

Simplicity of Mellin amplitudes for $\text{AdS}_3 \times \text{S}^3$ F. Aprile^{1,*}, C. Behan^{2,†}, R. S. Pitombo^{2,3,‡} and M. Santagata^{4,§}¹*Departamento de Física Teórica and IPARCOS, Facultad de Ciencias Físicas, Universidad Complutense, 28040 Madrid, Spain*²*ICTP South American Institute for Fundamental Research,**Instituto de Física Teórica UNESP, Rua Doutor Bento Teobaldo Ferraz 271, 01140-070 São Paulo, Brazil*³*Mathematical Institute, University of Oxford, Andrew Wiles Building, Radcliffe Observatory Quarter, Woodstock Road, Oxford OX2 6GG, United Kingdom*⁴*Department of Physics, National Taiwan University, Taipei 10617, Taiwan*

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The spectrum of half-Bogomol'nyi–Prasad–Sommerfeld (BPS) single-particle operators of the D1-D5 system in the supergravity regime is dual to the spectrum of Kaluza-Klein modes of tensor and graviton multiplets in $\text{AdS}_3 \times \text{S}^3$. We present *simple* formulas for all four-point tree-level correlators of scalar half-BPS primary operators in the spectrum, and in particular, we bootstrap the four-point correlator of the graviton multiplet for *all* KK levels. A key insight of our approach is the use of generalized Mellin space, and a decomposition of the correlators into distinct subsectors each one characterized by different on shell constraints among the Mellin variables. The resulting Mellin amplitudes involve a small set of building blocks with manifest properties under a six-dimensional symmetry.

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Introduction. Pulling the *strings* of gravity in AdS/CFT depends greatly on the ability to compute boundary correlation functions, especially in those superconformal field theories (SCFT) that have a weakly coupled gravitational dual. A paradigmatic example is that of $\mathcal{N} = 4$ super Yang-Mills (SYM); in the regime where the theory is dual to supergravity on $\text{AdS}_5 \times \text{S}^5$ all four-point correlators that describe tree-level scattering of Kaluza-Klein (KK) modes of the type IIB graviton are known in closed form thanks to the bootstrap approach pioneered in [1,2]. In this paper we extend these results to the D1-D5 system dual to $\text{AdS}_3 \times \text{S}^3$ in the supergravity regime, which is another paradigmatic example of AdS/CFT considered in the seminal paper [3]. Following the bootstrap approach, and previous work done in [4–10], our main purpose is to determine *all* tree-level half-BPS scalar four-point correlators in this theory.

The most novel and relevant aspect of the D1-D5 system is the fact that the spectrum of half-BPS primary operators consists of two distinct multiplets; the tensor multiplets

and the graviton multiplets. These are coupled to each other, and as a result, there are three types of four-point correlators that one could consider. The earliest studies focused on KK correlators of tensor multiplets showing that these can be organized into a single correlator of 6D scalar fields [4–6], closely resembling the structure that [11] found in $\text{AdS}_5 \times \text{S}^5$. However, the 6D origin of the scalar primary operators in the graviton multiplets is a self-dual three-form [7], rather than a scalar field. Dealing with the polarizations of this three-form introduces several challenges. For example, the mixed tensor-graviton correlators can be given in a 6D conformally covariant way [7], but the explicit expressions [9] for all KK levels are unwieldy, and therefore a similar approach has not been attempted for the four gravitons correlators.

The rigorous bootstrap approach of [10] yields explicit results for infinite families of graviton correlators, that could not be obtained otherwise, and therefore solves the problem of generating results for the graviton correlators. However, the bootstrap approach does not automatically give a formula with explicit dependence on the KK level, since the latter have to be specified on a case-by-case basis, and a bottleneck of this approach is the fact that the number of Witten diagrams needed to build the ansatz for the correlator grows fast with the KK levels, making the computation unfeasible. Thus, in order to organize the output, and prove the result for all KK levels, there is really the need for an additional principle. We shall see that this principle is naturally found when the correlators are written in

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generalized $\text{AdS}_3 \times S^3$ Mellin space, a version of Mellin space where both anti-de Sitter (AdS) and S are on the same footing.

