

Searching for single production of vectorlike quarks decaying into Wb at a future muon-proton collider

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This work investigates the discovery potential for singly produced vectorlike quarks (VLQs) T ($Q = +2/3e$) and Y ($Q = -4/3e$) decaying to Wb at future μp colliders with $\sqrt{s} = 5.29, 6.48$, and 9.16 TeV, analyzing both leptonic and hadronic W decay channels through detailed detector simulations. The hadronic channel demonstrates superior sensitivity, enabling 5σ discovery up to $m_T = 3750$ GeV ($m_Y = 4100$ GeV) at 9.16 TeV with 100 fb^{-1} , while exclusion limits reach $m_T = 4500$ GeV ($g^* \geq 0.06$) and $m_Y = 4800$ GeV ($\kappa_Y \geq 0.04$)—significantly beyond LHC capabilities. At 5.29 TeV, discovery regions cover $g^* \in [0.15, 0.5]$ for $m_T \in [1500, 2520]$ GeV and $\kappa_Y \in [0.12, 0.5]$ for $m_Y \in [1700, 2700]$ GeV, with exclusions extending to $m_T = 2750$ GeV and $m_Y = 3020$ GeV. These results, obtained within a simplified two-parameter framework ($m_{T/Y}$ and electroweak couplings g^*), establish μp colliders as uniquely powerful tools for probing high-mass VLQ states, particularly in the boosted jet regime.

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I. INTRODUCTION

Vectorlike quarks (VLQs) with masses at the TeV scale are generally predicted in a variety of extensions of the Standard Model (SM), such as the little Higgs models [1–4], composite Higgs models [5–8], two-Higgs-doublet models [9–14], and other extended models [15–18]. A common feature of these new particles is that the left- and right-handed chiral components transform in the same way under the electroweak (EW) symmetry group of the SM [19]. Unlike for chiral quarks, bare mass terms of VLQs are gauge invariant and therefore they can avoid the strict constraints by the Higgs boson data.¹ Moreover, VLQs have the potential to stabilize the EW vacuum [22,23], the so-called Cabibbo-Kobayashi-Maskawa (CKM) unitarity problem [24–28], and may also provide

explanations for various experimental anomalies, such as the W -mass one [29–33]. There exists three types of multiple VLQs, including EW singlet (T , B), doublets [(X, T) , (T, B) or (B, Y)], and triplets [(X, T, B) or (T, B, Y)]. In the models embedding these, VLQs are expected to couple preferentially to third-generation quarks and can give rise to a rich variety of phenomena at the Large Hadron Collider (LHC) and future high-energy colliders (for example, see [34–66]).

The VLQ T -state (VLQ- T , also denoted T in the remainder) has an electric charge of $+2/3e$ and occurs in any weak-isospin multiplet, whereas the VLQ Y -state (VLQ- Y , also denoted by Y in the remainder), which has an electric charge of $-(4/3)e$, only appears in doublet or triplet representations. The singlet VLQ- T has three possible decay modes: $T \rightarrow bW$, tZ , and th (unless an extended Higgs sector is present [48,51]). In the high-mass limit, the branching ratios (BRs) are $\text{BR}(T \rightarrow th) \approx \text{BR}(T \rightarrow tZ) \approx \frac{1}{2} \text{BR}(T \rightarrow Wb)$. Due to its charge, the VLQ- Y can decay only into Wb pairs with same charge with a BR of 100%. Up to now, the ATLAS and CMS Collaborations have conducted extensive VLQ searches for the (QCD induced) pair-production processes and the constraints on their masses have been obtained at a 95% confidence level (CL) [67–78]. For instance, the minimum mass of a singlet VLQ- $T(Y)$ is set at about 1.36 (1.7) TeV from direct searches by the ATLAS

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¹An extra fourth generation of SM-like quarks [20,21] should be much heavier due to the EW precision constraints.

