

Mesonic modes in a confining model at finite temperature

A. E. Radzhabov^{1,*} and X. L. Shang^{2,†}

¹*Matrosov Institute for System Dynamics and Control Theory SB RAS, Irkutsk 664033, Russia*

²*Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China*



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The mass spectrum of pseudoscalar and scalar mesons at finite temperature is studied within a nonlocal quark model with Polyakov loop extension. The analysis is performed in the leading order of the $1/N_c$ expansion. To suppress unphysical quark production thresholds in meson polarization loops in the hadronic phase, the quark propagator is modified after the inverse Laplace transform. In the deconfined phase, the quark pole is treated separately, while the same modification applies to the remaining part. For a consistent description of both confining and deconfining phases, an adjustment of the transformed propagator is proposed. At finite temperature, Lorentz symmetry breaking leads to the splitting of meson modes. The screening masses are computed over a wide temperature range, and the pole masses up to the deconfinement transition. The results are compared with lattice QCD data and show reasonable agreement.

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I. INTRODUCTION

One of the most interesting features of strong interactions is confinement, which prevents the appearance of fundamental color degrees of freedom in the physically observed mass spectrum. The only *ab initio* method describing confinement is lattice QCD calculations. As lattice QCD calculations produce numerical data, effective modeling provides a complementary framework for theoretical understanding of dynamics [1,2].

The symmetries of strong interactions provide the basis for constructing effective models such as the Nambu-Jona-Lasinio (NJL) model [3] and chiral perturbation theory (ChPT) [4]. One of the important symmetries of strong interactions is chiral symmetry, which is spontaneously broken by the quark condensate. The NJL model incorporates the feature of spontaneous and explicit chiral symmetry breaking by the quark condensate and the current quark masses, as in QCD. On the other hand, the NJL model operates with a constant constituent quark mass, which seems a rather rough approximation.

In the nonlocal version of the NJL model, e.g., [5–12], the quark mass function is momentum dependent, which makes it similar to Dyson-Schwinger studies of QCD with full quark and gluon (ghost) propagators and their

interactions [13,14]. In order to make calculations tractable, an approximation is required, typically the rainbow-ladder scheme for the Dyson-Schwinger equation, together with the corresponding Bethe-Salpeter equation for mesonic bound states. The feature of quark confinement can be implemented in the nonlocal NJL model by requiring that the quark propagator (or a part of it) be an entire function [15]. A similar scheme is also used in DSE studies [16–18]. However, the extension of this scheme to finite temperature is not a simple task since at some point the phase transition to quark matter should occur and quark should appear as a real degrees of freedom. This type of scheme is sometimes called technical or analytical confinement, since it does not follow from first principles [19].

In the previous work, a modification was suggested that involves a cutoff of a part of the quark propagator after the inverse Laplace transform [20]. On the other hand, to model the deconfinement phase transition at a certain temperature, this cutoff should be removed for the real quark pole. Due to this, a first-order phase transition occurs at the critical temperature. In order to remove this unwanted feature, it is suggested that the four-quark coupling constants be different in the hadronic phase and quark matter, so that the phase transition at finite temperature becomes second order. Here, another possibility is considered: to use the modification of the Laplace transform to synchronize different phases. The calculations are performed in the leading order of the $1/N_c$ expansion, i.e., no corrections from meson dressing to the quark propagator are included. Since the presented approach is aimed at removing quark thresholds, and the dynamics of the gluon is represented only via an effective four-quark interaction, the additional element of the model is the Polyakov loop potential [21], which describes the

*Contact author: AERadzhabov@yandex.ru

†Contact author: shangxinle@impcas.ac.cn

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dynamics of the gauge degrees of freedom at finite temperature. The background field is minimally coupled to the quark degrees of freedom and, at finite temperature, realizes the finite-temperature aspect of confinement: it suppresses the quark contribution to thermodynamic observables. As an approximation, the gap equation for the background field is taken in the same form as in the model without the confinement prescription for the quark propagator. As a result, the phase diagram is the same as in the nonlocal PNJL model without the confinement prescription for the quark propagator [22]. As predictions of the model, the mass spectra of pions and sigma mesons at finite temperature are discussed. Since Lorentz invariance is broken by the heat bath, different modes appear, which correspond to screening and temporal masses. The pion field should be renormalized in order for the screening mass to be smooth across the phase transition. The results are compared with lattice calculations for the screening masses and with other models for screening or temporal masses.

II. NONLOCAL MODEL

The $SU(2)$ nonlocal chiral quark model with separable interaction in the pseudoscalar-scalar channels is described by the Lagrangian:

$$\mathcal{L} = \bar{q}(x)(i\hat{d} - M_c)q(x) + \frac{G}{2} \left((J_S^a(x))^2 + (J_P^a(x))^2 \right), \quad (1)$$

$M_c = \text{diag}(m_c, m_c)$ is the current quark mass matrix, G is the four-quark coupling constant, and $J_{S,P}^a(x)$ denote the quark currents. Following the NJL model [3], the interaction kernel is taken to be chirally symmetric. However, its current-current interaction is taken in a nonlocal one-gluon-exchange-like (OGE) form [6,11,12,23]:

$$J_M^a(x) = \int d^4x_1 d^4x_2 \delta\left(x - \frac{x_1 + x_2}{2}\right) \times g((x_1 - x_2)^2) \bar{q}(x_1) \Gamma_M^a q(x_2), \quad (2)$$

with $M = S, P$ and $\Gamma_S^a = \lambda^a$, $\Gamma_P^a = i\gamma^5 \lambda^a$. For the $SU(2)$ model, the flavor matrices are Pauli matrices: $\lambda^a \equiv \tau^a$, $a = 0, \dots, 3$ with $\tau^0 = 1$. The nonlocal form factor is taken in the Gaussian form, $g(k^2) = \exp(-k^2/\Lambda^2)$ in momentum space. The parameter Λ is related to the characteristic scale of spontaneous symmetry breaking.¹

In the mean-field approximation, the thermodynamic potential of the nonlocal model in a medium is [24,25]

¹The numerical calculations are performed using the parameter set from scheme II of [11], which yields $\langle \bar{q}q \rangle = -(240 \text{ MeV})^3$. The corresponding values are $m_c = 5.8$, $m_d = 424$, and $\Lambda = 752.2 \text{ MeV}$.