First of all, Mellin space has long been recognized to be a powerful tool for studying holographic correlators [12,13], owing to the fact that, under a properly defined Mellin transform, a holographic correlator is dual to an amplitude whose structure makes the flat-space limit manifest [13]. Following [8,14–16], we will consider a generalization of Mellin space that includes a discrete Mellin transform on the R-symmetry coordinates, leading to the notion of an $\text{AdS}_3 \times S^3$ Mellin amplitude. In this setting the flat-space limit can be uplifted to 6D, by the same argument as in [14]. The $\text{AdS}_3 \times S^3$ Mellin amplitude, which generically depends on all Mellin variables and the KK levels, therefore encodes a 6D flat-space amplitude. Then, a symmetry enhancement takes place in all cases in which the $\text{AdS}_3 \times S^3$ Mellin amplitude only depends on those combinations of variables that reorganize into 6D Mandelstam invariants. This special feature of the amplitude trivializes the dependence on the KK levels. Crucially, even when the symmetry enhancement is approximate, it still serves as an organizing principle for an otherwise increasingly complex bottom-up ansatz of Witten diagrams. Making the 6D symmetry manifest is the basic idea that we shall exploit to organize the scalar correlators of the D1-D5 system. The result for the four tensor correlators [4–6] was given in [8]. Here we will consider the cases of the mixed tensor-graviton correlators and the four graviton correlators, since so far, in both these cases, the existing formalism to write down these correlators does not make both the symmetries and the KK level dynamics manifest.

Our main result in this letter is to demonstrate the *simplicity* of the three scalar types of single particle correlators of the D1-D5 SCFT₂ in $\text{AdS}_3 \times S^3$, by providing explicit and compact expressions for *all* KK levels, in particular, the complete four-point graviton correlators.

Four-point correlators. In the D1-D5 SCFT₂ dual to type IIB string theory on $\text{AdS}_3 \times S^3 \times M$, we will denote the primary operators dual to gravitons by σ_p where p is the KK level. The number of tensor multiplets arising from the reduction of R fields is equal to the Hodge number $h^{1,1}$ of M . There is also one from NS fields, which is on the same footing in supergravity (SUGRA), so we will denote these primary operators by s_p^i where i is a fundamental $SO(h^{1,1}+1)$ index. Note that $M = T^4$ has $h^{1,1} = 4$ and leads to (2, 2) SUGRA while $M = K3$ has $h^{1,1} = 20$ and leads to (2, 0) SUGRA. This paper will be specific to $K3$, closely following the approach of [6,10] that made use of the (2, 0) three-point couplings from [17]. However, we expect that similar results hold for T^4 as well. In particular, the extra supersymmetry in (2, 2) forces one

of the tensor multiplets [18] to have the same tree-level amplitudes of the (2, 0) theory [20], and therefore, as discussed in [5], this leads to agreement of four-point functions when all $SO(h^{1,1}+1)$ indices are the same.

The correlators we will study are

$$\langle s_{p_1}^{i_1} s_{p_2}^{i_2} s_{p_3}^{i_3} s_{p_4}^{i_4} \rangle \quad \langle s_{p_1}^{i_1} s_{p_2}^{i_2} \sigma_{p_3} \sigma_{p_4} \rangle \quad (1)$$

and

$$\langle \sigma_{p_1} \sigma_{p_2} \sigma_{p_3} \sigma_{p_4} \rangle. \quad (2)$$

An operator is inserted at spacetime coordinates, $x_i, \bar{x}_i$, and is specified by R-symmetry $SU(2)_L \times SU(2)_R$ spinors $(\begin{smallmatrix} 1 \\ y_i \end{smallmatrix}), (\begin{smallmatrix} 1 \\ \bar{y}_i \end{smallmatrix})$. Cross ratios $x, \bar{x}, y, \bar{y}$, are defined by

$$x = \frac{x_{12}x_{34}}{x_{13}x_{24}}, \quad y = \frac{y_{12}y_{34}}{y_{13}y_{24}}, \quad (3)$$

with $x_{ij} = x_i - x_j$, $y_{ij} = y_i - y_j$, and $\bar{x}, \bar{y}$ defined similarly.

Superconformal symmetry implies that correlators with external half-BPS scalar fields Φ , such as (1) and (2), can be written in the form [8,21],

$$\langle \Phi_1 \Phi_2 \Phi_3 \Phi_4 \rangle = k + (x-y)f + (\bar{x}-\bar{y})\bar{f} + (x-y)(\bar{x}-\bar{y})\mathcal{H}, \quad (4)$$

where k is constant in the cross ratios, $f, \bar{f}$ are single-variable functions determined by protected three-point couplings, for example through the chiral algebra twist discussed in [6], and the reduced correlator $\mathcal{H}$ is a two-variable function that splits into a plus and a minus sector,

$$\mathcal{H} = x_{13}^2 x_{24}^2 y_{13}^2 y_{24}^2 [\mathcal{H}^+(x_{ij}^2, y_{ij}^2) + \quad (5)$$

$$-(x-\bar{x})(y-\bar{y})x_{13}^2 x_{24}^2 y_{13}^2 y_{24}^2 \mathcal{H}^-(x_{ij}^2, y_{ij}^2)], \quad (6)$$

with $x_{ij}^2 \equiv x_{ij}\bar{x}_{ij}$ and $y_{ij}^2 \equiv y_{ij}\bar{y}_{ij}$. The reduced correlators $\mathcal{H}^\pm$ are symmetric functions of both $x, \bar{x}$ and $y, \bar{y}$. However, unlike higher-dimensional CFTs such as $\mathcal{N} = 4$ SYM in 4D, only the *simultaneous* exchange of $x \leftrightarrow \bar{x}$, $y \leftrightarrow \bar{y}$ leaves the full correlator invariant. See again (4)–(6).

