

**Erratum: Charged black-hole binary evolution
at second post-Newtonian order
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We thank Manfred Kraus and Allan Alonzo for pointing out several typos in the expressions for the 2PN observables, which are corrected here and now agree with the results of Ref. [1].

The corrected expressions for the observables appearing in Eqs. (43), (45), and (48) are given below:
Equation (43):

$$\begin{aligned}
 E_b(x_q) = & -\frac{x_q}{2} + \frac{x_q^2}{24} (1 - \eta_1 \eta_2)^{-2} \{9 + \nu - 2(2 + X_{12})\eta_1^2 - 2(3 + \nu)\eta_1 \eta_2 - 2(1 - X_{12})\eta_2^2 + (1 + \nu)\eta_1^2 \eta_2^2\} \\
 & + \frac{x_q^3}{48} (1 - \eta_1 \eta_2)^{-4} \{81 - 57\nu + \nu^2 - 18\eta_1^2(1 + X_{12}) - 2\nu(5 - X_{12})\eta_1^2 - 2(81 - 84\nu + 2\nu^2)\eta_1 \eta_2 \\
 & - 18(1 - X_{12})\eta_2^2 - 2\nu(5 + X_{12})\eta_2^2 - 2(1 + X_{12})^2\eta_1^4 + 4\eta_1^3 \eta_2[9 + 11\nu + (9 - \nu)X_{12}] \\
 & + (86 - 238\nu + 6\nu^2 - 4X_{12}^2)\eta_1^2 \eta_2^2 + 4\eta_1 \eta_2^3[9 + 11\nu - (9 - \nu)X_{12}] - 2(1 - X_{12})^2\eta_2^4 \\
 & + 2\eta_1^2 \eta_2^4(2\nu - 19 - 9X_{12}) - 2\eta_1^3 \eta_2^3(9 - 76\nu + 2\nu^2) + (1 - 25\nu + \nu^2)\eta_1^4 \eta_2^4 \\
 & + 2\eta_1^4 \eta_2^2[14(1 - X_{12}) - \nu(19 - X_{12})] + 2\eta_1^4 \eta_2^2(2\nu - 19 + 9X_{12}) \\
 & + 2\eta_1^2 \eta_2^4[14(1 + X_{12}) - \nu(19 + X_{12})]\}.
 \end{aligned}$$

Equation (45):

$$\begin{aligned}
 K = & 1 + \frac{x_q}{4} (1 - \eta_1 \eta_2)^{-2} [12 - \eta_1^2 - 12\eta_1 \eta_2 - \eta_2^2 + 2\eta_1^2 \eta_2^2 + X_{12}(\eta_2^2 - \eta_1^2)] \\
 & + \frac{x_q^2}{48} (1 - \eta_1 \eta_2)^{-4} \left\{ 648 - 1368\eta_1 \eta_2 + 204(1 + X_{12})\eta_1^3 \eta_2 - 120(1 - X_{12})\eta_2^2 - 120(1 + X_{12})\eta_1^2 - \frac{7}{2}(1 + X_{12})^2\eta_1^4 \right. \\
 & + (881 + 7X_{12}^2)\eta_1^2 \eta_2^2 - 70(1 + X_{12})\eta_1^4 \eta_2^2 + 204(1 - X_{12})\eta_1 \eta_2^3 - 192\eta_1^3 \eta_2^3 - \frac{7}{2}(1 - X_{12})^2\eta_2^4 + 10\eta_1^4 \eta_2^4 \\
 & - 70(1 - X_{12})\eta_1^2 \eta_2^4 + \nu[-336 - 20\eta_1^2 + 14\eta_1^4 + 1008\eta_1 \eta_2 + 112\eta_1^3 \eta_2 - 20\eta_2^2 - 1256\eta_1^2 \eta_2^2 - 92\eta_1^4 \eta_2^2 \\
 & \left. + 112\eta_1 \eta_2^3 + 688\eta_1^3 \eta_2^3 - 92\eta_1^2 \eta_2^2 - 104\eta_1^4 \eta_2^4 + 16X_{12}(\eta_1^2 - 2\eta_1^3 \eta_2 - \eta_2^2 + \eta_1^4 \eta_2^2 + 2\eta_1 \eta_2^3 - \eta_1^2 \eta_2^4)] \right\}.
 \end{aligned}$$

