesons $a_0(980)$ and $\kappa(700)$ induced by the lagrangian equation (2.17). This analysis provides the estimate (see [17,18])

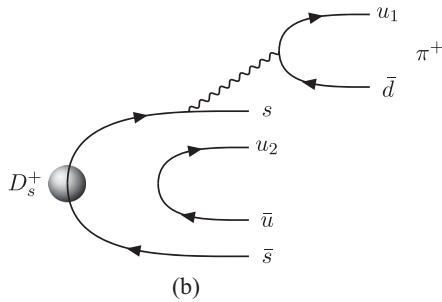
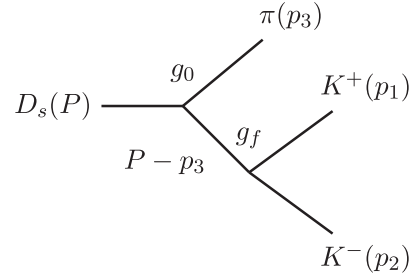
$$\gamma \simeq 0.7 \text{ GeV}^2. \quad (2.20)$$

III. CABIBBO ALLOWED S-WAVE DECAY:

$$D_s^+ \rightarrow K^+ K^- \pi^+$$

The relevant quark Feynman diagrams for Cabibbo allowed D_s^+ decays are reported in Fig. 1 and in Fig. 2, when a quark pair is created from vacuum. Considering the decay $D_s^+ \rightarrow K^+ K^- \pi^+$, we observe that

- (1) Diagram (a) in Fig. 1 may produce the f'' meson via its $q\bar{q}$ component (amplitude A_f) and the σ meson via its instanton mixing to the $s\bar{s}$ meson (amplitude A_I). The amplitudes A_I and A_f comprise the weak interaction pion production. The amplitudes A_f and A_I are generated by interactions of the $C_f O_f$ and $C_I O_I$ type, as in (2.15). However, A_I cannot be determined from the rates $\Gamma(D_s \rightarrow K\bar{K}\pi)$ since σ cannot decay into $K\bar{K}$.
- (2) Diagrams (b) and (c) in Fig. 2 produce f_0 via its dominant tetraquark component (amplitude B_f) and f' via its mixing generated tetraquark component, with subsequent decays in $K^+ K^-$ (amplitude B_I). The amplitudes B comprise the weak interaction pion production. The amplitudes B_f and B_I are

FIG. 2. D_s decay, Cabibbo allowed, with a quark pair from vacuum. A $u\bar{u}$ is created in (b) and a $d\bar{d}$ pair is created in (c).FIG. 3. Feynman diagram representation of the $D_s \rightarrow (K\bar{K})_f \pi$ decay amplitude.

generated by interactions of the $C_f O_f$ and $C_I O_I$ type, as in (2.15).

The amplitudes A_f , B_f , B_I are computed by comparison to data as follows. We will evaluate the Feynman diagram in Fig. 3 representing the $D_s \rightarrow f\pi \rightarrow K\bar{K}\pi$ decay. Its square amplitude is given by

$$|M|^2 = \frac{g_0^2 g_f^2}{(m_f \Gamma_f)^2 + (m_f^2 - m_{12}^2)^2} \quad (3.1)$$

and the rate $\Gamma(D_s \rightarrow f\pi \rightarrow K\bar{K}\pi)$ is computed applying the standard 3-body phase space formula

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32M^3} |M|^2 dm_{12}^2 dm_{23}^2 \quad (3.2)$$

with the m_{12}^2 and m_{23}^2 variables to be integrated over the Dalitz plot surface. The parameters m_f and Γ_f are the mass and the total width of the intermediate scalar resonance. The coupling g_f is determined by the partial width of the scalar meson to decay into $K^+ K^-$, when this channel is available

$$g_f = \sqrt{\frac{8\pi m_f^2 \mathcal{B}(f \rightarrow K\bar{K}) \Gamma_f}{p^*(m_f, m_K, m_K)}} \quad (3.3)$$

where p^* is the decay momentum of the particle of mass M into m_1 and m_2 , $p^*(M, m_1, m_2) = \sqrt{\lambda(M^2, m_1^2, m_2^2)}/2M$, λ being the Källén triangular function. The coupling g_0

TABLE II. Experimental parameters for the lightest scalar meson decays [12]. In the BR column, the first figure is the PDG branching ratio, which includes all charge states, to be multiplied by the figure next to it to obtain the charge combination indicated in the first column.

