

Electroweak phase transitions in a $U(1)_D$ extension of the standard model with dimension-six operators: Gravitational waves and LHC signatures

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We investigate the possibility of realizing strong first-order electroweak phase transition (SFOEWPT) in an effective field theory framework where the Standard Model is extended with a complex scalar singlet (ϕ) charged under a local $U(1)_D$ gauge group. The tree-level scalar potential contains a dimension-six term of the form $|H|^2|\phi|^4$. We show that this higher-dimensional operator plays a crucial role in the phase transition dynamics by weakening the correlation between the Higgs-singlet portal coupling and the scalar mixing angle that typically constrains singlet-extended models. Consequently, SFOEWPT can be achieved over a significantly extended region of parameter space. The strength of the phase transition is primarily driven by the vacuum expectation value of the singlet scalar, which plays a central role in this analysis. We analyze the phase transition in this model and identify regions of parameter space consistent with SFOEWPT. The resulting phase transition can generate stochastic gravitational-wave signals potentially observable at future interferometers. The extended scalar sector in presence of the dimension-six operator also leads to distinctive multiscalar production signatures at the LHC, intimately correlated with the singlet scalar VEV.

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I. INTRODUCTION

The transition of the Universe from the $SU(2)_L \times U(1)_Y$ gauge symmetric phase to a broken phase characterized by a nonzero vacuum expectation value (VEV) of the Higgs field is referred to as the electroweak phase transition (EWPT). It is a pivotal epoch in the early Universe when the electromagnetic force and the weak force split apart to become the two distinct forces we see today. There is a strong motivation to connect EWPT with baryogenesis, the mechanism of generating the observed matter-antimatter imbalance in the Universe. In particular, a strong first order electroweak phase transition (SFOEWPT) provides a suitable environment in the early Universe, which provides the “departure from equilibrium” condition [1,2], one of the three key criteria laid down by Sakharov [3] required for successful baryogenesis. These criteria suggest that sources of baryon number violation, C and CP violations, are also needed.

Within the standard model (SM), to have SFOEWPT one requires the mass of the Higgs to be below 40 GeV [4]. However, the Higgs-like scalar observed at the LHC with a mass of 125 GeV disfavors the realization of a SFOEWPT within the SM framework. In fact, in the SM the EWPT is a smooth crossover where the Universe always stays in the minima of the Higgs potential. Therefore, one needs to go beyond the SM in order to achieve a SFOEWPT, which is a necessary condition for sustainable electroweak baryogenesis.

The new physics effect can facilitate SFOEWPT by modifying the scalar potential. This can be achieved via introducing new degrees of freedom, adding higher dimensional operators in the scalar potential or both. Numerous studies have explored these possibilities. Scalar extension of the SM is one of the most popular and well-studied frameworks in this context. The real singlet scalar (S) extension of the SM with exact $\mathcal{Z}_2$ symmetry, for example, has been considered in Refs. [5–12] where the scalar field interacts with the SM via portal interaction of the form $|H|^2 S^2$, the so called $\mathcal{Z}_2$ symmetric Higgs-portal models. Such models can accommodate strong first order electroweak phase transition. Similarly, models with explicit $\mathcal{Z}_2$ symmetry breaking with a real or a complex scalar singlet have also been studied in the literature [7–9,13–25]. These scenarios typically contain a larger set of free parameters and predict viable regions of parameter space consistent

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with SFOEWPT. In singlet scalar extended theories with spontaneous $\mathcal{Z}_2$ symmetry breaking [26], a SFOEWPT can be achieved for sizable scalar mixing angle and relatively light additional singlet scalar masses. However, both of these parameters are strongly constrained by the Higgs signal strength measurements and limits on the invisible decay branching ratio of the Higgs boson. This setup also serves as a prototype for complex singlet scalar extension of the SM, charged under the gauge symmetry of the dark sector. For example, in [27] the author considered an Abelian $U(1)_D$ gauge symmetry in the dark sector, which is spontaneously broken once the complex scalar field acquires a nonzero VEV and different phase transition patterns accrued from such a scenario.

Models that extend beyond renormalizable dimension-four operators, i.e., incorporate higher dimensional operators, do also have important implications in the realization of a FOEWPT. In particular, FOEWPT within the Standard Model effective field theory (SMEFT) framework has been extensively explored by various authors [9,15,28–40]. Some of these studies considered operators beyond dimension six as well. For example, the contributions of dimension-eight operators in the context of EWPT in the SMEFT framework and their role in elevating the rather low cutoff scale required for EWPT have been investigated in [35,39]. An effective field theory (EFT) framework for the real singlet extension of the Standard Model has been studied in [41]. Notably, in this scenario, the singlet scalar field does not acquire a VEV. This analysis suggests a moderate improvement in terms of the allowed parameter space consistent with SFOEWPT in the low mass region without any significant changes in the high mass regime compared to the $\mathcal{Z}_2$ symmetric Higgs portal scenario. However, both of these scenarios demand large quartic coupling involving the Higgs and the additional scalar field.

In this work, we investigate the possibility of achieving SFOEWPT in an EFT framework where the particle content of the SM is extended by a complex scalar singlet (ϕ) charged under a local $U(1)_D$ symmetry. The gauge boson associated with this additional symmetry, commonly referred to as the dark photon, which receives mass via a Higgs-like mechanism in the dark sector. The zero temperature tree-level scalar potential of the model contains dimension six terms in addition to the renormalizable dimension four terms permitted by the symmetries of the theory. In particular, we concentrate on the dimension six operator of the form $\frac{1}{\Lambda^2}|H|^2|\phi|^4$ where Λ is the cutoff scale of the theory. The presence of dimension six operator helps to weaken the correlation between the scalar mixing angle ($\sin\theta$) and the scalar portal coupling (λ_{hs}), thus allowing a broader region of parameter space compatible with SFOEWPT compared to the case without the dimension-six contribution [26]. Moreover, SFOEWPT compatible parameter space is obtained for $|\lambda_{hs}| \sim 1$

without threatening the perturbative unitarity. Importantly, the phase transition pattern in this scenario is sensitive to the ratio $\frac{w}{\Lambda}$, where w is the VEV of the additional singlet scalar. Consequently, the effects of the higher-dimensional operator do not necessarily decouple for large Λ as long as the ratio $\frac{w}{\Lambda}$ is held fixed. This is in stark contrast to the SMEFT framework, where the electroweak VEV is fixed at 246 GeV, and from extensions with higher-dimensional operators involving multiple scalar fields, in which the additional scalar does not acquire a VEV [41] and consequently decouple in the large Λ limit. We identify the region of parameter space spanned by the scalar mixing angle, mass, and VEV of the singlet scalar that is not only consistent with theoretical and existing experimental constraints but also predicts SFOEWPT.

Theories predicting a first-order phase transition also have observable consequences in the form of stochastic gravitational waves (GW) generated by bubble nucleation, expansion, and subsequent collisions. With the rapid advancement of gravitational wave detectors, the detection of feeble and highly relic GW backgrounds may soon be within reach of next-generation interferometer experiments such as LISA [42], BBO [43], DECIGO [44], μ Ares [45], ultimate-DECIGO (UD), DECIGO-corr (DC), and ultimate-DECIGO-corr (UDC) [44]. The model presented in this work can also be confronted with future GW observations.

Moreover, this scenario is testable at the high luminosity LHC, through measurements of multiscalar production, in particular discalar (h_1h_1 , h_1h_2 , h_2h_2) and triple-Higgs ($h_1h_1h_1$) production. We show that the singlet scalar VEV plays a key role in this context and the cross section of the above processes are intimately correlated with it. This is unlike in the $\mathcal{Z}_2$ symmetric Higgs-portal framework with or without the effective operator considered in the literature where collider signal is challenging as the additional scalar does not mix with the SM Higgs.

The manuscript is organized as follows: in Sec. II, we introduce the model under consideration and discuss the relevant theoretical and experimental constraints on its parameter space. In Sec. III, we study the electroweak phase transition in this model, identify the region of parameter space that supports SFOEWPT, and examine its dependence on the model parameters. Section IV presents a discussion of the associated bubble nucleation process. Section V is dedicated to the analysis of the GW signals generated by the SFOEWPT. In Sec. VI, we study discalar and triple-Higgs production at the LHC. Finally, we summarize our findings and present our concluding remarks in Sec. VII.

II. THE MODEL

We have considered an effective field theory where the SM gauge group is extended with a local $U(1)_D$ symmetry. The local $U(1)_D$ symmetry is spontaneously broken by the nonzero VEV of an additional complex scalar field ϕ

singlet under SM gauge interactions. As a consequence the corresponding gauge boson, to be referred to as a dark photon, becomes massive. The scalar and the electroweak gauge sector of the low energy effective theory are given by

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}W_{\mu\nu}^i W^{i,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{\varepsilon}{2\cos\theta_W}B_{d,\mu\nu}B^{\mu\nu} - \frac{1}{4}B_{d,\mu\nu}B_d^{\mu\nu}, \quad (1)$$

$$\mathcal{L}_{\text{scalar}} = |D_\mu H|^2 + |D_\mu \phi|^2 - V_0(H, \phi), \quad (2)$$

where $W_{\mu\nu}^i$ with $i = 1, 2, 3$ are the field strength tensors of the $SU(2)_L$ gauge group, $B_{\mu\nu}$ is that of the $U(1)_Y$ and $B_{d,\mu\nu}$ corresponds to the field strength tensor of the additional local $U(1)_d$ group. The strength of the kinetic mixing involving the field strength tensors of two Abelian gauge groups is parametrized by $\frac{\varepsilon}{\cos\theta_W}$, where $\cos\theta_W$ is the cosine of the Weinberg angle.

The tree-level scalar potential introduced above, in presence of the dimension six term, which plays a crucial role in the context of the electroweak phase transition is given by

$$V_0(H, \phi) = -\mu_h^2|H|^2 + \lambda_h|H|^4 - \mu_s^2|\phi|^2 + \lambda_s\phi^4 + \lambda_{hs}|H|^2\phi^2 + \frac{c_6}{\Lambda^2}|H|^2\phi^4, \quad (3)$$

where Λ represents the cutoff scale of the theory. The Higgs doublet H and the complex singlet ϕ can be explicitly written as

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_1 + i\chi_2 \\ \phi_1 + i\chi_3 \end{pmatrix} \quad \phi = \frac{1}{\sqrt{2}}(\phi_2 + i\chi_4), \quad (4)$$

where χ_i 's ($i = 1, 2, 3, 4$) are the would-be Goldstone modes. Since we are working in a scenario where the low energy theory contains massive electroweak gauge bosons and a dark photon, therefore, the zero temperature ground state field configuration should be such that both the scalar fields always acquire nonzero vacuum expectation values. Hence, in this scenario the region of parameter space of interest to us is such that (v, w) with $v > 0$ and $w > 0$ corresponds to the global minima of the tree-level potential, where v and w represent the vacuum expectation values of the SM Higgs and additional scalar fields, respectively. In the unitary gauge the tree-level potential takes the form

$$V_0(\phi_1, \phi_2) = -\frac{1}{2}\mu_h^2\phi_1^2 + \frac{1}{4}\lambda_h\phi_1^4 - \frac{1}{2}\mu_s^2\phi_2^2 + \frac{1}{4}\lambda_s\phi_2^4 + \frac{1}{4}\lambda_{hs}\phi_1^2\phi_2^2 + \frac{1}{8\Lambda^2}\phi_1^2\phi_2^4, \quad (5)$$

assuming $c_6 = 1$. The minimization conditions of the above potential that determine v and w can be written as

$$\left. \frac{1}{v} \frac{\partial V_0(\phi_1, \phi_2)}{\partial \phi_1} \right|_{\phi_1=v, \phi_2=w} = -\mu_h^2 + \lambda_h v^2 + \frac{1}{2}\lambda_{hs}w^2 + \frac{w^4}{4\Lambda^2} = 0, \quad (6)$$

$$\left. \frac{1}{w} \frac{\partial V_0(\phi_1, \phi_2)}{\partial \phi_2} \right|_{\phi_1=v, \phi_2=w} = -\mu_s^2 + \frac{1}{2}\lambda_{hs}v^2 + \lambda_s w^2 + \frac{v^2 w^2}{2\Lambda^2} = 0. \quad (7)$$

In particular, we require $v = v_{\text{EW}} = 246$ GeV at zero temperature. Around this minima (v_{EW}, w) one can expand the field H and ϕ in terms of the fluctuations h and s , respectively, as (in the unitary gauge)

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{\text{EW}} + h \end{pmatrix} \quad \phi = \frac{1}{\sqrt{2}}(w + s). \quad (8)$$

The tree-level scalar mass matrix evaluated at this minima (v_{EW}, w) can be written as [using Eqs. (6) and (7)]

$$M^2 = \begin{pmatrix} 2v_{\text{EW}}^2\lambda_h & v_{\text{EW}}w\left(\frac{w^2}{\Lambda^2} + \lambda_{hs}\right) \\ v_{\text{EW}}w\left(\frac{w^2}{\Lambda^2} + \lambda_{hs}\right) & -v_{\text{EW}}^2\lambda_{hs} + 2\mu_s^2 \end{pmatrix}. \quad (9)$$

The basis in which the mass matrix is diagonal with corresponding eigenvalues $M_{h_1}^2$ and $M_{h_2}^2$, representing the physical masses of Higgs and additional scalar particle, is related to the $h-s$ basis by the following orthogonal transformation.

$$\begin{pmatrix} h \\ s \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \quad (10)$$

where $\sin\theta$ is the mixing angle between the two scalars.

The relevant physical parameters of this sectors are $\{M_{h_1}, M_{h_2}, \sin\theta, v_{\text{EW}}, w, \Lambda\}$. The Lagrangian parameters $\{\mu_h, \mu_s, \lambda_h, \lambda_s, \lambda_{hs}\}$ can be recast in terms of the physical parameters and listed in Appendix A.

A. Constraints

1. Theoretical constraints

- (i) We will now analyze various constraints on the Lagrangian parameters. First, we demand the tree-level scalar potential should be bounded from below at large field values or at least it should display a pattern consistent with it for field values less than the cutoff scale in an effective field theory framework. This is ensured in this model by the following requirement:

$$\lambda_s, \lambda_h > 0 \quad \text{and} \quad c_6 > 0. \quad (11)$$

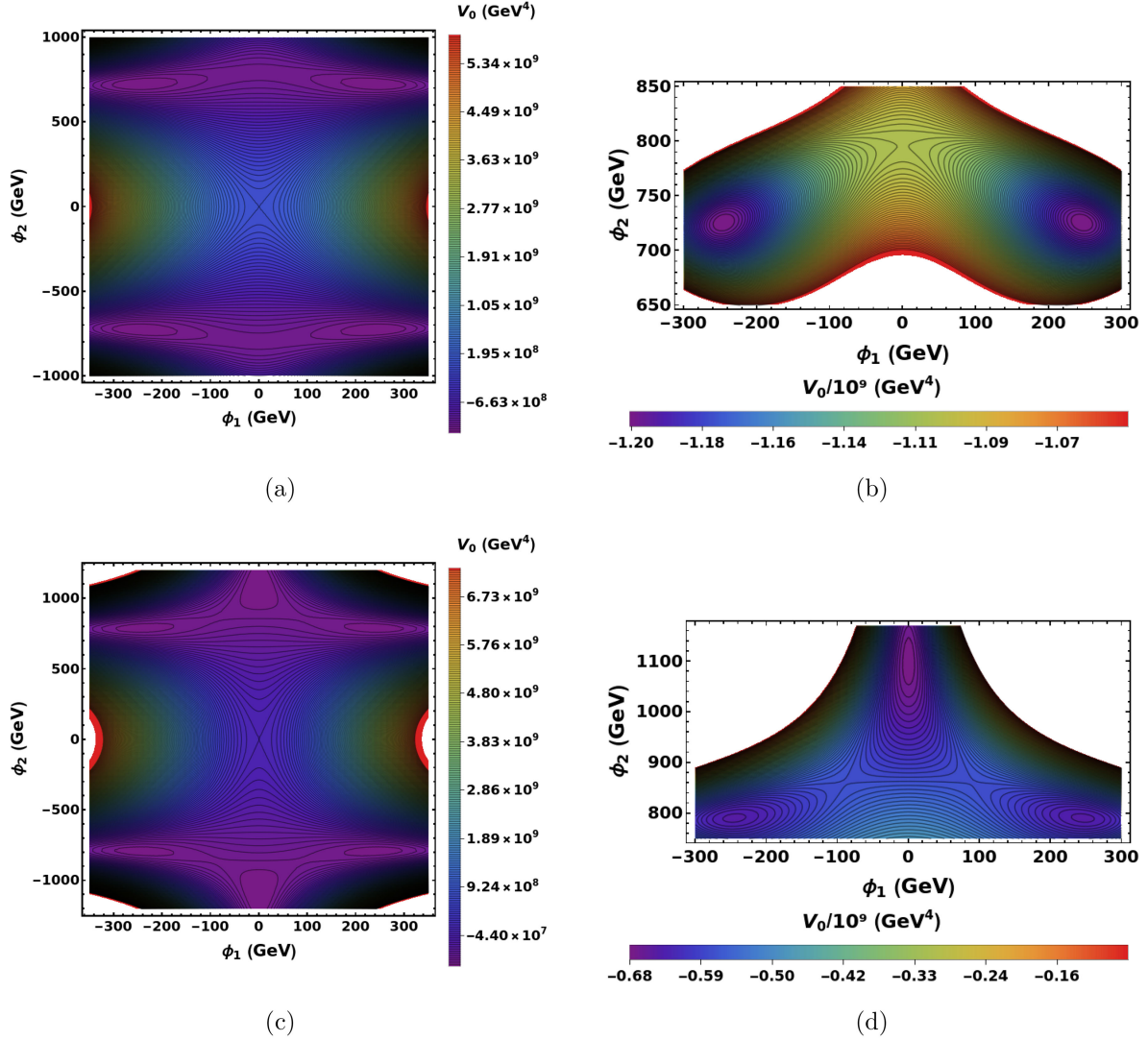


FIG. 1. Vacuum structure of the tree-level potential given in Eq. (5) for two representative sets of Lagrangian parameters. Panels (a) and (b) correspond to $(\mu_h^2 = -89746 \text{ GeV}^2, \mu_s^2 = 7002 \text{ GeV}^2, \lambda_h = 0.1378, \lambda_s = 0.011, \lambda_{hs} = -0.784)$ where $(246 \text{ GeV}, 725.7 \text{ GeV})$ is the global minima with a secondary extrema at $(0, 795.73 \text{ GeV})$. While panels (c) and (d) represent $(\mu_h^2 = -133321 \text{ GeV}^2, \mu_s^2 = 2231 \text{ GeV}^2, \lambda_h = 0.1378, \lambda_s = 0.00184, \lambda_{hs} = -0.942)$ where $(246 \text{ GeV}, 790.89 \text{ GeV})$ is a local minima with the global minima at $(0, 1101.65 \text{ GeV})$. The right panels show a rescaled ϕ_2 axis for better visualization. The value of the potential V_0 is indicated by the color gradient. The cutoff scale is fixed at $\Lambda = 800 \text{ GeV}$.

- (ii) The validity of the EFT approach additionally requires $v, w < \Lambda$ for $c_6 \sim 1$ [46]. For the rest of our discussions we will work with $c_6 = 1$.
- (iii) We require the tree-level potential to have a global minima at (v_{EW}, w) . However, the tree-level potential in principal can have minima at $(0, u = \sqrt{\frac{\mu_s^2}{\lambda_s}})$ or at $(0, 0)$ as well at zero temperature. We illustrate this in Fig. 1, where the upper panel [Figs. 1(a) and 1(b)] display a vacuum structure consisting of a global minima at (v_{EW}, w) , while the lower panel [Figs. 1(c) and 1(d)] display the same but with a global minima at $(0, u)$. To ensure that $(v = v_{\text{EW}}, w)$ is the global minima at zero temperature, the model parameters must satisfy the following conditions in addition to

the boundedness from below condition mentioned above:

$$2v_{\text{EW}}^4 \lambda_h - 2u^4 \lambda_s + 2w^4 \lambda_s + v_{\text{EW}}^2 \left(\frac{w^4}{\Lambda^2} + 2w^2 \lambda_{hs} - 4\mu_h^2 \right) + 4u^2 \mu_s^2 - 4w^2 \mu_s^2 < 0, \quad (12)$$

$$2v^4 \lambda_h + 2w^4 \lambda_s + v^2 \left(\frac{w^4}{\Lambda^2} + 2w^2 \lambda_{hs} - 4\mu_h^2 \right) - 4w^2 \mu_s^2 < 0. \quad (13)$$

The first one ensures $V_0(v_{\text{EW}}, w) < V_0(0, u)$ while the second one guarantees $V_0(v_{\text{EW}}, w) < V_0(0, 0)$.