Equation (48):

$$\begin{aligned}
\chi = & \frac{2GM}{p_\phi p_\infty} (1 - \eta_1 \eta_2) - \frac{2G^3 M^3}{3p_\phi^3 p_\infty^3} (1 - \eta_1 \eta_2)^3 \\
& + \frac{1}{c^2} \left\{ \frac{GM p_\infty}{p_\phi} (4 - \eta_1 \eta_2) + \frac{G^2 M^2 \pi}{p_\phi^2} \left[3 - \frac{\eta_1^2}{4} (1 + X_{12}) - 3\eta_1 \eta_2 - \frac{\eta_2^2}{4} (1 - X_{12}) + \frac{\eta_1^2 \eta_2^2}{2} \right] \right. \\
& + \frac{G^3 M^3}{p_\phi^3 p_\infty} [8 - \eta_1^2 (1 + X_{12}) - 15\eta_1 \eta_2 + \eta_1^3 \eta_2 (1 + X_{12}) - \eta_2^2 (1 - X_{12}) + 8\eta_1^2 \eta_2^2 + \eta_1 \eta_2^3 (1 - X_{12}) - \eta_1^3 \eta_2^3] \Big\} \\
& + \frac{1}{c^4} \left\{ \frac{G^2 M^2 \pi p_\infty^2}{p_\phi^2} \left[\frac{3}{4} (5 - 2\nu) + \frac{\eta_1^2}{8} (1 + X_{12}) (-3 + \nu) + \frac{3}{2} (-1 + \nu) \eta_1 \eta_2 + \frac{\eta_2^2}{8} (1 - X_{12}) (-3 + \nu) - \nu \frac{\eta_1^2 \eta_2^2}{4} \right] \right. \\
& + \frac{G^3 M^3 p_\infty}{p_\phi^3} \left[-16(-3 + \nu) - \frac{17}{2} \eta_1^2 (1 + X_{12}) + \frac{\eta_1^2}{2} (1 - 2\nu + X_{12}) + \nu \eta_1^2 (1 + X_{12}) + \frac{\eta_1 \eta_2}{4} (128\nu - 225) \right. \\
& + \eta_1^3 \eta_2 (1 + X_{12}) (6 - \nu) - \frac{3}{2} \eta_1^3 \eta_2 (1 - 2\nu + X_{12}) - \frac{17}{2} \eta_2^2 (1 - X_{12}) + \frac{\eta_2^2}{2} (1 - 2\nu - X_{12}) \\
& \left. \left. + \nu \eta_2^2 (1 - X_{12}) + (16 - 24\nu) \eta_1^2 \eta_2^2 + \eta_1 \eta_2^3 (1 - X_{12}) (6 - \nu) - \frac{3}{2} \eta_1 \eta_2^3 (1 - 2\nu - X_{12}) + \frac{\eta_1^3 \eta_2^3}{4} (-3 + 16\nu) \right] + \frac{GM p_\infty^3}{4p_\phi} \eta_1 \eta_2 \right\}.
\end{aligned}$$

We also correct minor typographical errors in the following equations:

Equation (21b):

$$\begin{aligned}
L_{2PN} = & \frac{Gm_1 m_2}{2r} + \frac{1}{c^2} \left\{ \frac{Gm_1 m_2}{2r} \left(3v_1^2 - \frac{1}{2} (\mathbf{n} \cdot \mathbf{v}_1)(\mathbf{n} \cdot \mathbf{v}_2) - \frac{7}{2} \mathbf{v}_1 \cdot \mathbf{v}_2 \right) - \frac{G^2 m_1^2 m_2}{2r^2} \right\} \\
& + \frac{1}{c^4} \left\{ \frac{Gm_1 m_2}{8} (14(\mathbf{a}_2 \cdot \mathbf{v}_1)(\mathbf{n} \cdot \mathbf{v}_1) + (\mathbf{n} \cdot \mathbf{a}_2)(\mathbf{n} \cdot \mathbf{v}_1)^2 + 7(\mathbf{n} \cdot \mathbf{a}_1)v_2^2) + \frac{G^3 m_1^2 m_2}{2r^3} \left(m_1 + \frac{19}{4} m_2 \right) \right. \\
& + \frac{G^2 m_1^2 m_2}{4r^2} (14(\mathbf{n} \cdot \mathbf{v}_1)^2 - 14(\mathbf{n} \cdot \mathbf{v}_1)(\mathbf{n} \cdot \mathbf{v}_2) + 2(\mathbf{n} \cdot \mathbf{v}_2)^2 + v_1^2 - 7\mathbf{v}_1 \cdot \mathbf{v}_2 + 7v_2^2) \\
& + \frac{Gm_1 m_2}{16r} (3(\mathbf{n} \cdot \mathbf{v}_1)^2(\mathbf{n} \cdot \mathbf{v}_2)^2 + 14v_1^4 + 12(\mathbf{n} \cdot \mathbf{v}_1)(\mathbf{n} \cdot \mathbf{v}_2)(\mathbf{v}_1 \cdot \mathbf{v}_2) - 32v_1^2(\mathbf{v}_1 \cdot \mathbf{v}_2) \\
& \left. + 2(\mathbf{v}_1 \cdot \mathbf{v}_2)^2 - 14v_2^2(\mathbf{n} \cdot \mathbf{v}_1)^2 + 15v_1^2 v_2^2) \right\} + (1 \leftrightarrow 2).
\end{aligned}$$