Processes	m_f (MeV)	Γ_f (MeV)	p^* (MeV)	BR
$\sigma \rightarrow \pi^+\pi^-$	475	275	192	$1 \times 2/3$
$\kappa^+ \rightarrow K^0\pi^+$	845	468	253	$1 \times 2/3$
$f_0 \rightarrow \pi^+\pi^-$	990	45	475	$0.75 \times 2/3$
$f_0 \rightarrow K^+K^-$	990	45	36	0.25/2
$a_0 \rightarrow \pi^0\eta$	980	75	317	0.85
$a_0 \rightarrow K^+K^-$	980	75	87–33	0.15/2

TABLE III. Experimental parameters [12] for the next-to-light scalar mesons. In the BR column, the first figure is the PDG branching ratio, which includes all charge states, to be multiplied by the figure next to it to obtain the charge combination indicated in the first column.

Processes	M_S (MeV)	Γ_S (MeV)	p^* (MeV)	BR
$f_0(1370) \rightarrow \pi^+\pi^-$	1345	350	672	$0.26 \times 2/3$
$f_0(1370) \rightarrow K^+K^-$	...	...	475	0.021/2
$a_0^0(1450) \rightarrow \pi^0\eta$	1439	258	606	0.093
$a_0^0(1450) \rightarrow \pi^0\eta'(958)$	...	...	383	0.033
$a_0^0(1450) \rightarrow K^+K^-$	...	...	523	0.083/2
$K_0^+(1430) \rightarrow K^0\pi^+$	1425	270	621	$0.93 \times 2/3$
$K_0^+(1430) \rightarrow K^+\eta$	...	...	490	0.086
$f_0(1710) \rightarrow \pi^+\pi^-$	1733	150	855	$0.039 \times 2/3$
$f_0(1710) \rightarrow K^+K^-$	...	...	711	0.36/2

contains the aforementioned $A_{f,I}, B_{f,I}$ amplitudes. We have access to

$$g_0 = (B_f\sqrt{2})\sqrt{m_{D_s}} \quad \text{for } D_s \rightarrow f_0\pi \rightarrow K\bar{K}\pi \quad (3.4)$$

$$g_0 = (B_I\sqrt{2})\sqrt{m_{D_s}} \quad \text{for } D_s \rightarrow f'\pi \rightarrow K\bar{K}\pi \quad (3.5)$$

$$g_0 = A_f\sqrt{m_{D_s}} \quad \text{for } D_s \rightarrow f''\pi \rightarrow K\bar{K}\pi \quad (3.6)$$

see the notation for scalar mesons introduced after (2.2). The introduction of the $\sqrt{m_{D_s}}$ dependency allows Γ, A, B to be *pure numbers*. The factors $\sqrt{2}$ on the first two equations arise because the sum of diagrams (b) + (c) corresponds to $\sqrt{2}f_0$, for B_f and $\sqrt{2}f'$, for B_I , see the Appendix. The physical parameters used in the previous calculation are extracted from Tables II and III.

The experimental data to confront with are reported in the following Table IV. Using Table IV and equating

TABLE IV. Branching fractions of resonant contributions to the decay: $D_s^+ \rightarrow \pi^+(K^+K^-)_S$, from PDG 2024 [12].

$D_s^+ \rightarrow \pi^+K^+K^-$	BR(%)	$(100/5.45) \cdot \text{BR}$
$\pi^+K^+K^-$	5.45	100
$f_0(980)\pi^+, f_0 \rightarrow K^+K^-$	1.1	20.5
$a_0^0(980)\pi^+, a_0^0(980) \rightarrow K^+K^-$	0.055	1.0
$f_0(1370)\pi^+, f_0(1370) \rightarrow K^+K^-$	7.1×10^{-2}	1.32
$f_0(1710)\pi^+, f_0(1710) \rightarrow K^+K^-$	6.7×10^{-2}	1.24

$\Gamma(D_s \rightarrow f_0\pi \rightarrow K\bar{K}\pi) = 20.5$, $\Gamma(D_s \rightarrow f'\pi \rightarrow K\bar{K}\pi) = 1.32$, and $\Gamma(D_s \rightarrow f''\pi \rightarrow K\bar{K}\pi) = 1.24$ we obtain³ the three parameters (expressed as pure numbers in the parametrization formulated above) A_f, B_f, B_I which we will be using to study the $D_{(s)} \rightarrow 3\pi$ decays

$$|A_f| = 46.1, \quad |B_f| = 55.2, \quad |B_I| = 96.7 \quad (3.7)$$

showing that instanton interactions in this scheme are of the same size of quark-swapping ones.

