

Bottomonia in an unquenched quark model

Ru-Hui Ni^{1,*}, Qian Deng,¹ Jia-Jun Wu^{2,3,†} and Xian-Hui Zhong^{1,4,‡}

¹*Department of Physics, Hunan Normal University, and Key Laboratory of Low-Dimensional Quantum Structures and Quantum Control of Ministry of Education, Changsha 410081, China*

²*School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China*

³*Southern Center for Nuclear-Science Theory (SCNT), Institute of Modern Physics, Chinese Academy of Sciences, Huizhou 516000, Guangdong Province, China*

⁴*Synergetic Innovation Center for Quantum Effects and Applications (SICQEA), Hunan Normal University, Changsha 410081, China*



(Received 7 February 2025; accepted 27 May 2025; published 23 June 2025)

The bottomonium spectrum is systematically studied within an unquenched quark model. Based on a good description of both the masses and widths for the well-established states, we further give predictions for the higher S -, P -, and D -wave bottomonium states up to a mass region of ~ 11.3 GeV. For the vector states, the S - D mixing and dielectron decays are studied. Additionally, to understand the role of the higher vector resonances in the e^+e^- annihilation reaction, we evaluate the cross section by combining our quark model predictions for the mass, dielectron, and strong decay properties. It is found that (i) The mass shifts of the high- $b\bar{b}$ states due to the coupled-channel effects are on the order of a few tens of MeV; most of the high-lying resonances contain significant non- $b\bar{b}$ components. (ii) The $\Upsilon_1(3D, 5D, 6D)$ states significantly mix with $\Upsilon(4S, 6S, 7S)$, respectively, which is mainly induced by the intermediate hadronic loops. (iii) The non- $b\bar{b}$ components will lead a significant suppression for the dielectron decay widths of some vector resonances. (iv) The threshold effects of open-bottom meson pairs can cause rich bump structures in the cross section of $e^+e^- \rightarrow b\bar{b}$. Our model shows that the $\Upsilon(10753)$ may arise from threshold effects due to the strong coupling between $\Upsilon(4S)$ and $\bar{B}^*B^*$.

DOI: [10.1103/6x3t-x15s](https://doi.org/10.1103/6x3t-x15s)

I. INTRODUCTION

The bottomonium ($b\bar{b}$) spectrum was initially established with the discovery of three vector states $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ by the E288 Collaboration at Fermilab in 1977 through the proton-nucleus collisions [1,2]. Subsequently, these vector states can be produced by the e^+e^- collisions at the Cornell Electron Storage Ring (CESR). In the early 1980s, the low-lying P -wave states $\chi_{bJ}(1P)$ [3,4] and $\chi_{bJ}(2P)$ [5,6] ($J = 0, 1, 2$) were established by the radiative decay processes of $\Upsilon(2S)$ and $\Upsilon(3S)$, respectively. Since then, more and more bottomonium states have been discovered below the $\bar{B}B$ threshold, including $\eta_b(1S, 2S)$, $h_b(1P, 2P)$, $\chi_{b1,2}(3P)$, and $\Upsilon_2(1D)$ as listed in the *Review of Particle Physics* by the

Particle Data Group (PDG) [7]. However, above the $\bar{B}B$ threshold, until now only four vector resonances $\Upsilon(10580)$, $\Upsilon(10750)$, $\Upsilon(10860)$, and $\Upsilon(11020)$ have been observed by the e^+e^- collision experiments carried out at CESR, CLEO, BABAR, and Belle [8–14].

Theoretically, the bottomonium spectrum has been studied by using many approaches and models, such as lattice QCD (LQCD) [15–17], QCD sum rules [18,19], Bethe-Salpeter equation [20], light-front quark model [21–23], relativistic flux tube model [24], various potential models [25–40], and so on. All of the observed $b\bar{b}$ states below the $\bar{B}B$ threshold together with the $\Upsilon(10580)$ slightly above the $\bar{B}B$ threshold can be well described in theory. However, for the resonances far above the $\bar{B}B$ threshold, $\Upsilon(10750)$, $\Upsilon(10860)$, and $\Upsilon(11020)$, there still exist some controversies about the nature. For example, the $\Upsilon(10860)$ and $\Upsilon(11020)$ are commonly explained as the $\Upsilon(5S)$ and $\Upsilon(6S)$, respectively; however, they were suggested as the $b\bar{b}$ $5S - 4D$ mixed states in a recent work [36] due to a congruent matching for both masses and leptonic widths. For the $\Upsilon(10753)$, one difficultly explains it as any conventional S -wave or D -wave $b\bar{b}$ states with $J^{PC} = 1^{--}$ [24,28,31,39,41–43]. Thus, many explanations for the $\Upsilon(10753)$, such as a mixed state via $\Upsilon(4S)$ - $\Upsilon_1(3D)$ or

* Contact author: niruhui@ucas.ac.cn

† Contact author: wujiajun@ucas.ac.cn

‡ Contact author: zhongxh@hunnu.edu.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

$\Upsilon(5S)$ - $\Upsilon_1(4D)$ mixing [43–46], a hybrid [47,48], a tetraquark state [49–52], have been raised in the literature.

The puzzle surrounding these high-lying states $\Upsilon(10750)$, $\Upsilon(10860)$, and $\Upsilon(11020)$ may mainly arise from neglecting the “unquenched coupled-channel effects” from the open-bottom meson pairs within in the conventional quark model. These effects can play a crucial role in the excited $b\bar{b}$ states above the $\bar{B}B$ threshold. In order to include the coupled-channel effects on the mass spectrum, some screened potential models have been proposed in the literature [30,31,39,42,53,54], where the widely used linear confinement potential is replaced by a screened form. However, a recent study of the charmonium spectrum [55] has revealed that the mass shifts predicted by a detailed treatment of coupled-channel effects do not follow the typical increasing trend from lower to higher states as often observed in screened potential models. A key aspect of the coupled-channel dynamics is the involvement of multiple channels with varying thresholds, which are unevenly spaced relative to the mass of the bare valence state. Additionally, states with different quantum numbers couple to distinct channels, each with different coupling strengths. As a result, the mass shifts for various bare states induced by these effects are generally unordered, posing a challenge for simple screened potential models to accurately capture these complexities. Thus, a systematic study of the bottomonium spectrum by seriously including the coupled-channel effects within an unquenched quark model is necessary.

The unquenched coupled-channel effects on hadron spectrum have been discussed for over four decades. Several pioneering works can be found in Refs. [56–60]. In the past decade, the coupled-channel effects on the bottomonium spectrum have been also widely discussed in the literature [41,61–68]. It should be emphasized that in these studies the coupled-channel effects mostly focus on the contributions from the ground meson pairs, $\bar{B}B$, $\bar{B}B^*$, $\bar{B}^*B^*$, $\bar{B}_s B_s$, $\bar{B}_s B_s^*$, and $\bar{B}_s^* B_s^{*1}$, while the high-lying channels involving excited bottom mesons are neglected. In this work, we further study the bottomonium spectrum within an unquenched quark model, which has achieved a good description of the both the heavy-light meson and charmonium spectrum, recently [55,69]. In this unquenched quark model, all of the Okubo-Zweig-Iizuka (OZI)-allowed channels with mass thresholds below the bare states, as well as nearby virtual channels, are included, while the contributions from the channels lying far above the bare states are subtracted from the dispersion relation by redefining the bare mass with the once-subtracted method [70]. Furthermore, a factor is adopted to suppress the unphysical contributions of the coupled-channel effects in

the high-momentum region as done in the literature [55,69,71–79].

On the other hand, the dielectron decay widths of vector $b\bar{b}$ states are crucial for understanding their production rates in the $e^+e^- \rightarrow \text{hadron}$ process and for revealing their intrinsic nature. Several studies [28,30,31,36,39,80–82] have analyzed the dielectron widths of vector $b\bar{b}$ states, many of which consider the S - D mixing effect, a phenomenon that significantly influences the dielectron decay width. However, these studies often overlook the underlying dynamic mechanisms responsible for S - D mixing, with the mixing angle typically introduced arbitrarily. The generation of this mixing angle, induced by intermediate mesonic loops, has been discussed by Törnqvist *et al.* [58,59] and Lu *et al.* [63]. However, their analyses are primarily focused on ground-state $B_{(s)}$ mesons, considering $\bar{B}_{(s)}^{(*)} B_{(s)}^{(*)}$ loops. Due to the limited number of coupling channels considered, these studies are restricted in scope and do not fully explore these effects for vector $b\bar{b}$ states.

Therefore, a comprehensive and systematic investigation of the mass spectra, strong decays, and dielectron decay properties of bottomonium, within a unified framework that incorporates unquenched coupled-channel effects, is currently essential. The main purposes of this work are as follows: (i) To obtain a more comprehensive understanding of the $b\bar{b}$ spectrum within the unquenched quark model. The strong decay properties, the mass shifts, and the $b\bar{b}$ -core components for the high-lying states are systematically evaluated in a unified unquenched framework. (ii) To better understand the S - D mixing dynamic mechanisms for the vector states. The S - D mixing induced by the tensor force and the intermediate hadronic loops is studied, and the mixing angles are evaluated by using these two different mechanisms. (iii) To deepen our understanding of the dielectron decay properties of vector states in the unquenched framework. The dielectron decay widths are evaluated by considering both the S - D mixing effects and the corrections from the non- $b\bar{b}$ components due to the coupled-channel effects. (iv) To explore the roles of the high-lying vector resonances and the threshold effects of the open-bottom meson pairs in the total cross section of the $e^+e^- \rightarrow b\bar{b}$ reaction. By combining the dielectron decay widths and spectral density functions derived within the unquenched framework, the cross section for $e^+e^- \rightarrow b\bar{b}$ is estimated. Then, the roles of the vector resonances and threshold effects are shown by the line shape of the cross section.

The paper is organized as follows. In Sec. II, we give an introduction of the framework of the unquenched quark model including coupled-channel effects. The mass spectrum of the S -, P -, and D -wave bottomonium states up to a mass region of ~ 11.3 GeV is presented, and related issues are discussed. In Sec. IV, the two-body OZI-allowed strong decay properties for the high-lying $b\bar{b}$ resonances are given

¹For convenience, throughout the paper we adopt abbreviations $\bar{B}B^*$, $\bar{B}_s B_s^*$, and etc. to stand for the $\bar{B}B^* + \text{c.c.}$, $\bar{B}_s B_s^* + \text{c.c.}$, and etc., respectively.

and discussed. In Secs. V and VI, the S - D mixing and electron decay widths for the vector resonances are evaluated, respectively. In Sec. VII, we further discuss the role of the high-lying $b\bar{b}$ vector resonances in the cross section of the e^+e^- annihilation reaction. Finally, a summary is given in Sec. VIII.

II. UNQUENCHED QUARK MODEL

A. Hamiltonian for bare states

The bare masses of $b\bar{b}$ states are calculated in a quenched semirelativistic potential model. In this model, the effective Hamiltonian is

$$H_0 = \sqrt{\mathbf{q}_1^2 + m_b^2} + \sqrt{\mathbf{q}_2^2 + m_{\bar{b}}^2} + V_0(r) + V_{sd}(r), \quad (1)$$

where $\mathbf{q}_1$ and $\mathbf{q}_2$ is the three momenta of quark and antiquark, respectively. r is the distance between quark and antiquarks, with m_b and $m_{\bar{b}}$ representing their respective masses. The $V_0(r)$ is spin-independent effective potential and adopts the well-known Cornell form [56]

$$V_0(r) = -\frac{4\alpha_s}{3r} + br + C_0, \quad (2)$$

which includes the color Coulomb interaction, linear confinement, and zero-point energy C_0 . The spin-dependent part $V_{sd}(r)$ adopts the widely used form [27,83]

$$V_{sd}(r) = \frac{32\pi\alpha_s \cdot \sigma^3 e^{-\sigma^2 r^2}}{9\sqrt{\pi^3} m_b m_{\bar{b}}} \mathbf{S}_q \cdot \mathbf{S}_{\bar{q}} + \frac{4}{3} \frac{\alpha_s}{m_b m_{\bar{b}}} \frac{1}{r^3} \left(\frac{3\mathbf{S}_q \cdot \mathbf{r} \mathbf{S}_{\bar{q}} \cdot \mathbf{r}}{r^2} - \mathbf{S}_q \cdot \mathbf{S}_{\bar{q}} \right) + H_{LS}. \quad (3)$$

This part includes the spin-spin contact hyperfine potential, tensor potential, and spin-orbit interaction. The spin-orbit interaction further decomposes into symmetric and antisymmetric terms, given as $H_{LS} = H_{\text{sym}} + H_{\text{anti}}$, with

$$H_{\text{sym}} = \frac{\mathbf{S}_+ \cdot \mathbf{L}}{2} \left[\left(\frac{1}{2m_b^2} + \frac{1}{2m_{\bar{b}}^2} \right) \left(\frac{4\alpha_s}{3r^3} - \frac{b}{r} \right) + \frac{8\alpha_s}{3m_b m_{\bar{b}} r^3} \right], \quad (4)$$

$$H_{\text{anti}} = \frac{\mathbf{S}_- \cdot \mathbf{L}}{2} \left(\frac{1}{2m_b^2} - \frac{1}{2m_{\bar{b}}^2} \right) \left(\frac{4\alpha_s}{3r^3} - \frac{b}{r} \right). \quad (5)$$

In these equations, $\mathbf{L}$ is the orbital angular momentum of the $b\bar{b}$ system; $\mathbf{S}_{\pm} \equiv \mathbf{S}_b \pm \mathbf{S}_{\bar{b}}$, $\mathbf{S}_b$ and $\mathbf{S}_{\bar{b}}$ are the spins of the quark and antiquark, respectively. It should be emphasized that the antisymmetric term H_{anti} has no contribution to the mass of $b\bar{b}$ states since the masses of quark and antiquark are equal. The parameter set $\{\alpha_s, b, m_{b/\bar{b}}, \sigma\}$ in the above

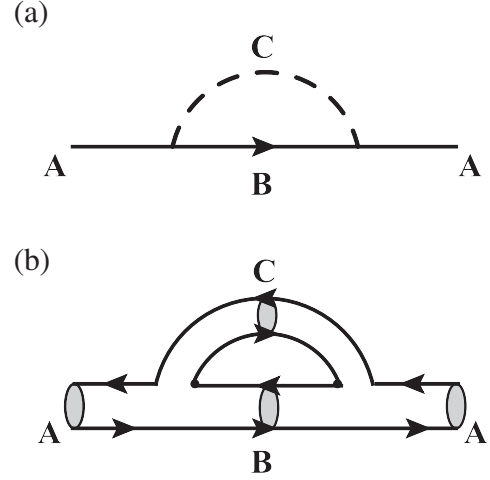


FIG. 1. The BC hadronic loop coupled to a bare meson state $|A\rangle$, drawn at (a) the hadronic level and (b) the quark level.

potentials is determined by fitting the spectrum of low-lying bottomonium states. In this work, we solve the radial Schrödinger equation using the Gaussian expansion method [84]. The detailed discussions on the numerical calculation techniques involving the Gaussian expansion method can be found in our previous work [78].

B. Coupled-channel effects

A bare state $|A\rangle$ described within the quenched quark model can further couple to the two-hadron continuum BC via hadronic loops, as illustrated in Fig. 1. The wave function of the physical state $|\Psi(M)\rangle$ with the mass eigenvalue M is written as

$$|\Psi(M)\rangle = c_A(M)|A\rangle + \sum_{BC} \int c_{BC}(\mathbf{p}, M) d^3\mathbf{p} |BC, \mathbf{p}\rangle, \quad (6)$$

where $\mathbf{p} = \mathbf{p}_B = -\mathbf{p}_C$ is final two-hadron relative momentum in the initial hadron static system; $c_A(M)$ and $c_{BC}(\mathbf{p}, M)$ denote the probability amplitudes of the bare valence state $|A\rangle$ and the continuum state $|BC, \mathbf{p}\rangle$, respectively.

The full Hamiltonian for the $|\Psi(M)\rangle$ is written as

$$H = H_0 + H_c + H_I, \quad (7)$$

where H_0 is the Hamiltonian for describing the bare state $|A\rangle$ based on the quark level, which has been given by Eq. (1). H_c is the Hamiltonian for describing the continuum state $|BC, \mathbf{p}\rangle$. In principle, H_c should include the kinetic energy of the continuum state and the interaction part between two continuum states. However, due to the interactions between continuum states involving many unknown parameters, such as the interaction potentials of the t and u channels, the currently very limited $b\bar{b}$ bottomonium spectrum data are insufficient to extract

information about these interactions. Theoretically, we could incorporate these interactions into H_I ; therefore, H_c here only includes the kinetic energy term as follows:

$$H_c|BC, \mathbf{p}\rangle = E_{BC}|BC, \mathbf{p}\rangle, \quad (8)$$

where $E_{BC} = \sqrt{m_B^2 + p^2} + \sqrt{m_C^2 + p^2}$ is the kinetic energy of the $|BC, \mathbf{p}\rangle$ continuum state.

The term H_I plays a crucial role in describing the coupling between the bare state $|A\rangle$ and the $|BC, \mathbf{p}\rangle$ continuum components. In this work, we adopt the widely used ${}^3P_0$ model [85–87] to depict the interaction H_I . Within the ${}^3P_0$ model, the operator is expressed as

$$H_I = -3\gamma\sqrt{96\pi}\sum_m \langle 1m1 - m|00\rangle \int d\mathbf{p}_3 d\mathbf{p}_4 \delta^3(\mathbf{p}_3 + \mathbf{p}_4) \times \mathcal{Y}_1^m\left(\frac{\mathbf{p}_3 - \mathbf{p}_4}{2}\right) \chi_{1-m}^{34} \phi_0^{34} \omega_0^{34} b_{3i}^\dagger(\mathbf{p}_3) d_{4j}^\dagger(\mathbf{p}_4), \quad (9)$$

where γ is a dimensionless constant that denotes the strength of the quark-antiquark pair creation with momentum $\mathbf{p}_3$ and $\mathbf{p}_4$ from vacuum; $b_{3i}^\dagger(\mathbf{p}_3)$ and $d_{4j}^\dagger(\mathbf{p}_4)$ are the creation operators for the quark and antiquark, with indexes i and j for the SU(3)-color indices of the created quark and antiquark, respectively; $\phi_0^{34} = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$ and $\omega_0^{34} = \delta_{ij}/\sqrt{3}$ correspond to flavor and color singlets, respectively; χ_{1-m}^{34} is a spin triplet state; and $\mathcal{Y}_{\ell m}(\mathbf{k}) \equiv |\mathbf{k}|^\ell Y_{\ell m}(\theta_k, \phi_k)$ is the ℓ th solid harmonic polynomial. More details of the ${}^3P_0$ model can be found in our previous works [88,89].

The Schrödinger-like equation of the composite hadron including coupled-channel effects is expressed as

$$\begin{pmatrix} H_0 & H_I \\ H_I & H_c \end{pmatrix} \begin{pmatrix} c_A(M)|A\rangle \\ \sum_{BC} \int c_{BC}(\mathbf{p}, M) d^3\mathbf{p} |BC, \mathbf{p}\rangle \end{pmatrix} = M \begin{pmatrix} c_A(M)|A\rangle \\ \sum_{BC} \int c_{BC}(\mathbf{p}, M) d^3\mathbf{p} |BC, \mathbf{p}\rangle \end{pmatrix}. \quad (10)$$

This equation gives rise to a system of coupled equations for c_A and $c_{BC}(\mathbf{p}, M)$:

$$c_A(M)M_A + \sum_{BC} \int c_{BC}(\mathbf{p}, M) d^3\mathbf{p} \langle A|H_I|BC, \mathbf{p}\rangle = c_A(M)M, \quad (11)$$

$$c_A(M) \langle BC, \mathbf{p}'|H_I|A\rangle + \int c_{BC}(\mathbf{p}, M) d\mathbf{p}^3 E_{BC} \delta^3(\mathbf{p}' - \mathbf{p}) = c_{BC}(\mathbf{p}', M)M. \quad (12)$$

Deriving $c_{BC}(\mathbf{p}, M)$ from Eq. (12), and substituting it into Eq. (11), we obtain the hadronic self-energy as follows:

$$\begin{aligned} g(M) &= \sum_{BC} g_{BC}(M) \\ &= \sum_{BC} \int_0^\infty \frac{|f(\mathbf{p})|^2}{M - E_{BC} + i\epsilon} p^2 dp d\Omega_p \\ &= \sum_{BC} \int_0^\infty \frac{|\mathcal{M}_{A \rightarrow BC}(\mathbf{p})|^2}{M - E_{BC} + i\epsilon} p^2 dp, \end{aligned} \quad (13)$$

where $f(\mathbf{p}) = \langle BC, \mathbf{p}|H_I|A\rangle$, and $g_{BC}(M)$ represents the self-energy of the partial channel BC .