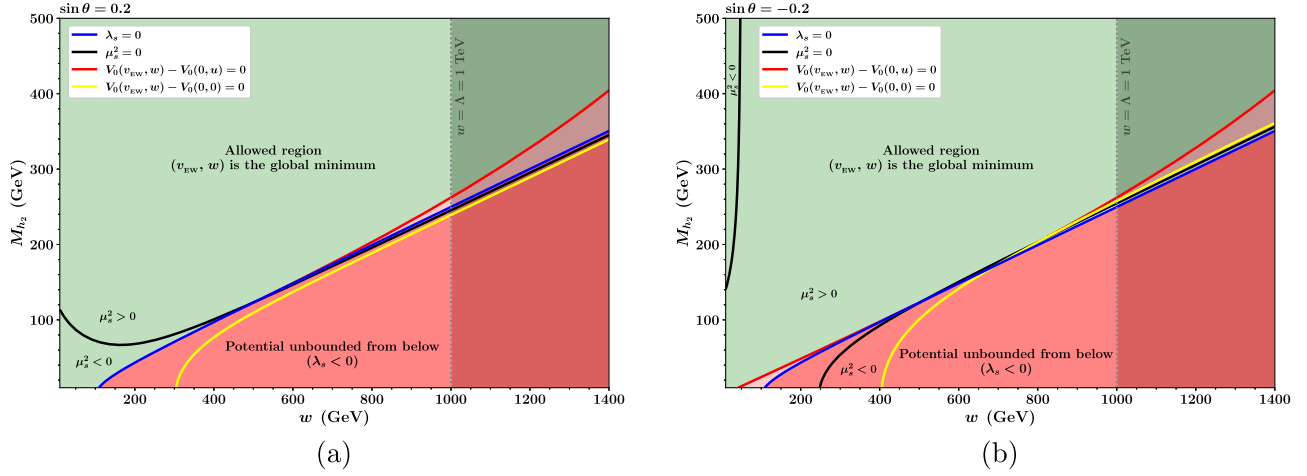


FIG. 2. Allowed parameter space in the M_{h_2} - w plane consistent with various theoretical constraints such as (v_{EW}, w) is the global minima, potential is bounded from below as discussed in Sec. II A 1. In the region shaded with green color (v_{EW}, w) is the global minima. The gray patch on the right is excluded by requirement $w < \Lambda$.

Out of these two conditions, the first one gives more stringent restrictions on the choice of parameter space. In terms of physical parameters $(M_{h_1}, M_{h_2}, v_{EW}, w, \sin \theta, \Lambda)$ Eq. (12) can be rewritten as (see Appendix A)

$$\begin{aligned}
 & V_0(v_{EW}, w) - V_0(0, u) \\
 &= \frac{v_{EW}^2(-M_{h_1}^2 M_{h_2}^2 + \frac{v_{EW}^2 w^2}{\Lambda^2}(M_{h_1}^2 \cos^2 \theta + M_{h_2}^2 \sin^2 \theta))}{8(M_{h_1}^2 \sin^2 \theta + M_{h_2}^2 \cos^2 \theta - \frac{v_{EW}^2 w^2}{\Lambda^2})} \\
 &< 0.
 \end{aligned} \tag{14}$$

For a given M_{h_2} and $\sin \theta$, the upper bound set on the w by the above equation supersedes that previously obtained by the restriction $w < \Lambda$.

In the limit either $M_{h_2} \rightarrow M_{h_1}$ or $\sin \theta \rightarrow 0$ tree-level potential difference $V_0(v_{EW}, w) - V_0(0, u)$ takes the simple form

$$\begin{aligned}
 & \lim_{M_{h_2} \rightarrow M_{h_1}} V_0(v_{EW}, w) - V_0(0, u) \\
 &= \lim_{\sin \theta \rightarrow 0} V_0(v_{EW}, w) - V_0(0, u) \\
 &= -\frac{M_{h_1}^2 v_{EW}^2}{8},
 \end{aligned} \tag{15}$$

which is constant for a given M_{h_1} and v_{EW} , and consistent with Eq. (14).

In Fig. 2, we show the different region of parameter space where one or more of these requirements mentioned above are met. The shaded region marked with red color is ruled out by the condition that the potential is bounded from below, in particular, in this region the quartic coupling λ_s becomes negative. The remaining region can be divided into

two parts. The region shaded with green color corresponds to (v_{EW}, w) is the global minima of the tree-level potential and a small strip between the red and green shaded region corresponding to $(0, u)$ or $(0, 0)$ as the global minima. In this parameter space we further display the boundary corresponding to $\mu_s^2 = 0$ with black solid line. The red and yellow solid lines are obtained when (v_{EW}, w) is degenerate with $(0, u)$ and $(0, 0)$, respectively, at tree level.

2. Experimental constraints

The parameter space of the local $U(1)_D$ extension of the standard model augmented with a scalar sector is constrained by various observations, such as Higgs signal-strength measurements, di-Higgs searches at the LHC. These constraints are more relevant and readily applicable in scenarios where the singlet scalar acquires a nonzero VEV and thus introduces Higgs and singlet scalar mixing. Below we discuss these constraints in the context of the model under consideration.

Higgs signal-strength measurements. The scalar mixing angle $\sin \theta$ is constrained by Higgs signal strength measurement at the LHC and also the invisible decay of the Higgs boson. For $M_{h_2} < \frac{M_{h_1}}{2}$ the measurement of the invisible decay branching ratio of the Higgs at the LHC sets a stringent limit on $\sin \theta$ and the current allowed value of $\sin \theta$ in this range is 0.01 for $w \sim 100$ GeV [47,48]. For M_{h_2} in the range 100–200 GeV Higgs signal strength measurement at the LHC rules out $|\sin \theta| > 0.1$ for $w \sim 0.1 v_{EW}$ [49]. For $M_{h_2} > 200$ GeV scalar mixing angle beyond 0.2 is disfavored by the LHC data [50–52].

Di-Higgs searches. The new physics contribution to the di-Higgs production can enter via either the modification of the trilinear Higgs coupling or due to the presence of new resonance or both. The LHC collaborations have

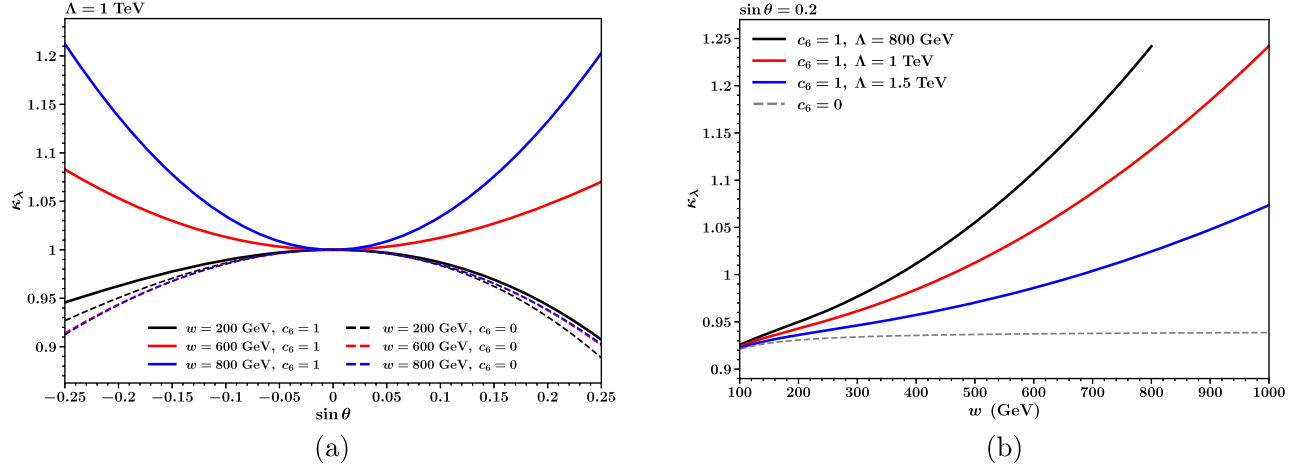


FIG. 3. Variation of κ_λ as a function of (a) $\sin \theta$ for various choices of w and fixed cutoff scale $\Lambda = 1$ TeV, and (b) w for different choices of the cutoff scale Λ and fixed $\sin \theta = 0.2$. The solid and dashed lines correspond to $c_6 = 1$ and $c_6 = 0$, respectively.

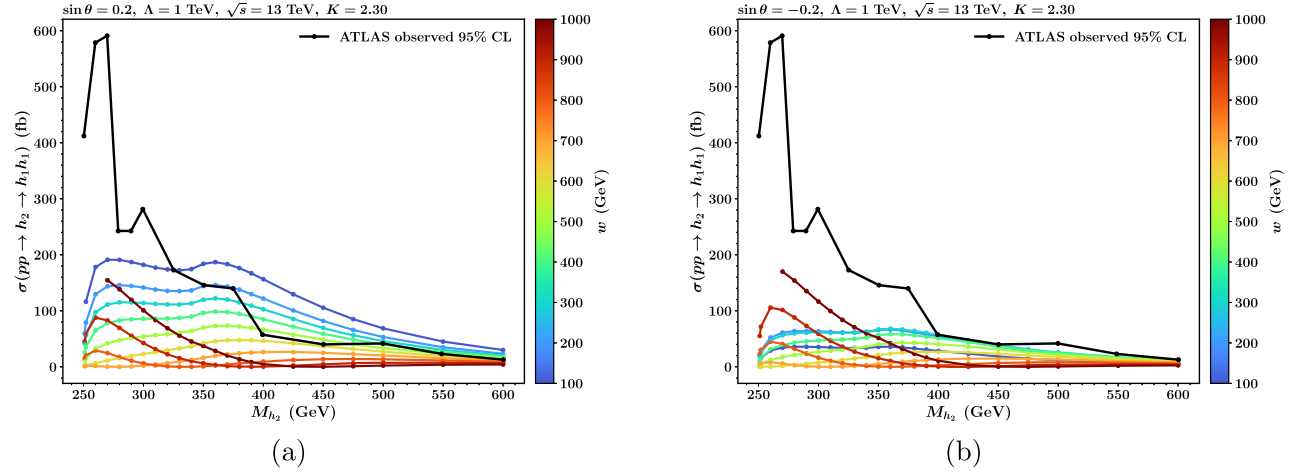


FIG. 4. Resonant di-Higgs production cross section $\sigma(pp \rightarrow h_2 \rightarrow h_1 h_1)$ as a function of M_{h_2} for different values of w (represented in color gradient) assuming $c_6 = 1$. The black curve represents the observed 95% CL upper limit on the resonant di-Higgs production set by ATLAS experiment.

independently explored the first two possibilities. The CMS data [53] corresponding to an integrated luminosity of 138 fb^{-1} and $\sqrt{s} = 13$ TeV sets a 95% confidence level (CL) on κ_λ , where κ_λ is defined as

$$\kappa_\lambda = \frac{\lambda_{h_1 h_1 h_1}}{\lambda_{h h h}^{\text{SM}}} = \left[\cos^3 \theta - \frac{v_{\text{EW}}}{w} \sin^3 \theta + 2 \frac{v_{\text{EW}}^2 w^2}{M_{h_1}^2 \Lambda^2} \cos \theta \sin^2 \theta \right], \quad (16)$$

which measures the deviation of trilinear Higgs coupling (see Appendix B) from its standard model value due to the presence of new physics contribution. The allowed values of κ_λ at 95% CL is $-1.39 < \kappa_\lambda < 7.02$. In Fig. 3(a), we plot κ_λ as a function of $\sin \theta$ for various choices of $w = 200, 600, 800$ GeV and fixed cutoff scale ($\Lambda = 1$ TeV). Figure 3(b)

illustrates the variation of κ_λ as a function of w for various choices of the cutoff scale $\Lambda = 800$ GeV, 1 TeV, 1.5 TeV, and $\sin \theta = 0.2$. Both of these plots suggest that the region of parameter space relevant for the present study is consistent with the above limit on κ_λ . Note that the effect of the dimension-six operator assumed for this analysis enters as a combination of $\frac{w}{\Lambda^2}$, both of which are free parameters of the theory and additionally suppressed by the square of the scalar mixing angle $\sin \theta$.

The ATLAS measurement of resonant di-Higgs production rate [54] at 13 TeV center of mass energy corresponding to an integrated luminosity of 139 fb^{-1} also restrict the allowed parameter space of our model under consideration. The ATLAS collaboration sets an observed upper limit on the di-Higgs rate as a function of the mass of the spin-0 resonance. In Fig. 4, we plot the resonant di-Higgs cross section, $\sigma(pp \rightarrow h_2 \rightarrow h_1 h_1)$ as a function of the scalar

mass M_{h_2} for various w assuming a K factor of 2.3 [55]. The cross section as a function of M_{h_2} in the range 250 to 600 GeV for a fixed w shows two local maxima with a minima in between. The first maxima corresponds to the resonance $M_{h_2} = 2M_{h_1}$ while the second maxima corresponds to on-shell top quark contribution in the loop. It turns out that for $\sin\theta = 0.2$, singlet scalar VEV in the range $600 < w < \Lambda$ is allowed for heavy scalar resonance of mass > 250 GeV. In the lower mass range of the scalar mass 250–370 GeV, $w > 200$ GeV is allowed by the resonance di-Higgs search. This limits are further relaxed for $\sin\theta = -0.2$, almost all values of the dark-Higgs VEV in the range $100 - \Lambda$ are allowed for scalar mass beyond 250 GeV.

III. PHASE TRANSITIONS

In this section, we will discuss the phase transition dynamics in the context of the model described in Sec. II by incorporating one loop finite temperature correction to the tree-level potential given in Eq. (5). Considering the fluctuations h, s around the background fields ϕ_1 and ϕ_2 one can write

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_1 + i\chi_2 \\ \phi_1 + h + i\chi_3 \end{pmatrix} \quad \phi = \frac{1}{\sqrt{2}}(\phi_2 + s + i\chi_4). \quad (17)$$

The field dependent mass-squared matrix of the scalars at zero temperature can be written as

$$m^2(\phi_1, \phi_2) = \begin{pmatrix} -\mu_h^2 + \frac{1}{2}\lambda_{hs}\phi_2^2 + 3\lambda_h\phi_1^2 + \frac{1}{4\Lambda^2}\phi_2^4 & \lambda_{hs}\phi_1\phi_2 + \frac{1}{\Lambda^2}\phi_1\phi_2^3 \\ \lambda_{hs}\phi_1\phi_2 + \frac{1}{\Lambda^2}\phi_1\phi_2^3 & -\mu_s^2 + \frac{1}{2}\lambda_{hs}\phi_1^2 + 3\lambda_s\phi_2^2 + \frac{3}{2\Lambda^2}\phi_1^2\phi_2^2 \end{pmatrix}. \quad (18)$$

Hence, the eigenvalues of the above matrix are,

$$\begin{aligned} m_{h_1, h_2}^2(\phi_1, \phi_2) = & \frac{1}{8} \left\{ -4\mu_h^2 - 4\mu_s^2 + 2(6\lambda_h + \lambda_{hs})\phi_1^2 + 2\left(\lambda_{hs} + 6\lambda_s + \frac{3}{\Lambda^2}\phi_1^2\right)\phi_2^2 + \frac{1}{\Lambda^2}\phi_2^4 \right. \\ & \pm \left[4(2\mu_h^2 - 2\mu_s^2 + (-6\lambda_h + \lambda_{hs})\phi_1^2)^2 + 8\left(7\lambda_{hs}^2\phi_1^2 - 6\left(2\lambda_s + \frac{1}{\Lambda^2}\phi_1^2\right)(-\mu_h^2 + \mu_s^2 + 3\lambda_h\phi_1^2) \right. \right. \\ & \left. \left. + \lambda_{hs}\left(-2\mu_h^2 + 2\mu_s^2 + 6(\lambda_h + \lambda_s)\phi_1^2 + \frac{3}{\Lambda^2}\phi_1^4\right)\right)\phi_2^2 \right. \\ & \left. + 4\left((\lambda_{hs} - 6\lambda_s)^2 + \frac{1}{\Lambda^2}(25\lambda_{hs} + 36\lambda_s)\phi_1^2 + \frac{9}{\Lambda^4}\phi_1^4 + \frac{2}{\Lambda^2}(-\mu_h^2 + \mu_s^2 + 3\lambda_h\phi_1^2)\right)\phi_2^4 \right. \\ & \left. + \frac{4}{\Lambda^2}\left(\lambda_{hs} - 6\lambda_s + \frac{13}{\Lambda^2}\phi_1^2\right)\phi_2^6 + \frac{1}{\Lambda^4}\phi_2^8 \right]^{1/2} \Big\}, \end{aligned} \quad (19)$$

which now receive contributions from the dimension six term.

The background field dependent masses for the other relevant fields, e.g., $W^\pm, Z, \gamma_d, t$ -quark and goldstone bosons (χ_i 's) are given by,

$$\begin{aligned} m_W^2(\phi_1, \phi_2) &= \frac{g_1^2}{4}\phi_1^2, & m_Z^2(\phi_1, \phi_2) &= \frac{g_1^2 + g_2^2}{4}\phi_1^2, & m_{\gamma_d}^2 &= g_d^2\phi_2^2, \\ m_{\chi_1}^2 &= m_{\chi_2}^2 = m_{\chi_3}^2 &= -\mu_h^2 + \lambda_h\phi_1^2 + \frac{1}{2}\lambda_{hs}\phi_2^2 + \frac{1}{4\Lambda^2}\phi_2^4, \\ m_{\chi_4}^2 &= -\mu_s^2 + \frac{1}{2}\lambda_{hs}\phi_1^2 + \lambda_s\phi_2^2 + \frac{1}{2\Lambda^2}\phi_1^2\phi_2^2, & m_t^2(\phi_1, \phi_2) &= \frac{y_t^2}{2}\phi_1^2, \end{aligned} \quad (20)$$

where g_1, g_2 and g_d are the gauge coupling constants of the gauge groups $SU(2)_L, U(1)_Y$, and $U(1)_D$, respectively. The degrees of freedom (n_i 's) associated with each of these species are

$$n_{h_1} = n_{h_2} = n_{\chi_{1,2,3,4}} = 1, \quad n_W = 6, \quad n_Z = n_{\gamma_d} = 3, \quad n_t = 12. \quad (21)$$

We now introduce the Coleman-Weinberg (CW) correction¹ in the $\overline{\text{MS}}$ renormalization scheme [4],

¹The UV divergent operators generated at one loop, $V_{\text{div}} = (-1)^N \sum_i -n_i \frac{m_i^4(\phi_1, \phi_2)}{64\pi^2} \left[\frac{1}{2-\epsilon} - \gamma_E + \log 4\pi \right]$ have been absorbed by appropriate counter terms in $\overline{\text{MS}}$ renormalization scheme to arrive at the form of the Coleman-Weinberg potential given in Eq. (22). Here, $N = 0(1)$ for bosons (fermions).

$$V_{\text{CW}}(\phi_1, \phi_2) = \frac{1}{64\pi^2} \left\{ \sum_B n_B m_B^4(\phi_1, \phi_2) \left[\log \left(\frac{m_B^2(\phi_1, \phi_2)}{Q^2} \right) - c_B \right] - \sum_F n_F m_F^4(\phi_1, \phi_2) \left[\log \left(\frac{m_F^2(\phi_1, \phi_2)}{Q^2} \right) - \frac{3}{2} \right] \right\}, \quad (22)$$

where the sum in the first term runs over all the bosons ($B = h_1, h_2, \chi_i$'s, $W^\pm, Z, \gamma_d$) and the second one runs over all the fermions of the theory that couple to the Higgs field. We have considered only the top quark contribution in the second term among all the SM fermions. Additionally, Q is the renormalization scale which we will set to 800 GeV without any loss of generality throughout our analysis.² The value of the parameter c_B in the equation is $3/2$ ($5/6$) for scalar (vector) bosons, respectively.

In addition, in order to ensure that the vacuum expectation values of the scalar fields and mass matrix at minima in presence of the CW term remain the same as that predicted at the tree level, we add additional counter terms or finite pieces to the above scalar potential and use the following set of renormalization conditions.

$$V_1 = V_{\text{CW}} + V_{\text{ct}}, \quad \left. \frac{\partial V_1}{\partial \phi_1} \right|_{\phi_1=v_{\text{EW}}, \phi_2=w} = \left. \frac{\partial V_1}{\partial \phi_2} \right|_{\phi_1=v_{\text{EW}}, \phi_2=w} = \left. \frac{\partial^2 V_1}{\partial \phi_1^2} \right|_{\phi_1=v_{\text{EW}}, \phi_2=w} = \left. \frac{\partial^2 V_1}{\partial \phi_2^2} \right|_{\phi_1=v_{\text{EW}}, \phi_2=w} = \left. \frac{\partial^2 V_1}{\partial \phi_1 \partial \phi_2} \right|_{\phi_1=v_{\text{EW}}, \phi_2=w} = \left. \frac{\partial V_1}{\partial \phi_2} \right|_{\phi_1=0, \phi_2=u} = 0 \quad (23)$$

where V_{ct} is given by

$$V_{\text{ct}}(\phi_1, \phi_2) = \frac{1}{2} a \phi_1^2 + \frac{1}{4} b \phi_1^4 + \frac{1}{2} c \phi_2^2 + \frac{1}{4} d \phi_1^2 \phi_2^2 + \frac{1}{4} e \phi_2^4 + \frac{1}{8} f \phi_1^2 \phi_2^4. \quad (24)$$

Here (v_{EW}, w) is the electroweak vacuum configuration corresponding to the tree-level scalar potential given in Eq. (5) and $(0, u)$ is the stationary field configuration in the $\phi_1 = 0$ direction as discussed earlier. The six unknown coefficients (a, b, c, d, e, f) are determined by solving the following six algebraic equations.

$$\begin{aligned} \left. \frac{\partial V_{\text{CW}}}{\partial \phi_1} \right|_{\phi_1=v_{\text{EW}}, \phi_2=w} + a v_{\text{EW}} + b v_{\text{EW}}^3 + \frac{d}{2} v_{\text{EW}} w^2 + \frac{f}{4} v_{\text{EW}} w^4 &= 0, & \left. \frac{\partial^2 V_{\text{CW}}}{\partial \phi_1^2} \right|_{\phi_1=v_{\text{EW}}, \phi_2=w} + a + 3b v_{\text{EW}}^2 + \frac{d}{2} w^2 + \frac{f}{4} w^4 &= 0, \\ \left. \frac{\partial V_{\text{CW}}}{\partial \phi_2} \right|_{\phi_1=v_{\text{EW}}, \phi_2=w} + c w + e w^3 + \frac{d}{2} v_{\text{EW}}^2 w + \frac{f}{2} v_{\text{EW}}^2 w^3 &= 0, & \left. \frac{\partial^2 V_{\text{CW}}}{\partial \phi_2^2} \right|_{\phi_1=v_{\text{EW}}, \phi_2=w} + c + 3e w^2 + \frac{d}{2} v_{\text{EW}}^2 + \frac{3f}{2} v_{\text{EW}}^2 w^2 &= 0, \\ \left. \frac{\partial^2 V_{\text{CW}}}{\partial \phi_1 \partial \phi_2} \right|_{\phi_1=v_{\text{EW}}, \phi_2=w} + d v_{\text{EW}} w + f v_{\text{EW}} w^3 &= 0, & \left. \frac{\partial V_{\text{CW}}}{\partial \phi_2} \right|_{\phi_1=0, \phi_2=u} + c u + e u^3 &= 0. \end{aligned} \quad (25)$$

When $(0, u)$ is not a stationary point, which is the case in a very small region of parameter space (see Fig. 2), we set $f = 0$ and just solve five equations for five unknowns. It is also important to note that the above choices of renormalization conditions are not unique. Alternative choices of renormalization conditions can have a mild, if not significant, effect on the parameter space consistent with either the strong EWPT or the requirement that (v_{EW}, w) is the electroweak vacuum at zero temperature [12].