IV. CABIBBO ALLOWED S-WAVE DECAY:

$$D_s^+ \rightarrow \pi^+\pi^+\pi^-$$

The quark diagrams are the same as (a), (b), (c) in Figs. 1 and 2. In this case, we observe that

- (1) Diagram (a) is related to the production of f'' proportionally to the amplitude A_f . The instanton interaction allows the σ in diagram (a) so that we can determine an upper value for the amplitude A_I by comparison to data on $D_s \rightarrow 3\pi$ decays, which indicate a very stringent upper bound on the σ in $D_s \rightarrow 3\pi$ decays.
- (2) Diagrams (b) and (c) are related to the production of f_0 through amplitude B_f . The production of f' is proportional to amplitude B_I .

Differently from the case $D_s \rightarrow K\bar{K}\pi$ (see Fig. 3), in $D_s \rightarrow 3\pi$ there are two interfering amplitudes (see Fig. 4)

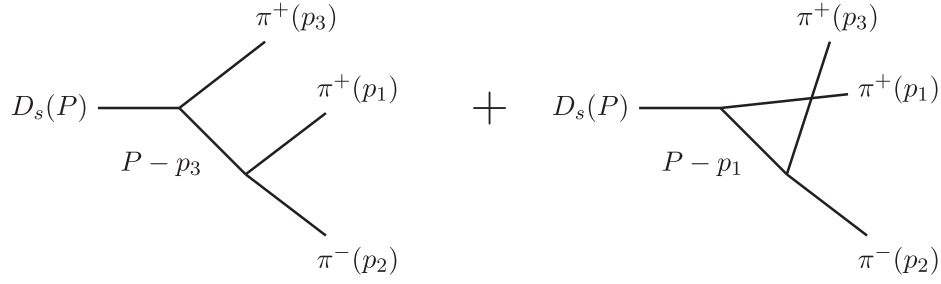
The amplitude M in (3.1) has to be substituted by the sum of the two unstable particle propagators

$$M = \frac{1}{m_f^2 - m_{12}^2 - i\gamma} + \frac{1}{m_f^2 - m_{23}^2 - i\gamma} \quad (4.1)$$

where again $\gamma = m_f\Gamma_f$. This is included in (3.2) to determine the rates of interest, with the integration measure in (3.2) divided by 2, for the identical particles.

First of all we can determine A_I by requiring that $\Gamma(D_s \rightarrow \sigma\pi \rightarrow 3\pi) : \Gamma(D_s \rightarrow f_0\pi \rightarrow 3\pi) \lesssim 0.1$ as reported by LHCb [3]. The rate in the denominator is determined by the B_f amplitude determined in the previous section, and

³We will compute only ratios of widths in the following.

FIG. 4. D_s decay into three pions via scalar mesons.TABLE V. Predicted rates, normalized to the rate of $f_0(980) \rightarrow \pi^+\pi^+\pi^-$ and observed branching fractions.

Channel	Predicted rates/Rate ($f_0(980)$)	Experimental [12]
$\pi^+\sigma(500)$	< 0.1 assumed for A_I	Not reported
$\pi^+f_0(980)$	1	$0.565 \pm 0.043 \pm 0.047$
$\pi^+f_0(1370)$	0.8	$0.324 \pm 0.077 \pm 0.017$
$\pi^+f_0(1710)$	1×10^{-2}	Not reported

we obtain

$$|A_I| \leq 15.4. \quad (4.2)$$

For the other channels of $D_s \rightarrow 3\pi$, we predict, see Table V,

$$\Gamma(D_s \rightarrow f'\pi \rightarrow 3\pi) : \Gamma(D_s \rightarrow f_0\pi \rightarrow 3\pi) \simeq 0.8 \quad (4.3)$$

$$\Gamma(D_s \rightarrow f''\pi \rightarrow 3\pi) : \Gamma(D_s \rightarrow f_0\pi \rightarrow 3\pi) \simeq 0.01. \quad (4.4)$$

The presence of a $u\bar{u} + d\bar{d}$ meson such as f' in the D_s^+ , Cabibbo allowed, decays with final states dominated by $s\bar{s}$ quark pairs, is quite remarkable.