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Collaboration with an integrated luminosity of 140 fb^{-1} [67]. The CMS Collaboration have excluded a singlet (doublet) VLQ- T mass below 1.46 (1.48) TeV at 95% CL by using 138 fb^{-1} of pp collision data in the leptonic final states [69]. In addition, such VLQ- T can be singly produced at the LHC via EW interactions and the corresponding processes are highly sensitive to the couplings between VLQs and SM quarks [79–81]. Searches performed recently by the ATLAS and CMS Collaborations set limits on VLQs masses and couplings using Run 2 recorded data [82–89]. For a benchmark signal prediction of a $SU(2)_L$ singlet VLQ- T with the mixing parameter $\kappa = 0.5$, which governs VLQ interactions with SM particles, masses below 2.1 TeV are excluded by ATLAS with an integrated luminosity of 139 fb^{-1} [85]. For the VLQ- Y , the strongest exclusion limit is set by ATLAS in the $bbbq'$ final state, where $\kappa \geq 0.3$ is excluded for mass near 2 TeV [86].

In this work, we consider a muon-proton collider with multi-TeV beam energies, which was proposed two decades ago [90–94], and more recently in Refs. [95–100]. Compared to a ep collider, a μp one can achieve a much higher center-of-mass energy and thus exotic particles production is more probable due to much larger scattering cross sections. Further, beyond the SM (BSM) studies at these types of machines usually suffer from smaller QCD backgrounds than at pp colliders. Recently, a lot of related phenomenological work has been carried out for a future μp collider [101–111]. Phenomenological studies of VLQs at an ep colliders can be found in Refs. [112–116]. In this study, we will focus on the observability of single T/Y production at a future μp collider via the $T/Y \rightarrow Wb$ decay channel both in the semileptonic and hadronic final states.

The paper is arranged as follows. In Sec. II, we consider a simplified model including the VLQ- T/Y and present the cross sections for the single production process at a μp collider with three different center-of-mass energies. In Sec. III, we discuss its observability via the decay modes $T/Y \rightarrow bW \rightarrow b\ell + E_T$ and $T/Y \rightarrow bW \rightarrow bj j$, where ℓ is a lepton, E_T is the missing transverse energy, and j is a jet (notice that the two jets will merge into a single fat one J). Finally, conclusions are presented in Sec. IV.

II. THE SIMPLIFIED MODEL AND SINGLE T/Y PRODUCTION AT A μp COLLIDER

Following the notation of Ref. [35], a generic parametrization of an effective Lagrangian for a singlet VLQ- T is given by

$$\mathcal{L}_T = \frac{gg^*}{2} \left\{ \frac{1}{\sqrt{2}} [\bar{T}_L W_\mu^+ \gamma^\mu b_L] + \frac{1}{2c_W} [\bar{T}_L Z_\mu \gamma^\mu t_L] - \frac{m_T}{2m_W} [\bar{T}_R h t_L] - \frac{m_t}{2m_W} [\bar{T}_L h t_R] \right\} + \text{H.c.}, \quad (1)$$

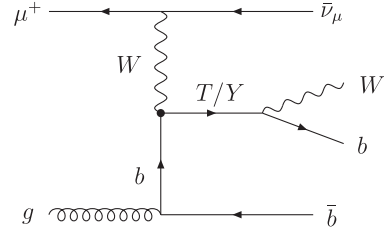


FIG. 1. Representative tree-level Feynman diagram for single production of VLQ- T/Y at a μp collider followed by the $T/Y \rightarrow bW$ decay channel.

where g is the $SU(2)_L$ gauge coupling constant, and θ_W is the Weinberg angle. Thus, there are only two model parameters: the VLQ- T quark mass m_T and the coupling strength to SM quarks in units of standard couplings, g^* . As mentioned, the singlet VLQ- T has three possible decay modes: $T \rightarrow bW$, tZ , and th . For $M_T \geq 1$ TeV, the BRs are $\text{BR}(T \rightarrow th) \approx \text{BR}(T \rightarrow tZ) \approx \frac{1}{2} \text{BR}(T \rightarrow Wb)$, which is a good approximation as expected from the Goldstone boson equivalence theorem [117–121].

Certainly, the coupling parameter, g^* , can also be described as other constants, i.e., s_L in [19] and κ_T [35]. A simple relationship among these coupling parameters is: $g^* = \sqrt{2}\kappa_T = 2s_L$ [122]. For $m_T = 1.5(1.8)$ TeV, g^* should be smaller than about 0.42 (0.56) [122]. In this work we take only a phenomenologically guided limit: $g^* \leq 0.5$, in the region $m_T \geq 1.5$ TeV.

