

Proton gravitational structure and mass decomposition on the light front

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(Received 16 June 2025; accepted 12 November 2025; published 1 December 2025)

Gravitational form factors (GFFs) of hadrons encode essential information about the internal distributions of mass, spin, pressure, and shear among their quark and gluon constituents. We compute the quark and gluon GFFs of the proton using a fully relativistic, nonperturbative framework based on a light-front quantized Hamiltonian with quantum chromodynamics (QCD) input. This allows us to quantify the impact of a dynamical gluon on the proton's mechanical properties, such as pressure and shear distributions. Our predictions agree well with recent lattice QCD results and experimental extractions. We also determine the proton's mass and mechanical radii and address the long-standing puzzle of its mass decomposition. At the scale $\mu^2 = 4 \text{ GeV}^2$, we find that quark energy, gluon field energy, the quark condensate, and the QCD trace anomaly contribute 31.5%, 34.7%, 11.3%, and 22.5%, respectively, which are consistent with lattice QCD findings.

DOI: [10.1103/79fs-5ghd](https://doi.org/10.1103/79fs-5ghd)

I. INTRODUCTION

Understanding the internal structure of the proton, a bound state of quarks and gluons governed by quantum chromodynamics (QCD), remains a central challenge in nuclear and particle physics [1–3]. Gravitational form factors (GFFs), defined via the proton matrix elements of the QCD energy-momentum tensor (EMT), provide access to fundamental mechanical properties, such as the internal distributions of mass, spin, pressure, and shear [4–6].

The GFFs are related to the second Mellin moments of generalized parton distributions (GPDs) and can be constrained through data from deeply virtual Compton scattering and meson production [7,8]. Quark GFFs have been extracted from Jefferson Lab (JLab) measurements [9],

with future constraints expected from PANDA at facility for antiproton and ion research [10], electron–ion colliders (EICs) [1,2], nucleon-based ion collider facility [11], international linear collider, and Japan proton accelerator research complex [12]. Theoretically, quark GFFs have been studied using lattice QCD, chiral perturbation theory, light cone sum rules, QCD-inspired models, and light-front quantization [13–18].

In contrast, the gluon GFFs remain poorly constrained, both experimentally and theoretically. Near-threshold charmonium and bottomonium production offers a promising probe of the gluonic structure of the proton [19–22], with recent J/ψ data from JLab and prospects at future EICs [23–25]. These processes are sensitive to the proton's gluon GPDs through two-gluon exchange, enabling indirect access to gluon GFFs under factorization [26,27]. Analyses based on GPD fits and lattice QCD suggest dipole- or tripole-like behaviors for these form factors [28,29].

While many theoretical models focus only on quark degrees of freedom, the role of gluons is essential for a complete picture of the proton's mechanical structure. In particular, the gluon contributions to the D -term, relevant for pressure and shear forces, depend on quark-gluon interactions that are encoded in the “bad” components of the EMT [30]. Existing theoretical work on gluon GFFs

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includes lattice QCD [31,32], holographic models [33–35], and recent Bethe-Salpeter equation approaches [36]. Improved theoretical determinations of these quantities are essential for refining future measurements and advancing our understanding of the proton's structure.

In this work, we address the proton's gravitational structure using basis light-front quantization (BLFQ), a fully relativistic and nonperturbative framework for solving quantum field theory problems [37–40]. We employ an effective light-front Hamiltonian incorporating QCD interactions [41] with explicit quark (q) and gluon (g) degrees of freedom in the $|qqq\rangle$ and $|qqqg\rangle$ Fock sectors, along with a complementary three-dimensional confinement potential [42]. By solving for the mass eigenstates at low-resolution scales, we obtain the light-front wave functions (LFWFs) and extract the quark and gluon GFFs of the proton.

This approach allows us to address two key aspects of the proton structure. First, it enables a quantitative assessment of the impact of a dynamical gluon on the proton's mechanical properties, including the pressure and shear force distributions. Second, it provides a nonperturbative determination of the proton's mass decomposition into contributions from quark energy, gluon field energy, the quark scalar condensate, and the QCD trace anomaly, which can only be reliably extracted through QCD-based calculations and experimental input.

II. PROTON WAVE FUNCTIONS FROM LIGHT-FRONT QCD HAMILTONIAN

In the light-front (LF) formalism, the proton state is expanded in terms of Fock sectors as $|\Psi\rangle = \psi_{(qqq)}|qqq\rangle + \psi_{(qqqg)}|qqqg\rangle + \dots$, where the LFWFs $\psi_{(\dots)}$ encode the probability amplitudes for different partonic configurations. These LFWFs are obtained by solving the Hamiltonian eigenvalue equation, $(P^+P^- - \vec{P}_\perp^2)|\Psi\rangle = M^2|\Psi\rangle$, where $P^\pm = P^0 \pm P^3$ defines the longitudinal momentum (P^+) and the LF Hamiltonian (P^-) with M^2 being the mass squared eigenvalue.

At the initial scale, where the proton consists of $|qqq\rangle$ and $|qqqg\rangle$ components, we use the LF Hamiltonian $P^- = P_{\text{QCD}}^- + P_{\text{I}}^-$, with P_{QCD}^- encoding the relevant QCD interactions and P_{I}^- modeling confinement [40]. In the LF gauge ($A^+ = 0$), with one dynamical gluon [43,44],

$$P_{\text{QCD}}^- = \int dx^- d^2x^\perp \left\{ \frac{1}{2} \bar{\psi} \gamma^+ \frac{m_0^2 + (i\partial^\perp)^2}{i\partial^+} \psi + \frac{1}{2} A_a^i [m_g^2 + (i\partial^\perp)^2] A_a^i + g_c \bar{\psi} \gamma_\mu T^a A_a^\mu \psi + \frac{g_c^2}{2} \bar{\psi} \gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi} \gamma^+ T^a \psi \right\}, \quad (1)$$

where ψ and A_a^i are quark and gluon fields, with bare masses m_0 , m_g , and coupling g_c . T denotes the generators of the

$SU(3)$ color gauge group, and γ^μ are the Dirac matrices. A phenomenological gluon mass m_g accounts for nonperturbative effects [45]. Quark masses are renormalized via $m_q = m_0 - \delta m_q$ [46,47], with additional nonperturbative contributions parametrized by effective vertex mass m_f [48].

Confinement is implemented in the leading Fock sector [42]

$$P_{\text{I}}^- P^+ = \frac{\kappa^4}{2} \sum_{i \neq j} \left[\vec{r}_{ij}^{\perp 2} - \frac{\partial_{x_i}(x_i x_j \partial_{x_j})}{(m_i + m_j)^2} \right], \quad (2)$$

with transverse separation $\vec{r}_{ij}^{\perp 2}$ and strength κ . We omit explicit confinement in the $|qqqg\rangle$ sector, anticipating that the restricted transverse basis and inclusion of a massive gluon effectively capture essential confinement features at this stage.