In the supergravity regime, the leading $O(1)$ contributions—in the large central charge c expansion—are disconnected two-point functions, when they exist. These can be rearranged into the form dictated by superconformal symmetry, thus giving a contribution to $k, f, \bar{f}$ and $\mathcal{H}^\pm$. In general, free-type contributions have a vanishing Mellin transform and we will not consider them further.

Generalized mellin space: We are interested in the tree-level connected $O(1/c)$ contribution to $\mathcal{H}^\pm$, and we will present results using the Mellin transform,

$$\mathcal{H}_{p_1, p_2, p_3, p_4}^{\pm} = \oint_{\check{s}} d\hat{s} \mathcal{M}^{\pm}(\hat{s}_{ij}, \check{s}_{ij}; p_1, p_2, p_3, p_4), \quad (7)$$

with the $\text{AdS}_3 \times \text{S}^3$ integration “measure” defined by

$$\oint_{\check{s}} d\hat{s} \equiv \sum_{\check{s}, \check{t}} \oint_{-i\infty}^{+i\infty} \frac{ds dt}{(2\pi i)^2} \prod_{i < j} x_{ij}^{2\hat{s}_{ij}} y_{ij}^{2\check{s}_{ij}} \frac{\Gamma[-\hat{s}_{ij}]}{\Gamma[\check{s}_{ij} + 1]}. \quad (8)$$

The Mellin variables are symmetric $\hat{s}_{ij} = \hat{s}_{ji}$, $\check{s}_{ij} = \check{s}_{ji}$, off-diagonal, $\hat{s}_{ii} = \check{s}_{ii} = 0$, and constrained by “on shell” relations that depend on p_i and have the form,

$$\sum_{j=1}^4 \check{s}_{ij} = p_i - \delta_S, \quad \sum_{j=1}^4 \hat{s}_{ij} = -p_i - \delta_{\text{AdS}}, \quad (9)$$

where $\delta_S, \delta_{\text{AdS}} \in \mathbb{N}$. Solving the constraints gives four independent variables, conjugate to the cross ratios. We shall call s and t the independent variables in the $\hat{s}_{ij}$, and $\check{s}$ and $\check{t}$ those in the $\check{s}_{ij}$. See (8).

We will always work with fixed values of δ_{AdS} ,

$$\delta_{\text{AdS}} = 1 \quad \text{for } \mathcal{H}^+, \quad \delta_{\text{AdS}} = 2 \quad \text{for } \mathcal{H}^-. \quad (10)$$

Regarding δ_S , we will fix it *a posteriori* and add this information to the symbol defined in (8). This is a crucial novelty of our formalism, and is motivated by the idea of simplifying the dependence on the R-symmetry variables in the Mellin amplitude. To see how, let us recall that $\mathcal{H}^{\pm}$ are

by construction polynomials in the y_{ij}^2 . This polynomiality shows up in the measure as a consequence of the fact that the $\Gamma[\check{s}_{ij} + 1]^{-1}$ vanish unless $\check{s}_{ij} \geq 0$ [which also shows that the sum (8) can be kept unbounded]. Then, because $\check{s}_{ij} \geq 0$, the corresponding on shell constraints in (9) imply that unless $p_i \geq \delta_S$ there is no contribution to the Mellin amplitude. Thus, the value of δ_S can be used as a knob to stratify the (numerator of the) Mellin amplitude in the R-symmetry variables. Note however that superconformal symmetry, see (4)–(6), gives from the start that $\delta_S \geq 1$ for $\mathcal{H}^+$ and $\delta_S \geq 2$ for $\mathcal{H}^-$. We shall see that $\delta_S = 1, 2, 3$ are the values that we need.

