

Three-field string inflation with perturbative corrections: Dynamics and implications

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In this work, we construct an explicit string motivated example of three-field inflation related to, yet distinct from the recently discovered perturbative large volume scenario (pLVS). In this setup, large volume is ensured by the interplay between the effects of α'^3 , logarithmic loop, and higher derivative F^4 corrections. After addressing full moduli stabilization, we move on to a detailed analysis of a three-field model of inflation while choosing a suitable canonical basis for the fields. Our model differs from previous setups in three key aspects: first, the interaction between subleading corrections that drive full moduli stabilization is different compared to pLVS for instance; second, the volume of the underlying Calabi-Yau is of “weak-Swiss-cheese” type; and third, in our three-field inflationary scenario, the second Hubble slow-roll parameter consistently dominates over the first by several orders of magnitude—primarily due to the steepness of the potential. When this hierarchy is aided by a brief deviation from slow roll, an enhancement in the primordial power spectrum can be naturally achieved. We have obtained two stages of inflation: the first stage mostly occurs when one of the inflatons roll along the steepest direction—completing 55 e-folds of inflation. Once the rolling inflaton falls off a ridge of the multifield potential, the other two fields become active, giving us a true multifield behavior in the second stage—adding few more e-folds of inflation. Inflation ends when all three of them roll toward the global minimum and settle there. We confirm this behavior by introducing the nonplanar torsion in the inflationary trajectory—this quantity becomes nontrivial in the second stage of inflation. Finally, we calculate the cosmological observables, which align with the Planck data, and discuss potential directions for future research.

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I. INTRODUCTION

Cosmic inflation refers to a rapid expansion of spacetime that occurred in the early Universe [1–3], which smoothed out any fluctuations in spacetime, resulting in a homogeneous and isotropic cosmos. Our current understanding of the Universe and its evolution is based on the six-parameter Λ -Cold Dark Matter model (Λ CDM) model. The model lacks an explanation for the horizon problem and the observed flatness of our Universe. Inflation can provide an explanation for the above two, besides, it can also give rise to the seed for the structure formation we observe today. In addition, the data from Planck [4] are fully consistent with the simplest inflationary scenario being the leading

mechanism to account for the observed anisotropies in the cosmic microwave background (CMB) radiation. Specifically, they align with two strong predictions of inflation: a nearly scale-invariant power spectrum of density perturbations and a stochastic background of gravitational waves.

According to the simplest inflationary scenario, inflation is driven by a scalar field minimally coupled to gravity, whose potential energy dominates over its kinetic energy, causing it to slowly roll over its potential. Once the kinetic energy begins to dominate, inflation comes to an end, providing sufficient spacetime expansion to resolve the horizon problem. However, recently proposed consistency conjectures [5–9] imposed on the 4D effective field theory (EFT) derived from a quantum theory of gravity, strongly disfavor vanilla single field inflation, although, it was hinted upon in [10] that multifield inflation with a nontrivial field space metric might be able to alleviate this tension.

Moreover, in this decade and the next, various experiments are being launched with the aim of detecting the B-mode polarization in the CMB induced by primordial gravitational waves. They are: CMB-S4 experiment [11],

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CLASS [12], LiteBIRD [13], and the Simon Observatory [14]. Moreover, upcoming large scale structure surveys, including DESI [15], LSST [16], Euclid [17], and SKA [18], may also detect primordial non-Gaussianities, which may unravel potential interactions during the inflationary era [19]. Both from a theoretical and experimental point of view, it thus becomes essential to go beyond the single field vanilla scenario for inflation. In particular, multifield models are generic in supergravity and string theory realizations.

Four-dimensional EFT emerging after dimensional reduction of a superstring theory features various scalar fields, and one/some of them might act as natural candidate/s to drive inflation. This forms the foundation of our three-field inflation model. However, these constructions also have a price to pay; most of the time, the existence of too many scalar fields becomes unavoidable, leading to the so-called cosmological moduli problem (see [20] and the references therein). One also needs to stabilize the scalar fields to prevent the appearance of fifth forces. Our aim in this paper is to display a concrete three-field inflationary scenario where the scalar potential is constructed out of the underlying low-energy effective string theory setup, in particular from a type-IIB flux compactification where the extra dimensions are compactified on a K3 fibered Calabi-Yau (CY). The first step toward string theory model building is to stabilize the scalar fields. Fortunately, string theory provides sufficient tools (such as Dp -branes pierced with magnetized fluxes) to help achieve this goal.

The scalar fields stemming from the closed string sector of type-IIB flux compactifications are of three types: complex structure (U^i) related to the shapes of the CY, Kähler moduli (T_α) related to the size of the CY, and a Universal scalar field known as axio-dilaton (s). Vacuum expectation value (vev) of the imaginary part of axio-dilaton determines the string coupling g_s . After dimensional reduction, in 4D supergravity, the scalar potential can be expressed in terms of a holomorphic superpotential and a real Kähler potential. The Type-IIB superpotential depends on the complex structure moduli and dilaton, however it is independent of the Kähler moduli, resulting in the so-called *no-scale structure*. Hence, to stabilize the Kähler moduli, one has to go beyond and add further quantum corrections, such as perturbative corrections to the Kähler potential and nonperturbative effects at the level of superpotential. Note that the complex structure and axio-dilaton are stabilized at tree level in the presence of fluxes and integrated out.

The most notable candidate models addressing full moduli stabilization are the Kachru-Kalosh-Linde-Trivedi construction (KKLT) [21–30] model and the large volume scenario (LVS) [31–34]. The former utilizes nonperturbative effects to be added in a holomorphic superpotential of the Gukov-Vafa-Witten form. The latter addresses the moduli stabilization by a careful interplay between perturbative effects at the level of real Kähler potential and nonperturbative effects to the superpotential.

This balance in LVS setup helps to achieve an exponentially large volume of the underlying 6D Calabi-Yau manifold spanned by the Kähler moduli.

KKLT and LVS being the main two examples to address full moduli stabilization, recently another class of string vacua also emerged [35–39] where it was shown that exponentially large volume can be achieved without the need of nonperturbative effects—this type of construction is known as pLVS. Nonperturbative effects are very restrictive and very specific toward the underlying CY geometry, additionally they may need to satisfy several constraints such as Witten’s unit arithmetic genus condition [40]. Therefore, if a construction ensures a large internal volume without relying on these effects, it becomes more well motivated. A recent study [41] has provided a global embedding of the pLVS framework and the robustness of the same against various subleading corrections, including leading-order corrections in inverse string tension, higher-derivative corrections, log-loop corrections from intersecting space-filling $D7$ -brane stacks, D-term effects, and Kaluza-Klein as well as winding-type loop corrections.

The present work studies the addition of perturbative effects such as leading α^3 correction [Becker-Becker-Haack-Louis (BBHL) term], corrections of log-loop type and from higher derivative terms, in a weak Swiss-cheese CY. We start deriving the multifield scalar potential and then discuss the stabilization of the scalars putting special emphasis on the higher derivative F^4 effect and the log-loop corrections. Then we move on to discuss their inflationary implications. Inflation takes place in two stages: initially along the steep direction of the potential, and then along the remaining two directions as all the fields move toward their global minimum from an initially displaced position. At this initially displaced position in the scalar potential, the minimum along one of the scalars disappears, producing a steep ridge in that direction. The inflaton then falls in the ridge and oscillates—in this stage the other two fields move relatively slower, giving rise to a quasi-single-field behavior in the first stage. In the later stage, the field that fell into the ridge moves relatively slowly. Since the fields are canonical, the nonplanar motion and deviation from a geodesic occur only when the field temporarily falls into a ridge and oscillates there. The oscillation can not only give rise to high turning rate and torsion, but it can also induce sizable entropic perturbation through transient deviation from slow roll. The backreaction of the entropic modes to the adiabatic modes may produce a significant enhancement in the primordial power spectrum signalling a copious production of primordial black holes. In this lore, it is also useful to study the effect of slow-roll violation in the secondary induced gravitational waves. We aim to come back to the implications of slow-roll violation in a future companion paper.

The layout of the paper is as follows: in Sec. II, we present a short overview of the quantum corrections to be

added in the Kähler potential along with their string origin and their effect in the stabilization of the Kähler moduli. We end the section by displaying several examples of de Sitter vacua. In Sec. III, we introduce the multifield setup where we introduce several concepts like turning rate and torsion. We also discuss their implications at the level of linear perturbation theory. Section IV mainly covers our detailed three-field inflationary analysis. After performing several consistency checks stemming from supergravity and the hierarchy of mass scales, we move on to checking the inflationary predictions dubbed in the value of spectral index, spectral tilt, and the amplitude of power spectrum. We also discuss how turning and torsion can affect them. We conclude with our future directions in Sec. V.

II. TOWARD MODULI STABILIZATION WITH PERTURBATIVE EFFECTS

The perturbative string loop corrections have been recently highlighted in [36] due to their major role in the moduli stabilization mechanism without the need for a nonperturbative effect like the large volume scenario [42]. In this section, we start by summarizing the world-sheet corrections, which are proportional to the Euler characteristic χ of the compactification, followed by an analysis of the origin of string loop corrections within the geometric framework of $D7$ -branes in type IIB string theory. After presenting a detailed derivation of the scalar potential, we shall showcase several examples of dS vacua where the correct mass hierarchy remains preserved.

A. α'^3 corrections

α'^3 corrections [43] are added at the level of the Kähler potential to stabilize the volume modulus $\mathcal{V}$. It acts as an additive factor to the tree-level Kähler potential, giving us

$$\mathcal{K} = -2 \log \left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}} \right). \quad (2.1)$$

This constant shift ξ of the volume is given in terms of the Euler characteristic of the manifold as

$$\xi = -\frac{\zeta(3)}{4(2\pi)^3} \chi. \quad (2.2)$$

B. Log-loop corrections

Now, the origin of string loop corrections can be traced back to higher derivative terms proportional to R^4 that appear in the effective action due to multigraviton scattering of the ten-dimensional theory. To make the discussion more precise, we can express the effective action as [35],

$$\begin{aligned} \mathcal{S}_{\text{grav}} = & \frac{1}{(2\pi)^7 \alpha'^4} \int_{M_4 \times \mathcal{X}_6} e^{-2\phi} \mathcal{R}_{(10)} - \frac{\chi}{(2\pi)^4 \alpha'} \\ & \times \int_{M_4} (-2\hat{\xi}(3)e^{-2\phi} \pm 4\hat{\xi}(2)) R_{(4)}, \end{aligned} \quad (2.3)$$

where ϕ is the string dilaton and $R_{(4)}$ denotes the Ricci scalar in four dimensions and the $\pm$ signs refers to the different type of string theory, type IIA/B theory, respectively [44–46]. In the last step, χ is defined as,

$$\chi = \frac{3}{4\pi^3} \int_{\mathcal{X}_6} R \wedge R \wedge R. \quad (2.4)$$

From the above, it is observed that an additional localized Einstein-Hilbert is induced in four dimensions with a coefficient proportional to χ . These terms correspond to localized gravity vertices in the bulk (in particular at points where $\chi \neq 0$), where their effect is to emit Kaluza-Klein states and gravitons. We assume a background with three intersecting $D7$ -branes, where we expect interactions between the closed string modes and Kaluza-Klein (KK) states in the bulk space. These interactions exhibit in general a logarithmic behavior in the large volume limit transverse to the $D7$ -branes. The new contributions to the $R_{(4)}$ term, which are going to be moduli dependent, can be summarized by the following formula [37]:

$$\delta = \frac{4\zeta(2)}{(2\pi)^3} \chi \int_{M_4} \left(1 - \sum_k e^{2\phi} T_k \ln(R_{\perp}^k/w) \right) R_{(4)}. \quad (2.5)$$

T_k is the tension of a $D7$ -brane, and $R_{\perp}$ stands for the size of the two-dimensional world volume in each brane. Moreover, w parametrizes a “width” utilized as an ultra-violet cutoff for the graviton KK modes.

