

Probing gluon TMD models with Drell-Yan structure functions

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We compute structure functions for the Drell-Yan process in proton-proton collisions at the center-of-mass energy $\sqrt{S} = 8$ TeV, both parity conserving and parity breaking. For this calculation, we use the high-energy factorization formalism. The hard scattering matrix elements used in our derivation consist of two channels, $q_{\text{val}}g^* \rightarrow qV^*$ and $g^*g^* \rightarrow q\bar{q}V^*$, both at the tree level. We consider four types of gluon transverse momentum distribution (TMD) models: Gaussian, Weizsäcker-Williams (WW), Kimber-Martin-Ryskin (KMR), and Jung-Hautmann. We also consider the models with phenomenological adjustments to improve the data description. We additionally compare the KMR model with its next-to-leading-order extension, called the Martin-Ryskin-Watt model, and the Kotikov-Lipatov result for the gluon TMD. We derive and compare the structure functions calculated for different gluon TMD models with the ATLAS 2016 data. Based on this comparison, we calculate χ^2 per number of degrees of freedom for each of the predictions. This assessment shows clear differences between the predictions obtained with different TMD models, both in the description of the full dataset and in the case of individual structure functions. The best description of the structure functions' data is obtained with one of the modified WW models. Our analysis can serve to identify the features of the TMD model that should be considered in future gluon TMD fits.

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I. INTRODUCTION

The Drell-Yan (DY) process [1] is one of the most informative probes of the internal structure of hadrons. In this process, a lepton-antilepton (dilepton) pair is produced through the decay of a neutral electroweak (EW) gauge boson, either a virtual photon γ^* or a Z^0 boson. The DY $W^\pm$ production can also occur, followed by the decay of the boson into a charged lepton-antineutrino (or charged antilepton-neutrino) pair. The dilepton pair angular distribution is described by functions of the dilepton angles (ϕ, ϑ) and angular coefficients A_i directly related to the DY structure functions (will refer to them as the structure functions interchangeably). They have been extensively studied within the parton model [2–5], in which nontrivial relations among them were derived in Refs. [6–8].

The structure functions depend on the dilepton invariant mass M , its total transverse momentum q_T , and its rapidity y . For the photon exchange, the only contribution comes from the parity-conserving structure functions, which,

including the differential cross section integrated over the dilepton angles, can be parametrized with four independent functions. For the Z^0 and $W^\pm$ bosons, there exist additionally five more functions that break parity. The Drell-Yan total cross section and the structure functions were measured accurately at the LHC [9,10]. The structure functions were measured at the Z^0 resonance, i.e., at $M = M_Z$, where the contribution of γ^* is negligible, as functions of the dilepton total transverse momentum q_T . Most of the data are well described by the next-to-leading-order (NLO) calculations within the collinear perturbative QCD (pQCD) [10]. Here, order denotes the perturbative order in the calculation of the differential distribution, with the leading contribution occurring at $\mathcal{O}(\alpha_s)$ and corresponding to parton plus EW boson production. One significant exception from the theory and data agreement is the Lam-Tung combination A_{LT} , which reaches values around $A_{LT} \approx 0.15$ at $q_T \approx 80$ GeV, while the NLO predictions give values around $A_{LT} \approx 0.09$. The Lam-Tung relation $A_{LT} = 0$ [6–8] is satisfied up to the leading order (LO) ($V^* + \text{jet}$ production) in the collinear pQCD, and its breaking at higher orders is related to real emissions that generate additional transverse momentum, causing the rotation of the parton plane with respect to the hadron plane [11,12].

The collinear factorization framework for high-energy scattering processes involving hadrons, such as deep inelastic scattering (DIS), semi-inclusive DIS, and the Drell-Yan

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process, is well established and rigorously formulated in pQCD [13–16]. At the leading twist in the collinear factorization framework, the hadronic cross section is given by a convolution of parton distribution functions (PDFs), which encode nonperturbative physics below the factorization scale μ_F , and hard scattering matrix elements computed in pQCD with on-shell initial partons. Within this formalism, a wide range of observables have been derived and studied, including the dilepton angular distributions in DY production. The DY structure functions were computed within the collinear framework at LO [3–5,17], NLO [18–20], and next-to-next-to-leading-order (NNLO) accuracy [21] in the perturbative α_s expansion, with the inclusion of NLO EW corrections [22]. Explicit phenomenological studies exist for $V^* + \text{jet}$ processes [23], in particular within Monte Carlo implementations [24]. In addition, the impact of resummation effects on the angular decomposition was investigated within the collinear framework [25].

The discrepancy between the NNLO prediction and the A_{LT} data, connected with the fact that the Lam-Tung relation breaks down due to partons' transverse momenta, induced the idea of incorporating partons' transverse momenta from the beginning. To account for these transverse momenta, we use the so-called high-energy factorization (or k_T factorization), derived and described in [26,27], where, in particular, the formulation of obtaining the cross section from hard matrix elements within this scheme was explained. The DY process within this approach was studied in, e.g., [28–36]. It was shown in [37] that within the k_T factorization approach, significant improvement in the description of A_{LT} can be reached, especially with the transverse momentum distributions (TMDs, alternatively called transverse momentum-dependent parton distribution functions—TMD PDFs) with slow k_T fall-off. This result is the main inspiration for this work. We consider the TMDs used in [37] together with other models to check if the A_{LT} description can be improved, and if the remaining structure functions' description stays in agreement with the data. It should be mentioned that DY structure functions have also been studied within high-energy factorization in an alternative formulation where both quarks and gluons are reggeized [38–41]. The DY structure functions have been studied within the k_T factorization framework at order $\mathcal{O}(\alpha_s)$ [32] and within the color glass condensate (CGC) formalism, with particular emphasis on the Lam-Tung combination [30,31]. Within the CGC framework, these observables have also been analyzed using the color-dipole formalism [42], including investigations of higher-twist effects [43,44].

An alternative framework that incorporates partonic transverse momentum effects is the transverse momentum-dependent factorization in the Collins-Soper-Sterman (CSS) formalism [45]. This approach applies in the region of small transverse momenta, $k_T^2 \ll M^2$, and resums logarithms of the form $\log(M^2/k_T^2)$. In contrast, the k_T

factorization is designed for kinematics with $k_T^2 \sim M^2 \ll S$ and resums the large logarithms of $1/x$, employing off-shell partons in the hard scattering matrix elements. The TMD factorization admits a rigorous operator formulation of TMDs and can be systematically matched to the collinear factorization. The DY structure functions in the small dilepton transverse momentum (q_T) region have been studied within the CSS framework and compared with experimental data in [46,47].

The parton TMDs parametrize the hadronic structure, providing the basis for QCD evolution in the transverse momentum scale and rapidity. In the small- x regime, the best known evolution equation that resums logarithms of $1/x$ and incorporates transverse momentum is the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation [48–51], which is linear and violates unitarity at very small x . The Catani-Ciafaloni-Fiorani-Marchesini (CCFM) [52–55] equation interpolates the BFKL evolution at small x and the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution in factorization scale, by taking into account effects of angular ordering and color coherence. At high parton densities, the gluon saturation effects become important and are taken into account in the Balitsky-Kovchegov equation [56,57] and, more generally, by the Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner evolution equations [58–62], which are nonlinear and consistent with unitarity, in contrast with the BFKL equation.

The gluon TMD denoted by $\mathcal{F}(x, \mathbf{k}_T^2, \mu_F^2)$ in general depends on the longitudinal momentum fraction x of the parent hadron, transverse momentum $\mathbf{k}_T$ with respect to hadron momentum, and the factorization scale μ_F . It provides information about the internal structure of hadrons, in particular of the proton. Since it depends on a larger number of parton kinematic variables, it should provide a more accurate description of the hadron structure in the region of applicability of the k_T factorization at a given order in perturbation expansion than the description given by the collinear PDFs. Therefore, the determination of the accurate TMD parametrization is important for obtaining precise hadron scattering predictions. There are many different TMD models in use today, each with different properties and origins. To distinguish and evaluate these models, it is important to find observables sensitive to transverse momentum effects.

In conclusion, we consider all the Drell-Yan structure functions with a separate consideration of the Lam-Tung combination A_{LT} for different TMD models. In our formalism, we take into account two channels, the scattering of two off-shell gluons $g^*g^* \rightarrow q\bar{q}V^*$ and the scattering of the collinear valence quark and off-shell gluon $q_{\text{val}}g^* \rightarrow qV^*$, both at the tree level, where V^* denotes a virtual electroweak gauge boson γ^* or Z^0 . We apply four different types of gluon TMD models, which come from QCD evolution equations or are inspired by QCD. These models are the quasicollinear Gaussian model, which

comes from the Golec-Biernat-Wüsthoff saturation model [63], the Jung-Hautmann model, which is the result of the CCFM evolution equation [52–55], the Kimber-Martin-Ryskin (KMR) model [64–66], which comes from the collinear DGLAP evolution equation [67–70], and the Weizsäcker-Williams model [37], which is a phenomenological model based on a concept of the gluon TMD to be driven by the Weizsäcker-Williams gluon emission from valence quarks, and should not be confused with Weizsäcker-Williams TMD used in the k_T factorization framework, see, e.g., [71]. We also proposed a modified Weizsäcker-Williams model, which preserves the k_T dependence of the original model, but adopts the dependence on x and the factorization scale from the collinear gluon PDF. For models with the collinear x dependence, we also consider a simple phenomenological modification relying on x rescaling, which should account for the additional generation of invariant mass in the process involving the transverse momenta of partons compared to the collinear process. Additionally, we compared the KMR model based on the LO DGLAP splitting with the NLO extension given by the Martin-Ryskin-Watt (MRW) prescription [72] and with the Kotikov-Lipatov (KL) model based on the analytical solution of DGLAP as input in the KMR procedure [73]. Some of the models underestimated the total Drell-Yan cross section, which led us to adjust these models by normalizing them to the total cross section data [9]. We observe that the Drell-Yan structure functions indeed differentiate between the gluon TMD models. The Weizsäcker-Williams model works quite well in comparison to the other models. Its modifications do not significantly change the description of the Lam-Tung combination, but their predictions for other structure functions are different—some are improved, while others are slightly worsened. We summarize the results in terms of χ^2 per number of degrees of freedom, and their ratio to the minimum χ^2 value.

II. KINEMATICS

We consider two colliding hadrons with momenta P'_1 and P'_2 that produce a lepton (l^-)-antilepton (l^+) (dilepton) pair. In the standard light cone coordinates, the hadronic momenta can be written as

$$\begin{aligned} P'_1 &= \left(\sqrt{S}, \frac{M_H^2}{\sqrt{S}}; \mathbf{0} \right) \approx (\sqrt{S}, 0; \mathbf{0}) =: P_1, \\ P'_2 &= \left(\frac{M_H^2}{\sqrt{S}}, \sqrt{S}; \mathbf{0} \right) \approx (0, \sqrt{S}; \mathbf{0}) =: P_2, \end{aligned} \quad (2.1)$$

where $S = (P_1 + P_2)^2$ is the center-of-mass collision energy, and we will consistently neglect the hadron mass M_H , because in the high-energy limit $M_H/\sqrt{S} \ll 1$. The other momenta will be represented using Sudakov decomposition

$$p = xP_1 + \frac{m^2 + \mathbf{p}_T^2}{xS}P_2 + p_T, \quad (2.2)$$

where $p_T = (0, 0; \mathbf{p}_T)$ is such that $p_T \cdot P_1 = p_T \cdot P_2 = 0$ and $m^2 = p^2$.

We denote the virtual boson momentum as q , its mass as M , and define its transverse mass by $M_T^2 = M^2 + \mathbf{q}_T^2$, so that its Sudakov decomposition reads

$$q = x_F P_1 + \frac{M_T^2}{x_F S} P_2 + q_T, \quad (2.3)$$

where x_F is the so-called Feynman x . The electroweak gauge boson rapidity y in the laboratory frame is related to x_F by the equation $y = \ln(x_F \sqrt{S}/M_T)$, or conversely $x_F = M_T e^y / \sqrt{S}$.

In general we define a coordinate system by the four-vectors (T, X, Y, Z) , which are orthogonal and normalized as $T^2 = 1, X^2 = Y^2 = Z^2 = -1$. We choose q to define the time direction as $T^\mu := q^\mu / M$ and X, Z coordinates to span the hadronic plane,

$$X^\mu := \alpha_+ \tilde{P}_{(+)}^\mu + \alpha_- \tilde{P}_{(-)}^\mu, \quad Z^\mu := \beta_+ \tilde{P}_{(+)}^\mu + \beta_- \tilde{P}_{(-)}^\mu, \quad (2.4)$$

where $\alpha_\pm, \beta_\pm$ are coefficients that define the specific frame and we defined the $\pm$ vectors as $P_{(\pm)} := P_1 \pm P_2$, with tilde meaning that they are projected onto a hyperplane orthogonal to q , i.e., $\tilde{P}_{(\pm)}^\mu := \tilde{g}^{\mu\nu} P_{(\pm)\nu}$, with the projection defined as

$$\tilde{g}^{\mu\nu} := g^{\mu\nu} - \frac{q^\mu q^\nu}{M^2}. \quad (2.5)$$

The Y coordinate is defined to complete the right-oriented orthonormal basis.