Boldface variables and 6D symmetry: A set of variables that will feature a special role are the boldface variables defined via

$$\mathbf{s}_{ij} = \check{s}_{ij} + \hat{s}_{ij}; \quad \sum_j \mathbf{s}_{ij} = -\delta, \quad (11)$$

where $\delta \equiv \delta_{\text{AdS}} + \delta_S$. Note that since the constraints do not depend on the p_i , there is really only one bold variable in each channel, as for scattering amplitudes,

$$\mathbf{s} \equiv \mathbf{s}_{12} = \mathbf{s}_{34}, \quad \mathbf{t} \equiv \mathbf{s}_{14} = \mathbf{s}_{23}, \quad \mathbf{u} \equiv \mathbf{s}_{13} = \mathbf{s}_{24}, \quad (12)$$

with $\mathbf{s} + \mathbf{t} + \mathbf{u} = -\delta$. A corresponding 6D Mellin transform in these variables is defined by using the measure,

$$\oint d\boldsymbol{\mu}_{\delta} \equiv \oint \frac{ds dt}{(2\pi i)^2} (\mathbf{z}_{12}^2 \mathbf{z}_{34}^2)^s (\mathbf{z}_{14}^2 \mathbf{z}_{23}^2)^t (\mathbf{z}_{13}^2 \mathbf{z}_{24}^2)^u \Gamma[-\mathbf{s}]^2 \Gamma[-\mathbf{t}]^2 \Gamma[-\mathbf{u}]^2, \quad (13)$$

where $\mathbf{z}_{ij}^2 = x_{ij}^2 - y_{ij}^2$ is now a 6D distance.

The $\text{AdS}_3 \times \text{S}^3$ measure (8) and the 6D measure (13) are closely related. In particular, for any $\text{AdS}_3 \times \text{S}^3$ Mellin amplitude that depends only on boldface variables, the KK reduction from 6D to $\text{AdS}_3 \times \text{S}^3$ is implemented by the following identity:

$$\sum_{p_i \geq \delta_S} \oint_{\check{s}} d\hat{s} \mathcal{M}(\mathbf{s}, \mathbf{t}) = \oint d\boldsymbol{\mu}_{\delta} \mathcal{M}(\mathbf{s}, \mathbf{t}), \quad (14)$$

which holds as an expansion in the y_{ij}^2 . This shows that if an $\text{AdS}_3 \times \text{S}^3$ Mellin amplitude depends only on boldface variables, then it corresponds to a 6D conformal integral.

Before presenting results for $\langle \sigma_{p_1} \sigma_{p_2} \sigma_{p_3} \sigma_{p_4} \rangle$, our first goal in the next sections will be to recast the known correlators [5–7,9] as $\text{AdS}_3 \times \text{S}^3$ Mellin amplitudes. The result for $\langle s_{p_1}^{i_1} s_{p_2}^{i_2} s_{p_3}^{i_3} s_{p_4}^{i_4} \rangle$ was given in [8], and since its 6D origin is a scalar amplitude the rewriting follows

immediately from the identity (14). This story is analogous to $\text{AdS}_5 \times \text{S}^5$ [11]. Instead, the rewriting for the tensor-graviton correlators $\langle s_{p_1}^{i_1} s_{p_2}^{i_2} \sigma_{p_3} \sigma_{p_4} \rangle$ is new. For the latter we expect interesting modifications, in addition to (14), because the 6D origin is not a scalar amplitude.

Tensor correlator. The four-tensor correlators are the simplest. For the plus-type correlators [8] found that

$$\mathcal{H}_{\pm}^{i_1 i_2 i_3 i_4; +} = \oint_{\sum_j \check{s}_{ij} = p_i - 1} d\hat{s} \mathcal{M}^{i_1 i_2 i_3 i_4}, \quad (15)$$

with the Mellin amplitude

$$\mathcal{M}^{i_1 i_2 i_3 i_4} = -\frac{\delta^{i_1 i_2} \delta^{i_3 i_4}}{\mathbf{s} + 1} - \frac{\delta^{i_1 i_4} \delta^{i_2 i_3}}{\mathbf{t} + 1} - \frac{\delta^{i_1 i_3} \delta^{i_2 i_4}}{\mathbf{u} + 1}, \quad (16)$$

where $\mathbf{s} + \mathbf{t} + \mathbf{u} = -2$. See Supplemental Material Sec. I [22] for the normalization we use. The 6D symmetry in (15) and (16) is completely manifest, since the amplitude depends only on bold variables. Using identity (14) we find

$$\sum_{p_i \geq 1} \mathcal{H}_{\underline{p}}^{i_1 i_2 i_3 i_4; +} = \oint d\mu_{\delta=2} \mathcal{M}^{i_1 i_2 i_3 i_4}. \quad (17)$$

This shows that the KK tower of correlators in (15) originates from a 6D correlator of dimension $\delta = 2$ scalar fields, as this is precisely the value appearing in the on shell constraints for the boldface variables.