The low-energy dynamics of the four-dimensional effective supergravity theory, emerging from type IIB superstring compactifications on CY orientifolds, can be described by a holomorphic superpotential (W) and a real Kähler potential (K). The tree-level superpotential and Kähler potential are functions of various chiral coordinates. These chiral coordinates are constructed by writing them as a combination of a modulus with a set of Ramond-Ramond (RR) axions, such as,

$$U^i = v^i - iu^i, \quad s = c_0 + ie^{-\phi}, \quad T_{\alpha} = c_{\alpha} - i\tau_{\alpha}, \quad (2.6)$$

where ϕ is the universal modulus known as dilaton, related to string coupling as $g_s = e^{\langle \phi \rangle}$; u^i is the saxion and v^i is the axion of the complex-structure modulus, τ_{α} 's are the Einstein frame four-cycle volume moduli defined as $\tau_{\alpha} = \partial_{\alpha} \mathcal{V}$, and $\mathcal{V}$ is the overall volume of the CY manifold in Einstein frame expressed as $\mathcal{V} = \frac{1}{3!} \kappa_{\alpha\beta\gamma} t^{\alpha} t^{\beta} t^{\gamma}$. Finally, c_0 and c_{α} 's are the universal RR axion and RR four-form axions, respectively.

The addition of these corrections changes the tree-level Kähler potential to,

$$K = -\log(-i(s - \bar{s})) - 2 \log \left[\mathcal{V} + \frac{\xi}{2} \left(\frac{s - \bar{s}}{2i} \right)^{3/2} + \left(\frac{s - \bar{s}}{2i} \right)^{-1/2} \delta \right],$$

$$\delta = \hat{\eta} \log[\tau_i]. \quad (2.7)$$

C. The geometrical setup

Taking into account the above discussions, we now turn to the derivation of the scalar potential and discuss the CY geometry. The fiber inflation model, proposed more than a decade ago in [42] is promising with respect to the embedding of inflation in string compactifications, since they provide a natural solution to the η problem of inflation [47]. Different variations of fiber inflation models have been proposed, where combinations of string loop corrections, Kaluza-Klein, and winding loops are considered, leading into inflationary potentials with a sufficient plateau-like form.

In all of these models, the presence of nonperturbative effects to stabilize the blow-up model and the overall volume were common. Recently, another aspect of fiber inflation was also proposed where the interplay of perturbative corrections assured a large volume [48] as well as full moduli stabilization. The absence of nonperturbative effects eliminates the need for an exceptional del Pezzo divisor. The novelty of this setup is that the usual drawback of fiber inflation where the field space excursion was bounded by Kähler cone conditions was alleviated. The use of perturbative corrections in the context of inflationary models can also be found in [49] where the perturbative effects were used in the stabilization of the blow-up mode. Later, from the perspective of brane-antibrane inflation, these effects are also proven useful in [50].

In this paper, we follow a similar approach as [48] but consider a different form of the volume, resembling that of K3-fibered Calabi-Yau models like fiber inflation. However, as will become clear in the following subsections, our inflationary predictions deviate significantly from those of the fiber models because we do not utilize nonperturbative effects to stabilize the overall volume and the blow-up modulus. To proceed with our analysis, the compactified space volume is given by,

$$\mathcal{V} = f_{3/2}(\tau_i) - \sum_j^{N_{rg}} \lambda_j \tau_j^{3/2} \quad i = 1, \dots, N, \quad (2.8)$$

where the $f_{3/2}$ is a homogeneous function of the Kähler moduli. Despite lacking a complete global embedding for our approach, we are going to adapt the following

volume form:

$$\mathcal{V} = \frac{1}{\sqrt{2\alpha}} \sqrt{\tau_7} \tau_6 - \frac{1}{3} \tau_1^{3/2}, \quad (2.9)$$

where α is a model dependent parameter, parametrizing the D-term constraint. Given this form, we cannot *a priori* characterize the hierarchy between the moduli since the log-loop corrections might affect the anticipated result for $\tau_1 \ll \tau_{6,7}$. Nevertheless, we are obliged to respect the requirement of having a large volume. It would be worth searching through the Kreuzer-Skarke list of Calabi-Yau manifolds embedded in toric varieties [51,52] for some representative examples where our geometric configuration of intersection $D7$ could lead to large log-loop corrections. τ_1 is the blow-up mode, τ_7 is the usual fiber modulus—the volume of the K3 manifold, and τ_6 is the volume of the $\mathbb{P}^2$ base. Their cumulative effect ensures that the model remains within the large volume scenario during inflation by adjusting their values.

D. The effective scalar potential and moduli stabilization

Having this volume-form as a starting point, our analysis will be focused on the addition of a BBHL term and logarithmic corrections in the Kähler potential without considering any nonperturbative corrections. The large volume in this model is ensured by a cumulative effect of the perturbative corrections. The Kähler potential upon the insertion of these corrections can be written as,

$$\mathcal{K} = -\log \left[\frac{s - \bar{s}}{i} \right] - 2 \log \left[\mathcal{V} + \frac{\xi}{2} \left(\frac{s - \bar{s}}{2i} \right)^{3/2} + \left(\frac{s - \bar{s}}{2i} \right)^{-1/2} \sum_i \hat{\eta}_i \log(\tau_i) \right] \quad (2.10)$$

where τ_i stands for the four-cycle moduli, related to the volume of the CY as $\mathcal{V} \sim \tau^{3/2}$, ξ denotes the α' correction term, and the logarithmic corrections $\sim \hat{\eta} \log(\dots)$ are induced along each four-cycle volume moduli. One important last observation we have to make is the fact that the stabilization will completely depend on perturbative corrections. The nonperturbative corrections, which will impose stringent limitation in our construction, could be prevented by considering certain world-volume fluxes that lift fermionic zero modes leading to zero contribution from these corrections [53–56]. Taking this into account, in this work, we will explore the case where τ_1 no longer remains at small values, but it is stabilized to large values but never taking us away from the domain of large volume. Despite the fact that this approach diverges from the typical paradigm of fibered models, our proposal focuses on the implications on the logarithmic correction on the weak-Swiss-cheese CY geometry of (2.9).

Regarding the complex structure moduli U^i as well as the axion dilaton, they are stabilized at tree level by the supersymmetric conditions,

$$D_{U^i} W = 0 = D_{\bar{U}^i} \bar{W}, \quad D_s W = 0 = D_{\bar{s}} \bar{W}. \quad (2.11)$$

We will also set,

$$\mathcal{W}_0 = \left\langle \int_{X_6} G_3 \wedge \Omega \right\rangle \quad (2.12)$$

where $G_3 = F_3 + isH_3$ with F and H denoting RR and Neveu-Schwarz–Neveu-Schwarz (NSNS) 3-form fluxes, respectively. Ω is the holomorphic $(3, 0)$ -form, which depends on the complex structure moduli U^i . Since the scalar potential is parametrized by the three moduli (τ_1, τ_7, τ_6) , we would like to trade one of them in order to introduce the volume $\mathcal{V}$ in the computations. This reparametrization helps us to perform the large volume expansion. So, in order to include the overall volume in our computations, we solve the Eq. (2.9) with respect to one modulus, e.g., τ_6 ,

$$\tau_6 = \frac{\sqrt{2\alpha}(\tau_1^{3/2} + 3\mathcal{V})}{3\sqrt{\tau_7}}. \quad (2.13)$$

Using the above definition, we are going to present the necessary computations needed to define the effective F-term scalar potential. Moreover, following [39], the functions in the Kähler potential can be written as follows:

$$Y = \mathcal{V} + \frac{\hat{\xi}}{2} + \hat{\eta} \log \left(\frac{2\alpha\tau_7}{9} (\tau_1^{5/2} + 3\mathcal{V}\tau_1)^2 \right), \quad (2.14)$$

where we have used the fact that the axio-dilaton s and the free parameters of the model $(\hat{\xi}, \hat{\eta})$ are recasted as,

$$s = c_0 + ie^{-\phi}, \quad d = e^{-\phi} \Rightarrow \hat{\xi} = \xi d^{3/2}, \quad \hat{\eta} = \eta d^{-1/2}, \quad d = \frac{1}{g_s}. \quad (2.15)$$

Imposing these redefinitions in (2.14), the Kähler potential can be rewritten as,

$$\mathcal{K} = -\log \left[\frac{s - \bar{s}}{i} \right] - 2\log[Y]. \quad (2.16)$$

The scalar potential can be computed by the $\mathcal{N} = 1$ supergravity formula,

$$e^{-\mathcal{K}} V_k = K^{A\bar{B}} (D_A W) (D_{\bar{B}} \bar{W}) - 3|W|^2, \quad W = \mathcal{W}_0. \quad (2.17)$$

The first term of the scalar potential (2.17) will be displayed below, where we expand in terms of $\mathcal{O}(\hat{\eta}, \hat{\xi}/\mathcal{V})$ keeping only the first order terms,

$$K^{A\bar{B}} (D_A W) (D_{\bar{B}} \bar{W}) = 3g_s \mathcal{W}_0^2 \left(1 + \frac{\hat{\xi} - 16\hat{\eta} + \hat{\eta}w}{4\mathcal{V}} + \frac{\hat{\xi}\hat{\eta}(1-2w)}{2\mathcal{V}^2} \right) + \mathcal{O}\left(\frac{\hat{\xi}}{\mathcal{V}^n}\right), \quad (2.18)$$

where we define $w = \log(\frac{2\alpha\tau_7}{9} (3\mathcal{V}\tau_1 + \tau_1^{5/2})^2)$. It is obvious that the second term in (2.17) cancels the first term above, and by turning off $\hat{\eta} \rightarrow 0$, it results in,

$$e^{-\mathcal{K}} V_{\text{eff}} = \frac{3g_s \mathcal{W}_0^2 \hat{\xi}}{4\mathcal{V}} + \mathcal{O}\left(\frac{\hat{\xi}}{\mathcal{V}^n}\right). \quad (2.19)$$

Based on the above, the effective scalar potential can be easily derived and it is shown below,

$$V_{\text{eff}} = \frac{3g_s \mathcal{W}_0^2 (\hat{\xi} - 16\hat{\eta} + \hat{\eta}w)}{4\mathcal{V}^3} + \mathcal{O}\left(\frac{\hat{\xi}}{\mathcal{V}^n}, \hat{\eta}^n\right). \quad (2.20)$$