We solve the LF Hamiltonian eigenvalue problem using the BLFQ framework [37], expanding the proton state in a basis composed of plane waves in the longitudinal direction (within a box of length $2L$ with antiperiodic/periodic boundary conditions for quarks/gluons), two-dimensional harmonic oscillator (2D-HO) functions $\Phi_{nm}(\vec{p}^\perp; b)$ in the transverse plane [49], and light-cone helicity spinors. Each single-particle basis state is labeled by $\vec{\alpha} = \{k, n, m, \lambda\}$, where k is the longitudinal mode (half-integer for quarks, integer for gluons, excluding zero mode), n and m are 2D-HO quantum numbers, and λ is helicity. Fock sectors with multiple color-singlet combinations, such as $|qqqg\rangle$, require an additional label.

The basis is truncated using N_{max} and $\mathcal{K}$ to render the Hamiltonian matrix finite. N_{max} limits the total 2D-HO energy: $\sum_i (2n_i + |m_i| + 1) \leq N_{\text{max}}$, while $\mathcal{K} = \sum_i k_i$ sets the longitudinal resolution, with momentum fractions given by $x_i = k_i/\mathcal{K}$.

Numerical truncations $\mathcal{K}$ and $\mathcal{N}_{\text{max}}$ define infrared and ultraviolet scales [49]. Diagonalizing the Hamiltonian gives the proton's momentum-space wave functions with helicity Λ ,

$$\Psi_{\{x_i, \vec{p}_{\perp i}, \lambda_i\}}^{N, \Lambda} = \sum_{\{n_i m_i\}} \psi^N(\{\alpha_i\}) \prod_{i=1}^N \Phi_{n_i m_i}(\vec{p}_{\perp i}, b), \quad (3)$$

where coefficients $\psi^{N=3}$ and $\psi^{N=4}$ are the components of the eigenvectors associated with $|uud\rangle$ and $|uudg\rangle$.

III. GRAVITATIONAL FORM FACTORS

The GFFs parametrize the EMT $\theta^{\mu\nu}$ for a spin-1/2 composite system as [5]

$$\begin{aligned} \langle P', \Lambda' | \theta_i^{\mu\nu}(0) | P, \Lambda \rangle &= \bar{u}(P', \Lambda') \left[A_i(Q^2) \frac{\vec{P}^\mu \vec{P}^\nu}{M} + J_i(Q^2) \frac{i \vec{P}^{\{\mu} \sigma^{\nu\} \alpha} q_\alpha}{2M} \right. \\ &\quad \left. + D_i(Q^2) \frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{4M} + \tilde{C}_i(Q^2) M g^{\mu\nu} \right] u(P, \Lambda), \end{aligned} \quad (4)$$

where $a^{\{\mu}b^{\nu\}} = (a^\mu b^\nu + a^\nu b^\mu)/2$. Here, $\bar{P}^\mu = \frac{1}{2}(P' + P)^\mu$, $\bar{u}(P', \Lambda')$ and $u(P, \Lambda)$ are Dirac spinors, M is the proton mass, and $Q^2 = -q^2$ is the squared momentum transfer in the transverse direction. Indices $i = (\varphi, \mathcal{Q})$ distinguish quark and gluon contributions, respectively, and helicities $(\Lambda, \Lambda') = (\uparrow, \downarrow)$ indicate spin orientations along the z -axis. The form factor $J_i(Q^2)$ relates to A_i and B_i by $J_i = (A_i + B_i)/2$.

The symmetric QCD EMT decomposes into quark and gluon parts, $\theta^{\mu\nu} = \sum_{i=(\varphi, \mathcal{Q})} \theta_i^{\mu\nu}$, with $\theta_q^{\mu\nu} = i\bar{\psi}\gamma^{\{\mu}\mathcal{D}^{\nu\}}\psi$ and $\theta_g^{\mu\nu} = -F^{\mu\lambda a}F_{\lambda a}^\nu + \frac{1}{4}g^{\mu\nu}(F_{\lambda\sigma a})^2$, where $\mathcal{D}^\mu = \partial^\mu + igA^\nu$ is the covariant derivative.

The matrix elements of the EMT, required to extract the GFFs, can be expressed compactly as $\mathcal{M}_{\Lambda\Lambda'}^{\mu\nu} = \langle P', \Lambda' | \theta_i^{\mu\nu}(0) | P, \Lambda \rangle$. The GFFs $A_i(Q^2)$ and $B_i(Q^2)$ are then obtained from the longitudinal components as follows:

$$\begin{aligned} \mathcal{M}_{\uparrow\uparrow}^{++} + \mathcal{M}_{\downarrow\downarrow}^{++} &= 2(P^+)^2 A_i(Q^2), \\ \mathcal{M}_{\uparrow\downarrow}^{++} + \mathcal{M}_{\downarrow\uparrow}^{++} &= \frac{iq^{(2)}}{M} (P^+)^2 B_i(Q^2). \end{aligned} \quad (5)$$

The transverse components yield the GFFs $D_i(Q^2)$ and $\bar{C}_i(Q^2)$,

$$\begin{aligned} \mathcal{M}_{\uparrow\uparrow}^{12} + \mathcal{M}_{\downarrow\downarrow}^{12} &= D_i(Q^2) q^{(1)} q^{(2)}, \\ q_\mu \mathcal{M}_{\uparrow\downarrow}^{\mu 1} + q_\mu \mathcal{M}_{\downarrow\uparrow}^{\mu 1} &= 2iM q^{(1)} q^{(2)} \bar{C}_i(Q^2). \end{aligned} \quad (6)$$

