

Amplitude-based analysis of QED radiative corrections to electroproduction of η -mesons on protons

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A formalism for radiative correction calculations in exclusive η electroproduction on the proton is presented, extending the treatment developed for the pion channel. The EXCLURAD code is used in the radiative correction procedure with EtaMAID-2023 multipole amplitudes. The cross-section correction factor δ varies by up to $\sim 30\%$ across the resonance region $W = 1.49\text{--}2.0$ GeV at $E_{\text{beam}} = 6.535$ GeV, with a local maximum near $W \simeq 1.66$ GeV driven by the $S_{11}(1535)$ and $S_{11}(1650)$ resonances. The beam-spin asymmetry is suppressed by 15%–25% at the same kinematics. Numerical results covering $Q^2 = 0.3\text{--}4.0$ GeV² and the full angular range are provided for kinematics relevant to CLAS12 experiments at Jefferson Lab.

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I. INTRODUCTION

Understanding the electromagnetic transition amplitudes from the nucleon's ground state to excited states provides valuable insight into the nucleon's electromagnetic structure. Exclusive pseudoscalar meson electroproduction is one of the major sources of direct information about the spatial and spin structures of excited states. With the development of a high-intensity, high-duty-factor electron beam possessing a high degree of polarization, this field has reached a new level of quality. For the past several years, exclusive electroproduction of mesons has been the focus of extensive studies at various accelerator laboratories, including Jefferson Lab and MAMI. Recent reviews provide broader context: baryon spectroscopy is surveyed in Refs. [1,2], while the electroexcitation of nucleon resonances and the role of meson photo- and electroproduction in mapping the excited-nucleon spectrum are reviewed in Refs. [3,4].

The η meson carries isospin $I = 0$, so the ηN system is a pure $I = 1/2$ state. Only nucleon resonances (N^*) contribute to η electroproduction; Δ states ($I = 3/2$) are excluded entirely [5,6]. Pion channels offer no equivalent filter since πN final states mix $I = 1/2$ and $I = 3/2$ components. Near threshold, η production is dominated by the $S_{11}(1535)$, whose $N\eta$ branching fraction of 30%–55% is comparable to its $N\pi$ decay [7]. The $S_{11}(1650)$ and $P_{11}(1710)$ also couple appreciably to $N\eta$, and the recent upgrade of $N(1895)1/2^-$ to four-star PDG status [7] has added further interest to the channel. These features have driven a sustained experimental program in η electroproduction at Jefferson Lab with the CLAS and CLAS12 detectors [8–26]. A general discussion of η electroproduction and its role in baryon spectroscopy can be found in Ref. [27] and the reviews cited therein.

The η production threshold, $W_{\text{th}} = m_p + m_\eta \approx 1.486$ GeV, lies only ~ 49 MeV below the $S_{11}(1535)$ peak. For comparison, the pion threshold sits ~ 159 MeV below the $\Delta(1232)$. This compressed gap limits the bremsstrahlung phase space near the dominant resonance: even moderate photon emission can shift the effective invariant mass below the η threshold, producing a steeper and more structured radiative correction than in the pion channel. Accurate radiative corrections are therefore required to interpret the measured cross sections and polarization asymmetries.

A formalism for radiative-correction (RC) calculations in exclusive pion electroproduction on the proton was developed over 20 years ago [28] and was later applied to pion electroproduction experiments at JLab and CERN (COMPASS). A code EXCLURAD was developed for the RC

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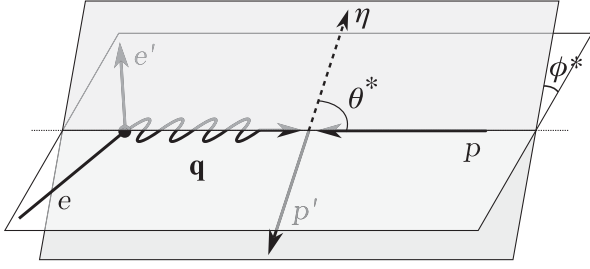


FIG. 1. Kinematics of η electroproduction in the center-of-mass frame, in the absence of any radiation, showing the angles θ^* and ϕ^* .

procedure. In this paper, we extend EXCLURAD to the η channel. The new contributions are: an explicit derivation of the leading-logarithm approximation for the η channel, which does not appear in closed form in Ref. [28]; an interface to EtaMAID-2023 multipole tables covering $W = 1.49\text{--}2.0$ GeV and $Q^2 = 0\text{--}5$ GeV²; and a numerical output of $\sim 265,000$ kinematic points made publicly available with an interactive browser-based explorer. The QED formalism is unchanged from Ref. [28]. Two-photon exchange corrections were not included in the present analysis; they can be applied independently using, e.g., a soft-photon approximation [29,30]. The numerical analysis is conducted in the kinematics of current experiments by the CLAS Collaboration [8–26].

II. KINEMATICS, OBSERVABLES, AND CONVENTIONS

We follow the kinematic conventions of EXCLURAD for exclusive electroproduction. Four independent variables define the kinematics of an event. Labeling the incoming and outgoing electron 4-momenta as k_1^μ and k_2^μ , these include the positive squared four-momentum transfer $Q^2 \equiv -(k_1^\mu - k_2^\mu)^2 > 0$, the invariant mass W of the virtual photon-nucleon system, and the angles θ^* and ϕ^* of the outgoing meson momentum with respect to $\mathbf{q} = \mathbf{k}_1 - \mathbf{k}_2$ in the cm frame of the virtual photon and the initial nucleon. The azimuthal angle ϕ^* is defined relative to the electron scattering plane. The geometry is depicted in Fig. 1.

As in Ref. [28], we consider the case where the outgoing electron and outgoing proton are detected while the outgoing meson is not detected. We define θ^* and ϕ^* based on the direction opposite to the momentum of the outgoing proton, following the convention used by EXCLURAD [28] and adopted in CLAS and CLAS12 analyses. The undetected outgoing 4-momentum is

$$\Lambda^\mu \equiv k_1^\mu + p^\mu - k_2^\mu - p'^\mu, \quad (1)$$

where p^μ and p'^μ are the 4-momenta of the incoming and outgoing nucleons, respectively. This includes the

4-momentum of the outgoing meson and any 4-momentum carried by a radiated photon.

Additional discussion of the kinematic variables and the response-function decomposition of the Born cross section can be found in Secs. II–III of Ref. [28].

In this work, we consider the radiative η electroproduction reaction

$$ep \rightarrow e' p \eta(\gamma), \quad (2)$$

where (γ) represents an undetected radiated photon and the meson takes the η -meson mass m_η . The production threshold is $W_{\text{th}} = m_p + m_\eta \approx 1.486$ GeV.