$$\Omega^{MF} = \frac{m_d^2}{2G} - 4 \sum_{i=0,\pm} \int_{k,n} \ln \left[k_{n,i}^2 + m^2(k_{n,i}^2) \right] + U(\Phi, \bar{\Phi}) + \Omega_0, \quad (3)$$

where $m(k^2) = m_c + m_d g(k^2)$, $k_{n,i} = (\omega_n^i - i\mu)^2 + \mathbf{k}^2$ and $\int_{k,n} \equiv T \sum_n \frac{d^3k}{(2\pi)^3}$. Fermionic Matsubara frequencies ω_n^i are partially shifted due to the presence of a Polyakov loop: $\omega_n^\pm = \omega_n \pm \phi_3$, $\omega_n^0 = \omega_n$ and $\omega_n = (2n+1)\pi T$. The thermodynamic potential is divergent due to the current quark contribution. The infinite normalization constant Ω_0 is chosen such that the pressure vanishes under vacuum conditions, i.e., at zero temperature, zero chemical potential, and the vacuum value of m_d . There are different versions of the Polyakov loop potential $U(\Phi, \bar{\Phi})$. In this paper, the modified logarithmic Polyakov loop potential is used [22].

In the presence of the Polyakov loop, the equations of motion for the mean fields are

$$\frac{\partial \Omega^{MF}}{\partial m_d} = 0, \quad \frac{\partial \Omega^{MF}}{\partial \phi_3} = 0. \quad (4)$$

The first of these equations (4), i.e., the gap equation for m_d , has the form

$$\frac{\partial \Omega^{MF}}{\partial m_d} = \frac{m_d}{G} - 8 \sum_{i=0,\pm} \int_{k,n} g(k_{n,i}^2) m(k_{n,i}^2) D(k_{n,i}^2), \quad (5)$$

where the scalar propagator is introduced

$$D(k^2) = \frac{1}{k^2 + m^2(k^2)}. \quad (6)$$

This is the complete model in the mean-field formulation, and in the next section the confining extension will be considered. In what follows, we use the results of this model: the temperature dependence of m_d , ϕ_3 , and the screening pion mass.

III. CONFINING PRESCRIPTION

The analytical structure of the scalar propagator is crucially dependent on the particular form of $g(k^2)$. For the Gaussian case, it has infinitely many singularities, which is similar to DSE studies [26,27]; for the other form factors the presence of cuts is also possible [9]. The scalar propagator for the case with only pole singularities can be represented in the form

$$D(k^2) = \sum_{i=1}^{\infty} \frac{R_{z_i}}{k^2 + z_i}, \quad (7)$$

where z_i are the pole positions and R_{z_i} are the corresponding residues, which can be found numerically once the form

of the quark propagator in (6) is fixed. For convenience, the poles are ordered starting from the smallest in absolute value. For different model parameters, there can be either infinitely many complex-valued singularities, or the first two poles can be real valued. The absence of real-valued solutions is sometimes interpreted as a sign of confinement [5,7,28], since a similar situation is present in the gauge sector in the (refined)-Gribov-Zwanziger scenario [29–31]. However, as discussed in our previous paper [20], the presence of complex-valued solutions leads to meson polarization loops with cusps, which does not seem compatible with confinement.

The general idea for modeling quark confinement is to modify this part of the quark propagator in order to remove singularities. In [20], this step is performed using the prescription of the inverse Laplace transform $\mathbf{D}(\alpha)$ of the momentum-space propagator $D(k^2)$. The exact form of $\mathbf{D}(\alpha)$ can be obtained by expanding (6) in powers of m_d and then taking the inverse Laplace transform of each term. The leading terms of such expansions are

$$D(k^2) \approx \frac{1}{k^2 + m_c^2} - 2m_c m_d \frac{\exp(-k^2/\Lambda_c^2)}{(k^2 + m_c^2)^2} + O(m_d^2), \quad (8)$$

and after the inverse Laplace transform lead to

$$\mathbf{D}(\alpha) \approx \exp(-\alpha m_c^2) - 2m_c m_d \alpha_1 \exp(-\alpha_1 m_c^2) \theta(\alpha_1) + O(m_d^2), \quad (9)$$

where $\alpha_m = \alpha - m/\Lambda^2$. The θ function in the second and higher-order terms implies that the small- α asymptotics is unaffected by nonperturbative corrections. The cut of the transform is

$$\mathbf{D}_R(\alpha) = \mathbf{D}(\alpha) \theta(1/\Lambda_c^2 - \alpha). \quad (10)$$

The scale parameter Λ_c can be associated with the confinement scale. For the Gaussian form factor $g(k^2)$, this implies that the number of terms in the expansion is limited by the condition $n < \Lambda^2/\Lambda_c^2$ due to the θ function in the higher-order terms. Performing the Laplace transform of each term, one obtains the series in momentum space

$$\begin{aligned} \mathbf{D}_R(k^2) \approx & \frac{1 - \exp(-(k^2 + m_c^2)/\Lambda_c^2)}{k^2 + m_c^2} \\ & - 2m_c m_d \frac{\exp(-k^2/\Lambda_c^2) - ((k^2 + m_c^2)(1/\Lambda_c^2 - 1/\Lambda^2) + 1) \exp(-(k^2 + m_c^2)/\Lambda_c^2 + m_c^2/\Lambda_c^2)}{(k^2 + m_c^2)^2} + O(m_d^2). \end{aligned} \quad (11)$$

One immediately sees that the denominators have only one singular point at $k^2 = -m_c^2$, while the numerators vanish there as well. Consequently, each term is finite at this point, and both the individual terms and the entire sum represent entire functions. Alternatively, one may apply the inverse Laplace transform to the sum in (7),

$$\mathbf{D}(\alpha) = \sum_{i=1}^{\infty} R_{z_i} \exp(-\alpha z_i). \quad (12)$$

After the prescription (10) and the Laplace transform, they lead to a different representation of the entire function $\mathbf{D}_R(k^2)$ [20]:

$$\mathbf{D}_R(k^2) = D(k^2) - \sum_{i=1}^{\infty} \frac{R_{z_i} Q(k^2 + z_i)}{k^2 + z_i}, \quad (13)$$

where $Q(k^2) = \exp(-k^2/\Lambda_c^2)$. Since $Q(0) \equiv 1$, all singularities in the sum in (7) of the original scalar propagator cancel out. The quark mass function $m_R(k^2)$ is defined as

$$m_R(k^2) = \sqrt{D_R^{-1}(k^2) - k^2}, \quad (14)$$

where the square root introduces branch points and cuts in the complex plane.