As discussed in [42,57], it is a unique feature of these models to preserve one flat direction. In [42,57], their approach is to stabilize first the internal volume and the blow-up divisor with an interplay of perturbative and nonperturbative effects, while the τ_7 remains flat. Our approach differs from the aforementioned due to the fact that the logarithmic corrections will be made sufficient to perform the stabilization while keeping track of the remaining flat direction. The previous studies of this kind of approach require only keeping terms that scale as $\sim \frac{1}{\mathcal{V}^3}$, but now we shall prove that even terms proportional to $\sim \frac{\hat{\eta}\hat{\xi}}{\mathcal{V}^4}$ could be significant. Firstly by inspecting the effective potential (2.20), we stabilize with respect to $\mathcal{V}$ while considering $\mathcal{V} \gg \tau_1$,

$$\begin{aligned} \partial_{\mathcal{V}} V_{\text{eff}} &= \frac{9\mathcal{W}_0^2 g_s}{4\mathcal{V}^4} \left(-\hat{\eta} \log \left(\frac{2\alpha\tau_7}{9} \right) - \hat{\xi} + 2\hat{\eta} \left(\frac{\mathcal{V}}{\tau_1^{3/2} + 3\mathcal{V}} + 8 \right) \right. \\ &\quad \left. - 2\hat{\eta} \log(\tau_1^{5/2} + 3\tau_1 \mathcal{V}) \right) = 0, \\ &\Rightarrow \mathcal{V} \cong \frac{e^{\frac{1}{6}(-3\log(\frac{2\alpha\tau_7}{9}) - \frac{3\hat{\xi}}{\hat{\eta}} + 50)} - \tau_1^{5/2}}{3\tau_1}. \end{aligned} \quad (2.21)$$

By substituting the above minimum in the scalar potential, the result is,

$$V_{\text{eff}}|_{\mathcal{V}_{\min}} = -\frac{27\sqrt{2}\hat{\eta}\tau_1^3 \mathcal{W}_0^2 (\alpha\tau_7)^{3/2} e^{\frac{3\hat{\xi}}{2\hat{\eta}} g_s}}{(\sqrt{2}\sqrt{\alpha\tau_1^5 \tau_7 e^{\frac{\hat{\xi}}{2\hat{\eta}}} - 3e^{25/3}})^3}, \quad (2.22)$$

where we can see that $\tau_{1,7}$ directions still remain flat. To remedy this, we now include the next to leading order term,

$$V_{\text{eff}} = \frac{3g_s \mathcal{W}_0^2 (\hat{\xi} - 16\hat{\eta} + \hat{\eta}w)}{4\mathcal{V}^3} - \frac{3g_s \mathcal{W}_0^2 \hat{\eta} \hat{\xi}(w)}{\mathcal{V}^4} + \mathcal{O}\left(\frac{\hat{\xi}^n}{\mathcal{V}^n}, \hat{\eta}^n\right). \quad (2.23)$$

This new correction will not modify the minimum with respect to the volume (at least to a degree that would lead to destabilization), since it is suppressed by $\sim \frac{1}{\mathcal{V}^4}$. Nevertheless, it would slightly lift the contributions to the transverse directions unraveling the importance of quantum corrections to the stabilization of the flat direction. So, the effective potential at $\mathcal{V}_{\min}$ is,

$$V_{\text{eff}}|_{\mathcal{V}_{\min}} = \frac{27\tau_1^4 \mathcal{W}_0^2 g_s \left(\frac{3\sqrt{2}\hat{\eta}e^{\frac{25}{3}-\frac{\hat{\xi}}{2\hat{\eta}}}}{\tau_1 \sqrt{\alpha\tau_7}} + 12\hat{\xi}(3\hat{\xi} - 50\hat{\eta}) - 2\hat{\eta}\tau_1^{3/2} \right)}{4(e^{\frac{1}{6}(-3\log(\frac{2\alpha\tau_7}{9}) - \frac{3\hat{\xi}}{\hat{\eta}} + 50)} - \tau_1^{5/2})^4}, \quad (2.24)$$

where minimizing with respect to τ_7 , there exists one extremum at,

$$\tau_7 = \frac{9e^{\frac{50}{3}-\frac{\hat{\xi}}{\hat{\eta}}}}{2\alpha(12\hat{\xi}\tau_1 + \tau_1^{5/2})^2}. \quad (2.25)$$

Given the above extremum, we achieve a saddle at,

$$V_{\text{eff}}|_{\mathcal{V}_{\min}}^{\langle\tau_7\rangle} = \frac{3\mathcal{W}_0^2 (\hat{\xi} - 16\hat{\eta})g_s}{256\hat{\xi}^3} > 0. \quad (2.26)$$

Despite the fact that the effective potential is positive at the minimum given $\hat{\xi} > 16\hat{\eta}$, not all the directions are stabilized. As we can see in (2.25), the extremum along τ_7 depends on τ_1 , which is yet to be fixed. Thus, we are obliged to investigate the possibility of additional corrections, whose effects could achieve this. The following correction can also be induced as an addition to the leading order α^3 corrections. They are summarized in the following contributions:

Higher-derivative string corrections: They appear at the level of α^3 corrections multiplying an F^4 term and they modify the 4D EFT scalar potential as [58],

$$V_{F^4} = -k^2 \frac{|\lambda| \mathcal{W}_0^4}{g_s^{3/2} \mathcal{V}^4} \Pi_i t^i, \quad (2.27)$$

where Π_i is a topological number depending on the intersection of the divisors and λ is an unknown combinatorial factor.

1. Inflationary potential and moduli stabilization

In the present model, we are going to utilize this subleading correction and study the implications both on

the stabilization procedure and on the inflationary analysis,

$$V_{\text{eff}} = \frac{3g_s \mathcal{W}_0^2 (\hat{\xi} - 16\hat{\eta} + \hat{\eta}w)}{4\mathcal{V}^3} - \frac{3g_s \mathcal{W}_0^2 \hat{\eta} \hat{\xi}(w)}{\mathcal{V}^4} + \frac{c}{\mathcal{V}^4} (t_1 + t_6 + t_7) + V_{\text{up}} + \mathcal{O}\left(\frac{1}{\mathcal{V}^5}\right), \quad (2.28)$$

where the c parameter quantifies the overall factor of F^4 corrections depending on the string coupling g_s and the fluxes $\mathcal{W}_0$. In addition, we have added a small constant V_{up} , which will be used to obtain an almost Minkowski/dS vacuum. Some efforts toward carefully computing this uplifting term have been analyzed in [59,60]. Now, for concreteness, we should stress the fact that winding loop corrections and KK corrections should also be considered in a concrete model as has been stressed in [41,48], but since we are agnostic about the value of their coefficients, in the current work we only study the case where only F^4 corrections are enough to stabilize the flat directions and produce a dS-shallow potential.

The relation between the two-cycle moduli and the four-cycle moduli is summarized below and it can be used to express the F^4 correction,

$$t_1 = \sqrt{\tau_1}, \quad t_7 = \frac{1}{\sqrt{2\alpha}} \frac{\tau_6}{\sqrt{\tau_7}}, \quad t_6 = \frac{1}{\sqrt{2\alpha}} \sqrt{\tau_7}. \quad (2.29)$$

$$V_{F^4} = \frac{c}{\mathcal{V}^4} \left(\frac{\sqrt{\tau_7}}{\sqrt{2\alpha}} + \sqrt{\tau_1} + \frac{\tau_1^{3/2} + 3\mathcal{V}}{3\tau_7} \right), \quad c = -k^2 \frac{|\lambda| \mathcal{W}_0^4}{g_s^{3/2}} \Pi_i. \quad (2.30)$$

Based on this addition, one can prove that the flat direction along $\{\tau_1, \tau_7\}$ is lifted. In order to connect this section with the inflationary analysis in the next section, it is easier to express the potential in the normalized field basis spanned by $(\varphi_1, \varphi_2, \varphi_3)$. To do so, we need to write down the leading order terms of the Kähler metric, which are,

$$K_{ij} = \begin{pmatrix} \frac{1}{4\tau_7^2} & 0 & 0 \\ 0 & \frac{1}{2\tau_6^2} & 0 \\ 0 & 0 & \frac{\sqrt{\alpha\tau_7}}{4\sqrt{2}\sqrt{\tau_1}\tau_6\tau_7} \end{pmatrix}. \quad (2.31)$$

Given the above form, the kinetic term is now written as,

$$K_{ij} \partial T_i \partial \bar{T}_j = \sum_i \frac{1}{2} (\partial \varphi_i)^2 + \dots \quad (2.32)$$

The new relation between the old and new bases where the off-diagonal entries are found small in the $\varphi \sim \mathcal{O}(1)$ limit and ignored, are the following:

$$\tau_7 = e^{\sqrt{2}\varphi_1}, \quad \tau_6 = e^{\varphi_2}, \quad (2.33)$$

$$\tau_1 = \left(\frac{3^4 \varphi_3^4 e^{(\sqrt{2}\varphi_1 + 2\varphi_2)}}{2^5 \alpha} \right)^{1/3}, \quad \mathcal{V} = \frac{(4 - 3\varphi_3^2) e^{\frac{\varphi_1}{\sqrt{2}} + \varphi_2}}{4\sqrt{2}\sqrt{\alpha}}. \quad (2.34)$$

Finally, we display the scalar potential in the new basis as

$$\begin{aligned} V_{\text{eff}} = V_{\text{up}} & - \frac{1024\alpha^2 \hat{\xi} \hat{\eta} g_s \mathcal{W}_0^2 e^{-2\sqrt{2}\varphi_1 - 4\varphi_2} \left(\log \left(\frac{9^4 \varphi_3^8}{2^{10} \alpha^2} \right) + 8\sqrt{2}\varphi_1 + 10\varphi_2 \right)}{(4 - 3\varphi_3^2)^4} \\ & - \frac{32\sqrt{2}\alpha^{3/2} g_s \mathcal{W}_0^2 e^{-\frac{3\varphi_1}{\sqrt{2}} - 3\varphi_2} (\hat{\eta} (\log(\frac{9^4 \varphi_3^8}{2^{10} \alpha^2}) + 8\sqrt{2}\varphi_1 + 10\varphi_2 - 48) + 3\hat{\xi})}{(3\varphi_3^2 - 4)^3} \\ & + \frac{512 \cdot 2^{1/6} \alpha^{3/2} c e^{-\frac{5\varphi_1}{\sqrt{2}} - 4\varphi_2} ((9\alpha\varphi_3^2 e^{2\sqrt{2}\varphi_1 + \varphi_2})^{1/3} + 2^{1/3} (e^{\sqrt{2}\varphi_1} + e^{\varphi_2}))}{(4 - 3\varphi_3^2)^4}. \end{aligned} \quad (2.35)$$

Note that the potential carries an asymptote along the φ_3 direction at $\varphi_3 = \sqrt{4/3}$. The implications of this will have a clear effect on the inflationary analysis as will be shown in Sec. IV.