It should be emphasized that the infinitesimal term $i\epsilon$ ($\epsilon \rightarrow 0^+$) is introduced in the self-energy function to properly deal with the kinematic region where $M > m_B + m_C$. In this case, the self-energy function becomes complex:

$$g(M) = \text{Reg}(M) - i \text{Im} g(M). \quad (14)$$

The real part, $\text{Reg}(M)$, represents the mass shift induced by the coupling to intermediate hadronic channels. It is given by

$$\Delta M(M) = \sum_{BC} \mathcal{P} \int_0^\infty \frac{|\mathcal{M}_{A \rightarrow BC}(\mathbf{p})|^2}{M - E_{BC}} p^2 dp, \quad (15)$$

where $\mathcal{P}$ is the principal part integral. It should be emphasized that calculating the mass shift while considering an infinite number of channels is practically insurmountable. In our calculations, we included all OZI-allowed two-body hadronic channels with mass thresholds below the bare $|A\rangle$ states. In addition, we include channels whose thresholds lie slightly above the bare mass, within an upper limit of 100 MeV. This choice is motivated by the fact that the bare masses predicted by potential models inherently carry uncertainties of several tens of MeV. Due to these uncertainties, certain channels may correspond either to open-flavor thresholds or to nearby virtual channels. Moreover, channels located close to the bare mass may couple to the bottomonium state via S wave and induce sizable mass shifts. Therefore, we include virtual meson pairs whose thresholds lie within 100 MeV above the predicted bare bottomonium mass to account for the most relevant coupled-channel effects. This upper limit is consistent with the typical uncertainty scale of potential models and ensures the inclusion of channels most likely to induce significant mass shifts. Unfortunately, with the current model and the limited experimental input, we cannot distinguish the contributions of all the hadron channels, and in this situation, the best optimal choice is to include the several closest channels. By contrast, virtual channels with thresholds far above the bare $|A\rangle$ state are treated separately using the once-subtracted dispersion relation formalism, following Refs. [55,69,70,90,91]. These high-lying channels are numerous, and their effects vary slowly

across different states. As a result, their cumulative contribution can be well approximated by a constant, which can be absorbed into a redefinition of the bare mass.

Adopting the once-subtracted approach, the hadronic mass shift can be rewritten as [55,69]

$$\Delta M(M) = \sum_{BC} \mathcal{P} \int_0^\infty \frac{(M_0 - M) |\mathcal{M}_{A \rightarrow BC}(\mathbf{p})|^2}{(M - E_{BC})(M_0 - E_{BC})} p^2 dp. \quad (16)$$

Here, M_0 represents the subtracted zero point, which is typically the mass of a $b\bar{b}$ state nearly unaffected by intermediate hadronic loops. In this work, we choose $\Upsilon(1S)$ as the subtracted zero point with a value of $M_0 = 9460$ MeV. It is important to emphasize that the choice of M_0 has minimal impact on our predictions of the mass shifts. The reason for this lies in the factor $F \equiv (M_0 - M)/[(M - E_{BC})(M_0 - E_{BC})]$ depending on M_0 in the mass shift as shown in Eq. (16). This factor can be expressed as $F = 1/[(M - E_{BC})(1 - \frac{M - E_{BC}}{M - M_0})]$. Note that the mass shift $\Delta M(M)$ is primarily influenced by the integral around the pole position $M \sim E_{BC}$, where $M - E_{BC} \simeq 0$. Since the subtracted zero-point M_0 is usually far below E_{BC} (i.e., $M - M_0 \sim E_{BC} - M_0 \gg 0$), it follows that $\frac{M - E_{BC}}{M - M_0} \sim 0$. Therefore, the factor F is not significantly sensitive to the selection of M_0 . In other words, the mass shift $\Delta M(M)$ determined by the coupled-channel equation is not sensitive to the chosen subtraction point.

The coupled-channel equation is obtained from Eq. (11), i.e.,

$$M = M_A + \Delta M(M). \quad (17)$$

Combining Eqs. (16) and (17), one can determine the physical mass M and the mass shift $\Delta M(M)$ simultaneously. However, it should be emphasized that when using the Eq. (16) to calculate the mass shift, the contribution in the higher- $\mathbf{p}$ region may be nonphysical because the quark pair-production rates via the nonperturbative interaction H_I should be suppressed [72,73,78]. The effective approach to overcome this difficulty is to introduce a suppressed factor into the transition amplitudes, i.e.,

$$\langle BC, \mathbf{p} | H_I | A \rangle \rightarrow \langle BC, \mathbf{p} | H_I e^{-p^2/(2\Lambda^2)} | A \rangle, \quad (18)$$

where Λ is the cutoff parameter. In this work, we adopt $\Lambda = 0.78$ GeV to be consistent with our recent studies of heavy-light meson [69] and charmonium spectra [55]. This value is also comparable to $\Lambda = 0.84$ GeV used in the literature [74,75,92]. In fact, a similar suppression factor, $\sqrt{\Lambda^2/(\Lambda^2 + \mathbf{q}^2)}$, is widely adopted in the study of hadron spectra and hadron-hadron interactions within the chiral quark model [54,93–97]. This suppression factor is related to the dynamics of quark-antiquark pair creation in the vacuum. The cutoff parameter Λ determines the scale at

which chiral symmetry is broken. The typical range of Λ is around 0.8 ± 0.2 GeV [54,93–97].

If the physical mass of the initial hadron exceeds the threshold of the final states B and C , i.e., $M > m_B + m_C$, the strong decay process $A \rightarrow BC$ will occur. The partial decay width for this process is given by

$$\Gamma_{BC} = 2\pi \frac{|\mathbf{p}| E_B E_C}{M} |\mathcal{M}_{A \rightarrow BC}(\mathbf{p})|^2. \quad (19)$$

According to the optical theorem, the imaginary part of the hadronic self-energy, $\text{Im } g(M)$, can be related to the decay process through the cutting of the hadronic loop diagram, as shown in Fig. 1. This imaginary part is determined by the same transition amplitude $\mathcal{M}_{A \rightarrow BC}(\mathbf{p})$ that governs the decay and satisfies the relation $\text{Im } g_{BC}(M) = \Gamma_{BC}/2$. It is important to emphasize that both the mass shift and the decay width are determined by the same strong transition mechanism, i.e., the $^3P_0$ model, and by using consistent wave functions and parameters. Therefore, in our model, the real and imaginary parts of the self-energy are not independent, but instead emerge from a unified theoretical framework.

For the low-lying $b\bar{b}$ states, whose mass is smaller than the threshold of the lowest open-bottom channel, the derivative of the self-energy is associated with the probability of finding the bare state within the wave function of the physical state. The component of the $b\bar{b}$ core is determined by [98]

$$P_{b\bar{b}} \equiv |c_A|^2 = \left(1 - \frac{\partial}{\partial M} g(M) \Big|_{M=M_{\text{phy}}}\right)^{-1}. \quad (20)$$

The above equation can also be readily derived from the normalization of the wave function of the bound state [99]. Furthermore, the probability of the presence of a BC hadron pair is determined by

$$P_{BC} \equiv |c_{BC}|^2 = -|c_A|^2 \frac{\partial}{\partial M} g_{BC}(M) \Big|_{M=M_{\text{phy}}}. \quad (21)$$

On the other hand, for the high-lying $b\bar{b}$ resonances, which have one or more OZI-allowed two-body strong decay channels, Eqs. (20) and (21) are invalid for calculating the components of physical states, because the wave function of the scattering state cannot be normalized directly. To address this, a method known as the “spectral density function” provides an effective scheme for estimating the probability density of the bare-state component. This approach was originally introduced in Ref. [100] and further developed in Refs. [99,101–106]. The spectral density near a resonance is given by

$$\omega_R(M) = 4\pi |\mathbf{p}| \frac{E_B E_C}{E_{BC}} |c_A(M)|^2. \quad (22)$$

Here, $c_A(M)$ represents the probability density of a bare state within the spectral density function, determined as

$$c_A(M) = \frac{f^*(p_M)}{M - M_A - g(M)}. \quad (23)$$

Substituting Eq. (23) into Eq. (22), one can obtain

$$\omega_R(M) = \frac{1}{2\pi} \frac{\Gamma_R}{(M - M_R)^2 + \Gamma_R^2/4}, \quad (24)$$

where M_R and $\Gamma_R \equiv \sum_{BC} \Gamma_{BC}$ denote the mass and width of the resonance, respectively. It is easy to see that the spectral function in this case is the well-known Breit-Wigner form. For a physical resonance state with a complex energy $M_R - i\Gamma_R/2$, the probability of the $b\bar{b}$ core is estimated to be

$$P_{b\bar{b}} = \int_{M_R - \Delta}^{M_R + \Delta} \omega(M) dM, \quad (25)$$

where Δ is the integral interval that we choose. If the spectral function around the resonance pole shows a typical Breit-Wigner line shape, we take $\Delta = \Gamma_R$ as usually adopted in the literature [99]. However, the line shape of the spectral function around the resonance pole is often distorted by the strong couplings of multiple channels to the bare $b\bar{b}$ state, which is not a typical Breit-Wigner form. In this case, to obtain more reliable predictions, the integration interval is chosen by combining the line shape of the spectral functions near the physical poles.

C. Model parameters

In our calculations, the potential model Hamiltonian H_0 incorporates six parameters, $\{b, \alpha_s, m_b, \sigma, C_0, r_c\}$, which define the interactions within the quenched framework. Additionally, the effective interaction H_I in the ${}^3P_0$ model includes a single parameter γ , representing the quark-antiquark pair-creation strength in vacuum.

The parameters of the potential model are specified as follows: $b = 0.180 \text{ GeV}^2$, $\alpha_s = 0.372$, $m_b = 4.80 \text{ GeV}$, $\sigma = 2.42 \text{ GeV}$, and $C_0 = -35.0 \text{ MeV}$. In our model, b is fixed at a typical value 0.180 GeV^2 , commonly used in relativistic potential models [27,107–109]. The bottom quark mass m_b is fixed at 4.80 GeV to ensure consistency with our previous study on B and B_s mesons spectrum [69]. Consequently, only three parameters, α_s , σ , and C_0 , are treated as adjustable. These parameters are determined by minimizing a χ^2 function defined with respect to the masses of well-established $b\bar{b}$ states far below the open-bottom threshold:

$$\chi^2 = \sum_n \frac{(M_{\text{th}}(n) - M_{\text{exp}}(n))^2}{M_{\text{err}}(n)^2}. \quad (26)$$

TABLE I. The model parameter determination by fitting the masses of the well-established states with the minimum χ^2 method. The errors of the observed masses are uniformly taken as $M_{\text{err}} = 5 \text{ MeV}$. The unit of mass is MeV.

Number	$n^{2S+1}L_J$	Observed state	M_{exp} [7]	M_{err}	M_{th}
1	1^1S_0	$\eta_b(1S)$	9399	5	9399
2	1^3S_1	$\Upsilon(1S)$	9460	5	9460
3	2^1S_0	$\eta_b(2S)$	9999	5	9991
4	2^3S_1	$\Upsilon(2S)$	10023	5	10013
5	3^3S_1	$\Upsilon(3S)$	10355	5	10348
6	1^3P_0	$\chi_{b0}(1P)$	9859	5	9877
7	1^1P_1	$h_b(1P)$	9899	5	9916
8	1^3P_1	$\chi_{b1}(1P)$	9893	5	9910
9	1^3P_2	$\chi_{b2}(1P)$	9912	5	9930
10	2^3P_0	$\chi_{b0}(2P)$	10233	5	10233
11	2^1P_1	$h_b(2P)$	10260	5	10261
12	2^3P_1	$\chi_{b1}(2P)$	10255	5	10258
13	2^3P_2	$\chi_{b2}(2P)$	10269	5	10273
14	1^3D_2	$\Upsilon_2(1D)$	10164	5	10162
χ^2					61.5

Here, M_{th} , M_{exp} , and M_{err} represent the theoretical mass, experimental mass, and experimental uncertainty, respectively. During the fitting, α_s and C_0 mainly control the overall mass scale of low-lying bottomonium states. The parameter σ , which appears in the spin-spin potential, is primarily constrained by the hyperfine splitting $M[\Upsilon(1S) - \eta_b(1S)]_{\text{exp}} \simeq 61 \text{ MeV}$. The resulting comparison between theoretical predictions and experimental data is presented in Table I, yielding a total χ^2 value of 61.5.

It should be emphasized that stable solutions for some states cannot be obtained due to the singularity of the $1/r^3$ term in the spin-orbit and tensor potentials. To address this issue, we introduce a cutoff distance r_c , following the method adopted in Refs. [42,43,78,89,110,111]. Specifically, in the region $r \leq r_c$, the divergent term $1/r^3$ is replaced by a constant $1/r_c^3$. The 1^3P_0 state is particularly sensitive to the choice of r_c , due to its large spin-orbit factor $\langle \mathbf{S}_+ \cdot \mathbf{L} \rangle$. To determine the optimal value of r_c , we fit the observed fine splitting $M[\chi_{b1}(1P) - \chi_{b0}(1P)]_{\text{exp}} \simeq 33 \text{ MeV}$ using the perturbative approach as adopted in Refs. [89,111]. This procedure yields $r_c = 0.101 \text{ fm}$. In the perturbative approach, the spin-orbit and tensor interactions with the $1/r_c^3$ term are treated as small corrections. The procedure involves four steps: (i) solving the system without $1/r_c^3$ terms to obtain the zeroth-order mass M_0 and the corresponding wave function; (ii) computing the mass correction δM using the perturbative terms; (iii) adding the correction to obtain the total mass $M = M_0 + \delta M$; and (iv) determining the cutoff parameter r_c by fitting the total mass M . While the perturbative method provides accurate mass estimates, it neglects the feedback of spin-dependent interactions on the

wave function. By introducing the cutoff r_c , these effects can be included nonperturbatively. This allows both the mass and the wave function to be consistently corrected by the spin-orbit and tensor potentials.

In our calculations of strong decay widths, the wave functions of the initial $b\bar{b}$ states are adopted from our quark model predictions. For the final B and B_s mesons in a decay process, the wave functions are adopted from the predictions of our previous work [69]. To be consistent with the mass calculations within the potential model, the bottom quark mass is adopted as $m_b = 4.80$ GeV. For the light quark masses, we adopt $m_{u,d} = 0.35$ GeV and $m_s = 0.55$ GeV, which are consistent with those adopted for the calculations of the B - and B_s -meson spectra [69]. For the well-established hadrons involved in the final states, the masses are adopted from the averaged values from the PDG [7], while for the missing states, the masses are adopted from the theoretical predictions in a recent work [69]. The creation quark pair strength $\gamma = 0.482$ within the $^3P_0$ model is determined by fitting the measured width of $\Upsilon(10580) \rightarrow \bar{B}B = 20.5$ MeV [7].

III. MASS SPECTRUM

The mass spectrum obtained from the unquenched quark model is given Table II, where some other model predictions are also listed for comparison. Furthermore, the theoretical mass spectrum compared with the data is plotted in Fig. 2. It is shown that the observed bottomonium spectrum below the $\bar{B}B$ threshold can be well described within the quark model. Above the $\bar{B}B$ threshold, four vector resonances $\Upsilon(10580)$, $\Upsilon(10753)$, $\Upsilon(10860)$, and $\Upsilon(11020)$ are observed in experiments. The masses of the $\Upsilon(10580)$ and $\Upsilon(11020)$ can be well understood in theory with the $\Upsilon(4S)$ and $\Upsilon(6S)$ assignments in the unquenched quark model framework. Assigning the $\Upsilon(10860)$ to the $\Upsilon(5S)$ state, our predicted mass is about 30 MeV lower than the PDG average value $M_{\text{exp}} = 10885$ MeV. At last, the $\Upsilon(10753)$ cannot be reasonably explained with any vector bottomonium states. If assigning it to the 3^3D_1 state, our predicted mass is about 100 MeV lower than the measured value.

Furthermore, in Table III, we present our predictions for the hyperfine splittings of some S -wave states and the fine splittings of some P -wave states. For comparison, we also list the data and the predictions from other works [28–30,39,42] in the same table. It is seen that our predicted mass splittings are in good agreement with the observations. It is worth noting that after including the unquenched coupled-channel effects, our predicted splittings of $m(4^3S_1) - m(3^3S_1)$, $m(5^3S_1) - m(4^3S_1)$, and $m(6^3S_1) - m(5^3S_1)$ are more in agreement with the observations.

A. Mass shift due to coupled-channel effects

The bare masses of the high- $b\bar{b}$ states may be significantly shifted to the physical points due to some strong

couplings to their nearby strong decay channels. In Table II, we give our predictions of the mass shifts contributed by these coupled-channel effects. It is found that these effects may contribute a sizable mass correction ($\sim$ tens of MeV) to some high-lying states. It should be highlighted for understanding the $\Upsilon(10580)$ and $\Upsilon(11020)$ states. It is based on mass shifts due to coupled-channel effects; the masses of $\Upsilon(4S)$ and $\Upsilon(6S)$ experience shifts of about -40 and -60 MeV, respectively. In contrast to the original quark model predictions that did not account for the coupled-channel effects [27,29], these mass shifts align much more closely with experimental measurements, providing a solid theoretical foundation for understanding these two states. Later, from their component analysis, $\Upsilon(11020)$ indeed shows rich continuum components. The contributions of each coupled channel to the mass shifts of the bare states are given in Tables IV–IX.

B. Components of physical states

The physical states with high masses may contain significant continuum components of open-bottom meson pairs. Within the unquenched framework, we estimate the $b\bar{b}$ -core components for the nS ($n = 4, 5, 6, 7$), nP , and nD ($n = 3, 4, 5, 6$) states. The results are given in Table II. It should be pointed out that for the $\eta_b(4S)$ and $3P$ states, since the masses are slightly below the lowest threshold of their opened-bottom decay channels, their $b\bar{b}$ -core and -continuum components are estimated with Eqs. (20) and (21), while for the other high-mass resonances which have OZI-allowed two-body strong decay channels, their $b\bar{b}$ -core components are evaluated with Eq. (25) by combining the spectral-density functions.

Our results show that for the well-established resonance $\Upsilon(10580)$, besides the dominant $b\bar{b}$ component, $P_{b\bar{b}} \simeq 60\%$, the $\bar{B}B^* + \bar{B}^*B^*$ component may reach up to a fairly large value $\sim 40\%$. While the $\Upsilon(11020)$ resonance may be a continuum-rich state, the $b\bar{b}$ -core component is only $\sim 40\%$. Generally, from Table II, it is found that the $4S$, $5S$, and $7S$ states may be $b\bar{b}$ -dominant states with a component of $P_{b\bar{b}} \simeq 55\text{--}75\%$; however, for the $6S$ states the $b\bar{b}$ component is estimated to be only a small value $\sim 40\%$. The small $b\bar{b}$ -core components are also predicted for the $6P$ and $5D$ states, which are in the range of 10–40%.

C. Two-pole structures

It is interesting to note that two-pole structures arise from the $\Upsilon_3(3D)$, $\Upsilon_3(5D)$, and $\eta_{b2}(5D)$ states when coupled-channel effects are considered. This phenomenon is similar to that observed in the case of $\chi_{c1}(2P)$ [55].

First, let us focus on the $\Upsilon_3(3D)$ state. Nearby the threshold of $\bar{B}^*B^*$ channel, since the $\Upsilon_3(3D)$ couples strongly to this channel, as shown in the upper panel of Fig. 8, there exists a notable dip of the mass shift function $\Delta M(M)$. By solving the coupled-channel equation (17), we

TABLE II. Our theoretical masses (MeV) are compared with experimental data and predictions from other quark models. The mass spectrum is presented within both quenched (Q) and unquenched (UQ) frameworks. The mass shifts due to the coupled-channel effects are denoted as ΔM . For the low-lying states having no strong decay two-body open-bottom meson channels, the mass corrections due to the coupled-channel effects are absorbed in the mass parameters with the once-subtracted method; thus, no mass shifts are given. In the unquenched pictures, the $b\bar{b}$ -core components evaluated for the resonances are denoted as $P_{b\bar{b}}(\%)$.