All the calculations are performed in the Collins-Soper frame [2], where the Z -axis is defined to bisect the angle formed by the hadrons' momenta directions in the particle rest frame. This definition leaves ambiguity due to two possible ways of choosing bisectors between two non-oriented directions. Such a definition fixes the Z -axis up to a sign. The orientation of the Z -axis is consistent with the Z -direction of the lepton pair in the laboratory reference frame. Therefore, it reverses the sign for the negative rapidity y , and due to the requirement of a right-handed basis, the Y -axis also reverses the sign. In this frame the coefficients $\alpha_\pm$ and $\beta_\pm$ are equal to

$$\alpha_\pm = \mp \frac{M(q \cdot P_\pm)}{q_T M_T S}, \quad \beta_\pm = \mp \frac{(q \cdot P_\mp)}{M_T S}. \quad (2.6)$$

We denote the leptons' momenta as l_1 and l_2 for the lepton l^- and antilepton l^+ , respectively. The angles of the dilepton in the center-of-momentum (COM) frame are

defined through the scalar products with the chosen coordinates

$$\begin{aligned} l_{1/2} \cdot X &= \mp \frac{M}{2} \sin \vartheta \cos \phi, & l_{1/2} \cdot Y &= \mp \frac{M}{2} \sin \vartheta \sin \phi, \\ l_{1/2} \cdot Z &= \mp \frac{M}{2} \cos \vartheta. \end{aligned} \quad (2.7)$$

Given the coordinates, we can define the polarization vectors of the electroweak boson,

$$\varepsilon_{(0)}^\mu := Z^\mu, \quad \varepsilon_{(\pm)}^\mu := \mp \frac{1}{\sqrt{2}} (X^\mu \pm iY^\mu). \quad (2.8)$$

III. THE DRELL-YAN PROCESS

In general, the Drell-Yan process is a process that describes lepton pair production in a collision of hadrons H_1 and H_2 . At the leading order in the electromagnetic coupling, the colliding hadrons produce an electroweak gauge boson V^* , which then decays into a dilepton pair l^+l^- . It can be written schematically as $H_1(P'_1) + H_2(P'_2) \rightarrow V^*(q) + X(p_X) \rightarrow l^+(l_2) + l^-(l_1) + X(p_X)$, where p_X denotes a set of n outgoing partons' momenta. The factorization formula, assuming the high-energy factorization for the DY cross section σ , is given by [74]

$$\begin{aligned} d\sigma &= \sum_{i,j} \int \frac{dx_1}{x_1} \int \frac{d^2k_{1T}}{\pi} \mathcal{F}_{i/H_1}(x_1, \mathbf{k}_{1T}^2, \mu_F^2) \\ &\times \int \frac{dx_2}{x_2} \int \frac{d^2k_{2T}}{\pi} \mathcal{F}_{j/H_2}(x_2, \mathbf{k}_{2T}^2, \mu_F^2) d\hat{\sigma}_{ij}, \end{aligned} \quad (3.1)$$

where $x_{1,2}$ are the fractions of the longitudinal momentum of the parent hadron carried by a parton, $\mathcal{F}_{i/H}(x, \mathbf{k}_T^2, \mu_F^2)$ is a transverse momentum-dependent parton distribution function, which gives the probability of finding the parton i with longitudinal momentum fraction x and transverse momentum $\mathbf{k}_T$ in the hadron H . Parton distributions depend also on the factorization scale μ_F , which separates nonperturbative effects captured by TMDs from the perturbative partonic cross section $d\hat{\sigma}_{ij}$. In the case of high-energy factorization, the partonic cross section $d\hat{\sigma}_{ij}$ is off-shell while remaining gauge invariant. TMDs are related to the standard collinear PDFs via

$$xf_i(x, \mu^2) = \int_0^{\mu^2} dk_T^2 \mathcal{F}_i(x, k_T^2, \mu^2). \quad (3.2)$$

The partonic cross section of a process $i(p_1)j(p_2) \rightarrow X(p_X)l^-(l_1)l^+(l_2)$ is given by the formula

$$\begin{aligned} d\hat{\sigma}_{ij} &= \frac{(2\pi)^4}{F(p_1, p_2)} L_{\mu\nu} \mathcal{M}_{ij}^{\mu\nu} |D_V(q^2)|^2 \\ &\times dPS_{n+2}(p_1, p_2; p_X, l_1, l_2), \end{aligned} \quad (3.3)$$

where PS_n is the phase space of n outgoing particles, $F(p_1, p_2) = \sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}$, where $m_i^2 = p_i^2$ is the incident flux of the initial partons, $D_V(q^2)$ is the propagator of boson V with mass M_V and decay width Γ_V given by $D_V(q^2) = i/(q^2 - M_V^2 + iM_V\Gamma_V)$, and $L_{\mu\nu}$ is the leptonic tensor,

$$\begin{aligned} L^{\mu\nu} &:= \sum_{s_1, s_2} \bar{u}_{s_1}(l_1) \Gamma_V^\mu v_{s_2}(l_2) \bar{v}_{s_2}(l_2) \Gamma_V^\nu u_{s_1}(l_1) \\ &= 2g_l^2 [q^\mu q^\nu - l^\mu l^\nu - q^2 g^{\mu\nu}] + 2i w_l^2 \varepsilon^{\mu\nu\rho\sigma} q_\rho l_\sigma, \end{aligned} \quad (3.4)$$

where the interaction of the boson V with dilepton l^+l^- is described by the vertex

$$\Gamma_V^\mu = (v_f^V + a_f^V \gamma_5) \gamma^\mu, \quad (3.5)$$

where v_f^V and a_f^V are the vector and axial couplings for the fermion with flavor f . We denote leptonic flavors by the index l and quark flavors by the index q . We also define $g_l^2 := v_l^2 + a_l^2$ and $w_l^2 := 2v_l a_l$ with suppressed boson designation V , which is obvious from the context. The formulas for the electroweak couplings are given in Appendix B. The boson momentum is $q = l_1 + l_2$, and we also use the momentum l^μ given by $l = l_1 - l_2$. We define the partonic amplitude squared with factorized boson polarizations as

$$\mathcal{M}_{ij}^{\mu\nu} = \sum \mathcal{A}^\mu \bar{\mathcal{A}}^\nu, \quad (3.6)$$

where the symbol $\sum$ means averaging over the set of indices for the incoming particles and summing over the indices of the outgoing particles, and $\mathcal{A}^\mu$ is the scattering amplitude for the process $i(p_1)j(p_2) \rightarrow X(p_X)V^*(q)$ with boson polarization removed. The contraction of the leptonic tensor and the above amplitude squared can be written as a sum over the boson basis polarizations,

$$L_{\mu\nu} \mathcal{M}_{ij}^{\mu\nu} = \sum_{r, r'=0, \pm} L^{(rr')} \mathcal{M}_{ij}^{(rr')}, \quad (3.7)$$

with

$$\begin{aligned} L^{(rr')} &:= \bar{\varepsilon}_\mu^{(r)}(q) L^{\mu\nu} \varepsilon_\nu^{(r')}(q), \\ \mathcal{M}_{ij}^{(rr')} &:= \varepsilon_\mu^{(r)}(q) \mathcal{M}_{ij}^{\mu\nu} \bar{\varepsilon}_\nu^{(r')}(q). \end{aligned} \quad (3.8)$$

The DY differential cross section can be written in the following form:

$$\begin{aligned} \frac{d\sigma}{dM^2 dy d^2q_T d\Omega} &= \frac{4}{(4\pi)^6} |D_V(M^2)|^2 \\ &\times \sum_{r, r'=0, \pm} L^{(rr')} \frac{d\sigma^{(rr')}}{dM^2 dy d^2q_T}, \end{aligned} \quad (3.9)$$

where the angular distribution of lepton angles $\Omega = (\vartheta, \phi)$ is given by the polarized leptonic tensor, and we define the polarized differential cross sections as

$$\begin{aligned} \frac{d\sigma^{(rr')}}{dM^2} &= \sum_{i,j} \int dx_1 \int \frac{d^2 k_{1T}}{\pi x_1} \mathcal{F}_i(x_1, \mathbf{k}_{1T}^2, \mu_F^2) \\ &\times \int dx_2 \int \frac{d^2 k_{2T}}{\pi x_2} \mathcal{F}_j(x_2, \mathbf{k}_{2T}^2, \mu_F^2) \\ &\times \frac{(2\pi)^4}{F(p_1, p_2)} \mathcal{M}_{ij}^{(rr')} dPS_{n+1}(p_1, p_2; p_X, q). \end{aligned} \quad (3.10)$$

A. The helicity structure functions

The DY differential cross section (3.9) can be reduced to the form

$$\begin{aligned} \frac{d\sigma}{dM^2 dy d^2 q_T d\Omega} &= \frac{4g_l^2 M^2}{(4\pi)^6} |D_V(M^2)|^2 \\ &\times \sum_{\tau \in \mathfrak{P}} g_\tau(\vartheta, \phi) \frac{d\sigma^\tau}{dM^2 dy d^2 q_T}, \end{aligned} \quad (3.11)$$

where $\mathfrak{P} := \{U+L, L, TT, LT, A, P, 7, 8, 9\}$, and with the angular coefficients derived from the leptonic tensor,

$$\begin{aligned} g_{U+L}(\vartheta, \phi) &= 1 + \cos^2 \vartheta, & g_L(\vartheta, \phi) &= 1 - 3\cos^2 \vartheta, & g_{TT}(\vartheta, \phi) &= 2\sin^2 \vartheta \cos(2\phi), \\ g_{LT}(\vartheta, \phi) &= 2\sqrt{2} \sin(2\vartheta) \cos \phi, & g_A(\vartheta, \phi) &= 4\sqrt{2} \sin \vartheta \cos \phi, & g_P(\vartheta, \phi) &= 2 \cos \vartheta, \\ g_7(\vartheta, \phi) &= 2\sin^2 \vartheta \sin(2\phi), & g_8(\vartheta, \phi) &= 2\sqrt{2} \sin(2\vartheta) \sin \phi, & g_9(\vartheta, \phi) &= 4\sqrt{2} \sin \vartheta \sin \phi, \end{aligned} \quad (3.12)$$

and the helicity differential cross sections $d\sigma^\tau$ are

$$d\sigma^{U+L} = d\sigma^{(00)} + d\sigma^{(++)} + d\sigma^{(--)}, \quad d\sigma^L := d\sigma^{(00)}, \quad d\sigma^{TT} = \frac{1}{2} (d\sigma^{(+-)} + d\sigma^{(-+)}), \quad (3.13)$$

$$d\sigma^{LT} = \frac{1}{4} (d\sigma^{(+0)} + d\sigma^{(0+)} - d\sigma^{(-0)} - d\sigma^{(0-)}), \quad d\sigma^P = c_l (d\sigma^{(++)} - d\sigma^{(--)}), \quad (3.14)$$

$$d\sigma^A = \frac{c_l}{4} (d\sigma^{(+0)} + d\sigma^{(0+)} + d\sigma^{(-0)} + d\sigma^{(0-)}), \quad d\sigma^7 = \frac{1}{2i} (d\sigma^{(+-)} - d\sigma^{(-+)}), \quad (3.15)$$

$$d\sigma^8 = \frac{1}{4i} (d\sigma^{(+0)} - d\sigma^{(0+)} + d\sigma^{(-0)} - d\sigma^{(0-)}), \quad d\sigma^9 = \frac{c_l}{4i} (d\sigma^{(+0)} - d\sigma^{(0+)} - d\sigma^{(-0)} + d\sigma^{(0-)}), \quad (3.16)$$

where $c_l := w_l^2/g_l^2 = 2v_l a_l/(v_l^2 + a_l^2)$.

The differential cross section can then be written as

$$\begin{aligned} \frac{d\sigma}{dM^2 dy d^2 q_T d\Omega} &= \frac{3}{16\pi} \frac{d\sigma}{dM^2 dy d^2 q_T} \left[1 + \cos^2 \vartheta + \frac{1}{2} A_0 (1 - 3\cos^2 \vartheta) + A_1 \sin(2\vartheta) \cos \phi + \frac{1}{2} A_2 \sin^2 \vartheta \cos(2\phi) \right. \\ &\quad \left. + A_3 \sin \vartheta \cos \phi + A_4 \cos \vartheta + A_5 \sin^2 \vartheta \sin(2\phi) + A_6 \sin(2\vartheta) \sin \phi + A_7 \sin \vartheta \sin \phi \right], \end{aligned} \quad (3.17)$$

where the differential cross section integrated over the lepton angles is

$$\frac{d\sigma}{dM^2 dy d^2 q_T} = \frac{16\pi 4g_l^2 M^2}{3 (4\pi)^6} |D_V(M^2)|^2 \frac{d\sigma^{U+L}}{dM^2 dy d^2 q_T}, \quad (3.18)$$

and the A_i structure functions are defined as follows:

$$\begin{aligned} A_0 &= \frac{2W_L}{W_{U+L}}, & A_1 &= \frac{2\sqrt{2}W_{LT}}{W_{U+L}}, & A_2 &= \frac{4W_{TT}}{W_{U+L}}, & A_3 &= \frac{4\sqrt{2}W_A}{W_{U+L}}, & A_4 &= \frac{2W_P}{W_{U+L}}, \\ A_5 &= \frac{2W_7}{W_{U+L}}, & A_6 &= \frac{2\sqrt{2}W_8}{W_{U+L}}, & A_7 &= \frac{4\sqrt{2}W_9}{W_{U+L}}, \end{aligned} \quad (3.19)$$

with the designation $W_\tau = \frac{d\sigma^\tau}{dM^2 dy d^2 q_T}$.

The structure functions obey the so-called Lam-Tung relation [6,8]

$$A_{LT} := A_0 - A_2 = 0, \quad (3.20)$$

which holds up to the NLO accuracy in the collinear factorization [12].

IV. THE DRELL-YAN CROSS SECTIONS

We adopt the approximation concerning the hierarchy of the scattering processes following Ref. [37]—we assume that the sea quarks entering the hard scattering are generated either directly in the matrix element or in the last step of the parton evolution via gluon splitting. In this approach we consider two partonic channels, namely, the $q_{\text{val}}g^*$ channel with polarized cross sections $d\sigma_{(qg^*+g^*q)}^{(r_1 r_2)} := d\sigma_{(qg^*)}^{(r_1 r_2)} + d\sigma_{(g^*q)}^{(r_1 r_2)}$ and the g^*g^* channel with polarized cross sections $d\sigma_{(g^*g^*)}^{(r_1 r_2)}$. The total cross section is a sum of the cross sections for these two channels,

$$d\sigma^{(r_1 r_2)} = d\sigma_{(qg^*+g^*q)}^{(r_1 r_2)} + d\sigma_{(g^*g^*)}^{(r_1 r_2)}. \quad (4.1)$$

Moreover, we are using the so-called nonsense polarization [74] for the initial off-shell gluons,

$$\epsilon^\mu(k_i) = \frac{x_i}{\sqrt{-k_{iT}^2}} P_i^\mu, \quad (4.2)$$

where the gluons' momenta are decomposed as $k_i = x_i P_i + k_{iT}$ for $i = 1, 2$.

A. Cross section for $q_{\text{val}}g^*$ channel

For the $q_{\text{val}}g^*$ channel, depicted in Fig. 1(a), we are using the so-called hybrid factorization, where the initial gluon g^* is off-shell and carries the transverse momentum k_T , while the valence quark q_{val} is assumed to be collinear. This contribution was derived earlier within the k_T factorization approach in Refs. [28–30]. According to (3.10), taking the collinear limit (3.2) for the valence quark, we obtain the polarized cross section for this channel, expressed as

$$\begin{aligned} \frac{d\sigma_{(qg^*)}^{(r_1 r_2)}}{dM^2 dy d^2 q_T} &= \sum_q \int dx_q f_{q,\text{val}}(x_q, \mu_F) \\ &\times \int \frac{d^2 k_T}{\pi \mathbf{k}_T^2} \mathcal{F}(x_g, \mathbf{k}_T^2, \mu_F^2) \frac{4\pi\alpha_s(\mu_F)}{N} \\ &\times \frac{z^2}{(8\pi)^2 x_F^2 (1-z) S^2} \Phi_{r_1 r_2}^{(q)}, \end{aligned} \quad (4.3)$$

where $f_{q,\text{val}}$ is a PDF for a valence quark q , the gluon momentum fraction x_g is fixed from the energy-momentum conservation as $x_g = ((1-z)M_T^2 + z(\mathbf{k}_T - \mathbf{q}_T)^2)/(x_F(1-z)S)$ with $z = x_F/x_q$, and the impact factor is defined as

$$\Phi_{r_1 r_2}^{(q)} := \sum_{\sigma_1, \sigma_2} A_{\sigma_1 \sigma_2}^{(r_1)} \bar{A}_{\sigma_1 \sigma_2}^{(r_2)}, \quad (4.4)$$