In our scheme, f' production arises quite naturally from the diagrams of Figs. 2(b) and 2(c) through its instanton generated four-quark component, see Eqs. (2.17) and (2.19). In the molecular model, it would require a sizeable violation of the Zweig rule, similar to the violation required to explain the large $f_0 \rightarrow \pi\pi$ decay of the $K\bar{K}$ molecule.

V. CABIBBO SUPPRESSED S-WAVE DECAY:

$$D^+ \rightarrow \pi^+\pi^+\pi^-$$

We have a different set of quark diagrams for this case. They are reported in Figs. 5 and 6. The basic features of these diagrams are the following:

- (1) In (a), f' is produced (amplitude A_f) whereas f_0 is produced via mixing (amplitude A_I).
- (2) In (b), σ is produced (amplitude B_f) and, via mixing, f'' (amplitude B_I).
- (3) The amplitude in (c) is shared between f_0 (amplitude B_f) and a_0 , which, however, does not decay in $\pi^+\pi^-$.
- (4) The diagram in (c) gives rise to the most important contribution to f' (amplitude B_I).

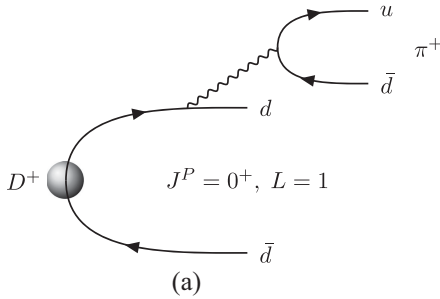
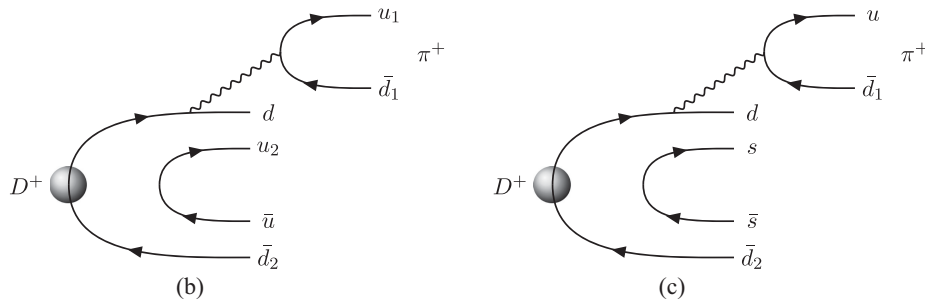
FIG. 5. D decay, Cabibbo forbidden.FIG. 6. D decay, Cabibbo forbidden, with a quark pair from the vacuum. A $u\bar{u}$ is created in (b) and a $s\bar{s}$ pair is created in (c).

TABLE VI. $D^+ \rightarrow 3\pi$. Predicted rates, normalized to the rate of $\sigma(500) \rightarrow \pi^+\pi^+\pi^-$ and observed branching fractions.

Channel	Predicted rates/Rate ($\sigma(500)$)	Expt. (10^{-3}) [12,19]
$\pi^+\sigma(500)$	1	1.38 ± 0.1
$\pi^+f_0(980)$	0.17	0.157 ± 0.032
$\pi^+f_0(1370)$	0.07	0.08 ± 0.04
$\pi^+f_0(1710)$	0.02	Not reported

We will now use the effective couplings

$$g_0 = B_f \sqrt{m_D} \quad \text{for } \sigma$$

$$g_0 = (B_f/\sqrt{2} \pm A_I/\sqrt{2})\sqrt{m_D} \quad \text{for } f_0 \quad (5.1)$$

$$g_0 = (B_I/\sqrt{2} \pm A_f/\sqrt{2})\sqrt{m_D} \quad \text{for } f'$$

$$g_0 = B_I \sqrt{m_D} \quad \text{for } f'' \quad (5.2)$$

where the amplitudes A, B have been determined in the previous section. The following ratios are determined by taking the four sign alternatives in (5.1) and (5.2). Taking the $-$ sign in both (5.1) and (5.2) we obtain

$$\Gamma(D \rightarrow f_0 \pi \rightarrow 3\pi) : \Gamma(D \rightarrow \sigma \pi \rightarrow 3\pi) \simeq 0.17 \quad (5.3)$$

$$\Gamma(D \rightarrow f' \pi \rightarrow 3\pi) : \Gamma(D \rightarrow \sigma \pi \rightarrow 3\pi) \simeq 0.07 \quad (5.4)$$

$$\Gamma(D \rightarrow f'' \pi \rightarrow 3\pi) : \Gamma(D \rightarrow \sigma \pi \rightarrow 3\pi) \simeq 0.01 \quad (5.5)$$

