

Could the 650 GeV excess be a pseudoscalar of a three-Higgs-doublet model?

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In this study, we propose the interpretation of a 650 GeV excess observed at the Large Hadron Collider (LHC) by the CMS Collaboration in terms of the production of a CP -odd (or pseudoscalar) Higgs boson A , with mass around 650 GeV, decaying into the Standard Model (SM)-like Higgs state h_{125} (in turn decaying into $\gamma\gamma$) and a Z boson (in turn decaying into $b\bar{b}$), within a three-Higgs doublet model (3HDM) featuring two active and one inert doublet, known as the $I(1+2)$ HDM. This theoretical structure features a spectrum with both the SM-like Higgs boson (with a 125 GeV mass) and a lighter CP -even (or scalar) Higgs state with mass around 95 GeV, h_{95} , which is present in this scenario for the purpose of simultaneously explaining anomalies seen in the $b\bar{b}$, $\gamma\gamma$, and $\tau^+\tau^-$ final states in searches for additional light Higgs states at the Large Electron-Positron collider and LHC itself. It should be noted that, in the $I(1+2)$ HDM, the inert sector presents loop-induced enhancements to the $h_{95} \rightarrow \gamma\gamma$ width via additional scalar charged states, providing a viable mechanism to explain, in particular, the observed (and most significant) diphoton excess at 95 GeV. Taking into account both experimental and theoretical constraints, our results can not only explain the aforementioned anomalies (possibly, aside from the $\tau^+\tau^-$, which is the most marginal one) but also predict, as collateral signals, resonant production of the same CP -odd scalar A followed by the decays: (i) $A \rightarrow h_{95}Z$, leading to the same $\gamma\gamma b\bar{b}$ final state displaying the original 650 GeV anomaly and (ii) $A \rightarrow t\bar{t}$, leading to a well-known and studied signature. Both of these signals are potentially explorable at Run 3 of the LHC and most possibly so at the High-Luminosity LHC, while being consistent with current data at a significance level of 2.5σ .

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I. INTRODUCTION

The discovery of a Higgs boson with a mass of 125 GeV by the ATLAS [1] and CMS [2] Collaborations marked a triumph for the Standard Model (SM) of particle physics and has opened the door to precision Higgs physics, while at the same time motivating searches for additional (psudo) scalar states Beyond the SM (BSM). Intriguingly, several experimental hints of BSM physics have emerged in recent years, notably, a local excess near 95 GeV in the $b\bar{b}$ channel

at the Large Electron-Positron (LEP) collider as extracted by all Collaborations therein (ALEPH, DELPHI, L3, and OPAL) [3–5] plus in the $\gamma\gamma$ and $\tau^+\tau^-$ decay modes at the LHC as seen by CMS [6,7] and ATLAS [8,9]. Recently, a CMS search has further reported an excess in events featuring a diphoton and a bottom quark pair (i.e., a $\gamma\gamma b\bar{b}$ final state), which could indicate a heavier resonance X with a mass of approximately 650 GeV decaying into a secondary object Y with a mass in the range of 90–100 GeV and the SM-like Higgs boson. Such an anomaly was observed with a local excess of 3.8σ and the best-fit value given for the cross section of this excess $\gamma\gamma b\bar{b}$ final state was as follows [10]:

$$\sigma(pp \rightarrow X_{650} \rightarrow h_{125}Y \rightarrow \gamma\gamma b\bar{b}) = 0.333^{+0.17}_{-0.13} \text{ (fb)}. \quad (1)$$

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Such anomalies, although not yet conclusive in terms of significance individually, taken together, actually stand out as compelling candidates for new phenomena, and their collective persistence motivates the search for theoretical frameworks that can simultaneously explain them all.

Extensive theoretical work has been devoted to explaining the observed excesses around 95 GeV. These include non-supersymmetric models such as the 2-Higgs doublet model (2HDM) [11–14], its singlet (next-to-2HDM) and triplet extensions [15–21], the Georgi-Machacek model [22–24], the minimal gauged two-Higgs-doublet model [25], the 3HDM containing one inert doublet [or $I(1+2)$ HDM] [26] as well as supersymmetric frameworks such as the next-to-minimal supersymmetric SM (NMSSM) [5,27–34] and the so-called μ LSSM [35]. Meanwhile, the anomaly observed around 650 GeV has also attracted attention and has been investigated within some BSM scenarios, including the NMSSM [36] and 2HDM [37].

In this paper, we examine yet another possibility to explain the 650 GeV anomaly, by using as a theoretical scenario the one of [26]: i.e., the $I(1+2)$ HDM. In fact, in doing so, we seek simultaneously a solution to the 95 GeV anomalies. As seen in [26], the advantage of this BSM framework is that it provides an additional pair of charged scalar states, $\chi^\pm$, which enhances the one-loop decay into two photons of the lightest CP -even Higgs state, h_{95} , thereby relieving the tension existing in the 2HDM Type-I when attempting to maximize simultaneously the $b\bar{b}$, $\gamma\gamma$, and $\tau^+\tau^-$ decay rates [14]. In fact, the $I(1+2)$ HDM that we use here is of Type-I, just like in Ref. [26]. Herein, we assume that the X_{650} is the pseudoscalar Higgs state, A , with a mass of 650 GeV, decaying into the SM-like Higgs boson h_{125} and a Z boson, followed by the subsequent decays $h_{125} \rightarrow \gamma\gamma$ and $Z \rightarrow b\bar{b}$, respectively. In particular, we are suggesting that the observed $b\bar{b}$ pair originates from a Z boson decay, which would then take the place of the corresponding decay of the lightest Higgs boson with a mass around 95 GeV (the one used in the NMSSM explanation), indeed, like in Ref. [37].

Notice that this interpretation is well motivated. In fact, although the CMS analysis [10] originally models the excess as a decay $X_{650} \rightarrow h_{125}Y_{95}$, with Y_{95} being a spin-0 state, at $\sqrt{s} = m_A \simeq 650$ GeV, where $m_Z^2/s \ll 1$, the equivalence theorem ensures that the pseudoscalar polarization of the Z boson behaves like the corresponding neutral Goldstone mode, with deviations suppressed by $\mathcal{O}(m_Z^2/s) \lesssim 2\%$. Hence, also considering the limited mass resolution of $b\bar{b}$ pairs, of some 10 GeV or so, and the fact that none of the selection cuts used by CMS has a marked spin dependence, it is conceivable that the Z boson of the SM is behind the 95 GeV component of the 650 GeV anomaly.

Additionally, while assuming this excess, one should bear in mind that the $\tau^+\tau^-b\bar{b}$ channel serves as a valuable complementary probe of the same underlying dynamics.