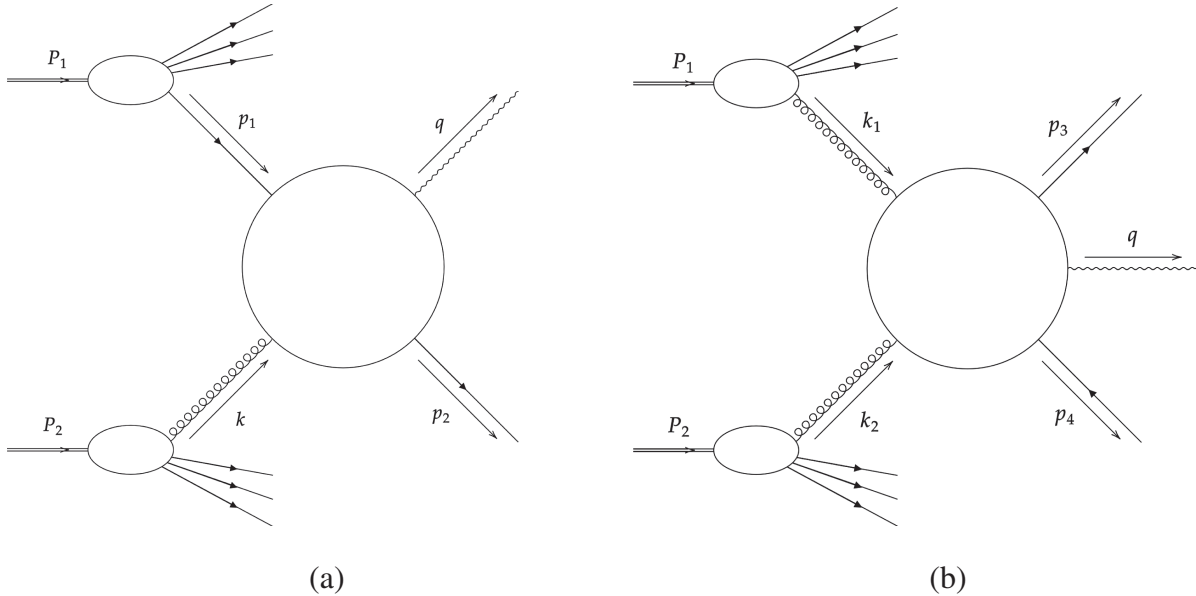


FIG. 1. General form of the diagrams for the DY process. Big circles represent all possible QCD LO subdiagrams. P_1 and P_2 are the momenta of the incoming hadrons and q is the momentum of the emitted electroweak gauge boson. (a) Diagram for the subprocess $q_{\text{val}}g^* \rightarrow qV^*$, where p_1 is the momentum of the incoming valence quark, k is the momentum of the off-shell incoming gluon and p_2 is the momentum of the outgoing quark and (b) diagram for the subprocess $g^*g^* \rightarrow q\bar{q}V^*$, where k_1, k_2 are the momenta of two off-shell gluons coming from each hadron and p_3, p_4 are the momenta of the outgoing quark and antiquark, respectively.

where $A_{\sigma_1\sigma_2}^{(r_1)} = \varepsilon_{(r_1)\mu} A_{\sigma_1\sigma_2}^\mu$ also carries the information about the quark flavor q . The scattering amplitudes for this channel are derived in Appendix A. The $d\sigma_{(g^*q)}^{(r_1r_2)}$ contribution can be obtained by swapping the proton beams.

B. Cross section for g^*g^* channel

In the g^*g^* channel, we consider the scattering of two off-shell gluons at the tree level $g^*g^* \rightarrow q\bar{q}V^*$. The general form of a diagram for such a process is illustrated in Fig. 1(b).

The contribution for this channel was calculated earlier, e.g., [37,75], and applied to the $Z + b\bar{b}$ production [35]. From formula (3.10), we obtain the cross section for this channel in the form

$$\begin{aligned} d\sigma_{(g^*g^*)}^{(r_1r_2)} &= \int dx_1 \int \frac{d^2k_{1T}}{\pi\mathbf{k}_{1T}^2} \mathcal{F}(x_1, \mathbf{k}_{1T}^2, \mu_F^2) \\ &\times \int dx_2 \int \frac{d^2k_{2T}}{\pi\mathbf{k}_{2T}^2} \mathcal{F}(x_2, \mathbf{k}_{2T}^2, \mu_F^2) \\ &\times \frac{(2\pi)^4}{2S} \mathcal{M}_{(g^*g^*)}^{(r_1r_2)} dPS_3(k_1, k_2; p_3, p_4, q), \end{aligned} \quad (4.5)$$

where $\mathcal{M}_{(g^*g^*)}^{(r_1r_2)}$ is polarized amplitude squared (with pulled out $x_i^2/\mathbf{k}_{iT}^2$ factors from gluon polarizations), which is given by (A17), and dPS_3 is a three-body phase space,

$$dPS_3(k_1, k_2; p_3, p_4, q) = \frac{dz d\phi_\kappa dy d^2q_T}{8(2\pi)^9}.$$

The chosen integration variables z, ϕ_κ are defined through the quark and antiquark momenta p_3 and p_4 decomposition,

$$\begin{aligned} p_3 &= zx_{q\bar{q}}P_1 + \frac{\mathbf{p}_3^2}{zx_{q\bar{q}}S}P_2 + p_{3T}, \\ p_4 &= (1-z)x_{q\bar{q}}P_1 + \frac{\mathbf{p}_4^2}{(1-z)x_{q\bar{q}}S}P_2 + p_{4T}, \end{aligned} \quad (4.6)$$

where $\mathbf{p}_3$ and $\mathbf{p}_4$ are transverse momenta, which can be written in terms of the COM transverse momentum Δ and momentum $\kappa = \kappa(\cos\phi_\kappa, \sin\phi_\kappa)$ as

$$\mathbf{p}_3 = z\Delta + \kappa, \quad \mathbf{p}_4 = (1-z)\Delta - \kappa. \quad (4.7)$$

By the energy-momentum conservation Dirac deltas, we get the following constraints:

$$\begin{aligned} x_{q\bar{q}} &= x_1 - x_F, \quad \Delta = \mathbf{k}_{1T} + \mathbf{k}_{1T} - \mathbf{q}_T, \\ \kappa^2 &= z(1-z) \left(x_2 x_{q\bar{q}} S - \frac{x_{q\bar{q}} M_T^2}{x_F} - \Delta^2 \right). \end{aligned} \quad (4.8)$$

V. MODELS FOR GLUON TMD

We consider four models of gluon TMD proposed earlier, and we introduce some modifications to them. The models were applied earlier in calculations of the DY structure functions measured by ATLAS collaboration [10].

Thus, we consider the following TMD models:

- (i) the Gaussian (Gauss.) model,
- (ii) the Jung-Hautmann (JH) model [76],
- (iii) the KMR model [65],
- (iv) the ‘‘Weizsäcker-Williams’’ (WW) model [37].

We will briefly summarize their main characteristics, underlying ideas, and the proposed modifications, along with their justifications.

A. The Gaussian model

The Gaussian gluon distribution is inspired by the Golec-Biernat–Wüsthoff (GBW) saturation model [77] and extracted from the color-dipole cross section. It is referred to as quasicollinear due to its very narrow k_T dependence of the form

$$\mathcal{F}_G(x, k_T^2) = \frac{N_2}{k_0^2} (1-x)^7 \exp \left[- \left(\frac{x}{x_0} \right)^\lambda \frac{k_T^2}{k_0^2} \right], \quad (5.1)$$

where we take the values of parameters used in Ref. [37], $N_2 = 68.4$, $k_0 = 1$ GeV, $x_0 = 3 \times 10^{-4}$, and $\lambda = 0.29$. The parameters λ and x_0 were fitted to inclusive DIS data on F_2 [63] and normalization was adjusted in Ref. [37]. It provides a simple, phenomenological description of the gluon density that captures saturation effects at small x , although it neglects QCD evolution with the factorization scale other than gluon k_T .

B. The Jung-Hautmann model

The JH TMD is obtained from the CCFM evolution equation [52–54], which attempts to unify the small- x BFKL evolution equation [48–51] and the collinear DGLAP evolution [69,70], thus providing a consistent treatment of both $\log(1/x)$ and the logarithm in the hard scale.

Currently, there are a few available grids in the TMDlib library that differ from each other. We chose to compare three sets of gluon TMD grids: JH-2013-set1 [78], PB-NLO-HERAI+II-2018-set1 [79], and PB-NLO-HERAI+II-2023-set2-qs=1.04 [80], which we will call JH2013, PB2018, and PB2023, respectively, and we will collectively call them the JH models. All mentioned grids were obtained using the same Monte Carlo method for solving the evolution equation. However, the evolution kernel used for the JH2013 was dominated by the gluon channel, while the kernel used for the two remaining grids has a fully coupled flavor structure [81].

In order to obtain intermediate TMD values from the grid files, we applied linear interpolation. One should be aware

of the limited range of the JH-2013-set1 grid, i.e., the maximum value for the transverse momentum is $k_T = 200$ GeV.

C. The Kimber-Martin-Ryskin model

The KMR model was derived by resumming the virtual contribution from the DGLAP equation by including the last splitting as nonintegrated. The KMR distribution takes the form

$$\mathcal{F}_{\text{KMR}}(x, k_T^2, \mu^2) = \frac{\partial}{\partial k_T^2} [x f_g(x, k_T^2) T_g(k_T, \mu)], \quad (5.2)$$

where f_g is the collinear gluon PDF and

$$T_g(k_T, \mu) = \exp \left[- \int_{k_T^2}^{\mu^2} \frac{dp^2}{p^2} \frac{\alpha_s(p^2)}{2\pi} \int_0^{1-\delta(p)} \times dz z \left(P_{gg}(z) + \sum_q P_{qg}(z) \right) \right] \quad (5.3)$$

is the Sudakov form factor that gives the probability of the gluon with transverse momentum $\mathbf{k}_T$ remaining unchanged by the evolution up to the factorization scale μ . The functions P_{ij} are the first-order Altarelli-Parisi splitting functions of the form

$$P_{gg}(z) = 2C_A \frac{(1-z(1-z))^2}{1-z}, \quad (5.4a)$$

$$P_{qg}(z) = \frac{1}{2} (z^2 + (1-z)^2), \quad (5.4b)$$

$$P_{gq}(z) = C_F \frac{1 + (1-z)^2}{z}, \quad (5.4c)$$

and $\delta(p) = p/(p + \mu)$ is the parameter that provides the correct angular ordering of the real gluon momenta. The form factor is set to $T_g(Q, \mu) = 1$ for $Q > \mu$. Because of that, the differential form of the KMR TMD (5.2) has a discontinuity due to the nondifferentiability of the differentiated function at $Q = \mu$. However, one can eliminate this problem by using the integral form [82],

$$\begin{aligned} \mathcal{F}_{\text{KMR}}(x, k_T^2, \mu^2) &= \frac{T_g(k_T, \mu)}{k_T^2} \frac{\alpha_s(k_T^2)}{2\pi} \int_x^{1-\delta(k_T)} \times dz \left[P_{gg}(z) \frac{x}{z} f_g\left(\frac{x}{z}, k_T^2\right) \right. \\ &\quad \left. + \sum_q P_{gq}(z) \frac{x}{z} f_q\left(\frac{x}{z}, k_T^2\right) \right], \quad (5.5) \end{aligned}$$

where the sum $\sum_q$ runs over all active quark and antiquark flavors. Equations (5.2) and (5.5) agree for $k_T < \mu$ and start to deviate for $k_T > \mu$. This behavior is due to the use of a

regularized gluon PDF and can be avoided by using a cutoff-dependent DGLAP solution for the gluon PDF in the TMD definition (5.2) [82]. However, the available grids for PDFs are cutoff free [$\delta(k_T) = 0$], therefore, we use formula (5.5) in our computations.

For the computations, we generated a grid for the KMR model using formula (5.5) and we defined the function as the linear interpolation on this grid. As the input PDF we employed the CT10nlo model [83] from the LHAPDF library, together with the one-loop running coupling $\alpha_s(\mu^2)$ given by

$$\alpha_s(\mu^2) = \frac{1}{\beta_f \log(\frac{\mu^2}{\Lambda_f^2})}, \quad (5.6)$$

where $\beta_f = \frac{33-2n_f}{12\pi}$ for n_f being the number of active flavors and Λ_f is the QCD scale, which also depends on the number of active quark flavors n_f . The grid covers the ranges $x \in [10^{-7}, 1]$, $k_T \in [10^{-1} \text{ GeV}, 10^4 \text{ GeV}]$, and $\mu \in [2 \text{ GeV}, 1024 \text{ GeV}]$. Below a certain scale k_0 that parametrizes the onset of nonperturbative physics, which we chose to be $k_0 = 1$ GeV, we fix the TMD to be constant in k_T . Thus, for $k_T < k_0$, we set the TMD to be

$$\mathcal{F}_{\text{KMR}}(x, k_T^2, \mu^2) = \frac{x f_g(x, k_0^2)}{k_0^2} T_g(k_0, \mu) \quad (5.7)$$

to preserve the collinear limit.

We also considered modifications of this model. Initially, we observed that it did not reproduce the correct values for the total DY cross section with the virtual photon exchange measured by ATLAS [9]. Therefore, as an adjustment, we rescaled the TMD by the normalization constant fitted to the total DY cross section data.

As one can see from Eq. (5.2), the x dependence of the TMD is governed by that of the collinear PDF. However, the collinear and transverse momentum-dependent approaches are characterized by different kinematics. As a consequence, the nonzero $\mathbf{k}_T$ gluon momentum fraction x has to be larger than the collinear gluon x to reproduce the same invariant mass of the final state. Therefore, we introduce a simple phenomenological correction by rescaling x in the TMD by a constant factor. Together with an unscaled x , we considered three cases:

$$\mathcal{F}_{\text{KMR}}^{(1)}(x, k_T^2, \mu^2) = N_{\text{KMR}}^{(1)} \mathcal{F}_{\text{KMR}}(x, k_T^2, \mu^2), \quad (5.8a)$$

$$\mathcal{F}_{\text{KMR}}^{(2)}(x, k_T^2, \mu^2) = N_{\text{KMR}}^{(2)} \mathcal{F}_{\text{KMR}}(x/2, k_T^2, \mu^2), \quad (5.8b)$$

$$\mathcal{F}_{\text{KMR}}^{(3)}(x, k_T^2, \mu^2) = N_{\text{KMR}}^{(3)} \mathcal{F}_{\text{KMR}}(x/4, k_T^2, \mu^2), \quad (5.8c)$$

where each case was additionally rescaled by the overall normalization constant $N_{\text{KMR}}^{(i)}$ fitted to the total DY total

cross section data [9]. This should be understood as building a data-guided phenomenological model. This approach unfortunately loses the QCD direct connection to the collinear gluon PDF. We will denote the $\mathcal{F}_{\text{KMR}}^{(i)}$ TMDs as $\text{KMR}^{(i)}$.