The set of parameters used leads to only a complex-valued solution in vacuum. With increasing temperature, there appear two real solutions, say z_1 and z_2 . As the temperature increases further, z_1 tends to m_c^2 , while its residue R_{z_1} tends to 1, whereas z_2 seems unphysical since it tends to infinity at high temperatures and its residue has the wrong sign. Therefore, the appearance and temperature behavior of the pole z_1 correctly reflect the deconfinement feature of QCD, namely, that at high temperatures the quarks should be free particles. In order to realize deconfinement, one can keep the pole z_1 and omit the $i = 1$ term in the subtraction sum in (13), starting the sum from $i = 2$, since this pole corresponds to the physical quark pole. Thus, $\mathbf{D}_R(k^2)$ can be rewritten in the form

$$\begin{aligned} \mathbf{D}_R(k^2) = & \frac{R_{z_1}}{k^2 + z_1 - i\epsilon} \\ & + \left[D(k^2) - \frac{R_{z_1}}{k^2 + z_1} - \sum_{i=2}^{\infty} \frac{R_{z_i} Q(k^2 + z_i)}{k^2 + z_i} \right], \end{aligned} \quad (15)$$

where the bracketed expression is an entire function.

Therefore, at this point, there are two phase structures: in the hadronic phase, there are no pole singularities [(13) or (11)], while in the deconfined phase, there is one physical singularity (15). Naively, one can set a condition to switch between phases based on features of $D(k^2)$: for m_d with only complex-conjugated poles, the confined-phase propagator is (13), and the appearance of real poles corresponds to deconfinement with the deconfined propagator (15). This happens at $m_d \approx 319$ MeV, but at this point z_1 and z_2 are equal. In practice, it is better to use a criterion that the difference between the two real poles is of the order of the first one, i.e., in the deconfined phase, $z_2 - z_1 > z_1$. This corresponds to $m_d \approx 310$ MeV [20].

To obtain the thermodynamic potential compatible with the structure of this quark propagator, one can integrate the gap equation (5). However, this simple prescription leads to a problem. Specifically, near the phase transition point between the confining and deconfining phases, the subtracted part differs, and, consequently, the gap equation yields different results in this region. As a consequence, the thermodynamic potential exhibits two possible solutions near the transition, which results in a first-order phase transition at finite temperature. This is clearly an artifact of the approximation.

In previous work [20], two different values of the four-quark coupling constant were suggested to compensate for the difference between the integrands in the gap equations (5) for different phases. In the present paper, a different idea is suggested—namely, one can modify the Laplace transform of the scalar propagator in order to make the gap equation smooth as a function of m_d while passing the phase transition. The guiding principle is related to the normalization of the mean-field gap equation of the original model without the confining extension.

In the confined phase, as follows from (13), the subtraction in the scalar propagator leads to a decrease in the gap equation. Therefore, to preserve its magnitude, this contribution must be compensated. The most convenient way to achieve this is to add an extra function directly to the Laplace-parameter function $\mathbf{D}_R(\alpha)$. This is illustrated in Fig. 1 for the parameter set with $\Lambda^2/\Lambda_c^2 = 5$. The red dashed curve represents $\mathbf{D}_R(\alpha)$ and exhibits the following properties: it behaves as $\exp(-m_c^2\alpha)$ at small α and possesses a cut at $1/\Lambda_c^2$. The additional function is required to lie between $\alpha_{\min}$ and $\alpha_{\max}$. These limits are constrained by the requirement not to spoil the asymptotic freedom and confinement properties, i.e., $\alpha_{\min} > 1/\Lambda^2$ and $\alpha_{\max} < 1/\Lambda_c^2$. As the simplest candidate, we consider a triangular pulse function: it rises linearly from zero to a peak value H_{Add} on $[\alpha_{\min}, \alpha_{\text{cen}}]$, then decreases linearly back to zero on $[\alpha_{\text{cen}}, \alpha_{\max}]$, and vanishes elsewhere,

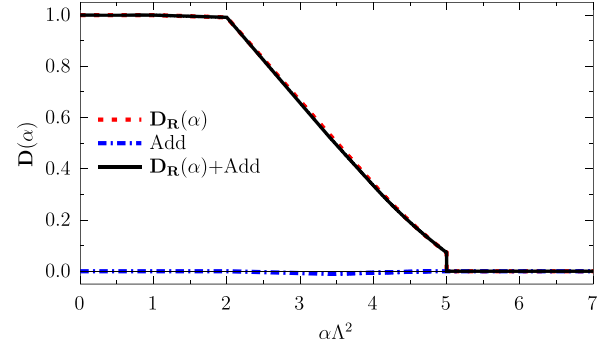


FIG. 1. Alpha-space functions obtained via the inverse Laplace transform of the scalar propagator in vacuum ($m_d = 424$ MeV): the red dashed line represents $\mathbf{D}_R(\alpha)$ with $\Lambda_c^2 = \Lambda^2/5$, the blue dash-dotted line shows the additional contribution, and the solid line corresponds to the total expression.

$$\text{Add}(\alpha) = \begin{cases} 2H_{\text{Add}} \frac{\alpha - \alpha_{\min}}{\alpha_{\max} - \alpha_{\min}}, & \alpha_{\min} \leq \alpha \leq \alpha_{\text{cen}}, \\ 2H_{\text{Add}} \frac{\alpha_{\max} - \alpha}{\alpha_{\max} - \alpha_{\min}}, & \alpha_{\text{cen}} < \alpha \leq \alpha_{\max}, \\ 0, & \text{otherwise,} \end{cases} \quad (16)$$

The pulse is taken to be symmetric, i.e., $\alpha_{\text{cen}} = (\alpha_{\max} + \alpha_{\min})/2$. Specifically, the values $\alpha_{\min} = 2/\Lambda^2$ and $\alpha_{\max} = 4.8/\Lambda^2$ are chosen. The height H_{Add} is fitted for a given m_d to reproduce the value of the original gap equation (5) in vacuum or in medium without applying the prescription (10). Thus, in vacuum H_{Add} is found to be -0.01 , which means that this contribution is tiny since the overall scale is set by $\mathbf{D}_R(0) = 1$. This can be clearly seen in Fig. 1. At finite temperature, the role of the additional contribution increases and becomes maximal near the phase transition point on the confined side, i.e., slightly above 310 MeV. Here H_{Add} is around 0.08, and immediately after the transition it drops to only -0.009 . This is illustrated in Fig. 3 for $m_d = 311$ and 309 MeV. The resulting temperature behavior of H_{Add} is shown in Fig. 2.