The GFF $\bar{C}_i(Q^2)$ in Eq. (6) arises from the nonconservation of the partial EMT [4]. These four GFFs can be expressed explicitly in terms of overlaps of the LFWFs as

$$\begin{aligned} A_i(Q^2) &= \frac{1}{2} \int_N x_1 \Psi'^{\Lambda*} \Psi^\Lambda, \\ iq^{(2)} B_i(Q^2) &= M \int_N x_1 \Psi'^{\Lambda*} \Psi^{-\Lambda}, \\ q^{(1)} q^{(2)} C_i(Q^2) &= \int_N \frac{\mathcal{O}_{i,\lambda\lambda'}^{12}}{x_1} \Psi'^{\Lambda*} \Psi^\Lambda, \\ 2iM q^{(1)} q^{(2)} \bar{C}_i(Q^2) &= \int_N \frac{q^{(k)} \mathcal{O}_{i,\lambda\lambda'}^{1k}}{x_1} \Psi'^{\Lambda*} \Psi^{-\Lambda}, \end{aligned} \quad (7)$$

where the struck parton's momenta are $x'_1 = x_1$ and $\vec{p}'_{i\perp} = \vec{p}_1^\perp + (1 - x_1)\vec{q}^\perp$, while for spectators $x'_i = x_i$ and $\vec{p}'_{i\perp} = \vec{p}_i^\perp - x_i\vec{q}^\perp$. The initial and final LFWFs are denoted $\Psi^\Lambda = \Psi_{\{x_i, \vec{p}_i^\perp, \lambda_i\}}^{N, \Lambda}$ and $\Psi'^{\Lambda*} = \Psi_{\{x'_i, \vec{p}'_{i\perp}, \lambda'_i\}}^{N, \Lambda*}$ and the form of $\mathcal{O}_{i,\lambda\lambda'}$ is expressed as

$$\begin{aligned} \mathcal{O}_{q,\lambda\lambda'}^{12} &= \delta_{\lambda\lambda'} \left[2\mathcal{R}^{(1)}\mathcal{R}^{(2)} + i\lambda(\mathcal{R}^{(1)}q^{(1)} - \mathcal{R}^{(2)}q^{(2)}) \right], \\ \mathcal{O}_{q,\lambda\lambda'}^{11} &= \delta_{\lambda\lambda'} \mathcal{R}^{(1)} \left(\mathcal{R}^{(1)} - i\lambda q^{(2)} \right), \\ \mathcal{O}_{g,\lambda\lambda'}^{12} &= \frac{1}{2} \delta_{\lambda\lambda'} \left[\mathcal{R}^{(1)}\mathcal{R}^{(2)} + \mathcal{R}^{(1)}q^{(2)} + q^{(1)}\mathcal{R}^{(2)} + q^{(1)}q^{(2)} \right] \\ &\quad + \frac{1}{2} i\lambda \left[(\mathcal{R}^{(2)} + q^{(2)})q^{(2)} - (\mathcal{R}^{(1)} + q^{(1)})q^{(1)} \right], \\ \mathcal{O}_{g,\lambda\lambda'}^{11} &= \frac{1}{2} \delta_{\lambda\lambda'} \left(\mathcal{R}^{(1)} + q^{(1)} \right)^2 + i\lambda \left(p^{(2)}q^{(1)} - p^{(1)}q^{(2)} \right) \\ &\quad - i\lambda \left(\mathcal{R}^{(1)}q^{(2)} + \mathcal{R}^{(2)}q^{(1)} \right) \delta_{\lambda\lambda'}, \end{aligned} \quad (8)$$

where $\mathcal{R}^{(j)} \equiv 2p^{(j)} + (1 - x_1)q^{(j)}$. The integration measure in Eq. (7) is expressed as $\int_N \equiv \sum_{N, \Lambda, \lambda_i, \lambda'_i} \prod_{i=1}^N \frac{[dx_i d^2 p_i^\perp]}{(16\pi^3)^{N-1}} \delta(1 - \sum_j x_j) \delta^{(2)}(\sum_j \vec{p}_j^\perp)$.

All calculations are performed with basis truncations $\mathcal{N}_{\max} = 9$, $\mathcal{K} = 16.5$, HO scale $b = 0.70$ GeV, and UV cutoff for the instantaneous interaction $b_{\text{inst}} = 3.00$ GeV. The Hamiltonian parameters $\{m_u, m_d, m_g, \kappa, m_f, g_c\} = \{0.31, 0.25, 0.50, 0.54, 1.80, 2.40\}$ (in GeV, except g_c) reproduce the proton mass and its electromagnetic properties [43]. We find quark probabilities of $\sim 44\%$ in $|qqq\rangle$ and $\sim 56\%$ in $|qqqg\rangle$ at the model scale.

A. Systematic uncertainty and sensitivity to model parameters

To quantify model dependence, we vary the six Hamiltonian parameters $\mathcal{P} = \{m_u, m_d, m_g, m_f, \kappa, g_c\}$ independently by a fixed fractional amount $\varepsilon = 0.10$ and propagate the induced changes to the GFFs. Let $y_0(Q^2)$ denote the central prediction of an observable evaluated on a grid $\{Q_1^2, \dots, Q_N^2\}$. For each parameter $p \in \mathcal{P}$ we compute two perturbed predictions with only p varied,

$$y_p^+(Q^2) \text{ for } p \rightarrow p(1 + \varepsilon), \quad y_p^-(Q^2) \text{ for } p \rightarrow p(1 - \varepsilon).$$

The symmetric finite difference response at each grid point is

$$\Delta_p(Q_i^2) = \frac{1}{2} |y_p^+(Q_i^2) - y_p^-(Q_i^2)|. \quad (9)$$

Assuming independent responses, we combine them in quadrature to obtain the absolute systematic uncertainty at Q_i^2 ,

$$\sigma_{\text{sys}}(Q_i^2) = \left(\sum_{p \in \mathcal{P}} \Delta_p(Q_i^2)^2 \right)^{1/2}. \quad (10)$$

The uncertainty band around the central prediction is

$$\begin{aligned} y_{\text{upper}}(Q_i^2) &= y_0(Q_i^2) + \sigma_{\text{sys}}(Q_i^2), \\ y_{\text{lower}}(Q_i^2) &= y_0(Q_i^2) - \sigma_{\text{sys}}(Q_i^2). \end{aligned} \quad (11)$$

This procedure treats the $\pm 10\%$ variations as a local linear sensitivity measure and neglects correlations among parameters. In addition to the combined uncertainty band, it is useful to quantify how sensitive the observable is to each individual parameter within the error estimate. To this end, we define a dimensionless sensitivity coefficient that measures the fractional change in the observable per unit fractional change in p ,

$$S_p(Q_i^2) = \frac{|\Delta_p(Q_i^2)|}{\varepsilon|y_0(Q_i^2)|}. \quad (12)$$

To obtain a single summary statistic for each parameter across the full Q^2 range, we compute the root mean square using trapezoidal weights on the fixed Q^2 grid,

$$\begin{aligned} w_1 &= \frac{1}{2}(Q_2^2 - Q_1^2), \\ w_i &= \frac{1}{2}(Q_{i+1}^2 - Q_{i-1}^2) \quad (2 \leq i \leq N-1), \\ w_N &= \frac{1}{2}(Q_N^2 - Q_{N-1}^2), \end{aligned} \quad (13)$$

and

$$S_p^{\text{RMS}} = \sqrt{\frac{\sum_{i=1}^N w_i S_p(Q_i^2)^2}{\sum_{i=1}^N w_i}}. \quad (14)$$

In Table I we report the parameter sensitivities S_p^{RMS} for the GFFs A , D , and $\bar{C}$ on a fixed grid with $0 \leq Q^2 \leq 3.61$. Sensitivity is the fractional response of the observable to a unit fractional change in a parameter. Values close to 1 mean that a 10% change in the parameter produces a change of about 10% in the observable. Values much smaller than 1 indicate weak influence, whereas values larger than 1 indicate strong influence.