We would like to emphasize that even though the tree-level potential in our analysis is truncated at $\frac{1}{\Lambda^2}$ in the EFT expansion (with only one particular dimension-six term,

$\frac{c_6}{\Lambda^2} |H|^2 |\phi|^4$ at this order), the CW potential includes a partial resummation of all the higher order effects consistently renormalized at one loop in the presence of the dimension-six operator.

Finally the finite temperature correction to the effective potential at one loop is given by [4],

$$V_1^T(\phi_1, \phi_2, T) = \frac{T^4}{2\pi^2} \left[\sum_B n_B J_B \left(\frac{m_B^2(\phi_1, \phi_2)}{T^2} \right) + n_F J_F \left(\frac{m_F^2(\phi_1, \phi_2)}{T^2} \right) \right] \quad (26)$$

²Dependence of various phase transition parameters on the choice renormalization scale has been presented in Appendix C.

where the thermal bosonic and fermionic function J_B and J_F are defined as,

$$J_B(\theta) = \int_0^\infty y^2 \log(1 - e^{-\sqrt{y^2 + \theta}}) dy$$

$$J_F(\theta) = \int_0^\infty -y^2 \log(1 + e^{-\sqrt{y^2 + \theta}}) dy. \quad (27)$$

We have also taken care of the daisy resummation by redefining the field dependent masses of the scalars and

longitudinal polarization of the gauge bosons as follows,

$$m_i^2(\phi_1, \phi_2, T) = m_i^2(\phi_1, \phi_2) + \Pi_i(T) \quad (28)$$

where i belongs to all the relevant bosonic degrees of freedom only. The one-loop thermal self energy contributions ($\Pi_i(T)$) are given by [27,56,57],

$$\Pi_{hh} = \Pi_{\chi_{1,2,3}} = \left(\frac{3g_1^2}{16} + \frac{g_2^2}{16} + \frac{y_t^2}{4} + \frac{\lambda_h}{2} + \frac{\lambda_{hs}}{12} + \frac{1}{6\Lambda^2} w^2 \right) T^2, \quad \Pi_{ss} = \Pi_{\chi_4} = \left(\frac{g_d^2}{4} + \frac{\lambda_{hs}}{6} + \frac{\lambda_s}{3} + \frac{1}{6\Lambda^2} v^2 + \frac{1}{2\Lambda^2} w^2 \right) T^2,$$

$$\Pi_{hs} \approx 0, \quad \Pi_{A_{1L}} = \Pi_{A_{2L}} = \Pi_{A_{3L}} = \frac{11}{6} g_1^2 T^2, \quad \Pi_{B_L} = \frac{11}{6} g_2^2 T^2, \quad \Pi_{B_{dL}} = \frac{1}{3} g_d^2 T^2. \quad (29)$$

The field dependent squared masses of the scalars at finite temperature are the eigenvalues of the following thermally corrected Hessian matrix,

$$m^2(\phi_1, \phi_2) + \begin{pmatrix} \Pi_{hh} & 0 \\ 0 & \Pi_{ss} \end{pmatrix}. \quad (30)$$

The mass matrix involving the longitudinal components of the gauge bosons at finite temperature in the (A_1, A_2, A_3, B, B_d) basis is given by

$$\begin{pmatrix} \frac{1}{4} g_1^2 \phi_1^2 + \Pi_{A_{1L}} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} g_1^2 \phi_1^2 + \Pi_{A_{2L}} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} g_1^2 \phi_1^2 + \Pi_{A_{3L}} & -\frac{1}{4} g_1 g_2 \phi_1^2 & 0 \\ 0 & 0 & -\frac{1}{4} g_1 g_2 \phi_1^2 & \frac{1}{4} g_2^2 \phi_1^2 + \Pi_{B_L} & 0 \\ 0 & 0 & 0 & 0 & g_d^2 \phi_2^2 + \Pi_{B_{dL}} \end{pmatrix} \quad (31)$$

where we have neglected the kinetic mixing. It turns out that the photon develops a nonvanishing longitudinal component at finite temperature.

The complete one loop effective potential at finite temperature is given by,

$$V_{\text{eff}}(\phi_1, \phi_2, T) = V_0(\phi_1, \phi_2) + V_{\text{CW}}(\phi_1, \phi_2) + V_{\text{ct}}(\phi_1, \phi_2) + V_1^T(\phi_1, \phi_2, T). \quad (32)$$

We are interested in studying the phase transition pattern and its strength for the above effective potential keeping in mind its implications for electroweak baryogenesis. As mentioned earlier, for successful electroweak baryogenesis in this scenario required baryon number violation is induced by sphaleron transition processes like in the SM, while a SFOEWPT ensures departure from thermal equilibrium. During the electroweak phase transition, bubbles of the broken electroweak phase nucleate and expand within the surrounding electroweak-symmetric vacuum. The critical temperature (T_c) is defined as the temperature at which the electroweak symmetry preserving vacuum becomes degenerate with that corresponding to the broken phase.

Any net baryon asymmetry generated at the critical temperature is efficiently washed out by sphaleron transitions in the symmetric phase. In contrast, it is preserved

in the broken phase, where the sphaleron transition rate is exponentially suppressed as $\propto \exp(-\frac{E_{\text{sph}}(T)}{T})$ [4]. In particular, one requires $\frac{E_{\text{sph}}(T_c)}{T_c} \gtrsim 45$ in order to avoid washout of the generated baryon asymmetry in the broken phase. This condition is approximately satisfied if $\frac{\phi_c}{T_c} \gtrsim 1$, where ϕ_c denotes the discontinuity in the order parameter ϕ_1 at the critical temperature, i.e., $\phi_c = \Delta\phi_1(T_c)$. However, the precise lower bound on $\frac{\phi_c}{T_c}$ is subject to theoretical uncertainties and may vary in the range 0.6–1.4 [58,59]. In this analysis, we adopt the following conservative criterion for a strong first-order phase transition,

$$\frac{\phi_c}{T_c} \geq 0.8.$$

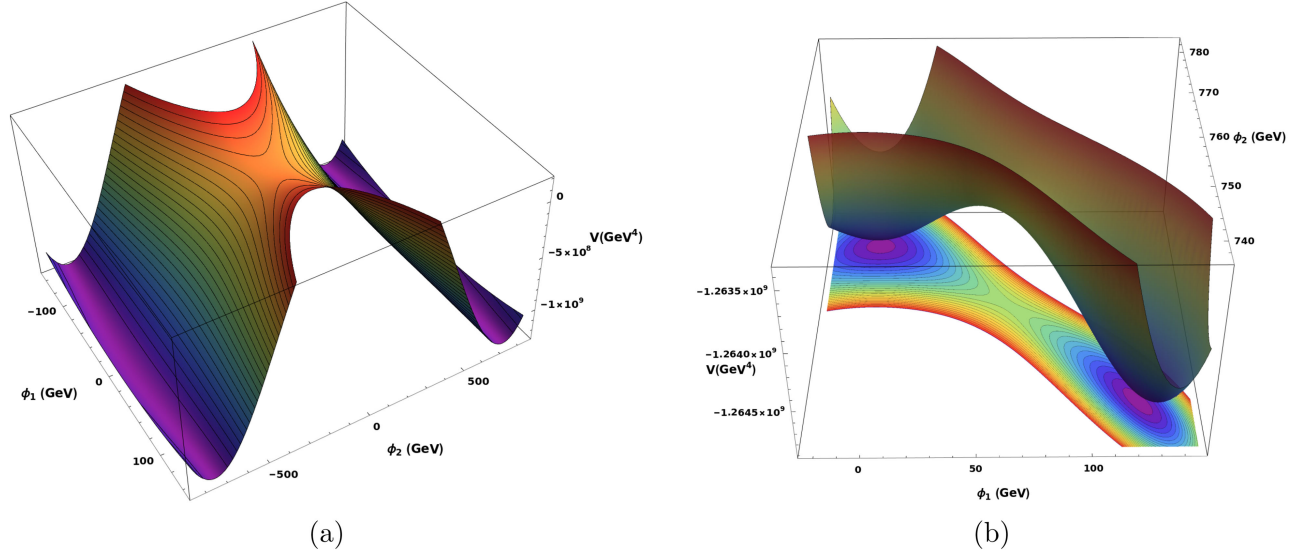


FIG. 5. 3D plot of the one loop finite temperature corrected effective potential at high temperature approximation [see Eq. (33)] in the $\phi_1 - \phi_2$ plane at $T_c = 106.2$ GeV. This plot is obtained neglecting the CW and daisy correction. Here the color gradient is representing the value of the potential. The figure in the right (b) represents the appropriate scaled version of the figure on the left (a).

A. High temperature approximation

In this subsection, we try to understand the phase transition pattern of the effective potential at high temperature approximation ($T > m$) neglecting the CW and daisy corrections. In this approximation, with leading order temperature dependent terms, the effective potential has the following form³

$$V_{\text{eff}}(\phi_1, \phi_2, T) \approx -\frac{1}{2}(\mu_h^2 - c_h T^2)\phi_1^2 + \frac{\lambda_h}{4}\phi_1^4 - \frac{1}{2}(\mu_s^2 - c_s T^2)\phi_2^2 + \left(\frac{\lambda_s}{4} + \frac{1}{24\Lambda^2}T^2\right)\phi_2^4 + \left(\frac{\lambda_{hs}}{4} + \frac{1}{12\Lambda^2}T^2\right)\phi_1^2\phi_2^2 + \frac{1}{8\Lambda^2}\phi_1^2\phi_2^4 \quad (33)$$

where,

$$c_h = \frac{1}{48}(9g_1^2 + 3g_2^2 + 2(6y_t^2 + 12\lambda_h + 2\lambda_{hs})), \quad c_s = \frac{1}{12}(2\lambda_{hs} + 4\lambda_s + 3g_d^2). \quad (34)$$

In order to illustrate that the above potential gives rise to a barrier that facilitates a FOEWPT (even in the absence of $T\phi_i^3$ term in the effective potential), we choose a representative benchmark point (BP) $M_{h_2} = 250$ GeV, $\sin \theta = 0.15$, $w = 725.7$ GeV, $M_{\gamma_d} = 60$ GeV, and $\Lambda = 800$ GeV. This corresponds to a critical temperature $T_c = 106.2$ GeV and the effective potential displays degenerate ground state configurations $(0, u(T_c))$ and $(v(T_c), w(T_c))$ (see Fig. 5).

In terms of numerical values at this benchmark point, the effective potential can be written as

$$V_{\text{eff}}(\phi_1, \phi_2, T \sim 106 \text{ GeV}) \approx \frac{\tilde{\mu}_h^2}{2}\phi_1^2 + \frac{\tilde{\lambda}_h}{4}\phi_1^4 + \frac{\tilde{\mu}_s^2}{2}\phi_2^2 + \frac{\tilde{\lambda}_{hs}}{4}\phi_1^2\phi_2^2 + \frac{\tilde{\lambda}_s}{4}\phi_2^4 + \frac{1}{8\Lambda^2}\phi_1^2\phi_2^4, \quad (35)$$

$$\text{with } \tilde{\mu}_h^2 = 9.36 \times 10^4, \quad \tilde{\lambda}_h = 1.38 \times 10^{-1}, \quad \tilde{\mu}_s^2 = -8.4 \times 10^3, \quad \tilde{\lambda}_{hs} = -7.8 \times 10^{-1}, \quad \tilde{\lambda}_s = 1.4 \times 10^{-2}, \quad (36)$$

and the values of the coefficients c_h and c_s are

$$c_h = 0.338, \quad c_s = -0.125. \quad (37)$$

³Strictly speaking, this approximation is not justified over the entire space of the background field values. Here, we use the high temperature approximation for the illustrative purpose only. The results presented in the following sections are based on exact numerical simulation and do not rely on this approximation.

Near the origin (0, 0) the leading terms are quadratic in ϕ_1 and ϕ_2 . Since the overall coefficient of ϕ_1^2 and ϕ_2^2 is positive and negative, respectively, at $T = T_c$ this corresponds to a saddle point of the scalar potential.

Along the $\phi_1 = 0$ direction the coefficient of ϕ_2^2 is negative and ϕ_2^4 is positive at critical temperature. Therefore one expects an extrema at $(0, u(T_c))$. In particular, this should correspond to a minima along the $\phi_1 = 0$ direction. The behavior of the potential around this point in the orthogonal direction is determined by the value of $u(T_c)$. Because $u(T_c)$ determines the overall coefficient of ϕ_1^2 around this point, which in this case turns out to be positive, it suggests that $(0, u(T_c))$ is actually a local minima. The potential along $\phi_2 = 0$ direction always increases as long as μ_h^2 is negative. Therefore, one does not encounter any further local extrema along the $\phi_2 = 0$ direction in this approximation.

Finally, the presence of the $\phi_1^2\phi_2^2$ term with a negative coefficient gives rise to the possibility of additional local minima at $(v(T_c), w(T_c))$. This is because for certain field values of ϕ_2 at critical temperature, i.e., $w(T_c)$, the overall coefficient of the ϕ_1^2 term becomes negative. The particular value of $w(T_c)$ for which this happens depends on the relative magnitude of the coefficients of ϕ_1^2 , $\phi_1^2\phi_2^2$, and $\phi_1^2\phi_2^4$. The overall sign of the coefficient of the $\phi_1^2\phi_2^2$ term is determined by the value of λ_{hs} as $\frac{T_c^2}{\Lambda^2}$ is a small number [see Eq. (33)]. In fact, it is crucial to have λ_{hs} negative in order to get a local minima at $(v(T_c), w(T_c))$ for a wide range of parameter space in this scenario.⁴ As we will see later $\lambda_{hs} \sim -1$ will facilitate a SFOEWPT.

As discussed in [26] the large λ_{hs} helps to enhance the thermal barrier via the ϕ_1^3 term in the singlet scalar extension of the SM with spontaneous $\mathcal{Z}_2$ breaking or in theories with spontaneous $U(1)_D$ breaking [27]. In both of these cases λ_{hs} is a function of the independent parameters, namely, the scalar mixing angle ($\sin\theta$), the VEV of the singlet scalar (w) along with the masses of the scalars. In fact, the magnitude as well as sign of λ_{hs} are strongly correlated with $\sin\theta$ in these models. For higher w one needs higher $\sin\theta$ value to achieve the same magnitude of λ_{hs} in usual singlet scalar extension models. However, in presence of the dimension six operator as considered in this work, this correlation is weakened. The same magnitude of λ_{hs} can be achieved even for smaller $\sin\theta$ assuming large w (alternatively for lower value of the cutoff scale Λ). This is due to the fact that λ_{hs} receives additional contribution ($\sim -\frac{w^2}{\Lambda^2}$) coming from the dimension-six term. This gives a greater flexibility in terms of the allowed region of parameter space consistent with SFOEWPT.

⁴Actually the relative sign and the magnitude among the coefficients of ϕ_1^2 , $\phi_1^2\phi_2^2$ and $\phi_1^2\phi_2^4$ play a crucial role to have a local minima at $(v(T_c), w(T_c))$.

The sign and magnitude of λ_{hs} also has important implications for symmetry restoration at high temperature. This dependency enters via c_h and c_s , the sign of which crucially controls the symmetry restoration at high temperature. Since c_h is always positive even for $\lambda_{hs} \sim -1$ the overall coefficient of the ϕ_1^2 term in Eq. (33) at high temperature becomes positive, which implies at high temperature electroweak symmetry is restored. On the other hand, the high temperature restoration of the $U(1)_D$ symmetry depends on the interplay between λ_{hs} and dark gauge coupling (g_d). The former one is mostly negative whenever the requirement of SFOEWPT is met. It turns out that c_s is negative due to $\lambda_s, g_d < |\lambda_{hs}|$. In this context, it is important to note the role of the dark-photon mass in the restoration of the $U(1)_D$ symmetry at high temperature. For low mass dark photons the value of g_d is small. However, as dark photon mass increases, the dark gauge coupling g_d also increase for a fixed w . Hence, c_s can in principle become positive, which in effect helps to restore the $U(1)_D$ symmetry at high temperature.

B. Exact numerical results

We have implemented the full one-loop finite temperature corrected effective potential, including CW and daisy corrections described by Eq. (32) in CosmoTransitions [60]. For the finite temperature correction, full numerical integration has been implemented using spline interpolation. The infrared divergences for the zero Matsubara modes of the bosonic degrees of freedom have been taken care of by leading order resummation and redefining the masses including the thermal correction (Parwani method [57]) as discussed in this section. To obtain the viable parameter space consistent with the requirement of SFOEWPT and its dependence on various parameters, we have performed a dedicated parameter space scan using CosmoTransitions.

In Fig. 6, we illustrate the evolution of the VEVs of the doublet and the singlet scalar fields with temperature for the previously mentioned benchmark point ($M_{h_2} = 250$ GeV, $\sin\theta = 0.15$, $w = 725.7$ GeV, $M_{\gamma_d} = 60$ GeV, $\Lambda = 800$ GeV). It shows that the ϕ_1 VEV ($\langle\phi_1\rangle$) has a discontinuity of $\Delta\phi_1 = 147.7$ GeV at the critical temperature $T = T_c = 133.6$ GeV leading to a FOEWPT of strength $\frac{\Delta\phi_1}{T_c} = \frac{\phi_c}{T_c} = 1.1$.

As discussed previously for low dark photon mass $M_{\gamma_d} = 60$ GeV, the value of c_s in Eq. (33) was negative resulting in nonrestoration of $U(1)_D$ symmetry even at high temperature as can be seen from Fig. 6(b). However, for high dark photon mass the ϕ_2 VEV at high temperature can in principle become zero, eventually restoring the $U(1)_D$ symmetry. To illustrate this, we plot the evolution of ϕ_1 and ϕ_2 VEVs with temperature for the same set of parameters $M_{h_2}, w, \sin\theta$ but now we take $M_{\gamma_d} = 400$ GeV, in Fig. 7. As a result of heavy dark photon mass the $U(1)_D$ symmetry is restored at high temperature [see Fig. 7(b)]. Moreover,

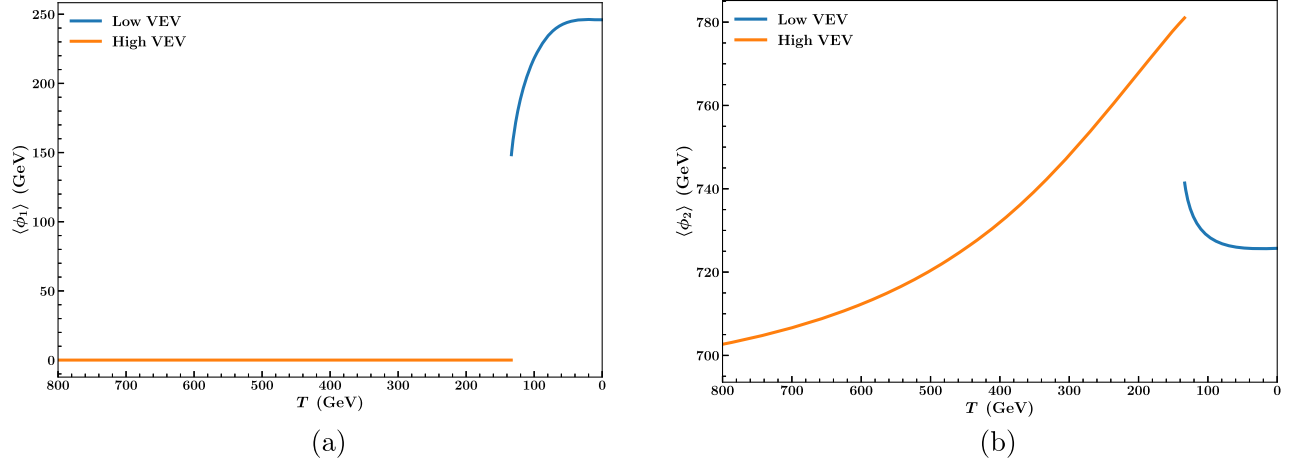


FIG. 6. Thermal evolution of (a) ϕ_1 and (b) ϕ_2 VEVs for $M_{h_2} = 250$ GeV, $\sin \theta = 0.15$, $w = 725.7$ GeV, $M_{\gamma_d} = 60$ GeV, and $\Lambda = 800$ GeV. The phase transition proceeds from $(0, 781 \text{ GeV}) \rightarrow (147.7 \text{ GeV}, 741.6 \text{ GeV})$ at $T = 133.6$ GeV.