$n^{2S+1}L_J$	J^{PC}	Q	ΔM	UQ	$P_{b\bar{b}}$	Experiment [7]	LC [30]	EFG [29]	WSLM [39]	DLGZ [42]	GI [27]
3^1S_0	0^{-+}	10334	—	10334	—	...	10330	10329	10336	10326	10336
4^1S_0	0^{-+}	10608	-37	10571	68	...	10595	10573	10597	10584	10623
4^3S_1	1^{--}	10619	-40	10579	61	10579	10611	10586	10612	10597	10635
5^1S_0	0^{-+}	10846	-6	10840	53	...	10817	10851	10810	10800	10869
5^3S_1	1^{--}	10856	-2	10854	75	10885	10831	10869	10822	10811	10878
6^1S_0	0^{-+}	11062	-55	11007	35	...	11011	11061	10991	10988	11097
6^3S_1	1^{--}	11071	-56	11015	37	11000	11023	11088	11001	10997	11102
7^1S_0	0^{-+}	11262	-8	11254	60	...	11183	...	11149	...	...
7^3S_1	1^{--}	11269	-10	11259	55	...	11193	...	11157	...	...
3^3P_0	0^{++}	10514	-16	10498	84	...	10502	10521	10513	10490	10522
3^1P_1	1^{+-}	10538	-17	10521	88	...	10529	10544	10530	10519	10541
3^3P_1	1^{++}	10536	-22	10514	86	10513	10524	10541	10527	10515	10538
3^3P_2	2^{++}	10550	-27	10523	85	10524	10540	10550	10539	10528	10550
4^3P_0	0^{++}	10758	+14	10772	53	...	10732	10781	10736	...	10775
4^1P_1	1^{+-}	10780	-1	10779	50	...	10757	10804	10751	...	10790
4^3P_1	1^{++}	10777	-2	10775	49	...	10753	10802	10749	...	10788
4^3P_2	2^{++}	10790	+1	10791	60	...	10767	10812	10758	...	10798
5^3P_0	0^{++}	10978	-21	10957	74	...	10933	...	10926	...	11004
5^1P_1	1^{+-}	10998	-40	10958	51	...	10955	...	10938	...	11016
5^3P_1	1^{++}	10996	-30	10966	58	...	10951	...	10936	...	11014
5^3P_2	2^{++}	11008	-32	10976	47	...	10965	...	10944	...	11022
6^3P_0	0^{++}	11182	+25	11207	46	...	...	...	11090	...	...
6^1P_1	1^{+-}	11199	+6	11205	44	...	...	...	11101	...	...
6^3P_1	1^{++}	11198	+5	11203	56	...	...	...	11099	...	...
6^3P_2	2^{++}	11209	+7	11216	37	...	...	...	11106	...	...
1^3D_1	1^{--}	10154	—	10154	—	...	10145	10154	10153	10146	10138
1^1D_2	2^{-+}	10163	—	10163	—	...	10152	10163	10163	10153	10148
1^3D_3	3^{--}	10167	—	10167	—	...	10156	10166	10170	10157	10155
2^3D_1	1^{--}	10440	—	10440	—	...	10432	10435	10442	10425	10441
2^1D_2	2^{-+}	10449	—	10449	—	...	10439	10445	10450	10432	10450
2^3D_2	2^{--}	10448	—	10448	—	...	10438	10443	10450	10432	10449
2^3D_3	3^{--}	10454	—	10454	—	...	10442	10449	10456	10436	10455
3^3D_1	1^{--}	10688	-18	10670	62	...	10670	10704	10675	...	10698
3^1D_2	2^{-+}	10697	-6	10691	91	...	10677	10713	10681	...	10706
3^3D_2	2^{--}	10696	-11	10685	76	...	10676	10711	10681	...	10705
3^3D_3	3^{--}	10702	-52/+16	10650/10718	30/60	...	10680	10717	10686	...	10711
4^3D_1	1^{--}	10911	-4	10907	68	...	10877	10949	10871	...	10928
4^1D_2	2^{-+}	10920	+6	10926	70	...	10883	10959	10876	...	10935
4^3D_2	2^{--}	10919	-6	10913	72	...	10882	10957	10876	...	10934
4^3D_3	3^{--}	10925	-7	10918	71	...	10886	10963	10880	...	10939
5^3D_1	1^{--}	11116	-18	11098	33	...	11060	...	11041	...	...
5^1D_2	2^{-+}	11126	-62/-38	11064/11088	10/33	...	11066	...	11046	...	...
5^3D_2	2^{--}	11125	-44	11081	34	...	11065	...	11045	...	...
5^3D_3	3^{--}	11131	-67/-41	11064/11090	12/21	...	11069	...	11049	...	...
6^3D_1	1^{--}	11308	-6	11302	70	...	...	...	...	...	...
6^1D_2	2^{-+}	11318	-7	11311	76	...	...	...	...	...	...
6^3D_2	2^{--}	11317	-5	11312	72	...	...	...	...	...	...
6^3D_3	3^{--}	11323	-11	11312	64	...	...	...	...	...	...

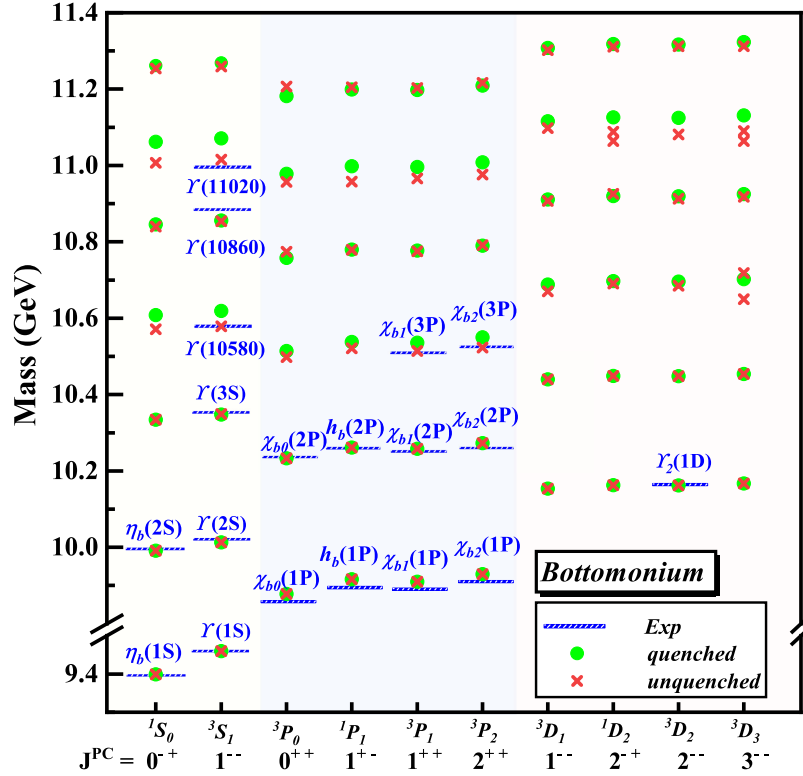


FIG. 2. Bottomonium spectrum compared with the observations. The well-established states are labeled with blue color. The results obtained within the unquenched and quenched quark models are labeled by red crosses and solid green circles, respectively.

obtain two physical solutions with masses 10650 and 10718 MeV; these two solutions correspond to the narrow and broad resonance structures in the spectral function as

shown in the lower panel of Fig. 8, respectively. This indicates that there may exist two physical states $\Upsilon_3(10650)$ and $\Upsilon_3(10718)$ arising from the bare state

TABLE III. Our predicted hyperfine and fine splittings (MeV) in both quenched (Q) and unquenched (UQ) frameworks compared with the data and other model predictions. The data are taken from the RPP [7].

Δm	Q	UQ	Experiment [7]	GM [28]	EFG [29]	LC [30]	DLGZ [42]	WSLM [39]
$m(1^3S_1) - m(1^1S_0)$	61	61	62.3 ± 3.2	63	62	71	70	65
$m(2^3S_1) - m(2^1S_0)$	22	22	24 ± 4	27	33	29	25	28
$m(3^3S_1) - m(3^1S_0)$	14	14		18	26	21	17	20
$m(4^3S_1) - m(4^1S_0)$	11	8		12	13	16	13	15
$m(5^3S_1) - m(5^1S_0)$	10	14		9	18	14	11	12
$m(6^3S_1) - m(6^1S_0)$	9	8		5	27	12	9	10
$m(7^3S_1) - m(7^1S_0)$	7	5		...	...	10	...	8
$m(3^3S_1) - m(2^3S_1)$	335	335	331.5 ± 0.13	351	332	335	328	339
$m(4^3S_1) - m(3^3S_1)$	271	231	224.3 ± 1.7	280	231	260	254	256
$m(5^3S_1) - m(4^3S_1)$	237	275	$305.8^{+3.8}_{-2.8}$	243	283	220	214	210
$m(6^3S_1) - m(5^3S_1)$	215	161	$114.8^{+6.6}_{-5.6}$	224	219	192	186	179
$m(7^3S_1) - m(6^3S_1)$	198	244		...	...	170	...	156
$m(1^3P_2) - m(1^3P_1)$	20	20	19.10 ± 0.25	21	20	21	18	21
$m(1^3P_1) - m(1^3P_0)$	33	33	32.5 ± 0.9	29	33	32	39	31
$m(2^3P_2) - m(2^3P_1)$	15	15	13.10 ± 0.24	15	13	18	15	14
$m(2^3P_1) - m(2^3P_0)$	25	25	23.5 ± 1.0	20	22	25	29	20
$m(3^3P_2) - m(3^3P_1)$	14	9	10.6 ± 1.5	12	9	16	13	12
$m(3^3P_1) - m(3^3P_0)$	22	16		16	20	22	25	14

$\Upsilon_3(3D)$. The narrow resonance $\Upsilon_3(10650)$ lies slightly below the $\bar{B}^*B^*$ threshold. This state is illustrated as a $\bar{B}^*B^*$ -dominant state, since the $b\bar{b}$ -core component is only $P_{b\bar{b}} \simeq 30\%$. On the other hand, the broad resonance $\Upsilon_3(10718)$ is a $b\bar{b}$ -dominant state; the $b\bar{b}$ -core component is estimated to be $P_{b\bar{b}} \simeq 60\%$. It should be noted that an unphysical solution can be found just above the $\bar{B}^*B^*$ threshold, as shown in the upper panel of Fig. 8. This solution does not correspond to any resonance structure in the spectral function, as evidenced by the lower panel of Fig. 8.

Then, we focus on the $\Upsilon_3(5D)$ and $\eta_{b2}(5D)$ states. Both $\Upsilon_3(5D)$ and $\eta_{b2}(5D)$ couple strongly to the $\bar{B}^*B_2^*(5747)$ channel; the strong coupled-channel effects from this channel will cause a notable dip in the mass shift function of each bare state nearby their corresponding mass thresholds. By solving Eq. (17) and combining the spectral function as shown in the third panel of Fig. 8, we obtain two physical states $\Upsilon_3(11064)$ and $\Upsilon_3(11090)$ arising from the $\Upsilon_3(5D)$, and other two physical states $\eta_{b2}(11064)$ and $\eta_{b2}(11088)$ arising from the $\eta_{b2}(5D)$. Furthermore, it can be seen that the $\Upsilon_3(11064)$ and $\eta_{b2}(11064)$ are narrow resonances, while the $\Upsilon_3(11090)$ and $\eta_{b2}(11088)$ are broad resonances. This is due to the opening of decay channels for the S -wave mode, which significantly contributes to the decay widths. Both the $\Upsilon_3(11064)$ and $\eta_{b2}(11064)$ may be $\bar{B}^*B^*(5747)$ -dominant states; their $b\bar{b}$ -core components are estimated to be $P_{b\bar{b}} \simeq 12$ and 10% , respectively. For the two broad states $\Upsilon_3(11090)$ and $\eta_{b2}(11088)$, the $b\bar{b}$ -core components are estimated to be $P_{b\bar{b}} \simeq 21$ and 33% , respectively; their components are dominated by the continuum states of various open-bottom meson pairs.

IV. STRONG DECAYS

For the high-lying states with a mass above the $B\bar{B}$ threshold, the OZI-allowed two-body strong decay channels are open. The partial and total strong decay widths for these high-lying nS ($n = 5, 6, 7$), nP ($n = 4, 5, 6$), and nD ($n = 3, 4, 5, 6$) states are estimated by using Eq. (19). Our results have been given in Tables IV–IX. It should be mentioned that in this work we take the measured width $\Gamma_{\text{exp}} \simeq 20.5$ MeV [7] of the $\Upsilon(10580)$ as an input to determine the strength of quark pair creation in the $^3P_0$ model.

A. S -wave states

1. $\Upsilon(11020)$

First, we give some discussions about a well-established resonance $\Upsilon(11020)$. It is often assigned to the $\Upsilon(6S)$ state classified in the quark model. In the present work, it is found that the $\Upsilon(11020)$ resonance may have significant hadron continuum components, while the $b\bar{b}$ -core component is only $\sim 40\%$. The decay width is predicted to be

$$\Gamma \simeq 16.5 \text{ MeV}, \quad (27)$$

which is consistent with the PDG-averaged value $\Gamma_{\text{exp}} = 24_{-6}^{+8}$ MeV [7]. Our results show that the $\Upsilon(11020)$ dominantly decays into the $\bar{B}B(1P_1)$ channel with a branching fraction of $\sim 75\%$. Thus, the $\Upsilon(11020)$ may provide a good source for looking for the missing orbitally excited state $B(1P_1)(5656)$ predicted in the quark model [69]. From Table IV, one finds that the decay rates of $\Upsilon(11020)$ into the $\bar{B}B$, $\bar{B}_s B_s$, $\bar{B}B^*$, and $\bar{B}_s B_s^*$ channels are predicted to be tiny; their branching fractions are on the order of 1% , which may be a natural explanation why $\Upsilon(11020)$ has not been established in these open-bottom channels.

2. $\Upsilon(10860)$

For the $\Upsilon(10860)$ resonance, as for the $\Upsilon(5S)$ assignment, the decay properties cannot be well explained in theory, although the observed mass is consistent with the quark model predictions. If taking the measured mass $M_{\text{exp}} = 10885$ MeV, the width is predicted to be

$$\Gamma \simeq 8 \text{ MeV}, \quad (28)$$

which is notably narrower than the observed one, $\Gamma_{\text{exp}} \simeq 37 \pm 4$ MeV [7]. It should be mentioned that the $\bar{B}^*B(1^3P_0)$ channel may open. By combining the predicted mass of $M = 5573$ MeV for the $B(1^3P_0)$ state in our previous work [69], the mass threshold of the $\bar{B}^*B(1^3P_0)$ channel is predicted to be ~ 10.9 GeV, which is very close to the mass of $\Upsilon(10860)$. If the $\bar{B}^*B(1^3P_0)$ channel opens, it can contribute a fairly large partial width of ~ 30 – 40 MeV due to its strong S -wave coupling to the $\Upsilon(5S)$. Then, the total width of $\Upsilon(5S)$ can reach up to $\Gamma \sim 40$ – 50 MeV, which is consistent with the measured width of $\Upsilon(10860)$. However, if the $\bar{B}^*B(1^3P_0)$ channel opens, the $\Upsilon(10860)$ should have a large decay rate ($\sim 70\%$) into the $\bar{B}^*B\pi$ channel via the cascade decay, which is inconsistent with the observation ($\sim 7.3\%$) by the Belle Collaboration [112].

On the other hand, if assigning $\Upsilon(10860)$ to the $\Upsilon(5S)$ state, there also exists a puzzle in the decay modes. According to our predictions, the $\Upsilon(5S)$ state mainly decays into the $\bar{B}_s^*B_s^*$, $\bar{B}_s B_s^*$, and $\bar{B}B^*$ channels if the $\bar{B}^*B(1^3P_0)$ channel does not open. However, the observation shows that the $\Upsilon(10860)$ resonance dominantly decays into the $\bar{B}^*B^*$, $\bar{B}_s^*B_s^*$, and $\bar{B}B^*$ channels with branching fractions of $\sim 38\%$, $\sim 18\%$, and $\sim 14\%$, respectively [7]. There have been many studies of the decay properties of the $\Upsilon(5S)$ in the literature [28,31,35,39,41,43,63,113]. In these works, the $\bar{B}B^*$ and $\bar{B}_s^*B_s^*$ are often predicted to be the main channels, while the decay rate into the $\bar{B}^*B^*$ channel is often small.

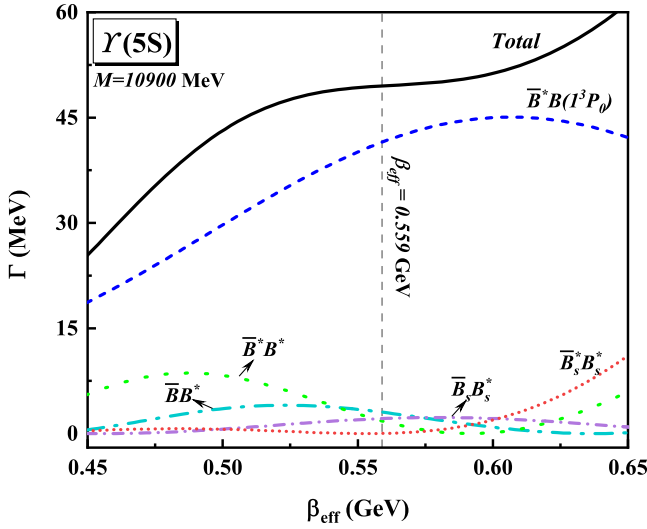


FIG. 3. The total and partial decay widths of $\Upsilon(5S)$ as a function of its wave-function size parameter β_{eff} . The $\bar{B}B$ and $\bar{B}_s B_s$ channels are neglected due to their tiny contributions.

The discrepancy between theoretical predictions and observations may arise from the uncertainty of the wave function of the $\Upsilon(5S)$ predicted in the quark model. To see the wave-function dependency of our predictions, we first fit the numerical wave function obtained from the potential model by using a simple harmonic oscillator (SHO) form, and obtain the size parameter $\beta_{\text{eff}} = 0.559$ GeV. Then, we calculate the decay properties with the SHO wave function by varying the size parameter around the region of $\beta_{\text{eff}} = 0.559$ GeV estimated from the potential model. When calculating the decay width, we adopted a mass of $M = 10900$ MeV, which is slightly above the $\bar{B}^*B(1^3P_0)$ threshold, to include this channel contribution. Our results have been shown in Fig. 3. It is seen that the partial widths of $\bar{B}^*B^*$, $\bar{B}B^*$, $\bar{B}_s^*B_s^*$, and $\bar{B}_s B_s^*$ are rather sensitive to the details of the wave function. If taking $\beta_{\text{eff}} = 0.500$ GeV, the $\bar{B}^*B^*$ and $\bar{B}B^*$ are the main decay channels of the $\Upsilon(5S)$, which seems to favor the observation. However, in this case the decay rates of $\Upsilon(5S) \rightarrow \bar{B}_s^*B_s^*, \bar{B}_s B_s^*$ are too small to be comparable with the data.

Finally, it should be mentioned that the $\Upsilon(10860)$ may be partly contributed by some exotic components as well. For example, in Ref. [114] the analysis of the data of $e^+e^- \rightarrow b\bar{b}$ cross section from the BABAR Collaboration [12] may indicate two overlapping resonances in the $\Upsilon(5S)$ mass region. One corresponds to the $\Upsilon(5S)$ state, while the other may correspond to a bottomonium-like tetraquark state. To uncover the puzzles of the $\Upsilon(10860)$ resonance, more observations are waiting to be carried out in future experiments.

3. Higher S-wave states

For the other missing S-wave states, n^1S_0 ($n = 5, 6, 7$) and 7^3S_1 , our predicted decay properties have been given in

Tables IV and V. These states may be very narrow states with a width of $\Gamma \simeq 1\text{--}20$ MeV. The 5^1S_0 state may have a mass of $M = 10840$ MeV, and dominantly decays into the $\bar{B}_s^*B_s^*$, $\bar{B}^*B^*$, and $\bar{B}_s B_s^*$ channels with branching fractions ~ 64 , ~ 25 , and $\sim 10\%$, respectively. The 7^1S_0 and 7^3S_1 states have a nearly degenerated mass of $M = 11255$ MeV and a comparable width of $\Gamma \simeq 22$ MeV. They mainly decay into the channels containing excited B-meson states, while their decay rates into the $\bar{B}^{(*)}B^{(*)}$ and $\bar{B}_s^{(*)}B_s^{(*)}$ are very tiny.

TABLE IV. The mass shifts and strong decay width of each channel for the 4S-, 5S-, and 6S-wave bottomonium states, in units of MeV. For the forbidden/unopened channels, the absence values are labeled with “—.” The mass shifts labeled with bold numbers are contributed by the nearby virtual channels whose mass thresholds are slightly above the bare state. The experimental data are taken from PDG [7].

