Higher-order Chern-Simons extensions to QED in 2 + 1 dimensions

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In this work, we investigate radiative corrections in the higher-order extension of the Maxwell-Chern-Simons model coupled to standard spinor matter in $2+1$ dimensions. We begin analyzing the higher-order gauge sector, where we find four canonical modes. One massive transverse photon, a ghost mode, a massless longitudinal mode and an additional degenerate massless mode associated with the double pole structure of the propagator. The higher-order gauge model is canonically quantized and as expected the resulting algebra of creation and annihilation operators exhibits an indefinite metric structure. Subsequently, we compute all relevant one-loop diagrams in the modified QED starting with the fermion self-energy. We show that the induced corrections to the fermion two-point function produce two independent fermionic degrees of freedom, which can be included in a redefined Lagrangian describing two decoupled fermions fields, one corresponding to a particle and the other to a negative-norm ghost state. We take advantage of this decomposition to compute the photon polarization operator and the vertex correction, both of which are found to be finite. Finally, we analyze the causal behavior of the model by computing the commutator of gauge fields at different spacetime points, and found that microcausality is preserved.

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I. INTRODUCTION

Chern-Simons (CS) theory is a topological quantum field theory that arises naturally in odd-dimensional spacetimes. It was first derived as an effective contribution from the high-temperature limit of QCD and electroweak models in four dimensions [1–3]. The CS theory provides a rich framework for exploring a wide range of physical phenomena. In particular, the CS term generates a gauge-invariant mass for the gauge field in $2+1$ spacetime dimensions [4,5]. Moreover, the definitions of various topological invariants, such as the Jones polynomial in knot theory, can be connected with the non-Abelian CS action [6], revealing a deep interplay between topological aspects of quantum field theory and lower dimensional conformal field theory. Furthermore, the CS term presents interesting possibilities in describing various physical systems, as the quantum Hall effect [7–10], topological materials including topological insulators [11,12], spectroscopic features in Weyl semimetals [13–16], and also extensions of gravity [5,17], vortex dynamics [18,19], Skyrme models [20], Higgs mechanism [21], topological gravity [22], supersymmetric CS [23] and anisotropic QED [24]; see also the reviews [25,26].

A topic that has attracted growing interest over the years is the quantum behavior of the Abelian CS term. Perturbative calculations in CS-modified QED are of particular interest because they show that topological gauge contributions can emerge dynamically through quantum fluctuations. In $(2+1)$ dimensions, fermion-loop corrections generate parity-odd Chern-Simons terms in the effective action, establishing a deep connection between topology, anomalies, and parity violation [25]. Furthermore, the coefficient of the induced CS term is known to depend sensitively on the regularization prescription, so that different schemes may produce distinct finite contributions or even a vanishing

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result [27]. This regularization ambiguity makes perturbative analyses especially relevant, since they provide a natural framework for investigating gauge invariance, anomaly consistency, and the ultraviolet structure of the theory. In the present higher-derivative extension, these issues become even more significant due to the modified propagator structure and the appearance of additional poles generated by the higher-order sector.

Quantum corrections to modified QED incorporating the CS term exhibit several noteworthy features, such as the smoothness of infrared divergencies resulting from the massive gauge mode generated by the gauge-invariant CS term. Moreover, in modified QED the polarization tensor receives only one-loop corrections, with all higher-order contributions vanishing due to a remarkable cancellation [27], a result known as the Coleman-Hill theorem [28]. The CS term has also been studied to extend to parity violating mass terms [29,30].

The higher-derivative extension of the CS term was proposed some years ago by S. Deser and R. Jackiw in order to describe the momentum expansion of the effective action in three-dimensional QED [31]. Some works on such an extension have focused on relic symmetries [32], thermal effects [33,34], Lorentz-violating planar electrodynamics [35–38], analysis of unitarity [39,40], and vector multiplets [41]. It is well known that higher time derivatives can improve the ultraviolet behavior of a theory by introducing higher powers of momentum in the denominators of propagators. However, they may also lead to an indefinite metric, with additional degrees of freedom associated to ghost fields [42,43]. In four dimensions, one can consider an analogue, which, however, is not topological in essence and necessarily breaks Lorentz invariance [44]. In this work, we explore the higher-order CS term in $2+1$ dimensions, with particular emphasis on quantum corrections induced by ghost states associated with the indefinite metric of the theory.

The work is organized as follows. In Sec. II, we present the QED model, finding the gauge propagator together with its polarization vectors. In Sec. III, we canonically quantize the model and derive the commutator algebra of creation and annihilation operators. In Sec. IV we compute the radiative correction up to one-loop level of the fermion self-energy, photon polarization operator, and vertex function. Also, we analyze the superficial degree of divergence of each one-loop diagram. In Sec. V, we study microcausality and prove that it is preserved. Finally, in Sec. VI we present our concluding remarks. In order to provide some details of the calculations, especially regarding microcausality and the canonical quantization formalism, we have included three Appendixes. Appendix A contains the calculation of a component of the gauge-field commutator for microcausality in a general frame. Appendix B is devoted to the Hamiltonian formulation of the constrained theory, while Appendix C provides details of the propagator calculation, including the emergence of a double-pole structure.

II. EXTENDED QED MODEL IN (2 + 1)-DIMENSIONS

Our model is based on the higher-derivative QED Lagrangian in $2+1$ dimensions, given by

$$\mathcal{L}=\bar{\psi}(i\partial\!\!\!/+eA\!\!\!/-m)\psi-\frac{1}{4\gamma}F_{\mu\nu}F^{\mu\nu}-\frac{1}{2\xi}(\partial_\mu A^\mu)^2+\mathcal{L}_{\mathrm{CSe}}, \tag{1}$$

where $\mathcal{L}_{\mathrm{CSe}}$ includes both the CS term and its higher-derivative extension [31], written as

$$\mathcal{L}_{\mathrm{CSe}}=\frac{1}{2}\epsilon^{\alpha\beta\gamma}A_\alpha(\mu+g\Box)\partial_\beta A_\gamma. \tag{2}$$

In the Lagrangian (1), we have coupled the photons with standard fermions and considered the usual interaction term. Also, we have included a covariant gauge-fixing term with $\xi>0$ and a constant γ in the standard Maxwell term, which allow us to obtain the pure CS theory by taking the limit $\gamma\to\infty$ together with $g\to 0$. Let us emphasize that the CS term has been included from the outset, as it is naturally generated from the standard photon polarization operator diagram [27].

Taking into account that the action is dimensionless, we find that the mass dimensions of the parameters and fields are

$$[\mu]=1,\qquad [g]=-1,\qquad [e]=\frac{1}{2},[\psi]=1,\qquad [A_\mu]=\frac{1}{2}, \tag{3}$$

while the constants γ and ξ are dimensionless.

In this work we employ the mostly minus sign convention for the metric signature, i.e., $\eta^{\mu\nu}=\mathrm{diag}(1,-1,-1)$, together with the Levi-Civita convention $\epsilon^{012}=1$ and the two-dimensional realization of the Dirac algebra

$$\begin{aligned}\gamma^0&=\sigma^3=\begin{pmatrix}1&0\\0&-1\end{pmatrix},\\ \gamma^1&=i\sigma^1=\begin{pmatrix}0&i\\i&0\end{pmatrix},\\ \gamma^2&=i\sigma^2=\begin{pmatrix}0&1\\-1&0\end{pmatrix},\end{aligned} \tag{4}$$

with σ^1, σ^2, σ^3 being the Pauli matrices. Furthermore, one can check that the Dirac matrices satisfy the Clifford algebra

$$\{\gamma^\mu,\gamma^\nu\}=2\eta^{\mu\nu}, \tag{5}$$

and the relations

$$\gamma^\mu\gamma^\nu = \eta^{\mu\nu}\mathbb{1}_{2\times2} - i\epsilon^{\mu\nu\alpha}\gamma_\alpha,$$
$$\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho) = -2i\epsilon^{\mu\nu\rho},$$
$$\gamma^\mu = \eta^{\mu\nu}\gamma_\nu, \tag{6}$$

where $\mathbb{1}_{2\times2}$ is the 2×2 unit matrix.

A. The gauge propagator

Let us consider the free Lagrangian in the gauge sector:

$$\mathcal{L}_A = -\frac{1}{4\gamma}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \frac{1}{2}\epsilon^{\alpha\beta\gamma}A_\alpha(\mu + g\Box)\partial_\beta A_\gamma. \tag{7}$$

The generalized Euler-Lagrange equation for the second-order Lagrangian $\mathcal{L}_A$ can be written as

$$-\partial_\kappa\partial_\lambda\frac{\partial\mathcal{L}_A}{\partial(\partial_\kappa\partial_\lambda A_\sigma)} + \partial_\rho\frac{\partial\mathcal{L}_A}{\partial(\partial_\rho A_\sigma)} - \frac{\partial\mathcal{L}_A}{\partial A_\sigma} = 0. \tag{8}$$

From the Lagrangian (7) and using (8), we have the equation of motion

$$\left[\frac{\eta^{\mu\nu}}{\gamma}\Box - \left(\frac{1}{\gamma} - \frac{1}{\xi}\right)\partial^\mu\partial^\nu + \epsilon^{\mu\beta\nu}(\mu + g\Box)\partial_\beta\right]A_\nu(x) = 0. \tag{9}$$

Contracting Eq. (9) with ∂_μ yields

$$\frac{1}{\xi}\Box(\partial\cdot A) = 0, \tag{10}$$

which, by imposing suitable boundary conditions at infinity, gives the condition $\partial\cdot A = 0$.

Now, we go to momentum space expanding the gauge field in Fourier modes

$$A_\mu(x) = \int \mathrm{d}^3k A_\mu(k)e^{-ik\cdot x}. \tag{11}$$

Replacing in the equation of motion (9), we obtain in momentum space

$$S^{\mu\nu}(k)A_\nu(k) = 0, \tag{12}$$

where we have defined the operator

$$S^{\mu\nu}(k) = -\frac{k^2}{\gamma}\eta^{\mu\nu} + \left(\frac{1}{\gamma} - \frac{1}{\xi}\right)k^\mu k^\nu - i\mathcal{M}(k)\epsilon^{\mu\beta\nu}k_\beta, \tag{13}$$

and we have introduced the function of momentum

$$\mathcal{M}(k) \coloneqq \mu - gk^2, \tag{14}$$

which has dimension of mass.

By inverting the operator we find the propagator

$$G_{\nu\rho}(k) = \frac{-i\gamma}{k^2 - \gamma^2\mathcal{M}^2(k)}T_{\nu\rho}(k), \tag{15}$$

which satisfies $S^{\mu\nu}G_{\nu\rho} = i\delta^\mu_\rho$, with the definition

$$T_{\nu\rho}(k) \coloneqq \eta_{\nu\rho} - \left[1 - \frac{\xi}{\gamma}\left(1 - \frac{\gamma^2\mathcal{M}^2}{k^2}\right)\right]\frac{k_\nu k_\rho}{k^2} - i\gamma\mathcal{M}\epsilon_{\nu\beta\rho}\frac{k^\beta}{k^2}. \tag{16}$$

The propagator in the Landau gauge setting $\xi = 0$ reads

$$\Delta_{\nu\rho}(k) = \frac{-i\gamma}{k^2 - \gamma^2\mathcal{M}^2}\left(\eta_{\nu\rho} - \frac{k_\nu k_\rho}{k^2} - i\gamma\mathcal{M}\epsilon_{\nu\beta\rho}\frac{k^\beta}{k^2}\right), \tag{17}$$

which avoids infrared divergencies and will prove most convenient to use in the subsequent sections.

The dispersion relation can be inferred from the pole structure of the propagators in Eqs. (15) and (17), which contain both massive and massless poles. In the first place, the massive transverse modes arise from

$$k^2 - \gamma^2\mathcal{M}^2 = 0. \tag{18}$$

Solving (18) yields two massive propagating modes

$$\omega_1 = \sqrt{\vec{k}^2 + m_1^2}, \tag{19a}$$

$$W_2 = \sqrt{\vec{k}^2 + M_2^2}, \tag{19b}$$

with masses

$$m_1 = \frac{\sqrt{1 + 4\gamma^2\mu g} - 1}{2\gamma g}, \tag{20a}$$

$$M_2 = \frac{\sqrt{1 + 4\gamma^2\mu g} + 1}{2\gamma g}. \tag{20b}$$

Both solutions are guaranteed to be real, ensuring the absence of tachyons in the theory. Also, the first solution ω_1 corresponds to a massive photon, while the second solution W_2 corresponds to a massive ghost state. This explicitly demonstrates the indefinite metric structure of the theory [42,43] as one can show by taking the low energy limit $g \to 0$. Hence, expanding our dispersion relations up to the second order in g, we have

$$\omega_1 \approx \sqrt{\vec{k}^2 + \gamma^2\mu^2} - \frac{g\gamma^4\mu^3}{\sqrt{\vec{k}^2 + \gamma^2\mu^2}} + \mathcal{O}(g^2), \tag{21a}$$

$$W_2 \approx \frac{1}{\gamma g} + \gamma\mu + g\left(\frac{\vec{k}^2\gamma}{2} - \gamma^3\mu^2\right) + \mathcal{O}(g^2), \tag{21b}$$

$$m_1 \approx \mu\gamma - \mu^2\gamma^3 g + \mathcal{O}(g^2), \tag{21c}$$

$$M_2 \approx \frac{1}{\gamma g} + \mu\gamma - \mu^2\gamma^3 g + \mathcal{O}(g^2), \tag{21d}$$

and we can see that in the limit $g \to 0$, the solution remains regular, as expected, while the ghost solution exhibits a singularity, consistent with its problematic nature, in particular, connected to stability and causality.

From (16) we also find solutions associated to massless modes with $k^2 = 0$, which give rise to both simple and double poles. In the Landau gauge, $\xi \to 0$, only simple poles remain in the propagator, whereas in the Feynman gauge, $\xi = \gamma$, double poles appear in addition to the simple ones.

Furthermore, the equations (21c) and (21d) reveal a significant mass gap between ghost and particle, given by $M_2 - m_1 = 1/(\gamma g)$. This gap disappears in the pure Chern-Simons limit by taking the limit $\gamma \to \infty$, at which both masses coincide $M_2 = m_1$. Throughout this work, and without loss of generality, we assume $\mu > 0$, $g > 0$ and $\gamma > 0$.

B. Polarization vectors

We begin by defining an orthogonal basis of $(2+1)$-dimensional Minkowski spacetime, given by the real vectors

$$e^{(0)\mu} = \frac{1}{\sqrt{k^2}}k^\mu, \tag{22a}$$

$$e^{(1)\mu} = \frac{1}{\sqrt{G}}\epsilon^{\mu\beta\gamma}k_\beta n_\gamma, \tag{22b}$$

$$\begin{aligned} e^{(2)\mu} &= -\frac{1}{\sqrt{k^2}}\epsilon^{\mu\beta\gamma}k_\beta e^{(1)}_\gamma \\ &= \frac{1}{\sqrt{k^2 G}}(k^2 n^\mu - k^\mu(k\cdot n)), \end{aligned} \tag{22c}$$

where $G \equiv (k\cdot n)^2 - k^2 n^2$ and n^μ is an auxiliary field.

The three-vector basis $e^{(a)\mu}$ are normalized according to

$$e^{(a)} \cdot e^{(b)} = g_{ab}, \tag{23}$$

with $a = 0,\ 1,\ 2$ and $g_{ab} = \mathrm{diag}\,(1,-1,-1)$. They also satisfy

$$\epsilon^{\mu\beta\gamma}k_\beta e^{(2)}_\gamma = \sqrt{k^2}e^{(1)\mu}, \tag{24}$$

$$\epsilon^{\mu\beta\gamma}k_\beta e^{(1)}_\gamma = -\sqrt{k^2}e^{(2)\mu}. \tag{25}$$

We define the complex basis

$$\varepsilon^{(0)\mu} = e^{(0)\mu}, \tag{26a}$$

$$\varepsilon^{(+)\mu} = \frac{1}{\sqrt{2}}(e^{(2)\mu} + ie^{(1)\mu}), \tag{26b}$$

$$\varepsilon^{(-)\mu} = \frac{1}{\sqrt{2}}(e^{(2)\mu} - ie^{(1)\mu}), \tag{26c}$$

and introduce the index $\lambda = 0, \pm$. Here the $\pm$ modes are orthogonal to the momentum, i.e., $k\cdot\varepsilon^{(\pm)} = 0$. The vectors defining the complex basis satisfy the relations

$$\varepsilon^{(\lambda)} \cdot \varepsilon^{(\lambda')*} = g_{\lambda\lambda'}, \tag{27}$$

$$\epsilon^{\mu\beta\sigma}k_\beta\varepsilon^{(\pm)}_\sigma = \mp\, i\sqrt{k^2}\varepsilon^{(\pm)\mu}, \tag{28}$$

with $g_{\lambda\lambda'} = \mathrm{diag.}(1,-1,-1)$.