Although it is common to use the LO KMR prescription with the NLO PDFs [82], one can apply the NLO version of the KMR framework. The NLO prescription, usually referred to as the MRW model [72], incorporates two DGLAP splittings in generating the k_T dependence of the distribution. The MRW formula differs from the KMR model not only by replacing the collinear PDFs, splitting functions, and α_s coupling with their NLO counterparts, but also by accounting for the dependence of the scale on the collinear splitting variable z . The MRW gluon TMD is given by [72]

$$k_T^2 \mathcal{F}(x, k_T^2, \mu^2) = \int_x^1 dz T_g(k, \mu) \frac{\alpha_s(k^2)}{2\pi} \theta\left(1 - \frac{k_T^2}{\mu^2} - z\right) \times \left[\tilde{P}_{gg}^{(1)}(z) \frac{x}{z} f_g\left(\frac{x}{z}, k^2\right) + \sum_q \tilde{P}_{gq}^{(1)}(z) \frac{x}{z} f_q\left(\frac{x}{z}, k^2\right) \right], \quad (5.9)$$

where now the scale is not simply the transverse momentum k_T , but rather the virtuality k of the emitted partons, given by $k^2 = k_T^2/(1-z)$. The upper integration bound $z < 1 - k_T^2/\mu^2$ in (5.9) results from the condition on virtuality $k^2 < \mu^2$. This condition also makes a crucial difference between the KMR and MRW prescriptions, as the KMR model k_T dependence can be extended above the factorization scale with the angular ordering, while the MRW model is nonzero only for $k_T < \mu$. Here, $\tilde{P}_{ij}^{(1)}(z)$ denotes the NLO DGLAP splitting kernels, for which we use the explicit expressions given in [84]. Following the MRW prescription [72], the soft-gluon contribution is treated using the subtraction procedure with an angular ordering constraint, and the resulting kernels are used accordingly in the Sudakov form factor.

We also consider the KMR model with gluon-driven DGLAP evolution, following a similar logic as for the JH models, where the JH2013 model was derived using purely gluonic evolution. A TMD obtained with the KMR procedure was considered by Kotikov and Lipatov in [73], where analytical solutions of the DGLAP evolution equations are used as input PDFs, with the relevant phenomenological parameters determined from a fit to DIS data for the proton structure function F_2 . For our analysis, we use the KL-3d-2025 grid [73], provided by the authors through the TMDlib library, and refer to this model as the KL model. Although the KL approach is general, this specific TMD was obtained with the DGLAP evolution dominated by gluons.

In the general comparison of TMD models, we will use the KMR TMDs (5.8), collectively referred to as KMR-based models. Then, we will separately discuss the differences between the KMR, MRW, and KL approaches.

D. The Weizsäcker-Williams model

The simple WW model proposed in [37] is given by the formula

$$\mathcal{F}_{\text{WW}}(x, k_T^2) = \frac{N_1}{k_0^2} (1-x)^7 x^{-\lambda b} \times \begin{cases} 1 & k_T^2 < k_0^2 \\ \left(\frac{k_0^2}{k_T^2}\right)^b & k_T^2 \geq k_0^2 \end{cases}, \quad (5.10)$$

where $N_1 = 0.6111$, $\lambda = 0.29$, $b = 1$, and $k_0 = 1$ GeV. The normalization constant N_1 is obtained by fitting the DY cross section with a global K factor for photon exchange to the data [9]. The transverse momentum cutoff k_0 is chosen as the scale that delimits the region where the confinement or parton coherence effects in a hadron become important.

To capture the collinear limit of TMD (3.2), we propose the modification of the Weizsäcker-Williams model (WW'), which uses the collinear gluon PDF. The formula for WW' TMD reads (for $b = 1$)

$$\mathcal{F}'_{\text{WW}}(x, k_T^2, \mu^2) = \frac{x f_g(x, \mu^2)}{k_0^2 [1 + \log(\frac{\mu^2}{k_0^2})]} \times \begin{cases} 1 & k_T^2 < k_0^2 \\ \frac{k_0^2}{k_T^2} & k_T^2 \geq k_0^2 \end{cases}. \quad (5.11)$$

This modification preserves the $1/k_T^2$ behavior of the original WW model and satisfies Eq. (3.2). Additionally, it introduces a double scale dependence present in the JH and KMR models.

Although we did not use them, the values of parameter b other than 1 have been explored before. For $b \neq 1$, the WW' model takes the form

$$\mathcal{F}'_{\text{WW}}(x, k_T^2, \mu^2) = \frac{(1-b)x f_g(x, \mu^2)}{\left(\frac{k_0^2}{\mu^2}\right)^b \mu^2 - b k_0^2} \times \begin{cases} 1 & k_T^2 < k_0^2 \\ \left(\frac{k_0^2}{k_T^2}\right)^b & k_T^2 \geq k_0^2 \end{cases}, \quad (5.12)$$

which reduces to (5.11) in the $b \rightarrow 1$ limit.

As in the case of the KMR model, the WW' model also does not describe well the data from Ref. [9]. Therefore, we consider modifications analogous to (5.8) for the WW' model,

$$\mathcal{F}_{\text{WW}}^{(1)}(x, k_T^2, \mu^2) = N_{\text{WW}}^{(1)} \mathcal{F}'_{\text{WW}}(x, k_T^2, \mu^2), \quad (5.13a)$$

$$\mathcal{F}_{\text{WW}}^{(2)}(x, k_T^2, \mu^2) = N_{\text{WW}}^{(2)} \mathcal{F}'_{\text{WW}}(x/2, k_T^2, \mu^2), \quad (5.13b)$$

$$\mathcal{F}_{\text{WW}}^{(3)}(x, k_T^2, \mu^2) = N_{\text{WW}}^{(3)} \mathcal{F}'_{\text{WW}}(x/4, k_T^2, \mu^2), \quad (5.13c)$$

where again $N_{\text{WW}}^{(i)}$ were fitted to the total DY cross section. We will denote the $\mathcal{F}_{\text{WW}}^{(i)}$ TMDs as $\text{WW}^{(i)}$.

As a collinear gluon PDF, we used the CT10n1o set from the LHAPDF library.

VI. RESULTS

To date, the best measurement of the Drell-Yan dilepton angular coefficients was performed by ATLAS collaboration [10], which is the main source of the data on which we base our analysis. We derived the structure functions within the k_T factorization formalism for the $q_{\text{val}}g^*$ and g^*g^* channels. We focused on the first three parity-conserving functions A_0 , A_1 , and A_2 , considering separately the Lam-Tung combination $A_{LT} = A_0 - A_2$, and two parity-violating functions A_3 and A_4 . The remaining three structure functions— A_5 , A_6 , and A_7 —vanish within the framework

of the considered model. The data show that these functions are indeed consistent with zero at current accuracy. In definition (3.19) we used the differential cross sections (4.3) and (4.5) fully integrated over all variables except the boson transverse momentum q_T , over which we take the average in the bins used in the experiment and in the Z^0 boson peak, i.e., at $M = M_Z = 91.1876$ GeV at the pp collision energy $\sqrt{S} = 8$ TeV. In our analysis, we considered four main gluon TMD models along with modifications of two of them. The gluon TMD models used are listed and briefly described in Sec. V. For a collinear valence quark PDF in (4.3) we used the CT10n1o set from the LHAPDF library. The phase space integration is performed using the VEGAS 6.2.1 package [85]. For a running strong coupling we used one-loop coupling (5.6) for the number of flavors $n_f = 5$, where the QCD scale has the value $\Lambda_5 = 98.61$ MeV that gives $\alpha_s(M_Z^2) = 0.12$. The factorization and renormalization scales are set to be the same and equal to the boson transverse mass $\mu_F = \mu_R = M_T$. To obtain the electroweak couplings, we need the Weinberg mixing angle θ_W for which we took the value $\sin^2 \theta_W = 0.23122$ [86].

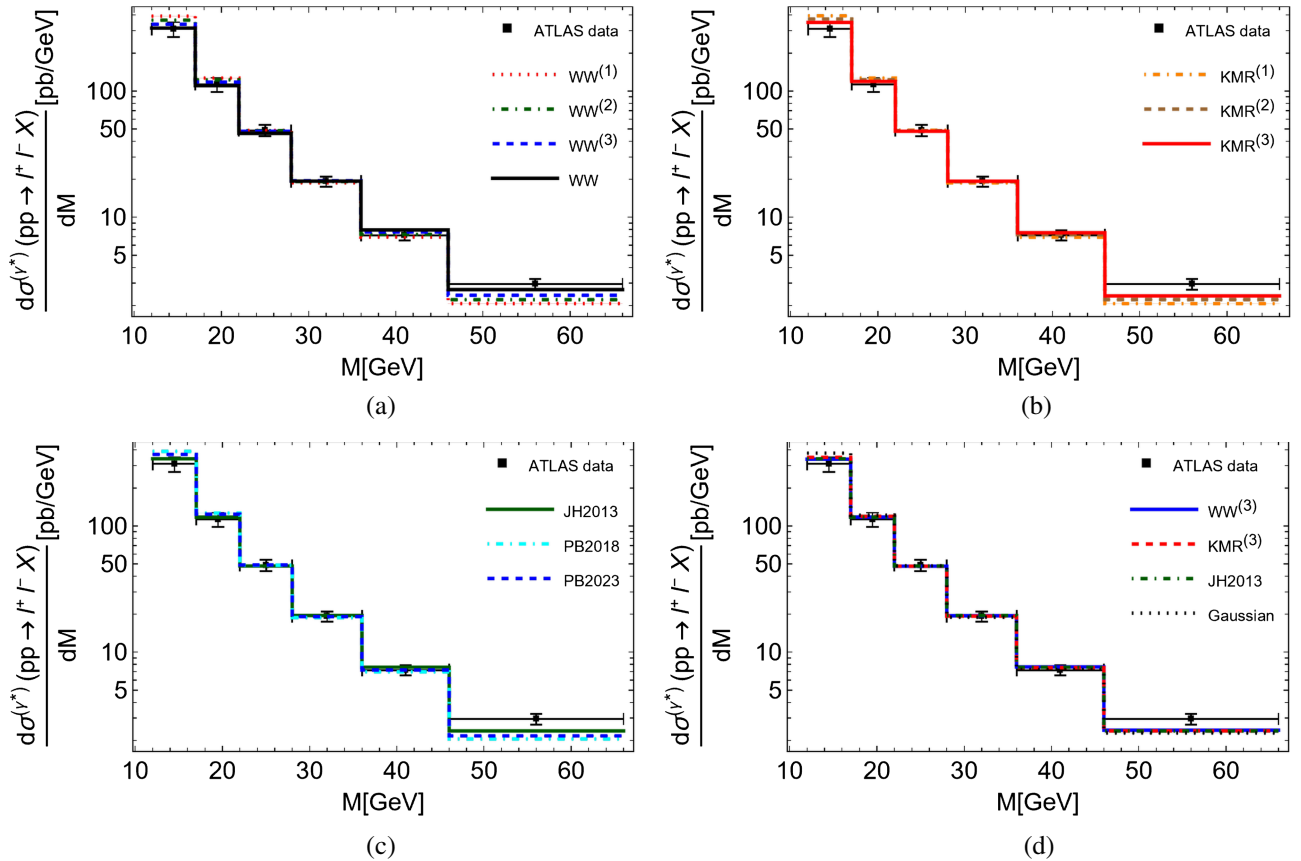


FIG. 2. The Drell-Yan total cross section for γ^* exchange in pp collision at the LHC at $\sqrt{S} = 7$ TeV. We compared the ATLAS data [9] to the cross sections calculated with different gluon TMD models: (a) the KMR-based TMD models, (b) the WW-based TMD models, (c) the CCFM-driven TMD models, (d) KMR⁽³⁾, WW⁽³⁾, JH2013, and Gaussian models as the models with the best overall data description (including the structure functions analysis) within each model family.

TABLE I. Normalization constants for different gluon TMD models.

Model	WW ⁽¹⁾	WW ⁽²⁾	WW ⁽³⁾	KMR ⁽¹⁾	KMR ⁽²⁾	KMR ⁽³⁾	JH2013	PB2018	PB2023	Gauss.
N_{TMD}	2.0848	1.3880	0.9484	2.6021	1.7619	1.2243	0.8728	2.2017	2.6453	0.4158

 TABLE II. The reduced χ^2 values for different TMD models.

Model	WW ⁽¹⁾	WW ⁽²⁾	WW ⁽³⁾	WW	KMR ⁽¹⁾	KMR ⁽²⁾	KMR ⁽³⁾	JH2013	PB2018	PB2023	Gauss.
χ^2	3.02	1.82	0.98	0.53	3.08	1.90	1.13	1.07	3.04	2.10	1.74

A. Normalization to the total Drell-Yan cross section

The basic versions of the gluon TMD models do not accurately describe the DY total cross section data given in Ref. [9], mostly because of the inaccurate overall normalization. Therefore, as a simple phenomenological adjustment, we normalized these functions to the data. In the case of the KMR model (5.5) and WW' model (5.11), the x dependence is based on the collinear PDFs. However, as mentioned in the previous section, different kinematics in the k_T factorization approach may result in a change in the shape of the distributions in the x variable. Thus, we also proposed versions of the discussed TMDs with the rescaled x variable. Ultimately, we consider the KMR-based TMD family given in (5.8), and the WW'-based TMD family given in (5.13), which together with the original WW TMD (5.10) will be referred to as WW-based TMDs.

We calculated the cross section $\frac{d\sigma^{(\gamma^*)}(pp \rightarrow l^+l^-X)}{dM}$ averaged over the dilepton mass bins, in the mass regions where the contribution from the Z^0 boson is negligible. The cross sections for the normalized TMDs are plotted in Fig. 2 and the normalization constants are given in Table I. The reduced χ^2 values, given in Table II, for this cross section were calculated according to the formula

$$\chi^2 = \frac{1}{N_{\text{pt}} - 1} \sum_{i=1}^{N_{\text{pt}}} \frac{1}{\sigma_{\text{data}}^2} \left[\left(\frac{d\sigma^{(\gamma^*)}}{dM} \right)_{\text{th}} - \left(\frac{d\sigma^{(\gamma^*)}}{dM} \right)_{\text{data}} \right]^2, \quad (6.1)$$

where $N_{\text{pt}} = 6$ and we fitted the overall normalization, and th/data correspond to the theoretical and experimental values, respectively.

In the calculations, we applied the K factor, which partially accounts for resummed higher-order QCD corrections. Use of this factor with an approximate form $K = \exp(\pi C_F \alpha_s(\mu_q^2)/2)$ with the optimal choice of the scale $\mu_q = (q_T M^2)^{1/3}$ in the DY cross section calculation was motivated in [87,88]. In our analysis, we simplified the calculations by assuming an average constant value of $K = 1.5$ instead of the full form depending on q_T and M , as was done in Ref. [37].

The behavior of the KMR-based and WW'-based TMDs is very similar. Both show a too steep decrease with increasing M , resulting in a slight discrepancy at the ends of the mass spectrum. This problem becomes less significant with stronger x rescaling. A similar discrepancy occurs for the JH models. Table II indicates that the WW model comes closest to describing the DY total cross section data, while models with the best structure function data description, i.e., WW⁽³⁾, KMR⁽³⁾, and JH2013, have similar cross section reduced χ^2 values close to 1.

The comparison of the KMR, MRW, and KL models, see Fig. 3, shows that the KMR and MRW prescriptions provide relatively similar descriptions of the DY cross section. As can be seen in Table III, the normalization factors obtained for these models are almost equal. The

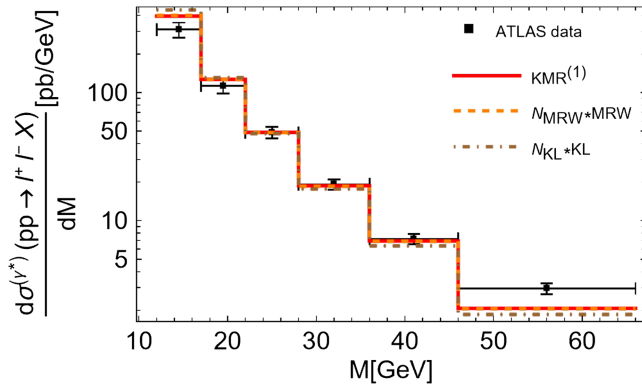


FIG. 3. The Drell-Yan total cross section for γ^* exchange in pp collision at the LHC at $\sqrt{S} = 7$ TeV. Comparison of the ATLAS data [9] to the cross sections calculated with KMR⁽¹⁾, MRW, and KL gluon TMD models.