The Lorentz-invariant inelasticity variable v parametrizes radiative effects,

$$v \equiv \Lambda^2 - m_\eta^2, \quad (3)$$

which vanishes at the Born point and increases with the energy of an unobserved bremsstrahlung photon. Experiments requiring exclusivity impose a cut $v < v_{\text{cut}}$ that controls how much of the radiative tail enters the exclusive sample. The value of v_{cut} is typically chosen well below v_{max} ,

$$v_{\text{max}} \equiv (W - m_p)^2 - m_\eta^2. \quad (4)$$

This limit corresponds to the configuration in which all available energy has been radiated away, and the η and proton are produced at rest in the cm frame, i.e., η production at threshold. The value of v_{cut} is chosen to exclude events in which an additional pion could have been produced in the undetected system. For the η channel with a detected proton, the lightest additional hadron that could appear alongside the η is a π^0 , so the natural cut corresponds to the π^0 production threshold in the missing mass: $v_{\text{cut}} = (m_\eta + m_{\pi^0})^2 - m_\eta^2 = m_{\pi^0}^2 + 2m_\eta m_{\pi^0} \approx 0.166$ GeV². Below this value, the only undetected particle kinematically allowed alongside the η is a radiated photon; above it, the missing mass is large enough to accommodate an unobserved pion, and the event can no longer be treated as exclusive η electroproduction. This is analogous to the logic applied in the pion channel in Ref. [28]. When v_{cut} exceeds v_{max} at a given W (which occurs for $W \lesssim 1.62$ GeV, where the compressed phase space near the η threshold restricts v_{max} below v_{cut}), the code uses v_{max} as the upper integration limit; no extrapolation beyond the physical phase space boundary is performed. Any departures from $v_{\text{cut}} = 0.166$ GeV² are stated explicitly (see, for example, the v_{cut} scan in Supplemental Material [31], Sec. S1).

We present results as two radiative-correction factors. For the unpolarized cross section,

$$\delta(W, Q^2, \cos \theta^*, \phi^*) \equiv \frac{\sigma_{\text{obs}}}{\sigma_0}, \quad (5)$$

where σ_{obs} includes radiative effects and σ_0 is the Born-level cross section; this matches Eq. (75) of Ref. [28]. For the beam spin asymmetry (BSA), A_{LU} , i.e., the asymmetry when scattering a longitudinally polarized electron beam from an unpolarized proton target,

$$R_A(W, Q^2, \cos \theta^*, \phi^*) \equiv \frac{A_{LU}^{\text{RC}}}{A_{LU}^{\text{Born}}}, \quad (6)$$

where A_{LU}^{RC} is the BSA including radiative effects, and A_{LU}^{Born} is the Born-level BSA. Because A_{LU} is a difference of cross sections, R_A need not equal δ ; the two are distinct radiative-correction factors.

III. UPDATED ETAMAID AMPLITUDES

EtaMAID is an isobar model for η photo- and electroproduction on the nucleon [27,35]. The model includes nucleon resonances in the s -channel parametrized by Breit-Wigner amplitudes, Born terms in the s - and u channels, and vector-meson exchange in the t channel. Because the η meson carries isospin $I = 0$, only nucleon resonances with $I = 1/2$ contribute to the ηN final state, excluding all Δ ($I = 3/2$) states that appear in pion electroproduction. This isospin filter provides a cleaner probe of the N^* spectrum than the pion channel, where $I = 1/2$ and $I = 3/2$ contributions overlap.

The EtaMAID framework has undergone several revisions. The original model (EtaMAID2001/2003 [35]) described the nonresonant background with Born terms and t -channel ρ and ω vector-meson exchanges and included eight nucleon resonances in Breit-Wigner form with partial waves up to $\ell = 3$, where ℓ denotes the orbital angular momentum between the η meson and the recoiling nucleon in the final state. In the EtaMAID2018 update [36], the background was replaced by a Regge-cut model incorporating ρ and ω Regge trajectories with Pomeron and f_2 rescattering contributions. The resonance count was expanded from 8 to 21 N^* states, partial waves were extended to $\ell = 5$ (through G_{17} and G_{19}), and energy-dependent unitarization phases were introduced for each resonance. Damping factors were applied to suppress double counting in the overlap region between Regge exchanges and s -channel resonances. The 2018 model was constrained by differential cross section and polarization data from MAMI, CLAS, GRAAL, and ELSA, covering η and η' photoproduction on protons and neutrons up to $W \approx 4.5$ GeV [36].

For this work, new EtaMAID-2023 lookup tables of the electromagnetic multipole amplitudes were generated: magnetic ($M_{\ell\pm}$), electric ($E_{\ell\pm}$), and scalar/longitudinal ($S_{\ell\pm}$) transitions for orbital angular momentum $\ell = 0$ through 5. The tables cover W from threshold up to 6 GeV and Q^2 up to 5 GeV². The present analysis uses only the region $W \leq 2$ GeV and $Q^2 \leq 5$ GeV², matching

the kinematic domain of CLAS12 η -electroproduction measurements at $E_{\text{beam}} = 6.535$ GeV [37]; the broader coverage of the tables is intended to support future applications at higher beam energies. The EtaMAID interface is publicly available at [38], with multipole tables accessible at [39]. These tables serve as the hadronic model input to EXCLURAD, as described in the following Section.

IV. η -CHANNEL IMPLEMENTATION IN EXCLURAD

The three technical additions relative to Ref. [28] are documented in this Section: the explicit leading-logarithm (LL) formula (Sec. IV C), the EtaMAID-2023 amplitude interface (Sec. IV D), and the η -channel adaptation (Sec. IV E). EXCLURAD [28] computes $\mathcal{O}(\alpha)$ QED radiative corrections to the fivefold differential cross section for $ep \rightarrow e' p \eta(\gamma)$ using the Bardin-Shumeiko covariant IR-cancellation scheme. The formalism of Ref. [28] is unchanged; this Section documents the computational structure in enough detail to make the results in Sec. V self-contained and derives the leading-logarithm approximation explicitly, as that expression does not appear in closed form in Ref. [28].

A. Born cross section

The Born-level fivefold differential cross section is decomposed as [28]

$$\begin{aligned} \frac{d^5 \sigma_0}{dW dQ^2 d\Omega_\eta^*} &= \Gamma_v [\sigma_T + \varepsilon \sigma_L + \varepsilon \sigma_{TT} \cos 2\phi^* \\ &\quad + \sqrt{\frac{\varepsilon(\varepsilon+1)}{2}} \sigma_{LT} \cos \phi^* \\ &\quad + h \sqrt{\frac{\varepsilon(1-\varepsilon)}{2}} \sigma_{LT'} \sin \phi^*], \end{aligned} \quad (7)$$

where Γ_v is the virtual photon flux factor, ε is the degree of transverse polarization of the virtual photon, $h = \pm 1$ is the longitudinal beam helicity, and ϕ^* is defined in Sec. II. The structure functions σ_T , σ_L , σ_{TT} , σ_{LT} , $\sigma_{LT'}$ depend on $(Q^2, W, \cos \theta^*)$ and are supplied by EtaMAID-2023 as described in Sec. IV D. The fifth structure function $\sigma_{LT'}$ is the longitudinal-transverse interference response function associated with the beam helicity; the prime distinguishes it from the helicity-independent σ_{LT} . It is odd in ϕ^* and is the source of the beam-spin asymmetry A_{LU} .