The $\mathcal{H}^-$ correlators vanish at this order. This can be shown both from the bootstrap [6,10] and from the Heavy-Heavy-Light-Light (HHLL) limit of [5]. From 6D it is also clear that it is not possible to construct a tree-level scalar function of the $\mathbf{z}_{ij}^2$ that gives $(x - \bar{x})(y - \bar{y})$. Nevertheless, the disconnected contributions to the minus sector are nontrivial, and similarly loop contributions.

Tensor-graviton correlator. The mixed tensor-graviton correlator was obtained in 6D notation in [7]. The scalar primaries in the graviton multiplets are KK modes of a self-dual three-form with polarizations projected appropriately. The bootstrap approach [10] gives results in agreement with the 6D derivation, thus providing conclusive evidence for the higher-dimensional symmetry.

Since the flavor symmetry is fully diagonal in this case, we will consider

$$\mathcal{H}_{\underline{p}}^{i_1 i_2} = \delta^{i_1 i_2} \mathcal{H}_{\underline{p}} \quad (18)$$

and present results for $\mathcal{H}^\pm$ using generalized Mellin space. In the Supplemental Material Sec. III [22] we explain how to go from the 6D formula of [7] to our new formula.

The minus-type correlator reads

$$\mathcal{H}_{\underline{p}}^- = \sum_{j, \tilde{s}_{ij}=p_i-2} \oint d\hat{s} \mathcal{M}_{stu} \quad (19)$$

with the Mellin amplitude

$$\mathcal{H}_{\underline{p}}^+ = \sum_{j, \tilde{s}_{ij}=p_i-1} \oint d\hat{s} \mathcal{M}_{1s} + \sum_{j, \tilde{s}_{ij}=p_i-2} \oint d\hat{s} (y_{12}^2 y_{34}^2 \mathcal{M}_{2s} + \mathcal{Y}_s \mathcal{M}_{tu} + \mathcal{Y}_t \mathcal{M}_{su} + \mathcal{Y}_u \mathcal{M}_{st}) - \sum_{j, \tilde{s}_{ij}=p_i-3} \oint d\hat{s} (y_{13}^4 y_{24}^4 (y - \bar{y})^2 \mathcal{M}_{stu}), \quad (23)$$

with $\mathcal{Y}_u = (y_{13}^2 y_{24}^2 - y_{23}^2 y_{14}^2 - y_{12}^2 y_{34}^2)$, and $\mathcal{Y}_s, \mathcal{Y}_t$ defined similarly, and the new generalized Mellin amplitudes defined as follows. First,

$$\mathcal{M}_{stu} = \frac{1}{(\mathbf{s} + 1)(\mathbf{t} + 1)(\mathbf{u} + 1)}, \quad (20)$$

where $\mathbf{s} + \mathbf{t} + \mathbf{u} = -4$. Interestingly, despite the fact that the mixed correlator is only invariant under the exchange of points $1 \leftrightarrow 2$ or $3 \leftrightarrow 4$, the Mellin amplitude in (20) exhibits full crossing symmetry as function of $\mathbf{s}, \mathbf{t}, \mathbf{u}$. Moreover, it is the same function that appears in $\text{AdS}_5 \times \text{S}^5$ [14]. To understand why let us recall that the superconformal block expansion of all correlators with external half-BPS scalar primaries, (4), when restricted to the $\mathcal{H}^-$ sector, is identical to that of $\mathcal{N} = 4$ SYM in the long sector [8,10]. This suggests that if a resummation like (14) exists for the minus-type correlators, it is likely to involve the same functions as in $\mathcal{N} = 4$ SYM, and indeed this is what we found,

$$\sum_{p_i \geq 2} \mathcal{H}_{\underline{p}}^- = \oint d\mu_{\delta=4} \mathcal{M}_{stu}. \quad (21)$$

We emphasize that the origin of this result is six-dimensional, and is a consequence of the fact that the 6D origin of $\mathcal{H}_{\underline{p}}$ is *not* a scalar amplitude. In particular, the 6D result of [7] not only generates the amplitude in (20), but the full result $x_{13}^2 x_{24}^2 y_{13}^2 y_{24}^2 (x - \bar{x})(y - \bar{y}) \mathcal{H}_{\underline{p}}^-$, including the prefactors!