Due to the complexity of the above potential, we can reduce its form by considering an expansion with respect to a small parameter y . One can readily see in the above form, that there is a factor of $4 - 3\varphi_3^2$, which can be recasted to,

$$\varphi_3^2 = \frac{y + 4}{3}. \quad (2.36)$$

Given the definitions in (2.34), we can see that φ_3 is bounded, since we want a positive volume. One can deduce that the y parameter has to be small in order to satisfy the above constrain. So, we can perform an expansion with respect to this parameter. The leading order terms of this expansion result into,

$$\begin{aligned} V_{\text{eff}}^{\text{appr}} & = \frac{32\alpha^{3/2} e^{-\frac{5\varphi_1}{\sqrt{2}} - 4\varphi_2}}{3y^4} [48\sqrt{2}c(e^{\sqrt{2}\varphi_1} + e^{\varphi_2}) + 4 \times 2^{5/6} (3\alpha)^{1/3} c(y + 12) e^{\frac{1}{3}(2\sqrt{2}\varphi_1 + \varphi_2)} \\ & - 3\mathcal{W}_0^2 y g_s e^{\sqrt{2}\varphi_1 + \varphi_2} (2\hat{\eta}(\sqrt{2}(-\log(2\alpha) + 5\varphi_2 - 24) + 8\varphi_1) + 3\sqrt{2}\hat{\xi}) \\ & - 96\sqrt{\alpha}\hat{\xi}\hat{\eta} e^{\frac{\varphi_1}{\sqrt{2}}} \mathcal{W}_0^2 g_s (-2\log(2\alpha) + 8\sqrt{2}\varphi_1 + 10\varphi_2 + y)] + V_{\text{up}}. \end{aligned} \quad (2.37)$$

We could further simplify our formula considering the regime where the α parameter is small $\alpha \ll 1$. The simpler formula for the effective potential is then given by,

$$\begin{aligned} V_{\text{eff}}^{\text{appr}} & = \frac{32\alpha^{3/2} e^{-\frac{5\varphi_1}{\sqrt{2}} - 4\varphi_2}}{3y^4} [4\sqrt{2}c(12(e^{\sqrt{2}\varphi_1} + e^{\varphi_2}) + (y + 12)(6\alpha e^{(2\sqrt{2}\varphi_1 + \varphi_2)})^{1/3}) \\ & - \mathcal{W}_0^2 y g_s e^{\sqrt{2}\varphi_1 + \varphi_2} (-\sqrt{2}\hat{\eta} \log(64\alpha^6) + 9\sqrt{2}\hat{\xi} + 6\hat{\eta}(8\varphi_1 + \sqrt{2}(5\varphi_2 - 24)))] + V_{\text{up}}. \end{aligned} \quad (2.38)$$

From this expression, one can easily minimize the potential with respect to the variable y . This can be approximated as

$$y \cong \frac{64\sqrt{2}c e^{-\sqrt{2}\varphi_1 - \varphi_2} ((6\alpha e^{(2\sqrt{2}\varphi_1 + \varphi_2)})^{1/3} + e^{\sqrt{2}\varphi_1} + e^{\varphi_2})}{\mathcal{W}_0^2 g_s (\sqrt{2}\hat{\eta}(-\log(64\alpha^6) + 30\varphi_2 - 144) + 9\sqrt{2}\hat{\xi} + 48\hat{\eta}\varphi_1)}. \quad (2.39)$$

By recalling the redefinition in (2.36), the minimum along the φ_3 direction is determined as,

$$\varphi_{3,\text{min}}^2 = \frac{1}{3}(y + 4). \quad (2.40)$$

As for the other two perpendicular directions, we could substitute the minimal value of y in the effective scalar potential of Eq. (2.38), which leads to the following minima along (φ_1, φ_2) ,

$$\varphi_{2,\min} \cong \frac{1}{30} \left(\log(64\alpha^6) - \frac{9\hat{\xi}}{\hat{\eta}} - 24\sqrt{2}\varphi_1 + 30W_{0/-1} \right. \\ \left. \times \left(\frac{2^{9/5} e^{\frac{1}{30}(\frac{9\hat{\xi}}{\hat{\eta}} + 54\sqrt{2}\varphi_1 - 184)}}{3\alpha^{1/5}} \right) + 184 \right), \quad (2.41)$$

$$\varphi_{1,\min} \cong \frac{6\hat{\eta}\log(\alpha) - 9\hat{\xi} + 2\hat{\eta} \left(144 + 15\log\left(\frac{13}{5}\right) + \log(8) \right)}{54\sqrt{2}\hat{\eta}}. \quad (2.42)$$

To prove the validity of our approximate formula of the potential in (2.38) and the exact potential in (2.35), we are going to sketch the potentials in the vicinity of the analytical and numerical global minimum in Fig. 1. It is readily seen that our approximated and numerical solutions match modulo tiny departure. Note that, for the inflationary analysis studied in Sec. IV, we work with the potential with the exact form as demonstrated in (2.28).

2. Mass scales

Let us now move on to discussing several mass scales associated with the underlying type-IIB flux compactification. The first mass scale of interest is the string scale, which for consistency of the low-energy string theory should be much less than the Planck scale with the relation¹ $M_s = \frac{g_s^{1/4}\sqrt{\pi}}{\sqrt{V}} M_{\text{pl}}$. Next, we define our Kaluza-Klein scale associated with the bulk, i.e., $\tau_{\text{bulk}}^{3/2} = \mathcal{V}$, $M_{KK} = \frac{\sqrt{\pi}}{\mathcal{V}^{2/3}} M_{\text{pl}}$. The gravitino mass is given by $m_{3/2} = \sqrt{\kappa} \frac{|W_0|}{\mathcal{V}} M_{\text{pl}}$. $m_{3/2}$ sets the scale of all complex structure moduli, the dilaton, and the Kähler moduli. Another important consistency requirement is $m_{3/2} < M_{KK}$, so that the gravitino mass of the 4D EFT is not integrated out. In order to trust the underlying EFT, we need to maintain the following hierarchy of mass scales,

$$m_{3/2} < M_{KK} < M_s < M_{\text{pl}}. \quad (2.43)$$

3. Example dS-vacua

In the following tables, several numerical examples are presented, where the values of the free parameters are displayed along with the values of the moduli/fields at their corresponding minimum.

¹Note that here, instead of using natural units, we have reintroduced the Planck scale to understand the hierarchy between various mass scales.

A very recent study [61] showed that the higher derivative corrections are still under consideration regarding the correct embedding in the supersymmetric field theories and their dimensional reduction. In this work it is argued that the inclusion of all the higher derivative terms derived from the ten-dimensional theory, i.e., an extension of the previous study [58], could in principle modify the strength of the F^4 corrections by affecting the values of the λ parameter. Thus, in the case of a constant superpotential, one could argue that the scale of λ parameter is expected to be enhanced $\lambda > \mathcal{O}(10^{-3})$. Our example could be seen as an example of the importance of these F^4 corrections in the moduli stabilization. One more crucial comment that needs to be highlighted is the fact that if the Kähler dependence of those corrections is changed due to the extended operators, the form of the effective potential is drastically changed. We refer for relevant discussion to [61]. In the following section, we continue our discussion on the fundamental concepts necessary for conducting a three-field inflationary analysis.

III. BASIC IDEA OF MULTIFIELD INFLATION

The purpose of this section is to set the stage for the three-field inflationary scenario that we are going to present in the next section. We begin by writing the Lagrangian for multiple scalar fields minimally coupled with gravity in $(3+1)$ -dimensional curved spacetime,

$$S = \int d^4x \sqrt{-g} \left[M_{\text{pl}}^2 \frac{R_4}{2} - \frac{\mathcal{G}_{ab}}{2} \partial_\mu \phi^a \partial^\mu \phi^b - V(\phi^a) \right], \quad (3.1)$$

where Greek letters label the 4D spacetime indices of the Friedman-Lemaître-Robertson-Walker (FLRW) metric with $(-, +, +, +)$ metric signature. Lower Latin indices denote field-space entries with $(a, b \equiv 1, 2, 3)$ and the metric is $\mathcal{G}_{ab}$. $V(\phi^a)$ is the scalar potential in terms of ϕ^a . $M_{\text{pl}}^2 = (8\pi G_N)^{-1}$ is the reduced Planck mass with G_N being the Newton's constant. We will work in natural units, hence from now on we will put $M_{\text{pl}} = 1$. In an FLRW spacetime, Einstein's equation of motion for a scalar field known as inflaton, takes the following form,

$$H^2 = \frac{1}{3} \left(\frac{\dot{\phi}^2}{2} + V(\phi^a) \right), \quad (3.2)$$

$$\ddot{\phi}^a + 3H\dot{\phi}^a + \Gamma_{bc}^a \dot{\phi}^b \dot{\phi}^c + \mathcal{G}^{ab} V_b = 0, \quad (3.3)$$

here

$$\dot{\phi}^2 \equiv \mathcal{G}_{ab} \dot{\phi}^a \dot{\phi}^b. \quad (3.4)$$

Note that H is the Hubble parameter overseeing the expansion of the spacetime during inflation and it is related to the scale factor of the FLRW metric. The Christoffel symbols in (3.3) are calculated from the field space metric

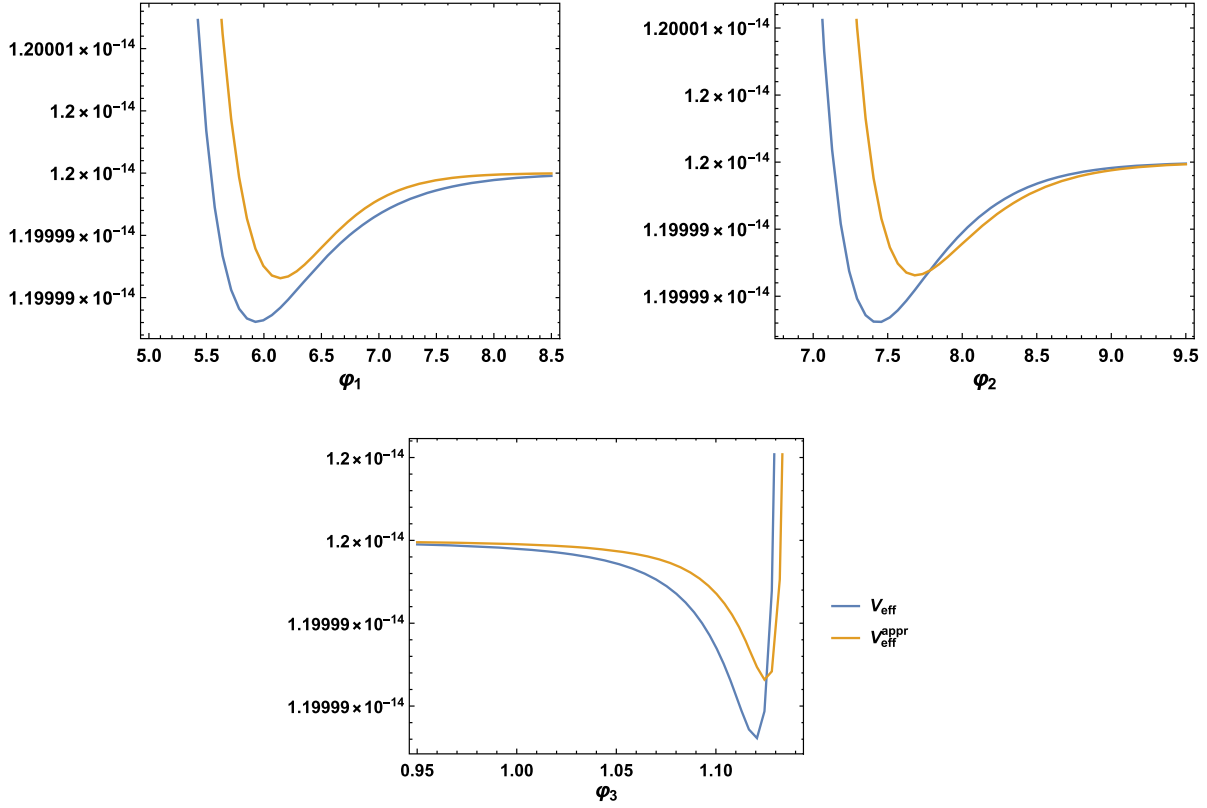


FIG. 1. Comparison between analytical and numerical computation of minimum along ϕ_1 , ϕ_2 and ϕ_3 directions. The parameters used in these plots correspond to Model 1 of Table II.

$\mathcal{G}_{ab}$ and V_a is the derivative of the potential with respect to the scalar field ϕ^a . In multifield inflationary models, (3.2) acts as a constraint and it is enough to solve just the field equations. The equations above are written in physical time but for our purpose of studying inflation, it is more convenient to switch to number of e-folds (N), which is related to time as $dN = Hdt$.