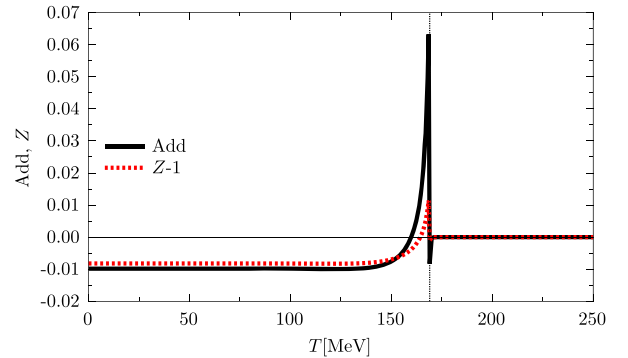


FIG. 2. Temperature dependence of the triangular pulse height H_{Add} of the additional contribution $\mathbf{D}_R^{\text{Add}}(\alpha)$ to the quark propagator and of the pion renormalization factor Z .

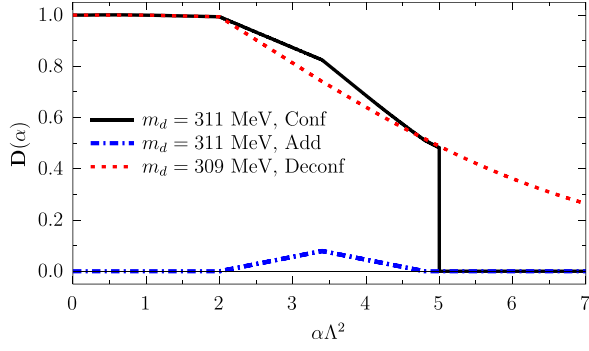


FIG. 3. Alpha-space function obtained via the inverse Laplace transform of the scalar propagator at finite temperature near the phase transition: the solid line represents the total expression in the confinement phase ($m_d = 311$ MeV) $\mathbf{D}_R(\alpha) + \text{Add}(\alpha)$, with the blue dash-dotted line showing the additional contribution; the red dashed line corresponds to the deconfinement phase $\mathbf{D}_R(\alpha) + \text{Add}(\alpha)$ ($m_d = 309$ MeV).

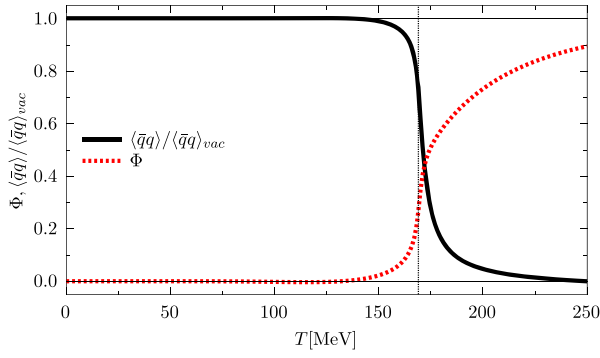


FIG. 4. Quark condensate normalized to its vacuum value (solid black line) and Polyakov loop (red dashed line) as functions of temperature. The thin vertical dashed line denotes the critical temperature.

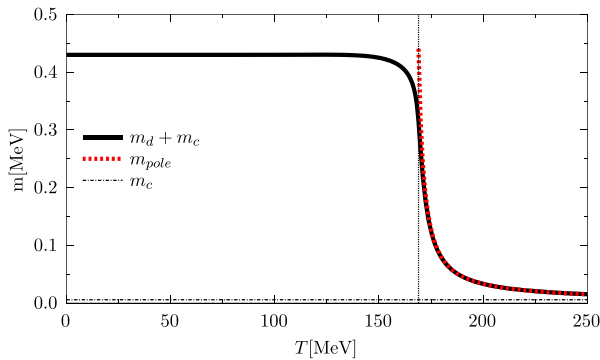


FIG. 5. Finite-temperature behavior of the current quark mass, $m_d + m_c$ (where m_d is the mass scale), and the quark pole mass. The thin vertical dashed line denotes the critical temperature.

Additionally, as an approximation, the value of ϕ_3 is taken to be the same as in the original model. The quark condensate and Polyakov loop are given in Fig. 4. Figure 5 shows the behavior of the sum $m_d + m_c$. Above the critical temperature, a pole appears in the quark propagator, and the corresponding behavior of the quark mass is also shown in Fig. 5.

IV. MESON LOOPS AT FINITE T

The pion polarization loop at finite T and μ has the form

$$\Pi_\pi((\nu_M - i\mu_{ab})^2, \mathbf{p}^2) = 8 \sum_{i=0,\pm} \int_{k,n} g^2(q^2) D(k_-^2) D(k_+^2) \times [(k_- \cdot k_+) + m(k_+^2)m(k_-^2)], \quad (17)$$

where quark momenta are

$$k_+ = (\omega_n^i - i\mu_a + \nu_M, \mathbf{k} + \zeta \mathbf{p}), \\ k_- = (\omega_n^i - i\mu_b, \mathbf{k} - (1 - \zeta) \mathbf{p}), \quad (18)$$

where a and b are quark flavors, and $\mu_{ab} = \mu_a - \mu_b$ is the meson chemical potential. The relations $p = k_+ - k_-$ and $q = (k_+ + k_-)/2$ for the four-momenta hold. In the general case, the flow of three-momentum in the diagram is arbitrary, with $0 < \zeta < 1$. For a scalar meson, the plus sign in brackets should be changed to a minus.