TABLE III. Normalization constants for KMR, MRW, and KL gluon TMD models.

Model	KMR ⁽¹⁾	MRW	KL
N_{TMD}	2.6021	2.6161	1.3379

 TABLE IV. The reduced χ^2 values for KMR, MRW, and KL TMD models.

Model	KMR ⁽¹⁾	MRW	KL
χ^2	3.08	3.26	5.94

cross section reduced χ^2 values, given in Table IV, are relatively close as well, although the KMR⁽¹⁾ model gives a slightly better description. The KL model provides a better overall magnitude of the gluon, which is reflected in the normalization constant close to 1. However, the DY cross section calculated with this model shows too steep a fall-off compared to data. Thus, the resulting reduced χ^2 is much larger.

B. The Drell-Yan structure functions

In our study, we compared the KMR-based models with one another in Fig. 4, the KMR, MRW, and KL models in Fig. 5, the WW-based models in Fig. 6, and the JH models in Fig. 7. Then, we compared the representatives with the minimal reduced χ^2 in each of the above-mentioned model families with one another and with the Gaussian model. This comparison is shown in Fig. 8.

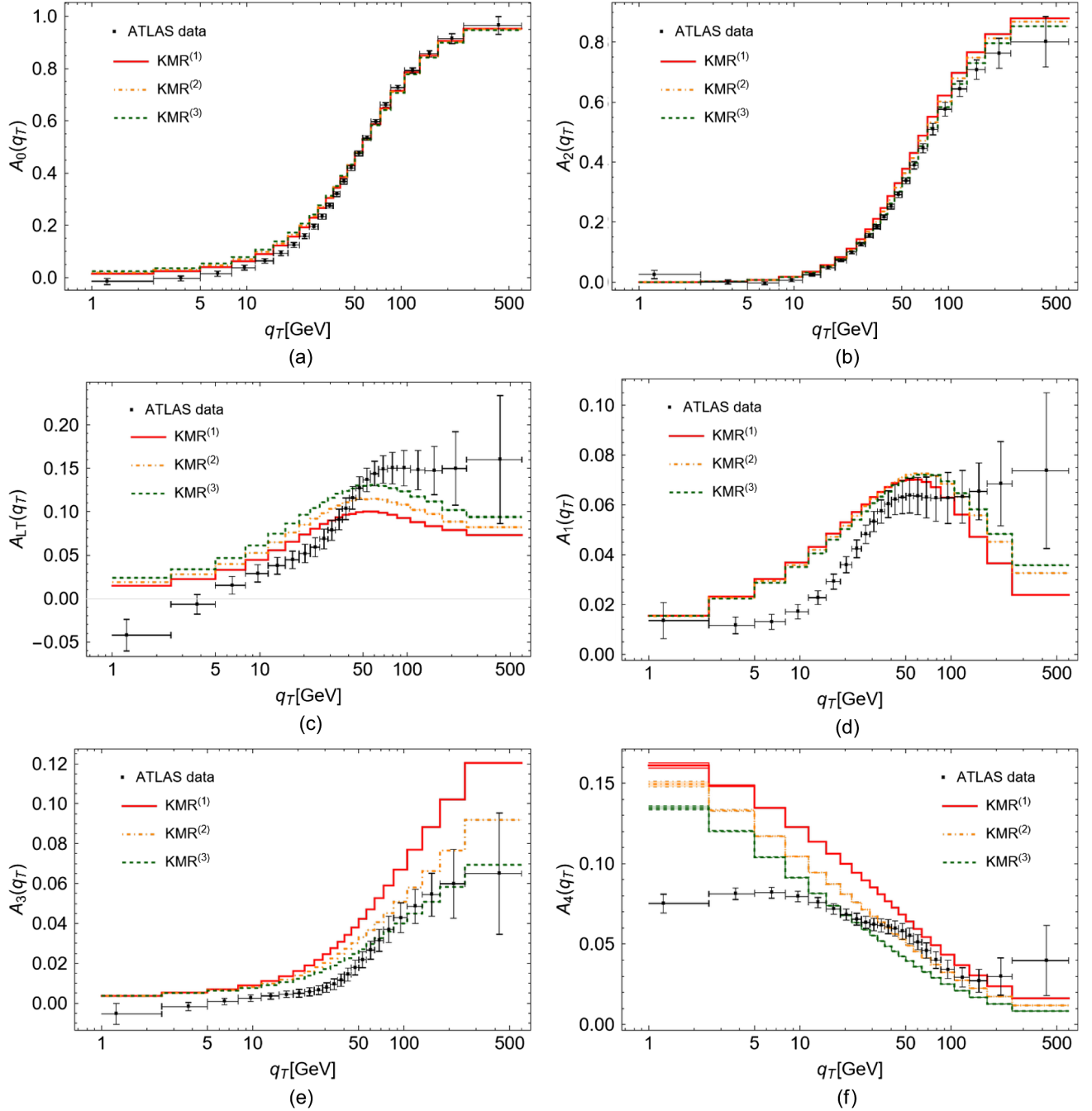


FIG. 4. Comparison of the structure functions: (a) A_0 , (b) A_2 , (c) A_{LT} , (d) A_1 , (e) A_3 , (f) A_4 , calculated with different modifications of the KMR TMD model with the ATLAS data [10]. The bands around the curves represent the estimated Monte Carlo integration uncertainties.

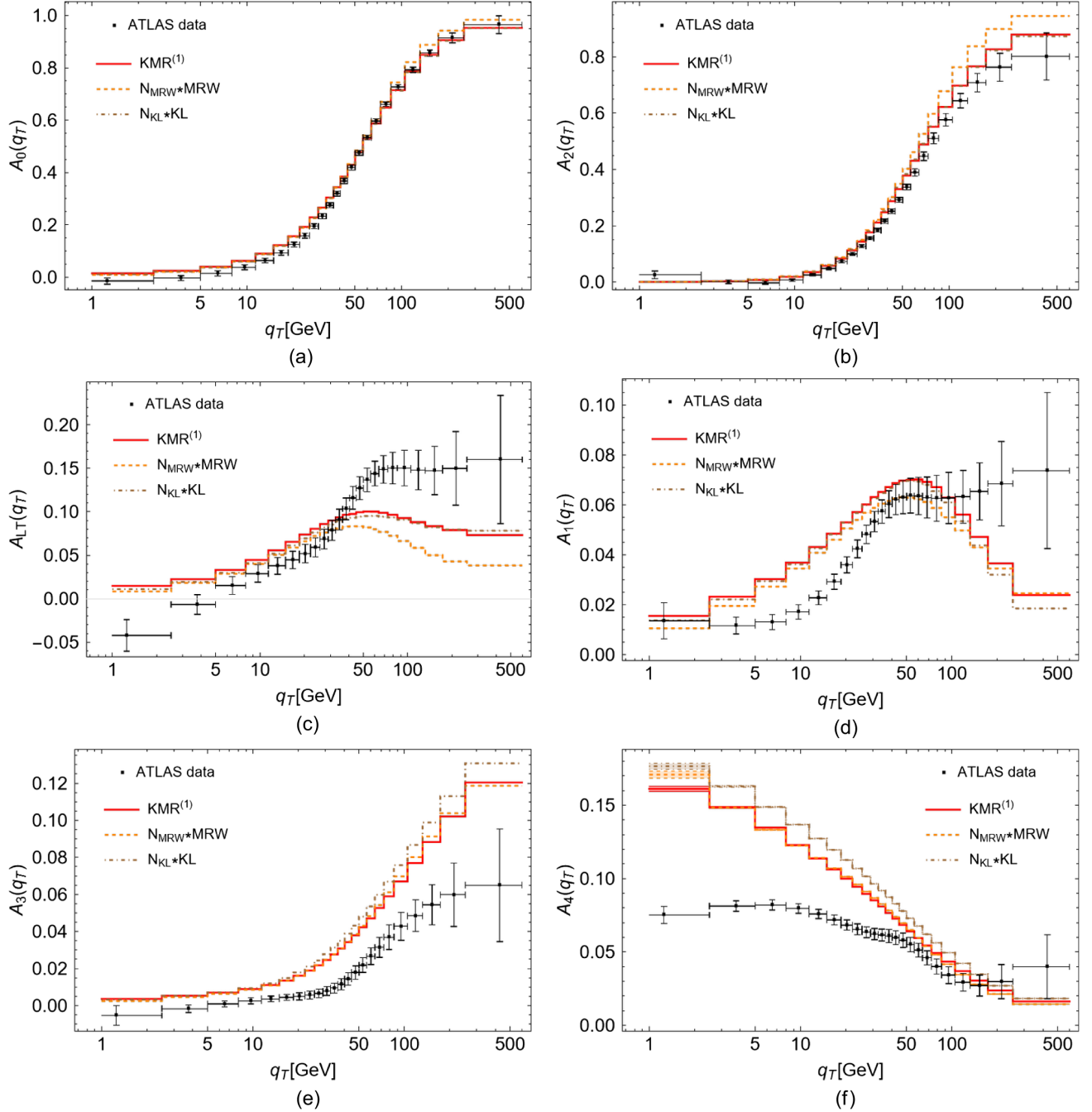


FIG. 5. Comparison of the structure functions: (a) A_0 , (b) A_2 , (c) A_{LT} , (d) A_1 , (e) A_3 , (f) A_4 , calculated using the KMR⁽¹⁾, MRW, and KL TMD models with the ATLAS data [10]. The bands around the curves represent the estimated Monte Carlo integration uncertainties.

The A_0 function is described very well by all of the models, with minor variations across TMD model families. The most problematic region for all the models is $q_T < 50$ GeV, where we can see a small gap between data and predictions. Although it is barely visible in the plots, this region largely contributes to the reduced χ^2 , as the errors are relatively small and points are quite densely distributed there. The x rescaling for WW- and KMR-based models increases this effect.

The A_2 function, on the other hand, provides a much clearer distinction between the model families. Here, the WW-based models are closest to the data at large q_T , and there is no significant difference between them for this observable. All the other models overestimate the data in that region. The KMR-based family overestimation, however, does not take the values outside the error bars very far. Here, we see significant improvement as we rescale the x dependence more. In the region $5 \text{ GeV} < q_T < 40 \text{ GeV}$,

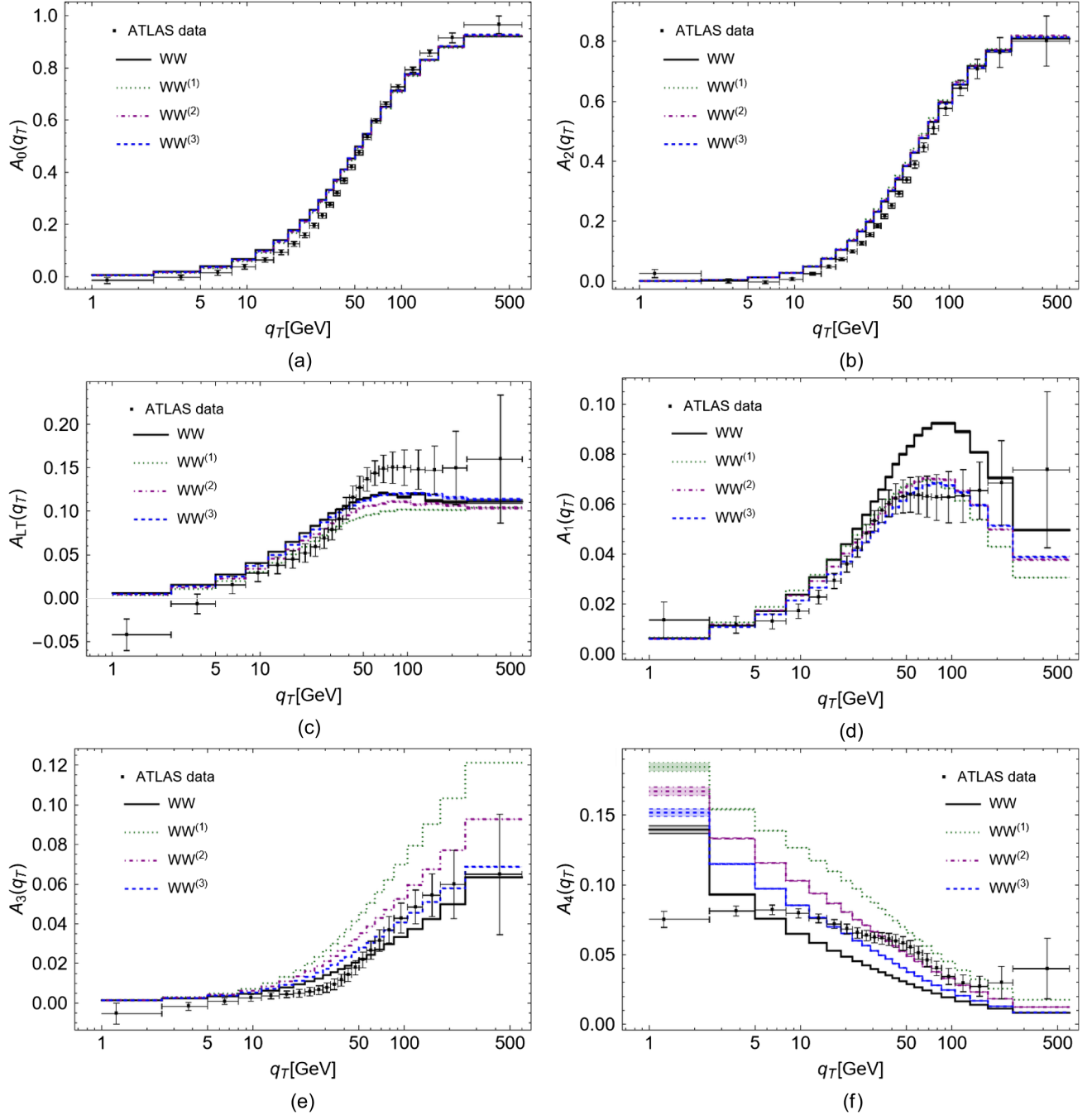


FIG. 6. Comparison of the structure functions: (a) A_0 , (b) A_2 , (c) A_{LT} , (d) A_1 , (e) A_3 , (f) A_4 , calculated with different modifications of the WW TMD model with the ATLAS data [10]. The bands around the curves represent the estimated Monte Carlo integration uncertainties.

one notices a small gap between the data and theory for almost all models except the KMR-based models. This region accounts for the main contribution to the reduced χ^2 for the WW-based and Gaussian models, similar to the case of A_0 , because the gap is more significant than for the other models, the errors are relatively small, and the points are quite densely distributed.

The features of the A_0 and A_2 described above manifest themselves in the Lam-Tung combination A_{LT} . In the

problematic region of $q_T < 50$ GeV, the A_{LT} is overestimated for all the models except for the Gaussian TMD. This effect is the most visible for the KMR-based models, where the deviation from data is present only for A_0 , while for the WW-based models, the effects in both functions cancel out. The deviation from data for large q_T is clearly visible for the JH models and especially for the Gaussian model, and it originates from a large discrepancy in A_2 at high q_T values.