Inflation is a nearly exponential evolution of the space-time. This is ensured by requiring the fractional change of the Hubble parameter per e-fold, denoted by $d \ln H / dN$ to be small. In the same spirit, one can define a series of Hubble slow-roll parameters ϵ_i s where $\epsilon_1 = -d \ln H / dN$. ϵ_1 is the first Hubble slow-roll parameter and for brevity we will drop the subscript. In order for inflation to last at least 60 e-folds, necessary to solve the horizon problem, ϵ should remain small for a sufficient number of Hubble times and this is measured by the second Hubble slow-roll parameter η , defined as,

$$\eta \equiv \frac{\dot{\epsilon}}{\epsilon H} = \frac{\ddot{H}}{H\dot{H}} + 2\epsilon = 2\delta_\phi + 2\epsilon \ll 1, \quad (3.5)$$

where a small η also requires a small δ_ϕ .

A. Kinematic basis decomposition

We are interested in a three-field inflation with turning and torsion in field space. Hence, it is natural to work on a

kinetic basis by decomposing the inflationary trajectory into a tangent, normal, and binormal direction as shown in Fig. 2. An extensive study on three-field inflation called helix-trajectory inflation using an hyperbolic metric is carried out in [62] and further extended in [63–65].

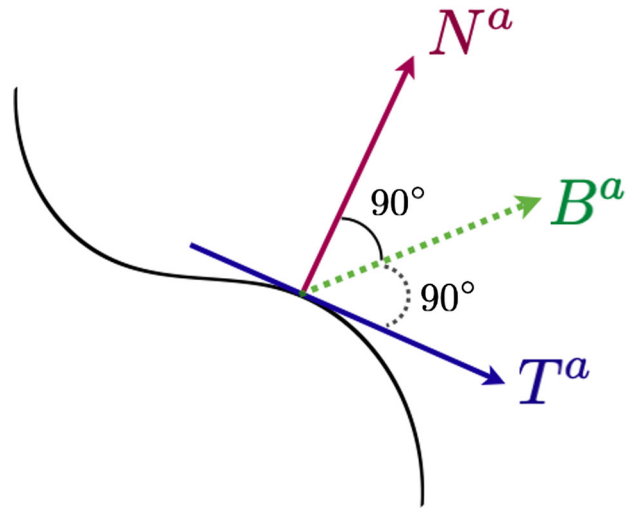


FIG. 2. Black line corresponds to the inflationary trajectory on which T^a is an unit vector tangential to it. N^a is another unit vector normal to the trajectory and B^a denotes the unit vector normal to both T^a and N^a .

In this lore, let us introduce unit tangent (T^a), normal (N^a), and binormal (B^a) vectors as following:

$$\begin{aligned} T^a &= \frac{\dot{\phi}^a}{\dot{\phi}}, & N^a &= \frac{\mathcal{D}_N T^a}{\Omega_1}, \\ B^a &= \frac{D_N N^a + \Omega_1 T^a}{\tau}, \\ T^a T_a &= B_a B^a = N_a N^a = 1, \\ N^a T_a &= B^a T_a = N^a B_a = 0, \\ \Omega_1 &= \frac{\Omega}{H}, & \Omega &= |\mathcal{D}_t T|, \\ \tau B^a &= \mathcal{D}_N N^a + \Omega_1 T^a. \end{aligned} \quad (3.6)$$

Ω_1 is known as the dimensionless *turning rate*, Ω is the dimensionful turning rate, and τ denotes the dimensionless *torsion* [63,66]. Both of them determine how much an inflationary trajectory deviates from a geodesic one ($\mathcal{D}_t T = 0, T = \sqrt{\mathcal{G}_{ab} T^a T^b}$). In a three-dimensional field-space (x, y, z) for example, Ω denotes how much the inflationary trajectory is bending in the x - y plane and τ denotes what is its motion along the vertical z direction. When $\Omega > 0$ the trajectory undergoes turning and when $\tau > 0$ the trajectory is nonplanar. If both (Ω, τ) are constant, the trajectory takes a helical path as found in [62]. Note that, the kinematic basis as well as the torsion are not well defined when $\Omega = 0$. In our work however, as presented in Sec. IV, the nongeodesic behavior will be manifested not in their kinetic terms but in the scalar potential through nontrivial coupling of scalar fields. Ω denotes the norm of the unit tangent vector and has one mass dimension. These can be summarized in terms of the Frenet-Serret system as follows,

$$\mathcal{D}_N \begin{pmatrix} T^a \\ N^a \\ B^a \end{pmatrix} = \begin{pmatrix} 0 & \Omega_1 & 0 \\ -\Omega_1 & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix} \begin{pmatrix} T^a \\ N^a \\ B^a \end{pmatrix}. \quad (3.7)$$

With the help of the equation of motion (3.3), the definitions of Ω_1 and τ can be related to the inflationary potential as [64],

$$\Omega = -\frac{V_N}{H\dot{\phi}'}, \quad ' = \frac{d}{dN}, \quad (3.8)$$

$$\tau = -\frac{V_{BT}}{\Omega_1 H^2}, \quad (3.9)$$

where $V_N = N^a V_a$, $V_{BT} = V_{ab} T^a B^b$ and $\nabla_a A_b \equiv \partial_a A_b - \Gamma_{ab}^c A_c$, for some vector A_b .

To study the masses of the scalar fields as well as to study multifield perturbations, we define two matrices. $\mathbb{M}_b^a$ is the

mass-matrix of the scalar fields computed from $V(\phi^a)$ and the projection of the $\mathbb{M}_b^a$ along the directions of the unit vectors is denoted by $\mathbb{M}$,

$$\mathbb{M}_b^a = \nabla^a \nabla_b V, \quad \mathbb{M} = \begin{pmatrix} V_{TT} & V_{TN} & V_{TB} \\ V_{NT} & V_{NN} & V_{NB} \\ V_{BT} & V_{BN} & V_{BB} \end{pmatrix}, \quad (3.10)$$

where $V_{TT} = T^a T^b \nabla_a \nabla_b V$.

B. Multifield perturbations

In the case of multifield inflation, it is standard to decompose the linear equation of motion for the Mukhanov-Sasaki variable in kinematic basis, i.e., along (T^a, N^a, B^a) . The projection of field fluctuations Q^a along the unit vectors are $Q_i = (Q_T, Q_N, Q_B) \equiv (Q^a T_a, Q^a N_a, Q^a B_a)$. The equations of motion in terms of Q_i are [64–67],

$$\mathcal{D}_N (Q')^i + F_j^i (Q')^j + C_j^i Q^j = 0,$$

$$F_j^i \equiv (3 - \epsilon) \delta_j^i - 2\Omega_j^i,$$

$$C_j^i = \left(\frac{k}{aH}\right)^2 \delta_j^i + \begin{pmatrix} 0 & -2(3 - \epsilon)\Omega_1 & 0 \\ 0 & \mathcal{M}_{NN} - \Omega_1^2 - \tau^2 & \mathcal{M}_{NB} - \tau(3 - \epsilon) \\ 0 & \mathcal{M}_{NB} + \tau(3 - \epsilon) & \mathcal{M}_{BB} - \tau^2 \end{pmatrix} + \mathcal{O}(\epsilon^2, \eta, \nu, \nu_\tau). \quad (3.11)$$

Primes denote a derivative with respect to conformal time $d\tau = dt/a(t)$. We defined $\nu = \Omega_1'/\Omega_1$ and $\nu_\tau = \tau'/\tau$. The mass terms appearing in (3.11) such as $(\mathcal{M}_{NN}, \mathcal{M}_{NB}, \mathcal{M}_{BB})$ can be obtained by contracting $\mathcal{M}_{ab}$ with (N^a, B^a) where $\mathcal{M}_{ab}$ is,

$$\begin{aligned} \mathcal{M}_{ab} &= \frac{V_{ab}}{H^2} - 2\epsilon R_{aTTb} + 2\epsilon(3 - \epsilon)T^a T^b \\ &\quad + \sqrt{2\epsilon} \frac{T_a V_b + T_b V_a}{H^2}. \end{aligned} \quad (3.12)$$

In order to further simplify the above set of equations one has to compare between the adiabatic mass and entropic masses. If the entropic masses are larger or comparable to the adiabatic mass, one has to consider the effect of all the fields in the perturbative analysis, which thus modify the spectral index and spectral tilt [62] through a parameter called speed of sound c_s . It is the propagation speed of the adiabatic perturbation, expressed in terms of the ratio of entropic masses. The study of perturbation with reduced speed of sound and nontrivial turns is studied for the two-field case in [68–70] and in the three-field scenarios in [71,72]. On the other hand, for the inflationary analysis done in this paper, the mass gap between the adiabatic and entropic masses are large enough that we integrate out the latter. In this case, we can study the inflationary

perturbation as a quasi-single-field effective theory, knowing that it is subject to $M_{TT}/M_{\text{entropic}}$ corrections.

IV. THREE-FIELD ANALYSIS

In this section, we shall thoroughly analyze the complete multifield inflationary evolution and the detailed predictions of the model. To achieve this, we begin by outlining the consistency checkpoints that must be met in viable string theory model building.

A. Consistency checks

- (1) To reside in the semiclassical regime, we need to achieve small g_s and large volume $\mathcal{V}$. From the examples presented in Tables I and II, we can observe that all of them meet the specified criteria with $g_s \sim \mathcal{O}(10^{-4})$ and $\mathcal{V} \sim \mathcal{O}(10^6)$. Due to the same reason, it is easier to trust the underlying EFT in our model by requiring,

$$V_{\alpha^3} > V_{F^4},$$

which means the leading order BBHL term proportional to $\hat{\xi}$ in the perturbative expansion in α' should be larger compared to the higher derivative F^4 term, proportional to λ . The above inequality simplifies to,

$$\mathcal{V} > \frac{W_0^2 \lambda}{2\pi^2 \sqrt{g_s} \hat{\xi}}, \quad (4.1)$$

where $\mathcal{V}$ is the volume in (2.9). For a typical set of parameters, one can show that the inequality poses a less strict constraint for the parameter sets presented in this paper. For the inflationary example as well as the models presented in Tables I and II, we get $\mathcal{V} > \mathcal{O}(10^2)\lambda$, with $\lambda \ll 1$.

- (2) High energy stringy effects can be neglected if each four cycle considered in this work, which are τ_1, τ_6, τ_7 , has volume larger than the string scale²: $\text{Vol}_s^{1/4} \gg \sqrt{\alpha'}$. The string frame and Einstein frame volume are related as: $\text{Vol}_s = g_s \text{Vol}_E = g_s \tau_E l_s$ with $l_s = 2\pi\sqrt{\alpha'}$. We arrive at the following relation:

$$\delta_{\tau_i} \equiv \frac{1}{g_s (2\pi)^4 \tau_i} \ll 1 \quad \forall i. \quad (4.2)$$

From Table I, we see that with $\tau_i \sim \mathcal{O}(10^3\text{--}10^4)$ and $g_s \sim \mathcal{O}(10^{-4})$, it is straightforward to satisfy this condition.

²For details on how the volumes and the 10D metric in Einstein and string frame are related see [73].

- (3) To trust the α' expansion, one has to satisfy: $\delta_{\xi} = \frac{\hat{\xi}}{2\mathcal{V}} \ll 1$. For $\hat{\xi} \sim \mathcal{O}(10)$ and $\mathcal{V} \sim \mathcal{O}(10^6)$, it is easily satisfied in our models.
- (4) Again from the consistency of the EFT, throughout the inflationary evolution, we need³:

$$m_{\varphi_i} < H < M_{kk}^{\text{bulk}} < M_s < M_{pl}. \quad (4.3)$$

- (5) Last but not least, the inflationary prediction should be in compliance with the Planck 2018 data [4],

$$\begin{aligned} P_s &= 2.105 \pm 0.03 \times 10^{-9}, \\ n_s &= 0.9649 \pm 0.0042, \\ r &< 0.036, \end{aligned} \quad (4.4)$$

at a pivot scale of 0.05 Mpc^{-1} .