Notably, CMS has performed a dedicated search for heavy resonances in this final state, setting a 95% confidence limit (CL) upper limit of approximately 3 fb on the cross section $\sigma(pp \rightarrow X_{650} \rightarrow \tau^+\tau^-b\bar{b})$ [38], which would then constrain the $pp \rightarrow A \rightarrow h_{125}Z$ (or $Zh_{125}Z \rightarrow \tau^+\tau^-b\bar{b}$) processes in our $I(1+2)$ HDM. We have thus verified that the parameter space of the $I(1+2)$ HDM, providing the illustrated explanation to all discussed anomalies, is also compliant with the aforementioned limit. The generated parameter space points were further tested against the most recent CMS upper limits on the cross section for the process $pp \rightarrow Y(\rightarrow \tau^+\tau^-)H(\rightarrow \gamma\gamma)$ [39] and these were found to constrain significantly the $I(1+2)$ HDM parameter space (see below). Further bearing in mind [40] alongside [39], the following production and decay patterns were also subject to scrutiny: $pp \rightarrow h_{125}(\rightarrow b\bar{b})Y(\rightarrow \gamma\gamma)$ and $pp \rightarrow h_{125}(\rightarrow \tau^+\tau^-)Y(\rightarrow \gamma\gamma)$. However, given that for our solution we have $Y \equiv Z$ [41], the (nonresonant) *transition* $Z^* \rightarrow \gamma\gamma$ is highly suppressed because of the Landau-Yang theorem [42,43] (note also Ref. [44]), the corresponding experimental limits quoted by CMS are irrelevant to our theoretical scenario (so we ignore these in the remainder).

Finally, in order to test our theoretical hypothesis, we also make predictions for two additional processes that would emerge over the same parameter space of the $I(1+2)$ HDM, explaining all the aforementioned anomalies, both of which may be testable at the present and/or future stages of the LHC. These are the channels $pp \rightarrow A \rightarrow h_{95}Z$ and $pp \rightarrow A \rightarrow t\bar{t}$.

The layout of the paper is as follows. In Sec. II, we briefly review the main features of the $I(1+2)$ HDM, along with the considered experimental and theoretical constraints. In Sec. III we present our main results by illustrating the correlations among Higgs boson masses and production cross sections relevant to searches for additional heavy resonances in various final states. In the last Section, we summarize our findings.

II. MODEL SETUP AND CONSTRAINTS

In this Section, we outline the $I(1+2)$ HDM model, which includes a scalar dark matter (DM) candidate. We begin with a recap of the scalar potential and the theoretical and experimental constraints affecting it. Next, we provide an analytical explanation for the $\gamma\gamma b\bar{b}$ excess within this model framework.

A. $I(1+2)$ HDM basics

The $I(1+2)$ HDM model is composed of two active Higgs doublets and one inert doublet. The extension of the SM adding only one extra Higgs doublet has been widely studied, with the 2HDM being one of the most explored frameworks for such an extension [45]. The $I(1+2)$ HDM discussed in Refs. [46,47] and adopted here features a discrete $\mathbb{Z}_2 \times \mathbb{Z}'_2$ symmetry, where the $\mathbb{Z}_2$ symmetry is

enforced upon the inert doublet, meaning that only the inert field η transforms as $\eta \rightarrow -\eta$, while the Higgs doublets Φ_1 and Φ_2 remain unaffected. In addition to this, a softly broken $\mathbb{Z}'_2$ symmetry is implemented for the active Higgs fields, following which Φ_1 remains unchanged whereas Φ_2 undergoes the transformation $\Phi_2 \rightarrow -\Phi_2$. This specific symmetry breaking,

as suggested by the Paschos-Glashow-Weinberg theorem [48], is crucial for suppressing flavour changing neutral currents at tree level, ensuring the consistency of the model.

The scalar potential that remains invariant under both the gauge group of the SM and the two discrete symmetries introduced above is given as follows:

$$\begin{aligned}
 V = & -\frac{1}{2}\{m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + [m_{12}^2\Phi_1^\dagger\Phi_2 + \text{H.c.}]\} + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 \\
 & + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{1}{2}[\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \text{H.c.}] \\
 & + m_\eta^2\eta^\dagger\eta + \frac{\lambda_\eta}{2}(\eta^\dagger\eta)^2 + \lambda_{1133}(\Phi_1^\dagger\Phi_1)(\eta^\dagger\eta) + \lambda_{2233}(\Phi_2^\dagger\Phi_2)(\eta^\dagger\eta) \\
 & + \lambda_{1331}(\Phi_1^\dagger\eta)(\eta^\dagger\Phi_1) + \lambda_{2332}(\Phi_2^\dagger\eta)(\eta^\dagger\Phi_2) + \frac{1}{2}[\lambda_{1313}(\Phi_1^\dagger\eta)^2 + \lambda_{2323}(\Phi_2^\dagger\eta)^2 + \text{H.c.}]
 \end{aligned} \quad (2)$$

where the λ_i 's denote the quartic coupling parameters while m_η^2 , m_{11}^2 , m_{22}^2 and m_{12}^2 are the mass-squared terms.

The two active Higgs doublets and the inert doublet under $SU(2)_L$ are parametrized as follows, respectively:

$$\Phi_k = \begin{pmatrix} \phi_k^\pm \\ (v_k + \eta_k + iz_k)/\sqrt{2} \end{pmatrix}, \quad \eta = \begin{pmatrix} \chi^\pm \\ (\chi + i\chi_a)/\sqrt{2} \end{pmatrix}. \quad (3)$$

Within this framework, v_1 and v_2 denote the nonzero vacuum expectation values (VEVs) developed by the neutral components of the two active scalar fields Φ_1 and Φ_2 . These VEVs determine the dynamics of the spontaneous electroweak symmetry breaking (EWSB) mechanism of the model, thereby generating masses for the gauge bosons and fermions (as appropriate). The η scalar doublet, however, remains inert due to the imposition of the discrete $\mathbb{Z}_2$ symmetry. This symmetry prevents any mixing between the (pseudo)scalar states of η and those of Φ_1 and Φ_2 , thus ensuring that η does not acquire a VEV or contribute to EWSB in the same way as the other doublets.

Consequently, the physical scalar spectrum arising from the two active Higgs doublets in this model is similar to the one in the pure 2HDM. This spectrum consists of four distinct Higgs particles: h (neutral and CP -even), H (neutral and CP -even), A (neutral and CP -odd) and $H^\pm$ (charged Higgs bosons), with masses m_h , m_H , m_A , and $m_{H^\pm}$, respectively [45,49]. In what follows, we shall refer to the lighter CP -even state with a 95 GeV mass, h , as h_{95} and to the heavier CP -even H state with a 125 GeV mass as h_{125} , the former being the candidate to explain the corresponding data anomalies and the latter being identified with the SM-like Higgs boson observed in experiments. Furthermore, the A state is the state that we propose being

behind the 650 GeV anomaly (hence, it will have a mass around such a value).