For A , the total sensitivities are modest. The dominant driver is the coupling g_c with $S_{g_c}^{\text{tot}} \approx 0.98$, followed by m_f with $S_{m_f}^{\text{tot}} \approx 0.27$ and κ with $S_{\kappa}^{\text{tot}} \approx 0.24$. Smaller totals are obtained for m_u (≈ 0.14) and m_d (≈ 0.06). Considering the

TABLE I. S_p^{RMS} sensitivities of GFFs A , D , and $\bar{C}$ on a fixed Q^2 grid, shown for quark (S_p^q), gluon (S_p^g), and total (S_p^{tot}) contributions.

Parameter	A			D			$\bar{C}$		
	S_p^q	S_p^g	S_p^{tot}	S_p^q	S_p^g	S_p^{tot}	S_p^q	S_p^g	S_p^{tot}
m_u	0.65	1.01	0.14	0.53	0.99	0.35	6.86	2.80	4.70
m_d	0.66	0.90	0.06	0.94	0.70	0.11	9.01	4.00	2.89
m_g	0.60	0.91	0.10	0.92	0.84	0.11	3.99	4.62	4.49
m_f	0.95	0.71	0.27	1.13	0.66	0.17	3.82	4.68	4.91
κ	0.88	0.67	0.24	0.97	0.65	0.20	8.41	2.08	1.98
g_c	0.60	1.68	0.98	2.86	2.51	2.66	19.05	16.15	6.30

quark and gluon contributions separately, the response to g_c is sizeable, with $S_{g_c}^q \approx 0.60$ and $S_{g_c}^g \approx 1.68$.

For D , the coupling g_c dominates with $S_{g_c}^{\text{tot}} \approx 2.66$. All other totals are much smaller: $S_{m_u}^{\text{tot}} \approx 0.35$, $S_{\kappa}^{\text{tot}} \approx 0.20$, $S_{m_f}^{\text{tot}} \approx 0.17$, and $S_{m_d}^{\text{tot}} \approx 0.11$. The quark contribution to the response for g_c is $S_{g_c}^q \approx 2.86$ and the gluon contribution is $S_{g_c}^g \approx 2.51$, indicating strong sensitivity of D to the interaction strength and weak dependence on the mass parameters within the tested range.

For $\bar{C}$, the sensitivities are large across the board, indicating substantial model dependence. The total is largest for g_c with $S_{g_c}^{\text{tot}} \approx 6.30$, followed by m_f with $S_{m_f}^{\text{tot}} \approx 4.91$, m_u with $S_{m_u}^{\text{tot}} \approx 4.70$, m_d with $S_{m_d}^{\text{tot}} \approx 2.89$, and κ with $S_{\kappa}^{\text{tot}} \approx 1.98$. The quark and gluon contributions for g_c are $S_{g_c}^q \approx 19.05$ and $S_{g_c}^g \approx 16.15$, and for several other parameters they lie between 1 and 10, underscoring that $\bar{C}$ is the most parameter-sensitive of the three GFFs.

To address confinement effects in the $|qqqg\rangle$ sector, which are implemented implicitly via the truncated transverse basis and an effective gluon mass m_g , we assess the impact of m_g by varying it by $\pm 10\%$ and recomputing the GFFs. The grid-averaged sensitivities in Table I show a small total response for A and D ($S_{m_g}^{\text{tot}} \approx 0.10$ and 0.11), but a pronounced response for $\bar{C}$ ($S_{m_g}^{\text{tot}} \approx 4.49$). Thus, a $\pm 10\%$ change in m_g translates to about $\pm 1\%$ in the totals of A and D , and roughly $\pm 45\%$ in the total of $\bar{C}$. These effects are propagated consistently to all figures and derived observables in the manuscript.

B. Numerical results

Figure 1 presents the proton GFFs $A(Q^2)$ and $D(Q^2)$ at $\mu^2 = 4 \text{ GeV}^2$, showing separate quark and gluon contributions. We compare our results with lattice QCD and experimental extractions based on various phenomenological approaches. Our $D(Q^2)$ and its quark and gluon components agree well with lattice QCD [29], while $A(Q^2)$, particularly the quark contribution, shows some deviation at high Q^2 [29,50]. The quark form factor $D_q(Q^2)$ agrees with JLab data [9,31], and the gluon form factor $D_g(Q^2)$ aligns with extractions from near-threshold vector meson photoproduction [22,23,51,52], as well as with string-based predictions [53] and Faddeev equation results [36]. Our gluon GFFs are consistent with experimentally extracted results based on the holographic QCD approach (Duran *et al.* I [23]), but differ from those obtained using a GPD-inspired analysis (Duran *et al.* II [23]). A recent GPD fit to GlueX data [22,52] shows better agreement with our $A_g(Q^2)$. Since $B(Q^2) \approx 0$, we have $J(Q^2) \approx A(Q^2)/2$ (see Appendix A). At $Q^2 = 0$, we find $A_q(0) = 0.54(9)$, $A_g(0) = 0.46(10)$, and $D_q(0) = -1.74(29)$, $D_g(0) = -2.04(43)$. Consequently, the total D -term at $Q^2 = 0$ is $D(0) = -3.78 \pm 0.52$. A detailed