B. Cosmological evolution

We now analyze the three-field cosmological evolution in a fully canonical basis spanned by $(\varphi_1, \varphi_2, \varphi_3)$. The field space metric being flat makes both the Christoffel symbols and the Ricci scalar zero. In this setup, the evolution equations of (3.3) in the canonical basis boil down to this simpler set of equations in number of e-folds,

$$\begin{aligned} \varphi_1'' &= -\left(\frac{1}{2}(\varphi_1'^2 + \varphi_2'^2 + \varphi_3'^2) + 3\right)\varphi_1' - \frac{V_{\varphi_1}}{H^2}, \\ \varphi_2'' &= -\left(\frac{1}{2}(\varphi_1'^2 + \varphi_2'^2 + \varphi_3'^2) + 3\right)\varphi_2' - \frac{V_{\varphi_2}}{H^2}, \\ \varphi_3'' &= -\left(\frac{1}{2}(\varphi_1'^2 + \varphi_2'^2 + \varphi_3'^2) + 3\right)\varphi_3' - \frac{V_{\varphi_3}}{H^2}, \\ ' &= \frac{d}{dN}. \end{aligned} \quad (4.5)$$

V is the scalar potential in (2.35), which consists of a leading order α'^3 BBHL term, log-loop corrections, higher derivative F^4 corrections, and a constant uplift term resulting in a Minkowski-like vacuum of the potential at its global minimum. We are agnostic about the exact computation of the uplift term, it can be a D-term uplift [33], a T-brane uplift [75], or an antibrane uplift [5,60,76,77].

We now present a three-field inflationary model where all the three fields $(\varphi_1, \varphi_2, \varphi_3)$ are displaced from their respective minimum. We get sufficient

³See [74] for a slightly different hierarchy of scales where the inequalities between inflaton mass m_{inf} and the underlying Hubble scale are interchanged to $m_{\text{inf}} > H$ throughout the slow-roll inflationary evolution in a multifield scenario.

TABLE I. Three different numerical solutions and the corresponding mass scales.

	τ_1	τ_7	τ_6	$\mathcal{V}$	φ_1	φ_2	φ_3
Model 1	1.35×10^5	4403	1690	1×10^6	5.93	7.43	1.12
Model 2	3.56×10^5	7120	2732	2×10^6	6.27	7.91	1.14
Model 3	3.79×10^6	44636	17191	1×10^8	7.57	9.75	1.13

	$m_{\langle\varphi_1\rangle}$	$m_{\langle\varphi_2\rangle}$	$m_{\langle\varphi_3\rangle}$	$m_{3/2}$	M_{KK}	M_s
Model 1	3.42×10^{-8}	4.63×10^{-10}	2.17×10^{-10}	5.72×10^{-8}	1.72×10^{-4}	1.73×10^{-4}
Model 2	3.2×10^{-8}	2.01×10^{-10}	9.27×10^{-11}	3.46×10^{-8}	1.1×10^{-4}	1.19×10^{-4}
Model 3	2.32×10^{-11}	2.28×10^{-13}	1.05×10^{-13}	2.87×10^{-10}	7.72×10^{-6}	9.51×10^{-6}

TABLE II. Free parameters used for the examples presented in Table I.

	g_s	$ \mathcal{W}_0 $	$\hat{\xi}$	$ \hat{\eta} $	α	$ \lambda $	V_{up}
Model 1	10^{-4}	30	11.5	0.5	2×10^{-5}	0.09	1.2×10^{-14}
Model 2	8.5×10^{-5}	38	13.5	0.5	5×10^{-5}	0.04	2.23×10^{-20}
Model 3	10^{-5}	50	19.5	0.5	1×10^{-6}	0.078	2.86×10^{-26}

TABLE III. Table contains the model parameters and initial conditions. The latter is labeled by superscript 0.

g_s	$ \mathcal{W}_0 $	$ \hat{\eta} $	α	$\hat{\xi}$	λ	C_{up}	φ_1^0	φ_2^0	φ_3^0	$\langle\varphi_1\rangle$	$\langle\varphi_2\rangle$	$\langle\varphi_3\rangle$
0.001	38	0.5	0.0001	16	0.008	2.45×10^{-14}	4.9	10.0	1.1	7.09	9.06	1.15437

e-folds as these fields complete their journey toward their minima and oscillate there to start the reheating process.⁴ Below, we discuss one benchmark model that uses the following set of parameters: Masses of the inflatons tabulated above are Eigenvalues of the Hessian matrix computed using (2.35) at their respective minimum. In accordance with the consistency checks [(2.43) and (4.3)] stated above, the hierarchy of various mass scales is satisfied, i.e.,

$$m_{3/2} < M_{\text{KK}} < M_s < M_{pl}, \quad \text{and} \quad M_{\varphi_i} \equiv M_{\text{inf}} < H.$$

For this benchmark model, the overall volume of (2.9) is $\mathcal{V} \sim \mathcal{O}(10^4)$. Residing in the large volume and the weak coupling limit, it is straightforward for this model to satisfy condition (4.1) and we obtain $\delta_{\xi} \sim \mathcal{O}(10^{-4})$ and $\delta_{\tau_i} \sim \mathcal{O}(10^{-5})$.

⁴Note that we do not intend to study the reheating dynamics in this article.

C. Slow-roll dynamics of the fields

We numerically evolve the set of equations presented in (4.5) with the values of the model parameters and initial conditions presented in Table III. We are focusing on vanishing initial velocities for the inflatons, i.e. $\{\varphi'_1(0), \varphi'_2(0), \varphi'_3(0)\} = 0$.

If we assess the initial conditions for the inflatons and their respective minima in Table III, we see that the initial path of the inflationary trajectory starts on the left side of $\langle\varphi_1\rangle$ and $\langle\varphi_2\rangle$. While both (φ_1, φ_2) are displaced away from their minima, if we choose φ_3 close to its minimum, we obtain an interesting three-field attractor where all three fields move together to give a sufficient amount of e-folds. As soon as the fields settle at their minima, inflation ends. In Fig. 3, although along φ_1 and φ_2 the displacement in fields ($\Delta\varphi_{1,2}$) remains relatively small in most of their evolution, they do exhibit some movement, as evident from the inset images. The potential along the φ_3 is the steepest among the rest, it dominates the field trajectory in the beginning when the (φ_1, φ_2) move relatively slower. Hence, we may conclude that most of the slow-roll dynamics up to $N = 55$ is mostly a cumulative effect of

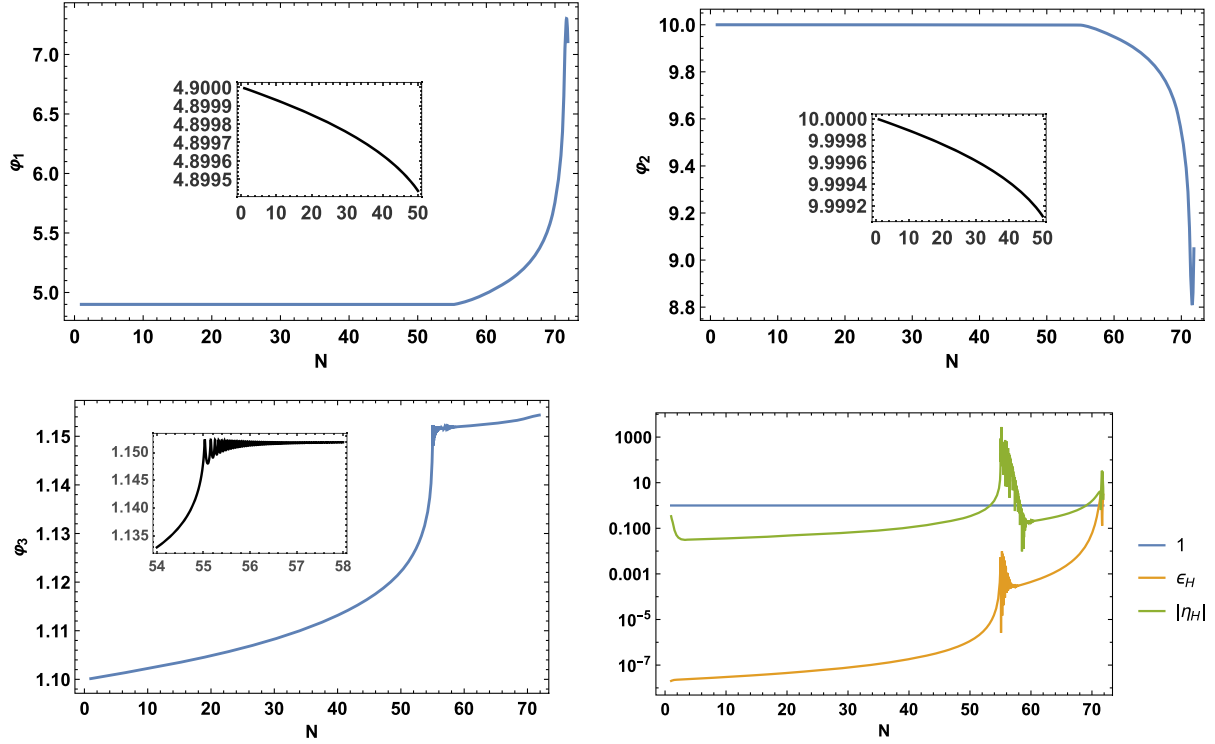


FIG. 3. Field evolutions of the three canonical scalar fields. ϵ_H stays less than 1 throughout the evolution but η_H crosses 1 at $N = 55$. The reasons for the same have been explained in the text.

these fields with contributions from φ_3 being dominant. An interesting observation here is that, although Table IV shows that φ_3 has the lowest mass at the global minimum, as the inflaton evolves from a shifted position, this hierarchy reverses, making the φ_3 direction the steepest among all.

The trajectory of φ_3 shows a peculiar behavior because if we notice and compare between the path φ_3 follows and its value of the minimum from Table III then we can see that the field is not directly going toward its minimum. Roughly $N = 55$ onward, we notice vigorous oscillations in both ϵ_H and η_H followed by a softening where η_H returns to slow roll after a brief period of slow-roll violation. We have $\epsilon_H < 1$ throughout but its amplitude increases beyond $N = 55$. The oscillation in (ϵ_H, η_H) is a manifestation of the oscillation in the φ_3 direction around $N \sim 55$.

We can understand this behavior by looking at the field trajectories as well as the steepness of the potential along φ_3 . The hierarchy between ϵ_H and η_H with $\eta_H > \epsilon_H$ is the manifestation of the fact that the potential has steep directions but is still shallow enough to sustain slow roll as ϵ_H remains less than one throughout. The oscillation in ϵ_H, η_H sets in when φ_3 falls in a ridge of the potential. We notice that along the inflationary trajectory where (φ_1, φ_2) are far away from their minima, the true minimum for φ_3 disappears and the ridge becomes unavoidable. Although, notice that the ridge is not at the point where the potential diverges, which is at

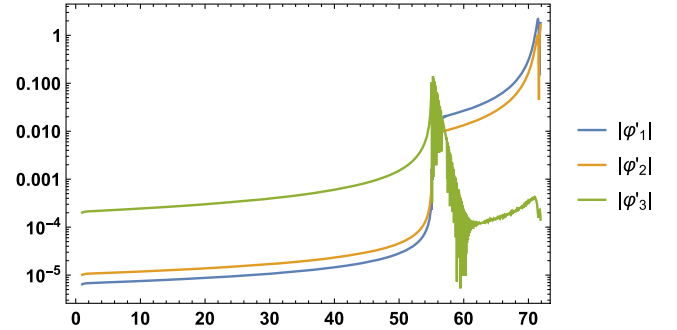


FIG. 4. Speed of the inflatons.