This set of complex basis vectors diagonalizes the $S^{\mu\nu}$ as follows:

$$S^\mu_\nu(p)\varepsilon^{(0)\nu} = \Lambda_0(p)\varepsilon^{(0)\mu}, \tag{29a}$$

$$S^\mu_\nu(p)\varepsilon^{(+)\nu} = \Lambda_+(p)\varepsilon^{(+)\mu}, \tag{29b}$$

$$S^\mu_\nu(p)\varepsilon^{(-)\nu} = \Lambda_-(p)\varepsilon^{(-)\mu}, \tag{29c}$$

where

$$\Lambda_0(k) = -\frac{k^2}{\xi}, \tag{30a}$$

$$\Lambda_+(k) = -\frac{k^2}{\gamma} - \sqrt{k^2}\mathcal{M}(k), \tag{30b}$$

$$\Lambda_-(k) = -\frac{k^2}{\gamma} + \sqrt{k^2}\mathcal{M}(k). \tag{30c}$$

The dispersion equation follows from the product of the three eigenvalues of $S^{\mu\nu}$

$$\prod_{\lambda=0,\pm}\Lambda_\lambda(p) = -\frac{(k^2)^2}{\xi\gamma^2}(k^2 - \gamma^2\mathcal{M}^2(k)) = 0. \tag{31}$$

One can show that

$$\varepsilon^{(\pm)}_\mu\varepsilon^{(\pm)*}_\nu = -\frac{1}{2}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \pm i\epsilon_{\mu\beta\nu}\frac{k^\beta}{\sqrt{k^2}}\right), \tag{32}$$

from where we make the connection with the propagator, by calculating

$$\sum_{\lambda,\lambda'=0,\pm} g_{\lambda\lambda'} \frac{\varepsilon_\mu^{(\lambda)} \varepsilon_\nu^{(\lambda')*}}{\Lambda_\lambda} = -iG_{\mu\nu}. \tag{33}$$

We can also make a connection with the equation of motion operator in momentum space

$$\sum_{\lambda,\lambda'=0,\pm} g_{\lambda\lambda'} \Lambda_\lambda \varepsilon_\mu^{(\lambda)} \varepsilon_\nu^{(\lambda')*} = S_{\mu\nu}(\xi, k). \tag{34}$$

Thus the set $\{\varepsilon^{(\lambda)\mu}\}$ corresponds to independent eigenvector solutions of the equation of motion operator.

III. CANONICAL QUANTIZATION

According to Ostrogradsky variational formalism [45], the momentum variables in our model can be written as

$$P^\mu := \frac{\partial \mathcal{L}_A}{\partial \dot{A}_\mu} - \frac{\partial \Pi^\mu}{\partial t}, \tag{35}$$

$$\Pi^\mu := \frac{\partial \mathcal{L}_A}{\partial \ddot{A}_\mu}. \tag{36}$$

The explicit form of momentum variables looks like

$$P^\mu = -\frac{1}{\gamma} F^{0\mu} - \frac{1}{\xi}(\partial_\lambda A^\lambda)\eta^{0\mu} + \frac{1}{2}\epsilon^{\alpha 0\mu}(\mu + g\Box)A_\alpha - \frac{g}{2}\epsilon^{\mu\beta\gamma}\partial_\beta \dot{A}_\gamma, \tag{37}$$

$$\Pi^\mu = \frac{g}{2}\epsilon^{\mu\beta\gamma}\partial_\beta A_\gamma. \tag{38}$$

We impose the equal-time commutation relations on the canonical variables

$$[A_\mu(t,\vec{x}), P_\nu(t,\vec{y})] = i\eta_{\mu\nu}\delta^{(2)}(\vec{x}-\vec{y}), \tag{39}$$

$$[\dot{A}_\mu(t,\vec{x}), \Pi_\nu(t,\vec{y})] = i\eta_{\mu\nu}\delta^{(2)}(\vec{x}-\vec{y}), \tag{40}$$

where all the others are defined to vanish.

As shown in Refs. [32,39], the higher-order theory constitutes a constrained system, leading to the appearance of Dirac brackets [46–48]. Accordingly, we perform the Hamiltonian analysis and determine the nonvanishing Dirac brackets in Appendix B. A direct counting of degrees of freedom, involving four second-class constraints, yields 4 total degrees of freedom that must be incorporated into the field expansion. To this end, the gauge field is expanded by means of the spectral decomposition of the equation-of-motion operator (30). This decomposition gives rise to three independent modes, encoded in the fields $\bar{A}_\mu$, G_μ, and A_μ^G. In addition, there exists a second type of solution associated with the degeneracy of the massless pole, which we denote by A_μ^D. This additional field accounts for the extra degree of freedom originating from the double multiplicity of the pole at $k^2 = 0$; see the propagator (15). We therefore expand the gauge field as

$$A_\mu(x) = \bar{A}_\mu(x) + G_\mu(x) + A_\mu^G(x) + A_\mu^D(x). \tag{41}$$

We see that the wave vector in the expansion must be a solution to the following eigenvector equation:

$$S_\nu^\mu(k)\varepsilon^{(\lambda)\nu}(k) = \Lambda_\lambda(k)\varepsilon^{(\lambda)\nu}(k) = 0. \tag{42}$$

Since $\bar{A}_\mu(x)$ is a massive photon with the positive energy ω_1 and mass m_1 satisfying $\Lambda_-(\omega_1) = 0$ we conclude that it belongs to the subspace $(-)$. Proceeding similarly for the ghost with $\Lambda_+(W_2) = 0$, we conclude that it is related to the $(+)$ subspace. Furthermore, we notice that $\bar{A}_\mu$ and G_μ satisfy the orthogonality condition for the A_μ field, $k^\mu \bar{A}_\mu = k^\mu G_\mu = 0$. Finally, the A_μ^G and A_μ^D fields are solutions associated with a polarization-vector basis that remains regular at $\omega_0 = |\vec{k}|$, as explicitly shown in Ref. [49].

Taking all this into account, we arrive at the expansion of the field

$$\bar{A}_\mu(x) = \int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{1}{2\omega_1\bar{\beta}}[\varepsilon_\mu^{(-)} a_k^{(-)} e^{-ik\cdot x} + \varepsilon_\mu^{(-)*} a_k^{(-)\dagger} e^{ik\cdot x}]_{k_0=\omega_1}, \tag{43}$$

$$G_\mu(x) = \int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{1}{2W_2\bar{\beta}} \times \left[\varepsilon_\mu^{(+)} b_k^{(+)} e^{-ik\cdot x} + \varepsilon_\mu^{(+)*} b_k^{(+)\dagger} e^{ik\cdot x}\right]_{k_0=W_2}, \tag{44}$$

$$A_\mu^G(x) = \int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{-ik_\mu}{2|\vec{k}|}[c_k e^{-ik\cdot x} - c_k^\dagger e^{ik\cdot x}]_{k_0=|\vec{k}|}, \tag{45}$$

$$A_\mu^D(x) = \int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{1}{2|\vec{k}|}[v_\mu(x_0,k) d_k e^{-ik\cdot x} + v_\mu^*(x_0,k) d_k^\dagger e^{ik\cdot x}]_{k_0=|\vec{k}|}, \tag{46}$$

where $\bar{\beta} = \frac{\sqrt{1+4\gamma^2\mu g}}{2\gamma}$ and we define the time-dependent polarization vector

$$v_\mu(x_0,k) = \mu x_0 k_\mu + i\mu n_\mu - \frac{2}{\xi}\epsilon_{\mu\rho\sigma}k^\rho n^\sigma, \tag{47}$$

with $n_\mu = (1,0,0)$.

The field A_μ^G is an ordinary eigenmode of the wave operator, whereas A_μ^D corresponds to a generalized eigenmode associated with the repeated massless root. In the space of solutions, these fields form a generalized eigenmode chain under successive applications of the wave operator,

$$A_\mu^D(x) \overset{\Box}{\mapsto} A_\mu^G(x) \overset{\Box}{\mapsto} 0. \tag{48}$$

Thus, A_μ^D is not annihilated by a single application of $\Box$, but rather satisfies $\Box^2 A_\mu^D = 0$. The term proportional to x_0 in A_μ^D is a characteristic consequence of this repeated root and provides the time-dependent contribution required by the generalized solution. Furthermore, one can check that the fields (43)–(46) satisfy the Dirac brackets given in Appendix B.

From the ETCR of modified commutators, we impose the creation and annihilation operator algebra

$$[a_k^{(-)}, a_q^{(-)\dagger}] = (2\pi)^2 2\omega_1 \bar{\beta}\delta^{(2)}(\vec{k}-\vec{q}), \tag{49}$$

$$[b_k^{(+)}, b_q^{(+)\dagger}] = -(2\pi)^2 2W_2 \bar{\beta}\delta^{(2)}(\vec{k}-\vec{q}), \tag{50}$$

$$[c_k, c_q^\dagger] = (2\pi)^2 \left(-\frac{2|\vec{k}|}{\gamma\mu^2} + \frac{\xi}{|\vec{k}|}\right)\delta^{(2)}(\vec{k}-\vec{q}), \tag{51}$$

$$[c_k, d_q^\dagger] = (2\pi)^2 \frac{\xi}{\mu}\delta^{(2)}(\vec{k}-\vec{q}), \tag{52}$$

$$[d_k, c_q^\dagger] = (2\pi)^2 \frac{\xi}{\mu}\delta^{(2)}(\vec{k}-\vec{q}). \tag{53}$$

Note that the algebra (50) admits negative-norm states, as evidenced by the sign of the $b_p^{(+)}$ sector, leading to an indefinite metric in Hilbert space. In addition, the nontrivial mixing between the operators (c_k, d_k) signals the presence of a dipole structure. Accordingly, we expect the emergence of a dipole ghost field [50,51]. This expectation is confirmed by diagonalizing the (c_k, d_k) sector, which admits a regular $\xi \to 0$ limit, as shown in Ref. [49].

As a consistency check, we have tested the field expansion by explicitly computing the Feynman propagator in Appendix C, confirming that the system correctly accounts for the four degrees of freedom identified in the Hamiltonian analysis.

IV. RADIATIVE CORRECTIONS

In this section, we compute the relevant one-loop radiative corrections of the higher-order QED model in $(2+1)$ dimensions. Since we introduce a higher-derivative extension of the standard CS-QED theory, our goal is to analyze how the new sector modifies the original model through the three fundamental quantum corrections of CS-QED. In this framework, the higher-derivative parameter g is treated as a fundamental parameter of the classical theory, and incorporated exactly into the gauge propagator, whereas the parameters μ and γ retain their usual roles as coefficients of the Chern-Simons and Maxwell sectors, respectively. Accordingly, the perturbative expansion is organized in the standard loop expansion associated with the gauge interaction, while the effects of the higher-derivative sector are fully encoded in the modified propagator. In the first subsection, we analyze the divergences of the model. Next, we compute the fermion self-energy and show that it induces a higher-order contribution proportional to $g\Box$, originating from the higher-order gauge sector. This result motivates the construction of a local higher-derivative extension of the fermionic Lagrangian, which is subsequently regarded as a classical effective theory with the same parameter g. Accordingly, we redefine the fermionic Lagrangian in terms of two decoupled fermion fields. In the last two subsections, we compute the vacuum polarization tensor and the vertex correction within this higher-derivative fermionic theory by employing the modified fermion propagator.

A. Analysis of divergences

Let us perform an analysis of the superficially divergent integrals in our modified QED. Using the UV asymptotic for the gauge propagator in the Landau gauge (17), one has the superficial degree of divergence D

$$D = 3L - 3P_A - P_\psi = 3 + 2P_\psi - 3V, \tag{54}$$

where L is the number of loops, and P_A and P_ψ are the numbers of gauge and spinor propagators respectively. Here we used the topological identity $L + V - P = 1$, together with the fact that the total number of propagators are $P = P_A + P_\psi$.

Then, we can relate numbers of vertices, propagators, and external legs (E_A for external photons and E_ψ for external fermions) calculating the numbers of all fields associated to all vertices. Since we have basically a QED theory with the standard gauge-spinor vertex, we have

$$V = 2P_A + E_A \to P_A = \frac{1}{2}(V - E_A), \tag{55}$$

$$2V = 2P_\psi + E_\psi \to P_\psi = \frac{1}{2}(2V - E_\psi), \tag{56}$$

for the gauge and fermion fields respectively, relating the number of vertices, propagators, and external legs. This allows us to write D as

$$D = 3 - E_\psi - V. \tag{57}$$

From the formula for D we find that in divergent diagrams, one has $E_\psi \leq 2$. In particular, we have for the photon polarization tensor graph $V = 2, E_\psi = 0$, and so $D = 1$. We will see that within the dimensional regularization scheme this contribution is finite; see Sec. IV. For the fermion one-loop self-energy graph, one has $V = 2$, $E_\psi = 2, D = -1$ which is finite. The fermion-photon triple vertex function is also finite, corresponding to $D = -2$. All

one-loop Feynman diagrams with more numbers of legs and vertices are finite as well. Performing a general analysis we have that the mass dimension in our interacting theory is $\Delta = 1/2$ [52,53].

We conclude that our theory is super-renormalizable and we can have divergences up to two loops, by straightforward consideration of Feynman diagrams. However, for gauge-symmetry reasons, the only potentially divergent contribution from the gauge sector, namely, the $A^\alpha A_\alpha$ term, cannot arise even at two loops, whereas derivative corrections (the CS and Maxwell terms) are finite at two loops. Therefore, the only possible divergences, in principle, are the two-loop corrections to the spinor kinetic term and the spinor-vector vertex.

B. Fermion self energy

Let us write down the two-point function for the spinor field:

$$i\Sigma(p) = (-ie)^2 \int \frac{\mathrm{d}^3k}{(2\pi)^3}\gamma^\alpha S_0(p-k)\gamma^\beta \Delta_{\alpha\beta}(k). \tag{58}$$

We work in the Landau gauge (17) and set $\gamma = 1$ ensuring a nontrivial impact of the Maxwell term.

The fermion propagator is the standard one

$$S_0(k) = \frac{i}{\not{k} - m}. \tag{59}$$

This expression can be written as a following sum of four terms:

$$\Sigma(p) = \Sigma^{(1)}(p) + \Sigma^{(2)}(p) + \Sigma^{(3)}(p) + \Sigma^{(4)}(p), \tag{60}$$

where the first one is

$$\Sigma^{(1)}(p) = ie^2 \int \frac{\mathrm{d}^3k}{(2\pi)^3}\gamma^\mu \left(\frac{\not{p}-\not{k}+m}{(p-k)^2-m^2}\right)\gamma_\mu \times \frac{1}{k^2-\mathcal{M}^2(k)}, \tag{61a}$$

the second one

$$\Sigma^{(2)}(p) = -ie^2 \int \frac{\mathrm{d}^3k}{(2\pi)^3}\gamma^\mu \left(\frac{\not{p}-\not{k}+m}{(p-k)^2-m^2}\right)\gamma^\nu \times \left(\frac{1}{k^2-\mathcal{M}^2(k)}\right)\frac{k_\mu k_\mu}{k^2}, \tag{61b}$$

the third one

$$\Sigma^{(3)}(p) = \mu e^2 \int \frac{\mathrm{d}^3k}{(2\pi)^3}\gamma^\mu \left(\frac{\not{p}-\not{k}+m}{(p-k)^2-m^2}\right)\gamma^\nu \times \left(\frac{\epsilon_{\mu\beta\nu}k^\beta}{k^2-\mathcal{M}^2(k)}\right)\frac{1}{k^2}, \tag{61c}$$

and the fourth one

$$\Sigma^{(4)}(p) = -ge^2 \int \frac{\mathrm{d}^3k}{(2\pi)^3}\gamma^\mu \left(\frac{\not{p}-\not{k}+m}{(p-k)^2-m^2}\right)\gamma^\nu \times \frac{\epsilon_{\mu\beta\nu}k^\beta}{k^2-\mathcal{M}^2(k)}. \tag{61d}$$

To simplify the calculation of the different pieces, we rewrite the pole structure of the gauge propagator into its particle and ghost contributions as

$$\frac{1}{k^2-\mathcal{M}^2(k)} = \frac{1}{g^2(M_2^2-m_1^2)}\left(\frac{1}{k^2-m_1^2} - \frac{1}{k^2-M_2^2}\right). \tag{62}$$

The first term is dominated by the particle mass m_1 and the second term by the ghost mass M_2. This decomposition turns out to be crucial for radiative corrections.

In order to compute the above corrections we shall use the Feynman parametrization

$$\frac{1}{AB} = \int_0^1 \mathrm{d}x \frac{1}{Ax + B(1-x)}. \tag{63}$$

With suitable choices of A and B together with a shift in the momenta, $k + xp \to l$, and parity properties, the first piece of the fermion self-energy in (61a) reduces to

$$\Sigma^{(1)}(p) = \frac{e^2}{8\pi}\int \mathrm{d}x \frac{\not{p}(1-x)-3m}{g^2(M_2^2-m_1^2)} \times \left[\frac{1}{\sqrt{\Delta(m_1)}} - \frac{1}{\sqrt{\Delta(M_2)}}\right], \tag{64}$$

with

$$\Delta(\alpha_i) = p^2x(x-1) + m^2x + \alpha_i^2(1-x), \tag{65}$$

where α_i is the notation for either m_1 or M_2.

In an analogous way, the second piece is

$$\Sigma^{(2)}(p) = \frac{e^2}{16\pi}\int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \frac{1}{g^2(M_2^2-m_1^2)} \times \left[3\alpha\left(\frac{1}{\sqrt{Q(m_1)}} - \frac{1}{\sqrt{Q(M_2)}}\right) - \beta\left(\frac{1}{(Q(m_1))^{3/2}} - \frac{1}{(Q(M_2))^{3/2}}\right)\right], \tag{66}$$

with

$$\alpha = \frac{\not{p}}{3}(1+5x) + m, \tag{67}$$

$$\beta = -\not{p}p^2(1-x)x^2 + p^2x^2m, \tag{68}$$

$$Q(\alpha_i) = p^2x(x-1) + m^2x + \alpha_i^2 y, \tag{69}$$

the third piece is

$$\Sigma^{(3)}(p) = -\frac{e^2}{16\pi}\int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \frac{1}{g^2(M_2^2 - m_1^2)} \times \left[3\mu\left(\frac{1}{\sqrt{Q(m_1)}} - \frac{1}{\sqrt{Q(M_2)}}\right) - \mu\lambda\left(\frac{1}{(Q(m_1))^{3/2}} - \frac{1}{(Q(M_2))^{3/2}}\right)\right], \quad (70)$$

and, finally, the fourth piece is

$$\Sigma^{(4)}(p) = \frac{ge^2}{2\pi^2}\int_0^1 \mathrm{d}x \frac{1}{g^2(M_2^2 - m_1^2)} \left[-\int_0^\infty \mathrm{d}r \frac{r^4}{(r^2+\Delta)^2} + \lambda\int_0^\infty \mathrm{d}r \frac{r^2}{(r^2+\Delta)^2}\right], \quad (71)$$

with

$$\lambda = mx\not{p} - 2p^2(1-x)x. \quad (72)$$

The first integral in (71) has to be analytically continued from arbitrary d dimensions to $d = 3$ in order to produce a finite result.