Channel	4^1S_0		4^3S_1	
	$\Delta M[10608]$	$\Gamma[10571]$	$\Delta M[10619]$	$\Gamma[as\Upsilon(10580)]$
$\bar{B}B$	—	—	−6.4	20.5
$\bar{B}_s B_s$	—	—	—	—
$\bar{B}B^*$	−22.0	—	−15.8	—
$\bar{B}^*B^*$	−15.2	—	−17.9	—
Total	−37.2	—	−40.1	20.5
$M_{\text{th}}, \Gamma_{\text{th}}$	10571, —		10579, 20.5	
$M_{\text{exp}}, \Gamma_{\text{exp}}$	—, —		10579, 20.5 ± 2.5	
Channel	5^1S_0		5^3S_1	
	$\Delta M[10846]$	$\Gamma[10840]$	$\Delta M[10856]$	$\Gamma[as\Upsilon(10860)]$
$\bar{B}B$	—	—	+0.1	0.8
$\bar{B}_s B_s$	—	—	~ 0	0.4
$\bar{B}B^*$	+1.4	0.2	+0.3	2.4
$\bar{B}_s B_s^*$	+1.8	1.2	+0.3	1.5
$\bar{B}^*B^*$	+5.1	3.0	+4.2	0.5
$\bar{B}_s^*B_s^*$	−5.9	7.8	−0.9	2.4
$\bar{B}B(1^3P_0)$	−8.2	—	—	—
$\bar{B}^*B(1^3P_0)$	—	—	−3.6	—
$\bar{B}B(1P_1)$	—	—	−2.7	—
Total	−5.8	12.2	−2.3	7.9
$M_{\text{th}}, \Gamma_{\text{th}}$	10840, 12.2		10854, 7.9	
$M_{\text{exp}}, \Gamma_{\text{exp}}$	—, —		$10885^{+3}_{-2}, 37 \pm 4$	
Channel	6^1S_0		6^3S_1	
	$\Delta M[11062]$	$\Gamma[11007]$	$\Delta M[11071]$	$\Gamma[as\Upsilon(11020)]$
$\bar{B}B$	—	—	+0.03	0.1
$\bar{B}_s B_s$	—	—	+0.02	0.04
$\bar{B}B^*$	+0.1	0.4	+0.06	0.2
$\bar{B}_s B_s^*$	~ 0	0.4	+0.02	0.2
$\bar{B}^*B^*$	+0.4	~ 0	+0.3	~ 0
$\bar{B}_s^*B_s^*$	+0.02	~ 0	−0.03	~ 0
$\bar{B}B(2^1S_0)$	—	—	−2.3	—
$\bar{B}(2^1S_0)B^*$	−4.8	—	−3.2	—

(Table continued)

TABLE IV. (Continued)

Channel	4^1S_0		4^3S_1	
	$\Delta M[10608]$	$\Gamma[10571]$	$\Delta M[10619]$	$\Gamma[as\Upsilon(10580)]$
$\bar{B}B(2^3S_1)$	-4.9	—	-3.3	—
$\bar{B}^*B(2^3S_1)$	-8.1	—	-9.4	—
$\bar{B}B(1^3P_0)$	+1.2	0.1	—	—
$\bar{B}_sB_s(1^3P_0)$	-0.5	—	—	—
$\bar{B}^*B(1^3P_0)$	—	—	+2.1	2.8
$\bar{B}_s^*B_s(1^3P_0)$	—	—	-0.3	—
$\bar{B}B_2^*(5747)$	-10.5	—	-6.1	—
$\bar{B}^*B_2^*(5747)$	-9.7	—	-13.3	—
$\bar{B}B(1P_1)$	—	—	+3.1	12.4
$\bar{B}_sB_s(1P_1)$	—	—	-0.3	—
$\bar{B}B_1(5721)$	—	—	-8.7	—
$\bar{B}^*B(1P_1)$	-9.2	3.2	-8.1	0.8
$\bar{B}^*B_1(5721)$	-8.9	—	-6.2	—
<i>Total</i>	-54.9	4.1	-55.6	16.5
M_{th}, Γ_{th}	11007, 4.1		11015, 16.5	
M_{exp}, Γ_{exp}	—, —		$11000 \pm 4, 24_{-6}^{+8}$	

TABLE V. (Continued)

Channel	7^1S_0		7^3S_1	
	$\Delta M[11262]$	$\Gamma[11254]$	$\Delta M[11269]$	$\Gamma[11259]$
$\bar{B}_sB_{s1}(5830)$	—	—	-0.1	0.01
$\bar{B}^*B(1P_1)$	+1.4	~ 0	+0.3	~ 0
$\bar{B}_s^*B_s(1P_1)$	-1.2	0.7	+0.2	0.6
$\bar{B}^*B_1(5721)$	~ 0	1.7	+0.7	1.5
$\bar{B}_sB_{s1}(5830)$	-2.2	1.3	-0.7	1.7
$\bar{B}B(1^3D_1)$	+1.7	3.6	+0.3	0.3
$\bar{B}B(1^3D_3)$	-4.9	—	-2.1	—
$\bar{B}B(1D_2)$	—	—	+1.5	3.6
$\bar{B}B(1D_2')$	—	—	-1.5	—
$\bar{B}^*B(1^3D_1)$	-5.0	—	-1.5	—
$\bar{B}^*B(1^3D_3)$	-0.5	—	-6.8	—
$\bar{B}^*B(1D_2)$	-3.5	—	-4.2	—
$\bar{B}(1^3P_0)B(1^3P_0)$	—	—	+0.08	0.09
$\bar{B}(1^3P_0)B_2^*(5747)$	—	—	-0.8	—
$\bar{B}(1^3P_0)B(1P_1)$	+1.2	3.5	+0.7	1.0
$\bar{B}(1^3P_0)B_1(5721)$	-0.9	—	-0.1	—
<i>Total</i>	-7.7	23.0	-10.3	21.7
M_{th}, Γ_{th}	11254, 23.0		11259, 21.7	

TABLE V. The mass shifts and strong decay width of each channel for the $7S$ -wave bottomonium states, in units of MeV. The rest of the caption is the same as that of Table IV.

Channel	7^1S_0		7^3S_1	
	$\Delta M[11262]$	$\Gamma[11254]$	$\Delta M[11269]$	$\Gamma[11259]$
$\bar{B}B$	—	—	+0.01	0.01
$\bar{B}_sB_s$	—	—	~ 0	~ 0
$\bar{B}B^*$	+0.1	0.1	+0.05	0.08
$\bar{B}_sB_s^*$	+0.02	~ 0	~ 0	~ 0
$\bar{B}^*B^*$	+0.06	0.2	+0.01	0.2
$\bar{B}_s^*B_s^*$	+0.03	0.03	+0.03	0.02
$\bar{B}B(2^1S_0)$	—	—	+0.4	1.0
$\bar{B}_sB_s(2^1S_0)$	—	—	-0.2	—
$\bar{B}(2^1S_0)B^*$	+0.05	1.6	-0.05	1.5
$\bar{B}_s(2^1S_0)B_s^*$	-0.5	—	-0.3	—
$\bar{B}B(2^3S_1)$	-0.07	2.5	-0.02	1.9
$\bar{B}_sB_s(2^3S_1)$	-0.5	—	-0.3	—
$\bar{B}^*B(2^3S_1)$	+6.3	1.9	+6.0	0.6
$\bar{B}_s^*B_s(2^3S_1)$	—	—	-0.9	—
$\bar{B}B(1^3P_0)$	+0.2	0.5	—	—
$\bar{B}_sB_s(1^3P_0)$	+0.4	0.7	—	—
$\bar{B}^*B(1^3P_0)$	—	—	+0.1	0.2
$\bar{B}_s^*B_s(1^3P_0)$	—	—	+0.2	1.2
$\bar{B}B_2^*(5747)$	+0.3	2.9	+0.2	1.4
$\bar{B}_sB_{s2}^*(5840)$	+0.9	1.2	+0.4	0.3
$\bar{B}^*B_2^*(5747)$	+0.7	0.6	+0.4	1.7
$\bar{B}_s^*B_{s2}^*(5840)$	-1.8	—	-2.6	0.3
$\bar{B}B(1P_1)$	—	—	+0.2	0.03
$\bar{B}_sB_s(1P_1)$	—	—	-0.1	1.4
$\bar{B}B_1(5721)$	—	—	+0.2	1.1

(Table continued)

B. P -wave states

Since the $3P$ states $\chi_{b1}(3P)$ and $\chi_{b2}(3P)$ have been established in experiments [7], the higher $4P$ states may have large potentials to be observed in future experiments. According to the quark model predictions, their masses scatter in the range of 10.77–10.79 GeV; thus, some OZI-allowed two-body strong decay channels open. We evaluate the strong decay properties of the $4P$ states, and list the results in Table VI. It is found that the $4P$ $b\bar{b}$ states may be narrow states. For the 0^+ state $\chi_{b0}(4P)$, the width is predicted to be $\Gamma \simeq 10$ MeV. This state mainly decays into $\bar{B}B$ and $\bar{B}^*B^*$ channels with branching fractions ~ 57 and $\sim 41\%$, respectively, while for the two $J^P = 1^+$ states, $\chi_{b1}(4P)$ and $h_b(4P)$, the widths are predicted to be only a few MeV. They may dominantly decay into the $\bar{B}B^*$ channel. The 2^+ state $\chi_{b2}(4P)$ has a narrow width of $\Gamma \simeq 11$ MeV. Besides the dominant decay channel $\bar{B}B^*$, it also has sizable decay rates into the $\bar{B}B$, $\bar{B}_sB_s$, and $\bar{B}^*B^*$ channels with a comparable branching fraction of $\sim 20\%$. To establish the $4P$ states, the decay channels $\bar{B}B$, $\bar{B}B^*$, $\bar{B}^*B^*$, and $\bar{B}_sB_s$ are worth observing in future experiments.

For the higher $5P$ and $6P$ excitations, the masses are predicted in the ranges of 10.96–10.98 and 11.20–11.22 GeV, respectively. By combining the predicted masses and wave functions, we further evaluate the strong decay properties. Our results are given in Tables VI and VII. The $5P$ may be very narrow states with a width of $\Gamma \simeq 6$ –20 MeV, while the $6P$ may be relatively broader states with a width of $\Gamma \simeq 40$ –60 MeV. The $5P$ and $6P$ states couple weakly to the common channels $\bar{B}^{(*)}B^{(*)}$ and

TABLE VI. The mass shifts and strong decay width of each channel for the $3P$ -, $4P$ -, and $5P$ -wave bottomonium states, in units of MeV. The rest of the caption is the same as that of Table IV.

Channel	3^3P_0		3^3P_2		3^1P_1		3^3P_1	
	ΔM [10514]	Γ [10498]	ΔM [10550]	Γ [10523]	ΔM [10538]	Γ [10521]	ΔM [10536]	Γ [10514]
$\bar{B}B$	-15.5	—	-3.5	—	—	—	—	—
$\bar{B}B^*$	—	—	-6.9	—	-17.3	—	-21.9	—
$\bar{B}^*B^*$	—	—	-16.5	—	—	—	—	—
<i>Total</i>	-15.5	—	-26.9	—	-17.3	—	-21.9	—
$M_{\text{th}}, \Gamma_{\text{th}}$	10498, —		10523, —		10521, —		10514, —	
Channel	4^3P_0		4^3P_2		4^1P_1		4^3P_1	
	ΔM [10758]	Γ [10772]	ΔM [10790]	Γ [10791]	ΔM [10780]	Γ [10779]	ΔM [10777]	Γ [10775]
$\bar{B}B$	-1.1	5.2	+0.6	2.0	—	—	—	—
$\bar{B}_s B_s$	+2.1	0.3	+0.1	1.8	—	—	—	—
$\bar{B}B^*$	—	—	-0.1	5.8	-0.4	5.5	-0.1	3.0
$\bar{B}_s B_s^*$	—	—	-2.2	0.5	-4.9	—	-5.9	—
$\bar{B}^*B^*$	+15.8	5.0	+5.1	1.2	+6.3	1.9	+6.2	0.5
$\bar{B}_s^*B_s^*$	-3.1	—	-2.8	—	-2.0	—	-1.7	—
<i>Total</i>	+13.7	10.5	+0.7	11.3	-1.0	7.4	-1.5	3.5
$M_{\text{th}}, \Gamma_{\text{th}}$	10772, 10.5		10791, 11.3		10779, 7.4		10775, 3.5	
Channel	5^3P_0		5^3P_2		5^1P_1		5^3P_1	
	ΔM [10978]	Γ [10957]	ΔM [11008]	Γ [10976]	ΔM [10998]	Γ [10958]	ΔM [10996]	Γ [10966]
$\bar{B}B$	+0.2	1.3	+0.1	0.2	—	—	—	—
$\bar{B}_s B_s$	+0.1	0.1	+0.02	~0	—	—	—	—
$\bar{B}B^*$	—	—	+0.4	0.9	+0.3	1.8	+0.2	2.0
$\bar{B}_s B_s^*$	—	—	+0.2	0.3	+0.2	0.7	+0.2	0.5
$\bar{B}^*B^*$	+0.4	1.4	+0.1	1.9	+0.1	1.1	+0.2	1.4
$\bar{B}_s^*B_s^*$	-0.1	0.7	+0.01	0.7	+0.1	0.4	-0.1	0.8
$\bar{B}B(2^1S_0)$	—	—	-0.1	—	—	—	—	—
$\bar{B}B(1^3P_0)$	—	—	—	—	+0.3	1.6	~0	~0
$\bar{B}^*B(1^3P_0)$	—	—	+0.1	0.8	~0	~0	+0.8	0.1
$\bar{B}B_2^*(5747)$	—	—	-3.6	—	-7.7	—	-7.0	—
$\bar{B}^*B_2^*(5747)$	—	—	-15.3	—	-9.3	—	-10.0	—
$\bar{B}B(1P_1)$	-0.3	17.8	+4.0	4.9	~0	~0	+3.2	10.6
$\bar{B}B_1(5721)$	-11.2	—	-3.5	—	-0	—	-2.7	—
$\bar{B}^*B(1P_1)$	-5.4	—	-9.1	—	-7.4	—	-6.5	—
$\bar{B}^*B_1(5721)$	-4.2	—	-4.9	—	-17.0	—	-8.2	—
<i>Total</i>	-20.5	21.3	-31.6	9.7	-40.4	5.6	-29.9	15.4
$M_{\text{th}}, \Gamma_{\text{th}}$	10957, 21.3		10976, 9.7		10958, 5.6		10966, 15.4	

$\bar{B}_s^{(*)}B_s^{(*)}$, while often couple strongly to the channels which contain an excited B meson. For example, the $\chi_{b2}(6P)$ state weakly couples to the $\bar{B}^{(*)}B^{(*)}$ and $\bar{B}_s^{(*)}B_s^{(*)}$ channels, while it strongly couples to the $\bar{B}^*B(2^3S_1)$ channel. To know more details of the strong decay properties of the $5P$ and $6P$ states, one can see Tables VI and VII.

C. D-wave states

From our model, the mass range of the $3D$ states is 10.67–10.72 GeV. They have phase spaces for decaying into the low-lying OZI-allowed channels, such as $\bar{B}B$, $\bar{B}B^*$, $\bar{B}^*B^*$, and so on. The strong decay properties are given in

Table VIII. For one case in $J^P = 1^-$ and two cases with different total quark spins in $J^P = 2^-$, there is only one solution for each case. The predicted widths of $\Upsilon_1(3D)(10670)$, $\Upsilon_2(3D)(10685)$, and $\eta_{b2}(3D)(10691)$ are $\Gamma \simeq 31$, 81, and 53 MeV, respectively. They mainly decay into both the $\bar{B}B^*$ and $\bar{B}^*B^*$ channels with a comparable ratio $\sim 1:2$. On the other hand, there are two physical solutions with $J^P = 3^-$, $\Upsilon_3(10650)$ and $\Upsilon_3(10718)$, obtained in the unquenched quark model. The lower mass resonance $\Upsilon_3(10650)$ may have a narrow width of $\Gamma \simeq 19$ MeV, and mainly decays into the $\bar{B}B$ ($\sim 45\%$) and $\bar{B}B^*$ ($\sim 55\%$) final states, while the higher mass resonance $\Upsilon_3(10718)$ has a relatively broader width

TABLE VII. The mass shifts and strong decay width of each channel for the $6P$ -wave bottomonium states, in units of MeV. The rest of the caption is the same as that of Table IV.

Channel	6^3P_0		6^3P_2		6^1P_1		6^3P_1	
	ΔM [11182]	Γ [11207]	ΔM [11209]	Γ [11216]	ΔM [11199]	Γ [11205]	ΔM [11198]	Γ [11203]
$\bar{B}B$	+0.1	0.1	+0.02	~ 0	—	—	—	—
$\bar{B}_s B_s$	~ 0	~ 0	~ 0	~ 0	—	—	—	—
$\bar{B}B^*$	—	—	+0.1	0.03	+0.2	0.2	+0.2	0.2
$\bar{B}_s B_s^*$	—	—	~ 0	0.02	~ 0	0.02	~ 0	0.03
$\bar{B}^* B^*$	+0.3	0.5	+0.2	0.3	+0.2	0.3	+0.2	0.2
$\bar{B}_s^* B_s^*$	+0.1	0.01	+0.02	~ 0	+0.03	~ 0	+0.04	~ 0
$\bar{B}B(2^1S_0)$	+1.8	6.9	+0.1	0.1	—	—	—	—
$\bar{B}_s B_s(2^1S_0)$	—	—	-0.2	—	—	—	—	—
$\bar{B}(2^1S_0)B^*$	—	—	-0.1	5.9	-0.3	4.7	+1.0	2.1
$\bar{B}B(2^3S_1)$	—	—	+0.6	6.1	-0.2	6.8	+0.2	4.1
$\bar{B}^* B(2^3S_1)$	+22.7	14.1	+10.8	6.5	+12.5	14.7	+13.2	9.0
$\bar{B}B(1^3P_0)$	—	—	—	—	-0.05	~ 0	~ 0	~ 0
$\bar{B}_s B_s(1^3P_0)$	—	—	—	—	+0.1	~ 0	~ 0	~ 0
$\bar{B}^* B(1^3P_0)$	-0.04	0.2	-0.04	0.2	~ 0	~ 0	~ 0	0.1
$\bar{B}_s^* B_s(1^3P_0)$	+0.1	0.7	+0.2	0.3	~ 0	~ 0	+0.09	0.6
$\bar{B}B_2^*(5747)$	—	—	+1.0	2.3	+1.4	6.2	+0.7	4.6
$\bar{B}_s B_{s2}^*(5840)$	—	—	-0.7	0.9	-1.8	—	-1.4	—
$\bar{B}^* B_2^*(5747)$	-1.5	12.6	+0.2	7.0	+0.2	4.6	-1.0	7.5
$\bar{B}_s^* B_{s2}^*(5840)$	—	—	-1.8	—	-1.2	—	-1.2	—
$\bar{B}B(1P_1)$	+0.2	~ 0	+0.1	~ 0	~ 0	~ 0	+0.2	~ 0
$\bar{B}_s B_s(1P_1)$	-0.2	0.8	~ 0	0.7	~ 0	~ 0	-0.2	0.6
$\bar{B}B_1(5721)$	+0.9	5.6	+1.1	1.2	~ 0	~ 0	+0.2	1.2
$\bar{B}_s B_{s1}(5830)$	-2.4	8.2	-0.7	0.6	~ 0	~ 0	-0.7	1.4
$\bar{B}^* B(1P_1)$	+0.7	0.6	+0.7	0.5	+1.0	1.0	+0.7	0.7
$\bar{B}_s^* B_s(1P_1)$	+0.6	2.3	+0.6	0.6	-0.5	0.6	+0.4	3.0
$\bar{B}^* B_1(5721)$	+0.2	0.7	+0.5	5.4	+3.6	4.2	+0.1	3.5
$\bar{B}_s^* B_{s1}(5830)$	+0.9	3.4	-0.6	—	-1.1	—	-0.9	—
$\bar{B}B(1^3D_1)$	—	—	-0.2	—	-1.1	—	-0.3	—
$\bar{B}B(1^3D_3)$	—	—	-1.7	—	-4.7	—	-3.3	—
$\bar{B}B(1D_2)$	—	—	-1.8	—	~ 0	—	-0.3	—
$\bar{B}^* B(1^3D_1)$	—	—	~ 0	—	-0.3	—	-0.9	—
$\bar{B}^* B(1D_2)$	—	—	-1.5	—	-2.2	—	-1.9	—
$\bar{B}(1^3P_0)B(1^3P_0)$	—	—	+0.1	0.01	—	—	—	—
Total	+24.5	56.7	+7.0	38.7	+5.8	43.3	+5.1	38.8
$M_{\text{th}}, \Gamma_{\text{th}}$	11207, 56.7		11216, 38.7		11205, 43.3		11203, 38.8	

of $\Gamma \simeq 63$ MeV, and dominantly decays into the $\bar{B}B^*$ ($\sim 40\%$) and $\bar{B}^*B^*$ ($\sim 53\%$) final states. Thus, to establish the $3D$ states, the $\bar{B}B$, $\bar{B}B^*$, and $\bar{B}^*B^*$ channels provide a nice place for observing in future experiments.