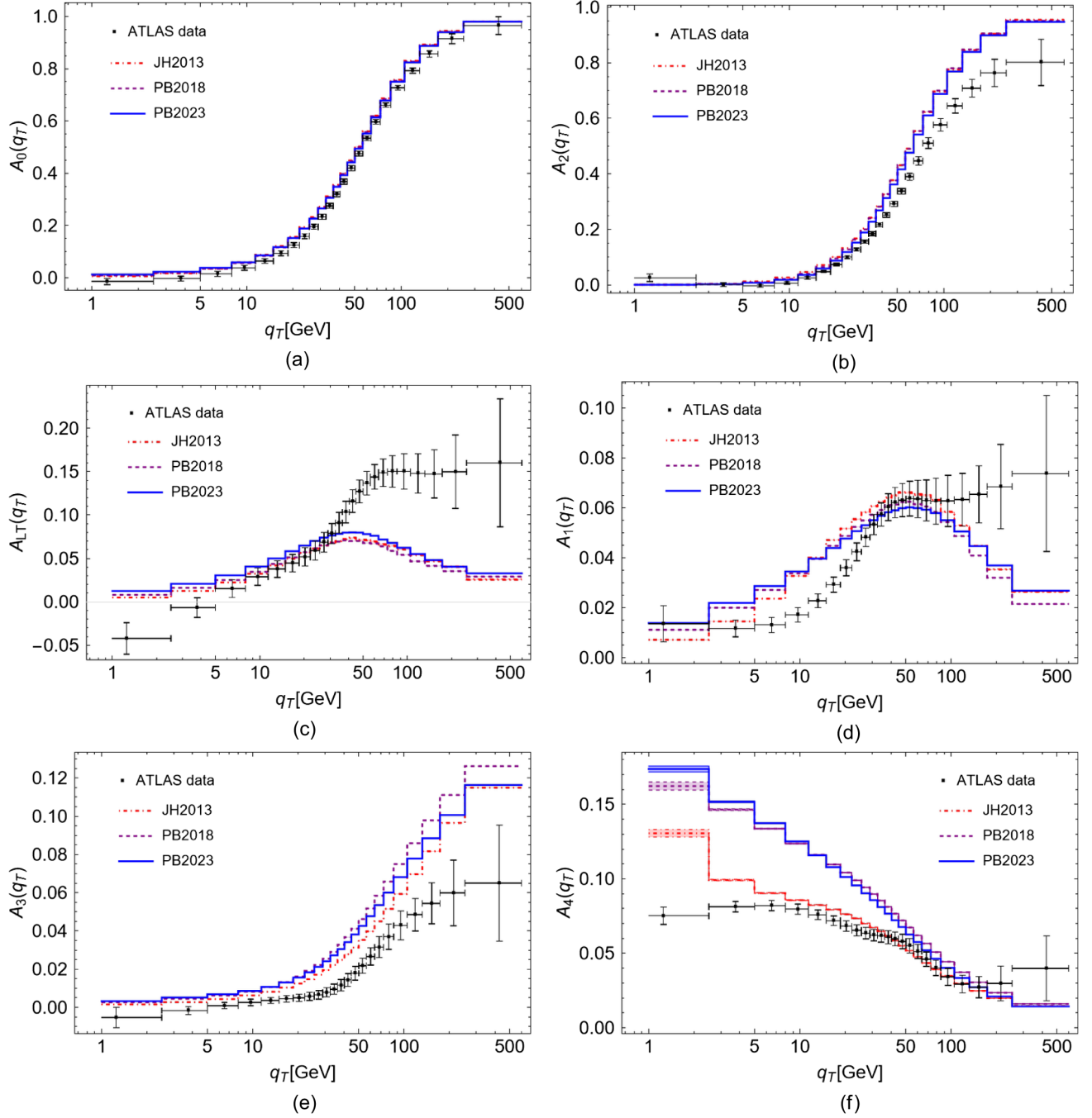


FIG. 7. Comparison of the structure functions: (a) A_0 , (b) A_2 , (c) A_{LT} , (d) A_1 , (e) A_3 , (f) A_4 , calculated with the JH TMD models with the ATLAS data [10]. The bands around the curves represent the estimated Monte Carlo integration uncertainties.

In the case of function A_1 , all models show a clear decline at very large q_T , contrary to the data trend. The KMR and JH models overestimate the data significantly for small transverse momenta $q_T < 30$ GeV, which constitutes the dominant contribution to the reduced χ^2 . On the other hand, they quite accurately describe the position of the local maximum of this function and approximately its value. The WW'-based models generally provide the best description of this observable with significant improvement with

stronger x rescaling. A distinct behavior can be noticed for the WW and Gaussian models, namely, they both highly overestimate the A_1 local maximum. The difference between WW- and WW'-based models indicates a strong effect of the TMD x dependence for this structure function.

The A_3 function is also overestimated at large q_T for the JH models and for the WW' and KMR models without x rescaling. However, the region where the deviation from the data is strongest is roughly $10 \text{ GeV} < q_T < 70 \text{ GeV}$. The

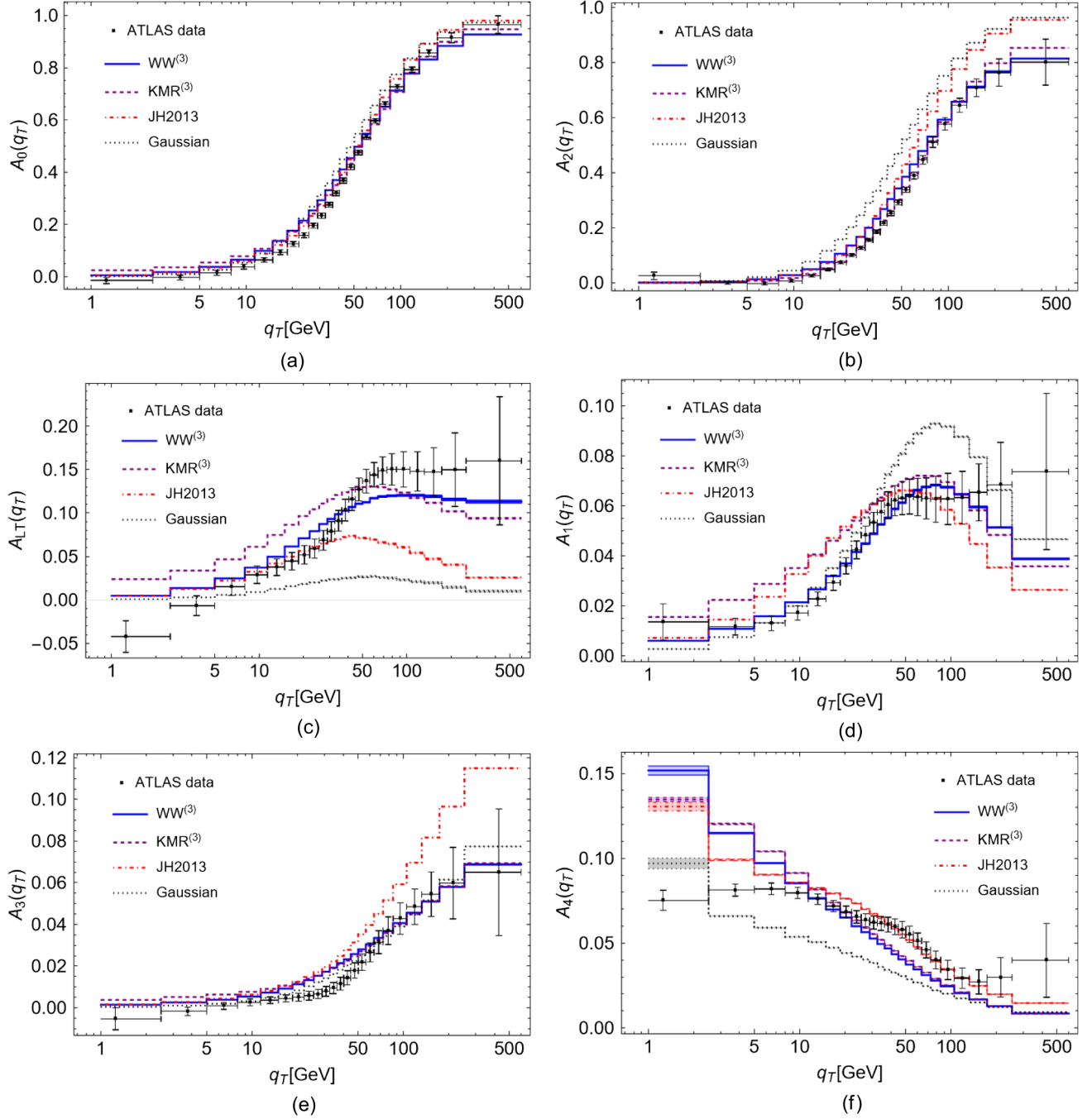


FIG. 8. Comparison of the structure functions: (a) A_0 , (b) A_2 , (c) A_{LT} , (d) A_1 , (e) A_3 , (f) A_4 , calculated with different TMD models with ATLAS data. For this purpose, we used KMR⁽³⁾, WW⁽³⁾, the JH2013 model, and the Gaussian model as representatives of each TMD class that provides the best description of the data within each class. The bands around the curves represent the estimated Monte Carlo integration uncertainties.

TABLE V. The ratios of reduced χ^2 values for each TMD to the minimal χ^2 value, $\chi^2_{\min}$.

Model	WW	WW ⁽¹⁾	WW ⁽²⁾	WW ⁽³⁾	KMR ⁽¹⁾	KMR ⁽²⁾	KMR ⁽³⁾	JH2013	PB2018	PB2023	Gauss.
$\chi^2/\chi^2_{\min}$	1.54	1.80	1.14	1.00	1.74	1.21	1.11	1.16	1.91	1.71	2.22

TABLE VI. The ratios of reduced χ^2 values for KMR, MRW, and KL TMDs to the minimal χ^2 value, $\chi_{\min}^2$.

Model	KMR ⁽¹⁾	MRW	KL
$\chi^2/\chi_{\min}^2$	1.74	1.74	2.15

experimental values slowly increase up to transverse momentum around $q_T \approx 35$ GeV, where the values start to increase more rapidly. All the predictions show, however, a much more gentle and monotonic increase in this region. The x rescaling has a big impact on this observable.

We also notice a large spread of predictions for the A_4 function. Similarly to the other parity-violating function, A_3 , the x rescaling leads to large changes in the description for both WW and KMR model families. In all the models, we observe a too steep decline, especially for small values of q_T , while the data points show a plateau. This feature is particularly evident in the WW model family, where the convexity of the curve appears to be opposite to that observed in the data and predictions of the other models.

The KMR, MRW, and KL models give considerably close descriptions of the DY structure functions. One prominent difference is the more pronounced overestimation of the A_2 function by the MRW prediction, which is reflected in a faster decrease of A_{LT} at high q_T . A minor detail is that, for the A_1 function, the MRW result better reproduces the data in the region of the function's local maximum. One can also notice that the KL results for parity-violating structure functions are slightly above those for the two other models.

To quantify these observations, we also evaluated a global reduced χ^2 for each TMD model. The χ^2 per number of degrees of freedom is computed with the formula

$$\chi_n^2 = \frac{1}{N_{\text{pt}}} \sum_{i=1}^{N_{\text{pt}}} \frac{(A_{n,i}^{\text{th}} - A_{n,i}^{\text{data}})^2}{(\sigma_{n,i}^{\text{data}})^2 + (\sigma_{n,i}^{\text{th}})^2}, \quad (6.2)$$

where $N_{\text{pt}} = 23$ is the number of points, $A_{n,i}^{\text{th/data}}$ is a theoretical/experimental value of the function A_n at the i th point, and $\sigma_{n,i}^{\text{th/data}}$ are the theoretical/experimental errors. We assumed a theoretical uncertainty of the order

$\alpha_s(M_Z) \simeq 0.12$, that is, a relative error of about 12%,

$$\sigma_{n,i}^{\text{th}} = 0.12 \cdot A_{n,i}^{\text{th}}. \quad (6.3)$$

The total reduced χ^2 of the results for each of the gluon TMDs is obtained by averaging the contributions of all nonvanishing structure functions,

$$\chi^2 = \frac{1}{5} \sum_{n=0}^4 \chi_n^2. \quad (6.4)$$

In Table V we show the ratios of the χ^2 values for each model to the minimal one, which is found in the WW⁽³⁾ model and is equal to $\chi_{\min}^2 := \min\{\chi^2\} = 2.50$. The comparison of the $\chi^2/\chi_{\min}^2$ ratios for KMR, MRW, and KL models is presented in Table VI.

As anticipated from the plots, the dominant contribution to the discrepancies comes from the parity-violating structure functions, which results in the wide range of χ^2 values listed in Table VII.

We considered the Lam-Tung combination separately, as it is not an independent structure function. In this case, χ^2 is calculated by an analogous formula to (6.2). The results are given in Tables IX and X, where the minimum χ_{LT}^2 value is found again for the WW⁽³⁾ model, and is equal to $\chi_{LT,\min}^2 := \min\{\chi_{LT}^2\} = 1.43$.

 TABLE VIII. The ratios of χ^2 per degree of freedom values for each structure function, χ_n^2 , for KMR, MRW, and KL TMD models to the minimal global reduced χ^2 value, $\chi_{\min}^2$.

Model	KMR ⁽¹⁾	MRW	KL
$\chi_0^2/\chi_{\min}^2$	0.67	0.53	0.56
$\chi_1^2/\chi_{\min}^2$	1.38	1.10	1.36
$\chi_2^2/\chi_{\min}^2$	0.50	0.86	0.58
$\chi_3^2/\chi_{\min}^2$	4.34	4.34	5.45
$\chi_4^2/\chi_{\min}^2$	1.80	1.87	2.83

 TABLE VII. The ratios of χ^2 per degree of freedom values for each structure function, χ_n^2 , for each TMD model to the minimal global reduced χ^2 value, $\chi_{\min}^2$.

Model	WW	WW ⁽¹⁾	WW ⁽²⁾	WW ⁽³⁾	KMR ⁽¹⁾	KMR ⁽²⁾	KMR ⁽³⁾	JH2013	PB2018	PB2023	Gauss.
$\chi_0^2/\chi_{\min}^2$	0.94	0.60	0.72	0.84	0.94	0.67	1.26	0.52	0.48	0.57	0.82
$\chi_1^2/\chi_{\min}^2$	0.76	0.38	0.22	0.15	1.38	1.24	1.10	0.98	1.06	1.05	0.70
$\chi_2^2/\chi_{\min}^2$	1.25	1.46	1.36	1.29	0.50	0.36	0.27	1.56	1.25	0.88	4.35
$\chi_3^2/\chi_{\min}^2$	0.79	4.47	2.48	1.34	4.34	2.66	1.78	2.40	4.72	4.21	0.55
$\chi_4^2/\chi_{\min}^2$	3.95	2.09	0.89	1.39	1.80	0.86	1.15	0.35	2.04	1.86	4.70

TABLE IX. The ratios of the Lam-Tung combination χ^2 values for each TMD to the minimal χ^2_{LT} value.