Assuming that the VLQ- Y only couples to the SM third generation quarks, an effective Lagrangian framework can be written as

$$\mathcal{L}_Y = \frac{g\kappa_Y^{L/R}}{\sqrt{2}} [\bar{Y}_{L/R} W_\mu^- \gamma^\mu b_{L/R}] + \text{H.c.} \quad (2)$$

Note that we can assume $\kappa_Y^L = 0$ for a (B, Y) doublet, and $\kappa_Y^R = 0$ for a (T, B, Y) triplet [19]. The cross sections for the single production process and the kinematics of the final states studied here are similar for left-handed and right-handed couplings. For simplicity, we consider a benchmark scenario with right-handed couplings only in this work: $\kappa_Y = \kappa_Y^R \neq 0$ and $\kappa_Y^L = 0$, as for example in the case of the (B, Y) doublet. For large mass values of the VLQ- Y , the decay width is approximated as $\Gamma_Y \sim \kappa_Y^2 m_Y^3 \times 3.28 \times 10^{-7} \text{ GeV}^{-2}$ [64].

From the above discussions, we know that the VLQ- T/Y can be singly produced in μp collisions via Wb fusion process with a subsequent decay $T/Y \rightarrow Wb$. An example of a leading order (LO) Feynman diagram is depicted in Fig. 1.

The LO cross sections are obtained using MadGraph5-aMC@NLO [123] with default NNPDF23L01 parton distribution functions (PDFs) [124] taking the default renormalization and factorization scales. The beam

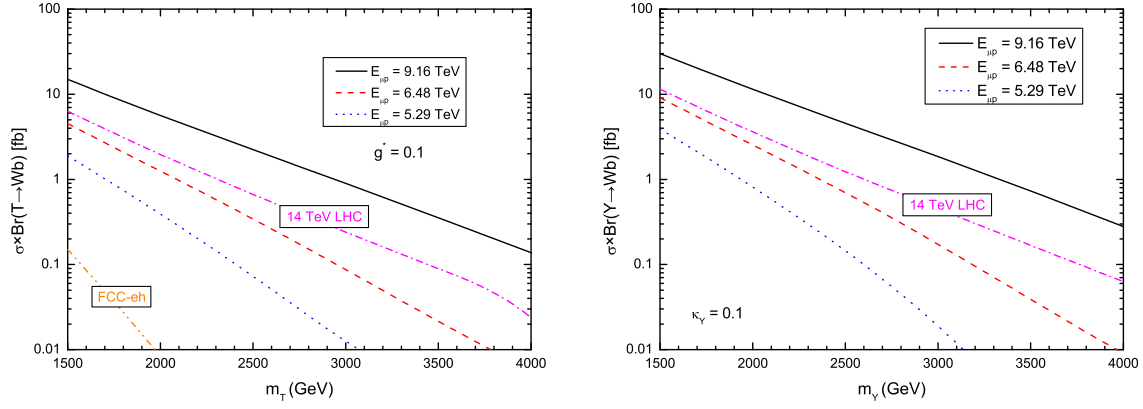


FIG. 2. Cross section of $\sigma \times \text{BR}(T/Y \rightarrow bW)$ as a function of the VLQ mass for $g^*/\kappa_Y = 0.1$ at a μp collider with three different c.m. energies. (The LHC and FCC-eh results for competing processes are also shown: see the text for details.)

energies are taken as three typical center-of-mass (c.m.) energies:

- (i) $\sqrt{s} = 5.29$ TeV, with $E_p = 7$ TeV, and $E_\mu = 1.0$ TeV,
 - (ii) $\sqrt{s} = 6.48$ TeV, with $E_p = 7$ TeV, and $E_\mu = 1.5$ TeV,
 - (iii) $\sqrt{s} = 9.16$ TeV, with $E_p = 7$ TeV, and $E_\mu = 3.0$ TeV,
- respectively.