At finite temperature, Lorentz invariance is broken due to the appearance of the heat bath collective motion vector u . Therefore, the dependence of correlations on the directions parallel and perpendicular to this vector should be studied. It is convenient to choose this vector as $u = (1, \mathbf{0})$, and therefore consider the zero component and three-momentum separately. One can consider two limiting cases:

- (i) In the first case, the meson has only the zero component of momentum (or the fourth component in the Euclidean formulation). It corresponds to an analytic continuation from real Matsubara frequencies to complex values $\nu_M = iM_\pi^{\text{pol}}$:

$$-G^{-1} + \Pi_\pi(-(M_\pi^{\text{pol}})^2, 0) = 0. \quad (19)$$

This case is referred to as the pole, temporal, or dynamical mass [32–35].

- (ii) The other case, that of screening masses of mesons, is connected with the zero energy component, i.e., the zero Matsubara mode, and the polarization loop is analytically continued to complex values of the three-momentum squared $\mathbf{p}^2 = -(M_\pi^{\text{scr}})^2$:

$$-G^{-1} + \Pi_\pi(0, -(M_\pi^{\text{scr}})^2) = 0. \quad (20)$$

This definition is connected with the asymptotic behavior of the correlation function and corresponds to the screening of the potential by meson exchange in coordinate space, $\exp(-M_\pi^{\text{scr}} r)$, [25,35–41].

In the limit of zero temperature, the masses should coincide: $M_\pi^{\text{pol}} = M_\pi^{\text{scr}}$.

Since the properties of the gap equation are closely related to those of the pion, the subtraction of some part from the gap equation means that the pion will change its mass. Unfortunately, the scheme with the additional contribution (16) does not resolve this issue. Therefore, in the model with prescriptions (10) and (16), the modification must also be applied to the meson sector. A possible solution is to adjust the normalization of the pion vertex as $g(p) \Rightarrow Zg(p)$. With a constant Z , the pion screening mass would exhibit a discontinuity at the phase transition, which seems unphysical even if it is small. Therefore, we consider the case where Z has an effective temperature dependence and determine it by fitting the pion screening mass to the mean-field result obtained with the quark propagator (6). Figure 2 shows the finite-temperature behavior of H_{Add} and $Z - 1$. Both quantities display a similar pattern: they are much smaller than unity, indicating that they represent only a correction; they remain at their vacuum values up to ~ 130 MeV, reach a maximum just before the phase transition, change sign at the transition point, and vanish afterwards. The same Z factor is used for the σ meson.

In lattice calculations, usually only screening masses are calculated. The pole masses are sensitive to the lattice size N_t , and it is hard to estimate them reliably. The calculation of pole masses in the nonlocal model is also not straightforward. Formally, one can perform the Matsubara sum in (17) numerically. However, for pole masses this procedure may be problematic, since the quark propagator exhibits singularities: in the confined phase, there are complex-plane poles [for (6)] or cuts [for (14)], whereas in the deconfined phase a real-valued pole emerges. One must therefore either verify that these singularities do not affect the result or devise a suitable way to handle them. A possible approach is to convert the Matsubara sum of a function $F(i\omega_n)$ into a contour integral [9,38]:

$$T \sum_{n=-\infty}^{\infty} F(i\omega_n) = \frac{i}{2\pi} \int_{-i\infty+\delta}^{i\infty+\delta} dz F(z) f(z) - \frac{i}{2\pi} \int_{-i\infty-\delta}^{i\infty-\delta} dz F(z) f(z), \quad (21)$$

where δ is an infinitesimal shift and the auxiliary function $f(z)$ is defined as

$$f(z) = \frac{1}{1 + \exp(z/T)}. \quad (22)$$

The function $F(z)$ also involves an integration over d^3k , and the sum is modified by the Polyakov loop shift. A potential issue arises if the singularities in the complex

plane cross the integration contour, i.e., move between quadrants $3 \leftrightarrow 4$ or $1 \leftrightarrow 2$, either due to the d^3k integration or as the external momentum M^{pol} increases. This occurs when M^{pol} exceeds half of the lowest threshold value, $\frac{1}{2} \min_i M_{\text{thr},i}$, where

$$M_{\text{thr},i}^2 = 2\text{Re}(-z_i) + 2\sqrt{\text{Re}(z_i)^2 + \text{Im}(z_i)^2}, \quad (23)$$

and the minus sign originates from the definition of the pole singularities in (6). Expression (23) generalizes the standard threshold condition for the onset of an imaginary part (when a particle mass exceeds twice the constituent mass) to the case of complex-valued singularities. For cuts, z_i

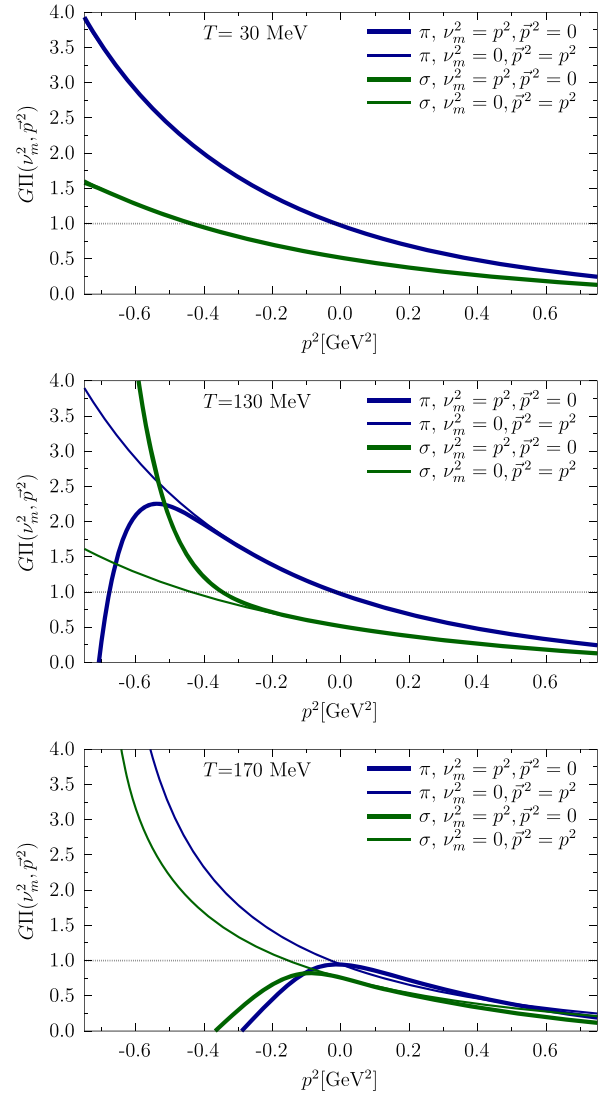


FIG. 6. Pion (blue) and σ -meson (green) polarization loops multiplied by the four-quark coupling constant at finite temperatures $T = 30, 130$, and 170 MeV. Thick (thin) lines correspond to temporal (spatial) correlations. The thin dashed horizontal line denotes 1, which corresponds to the meson pole.