The masses of the $4D$ states are estimated to be in the range of 10.91–10.93 GeV, which is about 200 MeV above the $3D$ states. With our predicted masses, the strong decay properties are analyzed, as listed in Table VIII. Here, it shows that the $4D$ states are narrow resonances with a width of $\Gamma \simeq 10$ –20 MeV. These states dominantly decay into the low-lying open-bottom channels, $\bar{B}^{(*)}B^{(*)}$ and $\bar{B}_s^{(*)}B_s^{(*)}$. For example, the $\Upsilon_1(4D)$ state may dominantly decay into $\bar{B}^*B^*$ and $\bar{B}_s^*B_s^*$ channels with a comparable decay rate of $\sim 35\%$, while it also has sizable decay rates

into the $\bar{B}B$ ($\sim 8\%$) and $\bar{B}B^*$ ($\sim 10\%$) channels. To establish the $4D$ states, the $\bar{B}B^*$, $\bar{B}^*B^*$, and $\bar{B}_s^*B_s^*$ channels are worth observing in future experiments; please note it is a bit different from the case of $3D$ states.

For the higher $5D$ and $6D$ states, the masses are predicted to be in the ranges of 11.06–11.10 and 11.30–11.31 GeV, respectively. By using the predicted masses, we further study the strong decay properties of these $5D$ and $6D$ states; the results are given in Tables VIII and IX, respectively. The widths of these state are significant larger than that of $4D$ states. These higher $5D$ and $6D$ states mainly decay into the open-bottom channels containing a B -meson excitation, while their decay rates into the $\bar{B}^{(*)}B^{(*)}$ and $\bar{B}_s^{(*)}B_s^{(*)}$ final states are tiny. Thus, the $5D$

TABLE VIII. The mass shifts and strong decay width of each channel for the $3D$ -, $4D$ -, and $5D$ -wave bottomonium states, in units of MeV. The rest of the caption is the same as that of Table IV.

Channel	3^3D_1		3^3D_3		3^1D_2		3^3D_2	
	ΔM [10688]	Γ [10670]	ΔM [10702]	Γ [10650/10718]	ΔM [10697]	Γ [10691]	ΔM [10696]	Γ [10685]
$\bar{B}B$	-0.8	2.1	-0.8/ + 2.5	8.6/4.0	—	—	—	—
$\bar{B}_s B_s$	-1.0	—	-0.5/ - 0.9	-/-	—	—	—	—
$\bar{B}B^*$	+8.0	8.6	-9.7/ + 2.7	10.6/25.3	-0.2	28.2	+4.5	20.7
$\bar{B}_s B_s^*$	—	—	-0.6/ - 0.9	-/-	-1.9	—	-2.0	—
$\bar{B}^* B^*$	-23.7	20.0	-40.4/ + 13.0	-/33.6	-4.2	52.5	-13.7	32.5
<i>Total</i>	-17.5	30.7	-52.0/ + 16.4	19.2/62.9	-6.3	80.7	-11.2	53.2
$M_{\text{th}}, \Gamma_{\text{th}}$	10670, 30.7		10650, 19.2/10718, 62.9		10691, 80.7		10685, 53.2	
Channel	4^3D_1		4^3D_3		4^1D_2		4^3D_2	
	ΔM [10911]	Γ [10907]	ΔM [10925]	Γ [10918]	ΔM [10920]	Γ [10926]	ΔM [10919]	Γ [10913]
$\bar{B}B$	+0.6	1.4	+0.3	0.1	—	—	—	—
$\bar{B}_s B_s$	+0.1	0.1	~ 0	0.04	—	—	—	—
$\bar{B}B^*$	+0.2	1.8	+1.1	1.3	+1.7	3.2	+1.4	4.1
$\bar{B}_s B_s^*$	+0.07	0.6	+0.4	0.3	+0.6	0.7	+0.6	1.4
$\bar{B}^* B^*$	+1.1	6.7	+0.7	5.4	+1.1	4.7	+1.0	4.8
$\bar{B}_s^* B_s^*$	-0.2	5.5	-0.04	2.3	+0.3	2.7	+0.1	3.4
$\bar{B}B(1^3P_0)$	—	—	—	—	+2.5	—	~ 0	—
$\bar{B}^* B(1^3P_0)$	-4.5	1.6	-3.8	5.1	~ 0	0.03	-4.6	3.8
$\bar{B}B(1P_1)$	-1.8	—	-2.6	—	~ 0	—	-2.0	—
$\bar{B}B(5721)$	—	—	-2.6	—	~ 0	—	-2.1	—
<i>Total</i>	-4.4	17.7	-6.5	14.5	+6.2	11.3	-5.6	17.5
$M_{\text{th}}, \Gamma_{\text{th}}$	10907, 17.7		10918, 14.5		10926, 11.3		10913, 17.5	
Channel	5^3D_1		5^3D_3		5^1D_2		5^3D_2	
	ΔM [11116]	Γ [11098]	ΔM [11131]	Γ [11064/11090]	ΔM [11126]	Γ [11064/11088]	ΔM [11125]	Γ [11081]
$\bar{B}B$	+0.1	0.1	+0.05/ + 0.04	0.02/ ~ 0	—	—	—	—
$\bar{B}_s B_s$	-0.01	0.03	~ 0 / ~ 0	0.01/0.02	—	—	—	—
$\bar{B}B^*$	+0.1	0.2	+0.2/ + 0.2	0.2/0.1	+0.3/ + 0.3	0.5/0.4	+0.3	0.5
$\bar{B}_s B_s^*$	~ 0	~ 0	+0.03/ ~ 0	~ 0 / ~ 0	+0.1/ + 0.02	~ 0	+0.04	~ 0
$\bar{B}^* B^*$	+0.5	0.8	+0.2/ + 0.3	0.9/0.9	+0.2/ + 0.3	0.7/0.7	+0.3	0.7
$\bar{B}_s^* B_s^*$	+0.1	0.1	+0.1/ + 0.1	0.3/0.1	+0.1/ + 0.1	0.2/0.1	+0.1	0.1
$\bar{B}B(2^1S_0)$	+6.9	9.8	-2.8/ - 4.8	-/7.3	-/-	—	—	—
$\bar{B}(2^1S_0)B^*$	-7.0	—	-2.2/ - 3.1	-/-	-5.1/ - 7.0	—	-6.9	—
$\bar{B}B(2^3S_1)$	-8.8	4.1	-2.3/ - 3.4	-/-	-5.4/ - 8.5	—	-8.1	—
$\bar{B}^* B(2^3S_1)$	-13.4	—	-9.2/ - 12.4	-/-	-7.4/ - 9.0	—	-7.8	—
$\bar{B}B(1^3P_0)$	—	—	-/-	-/-	+0.2/ + 0.06	—	~ 0	—
$\bar{B}_s B_s(1^3P_0)$	—	—	-/-	-/-	-0.4/ - 0.2	—	~ 0	—
$\bar{B}^* B(1^3P_0)$	+0.4	0.1	+0.4/ + 0.4	0.6/0.2	~ 0 / ~ 0	~ 0 / ~ 0	+0.5	0.3
$\bar{B}_s^* B_s(1^3P_0)$	-0.4	—	-0.1/ - 0.2	-/-	~ 0 / ~ 0	—	-0.2	—
$\bar{B}B_2(5747)$	+0.8	0.2	-1.6/ - 0.3	7.7/7.2	+1.6/ + 1.9	6.5/1.2	+0.8	16.6
$\bar{B}_s B_{s2}(5840)$	—	—	-0.2/ - 0.2	-/-	-0.1/ - 0.1	—	-0.5	—
$\bar{B}^* B_2(5747)$	+0.3	9.0	-42.0/ - 12.4	-/41.4	-36.6/ - 10.5	-/22.0	-17.4	18.0
$\bar{B}B(1P_1)$	+0.6	1.1	+0.1/ + 0.6	1.7/1.5	~ 0 / ~ 0	—	+0.4	1.4
$\bar{B}_s B_s(1P_1)$	-0.3	—	-0.2/ - 0.2	-/-	~ 0/ ~ 0	—	-0.2	—
$\bar{B}B_1(5721)$	+2.0	1.1	-2.8/ + 0.1	7.8/9.8	~ 0 / ~ 0	~ 0 / ~ 0	+1.2	0.2
$\bar{B}_s B_{s1}(5830)$	—	—	-0.2/ - 0.2	-/-	~ 0/ ~ 0	—	-0.1	—
$\bar{B}^* B(1P_1)$	+0.1	2.4	+2.5/ - 0.2	0.02/3.3	+2.2/ + 0.02	0.06/2.5	+0.3	0.9
$\bar{B}_s^* B_s(1P_1)$	—	—	-0.4/ - 0.5	-/-	-0.4/ - 0.5	—	-0.3	—
$\bar{B}^* B_1(5721)$	+0.1	23.9	-6.6/ - 4.6	3.7/12.4	-11.5/ - 5.2	8.0/21.8	-6.8	17.0
<i>Total</i>	-17.9	52.9	-67.0/ - 40.8	23.0/84.2	-62.2/ - 38.3	16.0/48.7	-44.4	55.7
$M_{\text{th}}, \Gamma_{\text{th}}$	11098, 52.9		11064, 23.0/11090, 84.2		11064, 16.0/11088, 48.7		11081, 55.7	

TABLE IX. The mass shifts and strong decay width of each channel for the $6D$ -wave bottomonium states, in units of MeV. The rest of the caption is the same as that of Table IV.

Channel	6^3D_1		6^3D_3		6^1D_2		6^3D_2	
	ΔM [11308]	Γ [11302]	ΔM [11323]	Γ [11312]	ΔM [11318]	Γ [11311]	ΔM [11317]	Γ [11312]
$\bar{B}B$	+0.02	0.01	~ 0	~ 0	—	—	—	—
$\bar{B}_s B_s$	~ 0	0.01	~ 0	~ 0	—	—	—	—
$\bar{B}B^*$	+0.03	0.02	~ 0	~ 0	+0.05	0.01	+0.06	0.02
$\bar{B}_s B_s^*$	~ 0	0.01	~ 0	~ 0	~ 0	0.03	~ 0	0.03
$\bar{B}^* B^*$	+0.1	0.06	+0.1	0.1	+0.08	0.05	+0.07	0.03
$\bar{B}_s^* B_s^*$	~ 0	~ 0	~ 0	~ 0	~ 0	~ 0	~ 0	0.02
$\bar{B}B(2^1S_0)$	+0.5	0.08	~ 0	~ 0	—	—	—	—
$\bar{B}_s B_s(2^1S_0)$	+0.3	3.2	-0.4	0.7	—	—	—	—
$\bar{B}(2^1S_0)B^*$	+0.5	1.0	+0.7	0.3	+1.4	1.2	+1.4	1.5
$\bar{B}_s(2^1S_0)B_s^*$	-0.2	—	-0.2	—	-0.5	—	-0.6	—
$\bar{B}B(2^3S_1)$	+0.5	0.7	+0.5	0.06	+1.1	0.6	+1.2	0.8
$\bar{B}_s B_s(2^3S_1)$	-0.2	—	-0.2	—	-0.6	—	-0.7	—
$\bar{B}^* B(2^3S_1)$	+3.1	13.1	+2.2	9.3	+2.3	7.9	+2.8	7.9
$\bar{B}_s^* B_s(2^3S_1)$	-0.9	—	-0.9	—	-0.4	—	-0.3	—
$\bar{B}B(1^3P_0)$	—	—	—	—	+0.05	0.2	~ 0	~ 0
$\bar{B}_s B_s(1^3P_0)$	—	—	—	—	~ 0	0.03	~ 0	~ 0
$\bar{B}^* B(1^3P_0)$	~ 0	0.2	+0.02	~ 0	~ 0	~ 0	+0.02	~ 0
$\bar{B}_s^* B_s(1^3P_0)$	+0.02	~ 0	+0.01	~ 0	~ 0	~ 0	+0.01	~ 0
$\bar{B}B_2^*(5747)$	+0.3	0.3	+0.2	0.07	+0.2	0.1	+0.6	0.4
$\bar{B}_s B_{s2}^*(5840)$	-0.04	0.2	+0.1	0.3	-0.01	0.3	+0.06	0.6
$\bar{B}^* B_2^*(5747)$	+0.1	0.6	+1.8	4.7	+1.5	2.9	+2.0	2.9
$\bar{B}_s^* B_{s2}^*(5840)$	+0.1	0.3	+0.6	2.1	+0.1	2.1	-0.2	2.6
$\bar{B}B(1P_1)$	+0.01	0.07	+0.02	0.1	~ 0	~ 0	~ 0	0.1
$\bar{B}_s B_s(1P_1)$	+0.06	0.02	+0.03	~ 0	~ 0	~ 0	+0.04	~ 0
$\bar{B}B_1(5721)$	+0.6	0.5	+0.1	0.1	~ 0	~ 0	+0.2	0.03
$\bar{B}_s B_{s1}(5830)$	-0.1	0.7	+0.1	0.1	~ 0	~ 0	+0.03	0.2
$\bar{B}^* B(1P_1)$	+0.2	~ 0	+0.3	~ 0	+0.3	~ 0	+0.2	~ 0
$\bar{B}_s^* B_s(1P_1)$	+0.1	0.6	+0.2	0.6	+0.2	0.6	~ 0	0.4
$\bar{B}^* B_1(5721)$	+0.6	1.7	+0.7	0.5	+1.0	1.0	+0.2	1.5
$\bar{B}_s^* B_{s1}(5830)$	+0.3	0.3	-0.05	1.1	~ 0	1.2	+0.2	0.9
$\bar{B}B(1^3D_1)$	+0.01	0.2	+0.1	0.1	+0.6	1.1	~ 0	0.4
$\bar{B}B(1^3D_3)$	-2.7	1.1	-2.0	3.4	-1.9	14.1	+2.2	10.9
$\bar{B}B(1D_2)$	+1.5	4.7	+0.7	2.3	~ 0	~ 0	+0.3	0.4
$\bar{B}B(1D_2')$	-2.7	—	-1.1	—	~ 0	—	-1.8	—
$\bar{B}^* B(1^3D_1)$	+1.6	2.6	-0.7	3.5	-0.1	0.9	~ 0	1.8
$\bar{B}^* B(1^3D_3)$	-5.7	—	-11.7	—	-6.2	—	-6.1	—
$\bar{B}^* B(1D_2)$	-2.3	3.2	+0.6	3.0	-0.5	6.5	-0.8	6.4
$\bar{B}^* B(1D_2')$	—	—	-1.9	—	-4.0	—	-3.4	—
$\bar{B}(1^3P_0)B(1^3P_0)$	+0.05	0.01	+0.03	0.05	—	—	—	—
$\bar{B}(1^3P_0)B_2^*(5747)$	-1.0	—	—	—	~ 0	—	-1.3	—
$\bar{B}(1^3P_0)B(1P_1)$	+0.05	—	+0.3	0.1	+0.3	0.6	+0.4	0.9
$\bar{B}(1^3P_0)B_1(5721)$	-0.4	0.5	-0.2	0.07	-0.9	3.4	-0.3	1.1
$\bar{B}(1P_1)B(1P_1)$	—	0.3	-0.9	—	-1.2	—	-1.8	—
Total	-5.6	36.3	-11.1	32.7	-7.1	44.8	-5.3	41.8
$M_{\text{th}}, \Gamma_{\text{th}}$	11302, 36.3		11312, 32.7		11311, 44.8		11312, 41.8	

and $6D$ states may be difficult to be established in the $\bar{B}^{(*)}B^{(*)}$ and $\bar{B}_s^{(*)}B_s^{(*)}$ final states. For example, to look for the $\Upsilon_1(5D)$ state one should select its dominant decay channel $\bar{B}^*B_1(5721)$.

V. $S-D$ MIXING

The n^3S_1 and m^3D_1 ($n = m + 1$) states with $J^{PC} = 1^{--}$ usually have a similar mass. A sizable mixing between the n^3S_1 and m^3D_1 should be induced by the tensor force and/or

the intermediate hadronic loops. Considering the $S-D$ mixing, the physical states $\Upsilon'(nS)$ and $\Upsilon'_1(mD)$, which are dominated by the n^3S_1 and m^3D_1 components, respectively, are defined by

$$\begin{pmatrix} \Upsilon'(nS) \\ \Upsilon'_1(mD) \end{pmatrix} = \begin{pmatrix} \cos \phi_S & \sin \phi_S \\ -\sin \phi_D & \cos \phi_D \end{pmatrix} \begin{pmatrix} \Upsilon(nS) \\ \Upsilon_1(mD) \end{pmatrix}, \quad (29)$$

where $\phi_{S/D}$ is the mixing angle. In the literature, the $S-D$ mixing are often artificially introduced to explain the dielectron decay widths of the vector resonances. However, the underlying dynamic mechanisms for the $S-D$ mixing are often overlooked. Here, we will introduce the mixing mechanisms based on this unquenched quark model.

A. Mixing induced by the tensor force

First, we consider the $S-D$ mixing induced by the following tensor force:

$$V_T(r) = \frac{4}{3} \frac{\alpha_s}{m_b m_{\bar{b}}} \frac{1}{r^3} \left(\frac{3 \mathbf{S}_q \cdot \mathbf{r} \mathbf{S}_{\bar{q}} \cdot \mathbf{r}}{r^2} - \mathbf{S}_q \cdot \mathbf{S}_{\bar{q}} \right). \quad (30)$$

The mixing angle can be determined by

$$\begin{pmatrix} M_{nS} & \tilde{V}_T \\ \tilde{V}_T & M_{mD} \end{pmatrix} \begin{pmatrix} a_S \\ a_D \end{pmatrix} = M \begin{pmatrix} a_S \\ a_D \end{pmatrix}, \quad (31)$$

where the M_{nS} and M_{mD} are the bare masses of the $\Upsilon(nS)$ and $\Upsilon_1(mD)$ states, respectively. The values of these masses have been determined within the potential model. The nondiagonal element $\tilde{V}_T$ is determined by $\tilde{V}_T = \langle n^3S_1 | V_T(r) | m^3D_1 \rangle$. By solving Eq. (31), one can obtain the physical masses and the eigenvectors of the mixed states. Finally, the mixing angle is given by $\phi_S = \phi_D = \arctan(a_D/a_S)$. Our calculations obtain a tiny mixing angle for this mechanism, $\phi_{S/D} \simeq -0.2^\circ$, for the $S-D$ mixing induced by the tensor term, which is consistent with the predictions in Refs. [63,81].

B. Mixing induced by hadronic loops

Then, we further consider the $S-D$ mixing induced by the intermediate hadronic loops. The mixing angle can be approximately determined by

$$\begin{pmatrix} M_S + \Delta M_S(M) & \Delta M_{SD}(M) \\ \Delta M_{DS}(M) & M_D + \Delta M_D(M) \end{pmatrix} \begin{pmatrix} a_S \\ a_D \end{pmatrix} = M \begin{pmatrix} a_S \\ a_D \end{pmatrix}. \quad (32)$$

The mass shifts $\Delta M_S(M)$ and $\Delta M_D(M)$ of bare states $\Upsilon(nS)$ and $\Upsilon_1(mD)$ are estimated by Eq. (16), respectively, while the off-diagonal elements $\Delta M_{SD}(M)$ and $\Delta M_{DS}(M)$ are determined by

TABLE X. The $\Upsilon(nS) - \Upsilon_1(mD)$ mixing coefficients c_S and c_D , as well as the mixed masses (in MeV), within the hadron loop-induced $S-D$ mixing framework.

	a_S	a_D	Mass	$\phi_{nS/mD}$
$\Upsilon'(4S)$	-0.9999	0.0139	10579	-0.79°
$\Upsilon'_1(3D)$	-0.2770	-0.9609	10676	-16.01°
$\Upsilon'(5S)$	-0.9999	0.0050	10854	-0.29°
$\Upsilon'_1(4D)$	-0.0672	-0.9977	10907	-3.85°
$\Upsilon'(6S)$	-0.9995	0.0310	11015	-1.78°
$\Upsilon'_1(5D)$	-0.3108	-0.9505	11104	-18.11°
$\Upsilon'(7S)$	-0.9933	0.1152	11258	-6.62°
$\Upsilon'_1(6D)$	-0.2408	-0.9706	11304	-13.93°

$$\Delta M_{SD}(M) = \Delta M_{DS}^*(M)$$

$$= \sum_{BC} \mathcal{P} \int \frac{(M_0 - M) \overline{\mathcal{M}_{S \rightarrow BC}^*(\mathbf{p})} \mathcal{M}_{D \rightarrow BC}(\mathbf{p})}{(M - E_{BC})(M_0 - E_{BC})} p^2 dp. \quad (33)$$

Equation (32) can be transformed into an eigenvalue problem, i.e.,

$$\det \begin{vmatrix} M_S + \Delta M_S(M) - M & \Delta M_{SD}(M) \\ \Delta M_{DS}(M) & M_D + \Delta M_D(M) - M \end{vmatrix} = 0. \quad (34)$$

With the constraint of Eq. (17), one can determine the physical masses, M , for the $\Upsilon'(nS)$ and $\Upsilon'_1(mD)$, and their corresponding eigenvector by diagonalizing Eq. (34). The mixing angles ϕ_S and ϕ_D for the $\Upsilon'(nS)$ and $\Upsilon'_1(mD)$ are given by $\phi_S = \arctan(a_D/a_S)$ and $\phi_D = -\arctan(a_S/a_D)$.