Finally, the total radiative corrections induce in the fermion self-energy the following higher-order operator of the form

$$\Sigma(p) = A(p^2, g, \mu)\not{p} - B(p^2, g, \mu)m - gp^2 C(p^2, \mu), \quad (73)$$

where A, B, and C are functions that do not contribute to the p^2 order. Motivated by the term proportional to gp^2, we phenomenologically identify the leading higher-derivative contribution with the local operator $g\bar{\psi}\Box\psi$ and consider the following higher-derivative extension of the fermionic Lagrangian,

$$\mathcal{L}_f = \bar{\psi}(i\partial\!\!\!/ - m)\psi + g\bar{\psi}\Box\psi, \quad (74)$$

where g denotes the same higher-derivative parameter introduced in the gauge sector. This identification also enables a direct comparison with the standard CS-QED results in the limit $g \to 0$.

We decompose the above Lagrangian in two parts

$$\mathcal{L}_f = \bar{\psi}_1(i\partial\!\!\!/ - \bar{m}_1)\psi_1 - \bar{\psi}_2(i\partial\!\!\!/ - \bar{m}_2)\psi_2, \quad (75)$$

where

$$\psi_1 = \tilde{\beta}(i\partial\!\!\!/ - \bar{m}_2)\psi, \quad (76)$$

$$\psi_2 = \tilde{\beta}(i\partial\!\!\!/ - \bar{m}_1)\psi, \quad (77)$$

with

$$\tilde{\beta} = \left(\frac{g}{\bar{m}_2 - \bar{m}_1}\right)^{1/2}. \quad (78)$$

The fermionic masses are

$$\bar{m}_1 = \frac{1 - \sqrt{1-4mg}}{2g}, \quad (79)$$

$$\bar{m}_2 = \frac{1 + \sqrt{1-4mg}}{2g}, \quad (80)$$

where the first corresponds to a fermion and the second to a ghost fermion of very high mass of the order $1/g$.

The two massive poles are real provided that

$$1 - 4mg \geq 0. \quad (81)$$

Hereafter, we assume this condition to ensure the reality of both poles.

It is interesting to note the analogy of the two fermion species that arises when including an odd mass term in the fermion Lagrangian [33].

Both propagators can be written as

$$S_1(p) = \frac{i}{\not{p} - \bar{m}_1}, \quad (82)$$

$$S_2(p) = \frac{i}{\not{p} - \bar{m}_2}. \quad (83)$$

Note that the fermion propagator described in (74) can be also obtained without resorting to the explicit decomposition of the fermion fields $\psi_{1,2}$, that is to say, we can consider

$$S(p) \equiv \frac{i}{\not{p} - m - gp^2} = \frac{\bar{m}_1 + \bar{m}_2}{\bar{m}_2 - \bar{m}_1}(S_1(p) - S_2(p)), \quad (84)$$

with $S_{1,2}(p)$ given in (82) and (83). In the next section, we shall use the decomposed expression (84) in order to evaluate the photon polarization operator.

C. Photon polarization operator

Now we evaluate the photon polarization operator (PPO)

$$\Pi^{\mu\nu}(q) = -ie^2\int \frac{\mathrm{d}^3k}{(2\pi)^3}\mathrm{tr}[\gamma^\mu S(k)\gamma^\nu S(k+q)]. \quad (85)$$

For this, we use the decomposed fermion propagator given in (84), and define the contributions

$$\Pi_{ij}^{\mu\nu}(q) = -i\bar{e}^2\int \frac{\mathrm{d}^3k}{(2\pi)^3}\mathrm{tr}[\gamma^\mu S_i(k)\gamma^\nu S_j(k+q)], \quad (86)$$

with

$$S_i(p) = \frac{i}{\not{p} - \bar{m}_i}, \tag{87}$$

$$\bar{e} = e\left(\frac{\bar{m}_1 + \bar{m}_2}{\bar{m}_2 - \bar{m}_1}\right), \tag{88}$$

where $i, j = 1, 2$. We also introduce the notation for the different contributions

$$\Pi^{\mu\nu}(q) \equiv \Pi^{\mu\nu}_{11}(q) - 2\Pi^{\mu\nu}_{12}(q) + \Pi^{\mu\nu}_{22}(q). \tag{89}$$

Since each contribution in (89) has a similar form, we compute the general contribution

$$\begin{aligned}\Pi^{\mu\nu}_{ij}(q) = 2i\bar{e}^2 \int \frac{\mathrm{d}^3k}{(2\pi)^3} \frac{1}{(k^2 - \bar{m}_i{}^2)((k+q)^2 - \bar{m}_j{}^2)} \\ \times [k^\mu (k+q)^\nu + k^\nu (k+q)^\mu - \eta^{\mu\nu}(k\cdot(k+q) - \bar{m}_i\bar{m}_j) \\ - i\epsilon^{\mu\nu\beta}(\bar{m}_i(k+q)_\beta - \bar{m}_j k_\beta)],\end{aligned} \tag{90}$$

where we have used the expressions (5) and (6) in the trace calculation.

Using the Feynman parametrization together with a shift in the momenta, $k + xq \to l$, and using parity properties, the PPO in (90) reduces in d dimensions to

$$\begin{aligned}\Pi^{\mu\nu}_{ij}(q) = 2i\bar{e}^2\mu^{3-d} \int \frac{\mathrm{d}^d l}{(2\pi)^d} \int_0^1 \mathrm{d}x \frac{1}{[l^2 - \Theta_{ij}]^2} \\ \times \left[\left(\frac{2}{d} - 1\right) l^2 \eta^{\mu\nu} - 2x(1-x)(q^\mu q^\nu - \eta^{\mu\nu} q^2) \right. \\ + \eta^{\mu\nu}(\Theta_{ij} + \bar{m}_i\bar{m}_j - (1-x)\bar{m}_i^2 - x\bar{m}_j^2) \\ \left. - i((1-x)\bar{m}_i + x\bar{m}_j) q_\beta \epsilon^{\mu\nu\beta}\right],\end{aligned} \tag{91}$$

where we have used dimensional regularization together with the symmetrization $l^\mu l^\nu \to \frac{1}{d} l^2 \eta^{\mu\nu}$ and introduced the element

$$\Theta_{ij} \equiv (1-x)\bar{m}_i^2 + x\bar{m}_j^2 - x(1-x)q^2. \tag{92}$$

In this way, the integral over momenta can be straightforwardly performed using the identities given in Ref. [53], arriving at

$$\Pi^{\mu\nu}_{ij}(q) = \left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\Pi^e_{ij}(q) + \eta^{\mu\nu}\Pi^\eta_{ij}(q) + i\epsilon^{\mu\nu\beta}\frac{q_\beta}{q}\Pi^o_{ij}(q), \tag{93}$$

with

$$\Pi^e_{ij}(q) = -\frac{2\bar{e}^2\mu^{3-d}q^2\Gamma(2-\frac{d}{2})}{(4\pi)^{d/2}} \int_0^1 \mathrm{d}x \frac{2x(1-x)}{\Theta_{ij}^{2-\frac{d}{2}}}, \tag{94}$$

$$\Pi^\eta_{ij}(q) = -\frac{2\bar{e}^2\mu^{3-d}\Gamma(2-\frac{d}{2})}{(4\pi)^{d/2}} \times \int_0^1 \mathrm{d}x \frac{\bar{m}_i\bar{m}_j - (1-x)\bar{m}_i^2 - x\bar{m}_j^2}{\Theta_{ij}^{2-\frac{d}{2}}}, \tag{95}$$

$$\Pi^o_{ij}(q) = \frac{2\bar{e}^2\mu^{3-d}q\Gamma(2-\frac{d}{2})}{(4\pi)^{d/2}} \int_0^1 \mathrm{d}x \frac{(1-x)\bar{m}_i + x\bar{m}_j}{\Theta_{ij}^{2-\frac{d}{2}}}. \tag{96}$$

To analyze the divergent behavior in the PPO in $2+1$ dimensions, let us introduce the dimensional regularization requiring the spacetime dimension to be $d = 3 - \epsilon$ and take the limit $\epsilon \to 0$, obtaining

$$\Pi^e_{ij}(q) = -\frac{4\bar{e}^2 q^2\sqrt{\pi}}{(4\pi)^{3/2}} \int_0^1 \mathrm{d}x \frac{x(1-x)}{\Theta_{ij}^{\frac{1}{2}}}, \tag{97}$$

$$\Pi^\eta_{ij}(q) = -\frac{2\bar{e}^2\sqrt{\pi}}{(4\pi)^{3/2}} \int_0^1 \mathrm{d}x \frac{\bar{m}_i\bar{m}_j - (1-x)\bar{m}_i^2 - x\bar{m}_j^2}{\Theta_{ij}^{\frac{1}{2}}}, \tag{98}$$

$$\Pi^o_{ij}(q) = \frac{2\bar{e}^2 q\sqrt{\pi}}{(4\pi)^{3/2}} \int_0^1 \mathrm{d}x \frac{(1-x)\bar{m}_i + x\bar{m}_j}{\Theta_{ij}^{\frac{1}{2}}}. \tag{99}$$

All these contributions are finite, as can be seen from the absence of $1/\epsilon$ terms.

After integrating over the Feynman parameter, we have

$$\begin{aligned}\Pi^e_{ij}(q) = -\frac{4\bar{e}^2\sqrt{\pi}q^2}{(4\pi)^{3/2}} \left[\frac{3(\bar{m}_i^2 - \bar{m}_j^2)^2 - 2(\bar{m}_i^2 + \bar{m}_j^2)q^2 - q^4}{8q^5} \right. \\ \left. \times \ln\left|\frac{\bar{m}_i + \bar{m}_j - q}{\bar{m}_i + \bar{m}_j + q}\right| + \frac{(\bar{m}_i + \bar{m}_j)(3(\bar{m}_i - \bar{m}_j)^2 - q^2)}{4q^4}\right],\end{aligned} \tag{100}$$

$$\begin{aligned}\Pi^\eta_{ij}(q) = 2\bar{e}^2 \frac{\pi^{1/2}}{(4\pi)^{3/2}} \frac{(\bar{m}_i + \bar{m}_j)(\bar{m}_i - \bar{m}_j)^2}{q^2} \\ \times \left[1 + \frac{(\bar{m}_i + \bar{m}_j)^2 - q^2}{2q(\bar{m}_i + \bar{m}_j)} \ln\left|\frac{\bar{m}_i + \bar{m}_j - q}{\bar{m}_i + \bar{m}_j + q}\right|\right],\end{aligned} \tag{101}$$

and

$$\begin{aligned}\Pi^o_{ij}(q) = 2\bar{e}^2 \frac{\pi^{1/2}}{(4\pi)^{3/2}} q \left[\frac{(\bar{m}_i - \bar{m}_j)^2}{q^2} \right. \\ \left. + \frac{(\bar{m}_i + \bar{m}_j)((\bar{m}_i - \bar{m}_j)^2 - q^2)}{2q^3} \ln\left|\frac{\bar{m}_i + \bar{m}_j - q}{\bar{m}_i + \bar{m}_j + q}\right|\right].\end{aligned} \tag{102}$$

Taking into account the above-defined notation, in an analogous way, we define for the pieces e, o, η the total contributions $\Pi^{e,\eta,o} = \Pi^{e,\eta,o}_{11} - 2\Pi^{e,\eta,o}_{12} + \Pi^{e,\eta,o}_{22}$, and we can see that

$$\Pi^e(q) = -4\bar{e}^2 \frac{\pi^{1/2}}{(4\pi)^{3/2}} \frac{q^2(\bar{m}_1+\bar{m}_2)^2}{(\bar{m}_2-\bar{m}_1)^2} \times \left\{ \frac{-4\bar{m}_1^2-q^2}{8q^3} \ln\left|\frac{2\bar{m}_1-q}{2\bar{m}_1+q}\right| - \frac{2\bar{m}_1}{4q^2} + \frac{-4\bar{m}_2^2-q^2}{8q^3} \ln\left|\frac{2\bar{m}_2-q}{2\bar{m}_2+q}\right| - \frac{2\bar{m}_2}{4q^2} - 2\frac{3(\bar{m}_1^2-\bar{m}_2^2)^2-2(\bar{m}_1^2+\bar{m}_2^2)q^2-q^4}{8q^5} \ln\left|\frac{\bar{m}_1+\bar{m}_2-q}{\bar{m}_1+\bar{m}_2+q}\right| - 2\frac{(\bar{m}_1+\bar{m}_2)(3(\bar{m}_1-\bar{m}_2)^2-q^2)}{4q^4} \right\}, \quad (103)$$

$$\Pi^\eta(q) = 2\bar{e}^2 \frac{\pi^{1/2}}{(4\pi)^{3/2}} \frac{(-2)(\bar{m}_1+\bar{m}_2)(\bar{m}_1-\bar{m}_2)^2}{q^2} \times \left\{ 1 + \frac{(\bar{m}_1+\bar{m}_2)^2-q^2}{2q(\bar{m}_1+\bar{m}_2)} \ln\left|\frac{\bar{m}_1+\bar{m}_2-q}{\bar{m}_1+\bar{m}_2+q}\right| \right\}, \quad (104)$$

and

$$\Pi^o(q) = 2\bar{e}^2 \frac{\pi^{1/2}}{(4\pi)^{3/2}} \frac{q(\bar{m}_1+\bar{m}_2)^2}{(\bar{m}_1-\bar{m}_2)^2} \times \left\{ -\frac{\bar{m}_1}{q} \ln\left|\frac{2\bar{m}_1-q}{2\bar{m}_1+q}\right| - \frac{\bar{m}_2}{q} \ln\left|\frac{2\bar{m}_2-q}{2\bar{m}_2+q}\right| - 2\frac{(\bar{m}_1-\bar{m}_2)^2}{q^2} - 2\frac{(\bar{m}_1+\bar{m}_2)((\bar{m}_1-\bar{m}_2)^2-q^2)}{2q^3} \times \ln\left|\frac{\bar{m}_1+\bar{m}_2-q}{\bar{m}_1+\bar{m}_2+q}\right| \right\}. \quad (105)$$

Let us explore the behavior of the contributions (103), (104), and (105), when g is very small. In this case the mass $\bar{m}_2$ becomes the dominant energy scale compared to $\bar{m}_1 \to m$, which yields

$$\Pi^e(q) \approx -4e^2 \frac{\pi^{1/2}}{(4\pi)^{3/2}} q^2 \left\{ -\frac{4m^2+q^2}{8q^3} \left(1+\frac{4m}{\bar{m}_2}\right) \times \ln\left|\frac{2m-q}{2m+q}\right| - \frac{2m}{4q^2} - \frac{11}{30\bar{m}_2} - \frac{2m^2}{q^2\bar{m}_2} \right\}, \quad (106)$$

$$\Pi^\eta(q) \approx 2e^2 \frac{\pi^{1/2}}{(4\pi)^{3/2}} \left\{ \frac{4}{3}\left(3m-\bar{m}_2-\frac{4m^2}{\bar{m}_2}\right) - \frac{4q^2}{15\bar{m}_2} \right\}, \quad (107)$$

and

$$\Pi^o(q) \approx 2e^2 \frac{\pi^{1/2}}{(4\pi)^{3/2}} q \left\{ -\frac{m}{q}\left(1+\frac{4m}{\bar{m}_2}\right) \ln\left|\frac{2m-q}{2m+q}\right| - \frac{1}{3} - \frac{4m}{\bar{m}_2} \right\}. \quad (108)$$

Another interesting scenario corresponds to the UV limit so that the higher-derivative terms have important contributions. In this case with $q \to \infty$, each coefficient has the form

$$\Pi^e(q) \overset{q\to\infty}{\approx} -4\bar{e}^2 \frac{\pi^{1/2}}{(4\pi)^{3/2}} q^2 \left\{ -i\pi \frac{3(\bar{m}_1+\bar{m}_2)^4}{4q^5} + \frac{16(\bar{m}_1+\bar{m}_2)^3(\bar{m}_1^2+\bar{m}_1\bar{m}_2+\bar{m}_2^2)}{3q^6} \right\}. \quad (109)$$

Note that Π^e in our case has a better ultraviolet behavior than in the case of the absence of higher-derivative contributions where it has the form

$$\Pi^e(q) \overset{q\to\infty}{\approx} -4e^2 \frac{\pi^{1/2}}{(4\pi)^{3/2}} q^2 \left\{ -i\pi \frac{q^2+4m^2}{8q^3} + \frac{8m^3}{3q^4} \right\}. \quad (110)$$

For the metric coefficient, we have

$$\Pi^\eta(q) \overset{q\to\infty}{\approx} 2\bar{e}^2 \frac{\pi^{1/2}}{(4\pi)^{3/2}} \left\{ i\pi \frac{(\bar{m}_1+\bar{m}_2)^2}{q} - \frac{4(\bar{m}_1+\bar{m}_2)^3}{q^2} \right\}. \quad (111)$$

Note that in the absence of higher-derivative contributions, this term does not exist.