denotes the position of the branch point. The factor $1/2$ arises because, at finite temperature, the external ν_M cannot be split arbitrarily without altering the fermion-loop statistics. In this work, we only verify that the cuts in (14) do not affect the calculation of the π and σ pole masses in the confined region. For the calculation of heavier particles, an additional scheme should be implemented.

The behavior of the meson polarization loops is shown in Fig. 6 for $T = 30, 130$, and 170 MeV. Two cases are considered: longitudinal and transverse with respect to the heat bath motion, i.e., the temporal and screening correlation functions. At low temperature ($T = 30$ MeV), Lorentz-breaking effects are almost invisible, and the corresponding masses coincide. As the temperature increases, the behavior of the temporal polarization loops for large timelike momentum starts to change, and as a result, the pole mass of the scalar meson is modified. In Fig. 6 for $T = 130$ MeV, one can see this situation, and, additionally, a second heavy solution appears in the pseudoscalar channel. However, since the considered model includes only the ground states of mesons, the appearance of a second solution cannot be discussed on such simple grounds. After deconfinement ($T = 170$ MeV), the temporal correlation functions change their behavior drastically, and no real-valued solution can be found. The complex-valued solution, i.e., unstable mesonic states after deconfinement, will be studied in a separate paper.

V. LATTICE

Screening correlations at finite temperatures have been extensively studied in lattice calculations; see, for example,

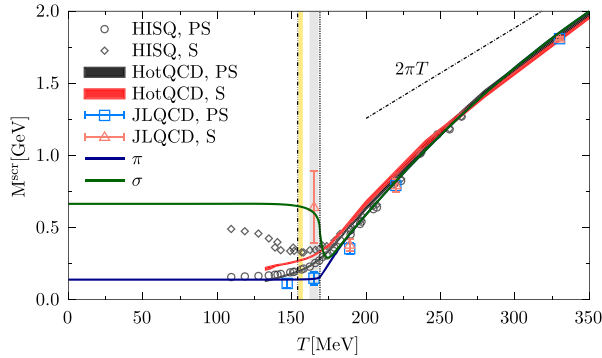


FIG. 7. Behavior of screening meson masses as a function of the temperature: blue lines correspond to the pion and green lines to the σ meson in the nonlocal model; shaded curves represent HotQCD calculations for pseudoscalar (black) and scalar (red) mesons; circles (pseudoscalar) and diamonds (scalar) correspond to calculations with the HISQ action, while boxes (pseudoscalar) and triangles (scalar) are the JLQCD results; the dash-dotted line is the asymptotic $2\pi T$. The thin vertical dashed line denotes the critical temperature in the nonlocal model; the gray shaded region corresponds to the JLQCD result, the yellow shaded region to the HotQCD result, and the dash-dotted line to the central value from calculations with the HISQ action.

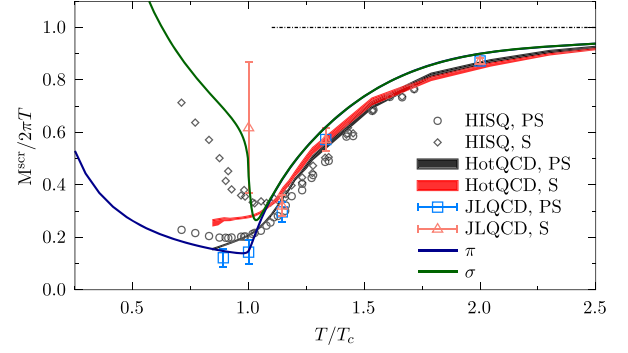


FIG. 8. Behavior of screening meson masses scaled to the asymptotic limit, $M/2\pi T$, as a function of the scaled temperature T/T_c : blue lines correspond to the pion and green lines to the σ meson in the nonlocal model; shaded curves represent HotQCD calculations for pseudoscalar (black) and scalar (red) mesons; circles (pseudoscalar) and diamonds (scalar) correspond to calculations with the HISQ action, while boxes (pseudoscalar) and triangles (scalar) are the JLQCD results.

[42–51]. A comparison of the model calculations of the pion and σ -meson screening masses with lattice results [47,49,51] is shown in Fig. 7 as a function of temperature. The continuum extrapolation was performed by the HotQCD collaboration [49], while results obtained with the HISQ action [47] and by the JLQCD Collaboration [51] are presented for different lattice sizes. The model reproduces the pion screening mass in reasonable agreement with lattice data. For the scalar meson, due to the restoration of chiral symmetry, the large- T behavior is similar to that of the pion and therefore consistent with lattice results, while the low- T results are found to agree with the JLQCD data, unlike the HotQCD data. The difference in the scalar channel on the lattice at the lowest simulated points, as discussed in [51] for the HotQCD calculations, tends to twice the pion mass in the low- T limit due to a lattice artifact of the staggered quark action [49]. In [51], which uses the Möbius domain-wall fermion formulation, this artifact is absent.

Since the critical temperature is slightly different for these calculations—HotQCD: 156 ± 1.5 MeV, HISQ: 154 ± 9 MeV [52], JLQCD: 165 ± 3 MeV, and 169 MeV in the nonlocal model—it is interesting to examine the situation as a function of the scaled temperature T/T_c in Fig. 8. Additionally, the masses are scaled by $2\pi T$, which represents the asymptotic value at high T . It can be seen that the difference in critical temperatures does not significantly affect the agreement between the model results and lattice calculations.