The obtained $S-D$ mixing angles induced by the hadronic loops are listed in Table X. It is interestingly found that there are fairly large mixing angles $\phi_D \simeq -(14-18)^\circ$ for the $\Upsilon'_1(mD)$ ($m = 3, 5, 6$) states. In other words, in the $b\bar{b}$ core of $\Upsilon'_1(mD)$ ($m = 3, 5, 6$), besides the dominant nD -wave component, there is a sizable $\Upsilon(nS)$ ($n = m + 1$) component due to the coupled-channel effects. However, for the S -wave dominant state $\Upsilon'(nS)$ ($n = 4, 5, 6, 7$), the D -wave $b\bar{b}$ -core component is small, with the mixing angle of only a few degrees.

In a previous work by our group [43], the $\Upsilon(10753)$ is suggested to be a $4D$ -wave dominant state via the $\Upsilon_1(4D) - \Upsilon(5S)$ mixing if the mixing angle reaches up to a fairly large value, $\phi_D = 20^\circ - 30^\circ$. However, with the hadronic loops and tensor forces we cannot explain such a fairly large mixing angle. There is only a small mixing angle, $\phi_D \simeq 4^\circ$, induced by the hadronic loops.

Finally, it should be mentioned that the strong decay properties of the $\Upsilon_1(4D)$ and $\Upsilon_1(5D)$ may be significantly changed by the $S-D$ mixing. Considering this mixing effect, the physical state $\Upsilon'_1(3D)$ has a broad width

$$\Gamma[\Upsilon'_1(3D)] = 57 \text{ MeV}, \quad (35)$$

which is nearly saturated by the $\bar{B}^*B^*$ channel ($\sim 92\%$). The decay width is about a factor of 2 broader than the case without mixing, while for the mixed state $\Upsilon'_1(5D)$, the decay width is predicted to be

$$\Gamma[\Upsilon'_1(5D)] \simeq 64 \text{ MeV}, \quad (36)$$

and it mainly decays into the $\bar{B}^*B_2^*(5747)$ ($\sim 53\%$), $\bar{B}^*B_2(5721)$ ($\sim 24\%$), and $\bar{B}B(2^1S_0)$ ($\sim 8\%$) final states. The S - D mixing significantly increases the decay width of $\Upsilon_1(5D)$.

VI. DIELECTRON DECAY OF VECTOR STATES

Taking into account QCD radiative corrections with the Van Royen–Weisskopf formula [115], the dielectron decay widths of the vector $b\bar{b}$ states are given by [116–118]

$$\Gamma_{ee}[\Upsilon(nS)] = \frac{4\alpha_0^2 e_b^2}{M_{nS}^2} |R_{nS}(0)|^2 \left(1 - \frac{16\alpha_s'}{3\pi}\right), \quad (37)$$

$$\Gamma_{ee}[\Upsilon_1(nD)] = \frac{4\alpha_0^2 e_b^2}{M_{nD}^2} \left| \frac{5}{2\sqrt{2}m_b^2} R''_{nD}(0) \right|^2 \left(1 - \frac{16\alpha_s'}{3\pi}\right), \quad (38)$$

where M_{nS} and M_{nD} are the masses of the $\Upsilon(nS)$ and $\Upsilon_1(nD)$, respectively; $e_b = -1/3$ is the charge of b quark, $\alpha_0 \simeq 1/137$ is the fine-structure constant, and α_s' is the strong coupling constant. Here, we take $\alpha_s' = 0.18$ as adopted in Refs. [30,39,45]. $R_{nS}(0)$ is the wave function of $\Upsilon(nS)$ at the zero point, while $R''_{nD}(0)$ is the second derivative of the wave function of $\Upsilon_1(nD)$ at the zero point.

It should be emphasized that the dielectron decay widths of some vector resonances may be significantly affected by the following two aspects: (i) The continuum components of a vector state at the long range should significantly suppress the dielectron width. As we know, the dielectron decay width is contributed by the zero-point wave function, which is governed by the short-range $q\bar{q}$ -core component rather than the long-range continuum components. The anomalously small dielectron width of $\Upsilon(10580)$ may be due to the suppression of the significant continuum components [63]. (ii) The S - D mixing induced by the intermediate hadronic loops can affect the dielectron decay width as well. Thus, in the unquenched quark model framework, and including the S - D mixing effects, the dielectron decay widths for the physical states are given by

$$\Gamma_{ee}[\Upsilon'(nS)] = \frac{4\alpha_0^2 e_b^2}{M_{nS}^2} \left(1 - \frac{16\alpha_s'}{3\pi}\right) \left| c_{nS} R_{nS}(0) \cos \phi + \frac{5}{2\sqrt{2}m_b^2} c_{mD} R''_{mD}(0) \sin \phi \right|^2, \quad (39)$$

$$\Gamma_{ee}[\Upsilon'_1(mD)] = \frac{4\alpha_0^2 e_b^2}{M_{mD}^2} \left(1 - \frac{16\alpha_s'}{3\pi}\right) \left| -c_{nS} R_{nS}(0) \sin \phi + \frac{5}{2\sqrt{2}m_b^2} c_{mD} R''_{mD}(0) \cos \phi \right|^2, \quad (40)$$

where c_{nS} and c_{mD} are the probability amplitudes of the $b\bar{b}$ -core components for the $\Upsilon(nS)$ and $\Upsilon_1(mD)$ states, respectively.

According to Eqs. (39) and (40), we evaluate the dielectron decay widths for the vector $\Upsilon(nS/nD)$ states. Our results are listed in column 9 of Table XI, labeled as case A. It is found that our predictions for the $\Upsilon(nS)$ ($n = 1-6$) are in good agreement with the observations [7]. For a comparison, we also give the quenched quark model results predicted with Eqs. (37) and (38) in column 11 of Table XI, labeled as case C. One can see that neglecting the affects of the continuum components, the dielectron decay widths for the high-lying resonances $\Upsilon(10580)$, $\Upsilon(10860)$, and $\Upsilon(11020)$ seem to be notably overestimated in the quenched quark model framework. Moreover, to examine the S - D mixing effects, we further evaluate the dielectron decay widths without mixing, i.e., $\phi = 0$ for Eqs. (39) and (40); the results are listed in column 10 of Table XI, labeled as case B. It is seen that the S - D mixing has small effects on the dielectron decay widths of the S -wave states, while it

has notable effects on those of the high-lying D -wave states $\Upsilon_1(nD)$ ($n = 3-6$).

Finally, it should be mentioned that recently, the Belle II Collaboration reported their new measurements of the $e^+e^- \rightarrow \bar{B}^*B^*$ cross sections [119]. There seems to be a rapid increase in the cross section around the $\bar{B}^*B^*$ threshold, which may be caused by the missing $\Upsilon'_1(3D)$ state predicted in the quark model. The reasons are given as follows. (i) The predicted mass, 10670 MeV, for the $\Upsilon'_1(3D)$ state is consistent with that of the observed structure in the cross section. (ii) For the $\Upsilon'_1(3D)$, there may be enough production rates via e^+e^- annihilation at Belle/Belle II. According to our prediction, the $\Upsilon'_1(3D)$ may have a relatively large dielectron decay width, $\Gamma_{ee} \simeq 0.028 \text{ keV}$, due to the significant S - D mixing induced by the hadronic loops. (iii) The $\Upsilon'_1(3D)$ has a relatively narrow width of $\Gamma \simeq 57 \text{ MeV}$, and dominantly decays into the $\bar{B}^*B^*$ final state with a branching fraction of $\sim 92\%$. This is also consistent with the Belle II observations.

TABLE XI. Our predicted dielectron widths (in units of keV) for the vector $b\bar{b}$ states compared with the data and predictions from other literature. Case A stand for the unquenched quark model results predicted with Eqs. (39) and (40), where the corrections from both the S - D mixing and the continuum components due to coupled-channel effects are included. Case B stands for the results neglecting the corrections from the S - D mixing. Case C stands for the quenched quark model results predicted with Eqs. (37) and (38).

State	BBD [80]	GVGV [82]	LC [30]	GM [28]	SOEF [31]	WSLM [39]	Exp [7]	Case A	Case B	Case C
$\Upsilon(1S)$	1.317	0.980	1.600	1.440	0.710	1.650	1.340 ± 0.018	1.344	1.344	1.344
$\Upsilon(2S)$	0.614	0.410	0.640	0.730	0.370	0.821	0.612 ± 0.011	0.557	0.557	0.557
$\Upsilon(3S)$	0.448	0.270	0.440	0.530	0.270	0.569	0.443 ± 0.008	0.408	0.408	0.408
$\Upsilon(10580)$	0.370	0.200	0.350	0.390	0.210	0.431	0.272 ± 0.029	0.209	0.209	0.343
$\Upsilon(10860)$	0.316	0.160	0.290	0.330	0.180	0.348	0.220 ± 0.120 [9]	0.224	0.224	0.298
$\Upsilon(11020)$	0.274	0.120	0.250	0.270	0.150	0.286	0.130 ± 0.030	0.100	0.101	0.273
$\Upsilon(7S)$	...	0.100	0.220	...	...	0.243	...	0.130	0.137	0.250
$\Upsilon_1(1D)$	0.62×10^{-3}	3.70×10^{-3}	...	1.38×10^{-3}	1.40×10^{-3}	1.88×10^{-3}	...	1.08×10^{-3}	1.08×10^{-3}	1.08×10^{-3}
$\Upsilon_1(2D)$	1.08×10^{-3}	5.80×10^{-4}	...	1.99×10^{-3}	2.50×10^{-3}	2.81×10^{-3}	...	2.13×10^{-3}	2.13×10^{-3}	2.13×10^{-3}
$\Upsilon_1(3D)$	1.44×10^{-3}	6.70×10^{-4}	...	2.38×10^{-3}	...	3.00×10^{-3}	...	0.028	1.95×10^{-3}	3.16×10^{-3}
$\Upsilon_1(4D)$	1.71×10^{-3}	7.90×10^{-4}	...	2.18×10^{-3}	...	3.00×10^{-3}	...	7.19×10^{-3}	2.80×10^{-3}	4.11×10^{-3}
$\Upsilon_1(5D)$	1.90×10^{-3}	7.10×10^{-4}	...	...	...	3.02×10^{-3}	...	0.019	1.66×10^{-3}	5.03×10^{-3}
$\Upsilon_1(6D)$	...	...	...	...	...	...	...	0.023	4.14×10^{-3}	5.92×10^{-3}

VII. THE ROLES OF VECTOR RESONANCES IN $e^+e^- \rightarrow \bar{B}_{(s)}^{(*)}B_{(s)}^{(*)}$ CROSS SECTION

In experiments, the production of $b\bar{b}$ vector resonances mainly depends on the e^+e^- annihilation reaction $e^+e^- \rightarrow b\bar{b}$. The high-lying $b\bar{b}$ vector resonances are most likely to be reconstructed in the open-bottom meson pair channels. Thus, the study of the cross section for the process $e^+e^- \rightarrow b\bar{b} \rightarrow \bar{B}_{(s)}^{(*)}B_{(s)}^{(*)}$ (where $B^{(*)}$ and $B_s^{(*)}$ denote both the ground-state $B^{(*)}$ and $B_s^{(*)}$ mesons, as well as their excitations) is essential for understanding the properties of $b\bar{b}$ vector resonances. If we neglect the contribution from the no-resonance backgrounds, the total cross section for the $e^+e^- \rightarrow \bar{B}_{(s)}^{(*)}B_{(s)}^{(*)}$ above the $B\bar{B}$

threshold could be roughly estimated by [120,121]

$$\sigma(e^+e^- \rightarrow \bar{B}_{(s)}^{(*)}B_{(s)}^{(*)}) \simeq \sum_R \frac{3\pi}{M^2} \frac{\Gamma_{e^+e^-}\Gamma_R}{(M - M_R)^2 + \Gamma_R^2/4}, \quad (41)$$

where M_R and Γ_R stand for the mass and width of the intermediate vector $b\bar{b}$ resonances and M is the center-of-mass energy.

By using Eq. (41), we calculate the cross section of $e^+e^- \rightarrow \bar{B}_{(s)}^{(*)}B_{(s)}^{(*)}$ in the center-of-mass energy range of (10.55, 11.35) GeV. In this energy range, 8 vector states, $\Upsilon(nS)$ ($n = 4, 5, 6, 7$) and $\Upsilon_1(mD)$ ($m = 3, 4, 5, 6$), are included according to our quark model prediction. Our results are plotted in Fig. 4. It should be mentioned that

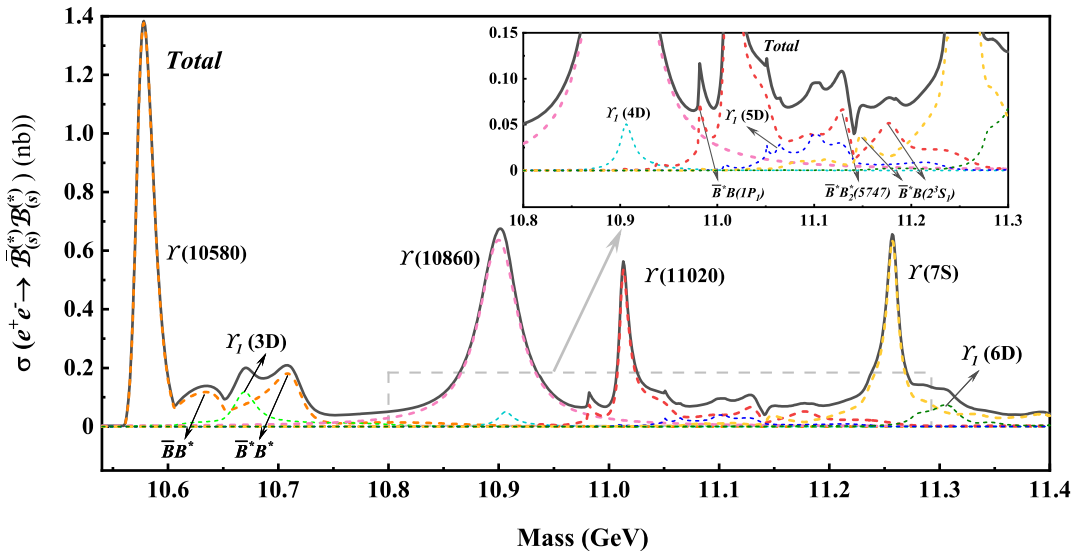


FIG. 4. Estimation of the cross section of the $e^+e^- \rightarrow \bar{B}_{(s)}^{(*)}B_{(s)}^{(*)}$ process (dashed curve). It should be emphasized that label $B_{(s)}^{(*)}$ includes both the ground-state $B_{(s)}^{(*)}$ mesons and their excited states. For clarity, the contributions from the vector resonances and some important threshold effects of coupled channels are labeled in the figure as well.

similar analysis was carried out recently by Ortega *et al.* [113]. In Fig. 4, one can see four prominent peak structures, contributed by four S -wave states $\Upsilon(4S)$, $\Upsilon(5S)$, $\Upsilon(6S)$, and $\Upsilon(7S)$, respectively. The $\Upsilon(4S)$, $\Upsilon(5S)$, and $\Upsilon(6S)$ have been established in experiments, which should correspond to the $\Upsilon(10580)$, $\Upsilon(10860)$, and $\Upsilon(11020)$, respectively. However, the $\Upsilon(7S)$ with a mass of ~ 11260 MeV is still missing: this state may be hard to observe in the $\bar{B}_{(s)}^{(*)}B_{(s)}^{(*)}$ channels due to their tiny decay rates; however, it may be observed in the $\bar{B}^{(*)}B_1(5721)$ channels.

For the $3D$ state, by including S - D mixing effect, the physical state $\Upsilon_1'(3D)(10670)$ has a relatively clear role in the cross-section distribution, which suggests to be observed by the Belle II Collaboration, recently [119]. However, it is notable that the high statistic data will be required to extract the clean signal of the $\Upsilon_1'(3D)(10670)$, since it would suffer from the interference due to the threshold effects of the $\bar{B}B^*$, $\bar{B}^*B^*$ channels. On the contrary, the high-lying D -wave states, $\Upsilon_1(mD)$ ($m = 4, 5, 6$), do not show a clear peak in the total cross-section distribution.

At last, let us make further discussion about the threshold effects of the coupled channels. As we know, the threshold effects can make a bump or cusp around the threshold. Here, as shown in Fig. 4, there are two bumps due to the threshold effects of $\bar{B}B^*$ and $\bar{B}^*B^*$ channels around 10.63 and 10.71 GeV, respectively. Actually, the bump near 10.71 GeV in our prediction provides a possible explanation for the bump, known as $\Upsilon(10753)$ resonance, observed at Belle/Belle II [13,122]. As discussed before, $\Upsilon(10753)$ resonance cannot be assigned as $\Upsilon_1(3D)$ or $\Upsilon_1(5S)$. Even a mixed state via $\Upsilon(4S)$ - $\Upsilon_1(3D)$ or $\Upsilon(5S)$ - $\Upsilon_1(4D)$ is ruled out in our current model due to the small mixing angle. Now, it is clear in our model that such bump is just the threshold effect of $\bar{B}^*B^*$ coupling with $\Upsilon(4S)$, rather than a genuine state. Similar phenomena are also reported by Ortega *et al.* in their recent work [113]. Of course, more evidence is still needed to confirm it. The interpolations of $\Upsilon(10753)$ as a hybrid [47,48] or a tetraquark state [49–52] are also possible.

VIII. SUMMARY

In this work, the bottomonium spectrum and the OZI-allowed two-body strong decay widths are systematically studied within an unquenched quark model. Furthermore, we study the S - D mixing and dielectron decay properties in the same framework to extract more detail of these vector states. Moreover, by combining the spectral-density functions and dielectron decay widths, we discuss the roles of the vector $b\bar{b}$ states in the cross section of the e^+e^- annihilation reaction. Some key results from this study are summarized as follows:

- (i) The mass shifts of the bare states due to the unquenched coupled-channel effects are estimated to be $\mathcal{O}(10)$ MeV. Including the coupled-channel effects, the theoretical description of the masses for the high-lying resonances, such as $\Upsilon(10580)$, $\Upsilon(11020)$, $\chi_{b1,2}(3P)$, is systematically improved.
- (ii) Most of the high-lying resonances may contain significant non- $b\bar{b}$ components. For example, for the $\Upsilon(11020)$ resonance the non- $b\bar{b}$ component may reach up to $\sim 60\%$.
- (iii) The intermediate hadronic loops can cause a significant S - D mixing for the high-lying vector states. The D -wave states, $\Upsilon_1(3D)$, $\Upsilon_1(5D)$, and $\Upsilon_1(6D)$, will significantly mix with their nearby S -wave states. The mixing angles are estimated to be $\phi \simeq -(14-18)^\circ$.
- (iv) The dielectron widths of some high-lying vector resonances may be significantly suppressed by their non- $b\bar{b}$ components due to the coupled-channel effects. Including these effects, the better description of the dielectron widths of the $\Upsilon(10580)$, $\Upsilon(10860)$, and $\Upsilon(11020)$ is achieved in this work.
- (v) The threshold effects of open-bottom meson pairs can cause rich bump structures in the total cross-section distribution of $e^+e^- \rightarrow b\bar{b}$. There are complex structures around the c.m. energy region of 10.62–10.75 GeV, which are mainly contributed by the $\Upsilon_1(3D)$ resonance together with the threshold effects of the $\bar{B}B^*$ and $\bar{B}^*B^*$. In the current model, we do not favor the existing $\Upsilon(10753)$ resonance, while the bump in the $e^+e^- \rightarrow b\bar{b}$ around 10.71 GeV is just due to the threshold effect of $\bar{B}^*B^*$.