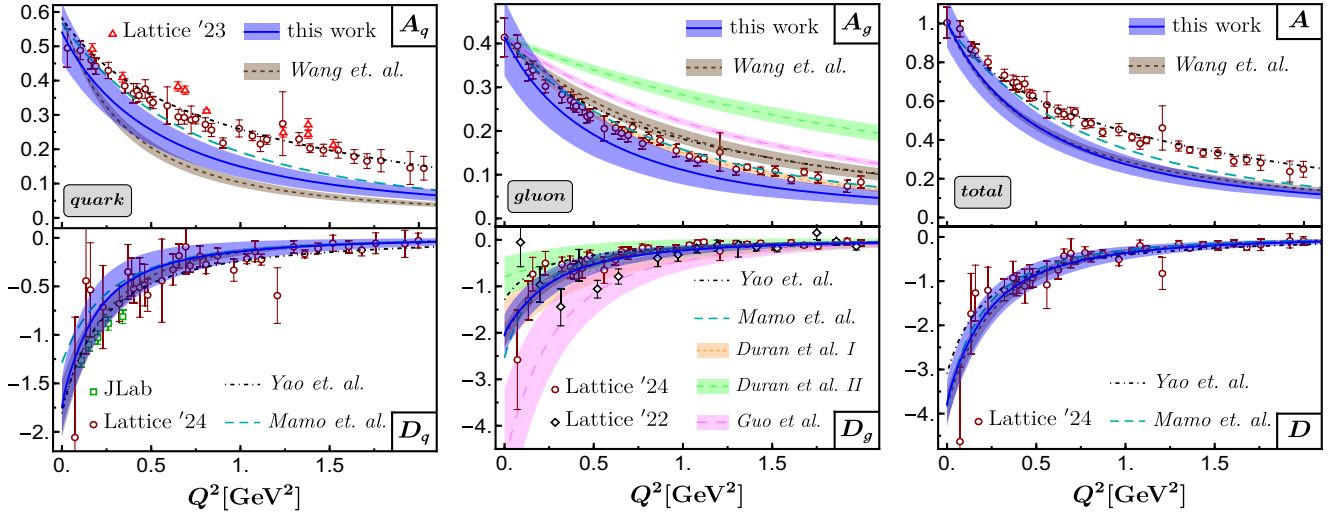


FIG. 1. Proton GFFs $A(Q^2)$ and $D(Q^2)$ at $\mu^2 = 4 \text{ GeV}^2$, separated into quark (q) and gluon (g) contributions. Our results (—) are compared with lattice QCD ($\diamond$ [28], $\triangle$ [50], and $\circ$ [29]), rescaled JLab data for $D_q(Q^2)$ ($\square$ [9,31]), and theoretical models: vector meson photoproduction (— [51]), Faddeev equation (--- [36]), string-based model (--- [53]), and multipole parametrizations (--- [23] and --- [52]). All $A_q(Q^2)$ curves—ours and all comparisons—are rescaled to $A_q(0) = 0.414$. The shaded band in our results indicates the 10% uncertainty arising from varying the Hamiltonian parameters, as described in Sec. III A.

comparison of our total $D(0)$ with lattice QCD and other theoretical studies is presented in Appendix A.

Figure 2 presents GFF $\tilde{C}(Q^2)$, highlighting opposite signs for quark ($\tilde{C}_q < 0$) and gluon ($\tilde{C}_g > 0$). The sum rule $\tilde{C}(Q^2) = 0$ is approximately satisfied at $Q^2 = 0$: $\tilde{C}(0) \approx 0$, with $\tilde{C}_q(0) = -0.11(52)$ and $\tilde{C}_g(0) = 0.11(51)$.

1. Mechanical densities and radii

Defined through the EMT and by analogy with classical mechanical systems, the GFF $D(Q^2)$ provides insight into the internal pressure and shear force distributions in the proton [5]. These mechanical distributions are expressed as [54]

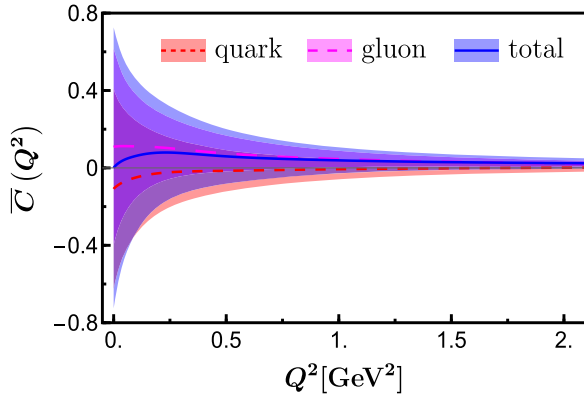


FIG. 2. Proton GFF $\tilde{C}(Q^2)$, together with its quark and gluon components, shown as functions of Q^2 . The shaded band shows the 10% model uncertainty obtained by varying the Hamiltonian parameters, as detailed in Sec. III A.

$$\mathcal{P}(b) = \frac{1}{16M} \frac{1}{b} \frac{d}{db} \left[b \frac{d}{db} \tilde{D}(b) \right],$$

$$\mathcal{S}(b) = -\frac{1}{8M} b \frac{d}{db} \left[\frac{1}{b} \frac{d}{db} \tilde{D}(b) \right], \quad (15)$$

where $\tilde{D}(b) = \frac{1}{(2\pi)^2} \int d^2 \vec{q}^\perp e^{-i\vec{q}^\perp \cdot \vec{b}^\perp} D(q^2)$ and $b = |\vec{b}^\perp|$.

Figure 3 shows the pressure $[2\pi b \mathcal{P}(b)]$ and shear $[2\pi b \mathcal{S}(b)]$ distributions, derived using the fitted GFF parameters provided in Appendix A (see Table III). The pressure distribution is positive at small b and negative at larger b , consistent with mechanical equilibrium [5], while the shear distribution remains positive, indicating local stability [5]. For comparison, we use lattice dipole fit parameters from Ref. [29] to perform the two-dimensional Fourier transform, with results labeled “lattice-fit” in Fig. 3. Our quark distributions have lower peak magnitudes compared to the lattice fit, whereas our gluon distributions exhibit higher peaks. This pattern follows from the behavior of the D -term: relative to the lattice fits, $D_q(Q^2)$ shows a steeper fall-off in Q^2 , while $D_g(Q^2)$ falls off more gently. After Fourier transform, a steeper fall-off produces a milder curvature of $\tilde{D}(b)$ near small b and hence a lower quark pressure peak, whereas a gentler fall-off leads to stronger curvature and a higher gluon peak—the same trend appears in the shear distributions. These differences reflect our current effective Hamiltonian and truncations (finite Fock sector and a limited transverse harmonic-oscillator basis) and are expected to diminish as the basis is enlarged and higher-order interactions are incorporated. Despite the magnitude differences, the qualitative behavior remains consistent with lattice-based results.

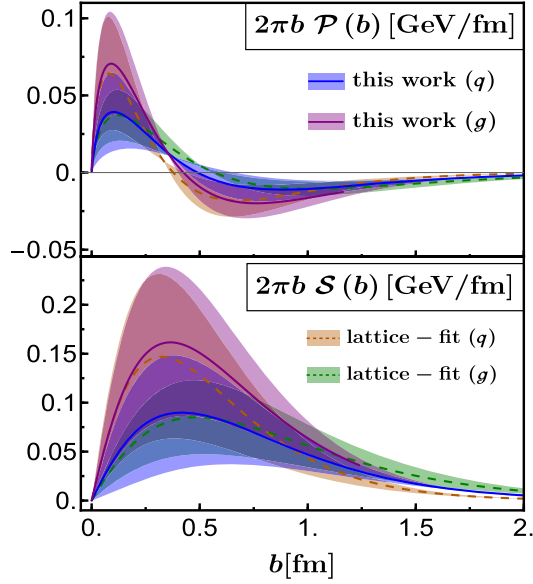


FIG. 3. Pressure (upper) and shear (lower) distributions compared with lattice QCD results. The “lattice-fit” curves are obtained using the dipole fit parameters from Ref. [29].