A new formula for the plus-type correlator: These correlators appear from the start to be nontrivial because the three-form polarizations give structure to the numerator of the amplitude. This is evident in the way the result is derived from 6D [7] [(4.1)]. The crucial idea is to organize this structure in terms of a convenient set of variables,

$$y_{ij}^2; \quad \frac{y_{ij}^2}{\mathbf{z}_{ij}^2}. \quad (22)$$

Our expectation is that poles in the boldface variables can only be accompanied by the y_{ij}^2 themselves or by the 6D “propagators” $y_{ij}^2/\mathbf{z}_{ij}^2$, which can be understood as operators acting on scalar Mellin amplitudes in 6D. As a result we find that $\mathcal{H}^\pm$ is naturally stratified over different R-symmetry factors, and given by

$$\mathcal{M}_{1s} = \frac{2f_{34}}{\mathbf{s} + 1}, \quad (24)$$

with

$$f_{ij} \equiv \left(\frac{p_i + p_j - 1}{2} - \check{s}_{ij} \right)^2 - \frac{p_i^2 + p_j^2 - 1}{4}. \quad (25)$$

Then,

$$\mathcal{M}_{2s} = -\frac{2}{\mathbf{s} + 1}, \quad (26)$$

$$\mathcal{M}_{st} = \frac{(\check{s}_{13} + \check{s}_{24} + 1)}{(\mathbf{s} + 1)(\mathbf{t} + 1)}, \quad (27)$$

$$\mathcal{M}_{su} = \frac{(\check{s}_{14} + \check{s}_{23} + 1)}{(\mathbf{s} + 1)(\mathbf{u} + 1)}, \quad (28)$$

$$\mathcal{M}_{tu} = \frac{(\check{s}_{12} + \check{s}_{34} + 1)}{(\mathbf{t} + 1)(\mathbf{u} + 1)}, \quad (29)$$

where $\mathbf{s} + \mathbf{t} + \mathbf{u} = -3$. Finally, $\mathcal{M}_{stu}$ is the same as in (20) with the same on shell constraints, $\mathbf{s} + \mathbf{t} + \mathbf{u} = -4$. Note that $-y_{13}^4 y_{24}^4 (y - \bar{y})^2 = \mathcal{Y}_s \mathcal{Y}_t + \mathcal{Y}_s \mathcal{Y}_u + \mathcal{Y}_t \mathcal{Y}_u$.

Our formula (23) is nontrivial, and what is crucial is that the $\check{s}$ -type Mellin variables satisfy different on shell constraints in correspondence of the different R-symmetry structures. Before moving on to the four-graviton correlator, we wish to emphasize some features of our result. First, much like the minus sector, the part containing simultaneous poles, namely $\mathcal{M}_{stu}$ and $(\mathcal{Y}_u \mathcal{M}_{st} + \text{crossing})$, are fully crossing symmetric. Another interesting property is that the correlator automatically vanishes when $p_3 = 1$ or $p_4 = 1$, in agreement with the fact that the KK tower

associated to the graviton starts from $p_i = 2$. When $p_1 = p_2 = 1$, the amplitude becomes proportional to $\frac{1}{\mathbf{s}+1}$, thus taking the same form as the s -channel contribution to the four-tensor amplitude, as pointed out in [10]. Finally, the subamplitude $\mathcal{M}_{1s}$ explicitly breaks a certain $\mathbb{Z}_2$ symmetry. We will come back to this later on.

Graviton correlator. Let us now consider the graviton correlators, for which the results here are *new*, and so far there is no 6D derivation. Inspired by the tensor-graviton results and guided by explicit results obtained using the bootstrap [10], which we recall in the Supplemental Material Sec. II [22], we were able to derive a closed-form expression for all KK levels. As we are going to see in a moment, the structure is remarkably similar to the tensor-graviton correlator.

Minus-type correlator: The minus sector is

$$\mathcal{H}_{\underline{p}}^- = d_{\underline{p}} \sum_j \check{s}_{ij}=p_i-2} d\hat{s} \mathcal{M}_{stu}, \quad (30)$$

where $\mathcal{M}_{stu}$ is exactly the same as (20), as suggested by the same argument used for the mixed tensor-graviton correlator, and the new normalization factor is given by

$$d_{\underline{p}} = \sum_i (p_i^2 - 1). \quad (31)$$

Plus-type correlator: The plus sector exhibits the same structural decomposition as the tensor-graviton correlator. Once again, the key observation is that the Mellin amplitude naturally splits into distinct subamplitudes, each one accompanied by a different prefactor:

$$\mathcal{H}_{\underline{p}}^+ = \sum_j \check{s}_{ij}=p_i-1} d\hat{s} (\mathcal{M}_{1s} + \text{crossing}) + \sum_j \check{s}_{ij}=p_i-2} d\hat{s} (y_{12}^2 y_{34}^2 \mathcal{M}_{2s} + \text{crossing} + d_{\underline{p}} \mathcal{Y}_u \mathcal{M}_{st} + \text{crossing}) \quad (32)$$

$$+ \sum_j \check{s}_{ij}=p_i-3} d\hat{s} (y_{12}^4 y_{34}^4 \mathcal{M}_{3s} + \text{crossing} + d_{\underline{p}} y_{13}^4 y_{24}^4 (y - \bar{y})^2 \mathcal{M}_{stu}), \quad (33)$$