Model	WW	WW ⁽¹⁾	WW ⁽²⁾	WW ⁽³⁾	KMR ⁽¹⁾	KMR ⁽²⁾	KMR ⁽³⁾	JH2013	PB2018	PB2023	Gauss.
$\chi^2_{LT}/\chi^2_{LT,min}$	1.18	1.58	1.17	1.00	2.39	2.32	2.80	5.14	5.83	4.73	17.67

TABLE X. The ratios of the Lam-Tung combination χ^2 values for KMR, MRW, and KL TMDs to the minimal χ^2_{LT} value.

Model	KMR ⁽¹⁾	MRW	KL
$\chi^2_{LT}/\chi^2_{LT,min}$	2.39	4.16	2.38

VII. DISCUSSION

After a brief look at the plots in Fig. 8, one recognizes that the structure functions are sensitive to the chosen TMD model over the entire spectrum of q_T . Although the main features of each structure function, such as the maximum position and monotonicity, are roughly preserved for each TMD, there is quite a large variation in magnitudes of the

theoretical predictions, especially for functions A_3 and A_4 . This variation is mostly due to the difference in the contribution of the $q_{val}g^*$ channel to the cross section, as only this channel contributes to these functions. This contribution is quantified by the ratio

$$R_{q_{val}g^*} := \left(\frac{d\sigma_{(qg^*+g^*q)}}{dM dq_T} \right) / \left(\frac{d\sigma}{dM dq_T} \right), \quad (7.1)$$

which is plotted for all the TMD models in Fig. 9. The different contributions of this channel to the cross section in different models' predictions arise from the size of the gluon TMD, which is especially evident when comparing the KMR- and WW-based models. As the gluon x decreases, TMD is enhanced over a wider range of phase

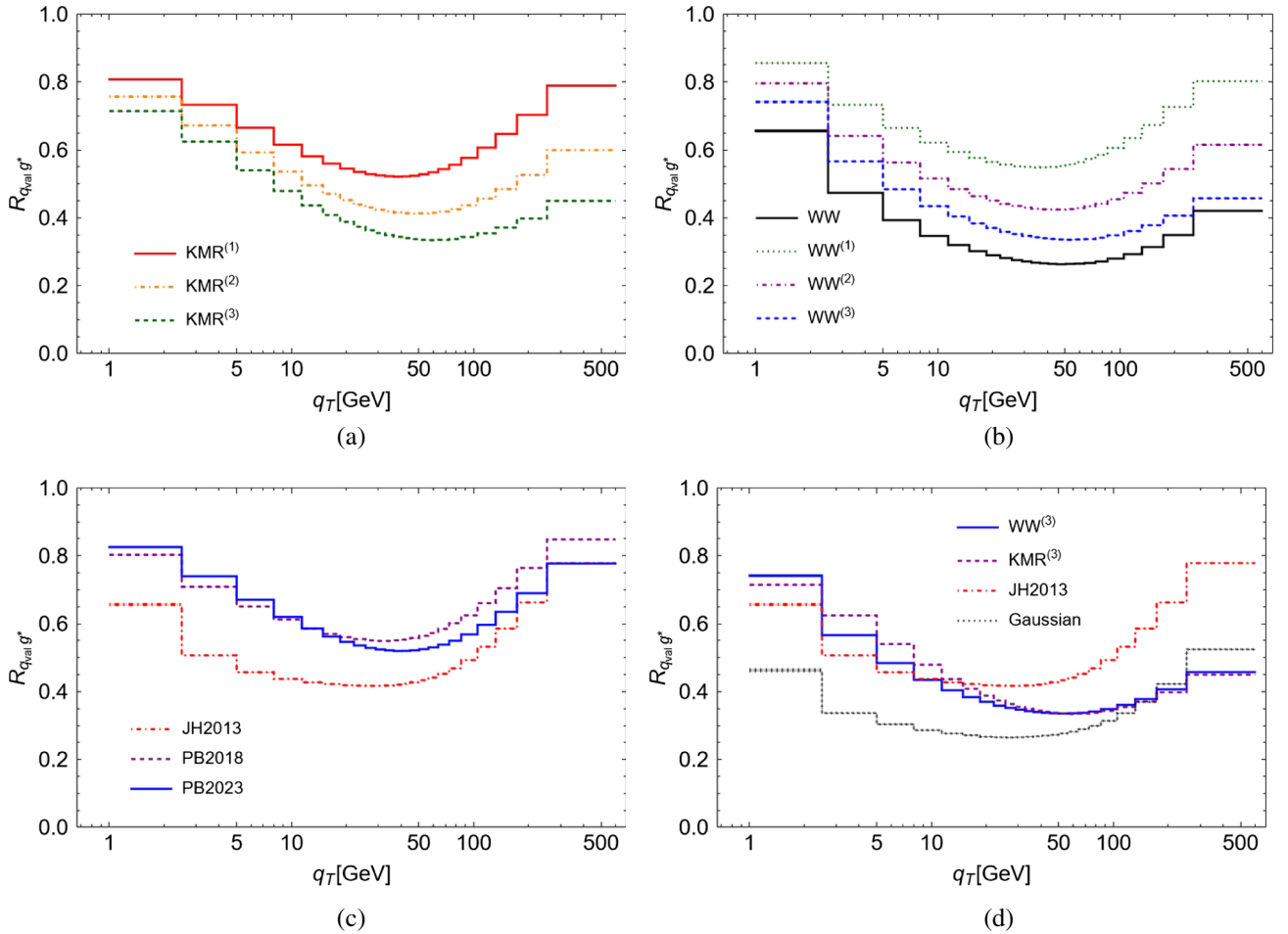


FIG. 9. The ratios of the $q_{val}g^*$ channel contributions to the total cross section $R_{q_{val}g^*}$ obtained for: (a) the KMR-based TMD models, (b) the WW-based TMD models, (c) the JH TMD models, (d) KMR⁽³⁾, WW⁽³⁾, JH2013, and Gaussian models.

space, and consequently, the contribution of the g^*g^* channel to the cross section is larger.

Another effect that occurs because of the change in the contribution of the $q_{\text{val}}g^*$ channel to the total differential cross section is the change of the shape of the function A_4 and slightly of the function A_3 , and the shift of the local maximum of the function A_1 . The $q_{\text{val}}g^*$ channel contribution is dominant at $q_T = 1$ GeV for all considered TMD models except the Gaussian, and decreases noticeably until it reaches the minimum somewhere in the region between 20 GeV and 70 GeV and then slowly grows. If one considers only $d\sigma_{(qg^*)}$ in definition (3.19) for the structure functions A_3 and A_4 , the shapes are similar to the shapes visible from the data (apart from the magnitude). However, when one includes the g^*g^* channel, which does not contribute to these functions but whose contribution to the cross section changes significantly with q_T , the shape of the discussed functions is altered. Similarly, the shift of the A_1 local maximum is related to the contribution of this channel. For the exclusive use of $d\sigma_{(qg^*)}$ in (3.19) the A_1 maximum tends to the left, while for the exclusive use of $d\sigma_{(g^*g^*)}$ in (3.19) the A_1 maximum tends to the right, and the total result is somewhere in between as a weighted average. Therefore, if the g^*g^* channel becomes more significant, the discussed maximum position tends to the higher values of q_T .

Table V shows that the model that describes the data best is the WW⁽³⁾ model. From Table VII one concludes that the advantage of WW⁽³⁾ is due to the rather uniform, relatively good description of all functions and the exceptionally good description of A_1 . As it can be seen from Table V, the models KMR⁽³⁾, WW⁽²⁾, JH2013, and KMR⁽²⁾ give quite close overall results, which suggests that the gluon contribution should be indeed dominant, as these models are the ones with the most dominant g^*g^* channel within each TMD class. The noticeable differences in the description of the structure functions within the WW and KMR families, as well as the differences between the WW'-based models and the WW model, highlight the sensitivity to the x dependence in the k_T factorization approach and its distinction from the collinear approach.

Tables VI and VIII show that results for KMR, MRW, and KL models behave similarly. A worse description of the A_2 function and, as a consequence, A_{LT} by the MRW model is caused by the strong cutoff on its k_T dependence, as these observables are known to be especially sensitive to the partons' transverse momenta [12]. Stronger overestimation of the KL model results for the A_3 and A_4 functions has its source in a higher $R_{q_{\text{val}}g^*}$ ratio, see Fig. 10, as discussed above.

An important note is that the ATLAS data used for comparison with the predictions are much denser in the central part of the q_T spectrum than at its edges. Additionally, the high- q_T region is subject to greater uncertainties. Therefore, the middle q_T region is weighted more in the χ^2 test. This feature is described in Sec. VI B

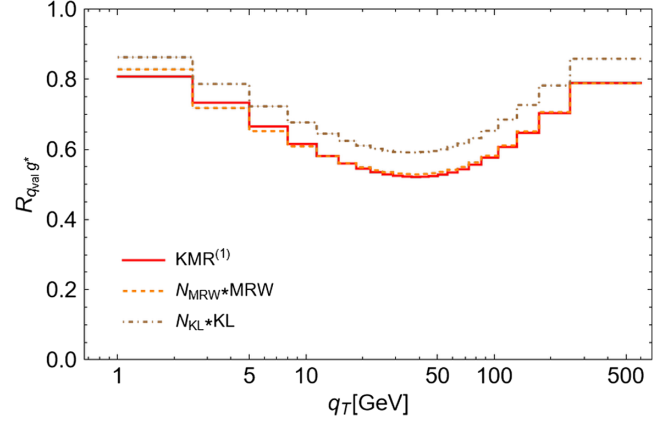


FIG. 10. Ratios of the $q_{\text{val}}g^*$ channel contribution to the total cross section for the KMR, MRW, and KL models.

and plays a role mostly for the A_0 , A_2 , and A_3 structure functions.

The behavior of functions A_3 and A_4 is not fully understood within our approach, and it requires further investigation. One possible solution may be the inclusion of higher-order corrections for the channel $q_{\text{val}}g^*$. In our model, this channel was considered to be subleading due to the dominant contribution of gluons. However, numerical calculations of the contribution of this channel to the full differential cross section clearly show that this assumption is not well fulfilled by some of the TMD models, as can be seen in Fig. 9.

It should also be emphasized that the applied high-energy factorization formalism is well suited for $q_T \sim M$, so it is justified for $q_T \lesssim 100$ GeV, and its use for higher values of q_T can be considered a slight stretching of the formalism. Our description should work better also at higher energies and more central rapidity regions, which gives hope for a good description for future LHC measurements at $\sqrt{S} = 13$ TeV and higher.

VIII. CONCLUSIONS

The DY structure functions are good probes for testing gluon TMD models. In our research, we found clear differences in the descriptions of DY structure functions both between gluon TMD model families and within individual families. The differences in the description are quantified by the reduced χ^2 , which can be read from Table V. They range from 2.5 to 5.56 and are approximately evenly distributed, indicating noticeable differences between the predictions obtained with different TMD models. The description of the individual structure functions, summarized in Table VII, shows even greater variation between the considered models. For example, we notice an exceptionally good description of the function A_1 by the WW'-based models and a comparably good description of the function A_2 by the KMR-based models. On the other hand, function A_3 is best described by TMD

models, for which channel g^*g^* makes the largest contribution in each model family. The situation is slightly different for function A_4 , but it also distinguishes between individual models within a given model class rather than between the model classes themselves.

The best overall description of the DY structure function is given by the WW⁽³⁾ model within our formalism, which also provides the best description of the Lam-Tung combination. This result indicates that the simple WW-like gluon TMD shape in k_T is a good approximation and can be taken as a leading behavior in k_T in future gluon TMD fits. The differences within the WW-based models and also within the KMR-based models show that the TMD shape in the x variable plays an important role, and that the difference from the collinear PDF x dependence needs to be taken into account in future studies of the gluon TMD model.

Within the CCFM-driven models, the best overall description is given by the oldest model—JH2013. The difference in the structure function description was additionally reduced by the TMD normalization procedure. The latter models incorporate in the integral kernel a coupling to the full flavor structure that includes sea quarks, which reduces the size of the gluon TMD compared to evolution driven solely by gluons. The procedure used for the JH2013 model might be better suited for the formalism used in our analysis, as in our approach, the sea quarks come exclusively from initial off-shell gluons, i.e., we do not consider processes with initial off-shell (anti)quarks. However, we do not observe a similar effect in the case of the KMR approach. Although the KL model results from the gluon-driven evolution, it does not provide an improvement in the description of the DY structure functions and the DY cross section compared to the KMR model, when the TMDs are normalized to the total DY cross section. It is worth noting, however, that the normalization factor for the KL TMD is closer to unity. Therefore, a comparison of the original, unnormalized TMDs would be expected to yield a more pronounced difference in favor of the KL model. We also conclude that there is no significant difference in the description of the DY structure functions between the KMR and MRW prescriptions.

Overall, our results demonstrate that the DY structure functions provide insightful probes of proton transverse momentum internal dynamics. In particular, the results obtained with different gluon TMD models indicate that both the k_T and longitudinal momentum dependence play crucial roles in obtaining results consistent with the experimental measurements of the DY structure functions.

In future work, the above-described formalism can be used to perform a dedicated fit of a gluon TMD to the structure functions data. Another direction for future research could be calculations of NLO corrections and

inclusion of additional parton channels with off-shell initial quarks. Such advancements should improve the accuracy of the k_T factorization approach used in this work and are expected to provide further constraints on the gluon TMD. Further investigation of the DY structure functions offers, therefore, a promising direction for advancing our understanding of the internal structure of hadrons.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [9,10].

APPENDIX A: THE AMPLITUDES

1. The amplitudes for $q_{\text{val}}g^* \rightarrow qV^*$

The process studied in this section is $q_{\text{val}}(p_1)g^*(k) \rightarrow q(p_2)V^*(q)$. We will consider massive boosted on-shell quarks even though, for all the structure function calculations, quarks were assumed to be massless. To restore the amplitudes used above, one simply has to take the limits $m_1, m_2 \rightarrow 0$, and $\mathbf{p}_{1T} \rightarrow \mathbf{0}$. We will apply the techniques and notation presented in [89]. The Sudakov decomposition of the initial quark momentum p_1 and the gluon momentum k is

$$p_1 = x_q P_1 + \frac{m_1^2 + \mathbf{p}_{1T}^2}{x_q S} P_2 + p_{1T}, \quad k = x_g P_2 + k_T, \quad (\text{A1})$$

where $p_{1T}^2 = -\mathbf{p}_{1T}^2$, then $p_1^2 = m_1^2$ and $k^2 = -\mathbf{k}_T^2$. The outgoing quark momentum is fixed by the momentum conservation as $p_2 = p_1 + k - q$ with the on-shell constraint $p_2^2 = m_2^2$. We are using the high-energy gluon polarization approximation defined in (4.2).