In Fig. 2, we show the dependence of the cross sections $\sigma \times \text{BR}(T/Y \rightarrow bW)$ on the VLQ- T/Y mass for $g^*/\kappa_Y = 0.1$. Note that the conjugate processes have also been included.² As the VLQ mass grows, the cross section of single production decreases slowly due to a smaller phase space. For comparison, we also display the cross sections for the single VLQ- T production processes at the 14 TeV LHC, $\sigma(pp \rightarrow Tbj) \times \text{BR}(T \rightarrow bW)$, and the electron-hadron Future Circular Collider (FCC-eh), $\sigma(ep \rightarrow \nu Tj) \times \text{BR}(T \rightarrow bW)$ with $\sqrt{s} = 3.46$ TeV, respectively. We also find that the cross sections can be up to about one order of magnitude larger than those at the FCC-eh, and even comparable to those at the LHC with $\sqrt{s} = 14$ TeV. For $g^*/\kappa_Y = 0.1$ and $m_{T/Y} = 2$ TeV, the cross section can reach 0.4 (0.82), 1.3 (2.54), and 5.6 (11.4) fb, respectively, at a μp collider with three different c.m. energies. Certainly, the single production cross section is proportional to the square of the coupling strength g^* or κ_Y .

III. COLLIDER SIMULATION AND ANALYSIS

Next we analyze the observation potential of the discussed μp processes by performing a Monte Carlo (MC) simulation of signals (and, eventually, also background) events generated by single VLQ- T/Y production and the $T/Y \rightarrow bW$ decay channel. Considering the case of the W boson decaying leptonically, the final events are required to have exactly one isolated identified lepton (specifically, an electron), at least one b -tagged jet and large missing

transverse energy from the escaping neutrino. (We do not consider the selection for the associated bottom quark from the splitting of an initial-stage gluon into a pair of b -quarks, which is often outside of the detector acceptance due to its typically low momentum.) To increase the signal rate, we also impose that the W boson decays into a pair of quarks where the emerging jets j can be collimated so as to appear as a fat jet (J), for which a representative Feynman diagram is given in Fig. 3.

Note that the kinematics of the final states are similar for VLQ- T and VLQ- Y , so that the acceptances for the two types of VLQs are found to be the same. Thus, the VLQ- Y signals were not simulated separately. Finally, since we are including charged conjugation and the charge of the jets is not reconstructed, the contributions to the signal of the T and Y mediated processes are indistinguishable so that, eventually, they can be summed over despite, in the remainder, they are considered separately (thus, by implicitly exploiting MC truth knowledge).

A. Expected discovery and exclusion reach for VLQ- T

Next we perform a the signal-to-background analysis for the above two final states, which signal patterns are (to recap)

- (i) $\mu^+ p \rightarrow \bar{\nu}_\mu T(\rightarrow bW^+) \bar{b} \rightarrow e^+ b \bar{b} + \cancel{E}_T$ for $W^+ \rightarrow \ell^+ \nu_\ell$ with $\ell = e$.
- (ii) $\mu^+ p \rightarrow \bar{\nu}_\mu T(\rightarrow bW^+) \bar{b} \rightarrow J + b \bar{b} + \cancel{E}_T$ for $W^+ \rightarrow q \bar{q}'$ with $q, q' \in \{u, d, s, c\}$.

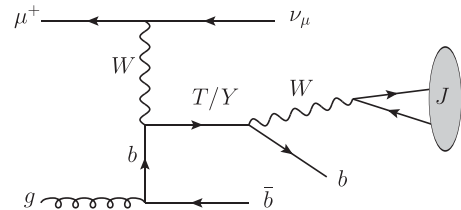


FIG. 3. Representative Feynman diagram for the $b + J$ final state.

²All calculations assume unpolarized initial states, providing conservative estimates that facilitate direct comparison with existing collider results.

TABLE I. Cross sections (in pb) for SM background processes, showing production cross sections multiplied by the relevant branching ratios ($\sigma \times \mathcal{B}$) for all specified decay channels.