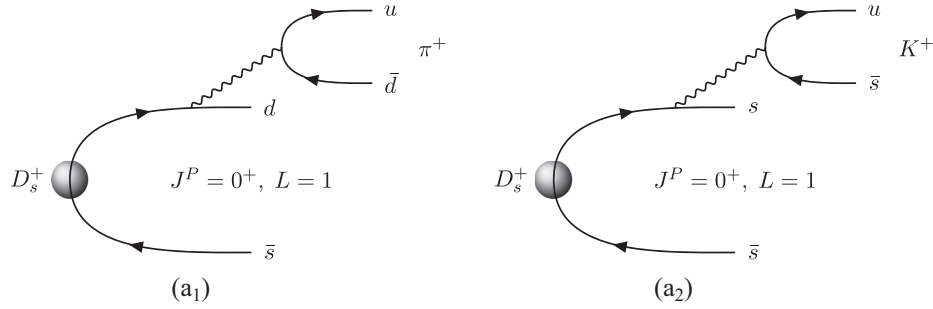


FIG. 7. D_s Cabibbo suppressed quark diagrams producing a π^+ in (a_1) and a K^+ in (a_2) .

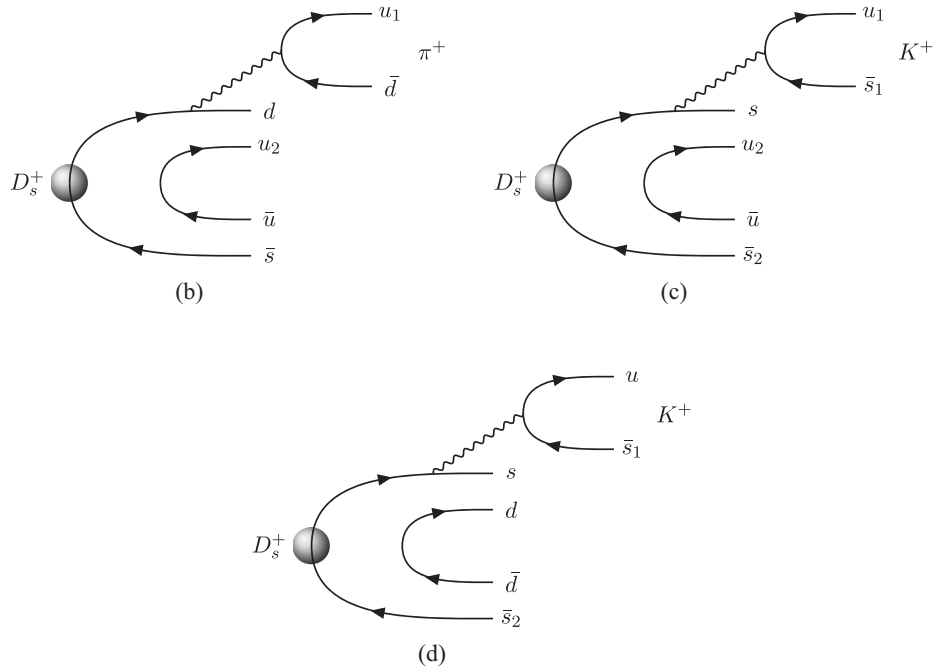


FIG. 8. D_s Cabibbo suppressed quark diagrams with pair production from the vacuum.

which happens to match very well the available experimental determination based on CLEO [19] data, as detailed in the following Table VI.⁴

Recent Dalitz plots from the LHCb experiment [4] suggest that the signals for σ and f_0 may have comparable intensities. This observation could potentially lead to a preference for a different sign choice in the Eqs. (5.1) and (5.2). However, as of now, there is no LHCb determination available to replace the CLEO values in the Table VI.