At the tree level, the full contribution to scattering amplitudes comes from two diagrams given by the expressions

$$\begin{aligned}\mathcal{A}_{1,\sigma_1\sigma_2}^{a,\mu} &= \frac{-ig}{v_1^2 - m_2^2} T_{ij}^a \bar{u}_{\sigma_2}(p_2) \hat{P}_2(\hat{v}_1 + m_2) \Gamma_V^\mu u_{\sigma_1}(p_1) =: -ig T_{ij}^a A_{1,\sigma_1\sigma_2}^\mu, \\ \mathcal{A}_{2,\sigma_1\sigma_2}^{a,\mu} &= \frac{-ig}{v_2^2 - m_1^2} T_{ij}^a \bar{u}_{\sigma_2}(p_2) \Gamma_V^\mu(\hat{v}_2 + m_1) \hat{P}_2 u_{\sigma_1}(p_1) =: -ig T_{ij}^a A_{2,\sigma_1\sigma_2}^\mu,\end{aligned}\quad (\text{A2})$$

where g is the strong coupling constant, T^a are generators of the color $SU(N)$ group in the fundamental representation, $u_\sigma(p)$ denotes the Dirac bispinor for fermion with spin σ and momentum p , and we use the notation $\hat{p} := p_\mu \gamma^\mu$ and define the momenta v_1 and v_2 as

$$v_1 = p_2 - k, \quad v_2 = p_1 + k.$$

The scattering amplitude for the process is given by the sum of the above expressions,

$$\begin{aligned}\mathcal{A}_{\sigma_1\sigma_2}^{a,\mu} &= \mathcal{A}_{1,\sigma_1\sigma_2}^{a,\mu} + \mathcal{A}_{2,\sigma_1\sigma_2}^{a,\mu} \\ &= -ig T_{ij}^a (A_{1,\sigma_1\sigma_2}^\mu + A_{2,\sigma_1\sigma_2}^\mu) =: -ig T_{ij}^a A_{\sigma_1\sigma_2}^\mu.\end{aligned}\quad (\text{A3})$$

We can further decompose each amplitude into right-handed R_n^μ and left-handed L_n^μ parts,

$$A_{n,\sigma_1\sigma_2}^\mu = (v_q^V + a_q^V) R_{n,\sigma_1\sigma_2}^\mu + (v_q^V - a_q^V) L_{n,\sigma_1\sigma_2}^\mu, \quad (\text{A4})$$

where in R_n^μ we replace Γ_V^μ by $(1 + \gamma_5)\gamma^\mu/2$ and in L_n^μ we replace Γ_V^μ by $(1 - \gamma_5)\gamma^\mu/2$.

Then, the amplitude squared for the qg^* channel is given by

$$\begin{aligned}\mathcal{M}_{(qg^*)}^{\mu\nu} &= \frac{1}{2N(N^2 - 1)} \sum_{\sigma_1, \sigma_2, a, i, j} \mathcal{A}_{\sigma_1\sigma_2}^{a,\mu} \bar{\mathcal{A}}_{\sigma_1\sigma_2}^{a,\nu} \\ &= \frac{g^2}{4N} \sum_{\sigma_1, \sigma_2} A_{\sigma_1\sigma_2}^\mu \bar{A}_{\sigma_1\sigma_2}^\nu.\end{aligned}\quad (\text{A5})$$

a. Formulas for diagrams

The formulas are presented in the spinorial notation

$$p^{\dot{A}A} := p_\mu (\bar{\sigma}^\mu)^{\dot{A}A} = \begin{pmatrix} p^{\dot{0}0} & p^{\dot{0}1} \\ p^{\dot{1}0} & p^{\dot{1}1} \end{pmatrix} = \begin{pmatrix} p^+ & p^\perp \\ p^\perp & p^- \end{pmatrix}, \quad (\text{A6})$$

where $\bar{\sigma} = (1, \boldsymbol{\sigma})$ for $\boldsymbol{\sigma}$, being the vector of Pauli matrices, and we are using double light cone coordinates defined as

$$p^\pm := p^0 \pm p^3, \quad p^\perp := p^1 + ip^2, \quad p^\perp := p^1 - ip^2. \quad (\text{A7})$$

Some formulas are written in terms of basis spinors,

$$\begin{aligned}o_A &= \begin{pmatrix} 0 \\ 1 \end{pmatrix}, & \iota_A &= \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \\ o^A &= \begin{pmatrix} 1 \\ 0 \end{pmatrix}, & \iota^A &= \begin{pmatrix} 0 \\ 1 \end{pmatrix}.\end{aligned}\quad (\text{A8})$$

We can reduce the number of independent amplitudes by a factor of 2 using the symmetries

$$\begin{aligned}R_{n,-+}^\mu &= -\bar{L}_{n,+-}^\mu, & L_{n,-+}^\mu &= -\bar{R}_{n,+-}^\mu, \\ R_{n,++}^\mu &= \bar{L}_{n,--}^\mu, & L_{n,++}^\mu &= \bar{R}_{n,--}^\mu.\end{aligned}\quad (\text{A9})$$

Therefore, for each diagram, we write only $R_{n,--}^\mu$, $L_{n,--}^\mu$, $R_{n,+-}^\mu$, and $L_{n,+-}^\mu$.

The first diagram is

$$\begin{aligned}R_{1,-}^{\dot{A}A} &= \frac{2\sqrt{S}}{v_1^2 - m_2^2} \frac{p_2^{\dot{0}0}}{\sqrt{p_1^+ p_2^+}} v_1^{\dot{A}0} p_1^{\dot{0}A}, \\ L_{1,-}^{\dot{A}A} &= \frac{2\sqrt{S}}{v_1^2 - m_2^2} \frac{m_1 m_2 p_2^{\dot{0}0}}{\sqrt{p_1^+ p_2^+}} \bar{t}^{\dot{A}} t^A,\end{aligned}\quad (\text{A10})$$

$$\begin{aligned}R_{1,+}^{\dot{A}A} &= -\frac{2\sqrt{S}}{v_1^2 - m_2^2} \frac{m_2 p_1^{\dot{0}0}}{\sqrt{p_1^+ p_2^+}} \bar{t}^{\dot{A}} p_1^{\dot{0}A}, \\ L_{1,+}^{\dot{A}A} &= \frac{2\sqrt{S}}{v_1^2 - m_2^2} \frac{m_1 p_2^{\dot{0}0}}{\sqrt{p_1^+ p_2^+}} \bar{t}^{\dot{A}} v_1^{\dot{0}A}.\end{aligned}\quad (\text{A11})$$

The second diagram is

$$\begin{aligned}R_{2,-}^{\dot{A}A} &= \frac{2\sqrt{S}}{v_2^2 - m_1^2} \frac{p_1^{\dot{0}0}}{\sqrt{p_1^+ p_2^+}} p_2^{\dot{A}0} v_1^{\dot{0}A}, \\ L_{2,-}^{\dot{A}A} &= \frac{2\sqrt{S}}{v_2^2 - m_1^2} \frac{m_1 m_2 p_1^{\dot{0}0}}{\sqrt{p_1^+ p_2^+}} \bar{t}^{\dot{A}} t^A,\end{aligned}\quad (\text{A12})$$

$$\begin{aligned}R_{2,+}^{\dot{A}A} &= -\frac{2\sqrt{S}}{v_2^2 - m_1^2} \frac{m_2 p_1^{\dot{0}0}}{\sqrt{p_1^+ p_2^+}} \bar{t}^{\dot{A}} v_2^{\dot{0}A}, \\ L_{2,+}^{\dot{A}A} &= \frac{2\sqrt{S}}{v_2^2 - m_1^2} \frac{m_1 p_1^{\dot{0}0}}{\sqrt{p_1^+ p_2^+}} \bar{t}^{\dot{A}} p_2^{\dot{0}A}.\end{aligned}\quad (\text{A13})$$

2. The amplitudes for $g^* g^* \rightarrow q \bar{q} V^*$

We list here the amplitudes that were used to define $\mathcal{M}_{(g^* g^*)}^{(r_1 r_2)}$ in (4.5):

$$\mathcal{A}_{1,\sigma_1\sigma_2}^{ab,\mu} = \frac{-ig^2}{(v_1^2 - m_4^2)(v_2^2 - m_4^2)} (T^a T^b)_{ij} \bar{u}_{\sigma_3}(p_3) \Gamma_V^\mu(\hat{v}_1 + m_4) \hat{P}_1(\hat{v}_2 + m_4) \hat{P}_2 v_{\sigma_4}(p_4), \quad (\text{A14a})$$

$$\mathcal{A}_{2,\sigma_1\sigma_2}^{ab,\mu} = \frac{-ig^2}{(v_3^2 - m_3^2)(v_2^2 - m_4^2)} (T^a T^b)_{ij} \bar{u}_{\sigma_3}(p_3) \hat{P}_1(\hat{v}_3 + m_3) \Gamma_V^\mu(\hat{v}_2 + m_4) \hat{P}_2 v_{\sigma_4}(p_4), \quad (\text{A14b})$$

$$\mathcal{A}_{3,\sigma_1\sigma_2}^{ab,\mu} = \frac{-ig^2}{(v_3^2 - m_3^2)(v_4^2 - m_3^2)} (T^a T^b)_{ij} \bar{u}_{\sigma_3}(p_3) \hat{P}_1(\hat{v}_3 + m_3) \hat{P}_2(\hat{v}_4 + m_3) \Gamma_V^\mu v_{\sigma_4}(p_4), \quad (\text{A14c})$$

$$\mathcal{A}_{4,\sigma_1\sigma_2}^{ab,\mu} = \frac{-ig^2}{(v_1^2 - m_4^2)(v_5^2 - m_4^2)} (T^b T^a)_{ij} \bar{u}_{\sigma_3}(p_3) \Gamma_V^\mu(\hat{v}_1 + m_4) \hat{P}_2(\hat{v}_5 + m_4) \hat{P}_1 v_{\sigma_4}(p_4), \quad (\text{A14d})$$

$$\mathcal{A}_{5,\sigma_1\sigma_2}^{ab,\mu} = \frac{-ig^2}{(v_6^2 - m_3^2)(v_5^2 - m_4^2)} (T^b T^a)_{ij} \bar{u}_{\sigma_3}(p_3) \hat{P}_2(\hat{v}_6 + m_3) \Gamma_V^\mu(\hat{v}_5 + m_4) \hat{P}_1 v_{\sigma_4}(p_4), \quad (\text{A14e})$$

$$\mathcal{A}_{6,\sigma_1\sigma_2}^{ab,\mu} = \frac{-ig^2}{(v_4^2 - m_3^2)(v_6^2 - m_3^2)} (T^b T^a)_{ij} \bar{u}_{\sigma_3}(p_3) \hat{P}_2(\hat{v}_6 + m_3) \hat{P}_1(\hat{v}_4 + m_3) \Gamma_V^\mu v_{\sigma_4}(p_4), \quad (\text{A14f})$$

$$\mathcal{A}_{7,\sigma_1\sigma_2}^{ab,\mu} = \frac{g^2}{(k_1 + k_2)^2 (v_1^2 - m_4^2)} f^{abc} T_{ij}^c \bar{u}_{\sigma_3}(p_3) \Gamma_V^\mu(\hat{v}_1 + m_4) \hat{V}_{\text{eff}} v_{\sigma_4}(p_4), \quad (\text{A14g})$$

$$\mathcal{A}_{8,\sigma_1\sigma_2}^{ab,\mu} = \frac{g^2}{(k_1 + k_2)^2 (v_4^2 - m_3^2)} f^{abc} T_{ij}^c \bar{u}_{\sigma_3}(p_3) \hat{V}_{\text{eff}}(\hat{v}_4 + m_3) \Gamma_V^\mu v_{\sigma_4}(p_4), \quad (\text{A14h})$$

where

$$\begin{aligned} v_1 &= p_3 + q, & v_2 &= k_2 - p_4, & v_3 &= p_3 - k_1, \\ v_4 &= -p_4 - q, & v_5 &= k_1 - p_4, & v_6 &= p_3 - k_2, \end{aligned} \quad (\text{A15})$$

and the form of V_{eff}^μ results from the contraction of the Lipatov vertex [90] with the approximated gluon polarizations,

$$\begin{aligned} V_{\text{eff}}^\mu &= \frac{S}{2} (k_2 - k_1)^\mu + \left(2P_2 \cdot k_1 + \frac{P_1 \cdot P_2}{P_1 \cdot k_2} k_1^2 \right) P_1^\mu \\ &\quad - \left(2P_1 \cdot k_2 + \frac{P_1 \cdot P_2}{P_2 \cdot k_1} k_2^2 \right) P_2^\mu \\ &= \frac{S}{2} (k_2 - k_1)^\mu + \left(x_1 S + \frac{k_1^2}{x_2} \right) P_1^\mu \\ &\quad - \left(x_2 S + \frac{k_2^2}{x_1} \right) P_2^\mu. \end{aligned} \quad (\text{A16})$$

These amplitudes were calculated from the spinor-helicity formalism in Ref. [89]. The amplitude squared is given by

$$\mathcal{M}_{(g^* g^*)}^{\mu\nu} = \frac{1}{(N^2 - 1)^2} \sum_{\sigma_1, \sigma_2, a, b, i, j} \mathcal{A}_{\sigma_1 \sigma_2}^{ab, \mu} \bar{\mathcal{A}}_{\sigma_1 \sigma_2}^{ab, \nu}, \quad (\text{A17})$$

where

$$\mathcal{A}_{\sigma_1 \sigma_2}^{ab, \mu} = \sum_{n=1}^8 \mathcal{A}_{n, \sigma_1 \sigma_2}^{ab, \mu}. \quad (\text{A18})$$

APPENDIX B: THE ELECTROWEAK COUPLINGS

The electroweak couplings for fermions with the Z^0 boson can be expressed through the Weinberg mixing angle θ_W . The formulas for leptons (l) and quarks (q) are

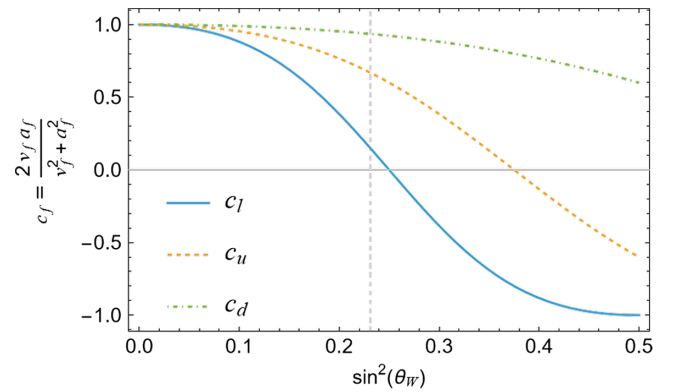


FIG. 11. The ratio of electroweak couplings c_f^Z for leptons and quarks. The dashed vertical line corresponds to the measured value of $\sin^2 \theta_W$.

$$v_l^Z = \frac{e}{\sin(2\theta_W)}(-1 + 4\sin^2\theta_W),$$

$$a_l^Z = \frac{e}{\sin(2\theta_W)}, \quad (\text{B1a})$$

$$v_q^Z = \frac{e}{\sin\theta_W \cos\theta_W}(T_{3q} - 2e_q\sin^2\theta_W),$$

$$a_q^Z = \frac{e}{\sin\theta_W \cos\theta_W}T_{3q}, \quad (\text{B1b})$$

where e is the value of electron charge, e_q are charge fractions of quarks ($e_u = 2/3$, $e_d = -1/3$), and T_3 is the

weak isospin with the component values $T_{3u} = 1/2$ and $T_{3d} = -1/2$. The most recent value of the mixing angle published by the Particle Data Group gives a value $\sin^2\theta_W = 0.23122(6)$ [86].

The accuracy of the mixing angle value is important, especially for parity-breaking structure functions due to the sensitivity of the coupling ratio c_l^Z in the region close to the measured values of $\sin^2\theta_W$. There is no such sensitivity in the case of quark couplings, as can be seen in the plot in Fig. 11, where the slope of c_l at the experimental value is quite large compared to the slopes of c_q .

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