where $\mathcal{M}_{st}, \mathcal{M}_{su}, \mathcal{M}_{tu}$ and $\mathcal{M}_{stu}$ are the same as in (20), (27)–(29), and the new generalized Mellin amplitudes are

$$\mathcal{M}_{1s} = -8 \frac{f_{12} f_{34}}{\mathbf{s} + 1}, \quad (34)$$

where f_{ij} was defined already in (25), then

$$\mathcal{M}_{2s} = 8 \frac{f_{12} + f_{34} + \check{s}_{12} + \check{s}_{34} + 2 - \frac{1}{2} \sum_i p_i}{\mathbf{s} + 1}, \quad (35)$$

and finally

$$\mathcal{M}_{3s} = -\frac{8}{\mathbf{s} + 1}. \quad (36)$$

As before, the on shell relations are different depending on the subamplitude. Note again that the correlators automatically vanish unless $p_i \geq 2$.

Properties of the amplitudes. Taking a closer look at our results we see that all three types of amplitudes have striking features. Firstly, all poles are given in boldface variables. Secondly, all amplitudes depend on a small set of building blocks; the subamplitudes contributing to simultaneous poles, $\mathcal{M}_{stu}$ and $(\mathcal{V}_u \mathcal{M}_{st} + \text{crossing})$, that appear in both the mixed tensor-graviton correlators (23) and the four graviton correlators (32) and (33), with the only difference given by the numerical factor d_p in (31), and the amplitudes contributing to single poles. The latter, $\mathcal{M}_{1s}$, $\mathcal{M}_{2s}$, and $\mathcal{M}_{1a}$, $\mathcal{M}_{2a}$, $\mathcal{M}_{3a}$, $a = s, t, u$, despite depending on the type of correlator, still follow a simple pattern.

The fact that all poles appear in boldface variables has a simple explanation from 6D; it follows from the structure of the propagators, see e.g., *identity 1, 2, 3* of Sec. III in the Supplemental Material [22]. From $\text{AdS}_3 \times S^3$, a similar conclusion follows from considering the large p limit [14], where p_i/n is fixed with $n \rightarrow \infty$. In this limit the measure (8) has a saddle point where $\hat{s}_{ij}/n$ and $\check{s}_{ij}/n$ are finite and recombine to become the 6D Mandelstam invariants of four (massless) geodesics meeting at a point. Thus $\mathbf{s}_{\text{flat}} + \mathbf{t}_{\text{flat}} + \mathbf{u}_{\text{flat}} = 0$, with the $\text{AdS}_3 \times S^3$ Mellin amplitude encoding a 6D flat-space scattering amplitude. Now, if $\mathcal{M}(\hat{s}_{ij}, \check{s}_{ij}; \underline{p})$ has a tree-level pole, say in the s -channel, this must become $\mathbf{s}_{\text{flat}} = 0$ in the large p limit. However, $\mathbf{s} = \text{constant}$ is the only possible deformation. Same for the t and u channel poles. Besides this, $\mathcal{M}^{i_1 i_2 i_3 i_4}$ is a scalar amplitude in 6D, therefore the large p limit fixes it completely at tree level. Instead, both $\langle s_{p_1}^{i_1} s_{p_2}^{i_2} \sigma_{p_3} \sigma_{p_4} \rangle$ in (23) and $\langle \sigma_{p_1} \sigma_{p_2} \sigma_{p_3} \sigma_{p_4} \rangle$ in (32) and (33) have more structure, and various subamplitudes have nontrivial numerators. In view of the flat-space amplitudes of [20], it would be very interesting to revisit the large p limit in these cases, and also, derive (32) and (33) from 6D.

$\mathbb{Z}_2$ -symmetry: Regarding the constructability of the subamplitudes, and their numerators, there is a symmetry that one might use. To begin with, note that the $\text{AdS}_3 \times S^3$ measure (8) is invariant under *variations* of the charges p_i such that the values $p_i + p_j$ swap in a given channel. Take the s -channel Gamma functions, for example, after solving the on shell constraints (9) one finds

$$\prod_{ij=12,34} \frac{\Gamma[\hat{s}_{ij}]}{\Gamma[1 + \check{s}_{ij}]} = \prod_{ij=12,34} \frac{\Gamma[\frac{p_i + p_j}{2} - s]}{\Gamma[1 + \frac{p_i + p_j}{2} + \check{s}]} \quad (37)$$