Equation (21c):

$$\begin{aligned}
L_{2PC} = & -\frac{q_1 q_2}{2r} + \frac{1}{c^2} \left\{ \frac{q_1 q_2}{4r} (\mathbf{v}_1 \cdot \mathbf{v}_2 + (\mathbf{n} \cdot \mathbf{v}_1)(\mathbf{n} \cdot \mathbf{v}_2)) \right\} \\
& + \frac{1}{c^4} \left\{ \frac{q_1 q_2}{8} (2(\mathbf{n} \cdot \mathbf{v}_2)(\mathbf{a}_1 \cdot \mathbf{v}_2) + (\mathbf{n} \cdot \mathbf{a}_1)(\mathbf{n} \cdot \mathbf{v}_2)^2 - v_2^2(\mathbf{n} \cdot \mathbf{a}_1)) \right. \\
& \left. + \frac{q_1 q_2}{16r} (2v_2^2(\mathbf{n} \cdot \mathbf{v}_1)^2 - 3(\mathbf{n} \cdot \mathbf{v}_1)^2(\mathbf{n} \cdot \mathbf{v}_2)^2 + 2(\mathbf{v}_1 \cdot \mathbf{v}_2)^2 - v_1^2 v_2^2) \right\} + (1 \leftrightarrow 2).
\end{aligned}$$

Equation (21d):

$$\begin{aligned}
L_{\text{mixed}} = & \frac{1}{c^2} \left\{ \frac{Gm_1 q_1 q_2}{r^2} - \frac{Gm_2 q_1^2}{2r^2} \right\} \\
& + \frac{1}{c^4} \left\{ \frac{21Gq_1^2 q_2^2}{8r^3} + \frac{q_1^2 q_2^2}{4m_2 r} (\mathbf{n} \cdot \mathbf{a}_1) - \frac{q_1^3 q_2^3}{8m_1 m_2 r^3} + \frac{G^2 m_2^2}{r^3} (q_1^3 - q_1 q_2) - \frac{2Gm_1 q_1 q_2}{r} (\mathbf{n} \cdot \mathbf{a}_1) \right. \\
& \left. + \frac{G^2 m_1 m_2 q_1}{8r} (4q_1 - 47q_2) + \frac{Gm_1 q_1 q_2}{2r^2} (v_1^2 - 2(\mathbf{n} \cdot \mathbf{v}_1)^2 - \mathbf{v}_1 \cdot \mathbf{v}_2) \right\} + (1 \leftrightarrow 2).
\end{aligned}$$

Equation (23):