B. Radiative correction formalism

Let $s = (k_1^\mu + p^\mu)^2$ and $x = (k_2^\mu + p^\mu)^2$ denote the Bardin-Shumeiko Lorentz invariants, where k_1^μ , k_2^μ , and p^μ are the four-momenta of the incoming electron, outgoing electron, and target proton, respectively, as defined in Sec. II.

The five diagrams contributing to the Born and $\mathcal{O}(\alpha)$ corrected cross sections are shown in Fig. 2: the Born process (a), initial- and final-state bremsstrahlung (b,c), the

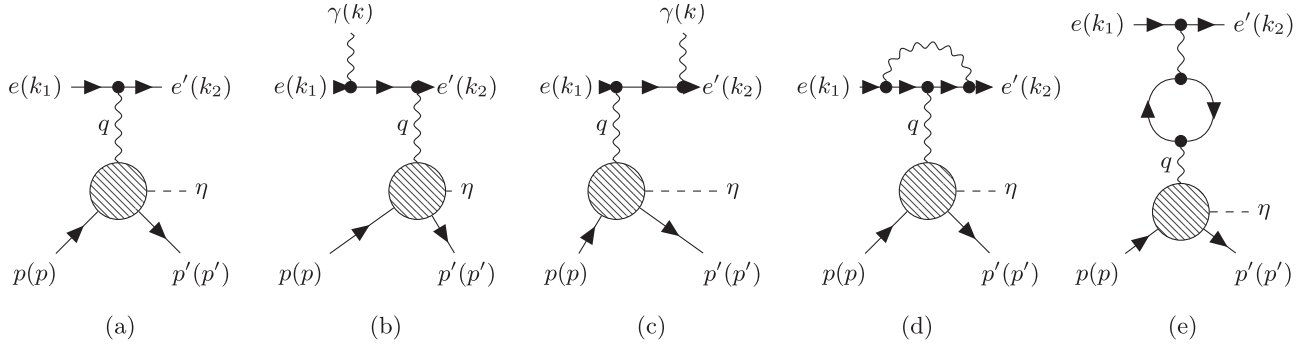


FIG. 2. Feynman diagrams contributing to the Born and $\mathcal{O}(\alpha)$ electroproduction cross sections. (a) Born process; (b) initial-state bremsstrahlung off the incoming electron; (c) final-state bremsstrahlung off the outgoing electron; (d) one-loop electron-photon vertex correction; and (e) vacuum polarization insertion on the virtual photon propagator. The hatched blob represents all hadronic structure (EtaMAID-2023); $q = k_1 - k_2$ is the virtual photon four-momentum. Diagrams produced with TikZ-Feynman [40].

one-loop vertex correction (d), and the vacuum polarization insertion (e).

The radiatively corrected cross section is (following Ref. [28])

$$\sigma_{\text{obs}} = \sigma_0 F_{\text{ext}} \left(1 + \frac{\alpha}{\pi} [\delta_{VR} + \delta_{\text{vac}}] \right) + \tau_{\text{hard}}, \quad (8)$$

where F_{ext} exponentiates the leading soft-photon IR contributions,

$$F_{\text{ext}} = \exp \left[\frac{\alpha}{\pi} (L_e - 1) \ln \frac{v_{\text{max}}^2}{\hat{s} \hat{x}} \right], \quad (9)$$

with $L_e = \ln \frac{Q^2}{m_e^2}$, and with Born-level invariants

$$\hat{s} = s - Q^2 - \tilde{v}_1^{(0)}, \quad (10)$$

$$\hat{x} = x + Q^2 - \tilde{v}_2^{(0)}, \quad (11)$$

(see Sec. IV C for the definition of $\tilde{v}_{1,2}^{(0)}$); δ_{VR} is the virtual-plus-soft finite remainder, given by

$$\delta_{VR} = \frac{3}{2} L_e - 2 - \frac{1}{2} \ln^2 \frac{\hat{x}}{\hat{s}} + \text{Li}_2 \left(1 - \frac{m_p^2 Q^2}{\hat{s} \hat{x}} \right) - \frac{\pi^2}{6}, \quad (12)$$

and δ_{vac} is the leptonic vacuum-polarization correction, summed over e, μ, τ , and light hadronic states.

The hard-bremsstrahlung contribution is the three-dimensional integral,

$$\tau_{\text{hard}} = -\frac{\alpha^3}{512\pi^4 s^2 W^4} \int_0^{v_{\text{max}}} dv \int_{-1}^1 d\cos\theta_\gamma \int_0^{2\pi} d\phi_\gamma \times \mathcal{F}(v, \cos\theta_\gamma, \phi_\gamma), \quad (13)$$

where $\mathcal{F}$ is the squared radiative amplitude contracted with the hadronic tensor, with the hadronic model evaluated at

the kinematically shifted arguments $(\tilde{Q}^2, \tilde{W}^2, \tilde{t})$ at each phase-space point. The Bardin-Shumeiko subtraction renders $\mathcal{F}$ finite as $v \rightarrow 0$, so no regularization of the lower limit is needed.

The angular integrand has two sources of near-singular behavior: collinear singularities when the radiated photon is emitted parallel to k_1 or k_2 ($\cos\theta_\gamma = \cos\theta_1$ or $\cos\theta_2$ in the hadronic cm frame), and forward singularities near $\phi_\gamma = 0$ and 2π . The code partitions the $(\cos\theta_\gamma, \phi_\gamma)$ domain into $5 \times 3 = 15$ subcells that isolate these peaks and applies adaptive quadrature to each subcell independently; a schematic of this partition is given in Supplemental Material S4.1 (Fig. 26). In the leading-log approximation (Sec. IV C), the $\cos\theta_\gamma$ and ϕ_γ integrals are performed analytically, reducing Eq. (13) to a one-dimensional integral over v .