Finally, in this limit the odd term gives

$$\Pi^o(q) \overset{q\to\infty}{\approx} 2\bar{e}^2 \frac{\pi^{1/2}}{(4\pi)^{3/2}} q \left\{ -i\pi \frac{(\bar{m}_1+\bar{m}_2)^3}{q^3} + \frac{4(\bar{m}_1+\bar{m}_2)^2(5\bar{m}_1^2+8\bar{m}_1\bar{m}_2+5\bar{m}_2^2)}{3q^4} \right\}, \quad (112)$$

from where we notice that the ultraviolet behavior is different to the standard CS QED result (see, e.g., [54]):

$$\Pi^o(q) \overset{q\to\infty}{\approx} 2\bar{e}^2 \frac{\pi^{1/2}}{(4\pi)^{3/2}} q \left\{ -i\pi \frac{m}{q} + \frac{4m^2}{q^2} \right\}. \quad (113)$$

D. The vertex diagram

Let us expand the propagator $S(k+p)$ given in (84) into a power series in the external momentum p up to the first order as

$$S(k+p) = S(k) - S(k)\not{p}S(k) + \ldots, \quad (114)$$

in such a way that the two-point function of the spinor field given in (58) can be expanded as

$$\Sigma(p) = \Sigma(0) + \Sigma_1(p) + \ldots, \quad (115)$$

with

$$\Sigma_1(p) = -e^2 \int \frac{d^3k}{(2\pi)^3} \gamma^\alpha S(k)\not{p}S(k)\gamma^\beta \Delta_{\alpha\beta}(k) + \ldots, \quad (116)$$

where dots are for irrelevant terms of zero, second, and higher orders.

On the other hand, the three-point vertex function, in zero order in external momenta and in the Landau gauge, looks like

$$\Gamma^{\mu}(0,0) = -e^2 \int \frac{\mathrm{d}^3k}{(2\pi)^3}\gamma^{\alpha}S(k)\gamma^{\mu}S(k)\gamma^{\beta}\Delta_{\alpha\beta}(k). \quad (117)$$

Comparing this expression with (116), we see that they are equal up to the overall factor. Therefore we see that the corrections to the kinetic term for the spinor field and the triple spinor-vector vertex are described by the same integral over momenta; hence, the gauge symmetry is not jeopardized by quantum corrections which is consistent with the Ward identities up to first order in external momenta. In other words, the "covariant derivative" term $\bar{\psi}D\psi$ receives the quantum correction $\bar{\psi}\Gamma^{\mu}D_{\mu}\psi$, where

$$\Gamma^{\mu}(0,0) = -e^2 \int \frac{\mathrm{d}^3k}{(2\pi)^3}\gamma^{\alpha}S(k)\gamma^{\mu}S(k)\gamma^{\beta}\Delta_{\alpha\beta}(k). \quad (118)$$

Now it is instructive to check the gauge invariance of the quantum correction generated on the spinor sector which as it is well known, is deeply related with the Ward identities. We proceed with the computation of the vertex function and take advantage of the previous computation.

Within our study, we consider the lower (zero) order contributions to the three-point function only. Therefore, we require $p = 0$ and $p' = 0$ and arrive at

$$\Gamma^{\mu}(0,0) = -e^2 \int \frac{\mathrm{d}^3k}{(2\pi)^3}\gamma^{\alpha}S(k)\gamma^{\mu}S(k)\gamma^{\beta}\Delta_{\alpha\beta}(k). \quad (119)$$

Similarly to the above PPO calculation, we use the decomposed fermion propagator (84) in (119), so the vertex contributions read

$$\Gamma^{\mu}_{ij}(0,0) = -\bar{e}^2 \int \frac{\mathrm{d}^3k}{(2\pi)^3}\gamma^{\alpha}S_i(k)\gamma^{\mu}S_j(k)\gamma^{\beta}\Delta_{\alpha\beta}(k), \quad (120)$$

where $i, j = 1, 2$. As before, we introduce the notation for the different contributions

$$\Gamma^{\mu}(0,0) = \Gamma^{\mu}_{11}(0,0) - 2\Gamma^{\mu}_{12}(0,0) + \Gamma^{\mu}_{22}(0,0). \quad (121)$$

Since all contributions have the same form, we calculate its general form given by

$$\Gamma^{\mu}_{ij}(0,0) = -i\bar{e}^2 \int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{\gamma^{\rho}(\not{k}-\bar{\not{m}}_i)\gamma^{\mu}(\not{k}-\bar{\not{m}}_j)\gamma^{\sigma}}{(k^2-\bar{m}_i^2)(k^2-\bar{m}_j^2)}\times\frac{\gamma T_{\rho\sigma}(k)}{k^2-\gamma^2\mathcal{M}^2}. \quad (122)$$

Using the identity

$$(\not{k}-\bar{\not{m}}_i)\gamma^{\mu}(\not{k}-\bar{\not{m}}_j) = -k^2\gamma^{\mu} + 2k k^{\mu} - \not{k}\gamma^{\mu}\bar{\not{m}}_j - \bar{\not{m}}_i\gamma^{\mu}\not{k} + \bar{\not{m}}_i\bar{\not{m}}_j\gamma^{\mu}, \quad (123)$$

together with (6), the three-point vertex function in (122) becomes

$$\Gamma^{\mu}_{ij}(0,0) = (\Gamma^{\mu}_{ij})^a + (\Gamma^{\mu}_{ij})^b, \quad (124)$$

where the $(\Gamma^{\mu}_{ij})^a$ and $(\Gamma^{\mu}_{ij})^b$ are the contributions without and with the Levi-Civita symbol coming from the gauge boson propagator, respectively. These are given by

$$(\Gamma^{\mu}_{ij})^a = -i\bar{e}^2 \int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{\gamma^{\rho}(-k^2\gamma^{\mu} + 2k k^{\mu} + \bar{\not{m}}_i\bar{\not{m}}_j\gamma^{\mu})\gamma^{\sigma}}{(k^2-\bar{m}_i^2)(k^2-\bar{m}_j^2)} \times\frac{\gamma(\eta_{\rho\sigma} - k_{\rho}k_{\sigma}/k^2)}{k^2-\gamma^2\mathcal{M}^2}, \quad (125)$$

and

$$(\Gamma^{\mu}_{ij})^b = \bar{e}^2 \int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{\gamma^{\rho}(\not{k}\gamma^{\mu}\bar{\not{m}}_j + \bar{\not{m}}_i\gamma^{\mu}\not{k})\gamma^{\sigma}}{(k^2-\bar{m}_i^2)(k^2-\bar{m}_j^2)}\times\frac{\gamma(\gamma\mathcal{M}\epsilon_{\rho\beta\sigma}k^{\beta}/k^2)}{k^2-\gamma^2\mathcal{M}^2}. \quad (126)$$

Now, we use (6) together with symmetry properties on the integral over momenta in the above equations, getting

$$(\Gamma^{\mu}_{ij})^a = i\bar{e}^2\gamma\gamma^{\mu}\times\int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{k^2 + \frac{1}{3}\bar{m}_i\bar{m}_j}{(k^2-\bar{m}_i^2)(k^2-\bar{m}_j^2)(k^2-\gamma^2\mathcal{M}^2)}, \quad (127)$$

and

$$(\Gamma^{\mu}_{ij})^b = i\bar{e}^2\gamma\gamma^{\mu}\times\int \frac{d^3k}{(2\pi)^3}\frac{\frac{10}{3}\gamma(\bar{m}_i + \bar{m}_j)\mathcal{M}}{(k^2-\bar{m}_i^2)(k^2-\bar{m}_j^2)(k^2-\gamma^2\mathcal{M}^2)}. \quad (128)$$

Once we carried out the Wick rotation, we have two integrals of the form

$$I_1 = \int_0^{\infty}\frac{r^2dr}{(\gamma^2g^2r^4 + \gamma^2 2g\mu r^2 + \gamma^2\mu^2 + r^2)} \times\frac{1}{(r^2+\bar{m}_i^2)(r^2+\bar{m}_j^2)}, \quad (129)$$

and

$$I_2 = \int_0^{\infty}\frac{r^4dr}{(\gamma^2g^2r^4 + \gamma^2 2g\mu r^2 + \gamma^2\mu^2 + r^2)} \times\frac{1}{(r^2+\bar{m}_i^2)(r^2+\bar{m}_j^2)}, \quad (130)$$

which can be easily performed using a standard procedure, yielding

$$I_1 = -\frac{\pi}{2g^2\gamma^2}\frac{m_1+M_2+\bar{m}_i+\bar{m}_j}{(m_1+\bar{m}_i)(m_1+\bar{m}_j)(M_2+\bar{m}_i)(M_2+\bar{m}_j)} \times \frac{1}{(\bar{m}_i+\bar{m}_j)(m_1+M_2)}, \tag{131}$$

and

$$I_2 = -\frac{\pi}{2g^2\gamma^2}\frac{(m_1+M_2)(\bar{m}_i\bar{m}_j)+(\bar{m}_i+\bar{m}_j)(m_1M_2)}{(m_1+\bar{m}_i)(m_1+\bar{m}_j)(M_2+\bar{m}_i)(M_2+\bar{m}_j)} \times \frac{1}{(\bar{m}_i+\bar{m}_j)(m_1+M_2)}. \tag{132}$$

With these results, we can see that $\Gamma^\mu = \Gamma^\mu_{11} - 2\Gamma^\mu_{12} + \Gamma^\mu_{22}$ has the form

$$\begin{aligned}\Gamma^\mu = \frac{\bar{e}^2}{4\pi g^2\gamma}\Bigg\{ & \frac{(2\bar{m}_1+m_1+M_2)(\bar{m}_1+20\gamma\mu)}{6(m_1+M_2)(\bar{m}_1+m_1)^2(\bar{m}_1+M_2)^2} \\ & -\frac{(\bar{m}_1(m_1+M_2)+2m_1M_2)(3+20\gamma g\bar{m}_1)}{6(m_1+M_2)(\bar{m}_1+m_1)^2(\bar{m}_1+M_2)^2} \\ & +\frac{(2\bar{m}_2+m_1+M_2)(\bar{m}_2+20\gamma\mu)}{6(m_1+M_2)(\bar{m}_2+m_1)^2(\bar{m}_2+M_2)^2} \\ & -\frac{(\bar{m}_2(m_1+M_2)+2m_1M_2)(3+20\gamma g\bar{m}_2)}{6(m_1+M_2)(\bar{m}_2+m_1)^2(\bar{m}_2+M_2)^2} \\ & -\frac{2}{(\bar{m}_1+\bar{m}_2)(m_1+M_2)(\bar{m}_1+m_1)} \\ & \times\Bigg[\frac{(\bar{m}_1+\bar{m}_2+m_1+M_2)(10\gamma\mu(\bar{m}_1+\bar{m}_2)+\bar{m}_1\bar{m}_2)}{3(\bar{m}_2+m_1)(\bar{m}_1+M_2)(\bar{m}_2+M_2)} \\ & +\frac{m_1M_2(\bar{m}_1+\bar{m}_2)(1+\frac{10}{3}\gamma g(\bar{m}_1+\bar{m}_2))}{(\bar{m}_2+m_1)(\bar{m}_1+M_2)(\bar{m}_2+M_2)} \\ & +\frac{\bar{m}_1\bar{m}_2(m_1+M_2)(1+\frac{10}{3}\gamma g(\bar{m}_1+\bar{m}_2))}{(\bar{m}_2+m_1)(\bar{m}_1+M_2)(\bar{m}_2+M_2)}\Bigg]\Bigg\}\gamma^\mu. \end{aligned} \tag{133}$$

Since this one-loop correction to the vertex has a simple γ^μ structure, its coefficient can be identified as the form factor $F_1(q^2=0)$, so the vertex can be written as

$$\Gamma^\mu = F_1(0)\gamma^\mu, \tag{134}$$

which is expected, as we set $p = p' = 0$, eliminating any other available tensor structure.

As (133) is a complete result for an arbitrary value of γ, to make contact with the literature, let us consider a pure CS theory by taking the limit $\gamma \to \infty$ in (133), getting

$$F_1(0) \overset{\gamma\to\infty}{=} \frac{e^2}{4\pi}\frac{5(\sqrt{\mu}-\sqrt{g}(m-3\mu))}{3(\sqrt{g}(\mu+m)+\sqrt{\mu})^3}, \tag{135}$$

where we use (20a), (20b), (79), and (80). Additionally, if we take off the higher-derivative contributions in (135) by setting $g \to 0$, we arrive at [27]

$$F_1(0) \overset{g\to 0}{=} \frac{e^2}{4\pi}\frac{5}{3\mu}. \tag{136}$$

V. MICROCAUSALITY IN THE GAUGE SECTOR

The causal structure of a quantum field theory is determined by the behavior of the commutator of fundamental fields at two distinct spacetime points, x and y, outside the light cone. In what follows, we analyze the implications of this criterion for causality in the gauge sector. Hence, let us consider the commutator

$$D_{\mu\nu}(x-y) = [A_\mu(x), A_\nu(y)], \tag{137}$$

and verify whether it vanishes outside the light cone, i.e., when $(x-y)^2 < 0$.

From the decomposition of the gauge field (41), we separate into two types of contributions. We consider a first contribution due to massive modes which we denote by

$$D^{(1)}_{\mu\nu}(x-y) = [\bar{A}_\mu(x), \bar{A}_\nu(y)] + [G_\mu(x), G_\nu(y)], \tag{138}$$

and a second contribution due to massless modes

$$\begin{aligned} D^{(2)}_{\mu\nu}(x-y) = {} & [A^D_\mu(x), A^G_\nu(y)] + [A^G_\mu(x), A^D_\nu(y)] \\ & + [A^G_\mu(x), A^G_\nu(y)], \end{aligned} \tag{139}$$

and define the total contribution

$$D_{\mu\nu}(x-y) = D^{(1)}_{\mu\nu}(x-y) + D^{(2)}_{\mu\nu}(x-y). \tag{140}$$

We begin to evaluate the first contribution, with

$$\begin{aligned}[\bar{A}_\mu(x), \bar{A}_\nu(y)] = \int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{1}{2\omega_1\bar{\beta}}(\varepsilon^{(-)}_\mu\varepsilon^{(-)*}_\nu e^{-ik\cdot(x-y)} \\ - \varepsilon^{(-)*}_\mu\varepsilon^{(-)}_\nu e^{ik\cdot(x-y)})_{k_0=\omega_1}, \end{aligned} \tag{141}$$

and

$$\begin{aligned}[G_\mu(x), G_\nu(y)] = -\int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{1}{2W_2\bar{\beta}}(\varepsilon^{(+)}_\mu\varepsilon^{(+)*}_\nu e^{-ik\cdot(x-y)} \\ - \varepsilon^{(+)*}_\mu\varepsilon^{(+)}_\nu e^{ik\cdot(x-y)})_{k_0=W_2}. \end{aligned} \tag{142}$$

Let us define $z = x - y$ and change $\vec{k} \to -\vec{k}$ in the second terms of Eq. (141) and (142), obtaining

$$[\bar{A}_\mu(x),\bar{A}_\nu(y)]=\int\frac{d^2\vec{k}}{(2\pi)^2}\frac{e^{i\vec{k}\cdot\vec{z}}}{2\omega_1\bar{\beta}}(\varepsilon_\mu^{(-)}(\omega_1,\vec{k})\times\varepsilon_\nu^{(-)*}(\omega_1,\vec{k})e^{-i\omega_1 z_0}-\varepsilon_\mu^{(-)*}(\omega_1,-\vec{k})\varepsilon_\nu^{(-)}(\omega_1,-\vec{k})e^{i\omega_1 z_0}),\quad(143)$$

and

$$[G_\mu(x),G_\nu(y)]=-\int\frac{d^2\vec{k}}{(2\pi)^2}\frac{e^{i\vec{k}\cdot\vec{z}}}{2W_2\bar{\beta}}(\varepsilon_\mu^{(+)}(W_2,\vec{k})\times\varepsilon_\nu^{(+)*}(W_2,\vec{k})e^{-iW_2 z_0}-\varepsilon_\mu^{(+)*}(W_2,-\vec{k})\varepsilon_\nu^{(+)}(W_2,-\vec{k})e^{iW_2 z_0}),\quad(144)$$

where we have used that the denominators are even functions of $\vec{k}$.