ACKNOWLEDGMENTS

We thank Qi Li and Yu Lu for useful discussions. This work is supported by the National Natural Science Foundation of China under Grants No. 12175065 and No. 12235018 (X.-H. Z.) and No. 12175239 and No. 12221005 (J.-J. W.), and by the National Key R&D Program of China under Contract No. 2020YFA0406400 (J.-J. W.), and supported by Chinese Academy of Sciences under Grant No. YSBR-101 (J.-J. W.).

DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: MASS SHIFT AND SPECTRAL-DENSITY FUNCTION

In this appendix, we present the total mass shifts as a function of the physical mass, along with the spectral-density function for the $b\bar{b}$ states, as shown in Figs. 5–8.

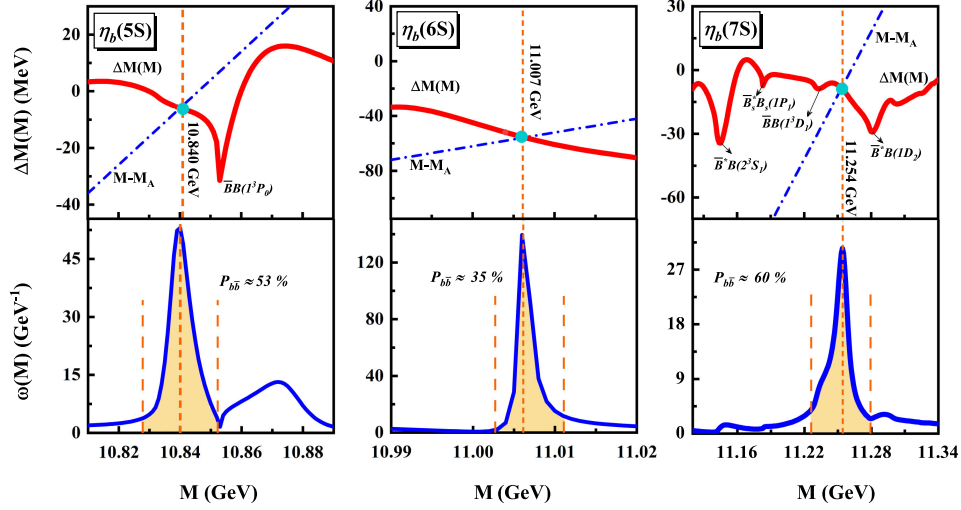


FIG. 5. The line shapes of the mass-shift function $\Delta M(M)$ (upper panel) and spectral-density function $\omega(M)$ (lower panel) for the $\eta_b(5S, 6S, 7S)$ states. The solution of the coupled-channel equation (17) corresponds to the intersection point between two lines $\Delta M(M)$ and $M - M_A$ in the upper panel. Some important coupled channels are also marked in the upper panel. The shaded area in the spectral-density function is the integral interval for estimating the $b\bar{b}$ component.

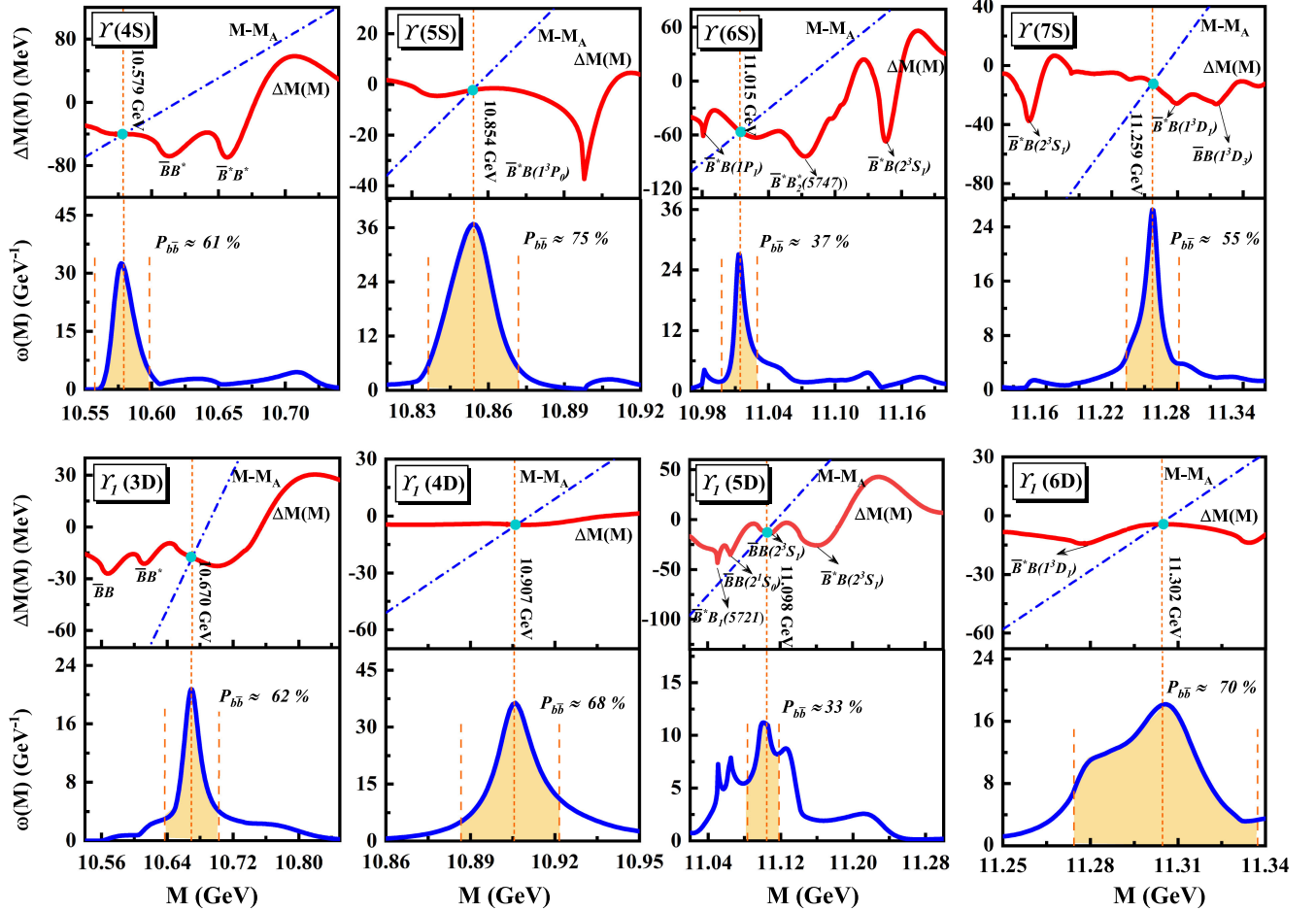


FIG. 6. The line shapes of the mass-shift function $\Delta M(M)$ (upper panel) and spectral-density function $\omega(M)$ (lower panel) for the vector $b\bar{b}$ states. The rest of the caption is the same as that of Fig. 5.

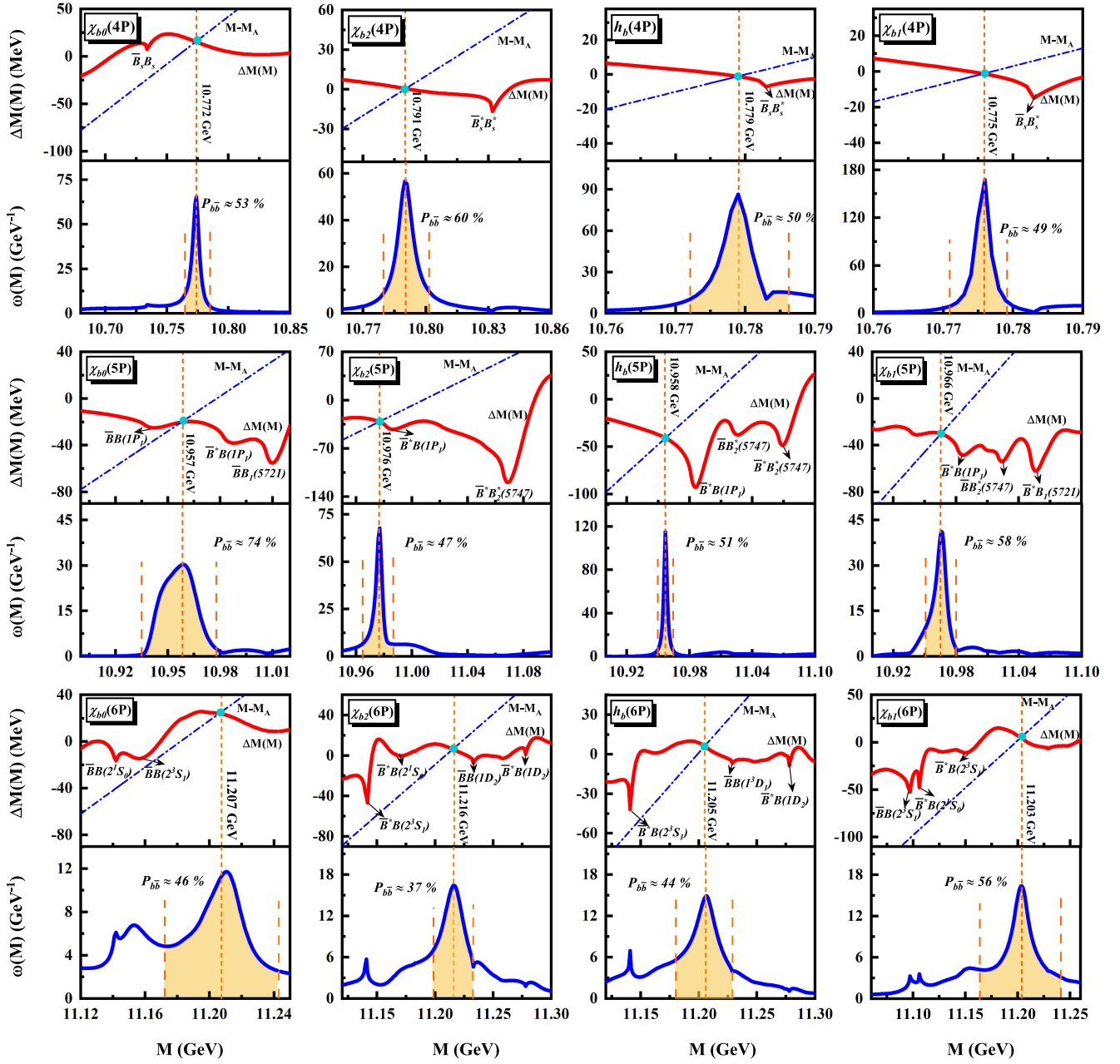


FIG. 7. The line shapes of the mass-shift function $\Delta M(M)$ (upper panel) and spectral-density function $\omega(M)$ (lower panel) for the 4P-, 5P-, and 6P-wave states. The rest of the caption is the same as that of Fig. 5.

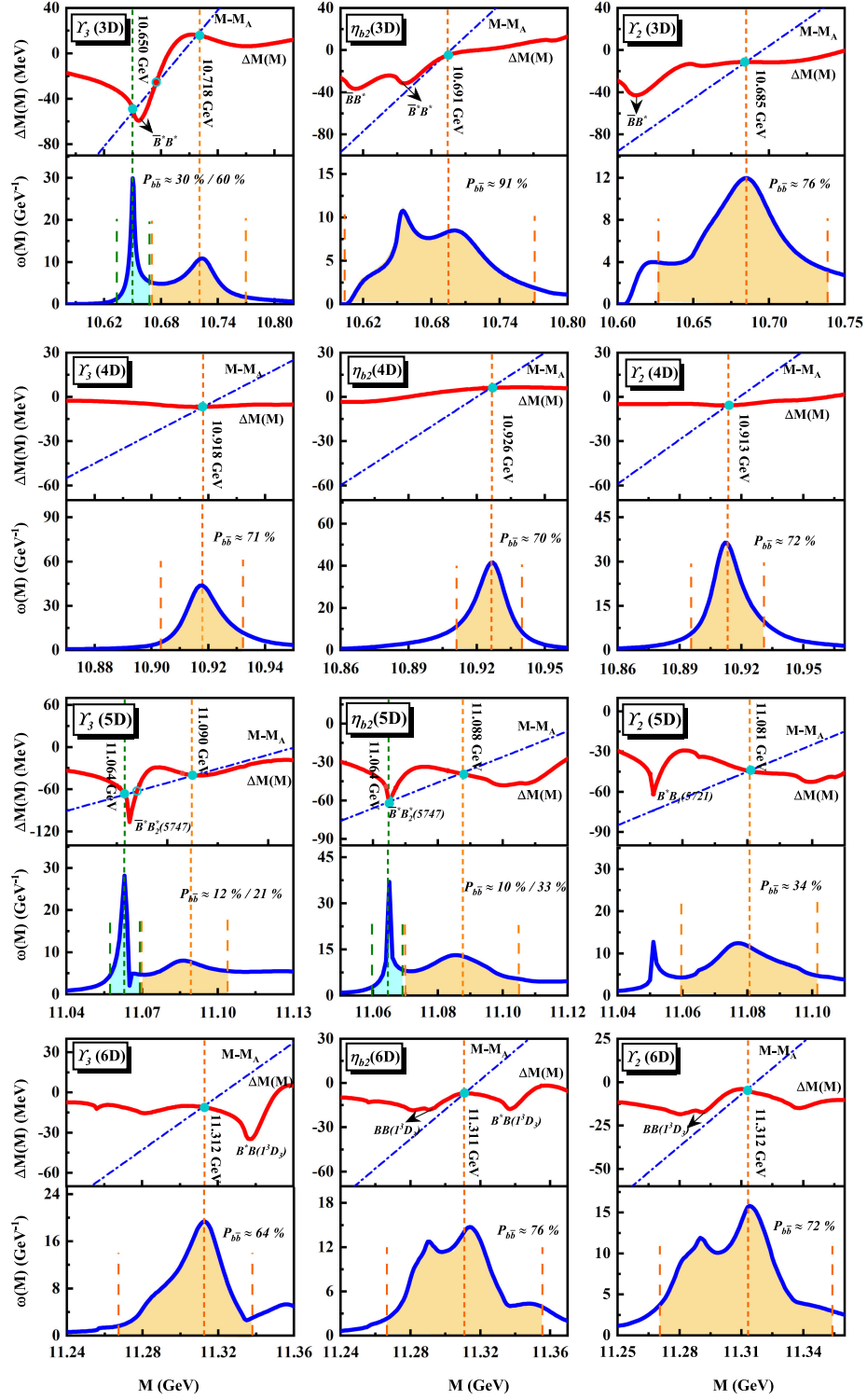


FIG. 8. The line shapes of the mass-shift function $\Delta M(M)$ (upper panel) and spectral-density function $\omega(M)$ (lower panel) for the $\Upsilon_{2,3}(nD)$ and $\eta_{b2}(nD)$ ($n = 3, 4, 5, 6$) states. The rest of the caption is the same as that of Fig. 5.

- [1] S. W. Herb *et al.* (E288 Collaboration), Observation of a dimuon resonance at 9.5-GeV in 400-GeV proton-nucleus collisions, *Phys. Rev. Lett.* **39**, 252 (1977).
- [2] W. R. Innes *et al.* (E288 Collaboration), Observation of structure in the Υ region, *Phys. Rev. Lett.* **39**, 1240 (1977); **39**, 1640(E) (1977).
- [3] C. Klopfenstein, J. E. Horstkotte, J. Lee-Franzini, R. D. Schamberger, M. Sivertz, L. J. Spencer, P. M. Tuts, P. Franzini, K. Han, E. Rice *et al.*, Observation of the lowest P wave $b\bar{b}$ bound states, *Phys. Rev. Lett.* **51**, 160 (1983).
- [4] F. Pauss, H. Dietl, G. Eigen, E. Lorenz, G. Mageras, H. Vogel, P. Franzini, K. Han, D. Peterson, E. Rice *et al.*, Observation of $\chi - b$ production in the exclusive reaction $\Upsilon' \rightarrow \gamma\chi_b \rightarrow \gamma\gamma\Upsilon \rightarrow \gamma\gamma(e^+e^- \text{ or } \mu^+\mu^-)$, *Phys. Lett.* **130B**, 439 (1983).
- [5] K. Han, T. Bohringer, P. Franzini, G. Mageras, D. Peterson, E. Rice, J. K. Yoh, J. E. Horstkotte, C. Klopfenstein, J. Lee-Franzini *et al.*, Observation of p wave $b\bar{b}$ bound states, *Phys. Rev. Lett.* **49**, 1612 (1982).
- [6] G. Eigen, G. Blunar, H. Dietl, E. Lorenz, F. Pauss, H. Vogel, T. Bohringer, P. Franzini, K. Han, G. Mageras *et al.*, Evidence for χ'_b production in the exclusive reaction $\Upsilon'' \rightarrow \gamma\chi''_b \rightarrow (\gamma\gamma\Upsilon' \text{ or } \gamma\gamma\Upsilon)$, *Phys. Rev. Lett.* **49**, 1616 (1982).
- [7] S. Navas *et al.* (Particle Data Group), Review of particle physics, *Phys. Rev. D* **110**, 030001 (2024).
- [8] D. M. J. Lovelock, J. E. Horstkotte, C. Klopfenstein, J. Lee-Franzini, L. Romero, R. D. Schamberger, S. Youssef, P. Franzini, D. Son, P. M. Tuts *et al.*, Masses, widths, and leptonic widths of the higher upilon resonances, *Phys. Rev. Lett.* **54**, 377 (1985).
- [9] D. Besson *et al.* (CLEO Collaboration), Observation of new structure in the e^+e^- annihilation cross-section above the $\Upsilon(4S)$, *Phys. Rev. Lett.* **54**, 381 (1985).
- [10] C. Bebek *et al.* (CLEO Collaboration), Exclusive decays and masses of the B mesons, *Phys. Rev. D* **36**, 1289 (1987).
- [11] B. Aubert *et al.* (BABAR Collaboration), A measurement of the total width, the electronic width, and the mass of the $\Upsilon(10580)$ resonance, *Phys. Rev. D* **72**, 032005 (2005).
- [12] B. Aubert *et al.* (BABAR Collaboration), Measurement of the $e^+e^- \rightarrow b\bar{b}$ cross section between $\sqrt{s} = 10.54\text{-GeV}$ and 11.20-GeV , *Phys. Rev. Lett.* **102**, 012001 (2009).
- [13] R. Mizuk *et al.* (Belle Collaboration), Observation of a new structure near 10.75 GeV in the energy dependence of the $e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$ ($n = 1, 2, 3$) cross sections, *J. High Energy Phys.* **10** (2019) 220.
- [14] I. Adachi *et al.* (Belle-II Collaboration), Observation of $e^+e^- \rightarrow \omega\chi_{bJ}(1P)$ and Search for $X_b \rightarrow \omega\Upsilon(1S)$ at $\sqrt{s}$ near 10.75 GeV, *Phys. Rev. Lett.* **130**, 091902 (2023).
- [15] S. Meinel, The Bottomonium spectrum from lattice QCD with $2 + 1$ flavors of domain wall fermions, *Phys. Rev. D* **79**, 094501 (2009).
- [16] T. Burch, C. DeTar, M. Di Pierro, A. X. El-Khadra, E. D. Freeland, S. Gottlieb, A. S. Kronfeld, L. Levkova, P. B. Mackenzie, and J. N. Simone, Quarkonium mass splittings in three-flavor lattice QCD, *Phys. Rev. D* **81**, 034508 (2010).
- [17] R. J. Dowdall *et al.* (HPQCD Collaboration), The upilon spectrum and the determination of the lattice spacing from lattice QCD including charm quarks in the sea, *Phys. Rev. D* **85**, 054509 (2012).
- [18] Z. G. Wang, Analysis of the heavy quarkonium states h_c and h_b with QCD sum rules, *Eur. Phys. J. C* **73**, 2533 (2013).
- [19] K. Azizi and J. Y. Süngü, On the mass and decay constant of the P -wave ground and radially excited h_c and h_b axial-vector mesons, *J. Phys. G* **46**, 035001 (2019).
- [20] C. S. Fischer, S. Kubrak, and R. Williams, Spectra of heavy mesons in the Bethe-Salpeter approach, *Eur. Phys. J. A* **51**, 10 (2015).
- [21] H. M. Choi, Decay constants and radiative decays of heavy mesons in light-front quark model, *Phys. Rev. D* **75**, 073016 (2007).
- [22] H. W. Ke, X. Q. Li, Z. T. Wei, and X. Liu, Re-Study on the wave functions of $\Upsilon(nS)$ states in LFQM and the radiative decays of $\Upsilon(nS) \rightarrow \eta_b + \gamma$, *Phys. Rev. D* **82**, 034023 (2010).
- [23] H. W. Ke, X. Q. Li, and X. Liu, Study on the radiative decays of $\Upsilon(nS) \rightarrow \eta_b + \gamma$, arXiv:1002.1187.
- [24] B. Chen, A. Zhang, and J. He, Bottomonium spectrum in the relativistic flux tube model, *Phys. Rev. D* **101**, 014020 (2020).
- [25] E. Eichten, The Υ family of resonances above threshold, *Phys. Rev. D* **22**, 1819 (1980).
- [26] E. J. Eichten and C. Quigg, Mesons with beauty and charm: Spectroscopy, *Phys. Rev. D* **49**, 5845 (1994).
- [27] S. Godfrey and N. Isgur, Mesons in a relativized quark model with chromodynamics, *Phys. Rev. D* **32**, 189 (1985).
- [28] S. Godfrey and K. Moats, Bottomonium mesons and strategies for their observation, *Phys. Rev. D* **92**, 054034 (2015).
- [29] D. Ebert, R. N. Faustov, and V. O. Galkin, Spectroscopy and Regge trajectories of heavy quarkonia and B_c mesons, *Eur. Phys. J. C* **71**, 1825 (2011).
- [30] B. Q. Li and K. T. Chao, Bottomonium spectrum with screened potential, *Commun. Theor. Phys.* **52**, 653 (2009).
- [31] J. Segovia, P. G. Ortega, D. R. Entem, and F. Fernández, Bottomonium spectrum revisited, *Phys. Rev. D* **93**, 074027 (2016).
- [32] T. Wei-Zhao, C. Lu, Y. You-Chang, and C. Hong, Bottomonium states versus recent experimental observations in the QCD-inspired potential model, *Chin. Phys. C* **37**, 083101 (2013).
- [33] N. Akbar, M. A. Sultan, B. Masud, and F. Akram, Higher hybrid bottomonia in an extended potential model, *Phys. Rev. D* **95**, 074018 (2017).
- [34] C. M. A. and R. Dhir, Spectroscopy and annihilation decay widths of charmonium and bottomonium excitations, *J. Phys. G* **51**, 115004 (2024).
- [35] I. Asghar and N. Akbar, Spectrum and decay properties of bottomonium mesons, *Eur. Phys. J. A* **60**, 58 (2024).
- [36] Z. Zhao, K. Xu, A. Limphirat, W. Sreethawong, N. Taksinsit, A. Kaewsnod, X. Liu, K. Khosonthongkee, S. Cheedket, and Y. Yan, Mass spectrum of 1^{--} heavy quarkonium, *Phys. Rev. D* **109**, 016012 (2024).
- [37] V. Kher, R. Chaturvedi, N. Devlani, and A. K. Rai, Bottomonium spectroscopy using Coulomb plus linear (Cornell) potential, *Eur. Phys. J. Plus* **137**, 357 (2022).