$\varphi_3 \equiv \sqrt{4/3}$. The ridge acts like a false vacuum for φ_3 and when it falls there it leads to such oscillations.⁵ Now, as soon as (φ_1, φ_2) move close to their minima along the trajectory, φ_3 recovers its actual minimum and φ_3 restarts its journey toward the true vacuum. Interestingly, it appears that after the oscillation the fields switch their roles, (φ_1, φ_2) move faster whereas φ_3 appears relatively slower, see Fig. 4. For the last few e-folds, all the fields

⁵Note that it may be possible that this oscillation along φ_3 may trigger an early preheating. However, since slow roll resumes after the end of oscillation, leading to the continued expansion of spacetime, any particles generated during this period will ultimately be diluted away.

TABLE IV. Mass scales follow the correct hierarchy. The Hubble parameter, derived from (3.2), remains nearly constant throughout the inflationary evolution, which is why a wiggle sign has been included.

$m_{(\varphi_1)}^2$	$m_{(\varphi_2)}^2$	$m_{(\varphi_3)}^2$	M_s/M_{pl}	M_{KK}/M_{pl}	$m_{3/2}/M_{pl}$	H/M_{pl}
1.54×10^{-6}	2.51×10^{-14}	5.33×10^{-15}	1.36×10^{-3}	1.25×10^{-3}	4.5×10^{-6}	$\sim 6.95 \times 10^{-8}$

become active to roll toward the global minimum, thereby increasing both ϵ_H and η_H , signaling the end of inflation. As expected, φ_3 being the steepest among them, as shown in Fig. 4, its velocity dominates over the other and as soon as it falls off the ridge the velocity decreases but does not become zero. When φ_3 oscillates at the ridge, (φ_1, φ_2) become dominant and move toward their minimum. In summary, this attractor solution we have discovered, gives a *quasi-single-field* behavior in the first part of inflation and *quasi-double-field* for the second period of inflation. We shall confirm this claim by looking at the projection of the masses of the scalar fields as well as perturbative masses of the modes throughout the inflationary evolution. We shall discuss this in greater detail in the following subsections.

To complete the analysis of this section, we also confirm that our benchmark model produces the correct hierarchy of scales as shown in Fig. 5.

D. Turning rate and torsion

Depending on the nature of turns and torsions, one can get a variety of different models with different predictions for cosmological observables. Models featuring sharp turns [78], smooth and large turns [79], and turns combined with nonplanar motion [62,65] have proven useful in the study of primordial black holes and particle production. For our multifield canonical model with vanishing Ricci curvature, it is expected that the fields follow a geodesic ($\mathcal{D}_t T = 0$) for most of their evolution producing small $\Omega/H \sim \mathcal{O}(10^{-8})$ and $\tau \sim \mathcal{O}(10^{-12})$. During the crossing of pivot-scale ($k_\star = 0.05 \text{ Mpc}^{-1}$), which roughly corresponds to a scale between $(N_{\epsilon_H=1} - 60) = 11.15$ to $(N_{\epsilon_H=1} - 50) = 21.15$, $(\Omega/H, \tau)$ remain small, however

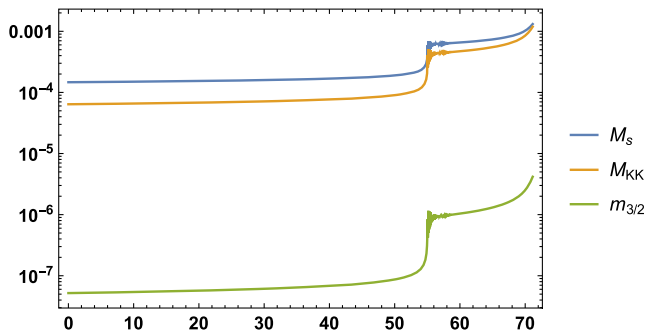


FIG. 5. Correct hierarchy of mass scales with $M_s > M_{KK} > m_{3/2}$. All the masses are expressed in Planck units.

if we look at Fig. 6, we notice that beyond $N = 55$, both $(\Omega/H, \tau)$ get an enhancement in magnitude. The rapid oscillation observed in the figure below arises from the oscillation in the φ_3 direction as it falls into its false vacuum (the ridge), as we can observe in Fig. 3.

The oscillations are present in both the Ω/H and τ direction but notice that postoscillation the hierarchy between the torsion and the turning rate changes. This signals the fact that, the field-motion was confined within a geodesic trajectory dominated by the φ_3 field but as soon as (φ_1, φ_2) find the valley toward its minima and start their journey, the whole inflationary trajectory becomes nonplanar.

Let us also point out that the oscillation and violation of slow-roll as depicted in the right-hand side plot of Fig. 3, can also be observed in single field models. One possible distinction between multi- and single-field models comes through possible detection of non-Gaussianities—future surveys may help unravel this mystery. Although, detailed multifield analysis of non-Gaussianities is beyond the scope of this paper, our attractor inflationary trajectory does feature a large turn, aided by the nonplanar motion (see Fig. 6), a characteristic of a multifield phenomenon and a key cause of the slow-roll violation in our model.

E. Cosmological parameters

The dynamics of the linear perturbation and cosmological predictions for an inflationary model depend on the hierarchies of the adiabatic and the entropic modes. The value of $(\Omega/H, \tau)$ play a pivotal role in understanding how much entropic perturbations affect the adiabatic curvature

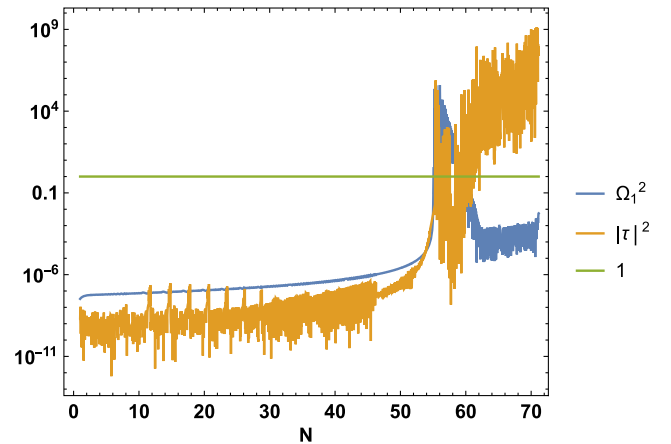


FIG. 6. Dimensionless turning rate and torsion.

perturbation. As we can see from the set of equations (3.11), $\tau \sim 0$ will simplify them to the standard Mukhanov-Sasaki mode equations in terms of two canonically normalized fields v_σ and v_s which depend on (Q_T, Q_N) in the following way:

$$v_\sigma = aQ_\sigma, \quad v_s = aQ_N. \quad (4.6)$$

In terms of these new variables, the mode equations in conformal time can be written as

$$v''_\sigma - \xi' v'_s + \left(k^2 - \frac{z''}{z}\right)v_\sigma - \frac{(z\xi')'}{z}v_s = 0, \quad (4.7)$$

$$v''_s + \xi' v'_\sigma + \left(k^2 - \frac{a''}{a} + a^2\mu_s^2\right)v_s - \frac{z'}{z}\xi'v_\sigma = 0, \quad (4.8)$$

$\xi' = 2a\Omega$ sources the coupling between curvature and entropic mode. $\Omega \ll 1$ naturally makes the coupling weak in our case. The masses of adiabatic and entropic perturbations can be derived from both (3.12) and the matrix (3.10). These quantities play a crucial role in determining the extent to which entropic perturbations influence the whole inflationary perturbations, guiding us either deeper into a multi-field regime or away from it. The masses are defined as follows:

$$\frac{m_T^2}{H^2} \equiv -\frac{3}{2}\eta - \frac{1}{4}\eta^2 + \frac{1}{2}\epsilon\eta - \frac{1}{2}\frac{\dot{\eta}}{H}, \quad \cdot = \frac{d}{dt}, \quad (4.9)$$

$$\frac{M^2}{H^2} = \frac{V_{NN}}{H^2} + M_{pl}^2\epsilon\mathbb{R} - \Omega_1^2, \quad (4.10)$$

$$\mathbb{R}_{bcd}^a = \Gamma_{bd,c}^a - \Gamma_{bc,d}^a + \Gamma_{ce}^a\Gamma_{bd}^e - \Gamma_{de}^a\Gamma_{bc}^e, \quad (4.11)$$

Below, we show a plot of (m_T^2, M^2) together with the mass along the binormal unit vector M_{BB}^2 and an effective entropy mass given as $\mathcal{M}_{\text{eff}}^2 = M^2 + 4\Omega^2$. The masses also help to define a speed of sound for the adiabatic perturbations via the relation [69,80,81],

$$c_s^{-2} = \frac{\mathcal{M}_{\text{eff}}^2}{M^2}.$$

We find that $\mathcal{M}_{\text{eff}} \sim M$, leading to $c_s \sim 1$ along the entire inflationary trajectory. Consequently, this results in negligible equilateral-type non-Gaussianity, with $f_{NL} \sim \mathcal{O}(10^{-8})$, as given by the relation [62],

$$f_{NL} = \frac{125\epsilon_H}{108c_s^2} + \frac{5}{81}\frac{c_s^2}{2}\left(1 - \frac{1}{c_s^2}\right)^2 + \frac{35}{108}\left(1 - \frac{1}{c_s^2}\right). \quad (4.12)$$

After carefully assessing the mass scales, we also move on to checking the adiabaticity condition [81],

$$\mathcal{A} = \left|\frac{\dot{\Omega}}{M\Omega}\right| \ll 1, \quad (4.13)$$

which holds true in our case with $\mathcal{A} \sim \mathcal{O}(10^{-5})$. Therefore, given the small value of $\mathcal{A}$ as well as the mass of the adiabatic perturbation being the dominant one, we use the standard formulas for the spectral index and spectral tilt in terms of the slow-roll parameters (3.5),

$$n_s = 1 - 2\epsilon_H - \eta_H, \quad r = 16\epsilon_H. \quad (4.14)$$

The amplitude of the scalar power spectrum (Δ_s^2) and tensor power spectrum (Δ_t^2) are given by,

$$\Delta_s^2 = \frac{1}{8\pi^2}\frac{H^2}{\epsilon}, \quad \Delta_t^2 = \frac{2H^2}{\pi^2}. \quad (4.15)$$

Figure 7 confirms that we are fairly within the Planck's bound presented in (4.4). The small value of r is a manifestation of the smallness of the scale of the potential in Planck units. Since r is directly related to the field excursion through Lyth bound, in our case, we also expect small field excursions. This makes our model in accordance with the swampland distance conjecture [82]. The running of the spectral index for this model, α_s is of the order $\mathcal{O}(10^{-3})$.