We recall the identity (32)

$$\varepsilon_\mu^{(\pm)}(k_0,\vec{k})\varepsilon_\nu^{(\pm)*}(k_0,\vec{k})=-\frac{1}{2}\left(\eta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\pm i\epsilon_{\mu\beta\nu}\frac{k^\beta}{\sqrt{k^2}}\right).\quad(145)$$

We take the complex conjugate

$$\varepsilon_\mu^{(\pm)*}(k_0,\vec{k})\varepsilon_\nu^{(\pm)}(k_0,\vec{k})=-\frac{1}{2}\left(\eta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\mp i\epsilon_{\mu\beta\nu}\frac{k^\beta}{\sqrt{k^2}}\right).\quad(146)$$

Thus, we conclude that

$$\varepsilon_\mu^{(\pm)*}(k_0,\vec{k})\varepsilon_\nu^{(\pm)}(k_0,\vec{k})=\varepsilon_\mu^{(\pm)}(-k_0,-\vec{k})\times\varepsilon_\nu^{(\pm)*}(-k_0,-\vec{k}),\quad(147)$$

which leads to the relations

$$\varepsilon_\mu^{(-)*}(\omega_1,-\vec{k})\varepsilon_\nu^{(-)}(\omega_1,-\vec{k})=\varepsilon_\mu^{(-)}(-\omega_1,\vec{k})\times\varepsilon_\nu^{(-)*}(-\omega_1,\vec{k}),\quad(148)$$

$$\varepsilon_\mu^{(+)*}(W_2,-\vec{k})\varepsilon_\nu^{(+)}(W_2,-\vec{k})=\varepsilon_\mu^{(+)}(-W_2,\vec{k})\times\varepsilon_\nu^{(+)*}(-W_2,\vec{k}).\quad(149)$$

Using these results, the total expression for the commutator is

$$D_{\mu\nu}^{(1)}(z)=\int\frac{d^2\vec{k}}{(2\pi)^2}\frac{e^{i\vec{k}\cdot\vec{z}}}{2\omega_1\bar{\beta}}(\varepsilon_\mu^{(-)}(\omega_1,\vec{k})\varepsilon_\nu^{(-)*}(\omega_1,\vec{k})\times e^{-i\omega_1 z_0}-\varepsilon_\mu^{(-)}(-\omega_1,\vec{k})\varepsilon_\nu^{(-)*}(-\omega_1,\vec{k})e^{i\omega_1 z_0})-\int\frac{d^2\vec{k}}{(2\pi)^2}\frac{e^{i\vec{k}\cdot\vec{z}}}{2W_2\bar{\beta}}(\varepsilon_\mu^{(+)}(W_2,\vec{k})\varepsilon_\nu^{(+)*}(W_2,\vec{k})\times e^{-iW_2 z_0}-\varepsilon_\mu^{(+)}(-W_2,\vec{k})\varepsilon_\nu^{(+)*}(-W_2,\vec{k})e^{iW_2 z_0}).\quad(150)$$

Let us recall some previous results (16), and by evaluating k_0 in a root of $k^2-\gamma^2\mathcal{M}^2$, we arrive at

$$T_{\mu\nu}(p)|_{\Lambda_\pm=0}=\eta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\pm i\epsilon_{\mu\beta\nu}\frac{k^\beta}{\sqrt{k^2}}.\quad(151)$$

We can establish a connection with the combination of polarization vectors

$$\varepsilon_\mu^{(-)}(\pm\omega_1,\vec{k})\varepsilon_\nu^{(-)*}(\pm\omega_1,\vec{k})=-\frac{1}{2}T_{\mu\nu}(k)|_{k_0=\pm\omega_1},\quad(152)$$

$$\varepsilon_\mu^{(+)}(\pm W_2,\vec{k})\varepsilon_\nu^{(+)*}(\pm W_2,\vec{k})=-\frac{1}{2}T_{\mu\nu}(k)|_{k_0=\pm W_2}.\quad(153)$$

Therefore

$$D_{\mu\nu}^{(1)}(z)=-\frac{1}{2\bar{\beta}}\int\frac{d^2\vec{k}}{(2\pi)^2}e^{i\vec{k}\cdot\vec{z}}\left(\frac{T_{\mu\nu}(k)|_{k_0=\omega_1}}{2\omega_1}\times e^{-i\omega_1 z_0}-\frac{T_{\mu\nu}(k)|_{k_0=-\omega_1}}{2\omega_1}e^{i\omega_1 z_0}\right)+\frac{1}{2\bar{\beta}}\int\frac{d^2\vec{k}}{(2\pi)^2}e^{i\vec{k}\cdot\vec{z}}\left(\frac{T_{\mu\nu}(k)|_{k_0=W_2}}{2W_2}\times e^{-iW_2 z_0}-\frac{T_{\mu\nu}(k)|_{k_0=-W_2}}{2W_2}e^{iW_2 z_0}\right).\quad(154)$$

Now we compute the terms for the second contributions, which give

$$[A_\mu^D(x),A_\nu^G(y)]=\int\frac{d^2\vec{k}}{(2\pi)^2}\frac{(ik_\nu)}{4|\vec{k}|^2}\frac{\xi}{\mu}(v_\mu(x_0,k)\times e^{-ik\cdot(x-y)}+v_\mu^*(x_0,k)e^{ik\cdot(x-y)})_{k_0=|\vec{k}|},\quad(155)$$

$$[A_\mu^G(x),A_\nu^D(y)]=\int\frac{d^2\vec{k}}{(2\pi)^2}\frac{(-ik_\mu)}{4|\vec{k}|^2}\frac{\xi}{\mu}(v_\nu^*(y_0,k)e^{-ik\cdot(x-y)}+v_\nu(y_0,k)e^{ik\cdot(x-y)})_{k_0=|\vec{k}|},\quad(156)$$

$$[A^G_\mu(x), A^G_\nu(y)] = \int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{k_\mu k_\nu}{4|\vec{k}|^2}\left[-\frac{2|\vec{k}|}{\gamma\mu^2}+\frac{\xi}{|\vec{k}|}\right] \times \left(e^{-ik\cdot(x-y)} - e^{ik\cdot(x-y)}\right)_{k_0=|\vec{k}|}. \tag{157}$$

Considering $z = x - y$, we arrive at

$$D^{(2)}_{\mu\nu}(z) = \int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{1}{(2|\vec{k}|)^2}[(K_{\mu\nu}(z_0,k) - \xi(n_\mu k_\nu + k_\mu n_\nu))e^{-ik\cdot z} - (K^*_{\mu\nu}(z_0,-k) - \xi(n_\mu k_\nu + k_\mu n_\nu))e^{ik\cdot z}]_{k_0=|\vec{k}|}, \tag{158}$$

with

$$K_{\mu\nu}(z_0,k) := i\xi z_0 k_\mu k_\nu - \frac{2i}{\mu}|\vec{k}|\epsilon_{\mu\beta\nu}k^\beta + k_\mu k_\nu\left(-\frac{2|\vec{k}|}{\gamma\mu^2}+\frac{\xi}{|\vec{k}|}\right). \tag{159}$$

Now we look for the individual components of the commutator (154). To find the contributions to $D^{(1)}_{\mu\nu}$ we use the following relations

$$T_{\mu\nu}(k)|_{k_0=\omega_1} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_1^2}\bigg|_{k_0=\omega_1} - i\epsilon_{\mu\beta\nu}\frac{k^\beta}{m_1}\bigg|_{k_0=\omega_1}, \tag{160}$$

$$T_{\mu\nu}(k)|_{k_0=-\omega_1} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_1^2}\bigg|_{k_0=-\omega_1} - i\epsilon_{\mu\beta\nu}\frac{k^\beta}{m_1}\bigg|_{k_0=-\omega_1}, \tag{161}$$

$$T_{\mu\nu}(k)|_{k_0=W_2} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{M_2^2}\bigg|_{k_0=W_2} + i\epsilon_{\mu\beta\nu}\frac{k^\beta}{M_2}\bigg|_{k_0=W_2}, \tag{162}$$

$$T_{\mu\nu}(k)|_{k_0=-W_2} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{M_2^2}\bigg|_{k_0=-W_2} + i\epsilon_{\mu\beta\nu}\frac{k^\beta}{M_2}\bigg|_{k_0=-W_2}, \tag{163}$$

where we have used (30b) and (30c).

For the contributions $D^{(1)}_{00}$, we have

$$\frac{T_{00}(\omega_1,\vec{k})}{2\omega_1}e^{-iz_0\omega_1} - \frac{T_{00}(-\omega_1,\vec{k})}{2\omega_1}e^{iz_0\omega_1} = -\frac{i}{\omega_1}\sin(z_0\omega_1)\left(1-\frac{\omega_1^2}{m_1^2}\right), \tag{164}$$

and

$$-\frac{T_{00}(W_2,\vec{k})}{2W_2}e^{-iz_0W_2} + \frac{T_{00}(-W_2,\vec{k})}{2W_2}e^{iz_0W_2} = \frac{i}{W_2}\sin(z_0W_2)\left(1-\frac{W_2^2}{M_2^2}\right). \tag{165}$$

For the contributions $D^{(1)}_{0i}$, we have

$$\frac{T_{0i}(k)|_{k_0=\omega_1}}{2\omega_1}e^{-iz_0\omega_1} - \frac{T_{0i}(k)|_{k_0=-\omega_1}}{2\omega_1}e^{iz_0\omega_1} = -\frac{k_i}{m_1^2}\cos(z_0\omega_1) - \frac{1}{\omega_1}\epsilon_{ji}\frac{k^j}{m_1}\sin(z_0\omega_1), \tag{166}$$

and

$$-\frac{T_{0i}(k)|_{k_0=W_2}}{2W_2}e^{-iz_0W_2} + \frac{T_{0i}(k)|_{k_0=-W_2}}{2W_2}e^{iz_0W_2} = \frac{k_i}{M_2^2}\cos(z_0W_2) - \frac{1}{W_2}\epsilon_{ji}\frac{k^j}{M_2}\sin(z_0W_2), \tag{167}$$

where we defined $\epsilon_{0ij} = \epsilon_{ij}$. The last ones, are the contributions $D^{(1)}_{ij}$

$$\frac{T_{ij}(\omega_1,\vec{k})}{2\omega_1}e^{-iz_0\omega_1} - \frac{T_{ij}(-\omega_1,\vec{k})}{2\omega_1}e^{iz_0\omega_1} = i\frac{\delta_{ij}}{\omega_1}\sin(z_0\omega_1) + i\frac{k_ik_j}{m_1^2\omega_1}\sin(z_0\omega_1) + i\epsilon_{ij}\frac{1}{m_1}\cos(z_0\omega_1), \tag{168}$$

and

$$-\frac{T_{ij}(W_2,\vec{k})}{2W_2}e^{-iz_0W_2} + \frac{T_{ij}(-W_2,\vec{k})}{2W_2}e^{iz_0W_2} = -i\frac{\delta_{ij}}{W_2}\sin(z_0W_2) - i\frac{k_ik_j}{M_2^2W_2}\sin(z_0W_2) + i\epsilon_{ij}\frac{1}{M_2}\cos(z_0W_2). \tag{169}$$

Note that the elements have the expected symmetry of $z \to -z$ and swapping $\mu \to \nu$.

Each element can be written as

$$D^{(1)}_{00}(z) = -\frac{i}{2\bar{\beta}} \times \int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}|\vec{k}|^2\left(\frac{\sin(z_0\omega_1)}{m_1^2\omega_1} - \frac{\sin(z_0W_2)}{M_2^2W_2}\right)e^{i\vec{k}\cdot\vec{z}}, \tag{170a}$$

$$D^{(1)}_{0i}(z) = \frac{1}{2\bar{\beta}}\times\int\frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\left[\left(\frac{\cos(z_0\omega_1)}{m_1^2} - \frac{\cos(z_0W_2)}{M_2^2}\right)k_i + \left(\frac{\sin(z_0\omega_1)}{m_1\omega_1} + \frac{\sin(z_0W_2)}{M_2W_2}\right)\epsilon_{ij}k_j\right]e^{i\vec{k}\cdot\vec{z}}, \tag{170b}$$

and

$$D_{ij}^{(1)}(z) = -\frac{i}{2\bar{\beta}} \times \int \frac{d^2\vec{k}}{(2\pi)^2} \left[\left(\frac{\sin(z_0\omega_1)}{\omega_1} - \frac{\sin(z_0 W_2)}{W_2} \right) \delta_{ij} + \left(\frac{\sin(z_0\omega_1)}{m_1^2\omega_1} - \frac{\sin(z_0 W_2)}{M_2^2 W_2} \right) k_i k_j + \left(\frac{1}{m_1}\cos(z_0\omega_1) + \frac{1}{M_2}\cos(z_0 W_2) \right) \epsilon_{ij} \right] e^{i\vec{k}\cdot\vec{z}}. \tag{170c}$$

In the same way the components of the second contribution are

$$D_{00}^{(2)}(z) = \int \frac{d^2\vec{k}}{(2\pi)^2} \left(\frac{i\xi z_0}{2}\cos(|\vec{k}|z_0) + \left[\frac{\xi}{2|\vec{k}|} + \frac{|\vec{k}|}{\gamma\mu^2} \right] i\sin(|\vec{k}|z_0) \right) e^{i\vec{k}\cdot\vec{z}}, \tag{171}$$

$$D_{0i}^{(2)}(z) = -\int \frac{d^2\vec{k}}{(2\pi)^2} \left(\frac{k_i}{\gamma\mu^2}\cos(|\vec{k}|z_0) - \left[\frac{\xi z_0 k_i}{2|\vec{k}|} - \frac{\epsilon_{ji}k^j}{\mu|\vec{k}|} \right] \sin(|\vec{k}|z_0) \right) e^{i\vec{k}\cdot\vec{z}}, \tag{172}$$

and

$$D_{ij}^{(2)}(z) = \int \frac{d^2\vec{k}}{(2\pi)^2} \left(\left[\frac{i\xi z_0 k_i k_j}{2|\vec{k}|^2} + \frac{i\epsilon_{ij}}{\mu} \right] \cos(|\vec{k}|z_0) - \frac{ik_i k_j}{2|\vec{k}|^2} \left(-\frac{2|\vec{k}|}{\gamma\mu^2} + \frac{\xi}{|\vec{k}|} \right) \sin(|\vec{k}|z_0) \right) e^{i\vec{k}\cdot\vec{z}}. \tag{173}$$

Because of Lorentz covariance, the commutator can be evaluated in any convenient frame. Since we are interested in the spacelike region $z^2 = (x-y)^2 < 0$, we choose a frame where $z_0 = 0$. For completeness, a part of $D_{00}(z)$ is also evaluated in a general frame in Appendix A.

In the frame with $z_0 = 0$, it is not difficult to show that the first component contribution reduces to

$$D_{00}(0,\vec{z}) = 0. \tag{174}$$

The second component contribution gives

$$D_{0i}^{(1)}(0,\vec{z}) = \frac{1}{2\bar{\beta}} \left(\frac{1}{m_1^2} - \frac{1}{M_2^2} \right) \int \frac{d^2\vec{k}}{(2\pi)^2} k_i e^{i\vec{k}\cdot\vec{z}}, \tag{175}$$

and

$$D_{0i}^{(2)}(0,\vec{z}) = -\frac{1}{\gamma\mu^2} \int \frac{d^2\vec{k}}{(2\pi)^2} k_i e^{i\vec{k}\cdot\vec{z}}. \tag{176}$$

The last one is

$$D_{ij}^{(1)}(0,\vec{z}) = -\frac{i}{2\bar{\beta}} \left(\frac{1}{m_1} + \frac{1}{M_2} \right) \int \frac{d^2\vec{k}}{(2\pi)^2} \epsilon_{ij} e^{i\vec{k}\cdot\vec{z}}. \tag{177}$$

and

$$D_{ij}^{(2)}(0,\vec{z}) = \frac{i}{\mu} \int \frac{d^2\vec{k}}{(2\pi)^2} \epsilon_{ij} e^{i\vec{k}\cdot\vec{z}}. \tag{178}$$

Hence, we find

$$D_{0i}(0,\vec{z}) = D_{ij}(0,\vec{z}) = 0, \tag{179}$$

where we have used

$$\frac{1}{m_1^2} - \frac{1}{M_2^2} = \frac{2\bar{\beta}}{\gamma\mu^2}, \tag{180}$$

$$\frac{1}{m_1} + \frac{1}{M_2} = \frac{2\bar{\beta}}{\mu}. \tag{181}$$

We confirm that microcausality is preserved in our model.

VI. FINAL REMARKS

In this work, we have studied the higher-derivative extension of three-dimensional QED which incorporates gauge-invariant CS modifications. In particular, we have considered a QED with standard fermions, CS gauge invariant extensions including higher-order and a standard interaction term. While earlier gauge extensions were treated mostly at the tree level [39], now we studied the one-loop perturbative impact of the presence of the higher-derivative CS term.

We have found that the higher-order gauge field propagates through four different modes. We have two massive transverse modes corresponding to the subspaces Λ_+ and Λ_- of the spectral decomposition of the equation-of-motion operator. In addition to the two massive modes we find two massless solutions that arise from the multiplicity of the root at $k^2 = 0$ in the dispersion relations. This degeneracy is also reflected in the propagator through the corresponding pole. We have derived the polarization vectors for both Proca-like fields, each described by transverse vectors in momentum space $\{\varepsilon^{(\pm)}\}$, and also for the massless modes.

We have developed the Hamiltonian formulation of the theory and identified four second-class constraints, which give rise to Dirac brackets and consequently modify the standard Poisson structure. According to the corresponding constraint counting, yielding 4 degrees of freedom, we constructed the most general field expansion and verified the resulting Dirac brackets explicitly. In addition, we derived the propagators directly from the definition of time-ordered vacuum expectation values of the fields. At this stage, we have shown that the theory exhibits an indefinite-metric structure, manifested through the commutation relations of the creation and annihilation operators.

The focus of the present work is the operator-level formulation of the theory. A detailed analysis of the Hamiltonian, the Hilbert-space structure, and related aspects is left for future work.

An interesting feature of the canonical formalism is the emergence of double poles and the way in which they are incorporated into the field expansion. Such structures may be relevant for the quantization of gauge theories in the Landau gauge, particularly in higher-derivative theories, Lee-Wick-type models, dipole-field theories, and gauge theories exhibiting degenerate massless sectors. Exploring these possible applications constitutes an interesting direction for future work.

Microcausality has been verified by explicitly computing the commutator of gauge fields at spacelike-separated points, $z^2 < 0$. Interestingly, although our analysis is performed at the level of the gauge field rather than gauge-invariant observables, we find that the commutator vanishes identically when $z_0 = 0$. This behavior suggests a nontrivial distributional structure, distinct from that of conventional QED in $(2+1)$ dimensions, where the commutator is supported on the light cone. We leave for future work the investigation of this effect and its possible connection with the massive modes arising in the Landau gauge.

We have computed the two-point function of the spinor field which induces radiative correction that allows us to redefine a Lagrangian in terms of two decoupled standard fermions, one being a ghost, which has served to compute the rest of radiative corrections. With this decomposition, we have computed the PPO where the complete dependence on the external momentum was obtained, and calculated the two- and three-point functions involving the spinor field. We have seen that in the gauge sector a redefinition of variables to cast the theory into the sum of Proca-like fields may be prohibited since these will eventually destroy gauge invariance. Moreover, we explicitly checked that our theory is microcausal in the gauge sector. The results for the spinor sector can be used within the studies of the three-dimensional effective field theories applied within the condensed matter context, especially within studies of graphene which is a perfect three-dimensional model. We also leave for future work the finite renormalization of the model.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: THE CALCULATION OF $D_{00}^{(1)}$

Here we analyze a more general frame with $z_0 \neq 0$, starting from Eq. (170). The contribution $D_{00}^{(2)}$ in (171) has a similar form; however, we restrain from this calculation.