C. Leading-logarithm approximation

When only the terms enhanced by $L_e = \ln(Q^2/m_e^2)$ are retained, the three-dimensional integral in Eq. (13) collapses to a one-dimensional integral over v . The formula implemented in the code is

$$\tau_{\text{LL}} = \frac{\alpha}{2\pi} (L_e - 1) \int_0^{v_{\text{max}}} dv [\mathcal{I}_{\text{ISR}}(v) + \mathcal{I}_{\text{FSR}}(v)], \quad (14)$$

where the initial- and final-state radiation contributions are

$$\mathcal{I}_{\text{ISR}}(v) = \frac{(1 + z_1^2) c_s \sigma_{\text{Born}}^{(s)} - 2\tilde{\sigma}_0}{\hat{s}(v)(1 - z_1)}, \quad (15)$$

$$\mathcal{I}_{\text{FSR}}(v) = \frac{[(1 + z_2^2) c_x \sigma_{\text{Born}}^{(x)} - 2\tilde{\sigma}_0] z_2}{\hat{x}(v)(1 - z_2)}. \quad (16)$$

To define the remaining quantities, consider the η four-momentum shifted to inelasticity v , while its CM direction $(\cos\theta^*, \phi^*)$ is held fixed at the Born value. The shifted energy and momentum magnitude are

$$\tilde{E}_\eta(v) = \frac{W^2 + m_\eta^2 - m_p^2 - v}{2W}, \quad |\tilde{\vec{p}}_\eta(v)| = \frac{\tilde{\lambda}^{1/2}(v)}{2W}, \quad (17)$$

where

$$\tilde{\lambda}^{1/2}(v) = \sqrt{(W^2 + m_\eta^2 - m_p^2 - v)^2 - 4W^2 m_\eta^2}. \quad (18)$$

At $v = 0$, this reduces to the Born η kinematics, while $v = v_{\max}$ corresponds to the threshold. The lepton- η invariants at shifted kinematics are $\tilde{v}_1(v) = 2k_1 \cdot \tilde{\vec{p}}_\eta(v)$ and $\tilde{v}_2(v) = 2k_2 \cdot \tilde{\vec{p}}_\eta(v)$, with $\tilde{v}_{1,2}^{(0)} \equiv \tilde{v}_{1,2}(0)$ being the Born values appearing in Eqs. (9)–(12). The v -dependent kinematic denominators are

$$\hat{s}(v) = s - Q^2 - \tilde{v}_1(v), \quad \hat{x}(v) = x + Q^2 - \tilde{v}_2(v), \quad (19)$$

and the momentum fractions are

$$z_1 = 1 - \frac{v}{\hat{s}(v)}, \quad z_2 = \frac{\hat{x}(v)}{\hat{x}(v) + v}, \quad (20)$$

so that $\hat{s}(1 - z_1) = \hat{x}(1 - z_2)/z_2 = v$, the standard Altarelli-Parisi variable. The hadronic invariant masses at shifted kinematics are

$$W_s^2 = z_1 s - x - z_1 Q^2 + m_p^2, \quad W_x^2 = s - \frac{x}{z_2} - \frac{Q^2}{z_2} + m_p^2, \quad (21)$$

and the Born cross section is evaluated at two sets of shifted arguments,

$$\sigma_{\text{Born}}^{(s)} = \sigma_0(z_1 s, x, z_1 Q^2; z_1 \tilde{v}_1, \tilde{v}_2), \quad (22)$$

$$\sigma_{\text{Born}}^{(x)} = \sigma_0\left(s, \frac{x}{z_2}, \frac{Q^2}{z_2}; \tilde{v}_1, \frac{\tilde{v}_2}{z_2}\right), \quad (23)$$

for initial- and final-state radiation, respectively. The subtraction term $\tilde{\sigma}_0 = \sigma_0(s, x, Q^2; \tilde{v}_1, \tilde{v}_2)$ is the Born cross section at physical invariants (s, x, Q^2) but with the η momentum at inelasticity v , so that $\tilde{\sigma}_0 \rightarrow \sigma_0$ as $v \rightarrow 0$. This ensures the integrand vanishes at the lower limit: as $v \rightarrow 0$, $z_{1,2} \rightarrow 1$, and $c_{s,x} \rightarrow 1$, both numerators in Eqs. (15)–(16) vanish as $(1 + 1^2)\sigma_0 - 2\sigma_0 = 0$.

The factors c_s and c_x correct for the change in hadronic phase space when the invariant mass shifts from W^2 to W_s^2 or W_x^2 . The Born cross section carries the η cm momentum $\lambda^{1/2}(W^2)/(2W')$ as a flux factor in the numerator and W'^2 in the denominator [see Eq. (7) via Sec. IV D]. Correcting for both gives

$$c_s = \frac{\tilde{\lambda}^{1/2}(v)}{\lambda^{1/2}(W_s^2)} \cdot \frac{W_s^2}{W^2}, \quad c_x = \frac{\tilde{\lambda}^{1/2}(v)}{\lambda^{1/2}(W_x^2)} \cdot \frac{W_x^2}{W^2}, \quad (24)$$

where

$$\lambda^{1/2}(W_{s,x}^2) = \sqrt{(W_{s,x}^2 + m_\eta^2 - m_p^2)^2 - 4W_{s,x}^2 m_\eta^2}. \quad (25)$$

At $v = 0$, $\tilde{\lambda}^{1/2}(0) = \lambda^{1/2}(W^2)$, and $W_{s,x}^2 = W^2$, so $c_{s,x} = 1$ as required. The factors c_s and c_x correspond to C_s and C_x in Eq. (59) of Ref. [28], which correct the hadronic phase-space flux when the invariant mass shifts from W^2 to W_s^2 or W_x^2 under photon emission. Reference [28] writes the numerator using the Born η momentum $\lambda^{1/2}(W^2)$; the form given here in Eq. (24) instead uses $\tilde{\lambda}^{1/2}(v)$, the η momentum evaluated at inelasticity v , making the v dependence of the correction explicit. The two forms are identical at $v = 0$ and differ only in how the intermediate v dependence is distributed between the numerator and the Born cross section $\tilde{\sigma}_0$. Both forms are implemented in the EXCLURAD source code [41] and yield identical numerical results.

Equation (14) is evaluated by adaptive Simpson quadrature on 150 subintervals. Over the kinematic range in Sec. V, the leading logarithm approximation deviates from the exact result by 2.7%–8.8% while running approximately 600 times faster.