As for the inert sector, the $I(1+2)$ HDM introduces three additional scalar fields: χ , χ_a (which are neutral) and $\chi^\pm$ (which is charged). Furthermore, we adopt the *dark democracy* approach of Refs. [50–53] in what follows, in order to reduce the number of free parameters, thus making the numerical analysis more tractable while preserving the essential features of the $I(1+2)$ HDM. Consequently, by setting $\lambda_a = \lambda_{1133} = \lambda_{2233}$, $\lambda_b = \lambda_{1331} = \lambda_{2332}$, and $\lambda_c = \lambda_{1313} = \lambda_{2323}$, the inert squared masses can be expressed as,

$$\begin{aligned}
 m_{\chi^\pm}^2 &= m_\eta^2 + \frac{1}{2}\lambda_a v^2, \\
 m_\chi^2 &= m_{\chi^\pm}^2 + \frac{1}{2}(\lambda_b + \lambda_c)v^2, \\
 m_{\chi_a}^2 &= m_{\chi^\pm}^2 + \frac{1}{2}(\lambda_b - \lambda_c)v^2.
 \end{aligned} \quad (4)$$

Thus, the Higgs sector of the $I(1+2)$ HDM is described by 12 free parameters,

$$\begin{aligned}
 \Sigma = \{ & m_h, m_A, m_H, m_{H^\pm}, m_{12}^2, \tan\beta, \sin(\beta - \alpha), \\
 & m_\chi, m_{\chi_a}, m_{\chi^\pm}, m_\eta^2, \lambda_\eta \}.
 \end{aligned} \quad (5)$$

Here, α denotes the rotation angle in the CP -even sector, which governs the mixing between the CP -even neutral Higgs states h and H whereas the parameter β (the mixing angle for both CP -odd and charged sectors) is defined as $\tan\beta = v_2/v_1$.

After enforcing the symmetries outlined above, the most general Yukawa interactions in the $I(1+2)$ HDM resemble those of the 2HDM and the corresponding Lagrangian is given by,

TABLE I. $I(1+2)$ HDM Type-I Yukawa couplings of the neutral Higgs bosons h , H to the up quarks, down quarks, and leptons in the $I(1+2)$ HDM, normalized by the corresponding SM Yukawa couplings.

κ_h^u	κ_h^d	κ_h^ℓ	κ_H^u	κ_H^d	κ_H^ℓ
c_α/s_β	c_α/s_β	c_α/s_β	s_α/s_β	s_α/s_β	s_α/s_β

$$\mathcal{L}_{\text{Yukawa}} = -\bar{Q}_L Y_u \tilde{\Phi}_u u_R - \bar{Q}_L Y_d \Phi_d d_R - \bar{L}_L Y_\ell \Phi_\ell \ell_R + \text{H.c.}$$

$$\supset - \sum_{f=u,d,\ell} \left[\frac{m_f}{v} \kappa_h^f \bar{f} f h + \frac{m_f}{v} \kappa_H^f \bar{f} f H - i \frac{m_f}{v} \kappa_A^f \bar{f} \gamma_5 f A \right],$$

where the Y_f 's ($f = u, d$ or l) are 3×3 Yukawa matrices and $\tilde{\Phi}_{1,2} = i\sigma_2 \Phi_{1,2}^*$, with σ_2 being the Pauli matrix. In Table I we list all the Type-I Yukawa reduced couplings κ_h^f and κ_H^f of the CP -even Higgs bosons, h and H .

B. Theoretical and experimental constraints

In our random parameter scan, we retained only those points that are physically viable, i.e., consistent with both theoretical and experimental requirements. Below we provide a summary of the constraints applied to the $I(1+2)$ HDM Type-I (see [26] for more details):

- (1) Theoretical constraints
 - (a) Perturbativity [47].
 - (b) Unitarity [47].
 - (c) Vacuum stability [46].
- (2) Experimental constraints.
 - (a) Higgs boson signal strength measurements from the LHC, implemented via HiggsSignals-3 [54].
 - (b) Exclusion limits on non-SM-like Higgs bosons from direct searches at LEP, the Tevatron, and LHC, as implemented in HiggsBounds-6 [55].
 - (c) Constraints on the invisible Higgs decay width (in the presence of the inert sector), as automatically included within HiggsTools [56].
 - (d) Electroweak (EW) precision tests from S , T , and U parameters [57], derived following the general result of [58,59].
 - (e) DM searches via microOMEGAs [60].
 - (f) Flavor constraints as evaluated using the public SuperIso [61] program, specifically, several branching ratios (BRs) have been checked: $\text{BR}(B \rightarrow X_s \gamma)$ [62], $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ [63–65], $\text{BR}(B \rightarrow \tau \nu)$ [62], among others.

C. The $\gamma\gamma b\bar{b}$ anomaly

Here we consider the scenario where the CP -odd scalar A , with a mass around 650 GeV, decays via $A \rightarrow h_{125} Z$, where the h_{125} subsequently decays as $h_{125} \rightarrow \gamma\gamma$, and the Z boson decays as $Z \rightarrow b\bar{b}$. For the latter decay, we have incorporated the effect of a lower cut on the invariant mass

TABLE II. Scan ranges of the $I(1+2)$ HDM Type-I parameters. [Mass (squared) are in $\text{GeV}^{(2)}$.]

Parameter	m_h	m_H	m_A	$m_{H^\pm}$
Scan range	[94, 97]	125.09	[600, 700]	[90, 10^3]
Parameter	m_{12}^2	$\tan \beta$	$s_{\beta-\alpha}$	
Scan range	$[-10^3, 10^3]$	[0.5, 25]	$[-0.4, 0.1]$	
Parameter	m_η^2	$m_\chi, m_{\chi_a}, m_{\chi^\pm}$	λ_η	
Scan range	$[-10^5, 10^5]$	[90, 10^3]	[7, 12]	

of the Z decay products ($m_{b\bar{b}} > 70$ GeV) implemented in the CMS analysis of [10] through the appropriate integration of the Breit-Wigner distribution over the $b\bar{b}$ final state. This interpretation choice is both theoretically consistent and phenomenologically relevant. On one hand, the original CMS analysis [10] modeled the excess in terms of a decay of the form $X_{650} \rightarrow h_{125} Y$, where Y denotes a generic spin-0 resonance. On the other hand, their event selection does not rely on spin or polarization-sensitive observables. Instead, it targets invariant mass reconstruction and global event kinematics, yet, the phenomenological difference between a pseudoscalar decay and that of a (dominant) longitudinal vector boson is expected to have minimal impact on acceptance.

With all this in mind, we also explore other possible channels that would manifest themselves over the $I(1+2)$ HDM parameter space explaining all aforementioned anomalies, such as $A \rightarrow h_{95} Z$ and $A \rightarrow t\bar{t}$, which are indeed competitive with the decay $A \rightarrow h_{125} Z$. In particular, we will demonstrate in the following that there exist viable regions in parameter space where the BRs for these two channels are substantial and consistent with all current constraints.

III. RESULTS AND DISCUSSIONS

Following the previous discussion, we briefly recapitulate the impact of the applied constraints on the parameter space and present our interpretation of the observed $\gamma\gamma b\bar{b}$ excess within the $I(1+2)$ HDM Type-I. To this end, we perform a numerical scan over the free parameters in Eq. (5), as summarized in Table II. The resulting parameter points are subsequently tested against theoretical conditions and experimental data using a dedicated Python interface to the aforementioned packages or our own routines to test the most recent limits.