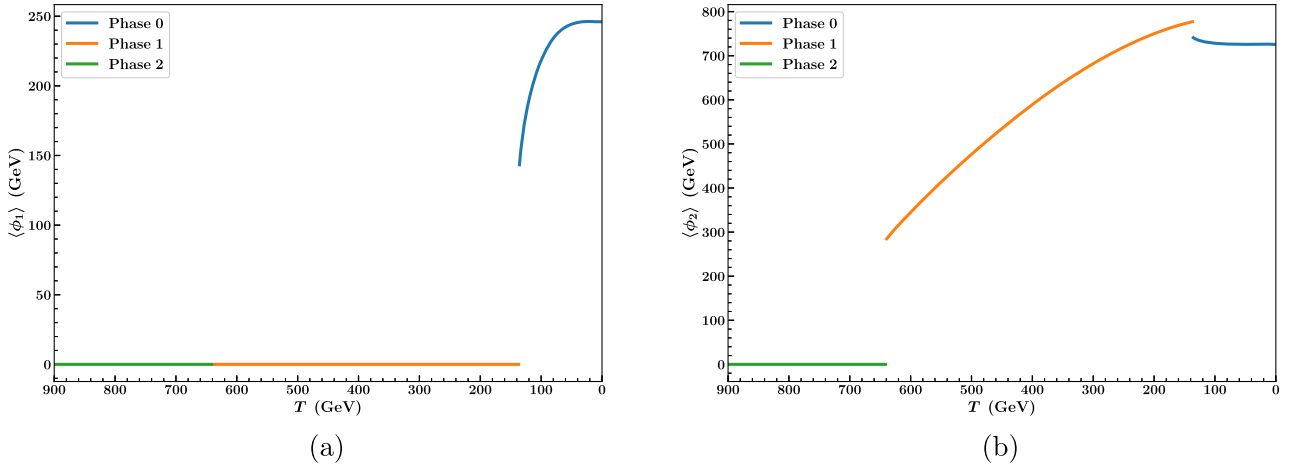


FIG. 7. Thermal evolution of (a) ϕ_1 and (b) ϕ_2 VEVs for $M_{h_2} = 250$ GeV, $\sin \theta = 0.15$, $w = 725.7$ GeV, $M_{\gamma_d} = 400$ GeV, and $\Lambda = 800$ GeV. The phase transition proceeds from $(0, 0) \rightarrow (0, 284.7 \text{ GeV})$ at $T = 639.4$ GeV, then from $(0, 777.4 \text{ GeV}) \rightarrow (142.3 \text{ GeV}, 741.1 \text{ GeV})$ at $T = 135.76$ GeV.

the resulting phase transition is strongly first order with a PT pattern $(0, 0) \rightarrow (0, u(T)) \rightarrow (v(T), w(T))$. Such phase transitions might be useful in the context of dark matter relic abundance calculation as claimed in [61,62] and GW signal. However, the strength of the EWPT remains almost unaffected or weakened for higher dark photon mass. Therefore, for the rest of our analysis we work with a low mass dark photon, e.g., $M_{\gamma_d} = 60$ GeV as a representative choice, consistent with existing low energy observations.

In Fig. 8, we display the region of parameter space in the $M_{h_2} - w$ plane consistent with the requirement of first order EWPT for various choices of scalar mixing angle and the cutoff scale. The value of $\frac{\phi_c}{T_c}$ has been shown in color gradient. The colored region that gives first order EWPT is bounded from the left by the requirement of SFOEWPT $\frac{\phi_c}{T_c} \geq 0.8$ and on the right by the requirement

that (v_{EW}, w) is the global minima of the tree-level potential [Eqs. (12) and (13)] and the potential is bounded from below [Eq. (11)]. The region on the right of the vertical black line that corresponds to $w > \Lambda$ is excluded by the requirement of validity of the EFT approach.

One can see that, for a given value of M_{h_2} , the ratio ϕ_c/T_c increases with increase in w within its allowed range. This behavior can be traced back to the fact that, as w increases, the zero-temperature tree-level potential difference $(\Delta V = V_0(0, u) - V_0(v_{EW}, w))$ between the extremum $(0, u)$, which preserves electroweak (EW) symmetry, and the true vacuum (v_{EW}, w) , corresponding to the broken EW phase, gradually decreases. Eventually, beyond a certain value of w , the point $(0, u)$ becomes the global minimum of the tree-level potential. This behavior can be understood from a careful inspection of Fig. 2 and clearly from Figs. 9(a) and 9(b). It is commonly understood that a

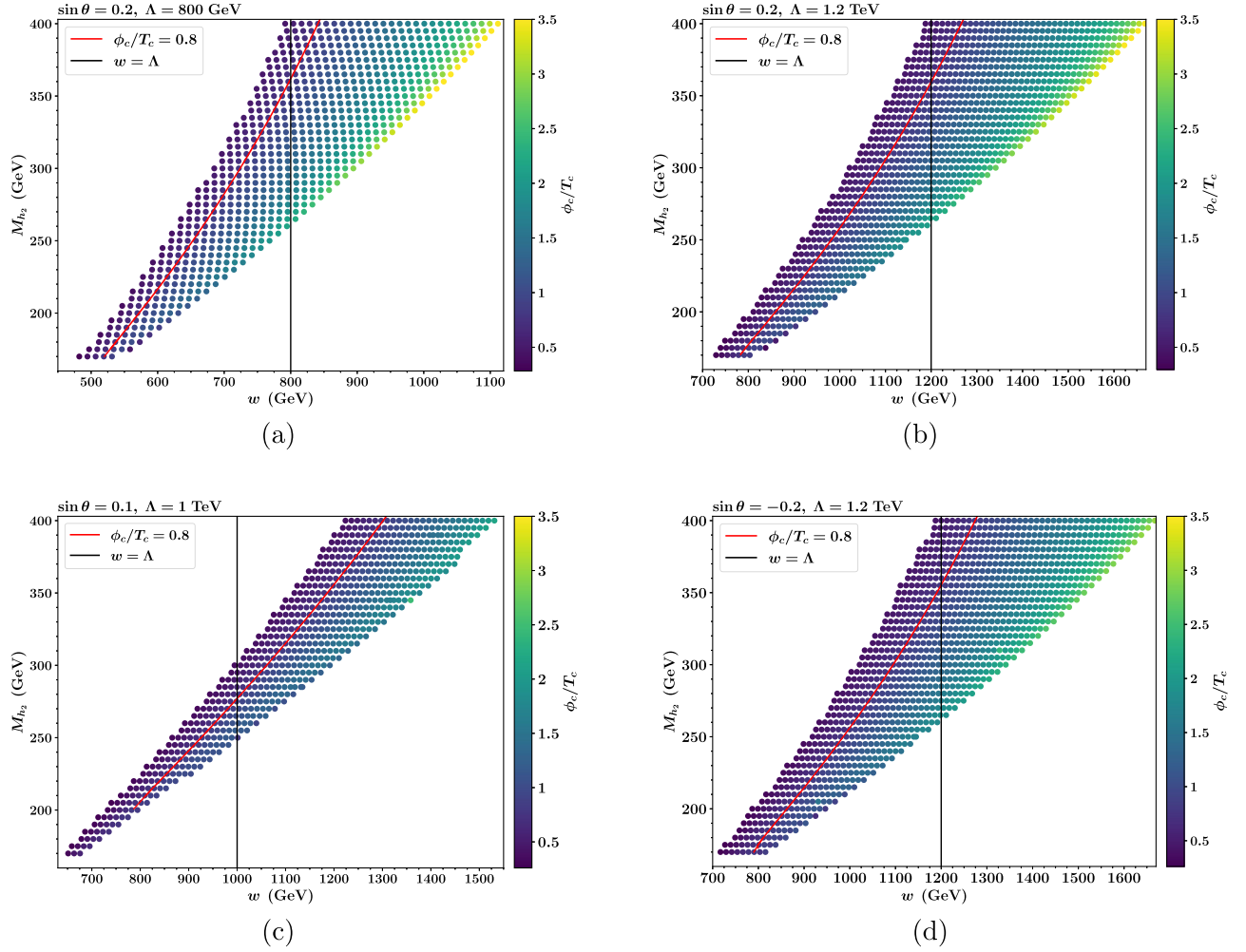


FIG. 8. Strength of the EWPT ($\frac{\phi_c}{T_c}$) in the $w - M_{h_2}$ plane for different choices of $\sin \theta$ and Λ . The color bar on the right of the plots represent the value of $\frac{\phi_c}{T_c}$. The points on the left of the red line correspond to $\frac{\phi_c}{T_c} < 0.8$. The black line corresponds to the equation $w = \Lambda$.

smaller tree-level potential difference ΔV leads to a lower critical temperature [26,63–65], which in turn enhances the strength of the EW phase transition. We explicitly illustrate this with the help of Figs. 9(c) and 9(d), respectively.

This feature can also be understood in terms of the Lagrangian parameter λ_{hs} . As w increases, λ_{hs} attains sizable negative values ($|\lambda_{hs}| \sim 1$) [see Eq. (A5) and the corresponding plot in Fig. 23(f) in Appendix A]. The tree-level potential difference (ΔV) decreases as λ_{hs} attains more and more negative values, resulting in an enhanced $\frac{\phi_c}{T_c}$. These findings are also supported by Figs. 9(e) and 9(f). Therefore, it is clear that negative λ_{hs} favors SFOEWPT in our scenario. Figures 9(c)–9(f) further indicate that the potential difference is larger, as a consequence $\frac{\phi_c}{T_c}$ is smaller, for negative $\sin \theta$ compared to the corresponding positive value of same magnitude for a fixed λ_{hs} . The fact that λ_{hs} and $\sin \theta$ are not strongly correlated is a characteristic feature of the particular model framework considered in this work.

In light of the above discussion, we now present the region of parameter space in the $M_{h_2} - \sin \theta$ plane consistent with SFOEWPT in Fig. 10. The parameter space can be divided into four regions. In the absence of the effective operator, it is well known that only for $M_{h_2} \ll M_{h_1}$ does one obtain an SFOEWPT-compatible parameter space with sizable $\sin \theta$ [26]. For higher values of M_{h_2} , it is difficult to accommodate a strong FOEWPT without assuming large $\sin \theta$, which is already ruled out by the Higgs signal strength measurements. This is in contrast with the present scenario, where, in the presence of the effective operator, the correlation between λ_{hs} and $\sin \theta$ is weakened, and a sizable λ_{hs} can be achieved even for small $\sin \theta$. One can see that there exists a wide range of SFOEWPT-compatible parameter space in the region $M_{h_2} > M_{h_1}$ for both positive and negative $\sin \theta$, as it is now facilitated by large w rather than $\sin \theta$. The slight asymmetry between the allowed regions corresponding to positive and negative $\sin \theta$ for $M_{h_2} > M_{h_1}$ can be traced back to Fig. 9(c), which suggests

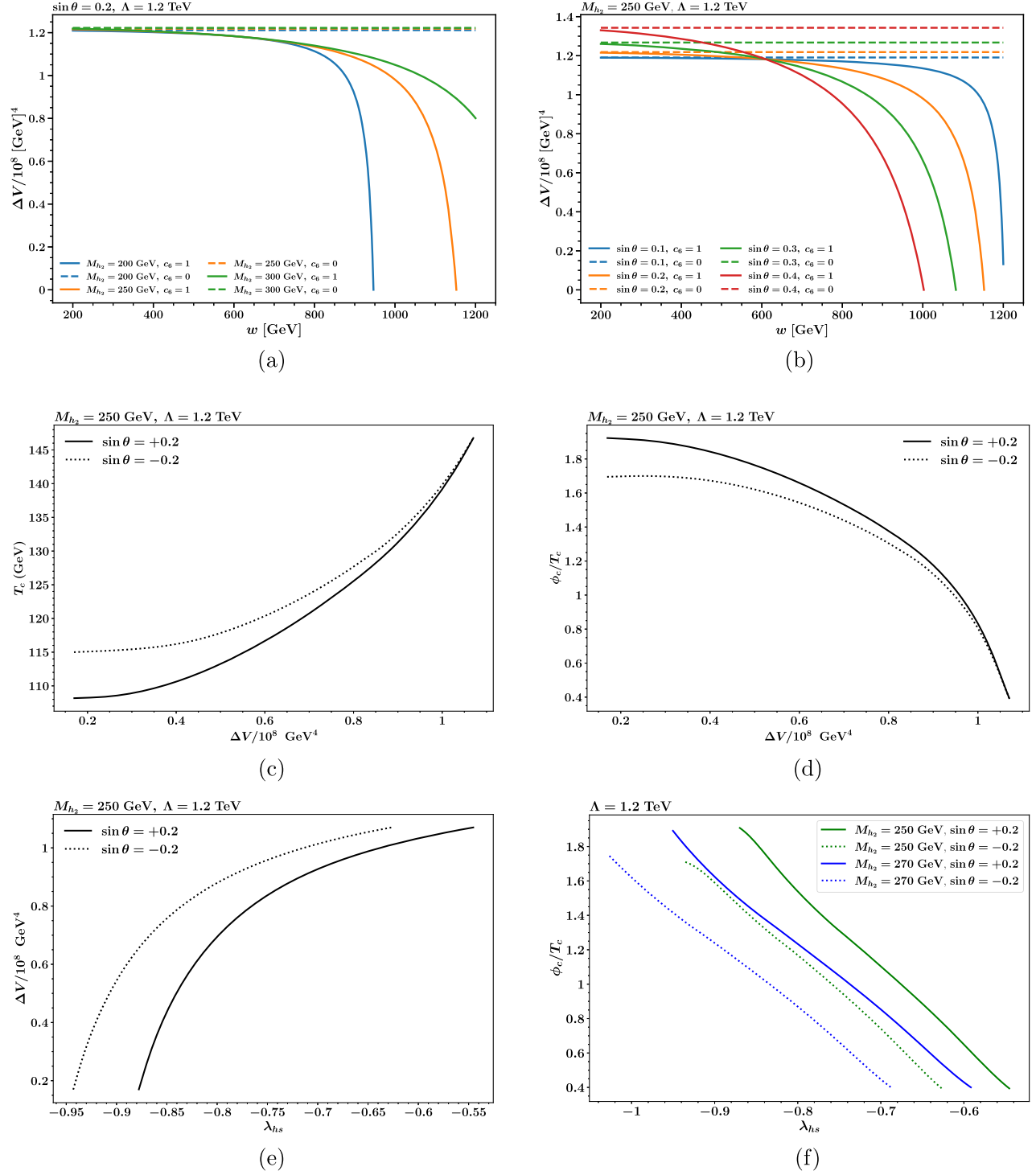


FIG. 9. Tree-level potential difference, $\Delta V = V_0(0, u) - V_0(v_{\text{EW}}, w)$ as a function of w (a) for fixed $\sin \theta$ and varying M_{h_2} , and (b) for fixed M_{h_2} and varying $\sin \theta$. Here, solid (dashed) lines correspond to $c_6 = 1$ ($c_6 = 0$). Panels (c) and (d) show the dependence of T_c and ϕ_c/T_c , respectively, on ΔV . Panels (e) and (f) display the variation of ΔV and $\frac{\phi_c}{T_c}$, respectively, with λ_{hs} . In panels (c)–(f), solid (dotted) lines denote $\sin \theta = +0.2$ (-0.2). The cutoff scale is assumed to be 1.2 TeV to obtain these plots.

that the tree-level potential difference ΔV is smaller for positive $\sin \theta$ for a fixed λ_{hs} .

In the left half of Fig. 10 (for $M_{h_2} < M_{h_1}$), there exist two competing phase transition patterns: $(0, 0) \rightarrow (v(T), w(T))$

or $(0, u(T)) \rightarrow (v(T), w(T))$. The former is more likely when $(0, u)$ is not an extremum of the tree-level potential. This occurs when μ_s^2 is negative. In fact, for very low M_{h_2} and positive $\sin \theta$, μ_s^2 is mostly negative [see Eq. (A3)].

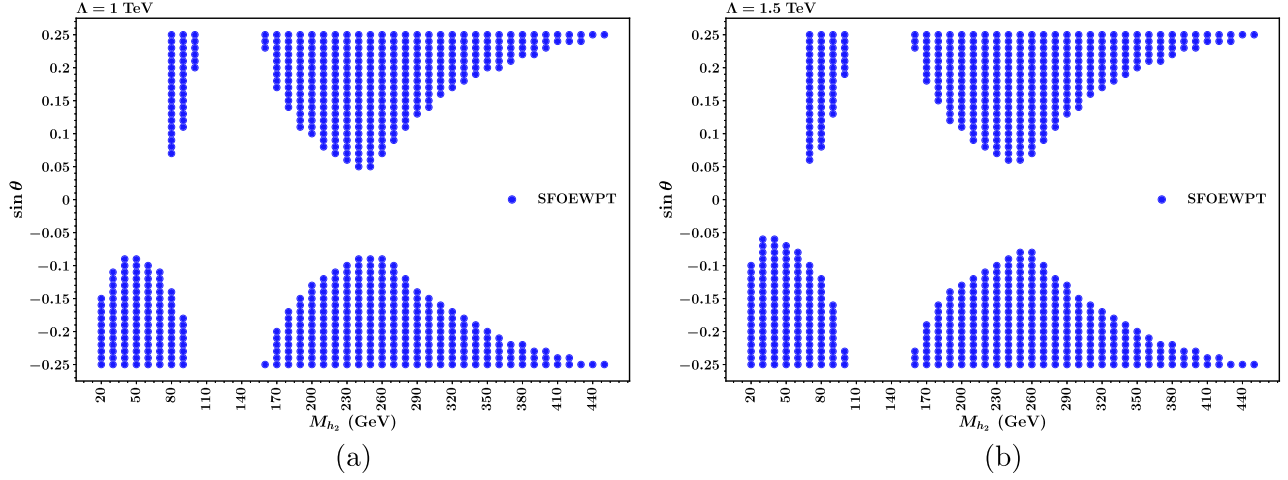


FIG. 10. SFOEWPT allowed parameter space ($\phi_c/T_c \geq 0.8$) in the $M_{h_2} - \sin \theta$ plane for two different choices of the cutoff scale (a) $\Lambda = 1$ TeV (left), (b) $\Lambda = 1.5$ TeV (right) assuming full one loop finite temperature corrected effective potential described by Eq. (32).

Transitions of the type $(0, 0) \rightarrow (v(T), w(T))$ are less likely as the tree-level potential difference between $V_0(0, 0)$ and $V_0(v, w)$ is large enough to rule out strong or even first-order phase transitions.⁵ For higher values of M_{h_2} , μ_s^2 becomes positive again [see Fig. 23(c) in Appendix A], and the phase transition proceeds via $(0, u(T)) \rightarrow (v(T), w(T))$, as usual. For negative $\sin \theta$ and very small M_{h_2} , the phase transition pattern is similar to that in the singlet scalar extension of the SM with spontaneous $\mathcal{Z}_2$ breaking, as the contribution from the dimension-six operator is suppressed by w^2/Λ^2 . As the value of M_{h_2} increases, the phase transition proceeds via the usual pattern for both positive and negative $\sin \theta$.

There are no SFOEWPT-compatible parameter space points in the $M_{h_2} - \sin \theta$ plane in the degenerate ($M_{h_1} \sim M_{h_2}$) and zero-mixing limit ($\sin \theta \rightarrow 0$). Even though in this limit (v, w) still remains the global minimum of the tree-level potential, the tree-level potential difference ΔV becomes independent of the singlet scalar VEV w [see Eq. (15)]. Therefore, one can not minimize this difference further by increasing w and consequently it does not help in lowering the critical temperature T_c . There are also no parameter space points consistent with SFOEWPT in the extreme right region of the plots in Fig. 10, as it requires the value of w to be greater than the cutoff scale Λ .

In Table I, we present few representative benchmark points $M_{h_2}, \sin \theta$ and corresponding minimum values of singlet scalar VEV (w) that lead to $\phi_c/T_c \geq 0.8$ assuming the cutoff scale $\Lambda = 1$ TeV. Figure 11 illustrates the dependency of ϕ_c/T_c on the singlet scalar VEV w for three different choices of the cutoff scale (800 GeV, 1 TeV, and 1.5 TeV). These plots also suggest that the ratio w/Λ

instead of the absolute value of the cutoff scale is a relevant factor in the context of EWPT.

Finally, we conclude this section with the following remark. Even though we have considered only the

TABLE I. Representative benchmark points $M_{h_2}, \sin \theta$ and corresponding minimum values of singlet scalar VEV (w) that yield $\phi_c/T_c \geq 0.8$ assuming the cutoff scale $\Lambda = 1$ TeV.

M_{h_2} (GeV)	$\sin \theta$	w (GeV)	ϕ_c/T_c
170	+0.2	650	0.80
	-0.2	660	0.80
180	+0.2	672	0.80
	-0.2	676	0.81
200	+0.2	716	0.80
	-0.2	719	0.80
220	+0.2	758	0.80
	-0.2	762	0.81
240	+0.2	798	0.80
	-0.2	802	0.80
260	+0.2	838	0.82
	-0.2	841	0.80
280	+0.2	873	0.81
	-0.2	878	0.80
300	+0.2	907	0.80
	-0.2	913	0.80
320	+0.2	940	0.81
	-0.2	947	0.81
340	+0.2	971	0.81
	-0.2	977	0.80
350	+0.2	985	0.80
	-0.2	993	0.81

⁵This can be understood by looking at the bottom left corner of Fig. 2(a), which suggests that the allowed green shaded region (with $\mu_s^2 < 0$) is well separated from the $V_0(v_{EW}, w) = V_0(0, 0)$ (yellow) line by the requirement $\lambda_s > 0$.