D. Hadronic amplitude interface

The five structure functions in Eq. (7) are built from EtaMAID-2023 multipole amplitudes $\{M_{\ell\pm}, E_{\ell\pm}, S_{\ell\pm}\}$ for partial waves $\ell = 0, \dots, 5$, tabulated on a discrete (Q^2, W) grid. At each grid node, the table stores the real and imaginary parts of all six amplitude types as (Re, Im) pairs, in units of $10^{-3}/m_{\pi^+}$ [36]; combinations forbidden by angular-momentum selection rules are set to zero. Intermediate grid points are obtained by bilinear interpolation in (Q^2, W) with no extrapolation beyond the grid boundary. The grid covers $W = 1.4856$ GeV and 1.490 – 2.000 GeV in 5 MeV steps, $Q^2 = 0.00$ – 5.01 GeV² in 0.01 GeV² steps. Where the public EtaMAID web interface is limited (nominally $W \leq 2$ GeV), extended tables from the EtaMAID-2023 update described in Sec. III were used. The amplitude chain that converts multipoles into structure functions is summarized in Fig. 3. It proceeds in four steps. (1) The multipole amplitudes are interpolated to the desired (Q^2, W) point. (2) The longitudinal multipoles $S_{\ell\pm}$ are rescaled by $\nu_{\text{cm}}/q_{\text{cm}}$ to convert from the L -multipole convention used in the tables to the S -multipole convention used in the cross-section formulas [27], and all amplitudes are converted from $10^{-3}/m_{\pi^+}$ to $\sqrt{\mu\text{b}}$. (3) A partial-wave sum constructs the Chew–Goldberger–Low–Nambu (CGLN) amplitudes [42] as functions of $\cos\theta^*$, parametrizing the hadronic electromagnetic current for pseudoscalar meson electroproduction off a spin-1/2 target [27]. For real photons, the decomposition yields four amplitudes F_1 – F_4 , built from the transverse multipoles $M_{\ell\pm}$ and $E_{\ell\pm}$; for virtual photons, two additional amplitudes F_5 and F_6 enter through the longitudinal coupling, constructed from the scalar multipoles $S_{\ell\pm}$ [27]. (4) The six CGLN amplitudes are projected onto helicity amplitudes H_1 – H_6 [27], from which

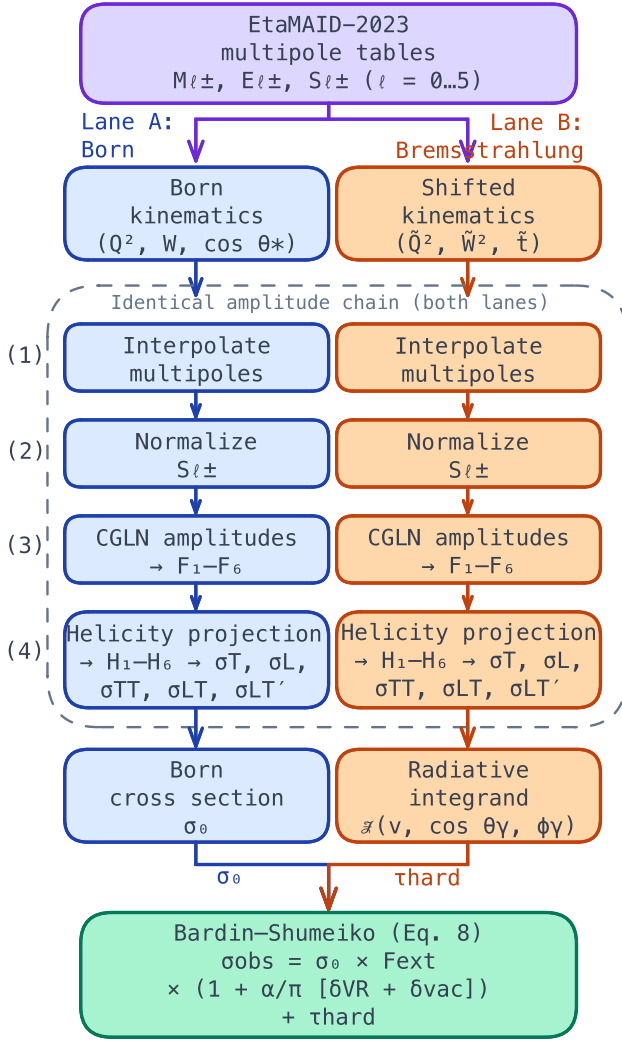


FIG. 3. From multipoles to σ_{obs} : the EtaMAID-2023 amplitude chain and its role in the radiative-correction calculation. The EtaMAID-2023 multipole tables (top, purple) store the electromagnetic multipole amplitudes $M_{\ell\pm}$, $E_{\ell\pm}$, $S_{\ell\pm}$ for $\ell = 0\text{--}5$ on a (Q^2, W) grid. The same four-step chain (dashed boundary) converts these multipoles into structure functions in two parallel lanes. Lane A (blue) runs at Born kinematics $(Q^2, W, \cos \theta^*)$ and produces σ_0 ; Lane B (orange) runs at the radiatively shifted kinematics $(\tilde{Q}^2, \tilde{W}^2, \tilde{t})$ for each quadrature point in the bremsstrahlung integral and produces the radiative integrand $\mathcal{F}$. The four shared steps are: (1) interpolation of the multipole amplitudes to the specified (Q^2, W) ; (2) normalization, including the $S_{\ell\pm} \rightarrow S_{\ell\pm} \times \nu_{\text{cm}}/q_{\text{cm}}$ rescaling and unit conversion [36]; (3) CGLN partial-wave summation to obtain $F_1\text{--}F_4$ from transverse multipoles and F_5, F_6 from scalar multipoles [27]; and (4) projection onto helicity amplitudes $H_1\text{--}H_6$, then construction of $\sigma_T, \sigma_L, \sigma_{TT}, \sigma_{LT}, \sigma_{LT'}$. The two outputs combine at the bottom via the Bardin-Shumeiko formula [Eq. (8)]: σ_0 enters as the Born prefactor and $\mathcal{F}$ enters through τ_{hard} , yielding σ_{obs} .

the five structure functions $\sigma_T, \sigma_L, \sigma_{TT}, \sigma_{LT}$, and $\sigma_{LT'}$ are constructed. Steps (2)–(4) are implemented in the EXCLURAD source code [41].

This chain runs in two kinematic contexts (Fig. 3, parallel lanes). Lane A evaluates it at the Born kinematics $(Q^2, W, \cos \theta^*)$ to produce the Born cross section σ_0 . Lane B evaluates the same chain at the radiatively shifted kinematics $(\tilde{Q}^2, \tilde{W}^2, \tilde{t})$ at every quadrature point in the bremsstrahlung integral over $(v, \cos \theta_\gamma, \phi_\gamma)$, producing the radiative integrand $\mathcal{F}$. Steps 1–4 are identical between lanes; only the input kinematics differ. The two outputs feed the Bardin-Shumeiko formula, Eq. (8): σ_0 enters as the prefactor and $\mathcal{F}$ enters through τ_{hard} . The QED formalism in Eqs. (8)–(24) is independent of this hadronic chain; the hadronic model is called at kinematically shifted arguments inside τ_{hard} and τ_{LL} , but the photon-emission kinematics and IR structure are those of Ref. [28].