Hereafter, for convenience, we shall use the shorthand notation for the relevant cross section,

$$\sigma_{\gamma\gamma b\bar{b}} = \sigma(pp \rightarrow A \rightarrow h_{125} Z \rightarrow \gamma\gamma b\bar{b}). \quad (6)$$

To start with, as mentioned above, we have explicitly used microOMEGAs to compute the relic density, Ωh^2 , and the (in)direct detection rates, the most constraining experiments

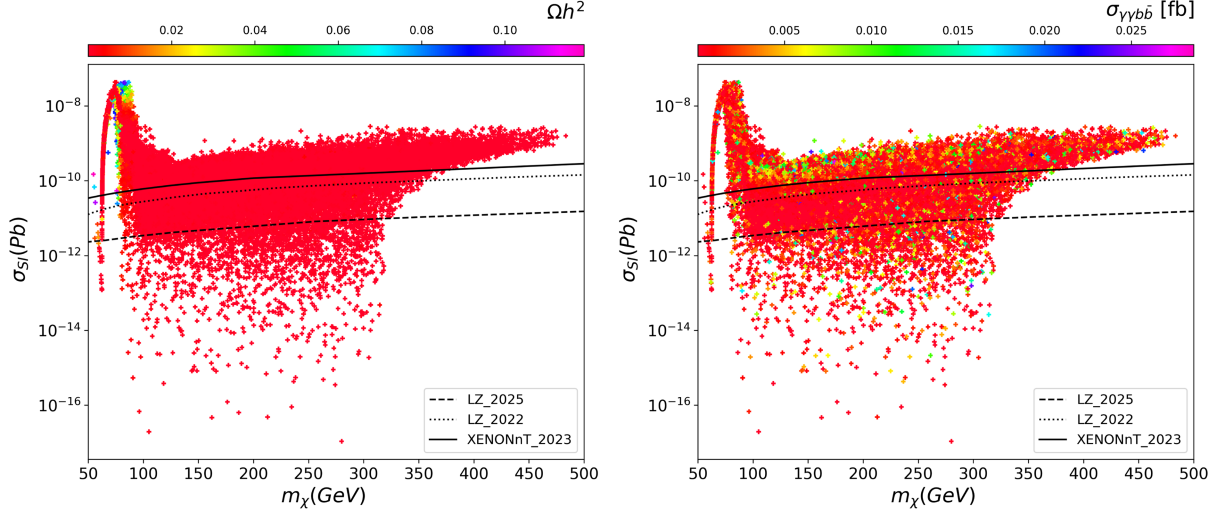


FIG. 1. Relic density Ωh^2 (left) and the cross section $\sigma_{\gamma b \bar{b}}$ (right) shown as a scatter plot in the $(m_\chi, \sigma_{\text{SI}})$ plane. The XENONnT [66] and LUX-ZEPLIN [67,68] upper limits are shown with solid, dotted, and dashed lines, respectively.

in the latter respect being XENONnT [66] and LUX-ZEPLIN [67,68], by taking into account a rescaling of the relevant cross sections in such a way that the computed spin-independent (SI) DM-nucleon one σ_{SI} reads as

$$\sigma_{\text{SI}} \rightarrow \sigma_{\text{SI}} \times \frac{\Omega h^2}{\Omega_c h^2}, \quad (7)$$

where $\Omega_c h^2 = 0.1200 \pm 0.0012$ is the DM relic density determined by the Planck collaboration [69].

Figure 1 displays σ_{SI} as a function of the DM mass m_χ with the map coding indicating the corresponding DM relic density, Ωh^2 (left), and the values of the $\gamma\gamma b \bar{b}$ signal rate, $\sigma_{\gamma\gamma b \bar{b}}$ (right). As can be seen from the left panel, all the sampled points remain below the Planck limit value while a broad region of parameter space simultaneously satisfies the latest direct detection bounds from XENONnT-2023, LZ-2022, and the projected LZ-2025. Additionally, the right panel in Fig. 1, except for very low DM masses ($m_\chi \leq 60$ GeV), where most points are excluded, confirms that the DM candidate in the $I(1+2)\text{HDM}$ not only survives the most stringent constraints from direct detection experiments but also yields values of $\sigma_{\gamma\gamma b \bar{b}}$ consistent with the CMS excess at 650 GeV at -2.5σ for larger masses ($100 \text{ GeV} \leq m_\chi \leq 350 \text{ GeV}$). Thus, the DM and collider sectors of the model are closely correlated. Most importantly, here, the relic density is not oversaturated and both direct and indirect constraints are satisfied.

Subsequently, we found it appropriate to show the possible dependencies of such an observable on the model parameters. As can be seen from Fig. 2 (left), a low $\tan\beta \leq 5$ is necessary to accommodate the $\sigma_{\gamma\gamma b \bar{b}}$ value corresponding to the excess. Additionally, the range of $\sin(\beta - \alpha)$ that matches the $\gamma\gamma b \bar{b}$ cross section value up to -2.5σ is relatively wide, i.e., 0.27–0.47 [see Fig. 2 (middle)].

Regarding the right panel in Fig. 2, it shows that the $\sigma_{\gamma\gamma b \bar{b}}$ decreasing with m_A happens for phase space reasons, so that larger values of m_A require larger values of the $\text{BR}(h_{125} \rightarrow \gamma\gamma)$. Unambiguously, though, our analysis indicates that the full mass range of m_A between 600 and 700 GeV remains viable in order to account for the observed excess.

In this connection, it should be noted here that inert model parameters, while not explicitly entering the discussion, do affect the phenomenology of this process indirectly, in particular, the additional scalar charged states with mass $100 \leq m_{\chi^\pm} \leq 500$ GeV alter the diphoton decay width of the SM-like Higgs boson, ultimately leading to $\sigma_{\gamma\gamma b \bar{b}}$ value close to the measured one. At the same time, they enter the $h_{95} \rightarrow \gamma\gamma$ decay process, enabling the $I(1+2)\text{HDM}$ studied here to explain the 95 GeV excesses, as explained in [26].

We then show in Fig. 3 the correlations between the predicted values for $\sigma_{\gamma\gamma b \bar{b}}$ and the signal strengths $\mu_{\gamma\gamma}(h_{95})$ and $\mu_{b \bar{b}}(h_{95})$, as defined in [26], of the lighter CP -even Higgs boson h_{95} , for different values of $\text{BR}(A \rightarrow h_{95}Z)$, $\text{BR}(A \rightarrow h_{125}Z)$ and $\text{BR}(A \rightarrow t \bar{t})$ [ignoring the signal strength $\mu_{\tau^+\tau^-}(h_{95})$, as the excess is here more marginal in comparison].

The figure highlights the $[-2.5\sigma, 0.5\sigma]$ horizontal region in orange corresponding to the $\gamma\gamma b \bar{b}$ anomaly (as in Fig. 2) and, as a vertical band depicted in cyan, the range of values of the $\mu_{\gamma\gamma}$ (upper panel) and $\mu_{b \bar{b}}$ (lower panel) excesses, both drawn at the $\pm 2\sigma$ level. From this figure, it can be seen that it is in general quite difficult to simultaneously accommodate the $\gamma\gamma b \bar{b}$ excess with the $\mu_{\gamma\gamma}(h_{95})$ and $\mu_{b \bar{b}}(h_{95})$ ones, but a nonzero overlapping region nevertheless exists.