$$\begin{aligned}
a_1^i = & -\frac{Gm_2 n^i}{r^2} \left(1 - \frac{q_1 q_2}{Gm_1 m_2} \right) \\
& + \frac{1}{c^2} \left\{ \frac{Gm_2 (v_1^i - v_2^i)}{r^2} \left[(\mathbf{n} \cdot \mathbf{v}_1) \left(4 - \frac{q_1 q_2}{Gm_1 m_2} \right) - 3(\mathbf{n} \cdot \mathbf{v}_2) \right] \right. \\
& + n^i \left[\frac{3Gm_2}{2r^2} (\mathbf{v}_1 - \mathbf{v}_2)^2 + \frac{Gm_2}{r^2} \left(1 - \frac{q_1 q_2}{Gm_1 m_2} \right) \left(\frac{3}{2} (\mathbf{n} \cdot \mathbf{v}_2)^2 + 4\mathbf{v}_1 \cdot \mathbf{v}_2 - v_1^2 + 2v_2^2 \right) \right. \\
& + \frac{1}{r^3} \left(5G^2 m_1 m_2 + 4Gm_2^2 + Gq_2^2 - 7Gq_1 q_2 - 5q_1 q_2 \frac{m_2}{m_1} + \frac{Gq_1^2 q_2^2}{m_1 m_2} \right) \left. \right] \left. \right\} \\
& + \frac{1}{c^4} \left\{ (v_1^i - v_2^i) \left[\frac{\mathbf{n} \cdot \mathbf{v}_2}{r^3} \left(\frac{55G^2 m_1 m_2}{4} - 2G^2 m_2^2 + \frac{4Gm_2 q_1^2}{m_1} - \frac{33Gq_1 q_2}{2} + \frac{4Gm_2 q_1 q_2}{m_1} + Gq_2^2 + \frac{7q_1^2 q_2^2}{4m_1 m_2} \right) \right. \right. \\
& + \frac{\mathbf{n} \cdot \mathbf{v}_1}{r^3} \left(\frac{41Gq_1 q_2}{2} - \frac{63G^2 m_1 m_2}{4} - 2G^2 m_2^2 - \frac{5Gm_2 q_1^2}{m_1} - 2Gq_2^2 \frac{Gm_2 q_1 q_2}{m_1} - \frac{11q_1^2 q_2^2}{4m_1 m_2} \right) \\
& + \frac{Gm_2}{2r^2} \left(9(\mathbf{n} \cdot \mathbf{v}_2)^3 + (\mathbf{n} \cdot \mathbf{v}_1) \left(\frac{q_1 q_2}{Gm_1 m_2} v_1^2 - 8(\mathbf{v}_1 \cdot \mathbf{v}_2) + 7v^2 - 9(\mathbf{n} \cdot \mathbf{v}_2)^2 \right) \right. \\
& + (\mathbf{n} \cdot \mathbf{v}_1) \left(1 - \frac{q_1 q_2}{Gm_1 m_2} \right) (v_2^2 - 3(\mathbf{n} \cdot \mathbf{v}_2)^2) + 2(\mathbf{n} \cdot \mathbf{v}_2) (v_1^2 + 4(\mathbf{v}_1 \cdot \mathbf{v}_2) - 5v_2^2) \left. \right] \left. \right\} \\
& + n^i \left[\frac{Gm_2}{8r^2} \left(\frac{q_1 q_2}{Gm_1 m_2} v_1^4 - 16(\mathbf{v}_1 \cdot \mathbf{v}_2)^2 + 2(\mathbf{v}_1 - \mathbf{v}_2)^2 \left(9(\mathbf{n} \cdot \mathbf{v}_2) - \frac{q_1 q_2}{Gm_1 m_2} \right) + 28v_2^2 (\mathbf{v}_1 \cdot \mathbf{v}_2) - 13v_2^4 \right. \right. \\
& + \left(1 - \frac{q_1 q_2}{Gm_1 m_2} \right) (4v_2^2 (\mathbf{v}_1 \cdot \mathbf{v}_2) - 15(\mathbf{n} \cdot \mathbf{v}_2)^2 + 6(\mathbf{n} \cdot \mathbf{v}_2) (3v_2^2 - 6(\mathbf{v}_1 \cdot \mathbf{v}_2) - v_1^2) - 3v_2^4) \\
& + \frac{Gm_2}{4r^3} \left(v_1^2 \left(\frac{28q_1 q_2}{Gm_2} - 15Gm_1 - \frac{2q_1^2}{m_1} + \frac{2q_1 q_2}{m_1} - \frac{5q_1^2 q_2^2}{Gm_1 m_2^2} - \frac{4q_2^2}{m_2} \right) \right. \\
& + v_2^2 \left(5Gm_1 + 16Gm_2 + \frac{2q_1^2}{m_1} - \frac{18q_1 q_2}{m_1} + 8 \frac{q_2^2}{m_2} - \frac{q_1 q_2}{Gm_1 m_2^2} \right) \\
& + (\mathbf{v}_1 \cdot \mathbf{v}_2) \left(\frac{36q_1 q_2}{m_1} - 10Gm_1 - 32Gm_2 - \frac{4q_1^2}{m_1} - \frac{16q_2^2}{m_2} + \frac{2q_1 q_2}{Gm_1 m_2} \right) \\
& + (\mathbf{n} \cdot \mathbf{v}_2)^2 \left(34Gm_1 - 24Gm_2 + \frac{4q_1^2}{m_1} - \frac{42q_1 q_2}{m_2} + \frac{40q_1 q_2}{m_1} - \frac{16q_2^2}{m_2} \right) \\
& + (\mathbf{n} \cdot \mathbf{v}_1)^2 \left(78Gm_1 + 8Gm_2 + \frac{16q_1^2}{m_1} - \frac{106q_1 q_2}{m_2} - \frac{8q_2^2}{m_2} + \frac{12q_1^2 q_2^2}{Gm_1 m_2^2} \right) \\
& + (\mathbf{n} \cdot \mathbf{v}_1) (\mathbf{n} \cdot \mathbf{v}_2) \left(\frac{212q_1 q_2}{m_2} - 16Gm_2 - \frac{32q_1^2}{m_1} - 156Gm_1 + \frac{16q_2^2}{m_2^2} - \frac{24q_1^2 q_2^2}{Gm_1 m_2^2} \right) \left. \right) \\
& + \frac{G^2 m_2}{4r^4} \left(\frac{q_1 q_2}{Gm_1 m_2} \left(10q_1^2 - 80q_1 q_2 + \frac{q_1^2 q_2^2}{m_1 m_2^2} (m_1 + 2m_2) + 6q_2^2 \right) + \frac{2m_2 q_1}{m_1} (17q_2 - 12q_1) \right. \\
& + \left. \left. \frac{q_1 q_2}{Gm_2^2} (101Gm_1 m_2 - 27q_1 q_2 + 10q_2^2) - 3G(19m_1^2 + 46m_1 m_2 + 6m_2^2) - 14q_1^2 + 132q_1 q_2 - 24q_2^2 \right) \right] \left. \right\}.
\end{aligned}$$