E. Adaptation to the η channel

Two changes were made to adapt EXCLURAD to the η -meson channel. First, the outgoing-hadron mass was set to $m_\eta = 0.54786$ GeV, which places the production threshold at $W_{\text{th}} \simeq 1.486$ GeV and sets v_{max} via Eq. (4). Second, the hadronic model input was replaced by the EtaMAID-2023 amplitude tables; because the table format is identical to the existing MAID pion tables, no changes to the table-reading or interpolation routines were needed. The multidimensional integration tolerances for τ_{hard} were tightened to a relative precision of 10^{-6} to improve convergence near the threshold. No changes were made to the QED formalism of Eqs. (8)–(24). All results in Sec. V use the exact $\mathcal{O}(\alpha)$ mode, which includes soft and hard bremsstrahlung, vertex and vacuum-polarization corrections, and soft-photon exponentiation. Unless otherwise noted, all figures use $v_{\text{cut}} = 0.166$ GeV²; individual figure captions state E_{beam} and any departures from this default. Convergence of τ_{hard} was verified by comparing exact and leading-log results at representative kinematic points (Supplemental Material S4. 1); the relative integration tolerance of 10^{-6} is sufficient that numerical integration error does not contribute meaningfully to any reported result. Full implementation details are in the repository (Supplemental Material S4).

V. RADIATIVE-CORRECTION RESULTS

All numerical results in this section use a beam energy of $E_{\text{beam}} = 6.535$ GeV, corresponding to the CLAS12 RG-K running period [37]; for comparison, Ref. [28] used $E_{\text{beam}} = 1.645$ GeV for the pion channel.

A. W -dependence of the correction factors

Figure 4 shows the cross-section RC factor δ versus W at fixed $(\cos \theta^*, \phi^*) = (-0.75, 90^\circ)$ for four values of Q^2 . All four curves rise from $\delta \approx 0.61\text{--}0.64$ at the threshold to a local maximum at $W \simeq 1.655$ GeV, where δ exceeds unity for all Q^2 values; the peak height increases from $\delta \approx 1.05$ at $Q^2 = 0.4105$ GeV² to $\delta \approx 1.25$ at $Q^2 = 4.0855$ GeV². The peak position at $W \approx 1.655$ GeV is the same for all four Q^2 values; it is pinned by the resonance content of the

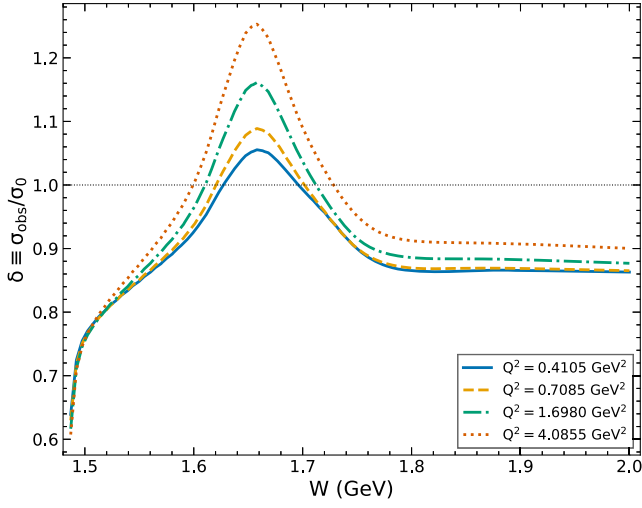


FIG. 4. Cross-section RC factor $\delta \equiv \sigma_{\text{obs}}/\sigma_0$ vs W at $\cos\theta^* = -0.75$, $\phi^* = 90^\circ$, $v_{\text{cut}} = 0.166$ GeV². Curves: $Q^2 = 0.4105, 0.7085, 1.6980, 4.0855$ GeV² (see legend). The horizontal line marks $\delta = 1$. All curves rise from $\delta \approx 0.61$ – 0.64 at threshold to a local maximum at $W \simeq 1.655$ GeV (peak $\delta \approx 1.05$ – 1.25 , increasing with Q^2), then fall and flatten to $\delta \approx 0.86$ – 0.90 for $W \gtrsim 1.85$ GeV.

hadronic amplitude input rather than by the photon phase space. Beyond the peak, the curves fall through unity and flatten to a plateau of $\delta \approx 0.86$ – 0.90 for $W \gtrsim 1.85$ GeV. Figure 5 shows the asymmetry ratio R_A at the same fixed angles. Near the threshold, the curves lie at $R_A \approx 0.88$ – 0.91 , fall to a local minimum of $R_A \approx 0.78$ – 0.82 at $W \simeq 1.66$ – 1.68 GeV, rise to a local maximum of $R_A \approx 1.02$ – 1.04 near $W \simeq 1.80$ – 1.82 GeV, and level off near $R_A \approx 1.00$ for $W \gtrsim 1.9$ GeV. The minimum is deepest at $Q^2 = 1.698$ GeV² ($R_A \approx 0.779$) rather than at the highest Q^2 value ($R_A \approx 0.800$ at $Q^2 = 4.0855$ GeV²), so the suppression at the R_A minimum is not monotonic across the Q^2 range shown. The spread across the four Q^2 curves is at most a few percent at any given W . The two RC factors δ and R_A are, in general, independent and coincide at a given kinematic point only by accident. Comparing Figs. 4 and 5, R_A reaches its minimum where δ peaks, and vice versa. This anticorrelation is consistent with the pion-channel behavior in Figs. 7–9 of Ref. [28], where the beam spin asymmetry RC changes sign between the $\Delta(1232)$ and $S_{11}(1535)$ resonance regions. The mechanism is the same here: the hard-bremsstrahlung integral enters the polarized and unpolarized cross-section pieces with different weights at kinematics where the hadronic amplitudes vary rapidly with W , so R_A departs from unity wherever δ is farthest from unity.

The W -dependence of δ in the η channel is more structured than the pion results in Figs. 3–8 of Ref. [28]. This follows from the EtaMAID resonance content: η production near threshold is dominated by the $S_{11}(1535)$ and $S_{11}(1650)$ resonances, whose contributions to the hadronic model input are concentrated in a narrow W interval. Pion production in the same W range involves

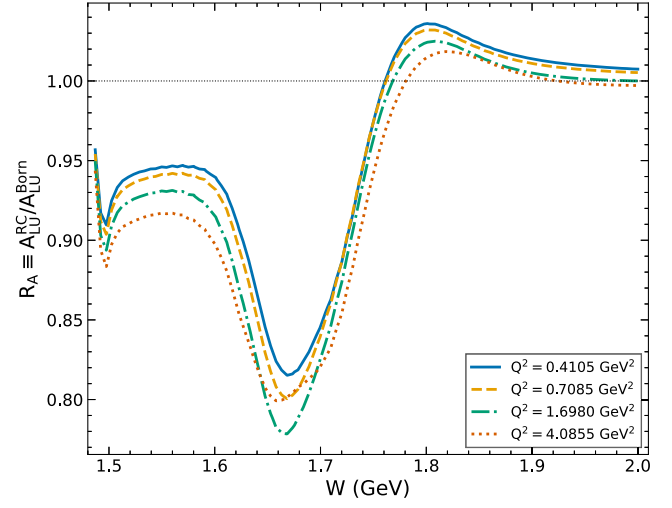


FIG. 5. Asymmetry ratio $R_A \equiv A_{LU}^{\text{RC}}/A_{LU}^{\text{Bom}}$ vs W at $\cos\theta^* = -0.75$, $\phi^* = 90^\circ$, $v_{\text{cut}} = 0.166$ GeV². Curves: $Q^2 = 0.4105, 0.7085, 1.6980, 4.0855$ GeV² (see legend). Near threshold: $R_A \approx 0.88$ – 0.91 . Local minimum $R_A \approx 0.78$ – 0.82 at $W \simeq 1.66$ – 1.68 GeV; local maximum $R_A \approx 1.02$ – 1.04 near $W \simeq 1.80$ – 1.82 GeV; plateau $R_A \approx 1.00$ for $W \gtrsim 1.9$ GeV.

several overlapping resonances, which smooth out the W dependence of δ . The sharper structure in the η channel is therefore a feature of the hadronic physics, not an artifact of the RC calculation.