Quite interestingly, the points most compatible with the CMS excess at 650 GeV lie within or near the

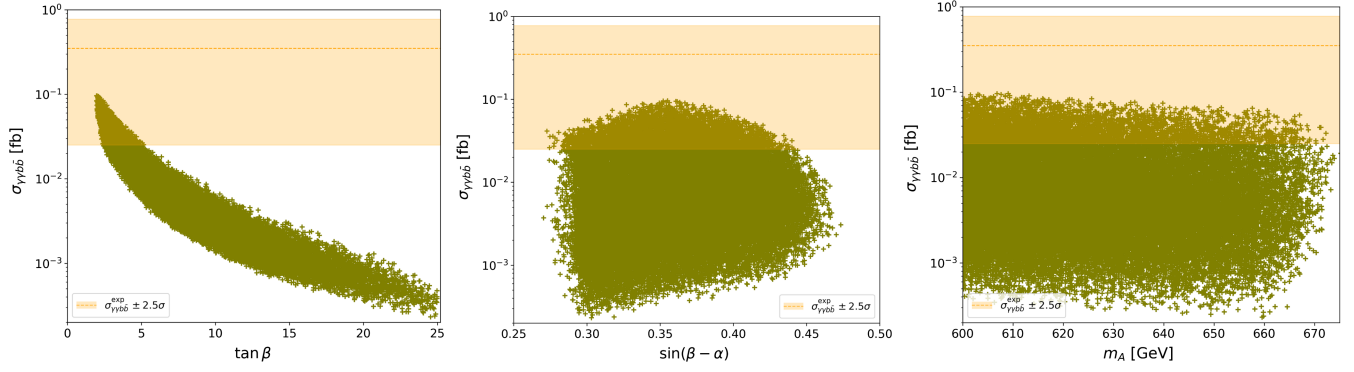


FIG. 2. $\sigma_{\gamma\gamma b\bar{b}}$ values as function of $\tan\beta$ (left), $\sin(\beta - \alpha)$ (middle), and m_A (right). The orange horizontal band corresponds to the value of the $\gamma\gamma b\bar{b}$ cross section consistent with the experimental measurement within $[-2.5\sigma, 0.5\sigma]$.

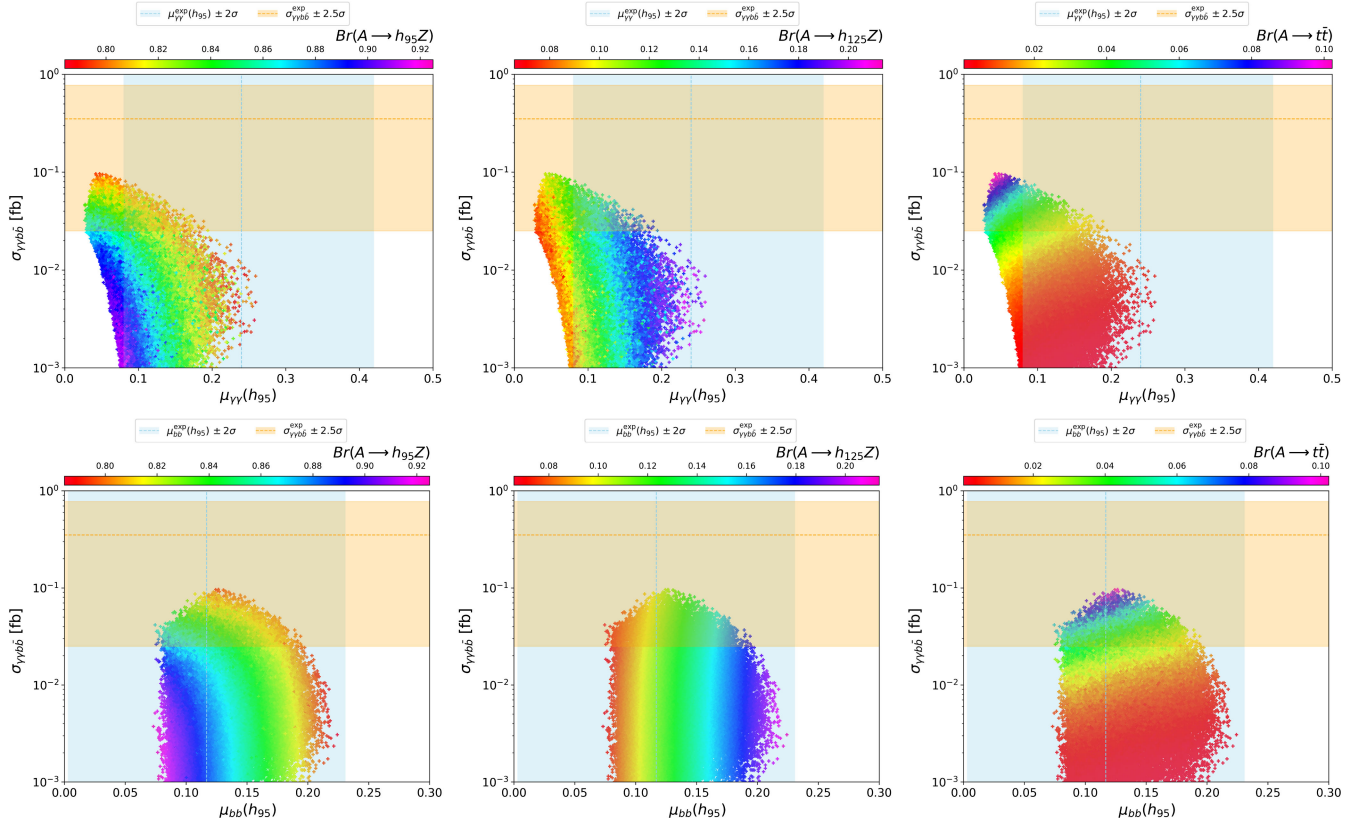


FIG. 3. The scatter plots of the production cross section $\sigma_{\gamma\gamma b\bar{b}}$, as a function of the signal strengths $\mu_{\gamma\gamma}(h_{95})$ (top) and $\mu_{b\bar{b}}(h_{95})$ (bottom). The color coding indicates one of the following BRs: $\text{BR}(A \rightarrow h_{95}Z)$ (left), $\text{BR}(A \rightarrow h_{125}Z)$ (middle), and $\text{BR}(A \rightarrow t\bar{t})$ (right). Furthermore, the orange (cyan) horizontal (vertical) band represents the measured value of $\sigma_{\gamma\gamma b\bar{b}}$ [signal strengths $\mu_{\gamma\gamma}(h_{95})$ and $\mu_{b\bar{b}}(h)$].