Equation (25b):

$$\begin{aligned}
\mathcal{Q}_1^i = & \frac{1}{c^4} \left\{ n^i \left[\frac{q_1 q_2}{8} ((\mathbf{n} \cdot \mathbf{v}_2)^2 + (\mathbf{n} \cdot \mathbf{a}_2)r - v_2^2) + \frac{Gm_1 m_2}{8} (7v_2^2 - (\mathbf{n} \cdot \mathbf{v}_2)^2) - \frac{7Gm_1 q_1 q_2}{4r} \right] \right. \\
& \left. - \frac{3}{8} r q_1 q_2 a_2^i - \frac{v_2^i}{4} (\mathbf{n} \cdot \mathbf{v}_2) (7Gm_1 m_2 - q_1 q_2) \right\}.
\end{aligned}$$

Equation (52):

$$(a^i)_{1.5\text{PN}} = \frac{2G}{3r^3 c^3} \frac{q_1(q_1 m_2 - q_2 m_1)}{m_1} \left(1 - \frac{q_1 q_2}{G m_1 m_2}\right) [3(\mathbf{n} \cdot \mathbf{v}) n^i - v^i].$$

Equation (B2b):

$$\begin{aligned} H_{2\text{PN}}^{\text{ADM-type}} = & -\frac{G m_1 m_2}{2R} + \frac{1}{c^2} \left\{ \frac{G}{4R} (\mathbf{N} \cdot \mathbf{P}_1)(\mathbf{N} \cdot \mathbf{P}_2) - \frac{3G m_2 P_1^2}{2R m_1} + \frac{7(\mathbf{P}_1 \cdot \mathbf{P}_2)}{4R} + \frac{G^2 m_1 m_2 (m_1 + m_2)}{2R^2} \right\} \\ & + \frac{1}{c^4} \left\{ \frac{5G^2 m_2^2 P_1^2}{2m_1 R^2} - \frac{G^3 m_1 m_2}{8R^3} (m_1^2 + 5m_1 m_2 + m_2^2) + \frac{5G m_2 P_1^4}{8m_1^3 R} \right. \\ & + \frac{G^2 (m_1 + m_2)}{8R^2} (19P_1^2 - 27(\mathbf{P}_1 \cdot \mathbf{P}_2) - 6(\mathbf{N} \cdot \mathbf{P}_1)(\mathbf{N} \cdot \mathbf{P}_2) + 19P_2^2) \\ & + \frac{G}{8m_1 m_2 R} (5P_1^2 (\mathbf{N} \cdot \mathbf{P}_2)^2 - 22P_1^2 P_2^2 - 4(\mathbf{P}_1 \cdot \mathbf{P}_2)^2 - 24(\mathbf{P}_1 \cdot \mathbf{P}_2)(\mathbf{N} \cdot \mathbf{P}_1)(\mathbf{N} \cdot \mathbf{P}_2) - 6(\mathbf{N} \cdot \mathbf{P}_1)^2 (\mathbf{N} \cdot \mathbf{P}_2)^2) \left. \right\} \\ & + (1 \leftrightarrow 2). \end{aligned}$$