F. Large turns and torsions on small scales

Before ending this section, let us drop our final remark on the implications of the enhancement in the magnitude of turning rate and torsion as has been shown in Fig. 6. It is widely known in the literature [78,79,83–87] that models featuring $\eta_H > \epsilon_H$, combined with a brief violation of the slow-roll conditions, can lead to an enhancement of the scalar power spectrum—potentially accounting for the production of primordial black holes. The field behavior reflected in our model reveals a similar behavior like double inflation. In double inflation, the process occurs in two stages. Initially, one field rolls down to its minimum while the second field remains frozen. In the later stage, the frozen field begins to roll, ultimately reaching its minimum and bringing inflation to an end. Similarly, in the case of the three-field inflation discussed in this paper, we observe a *quasidouble inflation*, where the first phase of inflation is mostly driven by one field, while the second phase is governed mostly by the other two fields as we can observe in Fig. 4. The first phase of slow roll takes place along the steep direction of the potential.

Referring to Fig. 8, we observe that during horizon crossing ($11.15 < N < 21.15$), the mass of the adiabatic mode, denoted as $\{M_{TT}^2, m_T^2\}$, remains dominant. But on a smaller scale however, when the inflationary trajectory enters its second stage, characterized by two-field dynamics, where (φ_1, φ_2) becomes more active compared to φ_3 —the masses of the entropic perturbations become dominant.

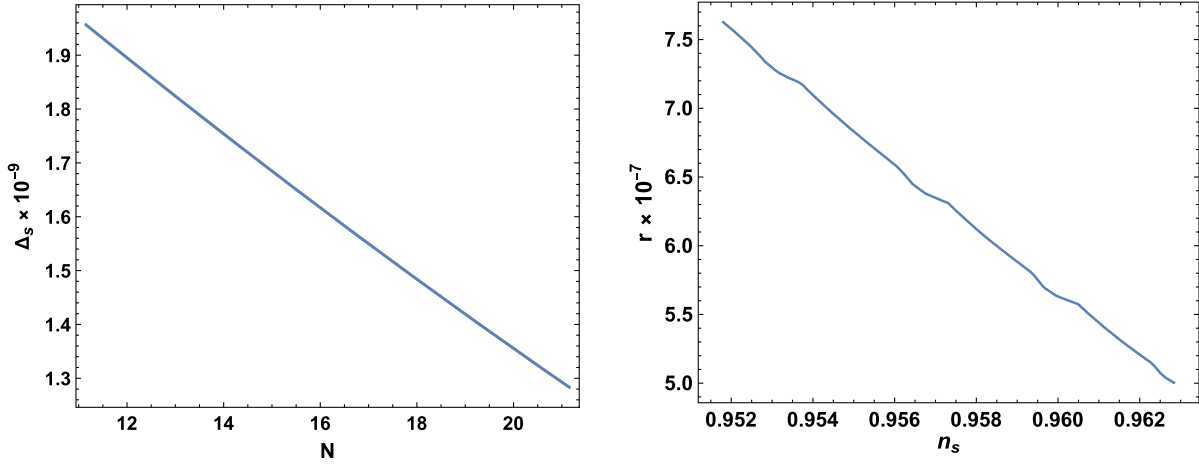


FIG. 7. Scalar power spectrum (Δ_s^2), spectral index (n_s) and spectral tilt (r) are calculated at horizon crossing, that is roughly 50–60 e-folds before the end of inflation, which is at $N_{e_H=1} = 71.15$.

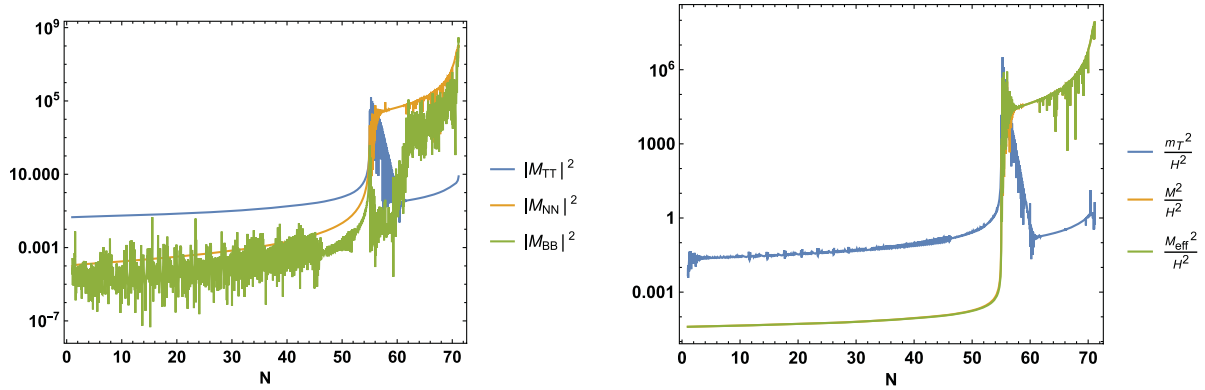


FIG. 8. Left: adiabatic mass is bigger than the entropic ones: $M_{TT} > M_{NN} > M_{BB}$. Right: similar hierarchy of scales are restored when the torsion is negligible. Also note that, we obtain $\mathcal{M}/\mathcal{M}_{\text{eff}} \sim 1$.

This implies that the effects of turns and torsions must be accounted for, requiring a simultaneous analysis of both adiabatic and all entropic perturbations to determine the power spectrum of scalar and tensor modes.

Primordial black holes, considered potential progenitors of dark matter, have been widely studied in recent years, including a recent investigation within a three-field model in [65] where the authors presented a toy model. We emphasize that our work is among the first to explore a three-field inflation scenario with nontrivial features at small scales—motivated by type-IIB string theory flux compactification where our inflatons are four cycles of a K3-fibered Calabi-Yau. The potential is a combination of a leading order BBHL term, a higher derivative F^4 term, and 1-loop correction terms. To ensure a nearly Minkowskian global minimum, a constant term has also been added. The fields are canonical and dimensionless in Planck units. We have performed a thorough consistency check to ensure the reliability of the 4D EFT. We plan to revisit the detailed study of primordial black holes

as well as secondary induced gravitational waves in a forthcoming companion paper.

V. CONCLUSIONS

In this paper, we have introduced a three-field inflationary model within the framework of string theory flux compactification, demonstrating that complete moduli stabilization can be achieved using only perturbative corrections in a weak-Swiss-cheese CY. After performing several consistency checks within the framework of the 4D low-energy supergravity approximation, we confirm the existence of an attractor three-field inflationary trajectory that remains consistent with observational constraints at large scales—specifically during the horizon crossing of the pivot scale. For the first time, this concrete string theory model features a hierarchy between first and second slow-roll parameters with a transient violation of slow roll through nontrivial turning rate and torsion, signaling a possible enhancement of the scalar power spectrum at small

scales. This enhancement can prompt the production of primordial black holes, which we plan to pursue in a companion paper. The scalar potential deviates from the usual pLVS [36] or fiber inflationary setup in their interplay of subleading corrections. The analysis is focused on the implications of α' corrections, logarithmic loop corrections, and higher derivative F^4 correction in the moduli stabilization procedure. We also added a small uplifting term to ensure the presence of a Minkowski vacuum.

The volume of the CY is stabilized to a large value of order $\mathcal{V} > \mathcal{O}(10^6)$ (see Table I) and hence we can trust the underlying 4D low energy EFT in making the correction terms subleading in α' and g_s expansions. The model reflects the correct hierarchy of mass scales, i.e., $M_{pl} > M_s > M_{KK} > M_{inf}$ in all the benchmark models displayed in Table I. As far as the inflationary setup is concerned, we present a multifield inflation model where all the scalar fields are active during inflation. Inflation occurs along $(\varphi_1, \varphi_2, \varphi_3)$ directions, which span the canonical field space metric, constructed out of several four cycles such as fiber modulus τ_7 , the volume of the base τ_6 , and the blow-up modulus τ_1 . Inflation starts at a position where all the scalars are shifted from their global minimum. However, it features a ridge valley along the φ_3 direction when the other fields are away from their minimum. The potential in that direction becomes steeper than in the other two directions (φ_1, φ_2) .

After analyzing the field trajectory, we confirm that inflation along this attractor occurs in two distinct stages. The first stage is mostly dominated by φ_3 because the potential in this direction is the steepest. The minimum along φ_3 disappears at $\{\varphi_1^{\text{shifted}}, \varphi_2^{\text{shifted}}\}$, but forms a ridge—compelling φ_3 to settle into it and oscillate, effectively behaving as a false vacuum for φ_3 . The ridge continues to stay until the other two fields start to roll near their respective minima—signaling the onset of the second stage of inflation. When comparing the speed of the inflatons, the inflationary trajectory in the first stage is largely dominated by φ_3 —leading to a behavior of *quasi-single-field*. The horizon crossing of the pivot scale (k_*) occurs in the first stage and the model is observationally viable.

In the second stage, as φ_3 stops its oscillation, the other two fields start their journey toward their minima. As soon as (φ_1, φ_2) start to get near their minima, the true minimum for φ_3 is restored and all the fields slowly roll toward their respective minimum as we can see in Fig. 4. Hence, in the second stage, the field velocity is mostly dominated by (φ_1, φ_2) . We can that these two stages exhibit a *quasi-double-inflation*-like behavior where the first stage is dominated by φ_3 and the second by (φ_1, φ_2) .

This oscillation demonstrate a nontrivial feature to which we now turn our attention. Due to the flat metric, as expected, during the first stage, the model exhibits minor deviation from a geodesic trajectory. However, when φ_3

oscillates and the other two fields start to roll, it leads to high value of turning rate. The torsion, which takes care of how much nonplanar behavior is present in the trajectory, despite being small for most of the evolution, increases right after the onset of oscillation.

We can notice the same even at the level of the masses of the adiabatic and entropic modes—they switch their roles as can be seen in Fig. 8. The domination of entropic mass at small scales indicates the fact that at the level of perturbation, three field analysis will become unavoidable. This type of feature is closely related to the production of primordial black holes at small scales—possibly at the time of radiation domination. Besides, it can also lead to nontrivial footprint on the scalar induced gravitational waves. We plan to return to this detailed study in a companion paper. We emphasize that this trajectory is very generic for this class of models, depending on the initial condition one can easily achieve this three-field attractor.

There exist a few areas in the inflationary model and the examples presented in Tables I and II that call for further refinement. The example in Table III, the vev of the blow-up modulus is calculated to be of order $\mathcal{O}(10^6)$ whereas the overall volume is $\mathcal{O}(10^5)$. However, we have made sure that the model still resides in the large volume regime throughout the inflationary evolution. Note that, this is not the case for the dS models presented in Tables I and II. The inflationary model is chosen so that the amplitude of the power spectrum is obtained without any fine-tuning or artificial scaling. Besides, the Kaluza-Klein and the string mass scales are almost comparable to each other in all the models—pushing us to the boundary of parametric control. In all the examples of de Sitter vacua presented in this paper, a large value of blow-up modulus invariably implies vanishing nonperturbative effects. Since these effects do not play an active role in moduli stabilization, we do not face any issue.

The log-loop corrections are presented in a conjectured form for a K3-fibered CY, based on calculations performed in the case of a torus. In addition, since the stabilization proceeds via loop corrections, the axionic moduli remains massless because the Kähler potential retains the axionic shift symmetry. Subsequently, these axions can play nontrivial roles in driving several postinflationary aspects such as reheating, and can source dark radiation and dark matter (see [20,88] for details). Moreover, it has been emphasized in [89] that if one considers the inflationary evolution of the axions, it usually gives rise to a greater variety of trajectories—with cases where the axionic evolution increases the effective number of e-folds. However, addressing the stabilization, evolution, and dynamics of the axions is beyond the scope of this paper. Furthermore, exact computation of the values of χ and λ is needed to unravel the robustness of the inflationary trajectories against additional string loop effects [90]. We save these questions for a future work.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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