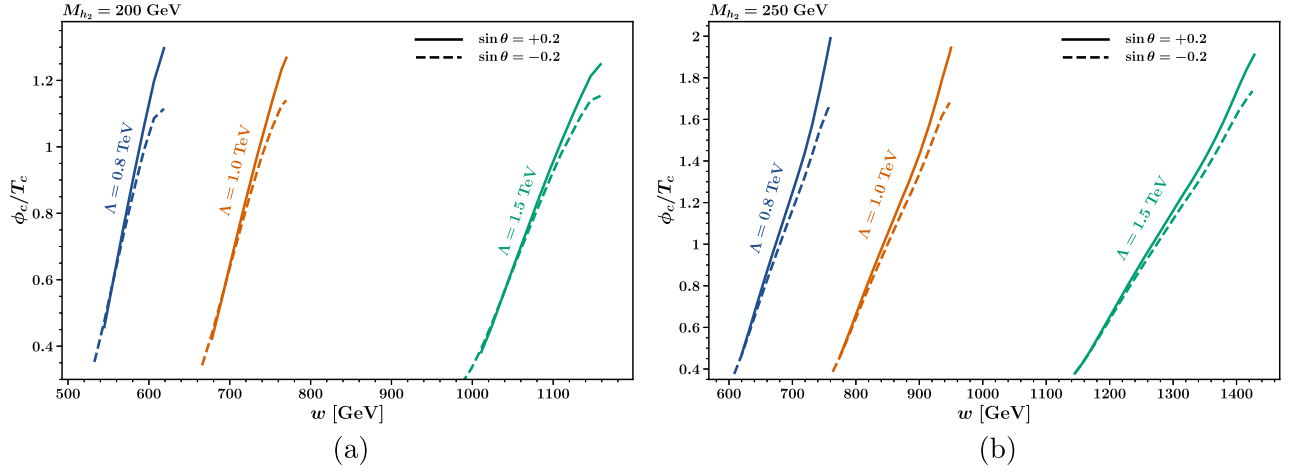


FIG. 11. Variation of $\frac{\phi_c}{T_c}$ with w for three different choices of the cut-off scale (800 GeV, 1 TeV, and 1.5 TeV) assuming (a) $M_{h_2} = 200$ GeV and (b) $M_{h_2} = 250$ GeV. The solid and dashed lines correspond to $\sin \theta = +0.2$ and -0.2 , respectively.

dimension-six term of the form $|H|^2|\phi|^4$ a complete analysis in the context of the electroweak phase transition and its signatures at GW detectors and at the LHC requires a comprehensive study of all possible dimension-six operators allowed by the all possible symmetries of the theory. In principle, the UV theories that can potentially generate the above operator at low energy after integrating out heavy degrees of freedom can also give rise to other dimension-six operators, such as $|H|^4|\phi|^2$, $|H|^6$, $|\phi|^6$ with Wilson coefficients of the same order ($\sim \frac{1}{\Lambda^2}$) as above. For example, the inclusion of the operator $|H|^4|\phi|^2$, in addition to the existing one, modifies Eqs. (A2), (A4), and (A5) (see Appendix A) in the following way (assuming the Wilson coefficients to be of the same order)

$$\lambda_h = \frac{M_{h_1}^2 \cos^2 \theta + M_{h_2}^2 \sin^2 \theta}{2v_{EW}^2} - \frac{w^2}{2\Lambda^2}, \quad (38)$$

$$\lambda_s = \frac{M_{h_1}^2 \sin^2 \theta + M_{h_2}^2 \cos^2 \theta}{2w^2} - \frac{v_{EW}^2}{2\Lambda^2}, \quad (39)$$

$$\lambda_{hs} = -\frac{v_{EW}^2}{\Lambda^2} - \frac{w^2}{\Lambda^2} + \frac{(-M_{h_1}^2 + M_{h_2}^2) \cos \theta \sin \theta}{v_{EW} w}. \quad (40)$$

One can see from Eq. (38) that the quartic Higgs coupling becomes negative for certain values of the ratio $\frac{w}{\Lambda}$ depending on the value of $\sin \theta$. The smaller the $\sin \theta$, the more stringent the upper limit becomes on the ratio $\frac{w}{\Lambda}$. However, a negative quartic Higgs coupling at tree level is perfectly acceptable if one includes all possible dimension-six operators in the scalar potential. The additional advantage one has in the presence of all possible dimension-six operators is both λ_h and λ_s are allowed to become negative. This eventually helps to create a barrier to facilitate

FOEWPT, a wider range of allowed parameter space, and at the same time helps to relax the upper bound on the cutoff scale, compared to that in the Higgs SMEFT scenario [35,37]. Moreover, λ_{hs} now receives an additional contribution of order $-\frac{v_{EW}^2}{\Lambda^2}$ [Eq. (40)], which can induce SFOEWPT even at lower w compared to the case where only the $|H|^2|\phi|^4$ term is present. A complete analysis of EWPT including the full list of dimension-six operators in the singlet scalar extension framework will be taken up in a future work.

In the present work, for simplicity, we restrict ourselves to a particular dimension-six operator as part of a phenomenological analysis. Since achieving a strong first-order phase transition in this setup typically requires $w \sim \Lambda$, higher-dimensional operators may have a significant impact on our results. The dimension-eight operators $\frac{c_8^{(1)}}{\Lambda^4} |H|^2|\phi|^6$, $\frac{c_8^{(2)}}{\Lambda^4} |H|^4|\phi|^4$, and $\frac{c_8^{(3)}}{\Lambda^4} |H|^6|\phi|^2$ provide a minimal representative corrections to the same operator tower considered at dimension six and in particular, λ_{hs} . The corresponding Wilson coefficients may be parametrically smaller, for example $c_8^{(i)} \sim x c_6$, with x in the range 0.1–0.5 in a particular UV completion due to UV matching or higher order in perturbative expansion. Since the additional expansion parameter (w/Λ) is relevant in this setup, the operator $\frac{c_8^{(3)}}{\Lambda^4} |H|^6|\phi|^2$ is expected to have less significant impact compared to the other two in the context of our analysis. In Appendix D, we present the effect of the dimension-eight operators on the tree level potential difference $\Delta V = V_0(0, u) - V_0(v_{EW}, w)$, neglecting the operator $\frac{c_8^{(3)}}{\Lambda^4} |H|^6|\phi|^2$. The inclusion of these operators does not alter the main conclusion of our analysis, instead their presence helps reduce the required value of w/Λ needed to achieve a strong first-order electroweak phase transition.

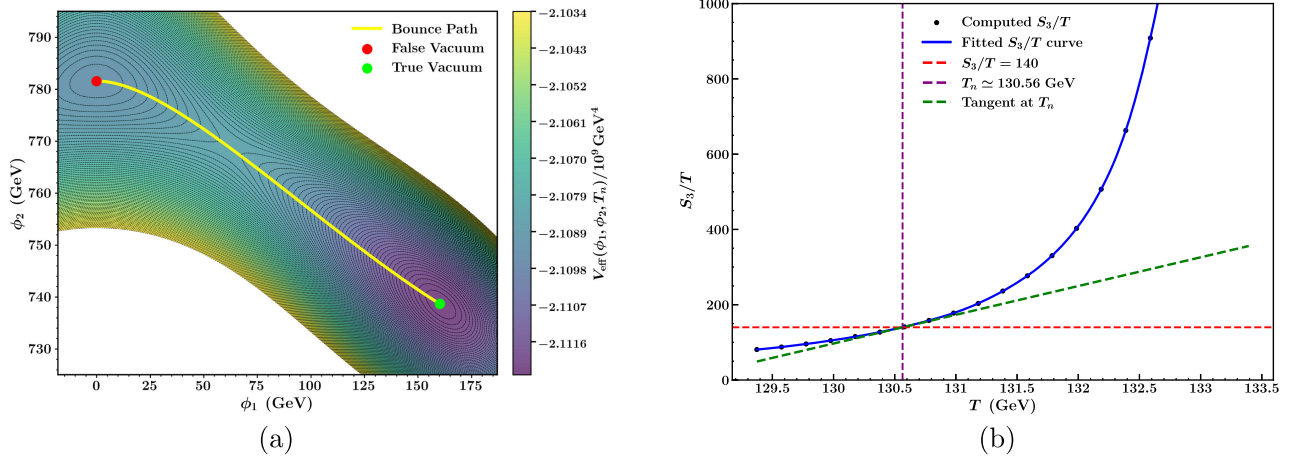


FIG. 12. (a) Bounce solution between false and true vacuum at the nucleation temperature. (b) Evolution of S_3/T with T . The numerical values of S_3/T computed using CosmoTransitions are shown by black dotted points. The green dashed line representing the slope of the fitted curve (blue) at $T = T_n$ determines the inverse phase transition duration β at $T = T_n$. Both plots are for $M_{h_2} = 250$ GeV, $\sin \theta = 0.15$, $w = 725.7$ GeV, $M_{\gamma_d} = 60$ GeV, and $\Lambda = 800$ GeV.

IV. NUCLEATION

During a strong first-order cosmological phase transition, the Universe evolves via thermal tunneling from a metastable false vacuum to the true vacuum via the nucleation of critical bubbles. The bubble nucleation rate per unit volume at temperature T is given by [66]

$$\Gamma(T) \simeq T^4 \left(\frac{S_3(T)}{2\pi T} \right)^{3/2} \exp \left[-\frac{S_3(T)}{T} \right], \quad (41)$$

where $S_3(T)$ denotes the three-dimensional Euclidean action of the critical bubble and is expressed as

$$S_3 = 4\pi \int_0^\infty r^2 dr \left[\frac{1}{2} \sum_i \left(\frac{d\phi_i}{dr} \right)^2 + V_{\text{eff}}(\phi_1, \phi_2, T) \right]. \quad (42)$$

The specific field configuration $(\phi_1(r), \phi_2(r))$ that minimizes this action is called the “bounce” solution. This is obtained by solving the Euler-Lagrange equation, which takes the form of a nonlinear differential equation with a “friction” term,

$$\frac{d^2 \phi_i}{dr^2} + \frac{2}{r} \frac{d\phi_i}{dr} = \frac{\partial V_{\text{eff}}(\phi_1, \phi_2, T)}{\partial \phi_i} \quad (43)$$

satisfying the boundary conditions,

$$\left. \frac{d\phi_i}{dr} \right|_{r=0} = 0, \quad \lim_{r \rightarrow \infty} \phi_i(r) = 0. \quad (44)$$

The nucleation temperature (T_n), is defined as the temperature where the probability of nucleating at least one bubble per Hubble volume is order unity,

$$\int_{T_n}^{T_c} \frac{dT}{T} \frac{\Gamma(T)}{H(T)^4} \sim 1 \Rightarrow \frac{S_3(T_n)}{T_n} \simeq \mathcal{O}(140), \quad (45)$$

with $H(T)$ the Hubble expansion rate. In models with two scalar fields (ϕ_1, ϕ_2) , like the one considered here, the bounce is a trajectory in the two-dimensional field space that extremizes the coupled equations, i.e., the solution to the coupled Euclidean equations for both fields. However, in our scenario the dominant field excursion takes place along the Higgs direction, as has been shown in Fig. 12(a). The evolution of the $O(3)$ -invariant Euclidean action (S_3) against temperature is displayed in Fig. 12(b). In particular we numerically evaluate the $\frac{S_3}{T}$ using CosmoTransitions [60].⁶

In order to achieve a sustainable electroweak baryogenesis during bubble nucleation one needs $\frac{\phi_c}{T_c} \gtrsim 0.6$ – 1.4 , as discussed in Sec. III. We have set $\frac{\phi_c}{T_c} \geq 0.8$, which in turn implies a larger value for $\frac{\phi_n}{T_n}$, where ϕ_n is the discontinuity of the order parameter ϕ_1 at nucleation temperature. This is also illustrated in Fig. 13 where we display $\frac{\phi_n}{T_n}$ outcomes for a given $\frac{\phi_c}{T_c}$ for various choices of M_{h_2} , $\sin \theta$, and w that correspond to SFOEWPT. While ϕ_c/T_c is often taken to be the standard measure of the transition strength, the growth of ϕ_n/T_n above the diagonal line illustrates that the nucleation typically starts only after significant supercooling.

⁶To determine the nucleation parameters, we compute the ratio S_3/T on a descending temperature grid beginning at T_c using CosmoTransitions. To mitigate numerical instabilities inherent in any bounce solver package, we reconstruct the discrete data using a smooth functional fit. The nucleation temperature T_n is then identified by the condition $S_3(T_n)/T_n = 140$, and the inverse phase transition duration β is extracted by evaluating the temperature derivative of the fitted curve at $T = T_n$.

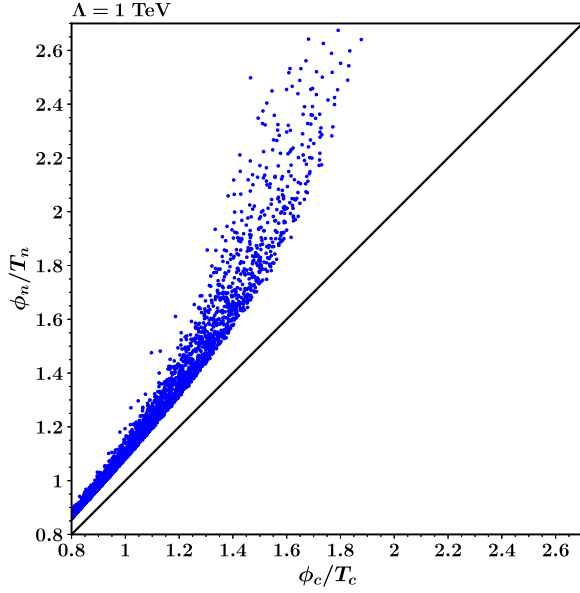
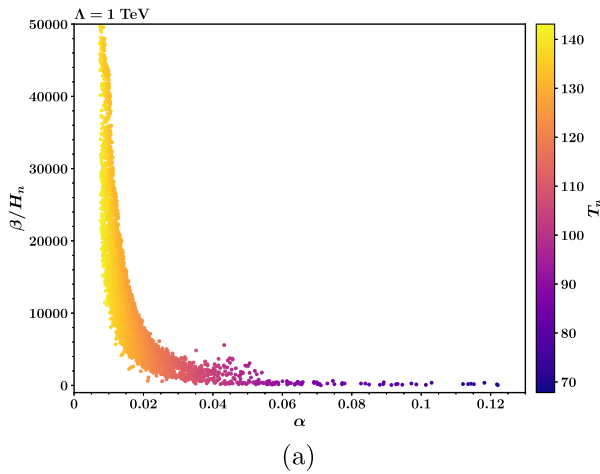


FIG. 13. Possible $\frac{\phi_n}{T_n}$ values for a given $\frac{\phi_c}{T_c}$ for various M_{h_2} , $\sin \theta$, and w values that predict SFOEWPT, assuming $\Lambda = 1$ TeV [see Fig. 10(a)].

Once nucleated, bubbles of the true vacuum expand due to the pressure difference between the two phases and eventually collide and percolate, completing the phase transition. The highly out-of-equilibrium dynamics associated with bubble expansion and collisions source a stochastic background of gravitational waves. The key parameters that determine the strength of the gravitational wave signal are the following:

$$\alpha \equiv \frac{\Delta\rho}{\rho_{\text{rad}}}, \quad (46)$$



(a)

where $\Delta\rho$ is the released latent heat and ρ_{rad} is the radiation energy density evaluated at T_n . The characteristic time scale of the transition is described by the inverse duration parameter

$$\frac{\beta}{H_n} = T_n \frac{d}{dT} \left(\frac{S_3}{T} \right) \Big|_{T=T_n}, \quad (47)$$

where $H_n \equiv H(T_n)$.

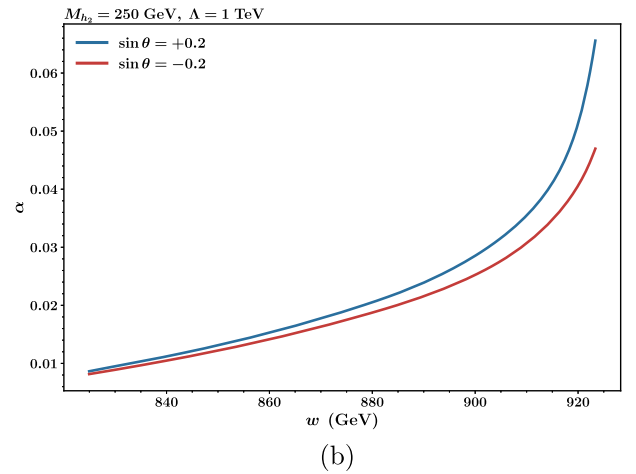
V. GRAVITATIONAL WAVE

The gravitational wave provides a robust probe of first-order electroweak phase transitions in the early Universe, with promising prospects for detection at future space-based interferometers. The strength of the GW signals are quantified by α , β/H_n , and the bubble wall velocity. A large α and a smaller β/H_n would typically result in a detectable signal over the background. From Fig. 14(a), we can see that large α and small β/H_n are only possible for super-cooled transitions that are characterized by small nucleation temperatures. The increase in α with the increase in the VEV of the singlet scalar (w) for a fixed cutoff scale Λ can be inferred from Fig. 14(b). There is a mild variation of this dependency due to the *sign* of the scalar mixing angle $\sin \theta$.

The resulting stochastic background is conventionally characterized by the present-day energy density spectrum,

$$\Omega_{\text{GW}}(f) \equiv \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}}{d \ln f}. \quad (48)$$

The stochastic GW generated during a first-order phase transition receives contributions primarily from three sources: collisions of the bubble wall, sound waves in the surrounding plasma, and magnetohydrodynamic (MHD) turbulence.



(b)

FIG. 14. Variations of the various GW signal quantifiers. The left panel: α vs β/H_n with color gradient as nucleation temperature T_n , for the SFOEWPT compatible parameter space points in Fig. 10(a). The right panel: variations of α is shown against the VEV of the singletlike scalar for an illustrative value $M_{h_2} = 250$ GeV. The blue (red) curve corresponds to the scalar mixing angle $\sin \theta = 0.2(-0.2)$. Both of these assume $\Lambda = 1$ TeV.

Most of the energy released during the transition is pumped into the surrounding plasma as heat and kinetic energy. Numerical simulations indicate that bubble collisions primarily create fluid perturbations in the form of sound waves, which act as a persistent and potent source of GWs until the expansion of the Universe (Hubble expansion) eventually dampens them. The total present-day GW energy density spectrum, in terms of the dimensionless Hubble parameter $h = \frac{H_0}{100} \text{ kms}^{-1} \text{ Mpc}^{-1}$ (where H_0 is the current Hubble constant) [67], can therefore be written as [68]

$$\Omega_{\text{GW}} h^2 \simeq \Omega_{\text{col}} h^2 + \Omega_{\text{sw}} h^2 + \Omega_{\text{tur}} h^2. \quad (49)$$

As a result, the shape and amplitude of the GW spectrum provide a direct observational window into the particle-physics dynamics underlying the phase transition.

The energy-momentum tensor, dominated by scalar fields, acts as the fundamental source for gravitational waves. The corresponding energy density spectrum as a function of frequency f can be estimated when the bubble walls expand pushing the surrounding plasma and eventually collide at near-relativistic speeds and is given by [69]

$$\Omega_{\text{col}} h^2 = 1.67 \times 10^{-5} \left(\frac{\beta}{H_n} \right)^{-2} \left(\frac{\kappa_c \alpha_n}{1 + \alpha_n} \right)^2 \left(\frac{100}{g_*} \right)^{1/3} \times \left(\frac{0.11 v_w^3}{0.42 + v_w^2} \right) \frac{3.8(f/f_{\text{col}})^{2.8}}{1 + 2.8(f/f_{\text{col}})^{3.8}}, \quad (50)$$

where v_w indicates the velocity of the bubble wall, α_n signifies the strength of the phase transition at the nucleation temperature, and κ_c is a factor quantifying the conversion of the vacuum energy into the kinetic energy of the expanding bubble walls. This efficiency factor can be parametrized as [70]

$$\kappa_c = \frac{0.715 \alpha_n + \frac{4}{27} \sqrt{\frac{3\alpha_n}{2}}}{1 + 0.715 \alpha_n}. \quad (51)$$

For the collision term, the frequency of the resulting GW spectrum peaks at

$$f_{\text{col}} = 16.5 \times 10^{-6} \left(\frac{f_*}{\beta} \right) \left(\frac{\beta}{H_n} \right) \left(\frac{T_n}{100 \text{ GeV}} \right) \left(\frac{g_*}{100} \right)^{1/6} \text{ Hz}, \quad (52)$$

where f_*/β is a fitted function and is given by [69]

$$\frac{f_*}{\beta} = \frac{0.62}{1.8 - 0.1 v_w + v_w^2}. \quad (53)$$

Assuming the bubbles reach relativistic speeds at the time of collision, we consider $v_w = 1$ [70,71] in the subsequent discussion.