experimentally allowed regions, particularly in the low-to-moderate diphoton signal strength, i.e., $\mu_{\gamma\gamma}(h_{95}) \lesssim 0.2$, see Fig. 3 (top frames), and remain consistent with the $\mu_{b\bar{b}}(h_{95})$ one at 2σ level, see Fig. 3 (lower frames). For the values of BRs shown, the maximum value for $\sigma_{\gamma\gamma b\bar{b}} \approx 0.096$ fb is reached for $\text{BR}(A \rightarrow h_{95}Z) \approx 79.16\%$, $\text{BR}(A \rightarrow h_{125}Z) \approx 10.57\%$, and $\text{BR}(A \rightarrow t\bar{t}) \approx 9.86\%$. Overall, we see that, for fixed $\mu_{\gamma\gamma}(h_{95})$ and/or $\mu_{b\bar{b}}(h_{95})$, $\sigma_{\gamma\gamma b\bar{b}}$ increases inversely

with $\text{BR}(A \rightarrow h_{95}Z)$, revealing that this decay mode dominates in the viable parameter space, with the largest BRs observed where the signal strengths $\mu_{b\bar{b}}(h_{95})$ and $\mu_{\gamma\gamma}(h_{95})$ remain consistent with experimental bounds, so that this is the key probe of the BSM scenario adopted here. Furthermore, notice that $\text{BR}(A \rightarrow t\bar{t})$ is generally not negligible, so that this channel may also be of relevance for testing the viability of the $I(1+2)\text{HDM}$ Type-I against

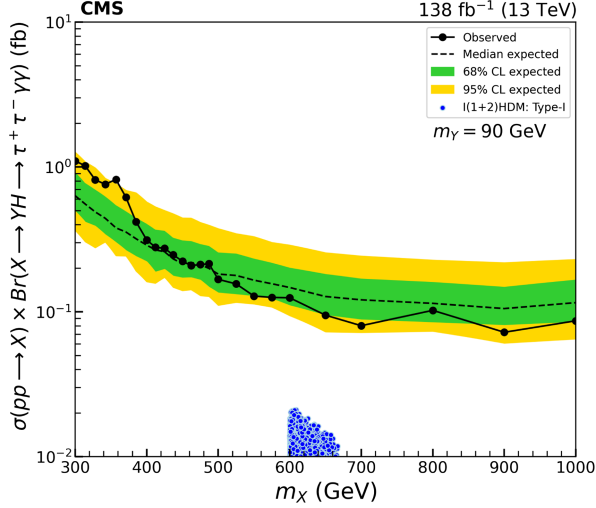


FIG. 4. $\sigma(pp \rightarrow X \rightarrow HY \rightarrow \gamma\gamma\tau^+\tau^-)$ values as function of m_X as obtained for the I(1+2)HDM points explaining the 650 and 95 GeV anomalies. Notice that, in such a scenario, one has $X = A$, $Y = Z$, and $H = h_{125}$.

all the studied datasets. All this is essentially exemplifying the fact that the $A \rightarrow h_{125}Z$ mode (i.e., the apparently anomalous one) is not the only available channel for probing our scenario, so the present 650 GeV anomaly may actually be accompanied by others.

As a final point, and as already intimated, we note that the CMS Collaboration has recently carried out a dedicated analysis focusing on a possible heavy spin-0 resonance X decaying into the SM-like Higgs boson and a lighter object Y (again, potentially but not necessarily a spin-0 one), with the distinctive final state $\gamma\gamma\tau^+\tau^-$ [39]. They studied in particular the mass regions $m_X \approx 650$ GeV and $m_Y \approx 95$ GeV and found no significant excess. This search is thus relevant for our case as it places stringent upper limits on the production cross section times BR for the channel $pp \rightarrow A \rightarrow h_{125}Z \rightarrow \gamma\gamma\tau^+\tau^-$ in our model, ranging from 0.69 to 15 fb as a function of the X mass. We have therefore accounted for this in our analysis. As shown in Fig. 4, our predictions for this process, over the region of I(1+2)HDM Type-I parameter space explaining the 650 and 95 GeV anomalies simultaneously, lie below the current sensitivity lines, yet not exceedingly so. Hence, while the $\gamma\gamma\tau^+\tau^-$ final state remains unconstrained presently in our model it also represents a promising probe of the I(1+2)HDM Type-I for upcoming searches at Run 3 and the HL-LHC [70].

IV. CONCLUSIONS

In this paper, we have tested the I(1+2)HDM Type-I as a possible theoretical scenario explaining a variety of

anomalies that have recently appeared in experimental data, the origin of which could well be in an extended Higgs sector, with respect to the one embedded in the SM. Specifically, excesses have been seen in the search for new Higgs bosons with masses both below and above the one of the SM-like Higgs state detected at the LHC in 2012, with a mass of 125 GeV. In fact, anomalies have appeared around 95 GeV in analyses targeting the following processes: $e^+e^- \rightarrow Z^* \rightarrow h_{95}Z \rightarrow b\bar{b} + \text{jets}$ at LEP and $pp \rightarrow h_{95} \rightarrow \gamma\gamma$ and $\tau^+\tau^-$ at the LHC. Furthermore, another quite significant excess has been extracted at the LHC around 650 GeV, potentially emerging from the process $pp \rightarrow A \rightarrow h_{125}Z \rightarrow \gamma\gamma b\bar{b}$ (with $m_A = 650$ GeV). All such production and decay channels are present in the I(1+2)HDM and, over sizable regions of the parameter space of such a BSM scenario, they can explain all aforementioned excesses at approximately the 2σ CL (possibly apart from the ditau excess, which is quite marginal, in fact). Crucially, this is made possible thanks to the presence of additional scalar charged states, a feature specific to our BSM setup [not existing, e.g., in more conventional (N)2HDMs], as they enable a perfect mapping of the 95 GeV anomaly in the diphoton channel, the most constraining one by far amongst all the discussed datasets.

In addition, alongside explaining existing anomalies, hence, through a data analysis done *a posteriori*, we have also proposed, over the parameter space of the I(1+2)HDM accommodating the various excesses, two collateral signals that should eventually appear in data. These should then be considered as *a priori* manifestations that one should expect from our theoretical framework for it to be the one underpinning nature. The new signatures are the processes $pp \rightarrow A \rightarrow h_{95}Z$ and $pp \rightarrow A \rightarrow t\bar{t}$, which can be searched for in a variety of final states. We thus encourage the experimental collaborations working at the current LHC and/or future HL-LHC to test such predictions, which constitute the hallmark manifestations of the I(1+2)HDM. For this purpose, we will make available upon request all our data.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