Process	Decay channel	$\sqrt{s} = 5.29$ TeV	6.48 TeV	9.16 TeV
$\mu p \rightarrow \nu_\mu tb$	$t \rightarrow b\ell\nu$	12.8	17.6	29.7
	$t \rightarrow bj\bar{j}$	38.2	52.9	89.1
$\mu p \rightarrow \nu_\mu Wj$	$W \rightarrow \ell\nu$	3.7	4.76	7.16
	$W \rightarrow jj$	11.1	14.3	21.5
$\mu p \rightarrow \nu_\mu Zj$	$Z \rightarrow q\bar{q}, b\bar{b}$	5.37	6.98	10.7
$\mu p \rightarrow \nu_\mu jj$	$\dots$	378	480	701.4

Note that, for the leptonic W decay (W_{lep}) mode, we consider the final leptons to be only electrons due to large SM background processes $\mu p \rightarrow \mu jj$ and $\mu p \rightarrow \mu b\bar{b}$ whereas, for the hadronic decay (W_{had}) mode, the W boson is boosted and thus its decay products have low angular separation and are reconstructed as a fat one J (as intimated). (In our simulation, the conjugate processes of all backgrounds have also been considered.) The dominant SM backgrounds for the first final states come from the following processes:

- (i) $\mu p \rightarrow \nu_\mu tb$ with $t \rightarrow bW \rightarrow b e \nu_e$;
- (ii) $\mu p \rightarrow \nu_\mu Wj$ with $W \rightarrow e \nu_e$.

For the second final states including fat jet, the dominant SM backgrounds come from the following processes:

- (i) $\mu p \rightarrow \nu_\mu tb$ with $t \rightarrow bW \rightarrow b j \bar{j}$;
- (ii) $\mu p \rightarrow \nu_\mu Wj$ with $W \rightarrow j \bar{j}$;
- (iii) $\mu p \rightarrow \nu_\mu Zj$ with $Z \rightarrow q\bar{q}$ and $Z \rightarrow b\bar{b}$;
- (iv) $\mu p \rightarrow \nu_\mu jj$.

The presented cross sections include both production and decay branching ratios ($\sigma \times \mathcal{B}$), accounting for all specified final states in Table I.

Signal and background events are generated at LO using MadGraph5-aMC@NLO with the aforementioned PDFs. Further, parton showers and hadronization are performed using PYTHIA8.3 [125]. Then, Delphes 3.4.2 [126] is used for fast detector simulation, using the standard LHeC detector card. The anti- k_t algorithm [127] with a jet radius of $R = 0.4$ is used for small-radius (small- R) jets. For fat jet studies, the jets are reconstructed by the Cambridge-Aachen algorithm [128,129], implemented in the FastJet package [130], assuming a cone radius $R = 0.8$. The b -tagging efficiency is set to 80%, while the misidentification rates are 0.1% for light-flavor jets (u, d, s, g) and 5% for charm jets. Finally, the reconstructed events are analyzed using MadAnalysis 5 [131,132].

To identify objects, the following basic (or generation) cuts are chosen

$$p_T^{\ell/\text{jet}} > 15 \text{ GeV}, \quad |\eta_\ell| < 2.5, \quad |\eta_{\text{jet}}| < 5, \quad \Delta R_{xy} > 0.4, \quad (3)$$

where $p_T^{\ell/\text{jet}}$ and $|\eta_{\ell/\text{jet}}|$ are the transverse momentum and pseudorapidity of the electrons ($\ell = e$) and jets b and J .

Here, $\Delta R(x, y) = \sqrt{\Delta\Phi^2 + \Delta\eta^2}$ is the separation in the pseudorapidity-azimuth plane between the pairs of objects x and y , where $x, y = e, b, J$, wherein b represents a b -tagged jet.

1. Analysis of signal events in the W_{lep} channel

The following three signal benchmark points are taken with the fixed parameter $g^* = 0.1$:

- (i) T_{1500} : $m_T = 1500$ GeV;
- (ii) T_{2000} : $m_T = 2000$ GeV;
- (iii) T_{2500} : $m_T = 2500$ GeV.

In Fig. 4, we plot some differential distributions for the three signal benchmark points (T_{1500} , T_{2000} , and T_{2500}) and SM backgrounds at $\sqrt{s} = 5.29$ TeV in the W_{lep} channel, including the transverse momentum distributions of the lepton (p_T^ℓ), the transverse momentum ($p_T^{b_{1,2}}$) and pseudorapidity ($\eta^{b_{1,2}}$) distributions for both the leading and sub-leading b -tagged jets, the pseudorapidity-azimuth separation between the leading b -tagged jet and the lepton of $\Delta R_{\ell, b_1}$, the missing transverse energy E_T and the invariant mass distribution M_{be} . Because of the larger mass of VLQ- T , the decay products are highly energetic, and thus the p_T^ℓ and p_T^b peaks of the signals are larger than those of the SM backgrounds. According to the behaviors of these distributions, we impose the following cuts to distinguish the signal from the SM backgrounds.