A continuous, low-frequency “hum” typically arises from the sound waves generated in the plasma during the expansion of the bubble after nucleation [72–74]. The corresponding GW energy density spectrum is

$$\Omega_{\text{sw}} h^2 = 2.65 \times 10^{-6} \Upsilon(\tau_{\text{sw}}) \left(\frac{\beta}{H_n} \right)^{-1} v_w \left(\frac{\kappa_{\text{sw}} \alpha_n}{1 + \alpha_n} \right)^2 \times \left(\frac{100}{g_*} \right)^{1/3} \left(\frac{f}{f_{\text{sw}}} \right)^3 \left[\frac{7}{4 + 3(f/f_{\text{sw}})^2} \right]^{7/2}. \quad (54)$$

The efficiency of converting the kinetic energy of the plasma (sound waves) into gravitational waves is characterized by a dimensionless parameter, κ_{sw} [70,71],

$$\kappa_{\text{sw}} = \frac{\sqrt{\alpha_n}}{0.135 + \sqrt{0.98 + \alpha_n}}. \quad (55)$$

The factor $\Upsilon(\tau_{\text{sw}})$ accounts for the finite lifetime of the sound wave period and is defined as

$$\Upsilon(\tau_{\text{sw}}) = 1 - \frac{1}{\sqrt{1 + 2\tau_{\text{sw}} H_*}}, \quad (56)$$

where τ_{sw} denotes the sound wave lifetime. Following Ref. [74], we approximate $\tau_{\text{sw}} \approx R_n / \bar{U}_f$, with the mean bubble separation $R_n = (8\pi)^{1/3} v_w \beta_n^{-1}$ and the root-mean-squared fluid velocity $\bar{U}_f = \sqrt{3\kappa_{\text{sw}} \alpha_n / 4}$. The redshifted peak frequency associated with the sound wave contribution is

$$f_{\text{sw}} = 1.9 \times 10^{-5} \left(\frac{1}{v_w} \right) \left(\frac{\beta}{H_n} \right) \left(\frac{T_n}{100 \text{ GeV}} \right) \left(\frac{g_*}{100} \right)^{1/6} \text{ Hz}. \quad (57)$$

Finally, the strong magnetic fields coupled to the ionized plasma creates turbulent motions during the bubble expansion, often called MHD turbulence, and can be another potential source for GWs [75]. The contribution to the GW energy density spectrum from such MHD turbulence is,

$$\Omega_{\text{tur}} h^2 = 3.35 \times 10^{-4} \left(\frac{\beta}{H_n} \right)^{-1} v_w \left(\frac{\kappa_{\text{tur}} \alpha_n}{1 + \alpha_n} \right)^{3/2} \left(\frac{100}{g_*} \right)^{1/3} \times \left[\frac{(f/f_{\text{tur}})^3}{[1 + (f/f_{\text{tur}})]^{11/3} (1 + \frac{8\pi f}{h_*})} \right], \quad (58)$$

where $h_* = 16.5 \times 10^{-6} \left(\frac{T_n}{100 \text{ GeV}} \right) \left(\frac{g_*}{100} \right)^{1/6} \text{ Hz}$ is the inverse Hubble time. The peak frequency of the MHD turbulence term appears as

$$f_{\text{tur}} = 2.7 \times 10^{-5} \frac{1}{v_w} \left(\frac{\beta}{H_n} \right) \left(\frac{T_n}{100 \text{ GeV}} \right) \left(\frac{g_*}{100} \right)^{1/6} \text{ Hz}. \quad (59)$$

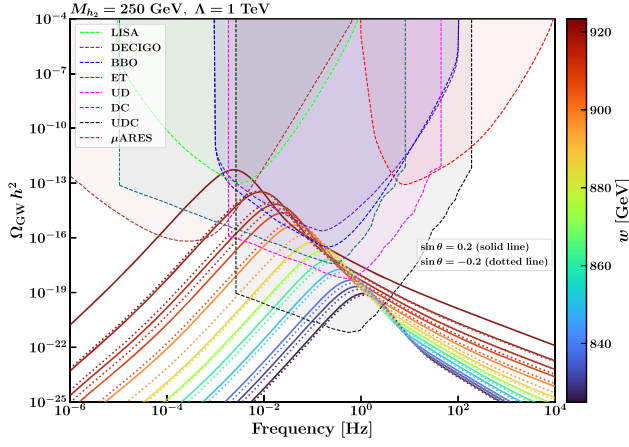


FIG. 15. Variations of the gravitational wave energy densities with the frequency for different values of w for $M_{h_2} = 250$ GeV and $\Lambda = 1$ TeV. The light shaded regions represent the expected sensitivity reach of the next generation space-based and ground-based detectors like, LISA [42], BBO [43], DECIGO [44], Einstein Telescope (ET) [77], μ Ares [45], UD, DC, and UDC [44]. The solid and dotted lines are for two distinct values of $\sin \theta = 0.2$ and -0.2 , respectively.

The turbulence efficiency factor is taken to be $\kappa_{\text{tur}} = \epsilon \kappa_{\text{sw}}$, where ϵ parametrizes the fraction of bulk kinetic energy converted into turbulent motion. Following previous studies [76], we adopt $\kappa_{\text{tur}} \simeq 0.1 \kappa_{\text{sw}}$ in our numerical analysis.

To illustrate the gravitational wave signals inherent to this model, we have estimated the spectral density of stochastic GW generated for $M_{h_2} = 250$ GeV and a fixed new physics scale $\Lambda = 1$ TeV with the singlet VEV w varying. The numerical results for some discrete values of w are synthesized in Fig. 15, which displays the spectral density across various frequency ranges for two values of

$\sin \theta = \pm 0.2$. We have considered the standard “runaway” scenario in which bubbles of a new vacuum state expand with nearly unrestrained speed in the absence of friction of the plasmic medium. The peak frequencies of the cumulative contributions of collision, sound, and MHD terms are within reach of the sensitivities of LISA [42], BBO [43], DECIGO [44], UD, DC, and UDC [44] as can be seen in Fig. 15. A few general comments on the patterns of the generated GW spectra for this scenario are as follows. The maximum energy density shifts toward a lower frequency and higher amplitude for larger values of the singlet VEV (w), which means greater chance of observing a GW signal over background noise. The pattern can be understood from Fig. 14(b), which shows the strength of the transition increases with the increase of the singlet VEV. Another observation would be the peak amplitude is marginally less for the negative values of the $\sin \theta$ compared to its positive counterpart.

In Fig. 16(a), we display the total GW signal coming from all three contributing sources generated during phase transitions, for all those parameter space points in the $M_{h_2} - \sin \theta$ plane satisfying SFOEWPT [see Fig. 10(a)]. The color gradient indicates the nucleation temperatures. The highest peak corresponds to the lowest nucleation temperature and is in accordance with the supercooled phase of the Universe, signifying a larger $(T_c - T_n)$, which in turn increases α , the strength of the transition.

The GW signals for a set of 12 selected BPs tabulated in Table II are also depicted in Fig. 16(b). The signals comprise all three said gravitating sources of that epoch. The peak frequencies vary by three orders of magnitudes. It should be noted that a wide range of interferometer based experiments can, in principle, detect such stochastic GW signals. In particular, LISA [42], the most imminent among

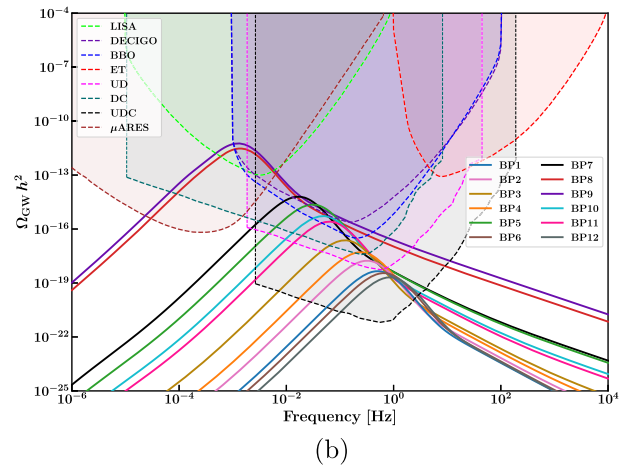
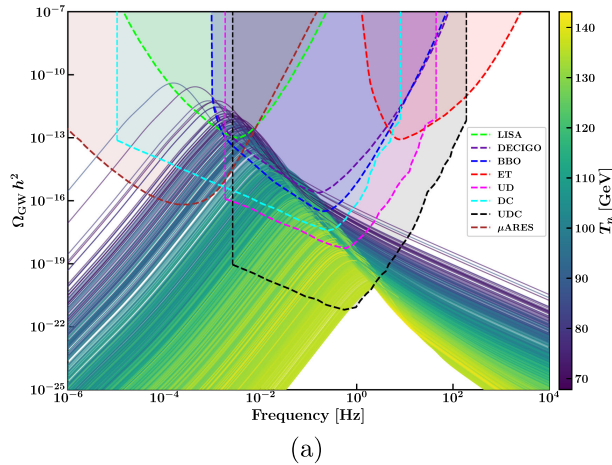


FIG. 16. Detectable GW signal coming from the cumulative contributions of the SW, bubble collision, and MHD turbulence, as a function of frequency. (a) The left panel shows the GW signals for all those SFOEWPT compatible parameter space points in the $M_{h_2} - \sin \theta$ plane [see Fig. 10(a)] have been plotted with color gradient as nucleation temperature. (b) The right panel displays the same but for certain selected benchmark points tabulated in Table II. A fixed cutoff scale $\Lambda = 1$ TeV has been considered for both of these figures.

TABLE II. Predicted values of $T_c, \frac{\phi_c}{T_c}, T_n, \frac{\phi_n}{T_n}, \alpha$, and $\frac{\beta}{H}$ for different representative benchmark points assuming $\Lambda = 1$ TeV.

BP	M_{h_2} (GeV)	$\sin \theta$	w (GeV)	T_c (GeV)	ϕ_c/T_c	T_n (GeV)	ϕ_n/T_n	α	β/H
BP1	170	0.20	655.0	143.1	0.84	141.3	0.93	0.009	18771
BP2	180	0.17	708.3	140.5	0.92	137.9	1.04	0.011	12211
BP3	200	-0.19	778.7	136.0	1.06	130.8	1.24	0.015	4976
BP4	220	0.18	803.9	135.3	1.07	132.3	1.19	0.014	9668
BP5	240	0.20	890.2	120.1	1.56	100.1	2.13	0.045	1635
BP6	240	-0.20	816.4	138.2	0.91	136.8	0.98	0.010	25325
BP7	250	-0.19	928.4	122.8	1.47	104.8	1.98	0.040	843
BP8	250	-0.23	907.2	116.7	1.66	76.7	2.90	0.103	94.1
BP9	260	0.20	956.0	113.8	1.75	67.8	3.34	0.145	97.8
BP10	280	0.20	975.4	121.4	1.50	112.0	1.80	0.031	2279
BP11	280	-0.20	975.4	124.2	1.41	116.1	1.66	0.027	2853
BP12	300	0.20	926.2	135.9	0.93	134.8	1.00	0.010	33558

all, should be able to see the gravitational waves predicted in this model for certain BPs in the midfrequency region. Even energy density peaks as small as $\sim 10^{-19}$ can be probed in μHz range interferometer experiments, like μARES [45].

VI. MULTISCALAR PRODUCTION AT THE LHC

As mentioned, one of the profound implications of the additional scalar field and dimension-six operator involving Higgs and singlet scalar fields described in Sec. II, is the modifications of the scalar potential, in particular, the latter one modifies the Higgs portal coupling and various triple and quartic scalar couplings (see Appendixes A and B). The detection of stochastic gravitation wave signals in any future experiments is definitely a novel way to gain knowledge on the structure of the scalar potential and its evolution in the early Universe. However, one can also explore the deviation of various trilinear and quartic scalar couplings that in turn can probe the phase transition dynamics, by investigating multiscalar production at the LHC. More specifically, we look for di-scalar bosons ($h_1 h_1$, $h_1 h_2$ and $h_2 h_2$) and triple-Higgs ($h_1 h_1 h_1$) production at the LHC. For the numerical estimation of multiscalar production cross sections, we implement the Lagrangian of the model described in Sec. II in FeynRules v2.3 [78,79] coupled with NLOCT [80] terms generated using FeynArts v3.12 [81] in order to handle the one-loop diagrams and counterterms. The universal FeynRules output [82,83] of FeynRules is then interfaced with MadGraph5_aMC@NLO [84] to generate multiscalar production in pp collision at one-loop level. The renormalization and the factorization scales have been set at the default dynamical scale of MadGraph5_aMC@NLO.

A. Discalar bosons production

Discalar bosons pair production ($h_i h_j$ with $i, j = 1, 2$) at the LHC proceeds primarily via gluon-gluon fusion process due to large flux of the initial state gluons in pp collisions at the LHC center of mass energies. The corresponding

Feynman diagrams for this process include the box and triangle diagrams, as shown in Fig. 17. Although the presence of the dimension-six operator does not introduce to any new Feynman diagrams, its effects enter through modifications of the $h_i - h_j - h_k$ vertex factors (Appendix B). The different Feynman diagrams in Fig. 17 can interfere with each other in a constructive or destructive manner depending on the overall sign of these triple scalar couplings ($\lambda_{h_i h_j h_k}$). Moreover, these couplings are also sensitive to the ratio $\frac{w}{\Lambda}$, as can be seen from Eq. (B2). In Fig. 24 (of Appendix B), we present the variation of $\lambda_{h_2 h_1 h_1}$ coupling as a function of the singlet scalar VEV, which is one of the crucial parameters in the context of FOEWPT as demonstrated in this manuscript. This plot also suggests that $\lambda_{h_2 h_1 h_1}$ has a zero at certain values of w , depending on M_{h_2} and $\sin \theta$. Therefore, to illustrate the intricate role that w plays in the context of di-Higgs cross sections from a collider perspective, we work with three different choices of w , namely w_1 , w_2 , and w_3 , with $w_1 > w_2 > w_3$. These three choices can be motivated as follows. The first choice, w_1 leads to SFOEWPT (as can be seen in Table I), while the choice, i.e., w_2 leads to $\lambda_{h_2 h_1 h_1} = 0$. The third choice w_3 , corresponds to comparable value of $\lambda_{h_2 h_1 h_1}$ coupling as that predicted by w_1 , but with opposite sign.

Figures 18 and 19 show normalized invariant mass distributions of the $h_i h_j$ pair to demonstrate the validity of the EFT framework. It is evident that most of the events correspond to $M_{h_i h_j} < \Lambda$ with a very small fraction of events (0.05%–5.5%) flowing beyond the cutoff scale Λ , that too depending on the mass of the additional singlet scalar and the type of the final states. The corresponding fraction of events is larger when the final state contains at least one heavy scalar. In particular, Fig. 18 also highlights the indirect effect of the dimension-six operator on the di-Higgs invariant mass distribution. Assuming the same set of $M_{h_2} - \sin \theta$ values, there exist two classes of distributions for different choices of w : one exhibiting a resonance peak 18(a) and the other without it 18(b) for $M_{h_2} > 2M_{h_1}$.

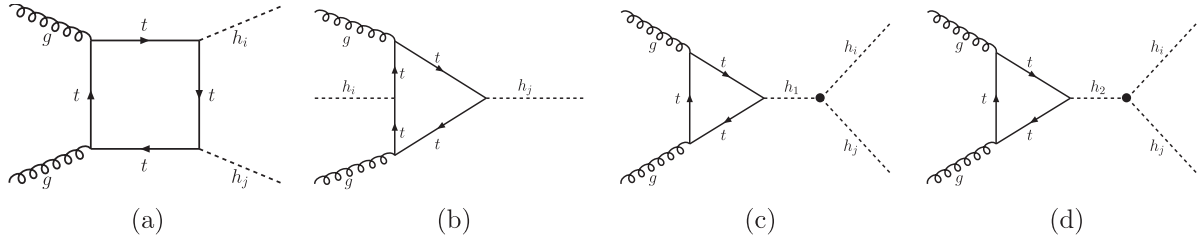


FIG. 17. Representative Feynman diagrams for $pp \rightarrow h_i h_j$ process. Here, the black dot in the above diagrams represent various modified triple scalar couplings in presence of the effective operator.

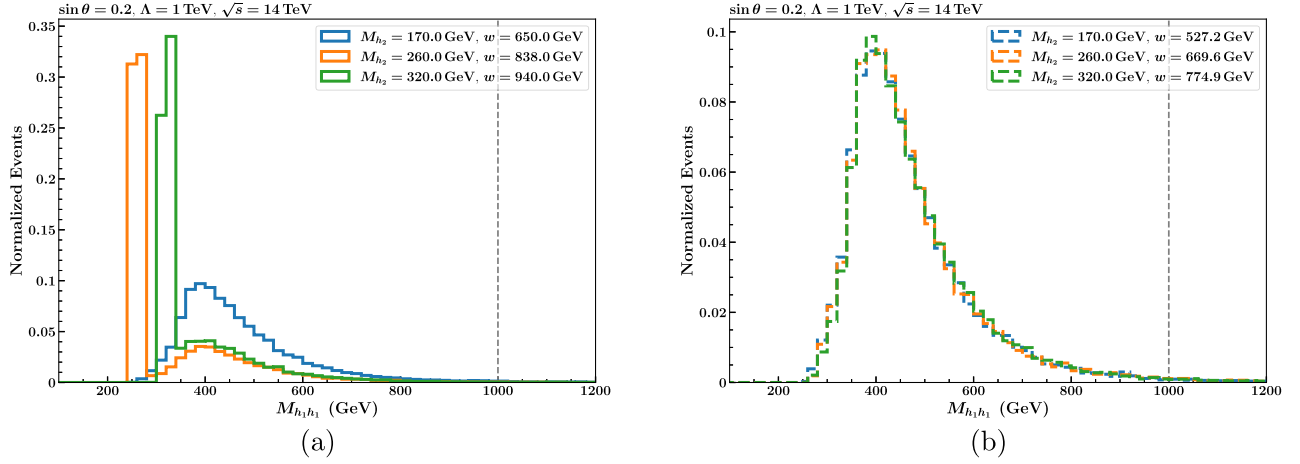


FIG. 18. (a) Normalized di-Higgs invariant mass ($M_{h_1 h_1}$) distributions obtained for $pp \rightarrow h_1 h_1$ process at $\sqrt{s} = 14$ TeV. (b) Represents the same but with different set of singlet scalar VEV that leads to vanishing $\lambda_{h_2 h_1 h_1}$ coupling. The gray dashed vertical line denotes the cutoff scale $\Lambda = 1$ TeV.

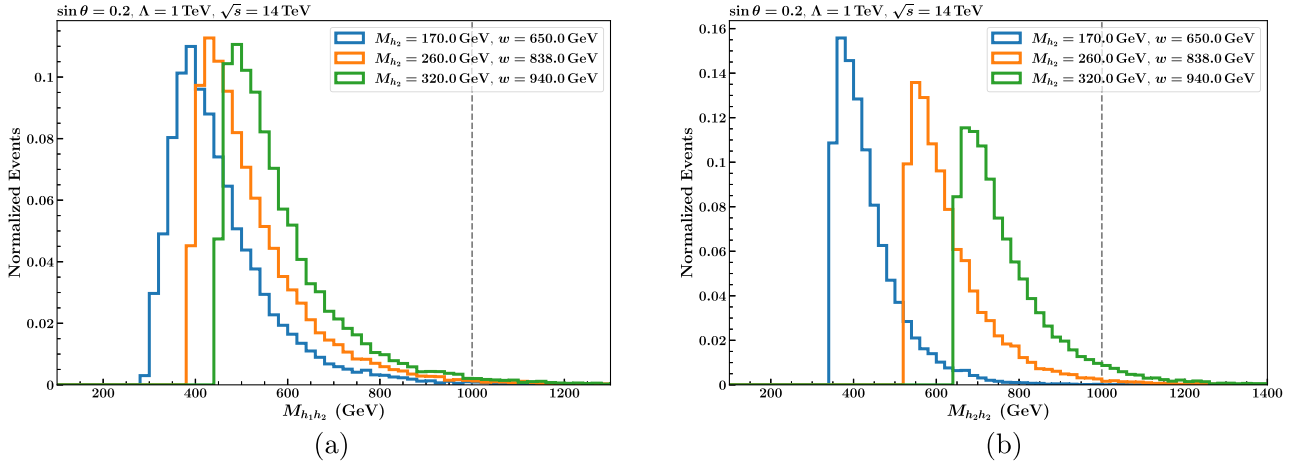


FIG. 19. Normalized discalar invariant mass ($M_{h_1 h_2}$ and $M_{h_2 h_2}$) distributions obtained for $pp \rightarrow h_1 h_2, h_2 h_2$ process at $\sqrt{s} = 14$ TeV. The gray dashed vertical line denotes the cutoff scale $\Lambda = 1$ TeV.

The absence of resonant peaks in some distributions can be attributed to the particular values of the singlet scalar VEV (w_2) for which $\lambda_{h_2 h_1 h_1}$ coupling vanishes (Fig. 24 in Appendix B). This phenomenon is a characteristic and distinguishing signature of the dimension-six operator considered in this work.

We have also tabulated the cross-sections for $pp \rightarrow h_i h_j$ production at the LHC for some representative benchmark points ($M_{h_2}, \sin \theta, w$) in Tables III and IV at 13 and 14 TeV LHC center of mass energies, respectively, with the requirement $M_{h_i h_j} < \Lambda$. The singlet scalar mass M_{h_2} is taken in the range 170 GeV to 340 GeV, with $\sin \theta = \pm 0.2$

TABLE III. Cross sections for $pp \rightarrow h_1 h_1$, $pp \rightarrow h_1 h_2$ and $pp \rightarrow h_2 h_2$ at $\sqrt{s} = 13$ TeV, requiring $M_{h_i h_j} < \Lambda = 1$ TeV. The quoted cross sections include a K factor of 2.3 [55].