- [1] G. Aad *et al.* (ATLAS Collaboration), Observation of a new particle in the search for the standard model Higgs boson with the ATLAS detector at the LHC, *Phys. Lett. B* **716**, 1 (2012).
- [2] S. Chatrchyan *et al.* (CMS Collaboration), Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC, *Phys. Lett. B* **716**, 30 (2012).
- [3] R. Barate *et al.* (LEP Working Group for Higgs Boson Searches, ALEPH, DELPHI, L3, and OPAL Collaborations), Search for the standard model Higgs boson at LEP, *Phys. Lett. B* **565**, 61 (2003).
- [4] S. Schael *et al.* (ALEPH, DELPHI, L3, OPAL Collaborations, and LEP Working Group for Higgs Boson Searches), Search for neutral MSSM Higgs bosons at LEP, *Eur. Phys. J. C* **47**, 547 (2006).
- [5] J. Cao, X. Guo, Y. He, P. Wu, and Y. Zhang, Diphoton signal of the light Higgs boson in natural NMSSM, *Phys. Rev. D* **95**, 116001 (2017).
- [6] A. Hayrapetyan *et al.* (CMS Collaboration), Search for a standard model-like Higgs boson in the mass range between 70 and 110 GeV in the diphoton final state in proton-proton collisions at $s = 13$ TeV, *Phys. Lett. B* **860**, 139067 (2025).
- [7] A. Tumasyan *et al.* (CMS Collaboration), Searches for additional Higgs bosons and for vector leptiquarks in $\tau\tau$ final states in proton-proton collisions at $\sqrt{s} = 13$ TeV, *J. High Energy Phys.* **07** (2023) 073.
- [8] Search for resonances in the 65 to 110 GeV diphoton invariant mass range using 80 fb⁻¹ of pp collisions collected at $\sqrt{s} = 13$ TeV with the ATLAS detector (2018).
- [9] Note that the best-fit mass actually differs from one channel to another, however, given the poor resolution in invariant mass of $b\bar{b}$ and $\tau^+\tau^-$ pairs, compared with to the $\gamma\gamma$ case (which is centered at 95 or so GeV), we collectively use the latter value throughout, including in our forthcoming tests of the various anomalies.
- [10] A. Tumasyan *et al.* (CMS Collaboration), Search for a new resonance decaying into two spin-0 bosons in a final state with two photons and two bottom quarks in proton-proton collisions at $\sqrt{s} = 13$ TeV, *J. High Energy Phys.* **05** (2024) 316.
- [11] G. Cacciapaglia, A. Deandrea, S. Gascon-Shotkin, S. Le Corre, M. Lethuillier, and J. Tao, Search for a lighter Higgs boson in two Higgs doublet models, *J. High Energy Phys.* **12** (2016) 068.
- [12] R. Benbrik, M. Boukidi, S. Moretti, and S. Semlali, Explaining the 96 GeV Di-photon anomaly in a generic 2HDM Type-III, *Phys. Lett. B* **832**, 137245 (2022).
- [13] R. Benbrik, M. Boukidi, S. Moretti, and S. Semlali, Probing a 96 GeV Higgs Boson in the Di-Photon Channel at the LHC, *Proc. Sci. ICHEP2022* (2022) 547 [arXiv:2211.11140].
- [14] A. Khanna, S. Moretti, and A. Sarkar, Explaining 95 (or so) GeV anomalies in the 2-Higgs doublet model Type-I, arXiv:2409.02587.
- [15] T. Biekötter, M. Chakraborti, and S. Heinemeyer, A 96 GeV Higgs boson in the N2HDM, *Eur. Phys. J. C* **80**, 2 (2020).
- [16] S. Heinemeyer, C. Li, F. Lika, G. Moortgat-Pick, and S. Paasch, Phenomenology of a 96 GeV Higgs boson in the 2HDM with an additional singlet, *Phys. Rev. D* **106**, 075003 (2022).
- [17] T. Biekötter, S. Heinemeyer, and G. Weiglein, Mounting evidence for a 95 GeV Higgs boson, *J. High Energy Phys.* **08** (2022) 201.
- [18] S. Banik, A. Crivellin, S. Iguro, and T. Kitahara, Asymmetric di-Higgs signals of the next-to-minimal 2HDM with a U(1) symmetry, *Phys. Rev. D* **108**, 075011 (2023).
- [19] T. Biekötter, S. Heinemeyer, and G. Weiglein, The CMS diphoton excess at 95 GeV in view of the LHC run 2 results, *Phys. Lett. B* **846**, 138217 (2023).
- [20] T. Biekötter, S. Heinemeyer, and G. Weiglein, 95.4 GeV diphoton excess at ATLAS and CMS, *Phys. Rev. D* **109**, 035005 (2024).
- [21] A. Kundu, P. Mondal, and G. Moulataka, Indications for new scalar resonances at the LHC and a possible interpretation, arXiv:2411.14126.
- [22] A. Ahrich, 95 GeV excess in the Georgi-Machacek model: Single or twin peak resonance, *Phys. Rev. D* **110**, 035010 (2024).
- [23] T.-K. Chen, C.-W. Chiang, S. Heinemeyer, and G. Weiglein, 95 GeV Higgs boson in the Georgi-Machacek model, *Phys. Rev. D* **109**, 075043 (2024).
- [24] X. Du, H. Liu, and Q. Chang, Interpretation of 95 GeV Excess within the Georgi-Machacek model in light of positive definiteness constraints, *Phys. Rev. D* **112**, 015019 (2025).
- [25] A. Arhrib, K. H. Phan, V. Q. Tran, and T.-C. Yuan, When the standard model Higgs meets its lighter 95 GeV twin, *Nucl. Phys. B* **1015**, 116909 (2025).
- [26] A. Hmissou, S. Moretti, and L. Rahili, Investigating the 95 GeV Higgs boson excesses within the I(1 + 2)HDM, arXiv:2502.03631.
- [27] J. Cao, X. Jia, Y. Yue, H. Zhou, and P. Zhu, 96 GeV diphoton excess in seesaw extensions of the natural NMSSM, *Phys. Rev. D* **101**, 055008 (2020).
- [28] T. Biekötter, A. Grohsjean, S. Heinemeyer, C. Schwanenberger, and G. Weiglein, Possible indications for new Higgs bosons in the reach of the LHC: N2HDM and NMSSM interpretations, *Eur. Phys. J. C* **82**, 178 (2022).
- [29] W. Li, H. Qiao, K. Wang, and J. Zhu, Light dark matter confronted with the 95 GeV diphoton excess, arXiv:2312.17599.
- [30] U. Ellwanger, C. Hugonie, S. F. King, and S. Moretti, NMSSM explanation for excesses in the search for neutralinos and charginos and a 95 GeV Higgs boson, *Eur. Phys. J. C* **84**, 788 (2024).
- [31] J. Lian, 95 GeV excesses in the Z3-symmetric next-to-minimal supersymmetric standard model, *Phys. Rev. D* **110**, 115018 (2024).
- [32] U. Ellwanger and C. Hugonie, Nmssm with correct relic density and an additional 95 GeV Higgs boson, *Eur. Phys. J. C* **84**, 526 (2024).
- [33] J. Cao, X. Jia, and J. Lian, Unified interpretation of the muon $g-2$ anomaly, the 95 GeV diphoton, and $b\bar{b}$ excesses in the general next-to-minimal supersymmetric standard model, *Phys. Rev. D* **110**, 115039 (2024).
- [34] A. Hammad, R. Ramos, A. Chakraborty, P. Ko, and S. Moretti, Explaining data anomalies over the NMSSM parameter space with deep learning techniques, arXiv:2508.13912.