- (i) Cut 1: Exactly one isolated electron with $p_T^\ell > 150$ GeV (i.e., events with final state muons or taus are vetoed).
- (ii) Cut 2: Events must contain at least one b -tagged jet ($N_b \geq 1$). The selected b -jet (defined as the highest- p_T b -tagged jet when multiple candidates exist) must satisfy $p_T^b > 200$ GeV and maintain a pseudorapidity-azimuth separation from the lepton of $\Delta R_{\ell, b} > 2.7$.
- (iii) Cut 3: The transverse missing energy is required to be $E_T > 300$ GeV.
- (iv) Cut 4: The be invariant mass is required to be $M_{be} > 1000$ GeV.

Since the behaviors of the relevant kinematic distributions at $\sqrt{s} = 6.48$ and 9.16 TeV are similar to the case of

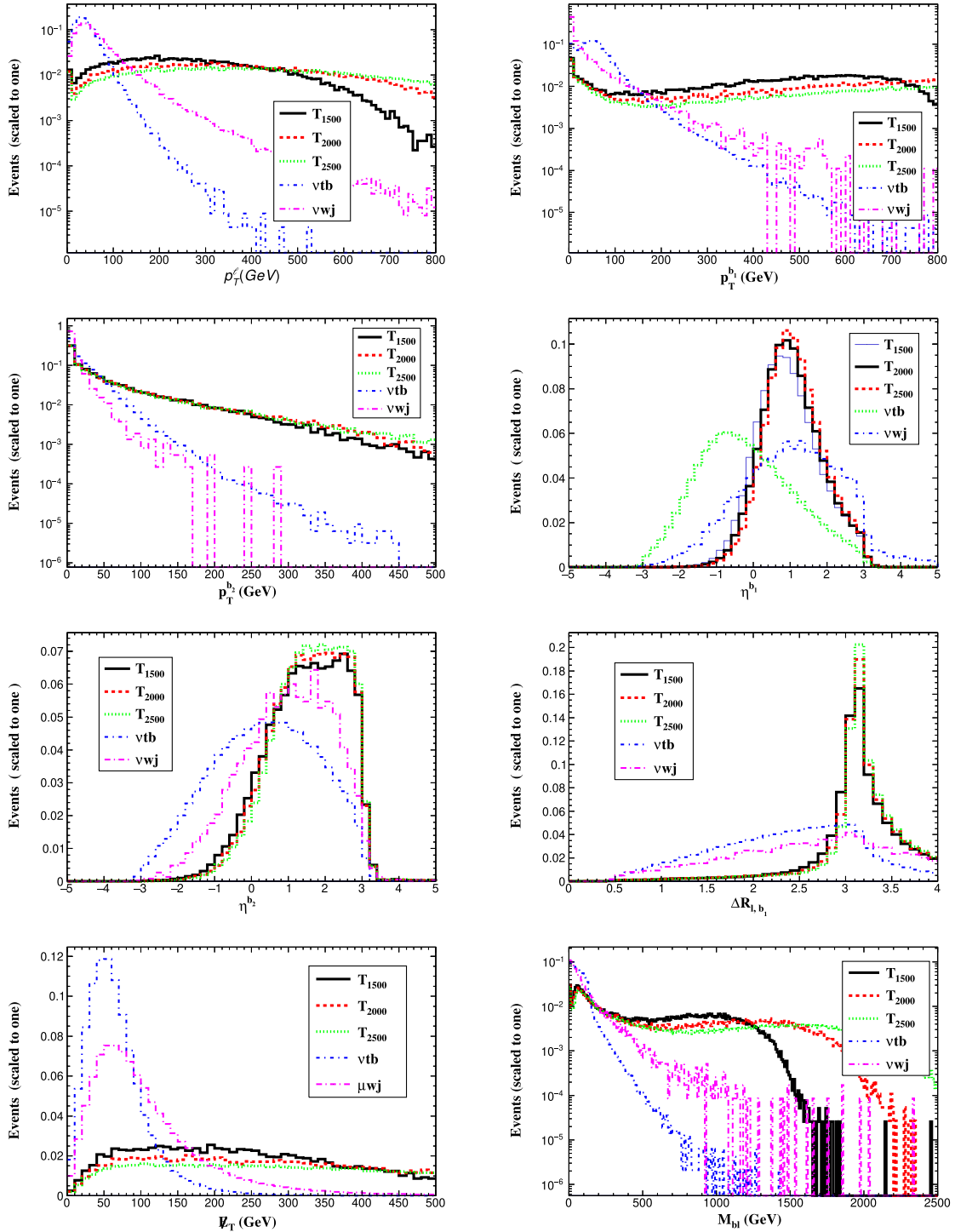


FIG. 4. Normalized distributions of p_T^l , $p_T^{b_{1,2}}$, $\eta^{b_{1,2}}$, $\Delta R_{e,b_1}$, E_T , M_{be} for the three signals [with $m_T = 1500$ GeV (solid), 2000 GeV (dashed), and 2500 GeV (dotted)] and SM backgrounds at $\sqrt{s} = 5.29$ TeV in the W_{lep} channel.

$\sqrt{s} = 5.29$ TeV, we do not display these here. Based on the behaviors of these distributions, we impose the same cuts to enhance the sensitivity also at other two c.m. energies.

The cross sections of the three typical signals and the relevant SM backgrounds are presented in Tables II–IV after imposing the aforementioned cuts. The cross sections for