The mechanical and mass radii are determined from the GFFs [5,23],

$$\begin{aligned} \langle r_{\text{mech}}^2 \rangle &= 6D(0) \left[\int_0^\infty dQ^2 D(Q^2) \right]^{-1}, \\ \langle r_{\text{mass}}^2 \rangle &= \frac{6}{A(0)} \frac{dA}{dt} \Big|_{t=0} - \frac{3}{2A(0)} \frac{D(0)}{M^2}. \end{aligned} \quad (16)$$

We obtain quark mechanical and mass radii of 0.78(16) fm and 0.76(4) fm, respectively, consistent with lattice and experimental results [9,29]. The gluon mechanical radius, 0.70(9) fm, lies between existing lattice estimates [28,29], while the gluon mass radius, 0.82(7) fm, agrees with recent lattice and phenomenological studies [23,29,52]. The total mechanical and mass radii, 0.73(7) fm and 0.79(4) fm, are in close agreement with lattice results [29] and vector meson photoproduction data [51], and are slightly below the proton charge radius [55]. See Fig. 4 for comparison.

IV. PROTON MASS DECOMPOSITIONS

Ji proposed a decomposition of the hadron mass, in analogy with the virial theorem, by splitting the QCD EMT into traceless and trace parts [56],

$$T^{\mu\nu} = \bar{T}^{\mu\nu} + \hat{T}^{\mu\nu}. \quad (17)$$

In the Hamiltonian formalism, the mass is the total energy in the rest frame, so the forward matrix elements of the temporal components satisfy

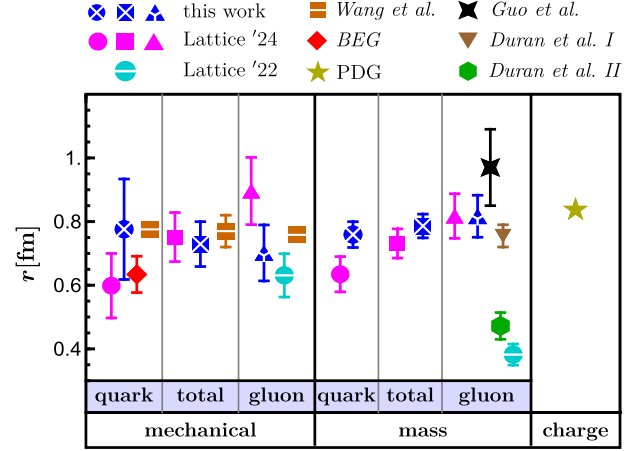


FIG. 4. Proton mechanical and mass radii compared with lattice QCD results [28,29]. The quark mechanical radius is shown alongside the BEG result [9], the gluon mass radius with extractions by Duran *et al.* [23] and Guo *et al.* [52], and the total mechanical radius with Wang *et al.* [51]. The charge radius is from PDG [55].

$$\langle \bar{T}^{00} \rangle = \frac{3}{4} M, \quad \langle \hat{T}^{00} \rangle = \frac{1}{4} M, \quad (18)$$

which means three quarters of M arise from the traceless part and one quarter from the trace.

The traceless part is further split into quark and gluon contributions,

$$\bar{T}^{\mu\nu} = \bar{T}_q^{\mu\nu} + \bar{T}_g^{\mu\nu}, \quad (19)$$

with forward matrix elements

$$\langle P | \bar{T}_q^{\mu\nu} | P \rangle = a(\mu^2) \frac{P^\mu P^\nu - \frac{1}{4} g^{\mu\nu} M^2}{M}, \quad (20)$$

$$\langle P | \bar{T}_g^{\mu\nu} | P \rangle = [1 - a(\mu^2)] \frac{P^\mu P^\nu - \frac{1}{4} g^{\mu\nu} M^2}{M}, \quad (21)$$

where

$$a(\mu^2) = \sum_f \int_0^1 dx x [q_f(x, \mu^2) + \bar{q}_f(x, \mu^2)] \quad (22)$$

is the quark momentum fraction from the deep inelastic scattering momentum sum rule. Consequently,

$$\langle \bar{T}_q^{00} \rangle = \frac{3}{4} a M, \quad \langle \bar{T}_g^{00} \rangle = \frac{3}{4} (1 - a) M. \quad (23)$$

For the trace part, define the quark mass fraction

$$b \equiv \frac{4 \langle \hat{T}_m^{00} \rangle}{M}, \quad (24)$$

so that the anomaly piece contributes

$$\langle \hat{T}_a^{00} \rangle = \frac{1}{4}(1-b)M. \quad (25)$$

Putting these ingredients together, Ji's four-term decomposition of the proton mass reads

$$M = M_q + M_g + M_m + M_a, \quad (26)$$

with the explicit contributions (at renormalization scale μ and for n_f active flavors)

$$M_q = \frac{3}{4} \left(a - \frac{b}{1 + \gamma_m(\mu)} \right) M, \quad (27)$$

$$M_g = \frac{3}{4}(1-a)M, \quad (28)$$

$$M_m = \frac{1}{4} \frac{4 + \gamma_m(\mu)}{1 + \gamma_m(\mu)} b M, \quad (29)$$

$$M_a = \frac{1}{4}(1-b)M, \quad (30)$$

where $\gamma_m(\mu)$ is the quark mass anomalous dimension. In the GFF notation used here,

$$a = A_q(0), \quad b = A_q(0) + 4\bar{C}_q(0), \quad (31)$$

so the four mass fractions are determined directly from the forward gravitational form factors $A_i(0)$ and $\bar{C}_i(0)$.

Following Lorcé [57], the forward matrix elements of the EMT contain two distinct pieces: internal energy and pressure-volume work. In the standard trace and Ji decompositions these contributions are mixed: the pressure terms governed by $\bar{C}_i$ can shift the apparent quark-gluon shares, and the traceless/trace split entangles the mass (internal energy) and pressure-volume budgets, which obscures the physical interpretation. To keep the picture clear, Lorcé advocates separating the mass (internal energy) budget from the pressure-volume budget and attributing the mass to two constituents only, quarks and gluons. In the forward limit this leads to the internal energy representation

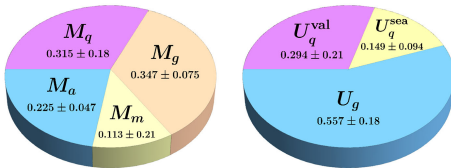


FIG. 5. Proton mass decomposition following Ji's four-term scheme [56] and the two-term scheme of Ref. [57]. Values are given in units of M at $\mu^2 = 4 \text{ GeV}^2$. In the two-term decomposition, the valence and sea quark contributions are shown separately, with $U_q = U_q^{\text{val}} + U_q^{\text{sea}}$.