Equation (B3):

$$\begin{aligned} a^i = & -\frac{GM}{r^2} n^i (1 - \eta_1 \eta_2) \\ & + \frac{1}{c^2} \left\{ n^i \left[\frac{G^2 M^2}{r^2} \left(4 - 5\eta_1 \eta_2 + \frac{1}{2} \eta_2^2 (1 - X_{12}) + \frac{1}{2} \eta_1^2 (1 + X_{12}) + 2\nu (1 - \eta_1 \eta_2)^2 \right) \right. \right. \\ & + \frac{GM}{r^2} \left(\frac{v^2}{2} \left(-1 - \frac{\eta_1 \eta_2}{2} + (-3 + 3\eta_1 \eta_2) \nu \right) + \frac{3}{2} \dot{r}^2 (1 - \eta_1 \eta_2) \nu \right) \left. \right] + v^i \dot{r} [4 - \eta_1 \eta_2 + (-2 + 2\eta_1 \eta_2) \nu] \left. \right\} \\ & - \frac{1}{c^3} \left\{ \frac{2G^2 M^4 \nu^2}{3r^4} (\eta_1 - \eta_2)^2 (1 - \eta_1 \eta_2)^2 \right\} \\ & + \frac{1}{c^4} \left\{ n^i \left[\frac{G^3 M^3}{r^4} \left(\frac{27}{2} \eta_1 \eta_2 - 9 - 3 \left(1 - \frac{1}{4} \eta_1 \eta_2 \right) (\eta_1^2 + \eta_2^2 - X_{12} (\eta_2^2 - \eta_1^2)) \right) \right. \right. \\ & + \nu \left(\frac{7}{4} X_{12} (\eta_2^2 - \eta_1^2) (1 - \eta_1 \eta_2) - \frac{87}{4} - \frac{7}{4} (\eta_1^2 + \eta_2^2) + \frac{203}{4} \eta_1 \eta_2 + \frac{1}{4} \eta_1 \eta_2 (\eta_1^2 - 107\eta_1 \eta_2 + \eta_2^2 + 3\eta_1^2 \eta_2^2) \right) \left. \right] \\ & + \frac{G^2 M^2}{r^3} \left(2\nu^2 (\dot{r}^2 - v^2) (1 - \eta_1^2 \eta_2^2)^2 + X_{12} (\eta_2^2 - \eta_1^2) \left(\dot{r}^2 (1 + \nu) - \frac{v^2}{2} (1 + 3\nu) \right) + \frac{v^2}{2} (\eta_1^2 + \eta_1 \eta_2 + \eta_2^2) \right. \\ & + \dot{r}^2 (2 - \eta_1^2 - \eta_2^2) + \nu \left(\dot{r}^2 (25 + 5\eta_1^2 - 39\eta_1 \eta_2 + 5\eta_2^2 + 4\eta_1^2 \eta_2^2) + \frac{v^2}{2} (13 - 12\eta_1 \eta_2 - \eta_1^2 \eta_2^2) \right) \left. \right] \\ & + \frac{GM}{r^2} \left(\nu^2 (1 - \eta_1 \eta_2) \left(4v^4 + \frac{45}{8} \dot{r}^4 - 6\dot{r}^2 v^2 \right) + 3\nu \left(1 - \frac{\eta_1 \eta_2}{2} \right) - \frac{15\nu}{8} \dot{r}^4 (1 - \eta_1 \eta_2) - \frac{v^4}{8} \eta_1 \eta_2 \right) \left. \right] \\ & + v^i \left[\frac{GM}{r^2} \left(\frac{1}{2} \dot{r} v^2 \eta_1 \eta_2 + \frac{\nu \dot{r}}{2} (-9\dot{r}^2 + 3v^2 (5 - \eta_1 \eta_2) + \nu^2 \dot{r} (1 - \eta_1 \eta_2) (2v^2 - 3\dot{r}^2)) \right) \right. \\ & + \frac{G^2 M^2}{r^3} \left(\dot{r} (1 - \nu) X_{12} (\eta_2^2 - \eta_1^2) - 4\nu^2 \dot{r} (1 - \eta_1 \eta_2)^2 + \dot{r} (\eta_1 \eta_2 - \eta_1^2 - \eta_2^2 - 2) \right. \\ & + \left. \left. \frac{\dot{r}}{2} (56\eta_1 \eta_2 - 4\eta_1^2 - 4\eta_2^2 - 7\eta_1^2 \eta_2^2 - 41) \right) \right] \left. \right\}. \end{aligned}$$