VI. CABIBBO SUPPRESSED S-WAVE DECAYS

$$D_s^+ \rightarrow K^+ \pi^+ \pi^-$$

The relevant quark diagrams are reported in Figs. 7 and 8. These are the main features to take into account

- (1) (a_1) produces $K_0(1430)$ via A_f , and κ via A_I .
- (2) (a_2) produces f'' via A_f and σ via A_I
- (3) (b) produces κ via B_f and $K_0(1430)$ via B_I .
- (4) (c),(d) produce f_0 via $\sqrt{2}B_f$ and f' via $\sqrt{2}B_I$ —compare to (3.4) and (3.5).

Using the same methods as in the previous sections we find

$$\Gamma(D \rightarrow \sigma K \rightarrow \pi\pi K): \Gamma_0 \simeq 0.05 \quad (6.1)$$

$$\Gamma(D \rightarrow \kappa\pi \rightarrow \pi\pi K): \Gamma_0 \simeq 0.28 \quad (6.2)$$

$$\Gamma(D \rightarrow f' K \rightarrow \pi\pi K): \Gamma_0 \simeq 0.43 \quad (6.3)$$

$$\Gamma(D \rightarrow f'' K \rightarrow \pi\pi K): \Gamma_0 \simeq 0.001 \quad (6.4)$$

$$\Gamma(D \rightarrow K_0(1430)\pi \rightarrow \pi\pi K): \Gamma_0 \simeq 0.31 \quad (6.5)$$

where we have used, in the order

$$g_0 = A_I \sqrt{m_{D_s}}, \quad (B_f - A_I) \sqrt{m_{D_s}}, \\ B_I \sqrt{2m_{D_s}}, \quad A_f \sqrt{m_{D_s}}, \quad (B_I - A_f) \sqrt{m_{D_s}}. \quad (6.6)$$

Ratios are taken with respect to Γ_0

$$\Gamma_0 = \Gamma(D \rightarrow f_0 K \rightarrow \pi\pi K) \quad (6.7)$$

where $g_0 = B_f \sqrt{2m_{D_s}}$. These results are compared to experimental data from [2] in Table VII, employing the same sign convention [see Eq. (6.6)] as utilized in the preceding section. No alternative sign convention facilitates a more favorable comparison to the data. We suspect that a significant portion of this discrepancy may be attributable to the fact that the κ is not even incorporated into the experimental fits.

⁴The $(+, +)$, $(+, -)$, $(-, +)$, $(-, -)$ sign choices correspond to values $(0.54, 0.56)$, $(0.54, 0.07)$, $(0.17, 0.56)$, $(0.17, 0.07)$, respectively.

TABLE VII. Predicted rates for $D^+ \rightarrow K^+ \pi^+ \pi^-$, normalized to the f_0 rate. The comparison of our predictions with data from [2,12] is not good despite the rather large experimental errors. Also, unlike previous cases, BESIII fits the low energy $K^+ \pi^-$ data by a continuum distribution, rather than the Breit-Wigner corresponding to the κ resonance, as would be required by our model.

Channel	Predicted rates/Rate ($f_0(980)$)	Expt. BR(10^{-4}) [2,12]
$K^+ \sigma(500)$	0.05	4.5 ± 3.0
$K^+ f_0(980)$	1	2.8 ± 1.1
$K^+ f_0(1370)$	0.43	12 ± 6
$K^+ f_0(1710)$	0.001	Not reported
$\pi^+ \kappa(700)$	0.28	Not considered in fit
$\pi^+ K_0(1430)$	0.31	9.4 ± 3.2

VII. CONCLUSIONS

In [7], it is proposed that 6-fermion instanton-induced interactions can significantly contribute to the dynamics of scalar mesons at low energies. We further investigate this possibility within the context of recently obtained LHCb experimental data on D and D_s decays into three pseudo-scalars. We substantially corroborate this picture, yet we encounter difficulties with the Cabibbo suppressed S -wave decays $D_s^+ \rightarrow K^+ \pi^+ \pi^-$ that may be attributed to the treatment of the $\kappa(700)$ resonance in data analyses or to characteristics of the model beyond our control. We believe that the present study represents a step in favor of a tetraquark interpretation of the lowest scalar meson nonet.