signal and SM backgrounds are calculated by $\sigma_{\text{after cut}} = \sigma_0 \times \epsilon_{\text{cut}}$, where σ_0 denotes the initial cross section and ϵ_{cut} represents the cumulative efficiency up to that cut. The efficiency for an individual cut can then be obtained from the ratio of consecutive cross section values in the table. The values in the last column of the cut-flow tables represent the

TABLE II. Cut flow of the cross sections (in fb) for the signals and SM backgrounds at $\sqrt{s} = 5.29$ TeV in the W_{lep} channel. Here, we set a benchmark value of $g^* = 0.1$.

Cuts	Signals			Backgrounds	
	T_{1500}	T_{2000}	T_{2500}	νtb	νWj
Basic	0.35	0.074	0.014	7296	2331
Cut 1	0.12	0.028	0.0054	26	102
Cut 2	0.084	0.02	0.004	5.1	0.42
Cut 3	0.06	0.015	0.0032	0.84	0.13
Cut 4	0.024	0.012	0.003	0.077	0.037
Total eff.	0.06	0.14	0.18	6.0×10^{-6}	1.0×10^{-5}

TABLE III. Cut flow of the cross sections (in fb) for the signals and SM backgrounds at $\sqrt{s} = 6.48$ TeV in the W_{lep} channel. Here, we set a benchmark value of $g^* = 0.1$.

Cuts	Signals			Backgrounds	
	T_{1500}	T_{2000}	T_{2500}	νtb	νWj
Basic	0.84	0.24	0.065	9328	2761
Cut 1	0.29	0.09	0.026	41	143
Cut 2	0.19	0.065	0.018	8.8	0.51
Cut 3	0.14	0.05	0.015	1.62	0.22
Cut 4	0.06	0.04	0.013	0.28	0.076
Total eff.	0.06	0.15	0.19	1.6×10^{-5}	1.6×10^{-5}

TABLE IV. Cut flow of the cross sections (in fb) for the signals and SM backgrounds at $\sqrt{s} = 9.16$ TeV in the W_{lep} channel. Here, we set a benchmark value of $g^* = 0.1$.