VI. SCREENING AND POLE MASSES

In effective models, simultaneous calculations of pole and screening masses are not a trivial task. This is due to the nonperturbative nature of QCD. There are the following

problems: in NJL-like models, the regularization should be Lorentz invariant from the beginning, i.e., one cannot use a three-dimensional cutoff scheme. In nonlocal models or DSE studies, the Matsubara summation is problematic due to the complicated structure of the quark propagator. Confinement should also be included in the hadronic phase, with a transition to quark matter at high temperature.

Therefore, in different models, only the temporal [53] or only the screening [25,36–41,54] masses are calculated. Both the pole and screening masses are estimated in the PV-regularized NJL model [33] (pion and σ), the PNJL model [35] (scalar and pseudoscalar mesons), and the quark-meson model [55] (for the pion). In the confining model, the calculation of both modes is possible.

The results of our calculations for the pion and sigma pole and screening masses are shown in Fig. 9. One can see that at low T , the pole and screening masses coincide. Around 100 MeV, the pole mass of the σ meson starts to decrease. Although the magnitude and sign of the effect cannot be predicted from general considerations, some Lorentz-breaking effect may nevertheless appear. The insensitivity of pion properties to temperature changes is a consequence of its Goldstone nature. Specifically, from a technical standpoint, the pion polarization loop contains two subtracted contributions that exhibit nearly identical temperature dependence. Consequently, the pion remains almost massless and largely insensitive to thermal effects. In contrast, the σ -meson polarization loop has a positive sign, which makes it strongly sensitive to temperature variations. Above ~ 140 MeV, the pole mass of the σ meson continues to decrease, while the pion pole mass begins to increase. The pion screening mass starts to rise near the phase transition. After the phase transition, the screening masses of the pion and sigma become degenerate. However, for the pole masses, no real solution can be

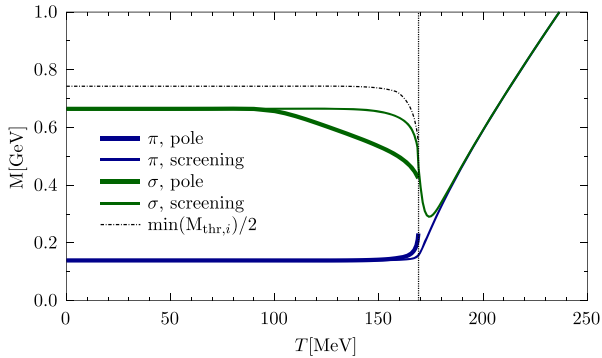


FIG. 9. Finite-temperature behavior of the pole (thick) and screening (thin) meson masses: blue lines correspond to the pion and green lines to the σ meson. The thin dash-dotted line indicates the threshold value from (23), below which the quark-mass branch points do not affect the pole masses of the mesons. The thin vertical dashed line denotes the critical temperature.

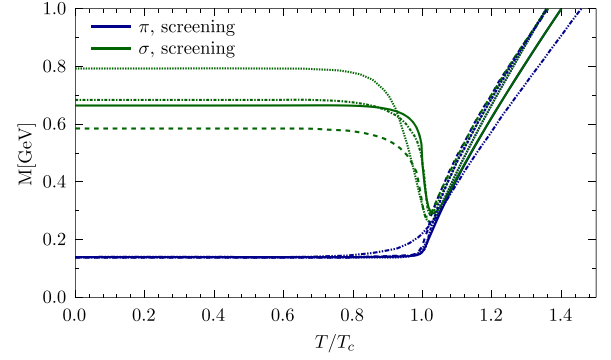


FIG. 10. Behavior of the screening meson masses as a function of the scaled temperature T/T_c in nonlocal models: blue lines correspond to the pion and green lines to the σ meson. Solid lines correspond to our model, dotted line to the $SU(2)$ PNJL model that includes wave function renormalization and vector interactions [25], dash-dotted line to the $SU(2)$ PNJL (set A) [38], dashed lines to the $SU(3)$ PNJL calculations [37], and dash-double-dotted line to the calculations in the rank-1 separable model [36].

found after T_c , and we plan to investigate the complex-valued poles in future work.

We begin by comparing our results with those of nonlocal models that are similar to the present one but without the confining extension. Since different approaches yield different critical temperatures, it is more convenient to plot the results as functions of the scaled temperature T/T_c . Figure 10 shows the comparison of our screening masses with the PNJL model in the $SU(2)$ [25] and $SU(3)$ [37] versions (set A from [38]) as well as with the separable model from DSE-BSE studies (rank-1) [36]. For the pion, only the results of [36] show a slightly earlier increase in mass; this is probably due to the inclusion of gauge dynamics via the Polyakov-loop extension in the other models. The vacuum mass of the σ meson is model

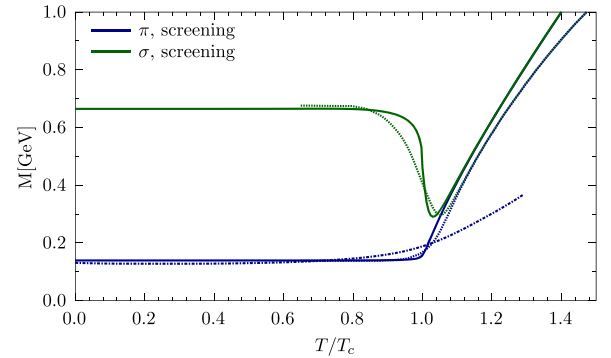


FIG. 11. Behavior of the screening meson masses as a function of the scaled temperature T/T_c : blue lines correspond to the pion and green lines to the σ meson. Solid lines correspond to the nonlocal model, dashed lines to the NJL calculations with PV regularization [35], and dash-dotted lines to the calculations in the quark-meson model [55].