TABLE II. Proton mass decomposition using Ji's four-term scheme [56] at $\mu^2 = 4 \text{ GeV}^2$, presented as percentages of the proton mass M . Our results are compared with those from lattice QCD [58].

	M_q (%)	M_g (%)	M_m (%)	M_a (%)
This work	31.5	34.7	11.3	22.5
Lattice [58]	32	36	9	23

$$U_i \equiv [A_i(0) + \bar{C}_i(0)]M, \quad M = U_q + U_g, \quad (32)$$

with the constraints $A_q(0) + A_g(0) = 1$ and $\bar{C}_q(0) + \bar{C}_g(0) = 0$ ensuring $U_q + U_g = M$. This two-term representation isolates the quark and gluon internal energy content of the proton mass. In the Lorcé framework, the pressure-volume work carried by each constituent is $p_i V = -\bar{C}_i(0)M$, and the total satisfies $\sum_i p_i V = 0$ for a stable hadron. Hence, the two-term mass budget $M = U_q + U_g$ contains only internal energy, while pressure-volume work is kept separate.

Figure 5 shows the proton mass decompositions computed from the forward-limit gravitational form factors, i.e., the values $A_i(0)$ and $\bar{C}_i(0)$. In Ji's four-term decomposition, the gluon, quark energy, quark mass, and trace anomaly contributions are 34.7%, 31.5%, 11.3%, and 22.5%, respectively. We adopt an anomalous quark mass dimension $\gamma_m \simeq -0.15$ for $n_f = 3$ [57]. In the two-term (internal energy) representation, the gluon share is 55.7%, and the quark share is 44.3%, split into valence (29.4%) and sea (14.9%) components. Table II compares our four-term results at $\mu^2 = 4 \text{ GeV}^2$ with lattice QCD findings [58], showing good agreement. In both schemes, the gluon contribution is dominant, followed by quark energy, trace anomaly, and quark mass terms.

It is worth noting that any direct, one-to-one comparison with lattice QCD is delicate because our Hamiltonian uses effective inputs (constituent masses and couplings). In this framework the mass decomposition is reconstructed from the forward gravitational form factors rather than computed independently: we use $A_i(0)$ and $\bar{C}_i(0)$, the $Q^2 = 0$ limits of A and $\bar{C}$, to assemble Ji's four-term decomposition and the two-term internal energy split in a fixed scheme at a specified scale. The trace anomaly contribution is then inferred from these forward limits together with the chosen $\gamma_m(\mu)$. We, therefore, do not claim a first-principles determination of the anomaly within this effective setup.

V. SUMMARY

We solved the light-front QCD Hamiltonian for the proton, including both the constituent three-quark and the three-quark–one-gluon Fock sectors. The resulting light-front wave functions were used to compute the quark and gluon contributions to the proton's GFFs and mass

decomposition. We quantified the role of a dynamical gluon in shaping the proton's mechanical properties, including pressure and shear distributions. Our predictions show good agreement with recent lattice QCD results and experimental extractions across several key aspects of the proton's internal structure.

We extracted the proton's mass and mechanical radii as 0.79(4) fm and 0.73(7) fm, respectively. The corresponding quark contributions are 0.76(4) fm and 0.78(16) fm, while the gluon contributions are 0.82(7) fm and 0.70(9) fm. These values align well with recent lattice and phenomenological analyses.

We also provided a nonperturbative determination of the proton's mass decomposition, finding contributions from quark energy (31.5%), gluon field energy (34.7%), the quark condensate (11.3%), and the QCD trace anomaly (22.5%), consistent with lattice QCD. Additionally, we analyzed the decomposition using an internal energy-based two-term scheme, confirming the dominant role of gluons (55.7%) in the proton mass.

Our long-term goal is a fully *ab initio* BLFQ framework that eliminates effective inputs and employs a systematically renormalized light-front QCD Hamiltonian. For a recent step toward this goal, see Ref. [59], which reports the first nucleon light-front wave functions as eigenstates of the QCD light-front Hamiltonian in a fully relativistic, nonperturbative framework without an explicit confining potential.

ACKNOWLEDGMENTS

C.M. acknowledges support from the new faculty startup funding provided by the Institute of Modern Physics, Chinese Academy of Sciences (Grant No. E129952YR0). X.Z. is supported by the new faculty startup funding at the Institute of Modern Physics, Chinese Academy of Sciences; the Key Research Program of Frontier Sciences, Chinese Academy of Sciences (Grant No. ZDB-SLY-7020); the Foundation for Key Talents of Gansu Province; the Central Funds Guiding Local Science and Technology Development of Gansu Province (Grant No. 22ZY1QA006); the National Natural Science Foundation of China (Grant No. 12375143); the National Key R&D Program of China (Grant No. 2023YFA1606903); the Strategic Priority Research Program of the Chinese Academy of Sciences (Grant No. XDB34000000). This research also received support from the Gansu International Collaboration and Talents Recruitment Base of Particle Physics (2023–2027), the Senior Scientist Program funded by Gansu Province (Grant No. 25RCKA008), and the International Partnership Program of the Chinese Academy of Sciences (Grant No. 016GJHZ2022103FN). J.P.V. is supported by the U.S. Department of Energy (Grant No. DE-SC0023692). Computational resources were primarily provided by the Sugon Advanced Computing Center.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX: GRAVITATIONAL FORM FACTORS: D -TERM AT $Q^2=0$, $B(Q^2)$ AND $J(Q^2)$, AND PARAMETRIC FITS

Figure 6 compares our calculated total proton D -term, $D(0) = -3.78 \pm 0.52$, with lattice QCD and other theoretical predictions. Most results are consistent within our uncertainty, particularly the lattice QCD results from the dipole fit [29], which align closely with our calculation.