B. Angular dependence

Figure 6 shows δ as a function of ϕ^* at $W = 1.660$ GeV, the $\delta(W)$ local maximum identified in Fig. 4, for three

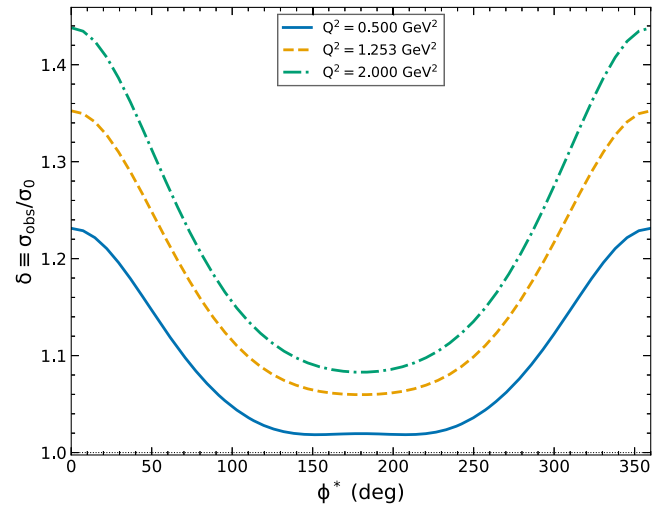


FIG. 6. $\delta \equiv \sigma_{\text{obs}}/\sigma_0$ vs ϕ^* at $W = 1.660$ GeV, $\cos\theta^* = -0.750$, $v_{\text{cut}} = 0.166$ GeV², for $Q^2 = 0.500, 1.253, 2.000$ GeV² (see legend). Cosinelike modulation with $\delta > 1$ throughout; both amplitude and offset increase with Q^2 . At $Q^2 = 0.500$ GeV², $\delta \in [1.02, 1.23]$; at $Q^2 = 2.000$ GeV², $\delta \in [1.08, 1.44]$. The horizontal dotted line marks $\delta = 1$.

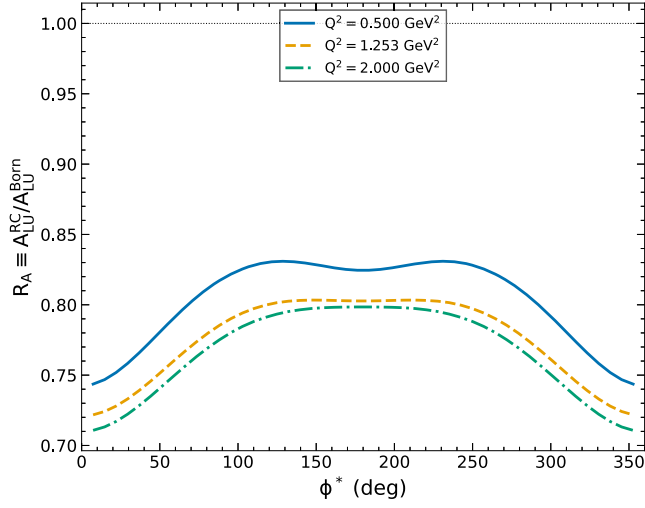


FIG. 7. Asymmetry ratio $R_A \equiv A_{LU}^{\text{RC}}/A_{LU}^{\text{Born}}$ vs ϕ^* at $W = 1.660$ GeV, $\cos \theta^* = -0.750$, $v_{\text{cut}} = 0.166$ GeV², for $Q^2 = 0.500, 1.253, 2.000$ GeV² (see legend). The dotted horizontal line marks $R_A = 1$ (no RC effect on the asymmetry). R_A is undefined near $\phi^* = 0^\circ$ and 180° where $A_{LU}^{\text{Born}} \rightarrow 0$. All curves lie below unity; suppression deepens with Q^2 : $R_A \in [0.74, 0.83]$ at $Q^2 = 0.500$ GeV² and $R_A \in [0.71, 0.80]$ at $Q^2 = 2.000$ GeV². The weak ϕ^* modulation, peaking near $\phi^* \approx 90^\circ$, indicates that the $\sin \phi^*$ shape of A_{LU} is preserved under radiative corrections.

values of Q^2 at fixed $\cos \theta^* = -0.750$. The correction follows a cosinelike modulation with $\delta > 1$ throughout; hard-photon emission enhances the observed cross section relative to the Born approximation at these kinematics. Both the amplitude and offset of the modulation increase with Q^2 : at $Q^2 = 0.500$ GeV² the correction ranges from $\delta \approx 1.02$ at $\phi^* \simeq 180^\circ$ to $\delta \approx 1.23$ at $\phi^* \simeq 0^\circ$, while at $Q^2 = 2.000$ GeV² the range extends to $\delta \approx 1.08$ – 1.44 . Figure 7 shows the asymmetry ratio $R_A \equiv A_{LU}^{\text{RC}}/A_{LU}^{\text{Born}}$ as a function of ϕ^* at the same kinematics as Fig. 6. R_A is undefined near $\phi^* = 0^\circ$ and 180° where $A_{LU}^{\text{Born}} \rightarrow 0$. All three curves lie below unity throughout the accessible range, confirming that RC systematically suppresses the asymmetry at $W = 1.660$ GeV. Each curve shows a weak ϕ^* modulation with a broad maximum near $\phi^* \approx 90^\circ$: R_A ranges from ~ 0.74 – 0.83 at $Q^2 = 0.500$ GeV² and from ~ 0.71 – 0.80 at $Q^2 = 2.000$ GeV². The suppression deepens with Q^2 , consistent with the larger δ at higher Q^2 in Fig. 6. The near-uniformity of R_A across ϕ^* confirms that the $\sin \phi^*$ form of A_{LU} is preserved under radiative corrections; the $\sin \phi^*$ moment extraction described in Supplemental Material S3 is therefore unaffected by RC at this level.