In polar coordinates, defined by

$$\mathrm{d}^2\vec{k} = |\vec{k}|\mathrm{d}|\vec{k}|\mathrm{d}\theta, \tag{A1}$$

$$\vec{k}\cdot\vec{z} = |\vec{k}||\vec{z}|\cos\theta, \tag{A2}$$

$$\vec{k} = (|\vec{k}|\cos\theta, |\vec{k}|\sin\theta), \tag{A3}$$

we obtain

$$D_{00}^{(1)}(z) = -\frac{i}{2\bar{\beta}}\int_0^{2\pi}\int_0^{\infty}\frac{|\vec{k}|^3\mathrm{d}|\vec{k}|\mathrm{d}\theta}{(2\pi)^2} \times\left(\frac{\sin(z_0\omega_1)}{m_1^2\omega_1} - \frac{\sin(z_0W_2)}{M_2^2W_2}\right)e^{i|\vec{k}||\vec{z}|\cos\theta}, \tag{A4a}$$

$$D_{0i}^{(1)}(z) = \frac{1}{2\bar{\beta}}\int_0^{2\pi}\int_0^{\infty}\frac{|\vec{k}|\mathrm{d}|\vec{k}|\mathrm{d}\theta}{(2\pi)^2} \times\left[\left(\frac{\cos(z_0\omega_1)}{m_1^2} - \frac{\cos(z_0W_2)}{M_2^2}\right)k_i + \left(\frac{\sin(z_0\omega_1)}{m_1\omega_1} + \frac{\sin(z_0W_2)}{M_2W_2}\right)\epsilon_{ij}k_j\right]e^{i|\vec{k}||\vec{z}|\cos\theta}, \tag{A4b}$$

and

$$D_{ij}^{(1)}(z) = -\frac{i}{2\bar{\beta}}\int_0^{2\pi}\int_0^{\infty}\frac{|\vec{k}|\mathrm{d}|\vec{k}|\mathrm{d}\theta}{(2\pi)^2} \times\left[\left(\frac{\sin(z_0\omega_1)}{\omega_1} - \frac{\sin(z_0W_2)}{W_2}\right)\delta_{ij} + \left(\frac{\sin(z_0\omega_1)}{m_1^2\omega_1} - \frac{\sin(z_0W_2)}{M_2^2W_2}\right)k_ik_j + \left(\frac{1}{m_1}\cos(z_0\omega_1) + \frac{1}{M_2}\cos(z_0W_2)\right)\epsilon_{ij}\right]e^{i|\vec{k}||\vec{z}|\cos\theta}.$$

Considering the following known integrals:

$$\int_0^{2\pi}\frac{\mathrm{d}\theta}{2\pi}e^{i|\vec{k}||\vec{z}|\cos\theta} = J_0(|\vec{k}||\vec{z}|), \tag{A5}$$

$$\int_0^{2\pi}\frac{\mathrm{d}\theta}{2\pi}\cos\theta e^{i|\vec{k}||\vec{z}|\cos\theta}=iJ_1(|\vec{k}||\vec{z}|), \tag{A6}$$

$$\int_0^{2\pi}\frac{\mathrm{d}\theta}{2\pi}\cos^2\theta e^{i|\vec{k}||\vec{z}|\cos\theta}=\frac{1}{2}(J_0(|\vec{k}||\vec{z}|)-J_2(|\vec{k}||\vec{z}|)), \tag{A7}$$

$$\int_0^{2\pi}\frac{\mathrm{d}\theta}{2\pi}\sin^2\theta e^{i|\vec{k}||\vec{z}|\cos\theta}=\frac{1}{2}(J_0(|\vec{k}||\vec{z}|)+J_2(|\vec{k}||\vec{z}|)), \tag{A8}$$

and using the fact that the following integrals vanish due to parity symmetry

$$\int_0^{2\pi}\frac{\mathrm{d}\theta}{2\pi}\sin\theta e^{i|\vec{k}||\vec{z}|\cos\theta}=0, \tag{A9}$$

$$\int_0^{2\pi}\frac{\mathrm{d}\theta}{2\pi}\cos\theta\sin\theta e^{i|\vec{k}||\vec{z}|\cos\theta}=0, \tag{A10}$$

we are led to the following set of integrals to evaluate. For $D_{00}^{(1)}(z)$

$$D_{00}^{(1)}(z)=-\frac{i}{2\bar{\beta}}\int_0^\infty\frac{|\vec{k}|^3\mathrm{d}|\vec{k}|}{2\pi}\times\left(\frac{\sin(z_0\omega_1)}{m_1^2\omega_1}-\frac{\sin(z_0W_2)}{M_2^2W_2}\right)J_0(|\vec{k}||\vec{z}|). \tag{A11}$$

For $D_{0i}^{(1)}(z)$ we have the components

$$D_{01}^{(1)}(z)=\frac{i}{2\bar{\beta}}\int_0^\infty\frac{|\vec{k}|^2\mathrm{d}|\vec{k}|}{2\pi}\times\left(\frac{\cos(z_0\omega_1)}{m_1^2}-\frac{\cos(z_0W_2)}{M_2^2}\right)J_1(|\vec{k}||\vec{z}|), \tag{A12a}$$

$$D_{02}^{(1)}(z)=-\frac{i}{2\bar{\beta}}\int_0^\infty\frac{|\vec{k}|^2\mathrm{d}|\vec{k}|}{2\pi}\times\left(\frac{\sin(z_0\omega_1)}{m_1\omega_1}+\frac{\sin(z_0W_2)}{M_2W_2}\right)J_1(|\vec{k}||\vec{z}|). \tag{A12b}$$

Finally for the $D_{ij}^{(1)}(z)$ components we have

$$D_{11}^{(1)}(z)=-\frac{i}{2\bar{\beta}}\int_0^\infty\frac{|\vec{k}|\mathrm{d}|\vec{k}|}{2\pi}\times\left[\left(\frac{\sin(z_0\omega_1)}{\omega_1}-\frac{\sin(z_0W_2)}{W_2}\right)J_0(|\vec{k}||\vec{z}|)+\frac{|\vec{k}|^2}{2}\left(\frac{\sin(z_0\omega_1)}{m_1^2\omega_1}-\frac{\sin(z_0W_2)}{M_2^2W_2}\right)\times(J_0(|\vec{k}||\vec{z}|)-J_2(|\vec{k}||\vec{z}|))\right], \tag{A13a}$$

$$D_{12}^{(1)}(z)=-\frac{i}{2\bar{\beta}}\int_0^\infty\frac{|\vec{k}|\mathrm{d}|\vec{k}|}{2\pi}\times\left(\frac{\cos(z_0\omega_1)}{m_1}+\frac{\cos(z_0W_2)}{M_2}\right)J_0(|\vec{k}||\vec{z}|), \tag{A13b}$$

$$D_{21}^{(1)}(z)=\frac{i}{2\bar{\beta}}\int_0^{2\pi}\int_0^\infty\frac{|\vec{k}|\mathrm{d}|\vec{k}|}{2\pi}\times\left(\frac{\cos(z_0\omega_1)}{m_1}+\frac{\cos(z_0W_2)}{M_2}\right)J_0(|\vec{k}||\vec{z}|), \tag{A13c}$$

$$D_{22}^{(1)}(z)=-\frac{i}{2\bar{\beta}}\int_0^\infty\frac{|\vec{k}|\mathrm{d}|\vec{k}|}{2\pi}\times\left[\left(\frac{\sin(z_0\omega_1)}{\omega_1}-\frac{\sin(z_0W_2)}{W_2}\right)J_0(|\vec{k}||\vec{z}|)+\frac{|\vec{k}|^2}{2}\left(\frac{\sin(z_0\omega_1)}{m_1^2\omega_1}-\frac{\sin(z_0W_2)}{M_2^2W_2}\right)\times(J_0(|\vec{k}||\vec{z}|)+J_2(|\vec{k}||\vec{z}|))\right]. \tag{A13d}$$

To illustrate how the case with $z_0\neq 0$ leads to the same conclusion as the $z_0=0$ frame, we explicitly compute the $D_{00}^{(1)}(z)$ integral given in Eq. (11). In this case, we encounter an integral of the form:

$$I(z)=\int_0^\infty\frac{k^2\sin(z_0\omega(k))}{\omega(k)}J_0(kz)k\mathrm{d}k, \tag{A14}$$

for $\omega=\omega_1,W_2$. We know that the Bessel functions satisfy the ordinary differential equation

$$x^2\frac{\mathrm{d}^2J_n(x)}{\mathrm{d}x^2}+x\frac{\mathrm{d}J_n(x)}{\mathrm{d}x}+(x^2-n^2)J_n(x)=0, \tag{A15}$$

that in our particular case means

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}+\frac{1}{z}\frac{\mathrm{d}}{\mathrm{d}z}\right)J_0(kz)=-k^2J_0(kz). \tag{A16}$$

By using this expression we can write the integral as

$$I(z)=-\left(\partial_z^2+\frac{1}{z}\partial_z\right)\left[\int_0^\infty\frac{\sin(z_0\omega(k))}{\omega(k)}J_0(kz)k\mathrm{d}k\right]. \tag{A17}$$

Now by changing the integration variable as $\tau=\omega(k),\mathrm{d}\tau=\frac{k\mathrm{d}k}{\omega(k)}$ then we obtain

$$\int_0^\infty \frac{\sin(z_0\omega(k))}{\omega(k)} J_0(kz)k\mathrm{d}k = \int_m^\infty \sin(z_0\tau)J_0(z\sqrt{\tau^2-m^2})\mathrm{d}\tau. \tag{A18}$$

From [55] (item 6.677-1), the previous integral has the form

$$\int_a^\infty J_0(b\sqrt{x^2-a^2})\sin(cx)\mathrm{d}x = \begin{cases} 0, & 0<c<b \\ \frac{\cos(a\sqrt{c^2-b^2})}{\sqrt{c^2-b^2}}, & 0<b<c \end{cases}. \tag{A19}$$

In our case $a=m$, $b=z$, and $c=z_0$, thus

$$\int_m^\infty \sin(z_0\tau)J_0(z\sqrt{\tau^2-m^2})\mathrm{d}\tau = \begin{cases} 0, & 0<z_0<z \\ \frac{\cos(m\sqrt{z_0^2-z^2})}{\sqrt{z_0^2-z^2}}, & 0<z<z_0 \end{cases}. \tag{A20}$$

We conclude that outside the light cone the integral $I(z)$ vanishes. The other integrals can be calculated in the same way.

APPENDIX B: THE HAMILTONIAN FORMALISM

To develop the Hamiltonian formalism it is more convenient to work with the integrated-by-parts Lagrangian (7)

$$\mathcal{L} = -\frac{1}{4\gamma}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \frac{1}{2}\epsilon^{\alpha\beta\gamma}\partial_\beta A_\gamma(\mu + g\Box)A_\alpha, \tag{B1}$$

which has only second-order derivatives.

Following the Ostrogradsky formalism momenta are

$$\Pi^\mu := \frac{\partial\mathcal{L}}{\partial\ddot{A}_\mu} = \frac{g}{2}\epsilon^{\mu\beta\gamma}\partial_\beta A_\gamma, \tag{B2}$$

$$P^\mu := \frac{\partial\mathcal{L}}{\partial\dot{A}_\mu} - \frac{\partial\Pi^\mu}{\partial t} = -\frac{1}{\gamma}(\dot{A}^\mu - \partial^\mu A^0) - \frac{1}{\xi}(\partial_\lambda A^\lambda)\eta^{0\mu} + \frac{1}{2}\epsilon^{\alpha 0\mu}(\mu + g\Box)A_\alpha - \frac{g}{2}\epsilon^{\mu\beta\gamma}\partial_\beta\dot{A}_\gamma. \tag{B3}$$

In components

$$P^0 = -\frac{1}{\xi}(\dot{A}_0 + \partial^i A_i) - \frac{g}{2}\epsilon^{ij}\partial_i\dot{A}_j, \tag{B4}$$

$$P^k = -\frac{1}{\gamma}(\dot{A}^k - \partial^k A_0) + \frac{1}{2}\epsilon^{ki}(\mu + g\partial^j\partial_j)A_i + g\epsilon^{ki}\ddot{A}_i - \frac{g}{2}\epsilon^{kj}\partial_j\dot{A}_0, \tag{B5}$$

$$\Pi^0 = \frac{g}{2}\epsilon^{ij}\partial_i A_j, \tag{B6}$$

$$\Pi^k = -\frac{g}{2}\epsilon^{ki}\dot{A}_i + \frac{g}{2}\epsilon^{ki}\partial_i A_0. \tag{B7}$$

Some momenta define a surface between extended phase space variables and hence are identified with primary constraints; these are denoted by

$$\varphi^0 := \Pi^0 - \frac{g}{2}\epsilon^{ij}\partial_i A_j, \tag{B8}$$

$$\varphi^i := \Pi^i + \frac{g}{2}\epsilon^{ij}\dot{A}_j - \frac{g}{2}\epsilon^{ij}\partial_j A_0, \tag{B9}$$

$$\psi^0 := P^0 + \frac{1}{\xi}(\dot{A}_0 + \partial^k A_k) + \frac{g}{2}\epsilon^{ij}\partial_i\dot{A}_j. \tag{B10}$$

The Hamiltonian can be obtained through the Legendre transform of extended phase space variables

$$\mathcal{H}_C = \dot{A}_\mu P^\mu + \ddot{A}_\mu\Pi^\mu - \mathcal{L}. \tag{B11}$$

For this we consider the identities

$$\partial_k\Pi^k = -\frac{g}{2}\epsilon^{ij}\partial_i\dot{A}_j, \tag{B12}$$

$$\dot{A}_0 = -\partial^i A_i - \xi P^0 + \xi\partial_k\Pi^k, \tag{B13}$$

$$\epsilon_{ij}\Pi^j = \frac{g}{2}(\dot{A}_i - \partial_i A_0), \tag{B14}$$

$$\epsilon_{ij}P^j = -\frac{1}{\gamma}\epsilon_{ij}(\dot{A}^j - \partial^j A^0) - \frac{1}{2}(\mu + g\partial^j\partial_j)A_i - g\ddot{A}_i - \frac{g}{2}\partial_i\dot{A}_0. \tag{B15}$$

These can be used to write

$$\dot{A}_0 = -\partial^i A_i - \xi P^0 + \xi\partial_k\Pi^k, \tag{B16}$$

$$\dot{A}_i = \partial_i A_0 + \frac{2}{g}\epsilon_{ij}\Pi^j, \tag{B17}$$

$$\ddot{A}_i = \frac{1}{g}\left[-\epsilon_{ij}P^j + \frac{2}{\gamma g}\Pi_i - \frac{1}{2}(\mu + g\partial^j\partial_j)A_i + \frac{g}{2}\partial_i(\partial^j A_j + \xi P^0 - \xi\partial_j\Pi^j)\right]. \tag{B18}$$

After some calculation we have the canonical Hamiltonian density

$$\mathcal{H} = \frac{\xi}{2}(-(P^0)^2 + (\partial_k \Pi^k)^2) + \frac{2}{\gamma g^2}\Pi^i \Pi_i + \frac{2}{g}\epsilon_{ij} P^i \Pi^j - \frac{1}{g}\Pi^i(\mu + g\partial^j \partial_j)A_i - \partial^i A_i P^0 + \partial_i A_0 P^i + \frac{1}{4\gamma}F_{ij}F^{ij} - \frac{1}{2}\epsilon^{kl}\partial_k A_l(\mu + g\partial^j\partial_j)A_0. \tag{B19}$$

We notice that the canonical Hamiltonian density only depends on spatial derivatives of the extended phase space variables (A_μ, P^μ, Π^μ). We incorporate the constraints to the Hamiltonian, obtaining

$$H = \int \mathrm{d}^2x \left(\frac{\xi}{2}(-(P^0)^2 + (\partial_k \Pi^k)^2) + \frac{2}{\gamma g^2}\Pi^i \Pi_i + \frac{2}{g}\epsilon_{ij} P^i \Pi^j - \frac{1}{g}\Pi^i(\mu + g\partial^j \partial_j)A_i - \partial^i A_i P^0 + \partial_i A_0 P^i + \frac{1}{4\gamma}F_{ij}F^{ij} - \frac{1}{2}\epsilon^{kl}\partial_k A_l(\mu + g\partial^j\partial_j)A_0 + \lambda_0\varphi^0 + \lambda_i\varphi^i + \theta_0\psi^0 \right). \tag{B20}$$

Considering the Poisson brackets

$$\{A_\mu(t,\vec{x}), P^\nu(t,\vec{y})\} = \delta^\nu_\mu \delta^{(2)}(\vec{x}-\vec{y}), \tag{B21}$$

$$\{\dot{A}_\mu(t,\vec{x}), \Pi^\nu(t,\vec{y})\} = \delta^\nu_\mu \delta^{(2)}(\vec{x}-\vec{y}), \tag{B22}$$

we have the constraints algebra

$$\{\varphi^0(t,\vec{x}), \varphi^0(t,\vec{y})\} = 0, \tag{B23}$$

$$\{\varphi^0(t,\vec{x}), \varphi^i(t,\vec{y})\} = 0, \tag{B24}$$

$$\{\varphi^0(t,\vec{x}), \psi^0(t,\vec{y})\} = -\frac{1}{\xi}\delta^{(2)}(\vec{x}-\vec{y}), \tag{B25}$$

$$\{\varphi^i(t,\vec{x}), \varphi^j(t,\vec{y})\} = g\epsilon^{ij}\delta^{(2)}(\vec{x}-\vec{y}), \tag{B26}$$

$$\{\varphi^i(t,\vec{x}), \psi^0(t,\vec{y})\} = -g\epsilon^{ik}\partial_k^{(x)}\delta^{(2)}(\vec{x}-\vec{y}), \tag{B27}$$

$$\{\psi^0(t,\vec{x}), \psi^0(t,\vec{y})\} = 0. \tag{B28}$$

Evolving in time all constraints we find four consistency relations

$$\theta_0 = \partial_i \Pi^i, \tag{B29}$$

$$\lambda_j = \partial_j \partial_i \Pi^i - \ddot{A}_j, \tag{B30}$$

$$\lambda_0 = -\partial_i P^i - \frac{2}{g}\left(1 + \frac{1}{\gamma}\right)\epsilon_{ij}\partial^i \Pi^j - \partial^i \partial_i A_0 + \partial_k \Pi^k + g\epsilon^{ki}\partial_k \ddot{A}_i, \tag{B31}$$

where $\ddot{A}_i$ is a function of phase space variables; see Eq. (B18).