Figure 7 presents the GFFs $B(Q^2)$ and $J(Q^2)$ as functions of Q^2 , separately indicating the quark and gluon contributions. At $Q^2 = 0$, we obtain the values $J_q(0) = 0.27(02)$, $J_g(0) = 0.22(02)$, $J(0) = 0.50(01)$,

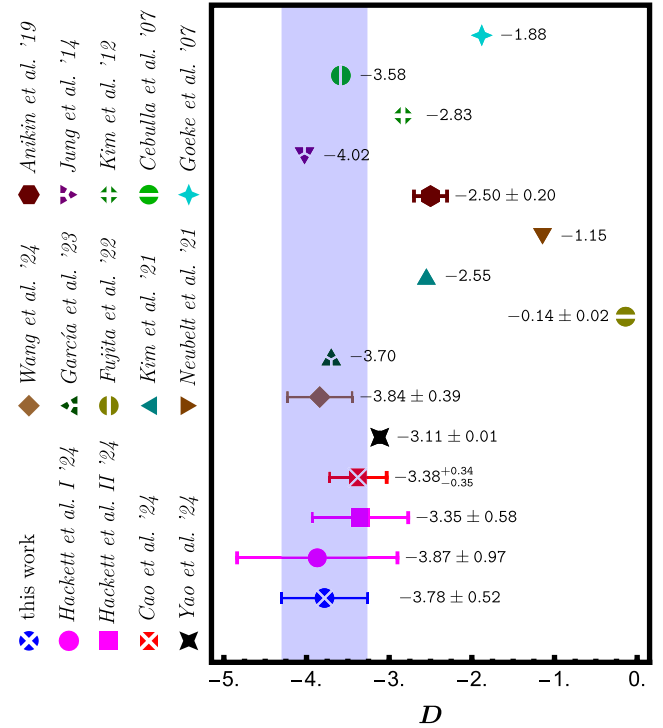


FIG. 6. Comparison of our total proton D -term at $Q^2 = 0$ (light blue error band) with lattice QCD calculations from Hackett *et al.* [29], labeled as “I” (dipole fit) and “II” (z-expansion fit), and predictions from other theoretical methods: dispersive analysis (Cao *et al.* [60]), Faddeev equation (Yao *et al.* [36]), vector meson photoproduction (Wang *et al.* [51]), holographic QCD (Fujita *et al.* [61]), bag model (Neubelt *et al.* [62]), light-cone sum rules (Anikin *et al.* [63]), Skyrme models (García *et al.* [64], Kim *et al.* [65], and Cebulla *et al.* [66]), and chiral quark soliton models (Kim *et al.* [67], Goeke *et al.* [68], and Jung *et al.* [69]).

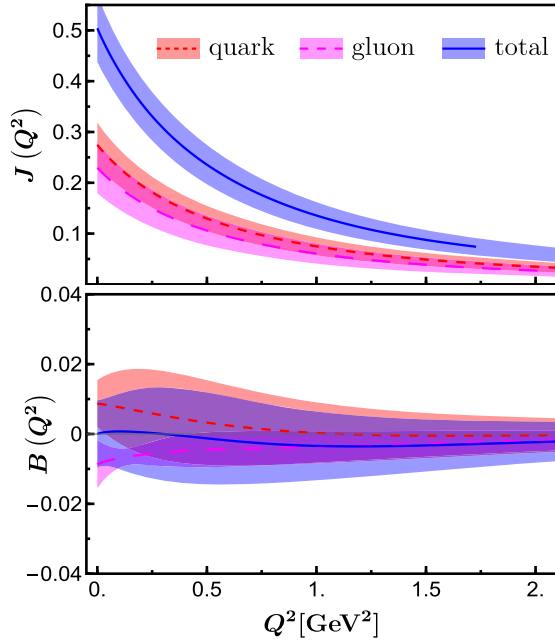


FIG. 7. Proton's GFFs $B(Q^2)$, $J(Q^2) = (A(Q^2) + B(Q^2))/2$, and their quark and gluon components as functions of Q^2 .

$$B_q(0) = 0.0087(15), \quad B_g(0) = -0.0087(13), \quad \text{and} \\ B(0) = 0.0000(20).$$

We fit the GFFs $A(Q^2)$, $D(Q^2)$, and $J(Q^2)$ using a dipole function

$$f_{\text{dipole}}(Q^2) = \frac{a_0}{(1 + a_1 Q^2)^2}. \quad (\text{A1})$$

The resulting fit parameters and reduced chi-square values (χ^2_{red}) are presented in Table III. The fit quality is evaluated using the reduced chi-square, computed as

$$\chi^2_{\text{red}} = \frac{1}{N - N_p} \sum_i \frac{(O_i - C_i)^2}{\sigma_{\text{tot},i}^2}, \quad (\text{A2})$$

where N is the number of data points, $N_p = 2$ is the number of fit parameters, O_i are the GFF values calculated within our BLFQ framework, C_i are the fitted values, and $\sigma_{\text{tot},i}^2$ represents the total variance obtained by adding the

TABLE III. Dipole fit parameters a_0 , a_1 , and reduced χ^2_{red} for the quark, gluon, and total contributions to the GFFs $A(Q^2)$, $D(Q^2)$, and $J(Q^2)$.

GFF	a_0	a_1	χ^2_{red}
$A_q(Q^2)$	0.537(40)	0.900(1)	0.019
$A_g(Q^2)$	0.456(29)	0.924(11)	0.046
$A(Q^2)$	0.996(11)	0.910(4)	1.441
$D_q(Q^2)$	-1.74(35)	2.559(10)	0.056
$D_g(Q^2)$	-2.04(42)	1.931(9)	0.514
$D(Q^2)$	-3.77(75)	2.181(0)	0.276
$J_q(Q^2)$	0.273(21)	0.911(1)	0.016
$J_g(Q^2)$	0.224(15)	0.932(10)	0.047
$J(Q^2)$	0.499(6)	0.920(5)	1.450

TABLE IV. Tripole fit parameters a_0 , a_1 , and reduced χ^2_{red} for the quark, gluon, and total contributions to the GFFs $A(Q^2)$, $D(Q^2)$, and $J(Q^2)$.

GFF	a_0	a_1	χ^2_{red}
$A_q(Q^2)$	0.530(40)	0.528(0)	1.448
$A_g(Q^2)$	0.450(29)	0.542(6)	1.146
$A(Q^2)$	0.979(49)	0.534(2)	2.624
$D_q(Q^2)$	-1.71(35)	1.494(6)	3.928
$D_g(Q^2)$	-2.01(41)	1.127(5)	0.334
$D(Q^2)$	-3.72(75)	1.272(0)	1.307
$J_q(Q^2)$	0.270(21)	0.534(1)	1.407
$J_g(Q^2)$	0.221(15)	0.547(6)	1.027
$J(Q^2)$	0.490(25)	0.540(1)	2.461

original data uncertainties and propagated fitting uncertainties in quadrature. For reference and comparison, Table IV shows the parameters obtained by fitting the same GFFs using a tripole form,

$$f_{\text{tripole}}(Q^2) = \frac{a_0}{(1 + a_1 Q^2)^3}. \quad (\text{A3})$$

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