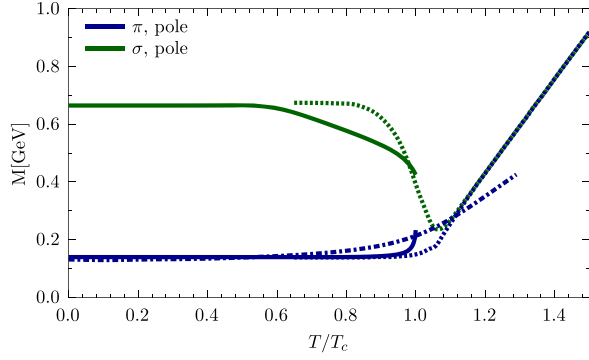


FIG. 12. Behavior of the pole meson masses: blue lines correspond to the pion and green lines to the σ meson. Solid lines correspond to the nonlocal model, dashed lines to the NJL calculations with PV regularization [35], and dash-dotted lines to the calculations in the quark-meson model [55].

dependent, so differences in absolute values are expected. The temperature dependence is qualitatively similar overall, with minor deviations possibly originating from wave-function renormalization [25] or mixing with strange quarks [37]. The closest agreement is found with Set A of [38], which is natural since our model shares the same parametrization of the quark sector. Above the phase transition, chiral symmetry is restored, and the π and σ masses become nearly degenerate. The curvature is also comparable; accordingly, the ordering of the curves at the top of the plot near the right end of the x -axis reflects the inverse ordering of the critical temperatures: the highest- T_c group corresponds to [25,37,38] (202–208 MeV), followed by our result (169 MeV), and finally that of [36] (146 MeV).

We then proceed to compare with models that provide both screening and pole masses. A comparison of the

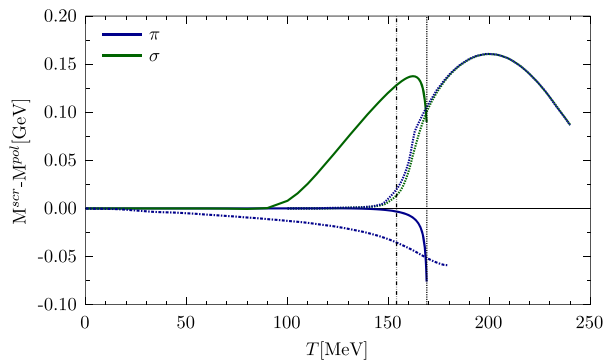


FIG. 13. Difference between the screening and pole meson masses at finite temperature: blue lines correspond to the pion and green lines to the σ meson. Solid lines correspond to the nonlocal model, dashed lines to the NJL calculations with PV regularization [35], and dash-dotted lines to the calculations in the quark-meson model [55]. The thin vertical dashed line denotes the critical temperature in the nonlocal model, while the dash-dotted line corresponds to the value used in [35].

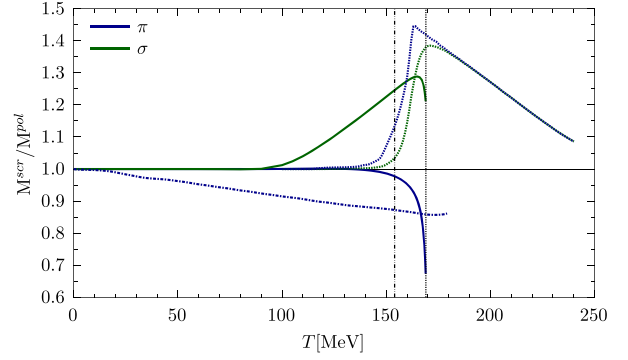


FIG. 14. Ratio of the screening to pole meson masses at finite temperature: blue lines correspond to the pion and green lines to the σ meson. Solid lines correspond to the nonlocal model, dashed lines to the NJL calculations with PV regularization [35], and dash-dotted lines to the calculations in the quark-meson model [55]. The thin vertical dashed line denotes the critical temperature in the nonlocal model, while the dash-dotted line corresponds to the value used in [35].

screening masses calculated in the confining model with those from the PNJL model with PV regularization [35] and the quark-meson model [55] is shown in Fig. 11. The corresponding comparison for the pole masses is presented in Fig. 12. One can see that in the PNJL model [35], in the scalar sector, the difference between the pole and screening masses below the critical temperature is much smaller—they both start to decrease smoothly around 130 MeV, i.e., about 20 MeV below T_c . In the PNJL model [35], the pion masses start to increase near the phase transition, while in the quark-meson model [55] the increase begins at $T \sim 100$ MeV. Figure 13 shows the difference between the screening and pole masses. One can see that in the local PNJL model, this difference is always positive, while in the nonlocal model it is positive for the σ meson but negative for the pion—i.e., the pole mass of the pion is larger than its screening mass. In the quark-meson model, this difference is also negative for the pion. The ratio of the screening to pole masses is shown in Fig. 14. In the nonlocal model, the mass difference is maximal near the phase transition, reaching up to about 30%. In the PNJL model, this occurs after the phase transition. In the quark-meson model, this ratio increases slowly in magnitude with temperature.

VII. CONCLUSIONS

In the previous paper, the confining extension of the nonlocal model was considered. It is based on a cut of the Laplace transform of the quark propagator with an additional dimensionful parameter and, to a large extent, leads to representing a part of the quark propagator as an entire function. In order to model the deconfinement phase transition at finite temperature, it was suggested to consider the quark propagator as a real quark pole with an additional

entire function contribution. However, in this case, an inconsistency appears between the hadronic and quark matter phases. The temperature dependence of the four-quark coupling constants was introduced to compensate for this inconsistency.

Here, a different scheme for this compensation is proposed: the Laplace-transformed function is modified to reproduce the gap equation in the model without the confining extension. As an approximation, the original ϕ_3 gap equation is used. The mass spectrum of the π and σ mesons at finite temperature is studied, including both the screening and pole masses. The screening and pole masses of the pion and σ meson are considered. The calculations of the pole masses are performed up to the deconfinement phase transition point, beyond which the mesons become unstable due to quark-antiquark dissociation, and no real-valued solution for the pole masses can be found. The pion pole mass is rather stable against temperature variations and only starts to increase in the vicinity of the phase transition. In contrast, the σ meson pole mass begins to decrease already around 100 MeV.

In the future, we plan to extend our model to finite chemical potential and include subleading $1/N_c$ terms. The $1/N_c$ corrections will introduce hadronic dressing with

corresponding widths for unstable hadrons and, possibly, the dressing of quarks in quark matter.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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