To continue with the Dirac algorithm we introduce the notation for the four constraints

$$\varphi_A = \{\varphi^0, \varphi^1, \varphi^2, \psi^0\}. \tag{B32}$$

We define the matrix

$$C_{AB}(t;\vec{x},\vec{y}) := \begin{pmatrix} \{\varphi^0(t,\vec{x}),\varphi^0(t,\vec{y})\} & \{\varphi^0(t,\vec{x}),\varphi^1(t,\vec{y})\} & \{\varphi^0(t,\vec{x}),\varphi^2(t,\vec{y})\} & \{\varphi^0(t,\vec{x}),\psi^0(t,\vec{y})\} \\ \{\varphi^1(t,\vec{x}),\varphi^0(t,\vec{y})\} & \{\varphi^1(t,\vec{x}),\varphi^1(t,\vec{y})\} & \{\varphi^1(t,\vec{x}),\varphi^2(t,\vec{y})\} & \{\varphi^1(t,\vec{x}),\psi^0(t,\vec{y})\} \\ \{\varphi^2(t,\vec{x}),\varphi^0(t,\vec{y})\} & \{\varphi^2(t,\vec{x}),\varphi^1(t,\vec{y})\} & \{\varphi^2(t,\vec{x}),\varphi^2(t,\vec{y})\} & \{\varphi^2(t,\vec{x}),\psi^0(t,\vec{y})\} \\ \{\psi^0(t,\vec{x}),\varphi^0(t,\vec{y})\} & \{\psi^0(t,\vec{x}),\varphi^1(t,\vec{y})\} & \{\psi^0(t,\vec{x}),\varphi^2(t,\vec{y})\} & \{\psi^0(t,\vec{x}),\psi^0(t,\vec{y})\} \end{pmatrix}, \tag{B33}$$

and find

$$C_{AB}(t;\vec{x},\vec{y}) = \begin{pmatrix} 0 & 0 & 0 & -1/\xi \\ 0 & 0 & g & -g\partial_2 \\ 0 & -g & 0 & g\partial_1 \\ 1/\xi & -g\partial_2 & g\partial_1 & 0 \end{pmatrix}\delta^{(2)}(\vec{x}-\vec{y}). \tag{B34}$$

The inverse matrix becomes

$$C^{-1}_{AB}(t;\vec{x},\vec{y}) = \begin{pmatrix} 0 & -\xi\partial_1 & -\xi\partial_2 & \xi \\ -\xi\partial_1 & 0 & -1/g & 0 \\ -\xi\partial_2 & 1/g & 0 & 0 \\ -\xi & 0 & 0 & 0 \end{pmatrix}\delta^{(2)}(\vec{x}-\vec{y}). \tag{B35}$$

After some calculation, the Dirac brackets for $A_0(t,\vec{x})$ are

$$\{A_0(t,\vec{x}), A_0(t,\vec{y})\}^* = 0, \tag{B36}$$

$$\{A_0(t,\vec{x}), A_j(t,\vec{y})\}^* = 0, \tag{B37}$$

$$\{A_0(t,\vec{x}), \dot{A}_0(t,\vec{y})\}^* = -\xi\delta^{(2)}(\vec{x}-\vec{y}) \tag{B38}$$

$$\{A_0(t,\vec{x}),\dot{A}_j(t,\vec{y})\}^* = 0, \tag{B39}$$

$$\{A_0(t,\vec{x}),P^0(t,\vec{y})\}^* = \delta^{(2)}(\vec{x}-\vec{y}), \tag{B40}$$

$$\{A_0(t,\vec{x}),P^j(t,\vec{y})\}^* = \frac{\xi g}{2}\epsilon^{jk}\partial_k\delta^{(2)}(\vec{x}-\vec{y}), \tag{B41}$$

$$\{A_0(t,\vec{x}),\Pi^0(t,\vec{y})\}^* = 0, \tag{B42}$$

$$\{A_0(t,\vec{x}),\Pi^j(t,\vec{y})\}^* = 0, \tag{B43}$$

for $A_i(t,\vec{x})$

$$\{A_i(t,\vec{x}),A_j(t,\vec{y})\}^* = 0, \tag{B44}$$

$$\{A_i(t,\vec{x}),\dot{A}_0(t,\vec{y})\}^* = 0, \tag{B45}$$

$$\{A_i(t,\vec{x}),\dot{A}_j(t,\vec{y})\}^* = 0, \tag{B46}$$

$$\{A_i(t,\vec{x}),P^0(t,\vec{y})\}^* = 0, \tag{B47}$$

$$\{A_i(t,\vec{x}),P^j(t,\vec{y})\}^* = \delta_i^j\delta^{(2)}(\vec{x}-\vec{y}), \tag{B48}$$

$$\{A_i(t,\vec{x}),\Pi^0(t,\vec{y})\}^* = 0, \tag{B49}$$

$$\{A_i(t,\vec{x}),\Pi^j(t,\vec{y})\}^* = 0, \tag{B50}$$

for $\dot{A}_0(t,\vec{x})$

$$\{\dot{A}_0(t,\vec{x}),\dot{A}_0(t,\vec{y})\}^* = 0, \tag{B51}$$

$$\{\dot{A}_0(t,\vec{x}),\dot{A}_j(t,\vec{y})\}^* = -\xi\partial_j\delta^{(2)}(\vec{x}-\vec{y}), \tag{B52}$$

$$\{\dot{A}_0(t,\vec{x}),P^0(t,\vec{y})\}^* = 0, \tag{B53}$$

$$\{\dot{A}_0(t,\vec{x}),P^j(t,\vec{y})\}^* = -\partial^j\delta^{(2)}(\vec{x}-\vec{y}), \tag{B54}$$

$$\{\dot{A}_0(t,\vec{x}),\Pi^0(t,\vec{y})\}^* = 0, \tag{B55}$$

$$\{\dot{A}_0(t,\vec{x}),\Pi^j(t,\vec{y})\}^* = 0, \tag{B56}$$

for $\dot{A}_i(t,\vec{x})$

$$\{\dot{A}_i(t,\vec{x}),\dot{A}_j(t,\vec{y})\}^* = -\frac{1}{g}\epsilon_{ij}\delta^{(2)}(\vec{x}-\vec{y}), \tag{B57}$$

$$\{\dot{A}_i(t,\vec{x}),P^0(t,\vec{y})\}^* = \frac{1}{2}\partial_i\delta^{(2)}(\vec{x}-\vec{y}), \tag{B58}$$

$$\{\dot{A}_i(t,\vec{x}),P^j(t,\vec{y})\}^* = -\frac{g\xi}{2}\epsilon^{kj}\partial_i\partial_k\delta^{(2)}(\vec{x}-\vec{y}), \tag{B59}$$

$$\{\dot{A}_i(t,\vec{x}),\Pi^0(t,\vec{y})\}^* = 0, \tag{B60}$$

$$\{\dot{A}_i(t,\vec{x}),\Pi^j(t,\vec{y})\}^* = \frac{1}{2}\delta_i^j\delta^{(2)}(\vec{x}-\vec{y}), \tag{B61}$$

for $P^0(t,\vec{x})$

$$\{P^0(t,\vec{x}),P^0(t,\vec{y})\}^* = 0, \tag{B62}$$

$$\{P^0(t,\vec{x}),P^j(t,\vec{y})\}^* = 0, \tag{B63}$$

$$\{P^0(t,\vec{x}),\Pi^0(t,\vec{y})\}^* = 0, \tag{B64}$$

$$\{P^0(t,\vec{x}),\Pi^j(t,\vec{y})\}^* = \frac{g}{4}\epsilon^{jk}\partial_k\delta^{(2)}(\vec{x}-\vec{y}), \tag{B65}$$

for $P^i(t,\vec{x})$

$$\{P^i(t,\vec{x}),P^j(t,\vec{y})\}^* = -\frac{g}{2}\epsilon^{ij}\partial^k\partial_k\delta^{(2)}(\vec{x}-\vec{y}), \tag{B66}$$

$$\{P^i(t,\vec{x}),\Pi^0(t,\vec{y})\}^* = \frac{g}{2}\epsilon^{ji}\partial_j\delta^{(2)}(\vec{x}-\vec{y}), \tag{B67}$$

$$\{P^i(t,\vec{x}),\Pi^j(t,\vec{y})\}^* = 0, \tag{B68}$$

and for $\Pi^0(t,\vec{x})$

$$\{\Pi^0(t,\vec{x}),\Pi^0(t,\vec{y})\}^* = 0, \tag{B69}$$

$$\{\Pi^0(t,\vec{x}),\Pi^j(t,\vec{y})\}^* = 0, \tag{B70}$$

$$\{\Pi^i(t,\vec{x}),\Pi^j(t,\vec{y})\}^* = -\frac{g}{4}\epsilon^{ij}\delta^{(2)}(\vec{x}-\vec{y}). \tag{B71}$$

A direct counting gives the number $\frac{1}{2}(6\times 2(\text{phase space variables})-4(\text{second-class constraints}))=4$ of degrees of freedom.

Imposing strongly the second-class constraints we have the Hamiltonian

$$\begin{aligned} H = \int \mathrm{d}^2x\Big(&-\frac{1}{2\xi}\dot{A}_0^2+\frac{1}{2\xi}(\partial^iA_i)^2-\frac{g}{2}\epsilon^{ij}\partial_i\dot{A}_j\dot{A}_0 \\ &+\frac{1}{2\gamma}(\dot{A}_i-\partial_iA_0)(\dot{A}^i-\partial^iA_0)+\frac{1}{2}\epsilon^{ij}\dot{A}_j(\mu+g\partial^k\partial_k)A_i \\ &-\epsilon^{ij}\partial_iA_j(\mu+g\partial_k\partial^k)A_0+\dot{A}_iP^i+\frac{1}{4\gamma}F_{ij}F^{ij}\Big). \end{aligned} \tag{B72}$$

1. Dirac commutators

Here, we verify the Dirac structure of the theory by explicitly evaluating several commutators using the field expansions. The first step is to promote the Dirac brackets to Dirac commutators according to

$$\{A(x),B(y)\}^* \leftrightarrow \frac{1}{i}[A(x),B(y)]_D. \tag{B73}$$

a. The commutator $[A_0(x), \dot{A}_0(y)]_D$

We begin to verify the commutator

$$[A_0(x), \dot{A}_0(y)]_D = -i\xi\delta^{(2)}(\vec{x}-\vec{y}). \tag{B74}$$

Replacing the field we have the contributions

$$\begin{aligned}[A_0(x), \dot{A}_0(y)]_D = {} & [A_0^D(x), \dot{A}_0^G(y)]_D + [A_0^G(x), \dot{A}_0^D(y)]_D \\ & + [A_0^G(x), \dot{A}_0^G(y)]_D + [\bar{A}_0(x), \dot{\bar{A}}_0(y)]_D \\ & + [G_0(x), \dot{G}_0(y)]_D. \end{aligned} \tag{B75}$$

We take first derivatives

$$\begin{aligned}\dot{A}_0^D(x) = \int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{\mu}{2}[(2-it|\vec{k}|)d_k e^{-ik\cdot x} \\ + (2+it|\vec{k}|)d_k^\dagger e^{ik\cdot x}]_{k_0=|\vec{k}|}, \end{aligned} \tag{B76}$$

$$\dot{A}_0^G(x) = \int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{-|\vec{k}|}{2}[c_k e^{-ik\cdot x} + c_k^\dagger e^{ik\cdot x}]_{k_0=|\vec{k}|}, \tag{B77}$$

$$\begin{aligned}\dot{\bar{A}}_0(x) = \int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{i}{2\bar{\beta}}[-\varepsilon_0^{(-)}a_k^{(-)}e^{-ik\cdot x} \\ + \varepsilon_0^{(-)*}a_k^{(-)\dagger}e^{ik\cdot x}]_{k_0=\omega_1}, \end{aligned} \tag{B78}$$

$$\begin{aligned}\dot{G}_0(x) = \int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{i}{2\bar{\beta}}[-\varepsilon_0^{(+)}b_k^{(+)}e^{-ik\cdot x} \\ + \varepsilon_0^{(+)*}b_k^{(+)\dagger}e^{ik\cdot x}]_{k_0=W_2}. \end{aligned} \tag{B79}$$

We obtain

$$[A_0^D(x), \dot{A}_0^G(y)]_D = \int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{-i\xi}{2}e^{ik\cdot(x-y)}, \tag{B80}$$

$$[A_0^G(x), \dot{A}_0^D(y)]_D = \int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}(-i\xi)e^{ik\cdot(x-y)}, \tag{B81}$$

$$[A_0^G(x), \dot{A}_0^G(y)]_D = \int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{i|\vec{k}|}{2}\left[-\frac{2|\vec{k}|}{\gamma\mu^2}+\frac{\xi}{|\vec{k}|}\right]e^{ik\cdot(x-y)}, \tag{B82}$$

$$[\bar{A}_0(x), \dot{\bar{A}}_0(y)]_D = \left(1-\frac{\omega_1^2}{m_1^2}\right)\int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{-i}{2\bar{\beta}}e^{ik\cdot(x-y)}, \tag{B83}$$

$$[G_0(x), \dot{G}_0(y)]_D = \left(1-\frac{W_2^2}{M_2^2}\right)\int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{i}{2\bar{\beta}}e^{ik\cdot(x-y)}. \tag{B84}$$

Adding all the contributions, we find

$$\begin{aligned}[A_0(x), \dot{A}_0(y)]_D = i\int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\left[-\frac{\xi}{2}-\xi+\frac{|\vec{k}|}{2}\left[-\frac{2|\vec{k}|}{\gamma\mu^2}+\frac{\xi}{|\vec{k}|}\right]\right. \\ \left. -\left(1-\frac{\omega_1^2}{m_1^2}\right)\frac{1}{2\bar{\beta}}+\left(1-\frac{W_2^2}{M_2^2}\right)\frac{1}{2\bar{\beta}}\right]e^{ik\cdot(x-y)}, \end{aligned} \tag{B85}$$

or

$$\begin{aligned}[A_0(x), \dot{A}_0(y)]_D = i\int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\left[-\xi-\frac{|\vec{k}|^2}{\gamma\mu^2}\right. \\ \left. +\frac{|\vec{k}|^2}{2\bar{\beta}}\left(\frac{1}{m_1^2}-\frac{1}{M_2^2}\right)\right]e^{ik\cdot(x-y)}, \end{aligned} \tag{B86}$$

and using (180), we arrive at

$$[A_0(x), \dot{A}_0(y)]_D = -i\xi\delta^{(2)}(\vec{x}-\vec{y}). \tag{B87}$$

b. The commutator $[A_0(x), P^i(y)]_D$

We continue to verify the commutator

$$[A_0(x), P^i(y)]_D = \frac{i\xi g}{2}\epsilon^{ij}\partial_j\delta^{(2)}(\vec{x}-\vec{y}). \tag{B88}$$

Using the definition of momentum (5), and the previous Dirac brackets, we have

$$\begin{aligned}[A_0(x), P^i(y)]_D = {} & -\frac{1}{\gamma}([A_0(x), \dot{A}^i(y)]_D - \partial^i_{(y)}[A_0(x), A_0(y)]_D) \\ & + \frac{1}{2}\epsilon^{ij}(\mu + g\partial^k_{(y)}\partial_k^{(y)})[A_0(x), A_j(y)]_D \\ & + g\epsilon^{ij}[A_0(x), \ddot{A}_j(y)]_D \\ & - \frac{g}{2}\epsilon^{ij}\partial_j^{(y)}[A_0(x), \dot{A}_0(y)]_D, \end{aligned} \tag{B89}$$

where the notation emphasizes the spatial derivatives acting on the second coordinate y. Some commutators are zero, and we obtain

$$\begin{aligned}[A_0(x), P^i(y)]_D = {} & g\epsilon^{ij}[A_0(x), \ddot{A}_j(y)]_D \\ & - i\xi\frac{g}{2}\epsilon^{ij}\partial_j\delta^{(2)}(\vec{x}-\vec{y})). \end{aligned} \tag{B90}$$

The second time derivatives on the fields are

$$\begin{aligned}\ddot{A}_j^D(x) = \int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{i}{2}\left[-\left(\mu(2-i|\vec{k}|t)k_j + i\frac{2|\vec{k}|}{\xi}\epsilon_{jk}k^k\right)d_k e^{-ik\cdot x}\right. \\ \left. + \left(\mu(2+i|\vec{k}|t)k_j - i\frac{2|\vec{k}|}{\xi}\epsilon_{jk}k^k\right)d_k^\dagger e^{ik\cdot x}\right]_{k_0=|\vec{k}|}, \end{aligned} \tag{B91}$$

$$\ddot{A}_j^G(x) = \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{-i|\vec{k}|k_j}{2} [-c_k e^{-ik\cdot x} + c_k^\dagger e^{ik\cdot x}]_{k_0=|\vec{k}|}, \quad \text{(B92)}$$

$$\ddot{\bar{A}}_j(x) = \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{-\omega_1}{2\bar{\beta}} [\varepsilon_j^{(-)} a_k^{(-)} e^{-ik\cdot x} + \varepsilon_j^{(-)*} a_k^{(-)\dagger} e^{ik\cdot x}]_{k_0=\omega_1}, \quad \text{(B93)}$$

$$\ddot{G}_j(x) = \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{-W_2}{2\bar{\beta}} [\varepsilon_j^{(+)} b_k^{(+)} e^{-ik\cdot x} + \varepsilon_j^{(+)*} b_k^{(+)\dagger} e^{ik\cdot x}]_{k_0=W_2}. \quad \text{(B94)}$$

The contributions to the bracket $[A_0(x), \ddot{A}_j(y)]_D$ are given by

$$\begin{aligned}[A_0(x), \ddot{A}_j(y)]_D = {} & [A_0^D(x), \ddot{A}_j^G(y)]_D + [A_0^G(x), \ddot{A}_j^D(y)]_D \\ & + [A_0^G(x), \ddot{A}_j^G(y)]_D + [\bar{A}_0(x), \ddot{\bar{A}}_j(y)]_D \\ & + [G_0(x), \ddot{G}_j(y)]_D. \end{aligned} \quad \text{(B95)}$$