- [35] J. Cao, X. Jia, J. Lian, and L. Meng, 95 GeV diphoton and bb^- excesses in the general next-to-minimal supersymmetric standard model, *Phys. Rev. D* **109**, 075001 (2024).
- [36] U. Ellwanger and C. Hugonie, Additional Higgs bosons near 95 and 650 GeV in the NMSSM, *Eur. Phys. J. C* **83**, 1138 (2023).
- [37] R. Benbrik, M. Boukidi, K. Kahime, S. Moretti, L. Rahili, and B. Taki, Exploring potential Higgs resonances at 650 GeV and 95 GeV in the 2HDM Type III, *Phys. Lett. B* **868**, 139688 (2025).
- [38] A. Tumasyan *et al.* (CMS Collaboration), Search for a heavy Higgs boson decaying into two lighter Higgs bosons in the $\tau\tau b\bar{b}$ final state at 13 TeV, *J. High Energy Phys.* **11** (2021) 057.
- [39] A. Hayrapetyan *et al.* (CMS Collaboration), Search for the nonresonant and resonant production of a Higgs boson in association with an additional scalar boson in the $\gamma\gamma\tau\tau$ final state in proton-proton collisions at $\sqrt{s} = 13$ TeV, *arXiv:2506.23012*.
- [40] A. Hayrapetyan *et al.* (CMS Collaboration), Search for a new scalar resonance decaying to a Higgs boson and another new scalar particle in the final state with two bottom quarks and two photons in proton-proton collisions at $\sqrt{s} = 13$ TeV, *arXiv:2508.11494*.
- [41] And the $I(1+2)$ HDM used here is CP -conserving, so that the decay $A \rightarrow h_{125}h_{95}$ is not possible.
- [42] L. D. Landau, On the angular momentum of a system of two photons, *Dokl. Akad. Nauk SSSR* **60**, 207 (1948).
- [43] C.-N. Yang, Selection rules for the dematerialization of a particle into two photons, *Phys. Rev.* **77**, 242 (1950).
- [44] S. Moretti, Variations on a Higgs theme, *Phys. Rev. D* **91**, 014012 (2015).
- [45] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher, and J. P. Silva, Theory and phenomenology of two-Higgs-doublet models, *Phys. Rep.* **516**, 1 (2012).
- [46] B. Grzadkowski, O. M. Ogreid, and P. Osland, Natural multi-Higgs model with dark matter and CP violation, *Phys. Rev. D* **80**, 055013 (2009).
- [47] S. Moretti and K. Yagyu, Constraints on parameter space from perturbative unitarity in models with three scalar doublets, *Phys. Rev. D* **91**, 055022 (2015).
- [48] S. L. Glashow and S. Weinberg, Natural conservation laws for neutral currents, *Phys. Rev. D* **15**, 1958 (1977).
- [49] J. F. Gunion and H. E. Haber, The CP conserving two Higgs doublet model: The approach to the decoupling limit, *Phys. Rev. D* **67**, 075019 (2003).
- [50] V. Keus, S. F. King, S. Moretti, and D. Sokolowska, Dark matter with two inert doublets plus one Higgs doublet, *J. High Energy Phys.* **11** (2014) 016.
- [51] V. Keus, S. F. King, S. Moretti, and D. Sokolowska, Observable heavy Higgs dark matter, *J. High Energy Phys.* **11** (2015) 003.
- [52] A. Cordero-Cid, J. Hernández-Sánchez, V. Keus, S. F. King, S. Moretti, D. Rojas, and D. Sokolowska, CP violating scalar dark matter, *J. High Energy Phys.* **12** (2016) 014.
- [53] A. Cordero, J. Hernandez-Sanchez, V. Keus, S. F. King, S. Moretti, D. Rojas, and D. Sokolowska, Dark matter signals at the LHC from a 3HDM, *J. High Energy Phys.* **05** (2018) 030.
- [54] P. Bechtle, S. Heinemeyer, T. Klingl, T. Stefaniak, G. Weiglein, and J. Wittbrodt, HiggsSignals-2: Probing new physics with precision Higgs measurements in the LHC 13 TeV era, *Eur. Phys. J. C* **81**, 145 (2021).
- [55] P. Bechtle, D. Dercks, S. Heinemeyer, T. Klingl, T. Stefaniak, G. Weiglein, and J. Wittbrodt, HiggsBounds-5: Testing Higgs sectors in the LHC 13 TeV era, *Eur. Phys. J. C* **80**, 1211 (2020).
- [56] H. Bahl, T. Biekötter, S. Heinemeyer, C. Li, S. Paasch, G. Weiglein, and J. Wittbrodt, HiggsTools: BSM scalar phenomenology with new versions of HiggsBounds and HiggsSignals, *Comput. Phys. Commun.* **291**, 108803 (2023).
- [57] M. Merchand and M. Sher, Constraints on the parameter space in an inert doublet model with two active doublets, *J. High Energy Phys.* **03** (2020) 108.
- [58] M. E. Peskin and T. Takeuchi, Estimation of oblique electroweak corrections, *Phys. Rev. D* **46**, 381 (1992).
- [59] W. Grimus, L. Lavoura, O. M. Ogreid, and P. Osland, The oblique parameters in multi-Higgs-doublet models, *Nucl. Phys. B* **801**, 81 (2008).
- [60] G. Alguero, G. Belanger, F. Boudjema, S. Chakraborti, A. Goudelis, S. Kraml, A. Mjallal, and A. Pukhov, micrOMEGAS 6.0: N-component dark matter, *Comput. Phys. Commun.* **299**, 109133 (2024).
- [61] F. Mahmoudi, SuperIso v2.3: A program for calculating flavor physics observables in supersymmetry, *Comput. Phys. Commun.* **180**, 1579 (2009).
- [62] Y. Amhis *et al.* (HFLAV Collaboration), Averages of b -hadron, c -hadron, and τ -lepton properties as of summer 2016, *Eur. Phys. J. C* **77**, 895 (2017).
- [63] R. Aaij *et al.* (LHCb Collaboration), Measurement of the $B_s^0 \rightarrow \mu^+\mu^-$ decay properties and search for the $B^0 \rightarrow \mu^+\mu^-$ and $B_s^0 \rightarrow \mu^+\mu^-\gamma$ decays, *Phys. Rev. D* **105**, 012010 (2022).
- [64] R. Aaij *et al.* (LHCb Collaboration), Analysis of neutral B-meson decays into two muons, *Phys. Rev. Lett.* **128**, 041801 (2022).
- [65] A. Tumasyan *et al.* (CMS Collaboration), Measurement of the $B_s^0 \rightarrow \mu^+\mu^-$ decay properties and search for the $B^0 \rightarrow \mu^+\mu^-$ decay in proton-proton collisions at $\sqrt{s} = 13$ TeV, *Phys. Lett. B* **842**, 137955 (2023).
- [66] E. Aprile *et al.* (XENON Collaboration), First dark matter search with nuclear recoils from the XENONnT experiment, *Phys. Rev. Lett.* **131**, 041003 (2023).
- [67] J. Aalbers *et al.* (LZ Collaboration), First dark matter search results from the LUX-ZEPLIN (LZ) experiment, *Phys. Rev. Lett.* **131**, 041002 (2023).
- [68] J. Aalbers *et al.* (LZ Collaboration), Dark matter search results from 4.2 tonne – years of exposure of the LUX-ZEPLIN (LZ) experiment, *Phys. Rev. Lett.* **135**, 011802 (2025).
- [69] N. Aghanim *et al.* (Planck Collaboration), Planck 2018 results. VI. Cosmological parameters, *Astron. Astrophys.* **641**, A6 (2020); **652**, C4(E) (2021).
- [70] F. Gianotti *et al.*, Physics potential and experimental challenges of the LHC luminosity upgrade, *Eur. Phys. J. C* **39**, 293 (2005).