M_{h_2} (GeV)	$\sin \theta = +0.2$				$\sin \theta = -0.2$			
	w (GeV)	$\sigma_{h_1 h_1}$ (fb)	$\sigma_{h_1 h_2}$ (fb)	$\sigma_{h_2 h_2}$ (fb)	w (GeV)	$\sigma_{h_1 h_1}$ (fb)	$\sigma_{h_1 h_2}$ (fb)	$\sigma_{h_2 h_2}$ (fb)
170	650.0	28.9	4.07	2.83	660.0	29.0	4.00	2.66
	527.2	29.3	3.37	1.18	476.5	29.4	2.95	0.535
	372.6	29.4	2.75	0.283	185.4	30.0	2.08	0.052
200	716.0	28.7	3.59	1.81	719.0	28.7	3.38	1.53
	571.8	28.8	2.93	0.729	521.2	29.1	2.52	0.305
	385.7	29.1	2.41	0.171	196.2	29.6	1.70	0.061
240	798.0	28.5	2.71	0.906	802.0	28.7	2.49	0.741
	636.0	28.3	2.26	0.382	585.4	28.6	1.89	0.154
	425.2	28.3	1.91	0.101	211.1	28.8	1.26	0.043
260	838.0	76.7	2.26	0.647	841.0	95.7	2.06	0.515
	669.6	27.9	1.92	0.280	619.1	27.8	1.60	0.114
	450.9	74.6	1.65	0.080	218.8	79.9	1.08	0.033
280	873.0	80.8	1.86	0.458	878.0	102	1.69	0.364
	703.1	27.5	1.61	0.206	653.5	27.9	1.35	0.085
	487.2	78.4	1.42	0.066	226.5	91.3	0.926	0.025
320	940.0	66.0	1.28	0.232	947.0	84.6	1.15	0.182
	774.9	26.9	1.15	0.117	724.4	27.3	0.968	0.050
	569.2	63.6	1.05	0.046	344.9	82.4	0.771	0.004
340	971.0	59.8	1.07	0.166	977.0	76.7	0.968	0.128
	811.1	26.5	0.986	0.088	760.7	26.9	0.844	0.039
	615.5	57.5	0.919	0.038	423.4	74.5	0.715	0.002

TABLE IV. Cross sections for $pp \rightarrow h_1 h_1$, $pp \rightarrow h_1 h_2$ and $pp \rightarrow h_2 h_2$ at $\sqrt{s} = 14$ TeV, requiring $M_{h_i h_j} < \Lambda = 1$ TeV. The quoted cross sections include a K factor of 2.3 [55].

M_{h_2} (GeV)	$\sin \theta = +0.2$				$\sin \theta = -0.2$			
	w (GeV)	$\sigma_{h_1 h_1}$ (fb)	$\sigma_{h_1 h_2}$ (fb)	$\sigma_{h_2 h_2}$ (fb)	w (GeV)	$\sigma_{h_1 h_1}$ (fb)	$\sigma_{h_1 h_2}$ (fb)	$\sigma_{h_2 h_2}$ (fb)
170	650.0	34.6	4.86	3.37	660.0	34.5	4.75	3.15
	527.2	34.9	4.01	1.40	476.5	35.2	3.51	0.636
	372.6	35.1	3.28	0.336	185.4	35.8	2.48	0.062
200	716.0	34.2	4.28	2.16	719.0	34.4	4.03	1.83
	571.8	34.4	3.49	0.874	521.2	34.6	3.00	0.365
	385.7	34.7	2.88	0.205	196.2	35.3	2.03	0.073
240	798.0	33.9	3.24	1.10	802.0	34.2	2.98	0.892
	636.0	33.9	2.70	0.458	585.4	34.1	2.27	0.186
	425.2	33.7	2.29	0.122	211.1	34.4	1.51	0.052
260	838.0	89.8	2.71	0.784	841.0	112	2.47	0.627
	669.6	33.4	2.30	0.339	619.1	33.1	1.92	0.139
	450.9	87.6	1.98	0.097	218.8	93.8	1.29	0.041
280	873.0	94.9	2.24	0.560	878.0	120	2.03	0.445
	703.1	33.0	1.94	0.252	653.5	33.3	1.62	0.105
	487.2	92.0	1.71	0.080	226.5	107	1.11	0.031

(Table continued)

TABLE IV. (Continued)

M_{h_2} (GeV)	$\sin \theta = +0.2$				$\sin \theta = -0.2$			
	w (GeV)	$\sigma_{h_1 h_1}$ (fb)	$\sigma_{h_1 h_2}$ (fb)	$\sigma_{h_2 h_2}$ (fb)	w (GeV)	$\sigma_{h_1 h_1}$ (fb)	$\sigma_{h_1 h_2}$ (fb)	$\sigma_{h_2 h_2}$ (fb)
320	940.0	77.9	1.54	0.286	947.0	99.9	1.39	0.223
	774.9	32.1	1.39	0.143	724.4	32.6	1.17	0.062
	569.2	75.0	1.27	0.056	344.9	97.2	0.929	0.005
340	971.0	70.7	1.29	0.205	977.0	90.6	1.17	0.157
	811.1	31.5	1.19	0.109	760.7	32.1	1.02	0.048
	615.5	68.0	1.11	0.047	423.4	87.9	0.864	0.002

and three different values of w are considered for each M_{h_2} and $\sin \theta$. As mentioned earlier, the first and third choice of w yield comparable values of the $\lambda_{h_2 h_1 h_1}$ coupling, but with opposite *sign* (Fig. 24). Therefore, the cross sections for these two different choices of w are determined by different interference patterns among various contributing Feynman diagrams. An additional interplay among these diagrams arises due to the variation of the $\lambda_{h_1 h_1 h_1}$ coupling as a function of w , as can be seen in Fig. 3(b). This feature is clearly visible in the quoted di-Higgs cross sections in Tables III and IV. One can see an enhancement in the di-Higgs cross sections by a factor 2–4 depending on the singlet scalar mass and the VEV of the scalar singlet. In contrast, the predicted di-Higgs cross sections remain close to the corresponding SM value for the choices of w that lead $\lambda_{h_2 h_1 h_1} = 0$. This behavior is also captured in Fig. 20, where we plot the ratio of the di-Higgs cross section as predicted in this scenario with respect to the corresponding SM value in $M_{h_2} - w$ plane. One can see that there exists a region of parameter space predicting close to SM-like di-Higgs rate even for $M_{h_2} > 2M_{h_1}$.

B. Triple-Higgs production

In this subsection, we discuss the triple-Higgs production which is an important channel to probe new physics beyond the SM, as a part of the multiscalar searches at the LHC. The next-to-leading-order triple Higgs production cross section in the SM is of the order of 0.103 fb at 14 TeV LHC center of mass energy [85,86]. Some representative Feynman diagrams for triple Higgs process in the context of the model described in Sec. II are shown in Fig. 21. This includes diagrams that are present in the SM plus additional diagrams due to the presence of the singlet scalar field. In Fig. 22, we plot the triple Higgs invariant mass distributions to validate the EFT framework and to obtain the region of phase space consistent with it. The corresponding cross sections for all events satisfying $M_{h_1 h_1 h_1} < \Lambda = 1$ TeV are quoted in Table V. A SM-like K factor is also assumed to estimate this cross section.

A close look at Table V indicates that the triple-Higgs cross sections also follow a pattern similar to that of the di-Higgs production at the LHC, in particular, with respect to the identical set of values chosen for the singlet scalar VEV.

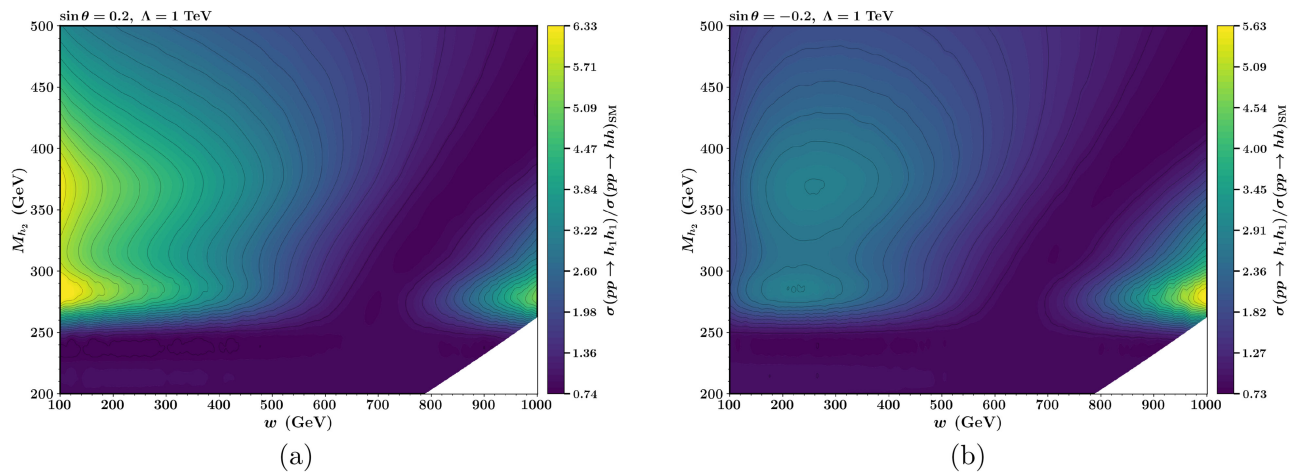


FIG. 20. Ratio of the di-Higgs cross section, $\frac{\sigma(pp \rightarrow h_1 h_1)}{\sigma(pp \rightarrow hh)_{\text{SM}}}$ in the $w - M_{h_2}$ plane for two different values of: (a) $\sin \theta = 0.2$ and (b) $\sin \theta = -0.2$, assuming $\Lambda = 1$ TeV. The value of the ratio is represented in the color gradient. The white shaded region in the bottom right corner is excluded by the theoretical constraints discussed in Sec. II A 1.

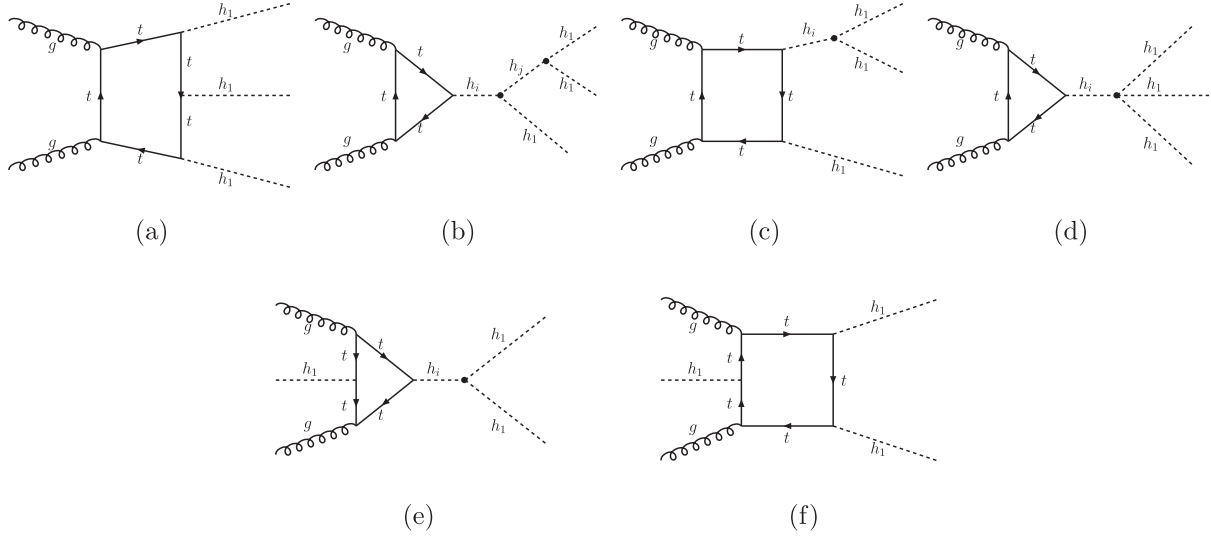


FIG. 21. Representative Feynman diagrams for $pp \rightarrow h_1 h_1 h_1$ process. Here, the black dot in the above diagrams represents various modified triple scalar couplings in presence of the effective operator.

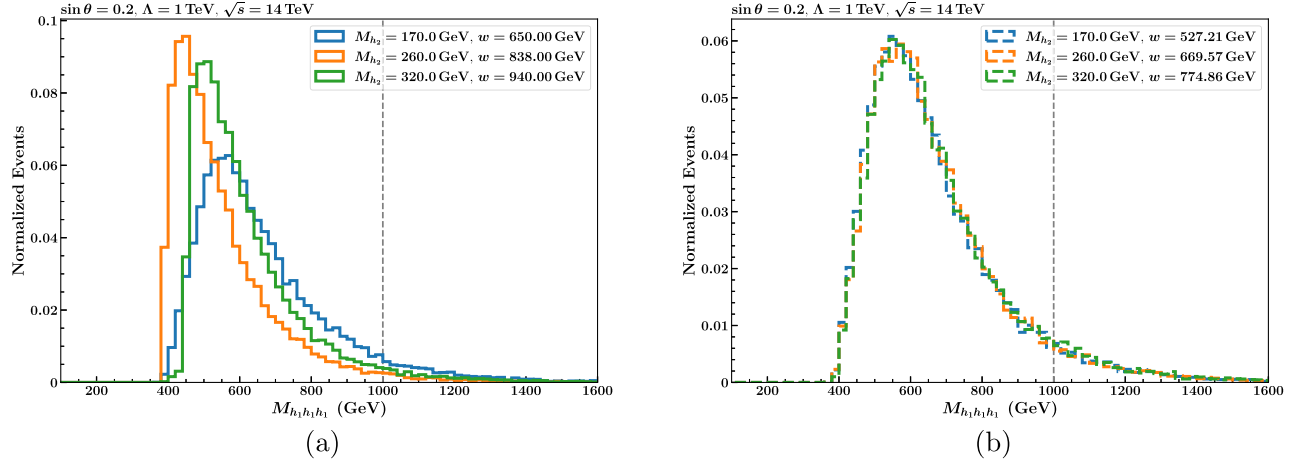


FIG. 22. (a) Normalized triple-Higgs invariant mass distributions ($M_{h_1 h_1 h_1}$) obtained for $pp \rightarrow h_1 h_1 h_1$ process at $\sqrt{s} = 14$ TeV. (b) Represents the same but with a different set of singlet scalar VEV that leads to vanishing $\lambda_{h_2 h_1 h_1}$ coupling. The gray dashed vertical line denotes the cutoff scale $\Lambda = 1$ TeV.

In fact, the behavior of the $pp \rightarrow h_1 h_1 h_1$ cross sections is also controlled by the same set of triple scalar couplings ($\lambda_{h_i h_1 h_1}$) that are relevant for di-Higgs production. The triple-Higgs cross sections for $M_{h_2} < 2M_{h_1}$ are close to the SM value, as expected, due to the absence of resonant contribution. However, for $M_{h_2} > 2M_{h_1}$ there is an enhancement in the triple Higgs cross section by almost a factor of 5 (6) for $\sin \theta = 0.2(-0.2)$ and certain choice of the singlet scalar VEV that ensures a resonance peak in the di-Higgs invariant mass distributions. In the singlet scalar mass range 260–340 GeV the cross section gradually decreases with an increase in M_{h_2} due to usual phase space suppression. For $M_{h_2} > 3M_{h_1}$ GeV, there is an additional enhancement in the cross section due to the resonant triple-Higgs

production, which can be attributed to the class of Feynman diagram depicted in Fig. 21(d). For example, at $M_{h_2} = 380$ GeV and $\sin \theta = 0.2$ the triple-Higgs cross section turns out to be 0.261 fb for $w = 500$ GeV. This point however does not give rise to SFOEWPT.

VII. CONCLUSIONS

We have considered an effective field theory framework where the SM is augmented with an additional singlet complex scalar field charged under a local $U(1)_D$ gauge group. The tree-level scalar potential contains additional dimension-six operator involving the Higgs and the singlet scalar fields of the form $|H|^2|\phi|^4$. The complex scalar field is also responsible for the generation of the mass of the

TABLE V. Cross sections (in fb) for $pp \rightarrow h_1 h_1 h_1$ at $\sqrt{s} = 14$ TeV, requiring $M_{h_1 h_1 h_1} < \Lambda = 1$ TeV. The quoted cross sections include a K factor of 2.34 to account for next-to-leading-order effects.

M_{h_2} (GeV)	$\sin \theta = +0.2$		$\sin \theta = -0.2$	
	w (GeV)	$\sigma_{h_1 h_1 h_1}$ (fb)	w (GeV)	$\sigma_{h_1 h_1 h_1}$ (fb)
170	650.0	0.077	660.0	0.077
	527.2	0.079	476.5	0.079
	372.6	0.080	185.4	0.081
200	716.0	0.076	719.0	0.077
	571.8	0.078	521.2	0.078
	385.7	0.078	196.2	0.079
240	798.0	0.077	802.0	0.078
	636.0	0.075	585.4	0.076
	425.2	0.074	211.1	0.075
260	838.0	0.373	841.0	0.457
	669.6	0.074	619.1	0.075
	450.9	0.309	218.8	0.239
280	873.0	0.362	878.0	0.445
	703.1	0.072	653.5	0.074
	487.2	0.326	226.5	0.277
320	940.0	0.227	947.0	0.283
	774.9	0.069	724.4	0.071
	569.2	0.236	344.9	0.249
340	971.0	0.174	977.0	0.217
	811.1	0.068	760.7	0.070
	615.5	0.196	423.4	0.217

$U(1)_D$ gauge boson, referred to as dark photon. We emphasize that the addition of the dimension-six operator allowed by all possible symmetries of the theory gives rise to distinctive features in the context of first order EWPT, resulting in GW signal and multiscalar productions at the LHC. This is in contrast to the conventional singlet scalar extension of the SM in absence of the dimension six term. Importantly, even when a dimension-six operator involving the singlet scalar is present, the phenomenology can differ substantially in terms of the phase transition pattern and the viable parameter space consistent with SFOEWPT depending on whether the singlet scalar acquires a VEV.

We identify the SFOEWPT compatible parameter space in the $M_{h_2} - \sin \theta$ plane while satisfying relevant theoretical and experimental constraints. We further present a detailed prediction of the GW power spectrum and asses their sensitivity at the future GW experiments. In addition, we also present cross sections for multiscalar production processes at the LHC. Our main findings can be summarized as follows:

- (i) In our setup, the phase transition proceeds predominantly through $(0, u(T)) \rightarrow (v(T), w(T))$. An important feature of our scenario is that the tree-level

potential difference between the local extremum $(0, u)$ and the global minima (v_{EW}, w) is controlled by the singlet scalar VEV (w) for a fixed cutoff scale Λ . With increasing w this difference decreases, which in turn helps to reduce the critical temperature, therefore, allowing a wider region of viable parameter space in the $M_{h_2} - \sin \theta$ plane consistent with SFOEWPT. In particular, we have shown that a completely new SFOEWPT compatible parameter space (for $M_{h_2} > M_{h_1}$) opens up in presence of the dimension six operator.

- (ii) In absence of the dimension-six operator one requires large Higgs-singlet portal coupling (λ_{hs}) or scalar mixing angle ($\sin \theta$) to achieve SFOEWPT [10,12,26]. The large value of the former might hamper perturbative unitarity while that of the latter is disfavored by the Higgs-signal strength measurements. The presence of the dimension-six operator helps to weaken the correlation between $\sin \theta$ and λ_{hs} in this scenario. Therefore, in this setup SFOEWPT can be achieved even for smaller $\sin \theta$ and $|\lambda_{hs}| \sim 1$.
- (iii) Additionally, the *dominant* effect of UV physics on the phase transition dynamics enters as a combination of $\frac{w}{\Lambda}$ and does not necessarily decouple in the large Λ limit as long as w is also large (but still $< \Lambda$), keeping the ratio $\frac{w}{\Lambda}$ fixed. This is in contrast to the scenarios with higher-dimensional operators involving only the Higgs field, or various combinations of the Higgs and additional singlet scalar fields where the latter does not acquire a VEV. It turns out that for $M_{h_2} = 200$ GeV (250 GeV) and $|\sin \theta| = 0.2$, the required minimum value of this ratio to achieve SFOEWPT is 0.71 (0.82).
- (iv) The parameter space consistent with SFOEWPT can be probed at future gravitational wave facilities as well. In particular, we have shown that the GW power spectrum generated during the phase transition is sensitive to LISA, BBO, DECIGO, μ Ares, ultimate-DECIGO, DECIGO-corr, and ultimate-DECIGO-corr gravitational wave detectors unlike usual singlet scalar extension of the SM with spontaneous $\mathbb{Z}_2$ symmetry breaking. For a fixed mass, mixing angle, and cutoff scale, the peak GW energy density shifts toward lower frequencies and attains a larger amplitude as w increases, thereby improving its sensitivity to future GW experiments.
- (v) This model also predicts a strong correlation between the SFOEWPT-viable parameter space and enhanced di-Higgs and multiscalar production rates at the LHC. This arises from the fact that achieving SFOEWPT requires a modified scalar potential, which in turn alters the trilinear and quartic scalar couplings. Consequently, measurements of di-Higgs and triple-Higgs cross sections at the LHC can probe

the nature of the EWPT. From a purely collider perspective, we also illustrate the sensitivity of the di-Higgs production cross section to the singlet scalar VEV (w) for a fixed cutoff scale Λ in the presence of the dimension-six operator. In particular, we emphasize that even in the presence of an additional scalar resonance in the mass range (250–500) GeV, a resonance peak in the di-Higgs invariant mass distribution may be absent for certain values of w . This is a very peculiar consequence of the dimension-six operator considered in this work. Moreover, for $M_{h_2} \geq 2M_{h_1}$ and $|\sin \theta| = 0.2$ the di-Higgs rate at the LHC can be 2–4 times that predicted in the SM in a region where SFOEWPT is achieved.