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX: RULES FOR AMPLITUDES

We summarize the rules for constructing the amplitudes of different processes, from the basic amplitudes A_f , A_I , B_f , B_I defined in Secs. III and IV. Diagrams in Sec. III:

- (1) Diagram Fig. 1(a): An f'' is produced from an $s\bar{s}$ pair through the A_f amplitude.
- (2) Diagram Fig. 2(b): The quark combination $[su][\bar{s}\bar{u}]$ produces a $f_0 = ([su][\bar{s}\bar{u}] + [sd][\bar{s}\bar{d}])/\sqrt{2}$ through the amplitude $B_f/\sqrt{2}$. If the same quark combination is

supposed to produce f' it will go through the instanton amplitude $B_I/\sqrt{2}$. Same for diagram Fig. 2(c).

Diagrams in Sec. IV: Same quark diagrams of Sec. III, same amplitudes to produce f'' and f_0, f' but now also σ is produced in Fig. 1(a) with amplitude A_I . Diagrams in Sec. V

- (1) Diagram Fig. 5(a): Gives f' through $A_f/\sqrt{2}$ and f_0 through $A_I/\sqrt{2}$.
- (2) Diagram of Fig. 6(b): The quark combination $[ud][\bar{u}\bar{d}]$ produces the σ through the B_f amplitude and f'' through the amplitude B_I .

- (3) Diagram of Fig. 6(c): Gives f_0 through the amplitude $B_f/\sqrt{2}$ and f' through the amplitude $B_I/\sqrt{2}$. Diagrams in Sec. VI:

- (1) Diagram Fig. 7(a1): Gives $K_0(1430)$ via $d\bar{s}$ through A_f and κ via A_I .
- (2) Diagram Fig. 7(a2): Gives f'' via A_f and σ via A_I .
- (3) Diagram Fig. 8(b): From $[ud][\bar{u}\bar{s}]$ gives κ via B_f and $K_0(1430)$ via B_I .
- (4) Diagram Fig. 8: (c) $f_0/\sqrt{2}$ through the amplitude $B_f/\sqrt{2}$ and $f'/\sqrt{2}$ through $B_I/\sqrt{2}$. Same for diagram (d).

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| <p>[1] M. Ablikim <i>et al.</i> (BESIII Collaboration), <i>Phys. Rev. D</i> 104, 012016 (2021).</p> <p>[2] M. Ablikim <i>et al.</i> (BESIII Collaboration), <i>J. High Energy Phys.</i> 08 (2022) 196.</p> <p>[3] R. Aaij <i>et al.</i> (LHCb Collaboration), <i>J. High Energy Phys.</i> 07 (2023) 204.</p> <p>[4] R. Aaij <i>et al.</i> (LHCb Collaboration), <i>J. High Energy Phys.</i> 06 (2023) 044.</p> <p>[5] R. L. Jaffe, <i>Phys. Rev. D</i> 15, 281 (1977).</p> <p>[6] L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, <i>Phys. Rev. Lett.</i> 93, 212002 (2004).</p> <p>[7] G. 't Hooft, G. Isidori, L. Maiani, A. D. Polosa, and V. Riquer, <i>Phys. Lett. B</i> 662, 424 (2008).</p> <p>[8] A. H. Fariborz, R. Jora, and J. Schechter, <i>Phys. Rev. D</i> 77, 094004 (2008).</p> <p>[9] G. 't Hooft, arXiv:hep-th/9903189.</p> | <p>[10] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, <i>Nucl. Phys.</i> B163, 46 (1980).</p> <p>[11] E. Borchini and R. Gatto, <i>Phys. Lett.</i> 14, 352 (1965).</p> <p>[12] S. Navas <i>et al.</i> (Particle Data Group), <i>Phys. Rev. D</i> 110, 030001 (2024).</p> <p>[13] C. Amsler and F. E. Close, <i>Phys. Rev. D</i> 53, 295 (1996).</p> <p>[14] X. y. Shen (BES Collaboration), arXiv:hep-ex/0209031.</p> <p>[15] F. E. Close and Q. Zhao, <i>Phys. Rev. D</i> 71, 094022 (2005).</p> <p>[16] F. E. Close and N. A. Tornqvist, <i>J. Phys. G</i> 28, R249 (2002).</p> <p>[17] D. Black, A. H. Fariborz, and J. Schechter, <i>Phys. Rev. D</i> 61, 074001 (2000).</p> <p>[18] A. Ali, L. Maiani, and A. Polosa, <i>Multiquark Hadrons</i> (Cambridge University Press, Cambridge, England, 2019).</p> <p>[19] G. Bonvicini <i>et al.</i> (CLEO Collaboration), <i>Phys. Rev. D</i> 76, 012001 (2007).</p> |
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