- [38] D. Molina, M. De Sanctis, C. Fernández-Ramírez, and E. Santopinto, Bottomonium spectrum with a Dirac potential model in the momentum space, *Eur. Phys. J. C* **80**, 526 (2020).
- [39] J. Z. Wang, Z. F. Sun, X. Liu, and T. Matsuki, Higher bottomonium zoo, *Eur. Phys. J. C* **78**, 915 (2018).
- [40] N. R. Soni, B. R. Joshi, R. P. Shah, H. R. Chauhan, and J. N. Pandya, $Q\bar{Q}$ ($Q \in \{b, c\}$) spectroscopy using the Cornell potential, *Eur. Phys. J. C* **78**, 592 (2018).
- [41] J. Ferretti and E. Santopinto, Higher mass bottomonia, *Phys. Rev. D* **90**, 094022 (2014).
- [42] W. J. Deng, H. Liu, L. C. Gui, and X. H. Zhong, Spectrum and electromagnetic transitions of bottomonium, *Phys. Rev. D* **95**, 074002 (2017).
- [43] Q. Li, M. S. Liu, Q. F. Lü, L. C. Gui, and X. H. Zhong, Canonical interpretation of $Y(10750)$ and $Y'(10860)$ in the Y family, *Eur. Phys. J. C* **80**, 59 (2020).
- [44] Z. Y. Bai, Y. S. Li, Q. Huang, X. Liu, and T. Matsuki, $Y(10753) \rightarrow Y(nS)\pi^+\pi^-$ decays induced by hadronic loop mechanism, *Phys. Rev. D* **105**, 074007 (2022).
- [45] Y. S. Li, Z. Y. Bai, Q. Huang, and X. Liu, Hidden-bottom hadronic decays of $Y(10753)$ with a $\eta^{(\prime)}$ or ω emission, *Phys. Rev. D* **104**, 034036 (2021).
- [46] Y. S. Li, Z. Y. Bai, and X. Liu, Investigating the $Y(10753) \rightarrow Y(1^3D_J)\eta$ transitions, *Phys. Rev. D* **105**, 114041 (2022).
- [47] N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C. P. Shen, C. E. Thomas, A. Vairo, and C. Z. Yuan, The XYZ states: Experimental and theoretical status and perspectives, *Phys. Rep.* **873**, 1 (2020).
- [48] J. Tarrús Castellà and E. Passemar, Exotic to standard bottomonium transitions, *Phys. Rev. D* **104**, 034019 (2021).
- [49] A. Ali, L. Maiani, A. Y. Parkhomenko, and W. Wang, Interpretation of $Y_b(10753)$ as a tetraquark and its production mechanism, *Phys. Lett. B* **802**, 135217 (2020).
- [50] P. Bicudo, N. Cardoso, L. Mueller, and M. Wagner, Computation of the quarkonium and meson-meson composition of the $Y(nS)$ states and of the new $Y(10753)$ Belle resonance from lattice QCD static potentials, *Phys. Rev. D* **103**, 074507 (2021).
- [51] P. Bicudo, N. Cardoso, L. Mueller, and M. Wagner, Study of $I = 0$ bottomonium bound states and resonances in S , P , D , and F waves with lattice QCD static-static-light-light potentials, *Phys. Rev. D* **107**, 094515 (2023).
- [52] Z. G. Wang, Vector hidden-bottom tetraquark candidate: $Y(10750)$, *Chin. Phys. C* **43**, 123102 (2019).
- [53] P. Gonzalez, Generalized screened potential model, *J. Phys. G* **41**, 095001 (2014).
- [54] J. Vijande, F. Fernandez, and A. Valcarce, Constituent quark model study of the meson spectra, *J. Phys. G* **31**, 481 (2005).
- [55] Q. Deng, R. H. Ni, Q. Li, and X. H. Zhong, Charmonia in an unquenched quark model, *Phys. Rev. D* **110**, 056034 (2024).
- [56] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T. M. Yan, Charmonium: The model, *Phys. Rev. D* **17**, 3090 (1978); **21**, 313(E) (1980).
- [57] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T. M. Yan, Charmonium: Comparison with experiment, *Phys. Rev. D* **21**, 203 (1980).
- [58] K. Heikkilä, S. Ono, and N. A. Tornqvist, Heavy $c\bar{c}$ and $b\bar{b}$ quarkonium states and unitarity effects, *Phys. Rev. D* **29**, 110 (1984); **29**, 2136(E) (1984).
- [59] S. Ono and N. A. Tornqvist, Continuum mixing and coupled channel effects in $c\bar{c}$ and $b\bar{b}$ quarkonium, *Z. Phys. C* **23**, 59 (1984).
- [60] S. Ono, A. I. Sanda, N. A. Tornqvist, and J. Lee-Franzini, Where are the $B\bar{B}$ mixing effects observable in the Y region?, *Phys. Rev. Lett.* **55**, 2938 (1985).
- [61] J. F. Liu and G. J. Ding, Bottomonium spectrum with coupled-channel effects, *Eur. Phys. J. C* **72**, 1981 (2012).
- [62] J. Ferretti and E. Santopinto, Threshold corrections of $\chi_c(2P)$ and $\chi_b(3P)$ states and $J/\psi\rho$ and $J/\psi\omega$ transitions of the $\chi(3872)$ in a coupled-channel model, *Phys. Lett. B* **789**, 550 (2019).
- [63] Y. Lu, M. N. Anwar, and B. S. Zou, Coupled-channel effects for the bottomonium with realistic wave functions, *Phys. Rev. D* **94**, 034021 (2016).
- [64] Y. Lu, M. N. Anwar, and B. S. Zou, How large is the contribution of excited mesons in coupled-channel effects?, *Phys. Rev. D* **95**, 034018 (2017).
- [65] M. N. Anwar, Y. Lu, and B. S. Zou, $\chi_b(3P)$ multiplet revisited: Hyperfine mass splitting and radiative transitions, *Phys. Rev. D* **99**, 094005 (2019).
- [66] R. Bruschini and P. González, Diabatic description of bottomoniumlike mesons, *Phys. Rev. D* **103**, 114016 (2021).
- [67] R. Bruschini and P. González, Coupled-channel meson-meson scattering in the diabatic framework, *Phys. Rev. D* **104**, 074025 (2021).
- [68] N. Hüsken, R. E. Mitchell, and E. S. Swanson, K -matrix analysis of e^+e^- annihilation in the bottomonium region, *Phys. Rev. D* **106**, 094013 (2022).
- [69] R. H. Ni, J. J. Wu, and X. H. Zhong, Unified unquenched quark model for heavy-light mesons with chiral dynamics, *Phys. Rev. D* **109**, 116006 (2024).
- [70] M. R. Pennington and D. J. Wilson, Decay channels and charmonium mass-shifts, *Phys. Rev. D* **76**, 077502 (2007).
- [71] D. Morel and S. Capstick, Baryon meson loop effects on the spectrum of nonstrange baryons, *arXiv:nucl-th/0204014*.
- [72] B. Silvestre-Brac and C. Gignoux, Unitary effects in spin orbit splitting of P wave baryons, *Phys. Rev. D* **43**, 3699 (1991).
- [73] H. H. Zhong, R. H. Ni, M. Y. Chen, X. H. Zhong, and J. J. Xie, Further study of $\Omega^*[1P_{3/2-}]$ within a chiral quark model, *Chin. Phys. C* **47**, 063104 (2023).
- [74] P. G. Ortega, J. Segovia, D. R. Entem, and F. Fernandez, Molecular components in P -wave charmed-strange mesons, *Phys. Rev. D* **94**, 074037 (2016).
- [75] P. G. Ortega, J. Segovia, D. R. Entem, and F. Fernández, Threshold effects in P -wave bottom-strange mesons, *Phys. Rev. D* **95**, 034010 (2017).
- [76] Z. Yang, G. J. Wang, J. J. Wu, M. Oka, and S. L. Zhu, The investigations of the P -wave B_s states combining quark model and lattice QCD in the coupled channel framework, *J. High Energy Phys.* **01** (2023) 058.
- [77] Z. Yang, G. J. Wang, J. J. Wu, M. Oka, and S. L. Zhu, Novel coupled channel framework connecting the quark model and lattice QCD for the near-threshold D_s states, *Phys. Rev. Lett.* **128**, 112001 (2022).

- [78] R. H. Ni, Q. Li, and X. H. Zhong, Mass spectra and strong decays of charmed and charmed-strange mesons, *Phys. Rev. D* **105**, 056006 (2022).
- [79] W. Hao, Y. Lu, and B. S. Zou, Coupled channel effects for the charmed-strange mesons, *Phys. Rev. D* **106**, 074014 (2022).
- [80] A. M. Badalian, B. L. G. Bakker, and I. V. Danilkin, On the possibility to observe higher n^3D_1 bottomonium states in the e^+e^- processes, *Phys. Rev. D* **79**, 037505 (2009).
- [81] A. M. Badalian, B. L. G. Bakker, and I. V. Danilkin, Dielectron widths of the S-, D-vector bottomonium states, *Phys. At. Nucl.* **73**, 138 (2010).
- [82] P. Gonzalez, A. Valcarce, H. Garcilazo, and J. Vijande, Heavy meson description with a screened potential, *Phys. Rev. D* **68**, 034007 (2003).
- [83] T. Barnes, S. Godfrey, and E. S. Swanson, Higher charmonia, *Phys. Rev. D* **72**, 054026 (2005).
- [84] E. Hiyama, Y. Kino, and M. Kamimura, Gaussian expansion method for few-body systems, *Prog. Part. Nucl. Phys.* **51**, 223 (2003).
- [85] L. Micu, Decay rates of meson resonances in a quark model, *Nucl. Phys.* **B10**, 521 (1969).
- [86] A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, Naive quark pair creation model of strong interaction vertices, *Phys. Rev. D* **8**, 2223 (1973).
- [87] A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, Naive quark pair creation model and baryon decays, *Phys. Rev. D* **9**, 1415 (1974).
- [88] L. C. Gui, L. S. Lu, Q. F. Lü, X. H. Zhong, and Q. Zhao, Strong decays of higher charmonium states into open-charm meson pairs, *Phys. Rev. D* **98**, 016010 (2018).
- [89] Q. Li, L. C. Gui, M. S. Liu, Q. F. Lü, and X. H. Zhong, Mass spectrum and strong decays of strangeonium in a constituent quark model, *Chin. Phys. C* **45**, 023116 (2021).
- [90] Z. Y. Zhou and Z. Xiao, Hadron loops effect on mass shifts of the charmed and charmed-strange spectra, *Phys. Rev. D* **84**, 034023 (2011).
- [91] M. X. Duan and X. Liu, Where are $3P$ and higher P -wave states in the charmonium family?, *Phys. Rev. D* **104**, 074010 (2021).
- [92] P. G. Ortega, J. Segovia, D. R. Entem, and F. Fernandez, Spectroscopy of B_c mesons and the possibility of finding exotic B_c -like structures, *Eur. Phys. J. C* **80**, 223 (2020).
- [93] K. Brauer, A. Faessler, F. Fernandez, and K. Shimizu, Nucleon-nucleon interaction in a quark model with pions, *Nucl. Phys.* **A507**, 599 (1990).
- [94] Z. Y. Zhang, A. Faessler, U. Straub, and L. Y. Glozman, The baryon baryon interaction in a modified quark model, *Nucl. Phys.* **A578**, 573 (1994).
- [95] Y. W. Yu, Z. Y. Zhang, P. N. Shen, and L. R. Dai, Quark quark potential from chiral symmetry, *Phys. Rev. C* **52**, 3393 (1995).
- [96] A. Valcarce, H. Garcilazo, F. Fernandez, and P. Gonzalez, Quark-model study of few-baryon systems, *Rep. Prog. Phys.* **68**, 965 (2005).
- [97] A. Valcarce, H. Garcilazo, and J. Vijande, Constituent quark model study of light- and strange-baryon spectra, *Phys. Rev. C* **72**, 025206 (2005).
- [98] S. Weinberg, Elementary particle theory of composite particles, *Phys. Rev.* **130**, 776 (1963).
- [99] Y. F. Wang, U. G. Meißner, D. Rönchen, and C. W. Shen, Examination of the nature of the N^* and Δ resonances via coupled-channels dynamics, *Phys. Rev. C* **109**, 015202 (2024).
- [100] L. N. Bogdanova, G. M. Hale, and V. E. Markushin, Analytical structure of the S matrix for the coupled channel problem $d + t \leftrightarrow n + \alpha d$ and the interpretation of the $J^\pi = \frac{3}{2}^+$ resonance in ^5He , *Phys. Rev. C* **44**, 1289 (1991).
- [101] V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova, and A. E. Kudryavtsev, Evidence that the $a_0(980)$ and $f_0(980)$ are not elementary particles, *Phys. Lett. B* **586**, 53 (2004).
- [102] Y. S. Kalashnikova, Coupled-channel model for charmonium levels and an option for $X(3872)$, *Phys. Rev. D* **72**, 034010 (2005).
- [103] Y. S. Kalashnikova and A. V. Nefediev, Nature of $X(3872)$ from data, *Phys. Rev. D* **80**, 074004 (2009).
- [104] V. Baru, C. Hanhart, Y. S. Kalashnikova, A. E. Kudryavtsev, and A. V. Nefediev, Interplay of quark and meson degrees of freedom in a near-threshold resonance, *Eur. Phys. J. A* **44**, 93 (2010).
- [105] C. Hanhart, Y. S. Kalashnikova, and A. V. Nefediev, Lineshapes for composite particles with unstable constituents, *Phys. Rev. D* **81**, 094028 (2010).
- [106] F. Giacosa and G. Pagliara, On the spectral functions of scalar mesons, *Phys. Rev. C* **76**, 065204 (2007).
- [107] J. Zeng, J. W. Van Orden, and W. Roberts, Heavy mesons in a relativistic model, *Phys. Rev. D* **52**, 5229 (1995).
- [108] D. Ebert, V. O. Galkin, and R. N. Faustov, Mass spectrum of orbitally and radially excited heavy-light mesons in the relativistic quark model, *Phys. Rev. D* **57**, 5663 (1998); **59**, 019902(E) (1999).
- [109] D. Ebert, R. N. Faustov, and V. O. Galkin, Heavy-light meson spectroscopy and Regge trajectories in the relativistic quark model, *Eur. Phys. J. C* **66**, 197 (2010).
- [110] W. J. Deng, H. Liu, L. C. Gui, and X. H. Zhong, Charmonium spectrum and their electromagnetic transitions with higher multipole contributions, *Phys. Rev. D* **95**, 034026 (2017).
- [111] Q. Li, M. S. Liu, L. S. Lu, Q. F. Lü, L. C. Gui, and X. H. Zhong, Excited bottom-charmed mesons in a nonrelativistic quark model, *Phys. Rev. D* **99**, 096020 (2019).
- [112] A. Drutskoy *et al.* (Belle Collaboration), Measurement of $\Upsilon(5S)$ decays to B^0 and B^+ mesons, *Phys. Rev. D* **81**, 112003 (2010).
- [113] P. G. Ortega, D. R. Entem, F. Fernández, and J. Segovia, Assessment of Υ states above $B\bar{B}$ threshold using a constituent-quark-model based meson-meson coupled-channels framework, *Phys. Rev. D* **109**, 114007 (2024).
- [114] A. Ali, C. Hambrock, I. Ahmed, and M. J. Aslam, A case for hidden $b\bar{b}$ tetraquarks based on $e^+e^- \rightarrow b\bar{b}$ cross section between $\sqrt{s} = 10.54$ and 11.20 GeV, *Phys. Lett. B* **684**, 28 (2010).
- [115] R. Van Royen and V. F. Weisskopf, Hadron decay processes and the quark model, *Nuovo Cimento Soc. Ital. Fis.* **50A**, 617 (1967); **51**, 583(E) (1967).
- [116] R. Barbieri, R. Gatto, R. Kogerler, and Z. Kunszt, Meson hyperfine splittings and leptonic decays, *Phys. Lett.* **57B**, 455 (1975).
- [117] V. A. Novikov, L. B. Okun, M. A. Shifman, A. I. Vainshtein, M. B. Voloshin, and V. I. Zakharov,

- Charmonium and gluons: Basic experimental facts and theoretical introduction, [Phys. Rep. **41**, 1 \(1978\)](#).
- [118] R. Barbieri, E. d’Emilio, G. Curci, and E. Remiddi, Strong radiative corrections to annihilations of quarkonia in QCD, [Nucl. Phys. **B154**, 535 \(1979\)](#).
- [119] I. Adachi *et al.* (Belle-II Collaboration), Measurement of the energy dependence of the $e^+e^- \rightarrow B\bar{B}, B\bar{B}^*$, and $B^*\bar{B}^*$ cross sections at Belle II, [J. High Energy Phys. **10** \(2024\) 114](#).
- [120] K. Konigsmann, Leptonic widths of ψ and upsilon resonances, Report No. DESY-87-046.
- [121] E. Klempt, Glueballs, hybrids, pentaquarks: Introduction to hadron spectroscopy and review of selected topics, [arXiv:hep-ph/0404270](#).
- [122] I. Adachi *et al.* (Belle-II Collaboration), Study of $\Upsilon(10753)$ decays to $\pi^+\pi^-\Upsilon(nS)$ final states at Belle II, [J. High Energy Phys. **07** \(2024\) 116](#).