Equation (B6):

$$\begin{aligned}
\frac{H_{\text{CoM}}^{\text{ADM-type}}}{\mu} = & -\frac{c^2}{\nu} + \frac{P^2}{2} - \frac{G}{R}(1 - \eta_1\eta_2) \\
& + \frac{1}{c^2} \left\{ \frac{P^4}{8}(3\nu - 1) + \frac{G^2}{4R^2}(2 + \eta_1^2 - 4\eta_1\eta_2 + \eta_2^2 + X_{12}(\eta_1^2 - \eta_2^2)) - \frac{GM}{2R}(3P^2 + \nu(1 - \eta_1\eta_2)(P^2 + P_R^2)) \right\} \\
& + \frac{1}{c^4} \left\{ \frac{G^3}{4R^3}(4\eta_1\eta_2 - \eta_1^2 - \eta_2^2 - 2 + X_{12}(\eta_2^2 - \eta_1^2)) + \nu(23\eta_1\eta_2 - 2\eta_1^2 - 2\eta_2^2 - 3\eta_1^2\eta_2^2 - \eta_1^3\eta_2^3 - 15) \right. \\
& + \frac{G^2}{8R^2}(X_{12}(\eta_1^2 - \eta_2^2)(P^2 - 2P_R^2)(1 - \nu) + 2P_R^2(\eta_1^2 + \eta_2^2 - 2) + P^2(\eta_1^2 + \eta_2^2 + 22) \\
& + \nu(P^2(58 + 7\eta_1^2 - 46\eta_1\eta_2 + 7\eta_2^2 + 2\eta_1^2\eta_2^2) - P_R^2(32 + 10\eta_1^2 + 10\eta_2^2 + 4\eta_1\eta_2))) \\
& + \frac{G}{8R}(5P^4 + 2\nu P^2((1 - \eta_1\eta_2)P_R^2 - P^2(11 + \eta_1\eta_2)) - \nu^2(1 - \eta_1\eta_2)(3P^4 + 2P^2P_R^2 + 3P_R^4)) \\
& + \frac{P^6}{16}(1 - 5\nu + 5\nu^2) + \frac{G}{4R^3}(1 - \eta_1\eta_2)(1 + 12\nu) + \frac{P_R^2}{2R^2} - \frac{P^2}{4R^2} + \frac{3\nu}{R^2}(2P_R^2 - P^2) \\
& - \frac{G\nu}{4R^2}(P^2 + 2P_R^2)(1 - \eta_1\eta_2) + \frac{\nu P^2}{4R}(P^2 - P_R^2) \\
& + \mathbf{A} \left[\frac{P^2}{MR}(P_R^2 - P^2) + \frac{G}{MR^2}(1 - \eta_1\eta_2)(2P_R^2 + P^2) \right] \\
& + \mathbf{B} \left[\frac{2P_R^2 - P^2}{M^2R^2} + \frac{G}{M^2R^3}(1 - \eta_1\eta_2) \right] \\
& + \mathbf{C} \left[\frac{3GP_R^2}{MR^2}(1 - \eta_1\eta_2) + \frac{3P_R^2}{MR}(P_R^2 - P^2) \right] \Bigg\}.
\end{aligned}$$

[1] A. Alonzo-Artiles and M. Kraus, [arXiv:2603.15933](https://arxiv.org/abs/2603.15933).