If the *values* of $p_1 + p_2$ and $p_3 + p_4$ swap, the rhs of (37) is invariant, and if at the same time the other values of $p_i + p_j$ do not change, then the measure is invariant. In terms of the $\check{s}_{ij}$ variables this is the $\mathbb{Z}_2$ -symmetry,

$$\check{s}_{12} \leftrightarrow \check{s}_{34}, \quad \hat{s}_{12} \leftrightarrow \hat{s}_{34}, \quad c_s \leftrightarrow -c_s, \quad (38)$$

where $c_s = \frac{p_1 + p_2 - p_3 - p_4}{2}$. A concrete case to have in mind is $\underline{p} = \{k+1, k+1, k-1, k+1\}$ and $\underline{p} = \{k, k, k, k+2\}$. Analogous considerations apply to the t and u channels, but note that this symmetry is not crossing symmetry [16,23]. To see what combination of p_i is invariant it is useful to write them in terms of $c_s, c_t, c_u, \Sigma = \frac{p_1 + p_2 + p_3 + p_4}{2}$. For example, d_p in (31) is quadratic and therefore invariant in all three channels. Now, a Mellin amplitude that depends only on boldface variables is obviously invariant under the $\mathbb{Z}_2$ -symmetries of the measure. This is the case for $\mathcal{M}^{i_1 i_2 i_3 i_4}$, for $\mathcal{M}_{2s}, \mathcal{M}_{3s}, \mathcal{M}_{3t}, \mathcal{M}_{3u}$, and for $\mathcal{M}_{stu}, \mathcal{M}_{st}, \mathcal{M}_{tu}, \mathcal{M}_{su}$. In particular, all subamplitudes contributing to simultaneous poles are invariant. The question is more subtle in the presence of f_{ij} . However, $\mathcal{M}_{2s}, \mathcal{M}_{2t}, \mathcal{M}_{2u}$ are $\mathbb{Z}_2$ -symmetric, as follows from the fact that no degree-2 polynomial can be invariant under crossing while breaking the $\mathbb{Z}_2$ -symmetry. In contrast, a single f_{ij} or a product $f_{ij} f_{kl}$ of degree 4, are not invariant. Consequently, $\mathcal{M}_{1s}$ and $\mathcal{M}_{1s}, \mathcal{M}_{1t}, \mathcal{M}_{1u}$ fail to be invariant. Thus, the sub-amplitudes contributing to simple poles with no R-symmetry factor are the only symmetry breaking terms.

Summary and outlook. We studied tree-level four-point correlators of the chiral primary operators s_p^i and σ_p of the D1-D5 SCFT₂ in the supergravity regime, in the tensor and graviton multiplets, respectively. For all three types of correlators we have shown that the $\text{AdS}_3 \times S^3$ Mellin space formalism, together with the idea of reducing the Mellin amplitude to a set of building blocks with given on shell constraints, dramatically simplifies their structure. This, and additional results from the bootstrap, has led us to the complete determination of $\langle \sigma_{p_1} \sigma_{p_2} \sigma_{p_3} \sigma_{p_4} \rangle$ in (32) and (33). Remarkably, all three types of correlators are given by simple formulas with manifest properties under a 6D symmetry.

While the existence of the 6D symmetry is not new, it was not understood how to make both symmetries and the KK level dynamics manifest. The first purpose of our work has been to establish a formalism that allows to do so. In particular, our approach reduces the bootstrap problem to an overconstrained problem allowing a rigorous proof of all our results, thus going beyond state-of-the-art. We believe that our formalism will be useful in other studies, and we list some future directions: the study of correlators with vector fields that sit in the same 6D three-form field as the σ_p ; the unmixing of long double-particle operators and the use of unitary methods as starting point to develop a bootstrap program at loop level, as done in higher-dimensional cases [11,24–26]; the study of correlators with multi-particle operators built out of the s_p^i [27,28] and/or the σ_p ; the

relation between $K3$ and T^4 compactifications, motivated by a possible generalization to AdS of the flat-space SUSY reduction $(2, 2) \rightarrow (2, 0)$ in [20]; and lastly, the study of stringy corrections, generalizing the progress made in [19].

Finally, it would be interesting to turn our attention to the 4D $\mathcal{N} = 2$ SCFT in [29] dual to $\text{AdS}_5 \times S^3$ whose spectrum contains both gluon and gravitational fields. The KK flavor currents from the gluons, analogous to the s_p^i from the tensors, sit in an 8D scalar field. Their correlators are known in terms of generalized Mellin amplitudes at four-points tree-level [29,30], in the α' expansion [31], and at five points [32]. The striking relation of these amplitudes with their flat-space counterparts has shed light on BCJ and double-copy relations in AdS [30,33]. Similar results for gravitons are not yet known and are complicated by the fact that the $\mathcal{N} = 2$ stress tensor is typically exchanged together with infinitely many long multiplets [34–36]. Instead, one might start with $SU(2)$ flavor

current multiplets analogous to σ_p . Progress computing their correlators was recently made in [37].

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