Although our analysis primarily focuses on the dimension-six operator of the form $|H|^2|\phi|^4$, additional dimension-six operators may also arise from the same UV completion. The inclusion of all possible dimension-six operators allowed by the symmetries of the theory can further enlarge the SFOEWPT-compatible parameter space and/or reduce the required minimum value of the ratio $\frac{w}{\Lambda}$ to achieve SFOEWPT.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: VARIATION OF LAGRANGIAN PARAMETERS

The Lagrangian parameters $\{\mu_h, \mu_s, \lambda_h, \lambda_s, \lambda_{hs}\}$ can be expressed in terms of the physical parameters $\{M_{h_1}, M_{h_2}, \sin \theta, v_{EW}, w, \Lambda\}$ as follows:

$$\begin{aligned} \mu_h^2 &= \frac{1}{2}(M_{h_1}^2 \cos^2 \theta + M_{h_2}^2 \sin^2 \theta) + (M_{h_2}^2 - M_{h_1}^2) \frac{w}{2v_{EW}} \\ &\times \sin \theta \cos \theta - \frac{w^4}{4\Lambda^2}, \end{aligned} \quad (\text{A1})$$

$$\lambda_h = \frac{M_{h_1}^2 \cos^2 \theta + M_{h_2}^2 \sin^2 \theta}{2v_{EW}^2}, \quad (\text{A2})$$

$$\begin{aligned} \mu_s^2 &= \frac{1}{2}(M_{h_1}^2 \sin^2 \theta + M_{h_2}^2 \cos^2 \theta) + (M_{h_2}^2 - M_{h_1}^2) \frac{v_{EW}}{2w} \\ &\times \sin \theta \cos \theta - \frac{v_{EW}^2 w^2}{2\Lambda^2}, \end{aligned} \quad (\text{A3})$$

$$\lambda_s = \frac{1}{2w^2}(M_{h_1}^2 \sin^2 \theta + M_{h_2}^2 \cos^2 \theta) - \frac{v_{EW}^2}{2\Lambda^2}, \quad (\text{A4})$$

$$\lambda_{hs} = -\frac{w^2}{\Lambda^2} + \frac{(-M_{h_1}^2 + M_{h_2}^2) \cos \theta \sin \theta}{v_{EW} w}. \quad (\text{A5})$$

The cutoff scale Λ that appears in the Lagrangian is one of the free parameters of the theory. In Fig. 23, we plot several Lagrangian parameters against the singlet scalar VEV (w) for different choices of its mass M_{h_2} and scalar mixing angle $\sin \theta$.

APPENDIX B: TRIPLE SCALAR COUPLINGS

1. $h_1 - h_1 - h_1$ coupling

The $h_1 - h_1 - h_1$ coupling can be expressed as

$$\lambda_{h_1 h_1 h_1} = 3 \frac{M_{h_1}^2}{v_{EW}} \left[\cos^3 \theta - \frac{v_{EW}}{w} \sin^3 \theta + 2 \frac{v_{EW}^2 w^2}{M_{h_1}^2 \Lambda^2} \cos \theta \sin^2 \theta \right], \quad (\text{B1})$$

highlighting the additional contribution due to the presence of the dimension-six operator.

2. $h_2 - h_1 - h_1$ coupling

Similarly, the modified $h_2 - h_1 - h_1$ coupling in presence of the dimension-six operator can be written as

$$\begin{aligned} \lambda_{h_2 h_1 h_1} &= (2M_{h_1}^2 + M_{h_2}^2) \sin \theta \cos \theta \left(\frac{\cos \theta}{v_{EW}} + \frac{\sin \theta}{w} \right) \\ &- 2v_{EW} \frac{w^2}{\Lambda^2} \sin \theta (2 \cos^2 \theta - \sin^2 \theta). \end{aligned} \quad (\text{B2})$$

The variation of this coupling as a function of w is shown in Fig. 24, for different choices of M_{h_2} and $\sin \theta$ with fixed cutoff scale $\Lambda = 1$ TeV.

APPENDIX C: DEPENDENCE ON THE RENORMALIZATION SCALE

In Fig. 25, we present the variation of ϕ_c/T_c , T_n , α , and β/H as a function of w and their dependency on the renormalization scale $Q = 500, 800, 1000$ GeV. It is clear that the sensitivity of the above quantities on the choice of

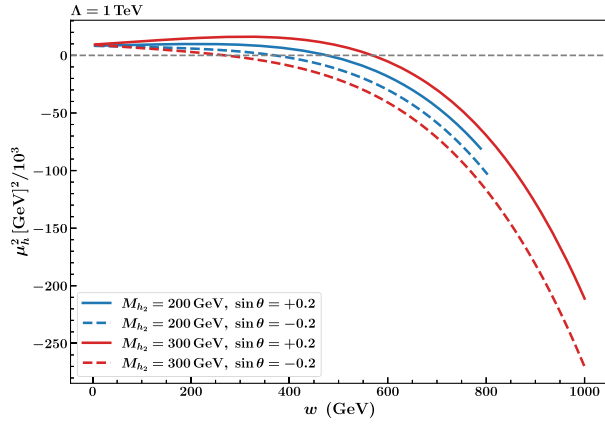
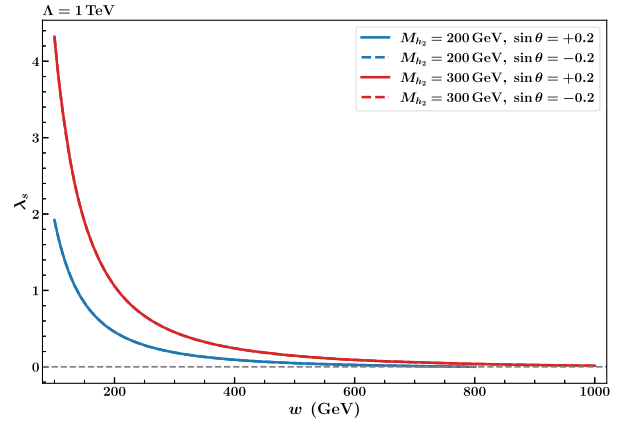
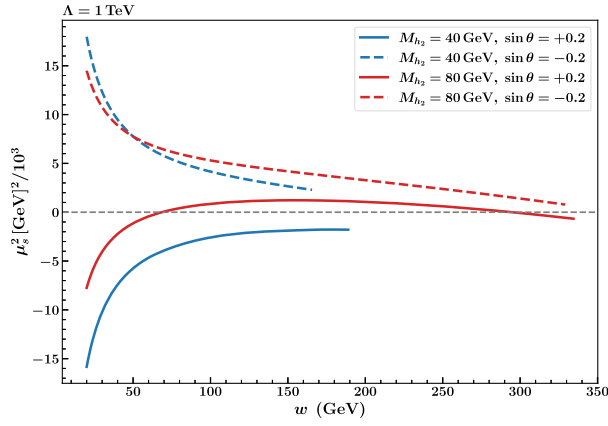
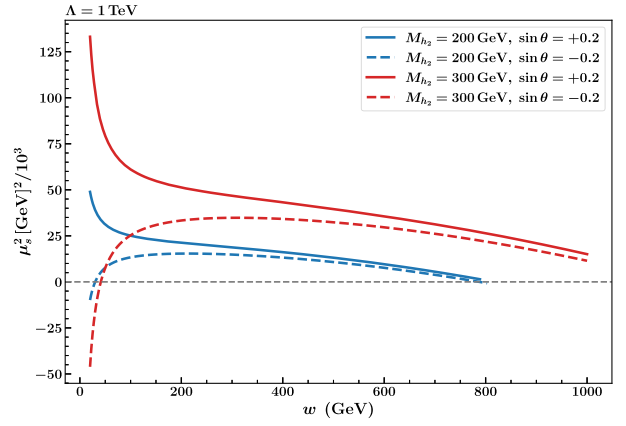
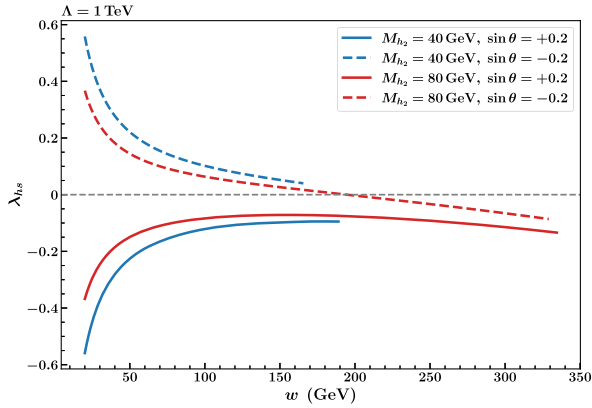
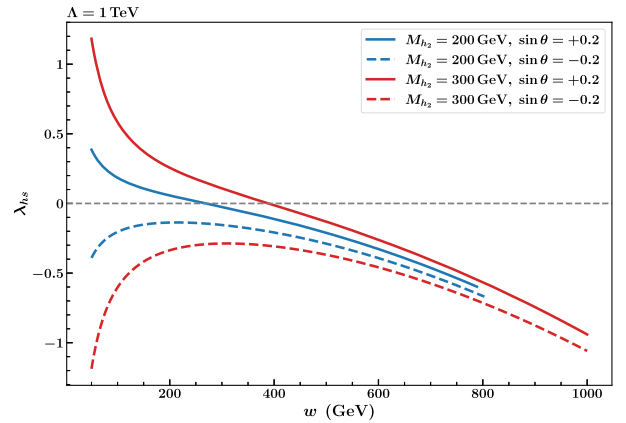
(a) μ_h^2 vs w (b) λ_s vs w (c) μ_s^2 vs w (d) μ_s^2 vs w .(e) λ_{hs} vs w (f) λ_{hs} vs w

FIG. 23. Variation of the relevant Lagrangian parameters as a function of the singlet scalar VEV (w) for different choices of M_{h_2} and $\sin \theta$ with fixed Λ (1 TeV).

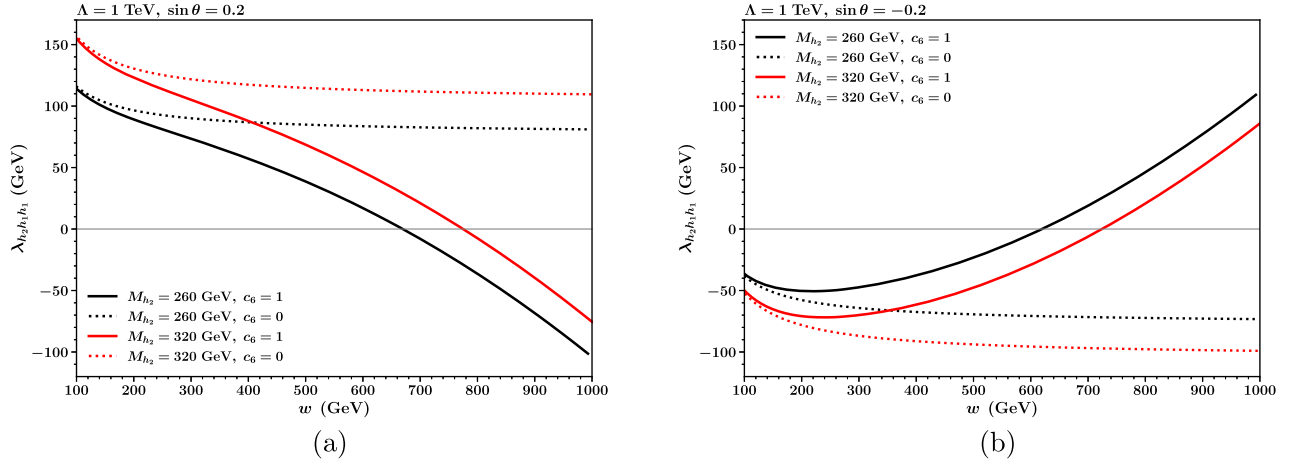


FIG. 24. Variation of $\lambda_{h_2 h_1 h_1}$ with singlet scalar VEV (w) for (a) $\sin \theta = 0.2$, (b) $\sin \theta = -0.2$ assuming $\Lambda = 1$ TeV. Here, the dotted and solid lines represent the variation of this coupling in absence ($c_6 = 0$) and presence ($c_6 = 1$) of the dimension-six operator, respectively.

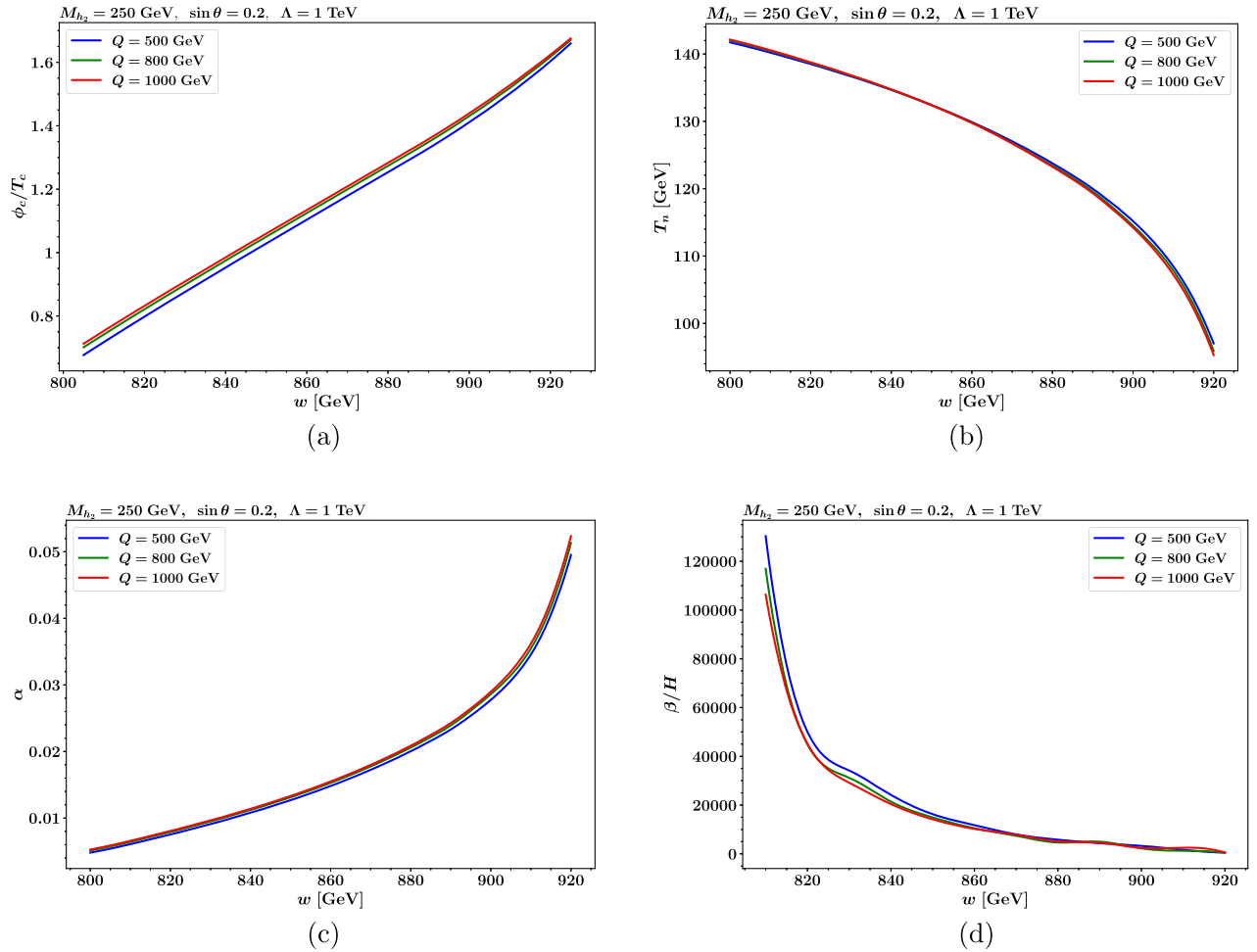


FIG. 25. Variation of (a) ϕ_c/T_c , (b) T_n , (c) α , (d) β/H with singlet scalar VEV w , for three different choices ($Q = 500, 800, 1000$ GeV) of renormalization scale in the CW potential.

renormalization scale is very mild owing to the on-shell-like renormalization used in our analysis, i.e., adding the finite pieces of Eq. (24) to the effective potential, satisfying the conditions of Eq. (23).

APPENDIX D: IMPACT OF DIMENSION-EIGHT OPERATORS

We extend the tree-level potential considered in Eq. (3) by the two additional dimension-eight operators

$$V_0(H, \phi) = -\mu_h^2 |H|^2 + \lambda_h |H|^4 - \mu_s^2 |\phi|^2 + \lambda_s |\phi|^4 + \lambda_{hs} |H|^2 |\phi|^2 + \frac{c_6}{\Lambda^2} |H|^2 |\phi|^4 + \frac{c_8^{(1)}}{\Lambda^4} |H|^2 |\phi|^6 + \frac{c_8^{(2)}}{\Lambda^4} |H|^4 |\phi|^4. \quad (\text{D1})$$

Since the tree-level potential difference ($\Delta V = V_0(0, u) - V_0(v_{\text{EW}}, w)$) plays a crucial role in determining ϕ_c/T_c , we examine its variation with the singlet scalar VEV (w) (Fig. 26) in presence of these additional dimension-eight

operators for a representative choice of singlet scalar mass $M_{h_2} = 250$ GeV, $\sin \theta = \pm 0.2$, $\Lambda = 1$ TeV, and different choices of the Wilson coefficients $c_8^{(1)}$ and $c_8^{(2)}$ corresponding to the dimension-eight operators.

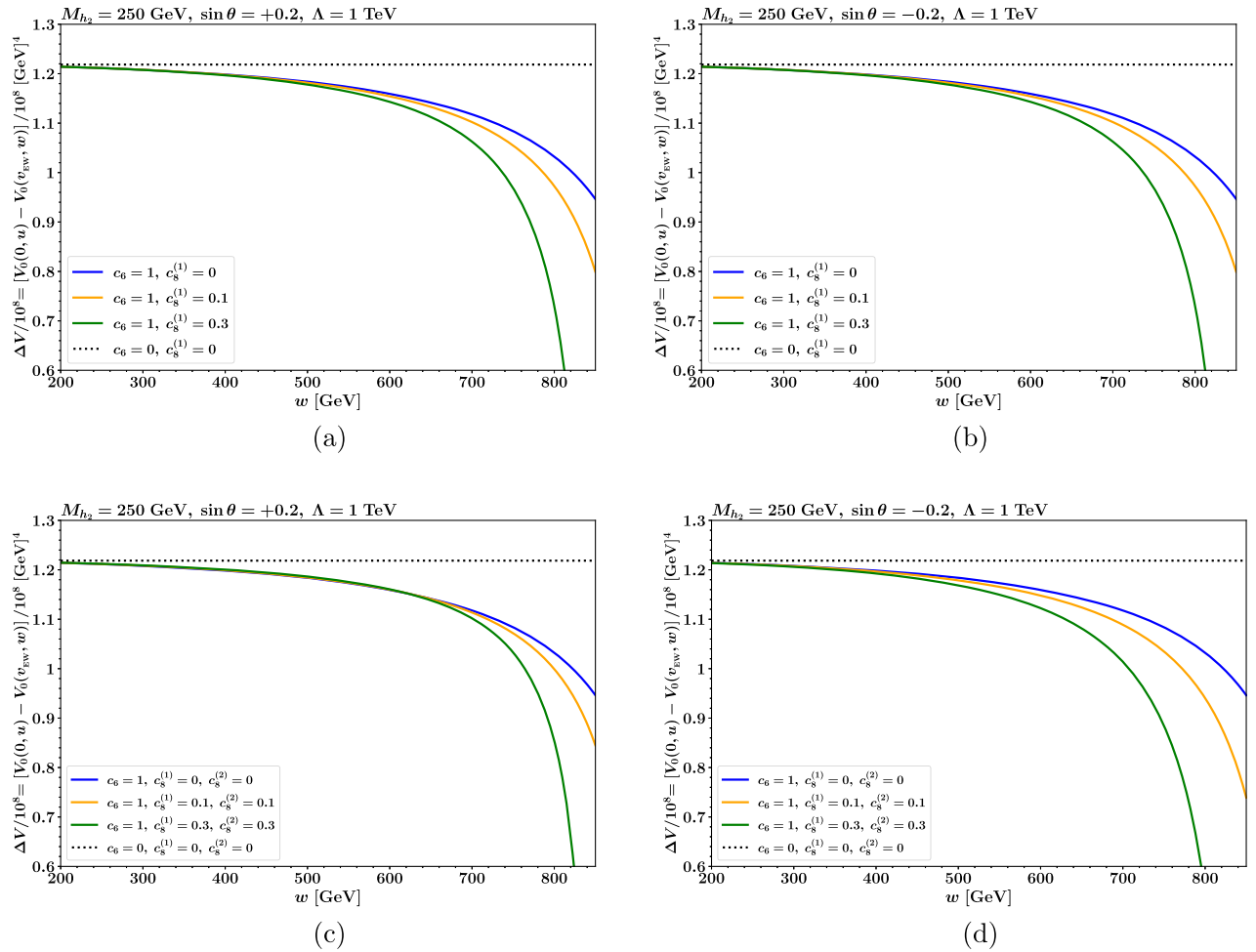


FIG. 26. Variation of tree-level potential difference $\Delta V = V_0(0, u) - V_0(v_{\text{EW}}, w)$ with and without the relevant dimension-eight operators.

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