Each term contributes with

$$[A_0^D(x), \ddot{A}_j^G(y)]_D = -\int \frac{d^2\vec{k}}{(2\pi)^2} \frac{\xi}{2} k_j e^{ik\cdot(x-y)}, \quad \text{(B96)}$$

$$[A_0^G(x), \ddot{A}_j^D(y)]_D = -\int \frac{d^2\vec{k}}{(2\pi)^2} \xi k_j e^{ik\cdot(x-y)}, \quad \text{(B97)}$$

$$[A_0^G(x), \ddot{A}_j^G(y)]_D = \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{|\vec{k}|}{2} \left[-\frac{2|\vec{k}|}{\gamma\mu^2} + \frac{\xi}{|\vec{k}|} \right] \times k_j e^{ik\cdot(x-y)}, \quad \text{(B98)}$$

$$[\bar{A}_0(x), \ddot{\bar{A}}_j(y)]_D = \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{1}{2\bar{\beta}} \frac{\omega_1^2}{m_1^2} k_j e^{ik\cdot(x-y)}, \quad \text{(B99)}$$

$$[G_0(x), \ddot{G}_j(y)]_D = -\int \frac{d^2\vec{k}}{(2\pi)^2} \frac{1}{2\bar{\beta}} \frac{W_2^2}{M_2^2} k_j e^{ik\cdot(x-y)}. \quad \text{(B100)}$$

The sum of all contributions give

$$\begin{aligned}[A_0(x), \ddot{A}_j(y)]_D = {} & -\int \frac{d^2\vec{k}}{(2\pi)^2} \xi k_j e^{ik\cdot(x-y)} \\ & - \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{|\vec{k}|^2}{\gamma\mu^2} k_j e^{ik\cdot(x-y)} \\ & + \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{|\vec{k}|^2}{2\bar{\beta}} \left[\frac{1}{m_1^2} - \frac{1}{M_2^2} \right] k_j e^{ik\cdot(x-y)}. \end{aligned} \quad \text{(B101)}$$

Using (180), we obtain

$$[A_0(x), \ddot{A}_j(y)]_D = -\int \frac{d^2\vec{k}}{(2\pi)^2} \xi k_j e^{ik\cdot(x-y)}. \quad \text{(B102)}$$

Thus, the Dirac bracket becomes

$$\begin{aligned}[A_0(x), P^i(y)]_D &= i\xi g \epsilon^{ij} \partial_j \delta^{(2)}(\vec{x}-\vec{y}) - i\xi \frac{g}{2} \epsilon^{ij} \partial_j \delta^{(2)}(\vec{x}-\vec{y}) \\ &= i\xi \frac{g}{2} \epsilon^{ij} \partial_j \delta^{(2)}(\vec{x}-\vec{y}). \end{aligned} \quad \text{(B103)}$$

c. The commutator $[A_i(x), P^j(y)]_D$

We verify the Dirac bracket

$$[A_i(x), P^j(y)]_D = i\delta_i^j \delta^{(2)}(\vec{x}-\vec{y}). \quad \text{(B104)}$$

Again using the definition of momentum P^j we have

$$[A_i(x), P^j(y)]_D = g\epsilon^{jk} [A_i(x), \ddot{A}_k(y)]_D. \quad \text{(B105)}$$

From (93)–(96), we have

$$\begin{aligned}[A_i(x), \ddot{A}_k(y)]_D = {} & [A_i^D(x), \ddot{A}_k^G(y)]_D + [A_i^G(x), \ddot{A}_k^D(y)]_D \\ & + [A_i^G(x), \ddot{A}_k^G(y)]_D + [\bar{A}_i(x), \ddot{\bar{A}}_k(y)]_D \\ & + [G_i(x), \ddot{G}_k(y)]_D. \end{aligned} \quad \text{(B106)}$$

We compute and obtain

$$[A_i^D(x), \dot{\ddot{A}}_k^G(y)]_D = \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{-i\xi}{2\mu} \left(\mu t k_i - \frac{2}{\xi} \epsilon_{il} k^l \right) \times k_k e^{ik\cdot(x-y)}, \quad \text{(B107)}$$

$$[A_i^G(x), \dot{\ddot{A}}_k^D(y)]_D = \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{i\xi}{2\mu} k_i \left(\mu t k_k - \frac{2}{\xi} \epsilon_{kl} k^l \right) \times e^{ik\cdot(x-y)}, \quad \text{(B108)}$$

$$[A_i^G(x), \ddot{A}_k^G(y)]_D = 0, \quad \text{(B109)}$$

$$[\bar{A}_i(x), \ddot{\bar{A}}_k(y)]_D = \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{i\omega_1^2}{2\bar{\beta} m_1} \epsilon_{ik} e^{ik\cdot(x-y)}, \quad \text{(B110)}$$

$$[G_i(x), \ddot{G}_k(y)]_D = \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{iW_2^2}{2\bar{\beta} M_2} \epsilon_{ik} e^{ik\cdot(x-y)}. \quad \text{(B111)}$$

Adding all the contributions

$$\begin{aligned}[A_i(x), \ddot{A}_k(y)]_D = {} & \int \frac{d^2\vec{k}}{(2\pi)^2} \left[\frac{i}{\mu} (\epsilon_{il} k^l k_k - k_i \epsilon_{kl} k^l) \right. \\ & \left. + \frac{i\omega_1^2}{2\bar{\beta} m_1} \epsilon_{ik} + \frac{iW_2^2}{2\bar{\beta} M_2} \epsilon_{ik} \right] e^{ik\cdot(x-y)}. \end{aligned} \quad \text{(B112)}$$

We consider the identity

$$\epsilon_{il}k^l k_k - k_i\epsilon_{kl}k^l = -\epsilon_{ik}|\vec{k}|^2. \tag{B113}$$

Replacing the expression above, we have

$$\begin{aligned}[A_i(x),\ddot{A}_k(y)]_D &= \int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\left[-\frac{i}{\mu}|\vec{k}|^2 + \frac{i}{2\bar{\beta}}\frac{2\gamma\bar{\beta}}{\gamma g}\right.\\ &\left.+\frac{i|\vec{k}|^2}{2\bar{\beta}}\frac{2\gamma\bar{\beta}}{\gamma\mu}\right]\epsilon_{ik}e^{ik\cdot(x-y)}\\ &= \int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{i}{g}\epsilon_{ik}e^{ik\cdot(x-y)}.\end{aligned} \tag{B114}$$

Finally

$$[A_i(x),P^j(y)]_D = g\epsilon^{jk}\int\frac{\mathrm{d}^2k}{(2\pi)^2}\frac{i}{g}\epsilon_{ik}e^{ik\cdot(x-y)}, = i\delta_i^j\delta^{(2)}(\vec{x}-\vec{y}). \tag{B115}$$

We have presented the details for the first three cases; the remaining ones can be checked analogously.

APPENDIX C: THE FEYNMAN PROPAGATOR

In this part we give the Feynman propagator using the definition. Recall the time ordered product of fields

$$\begin{aligned}D^F_{\mu\nu}(x-y) &= \theta(x^0-y^0)\langle 0|A_\mu(x)A_\nu(y)|0\rangle\\ &+\theta(y^0-x^0)\langle 0|A_\nu(y)A_\mu(x)|0\rangle.\end{aligned} \tag{C1}$$

We first consider the case $x_0 > y_0$

$$\begin{aligned}\langle 0|A_\mu(x)A_\nu(y)|0\rangle &= \langle 0|A^D_\mu(x)A^G_\nu(y)|0\rangle + \langle 0|A^G_\mu(x)A^D_\nu(y)|0\rangle\\ &+\langle 0|A^G_\mu(x)A^G_\nu(y)|0\rangle + \langle 0|\bar{A}_\mu(x)\bar{A}_\nu(y)|0\rangle\\ &+\langle 0|G_\mu(x)G_\nu(y)|0\rangle.\end{aligned} \tag{C2}$$

We compute each piece

$$\begin{aligned}\langle 0|A^D_\mu(x)A^G_\nu(y)|0\rangle &= \iint\frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{\mathrm{d}^2\vec{q}}{(2\pi)^2}\frac{iv_\mu(x_0,k)q_\nu}{4|\vec{k}||\vec{q}|}\\ &\times\langle 0|d_{\vec{k}}c^\dagger_{\vec{q}}|0\rangle e^{-i(k\cdot x-q\cdot y)}\\ &= \int\frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{iv_\mu(x_0,k)k_\nu}{4|\vec{k}|^2}\frac{\xi}{\mu}e^{-ik\cdot(x-y)},\end{aligned} \tag{C3}$$

also

$$\begin{aligned}\langle 0|A^G_\mu(x)A^D_\nu(y)|0\rangle &= \iint\frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{\mathrm{d}^2\vec{q}}{(2\pi)^2}\frac{(-ik_\mu)v^*_\nu(y_0,q)}{4|\vec{k}||\vec{q}|}\\ &\times\langle 0|c_{\vec{k}}d^\dagger_{\vec{q}}|0\rangle e^{-i(k\cdot x-q\cdot y)}\\ &= \int\frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{(-ik_\mu)v^*_\nu(y_0,k)}{4|\vec{k}|^2}\frac{\xi}{\mu}e^{-ik\cdot(x-y)},\end{aligned} \tag{C4}$$

and

$$\begin{aligned}\langle 0|A^G_\mu(x)A^G_\nu(y)|0\rangle &= \iint\frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{\mathrm{d}^2\vec{q}}{(2\pi)^2}\frac{(-ik_\mu)(iq_\nu)}{4|\vec{k}||\vec{q}|}\\ &\times\langle 0|c_{\vec{k}}c^\dagger_{\vec{q}}|0\rangle e^{-i(k\cdot x-q\cdot y)}\\ &= \int\frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\frac{k_\mu k_\nu}{4|\vec{k}|^2}\left[-\frac{2|\vec{k}|}{\gamma\mu^2}+\frac{\xi}{|\vec{k}|}\right]e^{-ik\cdot(x-y)}.\end{aligned} \tag{C5}$$

Thus the contributions coming from the massless sector correspond to

$$\begin{aligned}&\langle 0|A^D_\mu(x)A^G_\nu(y)|0\rangle + \langle 0|A^G_\mu(x)A^D_\nu(y)|0\rangle + \langle 0|A^G_\mu(x)A^G_\nu(y)|0\rangle\\ &= \int\frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\left(\frac{i}{4|\vec{k}|^2}\frac{\xi}{\mu}(v_\mu(x_0,k)k_\nu - k_\mu v^*_\nu(y_0,k))\right.\\ &\left.+\frac{k_\mu k_\nu}{4|\vec{k}|^2}\left[-\frac{2|\vec{k}|}{\gamma\mu^2}+\frac{\xi}{|\vec{k}|}\right]\right)e^{-ik\cdot(x-y)}.\end{aligned} \tag{C6}$$

Considering

$$\begin{aligned}v_\mu(x_0,k)k_\nu - k_\mu v^*_\nu(y_0,k) &= \mu(x_0-y_0)k_\mu k_\nu + i\mu(n_\mu k_\nu + k_\mu n_\nu)\\ &-\frac{2|\vec{k}|}{\xi}\epsilon_{\mu\beta\nu}k^\beta,\end{aligned} \tag{C7}$$

and replacing above, we have

$$\begin{aligned}&\langle 0|A^D_\mu(x)A^G_\nu(y)|0\rangle + \langle 0|A^G_\mu(x)A^D_\nu(y)|0\rangle + \langle 0|A^G_\mu(x)A^G_\nu(y)|0\rangle\\ &= \int\frac{\mathrm{d}^2\vec{k}}{(2\pi)^2}\left(\frac{(-1)}{(2|\vec{k}|)^2}\times\left(\left[-i\xi(x_0-y_0)-\frac{\xi}{|\vec{k}|}+\frac{2|\vec{k}|}{\gamma\mu^2}\right]k_\mu k_\nu\right.\right.\\ &\left.\left.+\xi(n_\mu k_\nu + k_\mu n_\nu)+i\frac{2|\vec{k}|}{\mu}\epsilon_{\mu\beta\nu}k^\beta\right)e^{-ik\cdot(x-y)}.\end{aligned} \tag{C8}$$

Finally we identify the residue of the propagator (15) times the plane wave at the double pole $k_0 = |\vec{k}|$

$$\begin{aligned}&\mathrm{Res}(G_{\mu\nu}(k)e^{-ik\cdot(x-y)}, k_0 = |\vec{k}|)\\ &= \frac{-ie^{-ik\cdot(x-y)}}{(2|\vec{k}|)^2}\left[\left((-i\xi)(x^0-y^0)-\frac{\xi}{|\vec{k}|}+\frac{2|\vec{k}|}{\gamma\mu^2}\right)k_\mu k_\nu\right.\\ &\left.+\xi(n_\mu k_\nu + k_\mu n_\nu)+i\frac{2|\vec{k}|}{\mu}\epsilon_{\mu\beta\nu}k^\beta\right].\end{aligned} \tag{C9}$$

This means that

$$\langle 0|A_\mu^D(x)A_\nu^D(y)|0\rangle + \langle 0|A_\mu^D(x)A_\nu^G(y)|0\rangle + \langle 0|A_\mu^G(x)A_\nu^D(y)|0\rangle + \langle 0|A_\mu^G(x)A_\nu^G(y)|0\rangle = \frac{1}{i}\int \frac{d^2\vec{k}}{(2\pi)^2}\mathrm{Res}(G_{\mu\nu}(k)e^{-ik\cdot(x-y)}, k_0 = |\vec{k}|). \tag{C10}$$

For the massive contributions we have

$$\langle 0|\bar{A}_\mu(x)\bar{A}_\nu(y)|0\rangle = \iint \frac{d^2\vec{k}}{(2\pi)^2}\frac{d^2\vec{q}}{(2\pi)^2}\frac{1}{4\omega_1(k)\omega_1(q)\bar{\beta}^2} \times \varepsilon_\mu^{(-)}\varepsilon_\nu^{(-)*}\langle 0|a_{\vec{k}}^{(-)}a_{\vec{q}}^{(-)\dagger}|0\rangle e^{-i(k\cdot x - q\cdot y)} = \int \frac{d^2\vec{k}}{(2\pi)^2}\frac{1}{2\omega_1(k)\bar{\beta}}\varepsilon_\mu^{(-)}\varepsilon_\nu^{(-)*}e^{-ik\cdot(x-y)}, \tag{C11}$$

together with

$$\langle 0|G_\mu(x)G_\nu(y)|0\rangle = \iint \frac{d^2\vec{k}}{(2\pi)^2}\frac{d^2\vec{q}}{(2\pi)^2}\frac{1}{4W_2(k)W_2(q)\bar{\beta}^2} \times \varepsilon_\mu^{(+)}\varepsilon_\nu^{(+)*}\langle 0|b_{\vec{k}}^{(+)}b_{\vec{q}}^{(+)\dagger}|0\rangle e^{-i(k\cdot x - q\cdot y)} = -\int \frac{d^2\vec{k}}{(2\pi)^2}\frac{1}{2W_2(k)\bar{\beta}}\varepsilon_\mu^{(+)}\varepsilon_\nu^{(+)*}e^{-ik\cdot(x-y)}.$$

We recognize the residue

$$\mathrm{Res}(G_{\mu\nu}(k)e^{-ik\cdot(x-y)}, k_0 = \omega_1) = \frac{i}{(2\omega_1\bar{\beta})} \times [\varepsilon_\mu^{(-)}\varepsilon_\nu^{(-)*}e^{-ik\cdot(x-y)}]_{k_0=\omega_1}, \tag{C12}$$

and

$$\mathrm{Res}(G_{\mu\nu}(k)e^{-ik\cdot(x-y)}, k_0 = W_2) = \frac{(-i)}{(2W_2\bar{\beta})} \times [\varepsilon_\mu^{(+)}\varepsilon_\nu^{(+)*}e^{-ik\cdot(x-y)}]_{k_0=W_2}. \tag{C13}$$

Thus we obtain

$$\langle 0|\bar{A}_\mu(x)\bar{A}_\nu(y)|0\rangle + \langle 0|G_\mu(x)G_\nu(y)|0\rangle = \frac{1}{i}\int \frac{d^2\vec{k}}{(2\pi)^2}[\mathrm{Res}(G_{\mu\nu}(k)e^{-ik\cdot(x-y)}, k_0 = \omega_1) + \mathrm{Res}(G_{\mu\nu}(k)e^{-ik\cdot(x-y)}, k_0 = W_2)]. \tag{C14}$$

The result for $x_0 > y_0$ of the propagator becomes

$$\langle 0|A_\mu(x)A_\nu(y)|0\rangle = \frac{1}{i}\int \frac{d^2\vec{k}}{(2\pi)^2} \times [\mathrm{Res}(G_{\mu\nu}(k)e^{-ik\cdot(x-y)}, k_0 = \omega_0) + \mathrm{Res}(G_{\mu\nu}(k)e^{-ik\cdot(x-y)}, k_0 = \omega_1) + \mathrm{Res}(G_{\mu\nu}(k)e^{-ik\cdot(x-y)}, k_0 = W_2)]. \tag{C15}$$

Finally, we arrive at

$$\langle 0|A_\mu(x)A_\nu(y)|0\rangle = -i\int \frac{d^2\vec{k}}{(2\pi)^2}\int_{\mathcal{C}_F}\frac{dk_0}{(2\pi i)}G_{\mu\nu}(k)e^{-ik\cdot(x-y)},$$

where the contour $\mathcal{C}_F$ is the usual path that has a positive slope and encloses the positive poles $k_0 = |\vec{k}|$, ω_1, and W_2 when $x_0 > y_0$. A similar analysis for $x_0 < y_0$ then leads to the propagator given in Eq. (15).

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