C. Dependence on the inelasticity cut

Figure 8 shows the dependence of the RC on the inelasticity cut v_{cut} . For η electroproduction with a detected proton, the kinematic maximum of the inelasticity is $v_m = v_{\text{max}}$ from Eq. (4). At $W = 1.900$ GeV, this gives

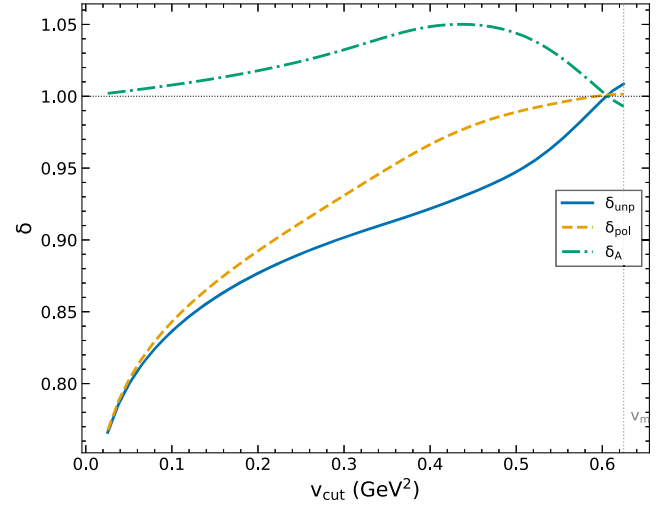


FIG. 8. RC factors vs inelasticity cut v_{cut} for η electroproduction at $W = 1.900$ GeV, $Q^2 = 0.4105$ GeV², $\cos \theta^* = -0.750$, $\phi^* = 90^\circ$. δ_{unp} : RC to the unpolarized cross section; δ_{pol} : RC to the polarized cross section; $\delta_A = \delta_{\text{unp}}/\delta_{\text{pol}}$: RC to the beam polarization asymmetry. Kinematic upper limit $v_m \approx 0.625$ GeV² (Eq. (4)). EtaMAID was used for structure functions.

$v_m \approx 0.625$ GeV². The figure shows the RC factors for the unpolarized cross section (δ_{unp}), the polarized cross section (δ_{pol}), and the beam spin asymmetry ($\delta_A = \delta_{\text{unp}}/\delta_{\text{pol}}$) as functions of v_{cut} . For small v_{cut} , only soft bremsstrahlung contributes; the corrections to the polarized and unpolarized pieces are similar, and δ_A stays close to unity. As v_{cut} increases, hard-photon emission enters, and the two corrections separate: δ_{unp} rises from ~ 0.77 toward unity. In contrast, δ_{pol} rises more slowly, producing $\delta_A > 1$ in the midrange before the three curves converge near $v_{\text{cut}} = v_m$. This pattern matches the pion-channel result in Fig. 9 of Ref. [28].

Extended kinematic scans are collected in Supplemental Material S2. Two results warrant attention: the $\cos \theta^*$ scan (Supplemental Material S2.1) shows that δ reaches ~ 1.76 at forward angles ($\cos \theta^* = +0.9$) at the δ -peak kinematics, while R_A falls to ~ 0.67 at the same point. The two effects are the largest deviations from unity in the full grid. The Q^2 scan (Supplemental Material S2.2) shows a localized nonmonotonic feature near $Q^2 \approx 0.55$ – 0.70 GeV² that appears coherently across all four kinematic combinations examined and is attributed to EtaMAID-2023 multipole structure. The full RC grid is available interactively at [43] (Supplemental Material S4).

VI. CONCLUSION

We extended EXCLURAD for exact $\mathcal{O}(\alpha)$ QED radiative corrections from exclusive pion electroproduction to the η channel $ep \rightarrow e'p\eta(\gamma)$. The modifications were restricted to kinematics ($m_\pi \rightarrow m_\eta$), numerical precision, and the

model-input interface; the underlying Bardin-Shumeiko formalism and all analytic expressions remain unchanged. An explicit leading-logarithm formula for the η channel, absent from Ref. [28], is derived in Sec. IV C. Structure functions were supplied by EtaMAID-2023 multipole tables covering $W = 1.486\text{--}2.0$ GeV and $Q^2 = 0\text{--}5$ GeV²; the model is most reliable for $W \lesssim 2$ GeV, where the s -channel resonance description applies.

The cross-section RC factor δ varies by up to $\sim 30\%$ across the resonance region, with a local maximum near $W \simeq 1.66$ GeV whose height increases with Q^2 , and a subunity plateau of $\delta \simeq 0.86\text{--}0.92$ at higher W . The asymmetry ratio R_A is anticorrelated with δ , consistent with the pion-channel behavior of Ref. [28] and arising from the different RC weights on the polarized and unpolarized pieces in the hard-bremsstrahlung integral. The W -dependence of δ is more structured than in the pion channel because the $S_{11}(1535)$ and $S_{11}(1650)$ resonances dominate η production over a narrow W interval, unlike the overlapping resonances in the pion case. The $\sin\phi^*$ form of A_{LU} is preserved under RC, and the v_{cut} dependence follows the pattern established in the pion channel. The nontrivial angular dependence of δ in both $\cos\theta^*$ and ϕ^* means that radiative corrections must be applied differentially across the full kinematic grid rather than as a single multiplicative factor; this requirement is more pronounced here than in the pion channel because of the compressed phase space between the η threshold and the dominant resonance.

Model dependence in δ is expected to be small in the resonance-dominated region, by analogy with the pion-channel study in Ref. [28], where three hadronic models agreed to within a few percent near the $\Delta(1232)$ peak. This estimate is a lower bound: the pion-channel spread reached $\sim 5\%$ even near the dominant resonance, and the η channel carries additional uncertainty given that EtaMAID-2023 predictions for A_{LU} differ in sign from measured values [37]. A quantitative model-comparison study, requiring

EXCLURAD-compatible multipole tables from the Jülich–Bonn–Washington (JBW) [44–47] or Bonn-Gatchina frameworks, is left for future work.

The η and η' channels share similar production dynamics. EtaMAID-2023 multipole tables for the $\eta'p$, $\eta'n$, and ηn channels are available from the same framework [36], so extending EXCLURAD to η' electroproduction requires only the mass substitution and the generation of the corresponding lookup tables.

The modified EXCLURAD source, EtaMAID-derived lookup tables, run scripts, and plotting code used in this work are available at [43] and archived at [48] (v1.0.0). An interactive browser-based tool for exploring the radiative-correction results across the full $(W, Q^2, \cos\theta^*, \phi^*)$ grid is deployed at [43] and described in Supplemental Material S4. The CERNLIB-free distribution of EXCLURAD is maintained at [49]; legacy materials and earlier documentation are available at [50]. File checksums and command-line recipes for reproducing all figures are provided in Supplemental Material S4.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [41]. The modified EXCLURAD source code, EtaMAID-2023 lookup tables, run scripts, and radiative-correction output are available at [43] and archived on Zenodo at [48] (v1.0.0). An interactive browser-based explorer for the full radiative-correction dataset is available at [43].

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