Cuts	Signals			Backgrounds	
	T_{1500}	T_{2000}	T_{2500}	νtb	νWj
Basic	2.72	1.03	0.41	13,055	3437
Cut 1	0.96	0.4	0.16	82	208
Cut 2	0.67	0.28	0.12	18	0.87
Cut 3	0.48	0.22	0.11	4.3	0.57
Cut 4	0.21	0.18	0.09	0.27	0.37
Total eff.	0.063	0.15	0.19	9.0×10^{-6}	5.2×10^{-5}

total cumulative efficiencies for both signal and background processes. The appendix include the details about the events and efficiencies before and after each cut at $\sqrt{s} = 5.29$ TeV in the W_{lep} channel. Notably, all background processes are very significantly suppressed at the end of the cut flow, and the cross section of the total SM background is about 0.11 fb at $\sqrt{s} = 5.29$ TeV, 0.36 fb at $\sqrt{s} = 6.48$ TeV, and 0.64 fb at $\sqrt{s} = 9.16$ TeV, respectively.

2. Analysis of signal events in the W_{had} channel

In Fig. 5, we show the normalized distributions of the transverse momentum of the fat jet (p_T^J) and leading b -tagged jet (p_T^b) as well as those of the fat jet mass (M_J)

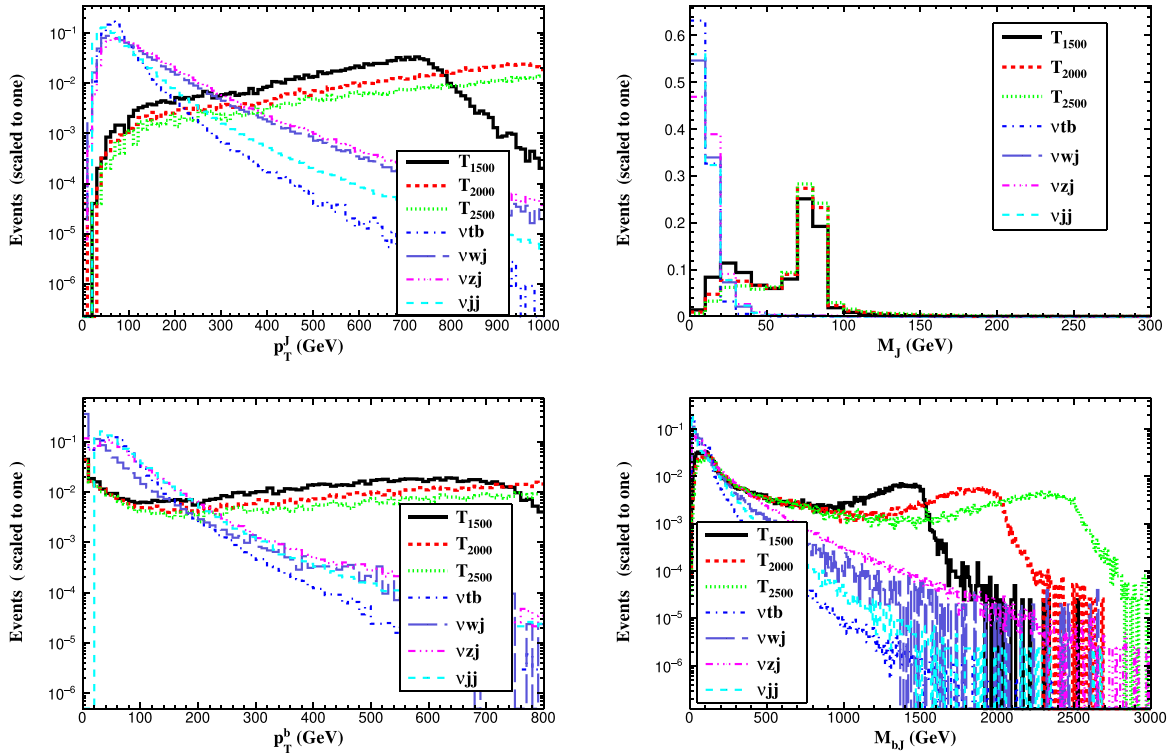


FIG. 5. Normalized distributions of p_T^J , M_J , p_T^b , M_{bJ} for the signals (with $m_T = 1500, 2000$, and 2500 GeV) and SM backgrounds at $\sqrt{s} = 5.29$ TeV in the W_{had} channel.

TABLE V. Cut flow of the cross sections (in fb) for the signals and SM backgrounds at $\sqrt{s} = 5.29$ TeV in the W_{had} channel. Here, we set a benchmark value of $g^* = 0.1$.

Cuts	Signals			Backgrounds			
	T_{1500}	T_{2000}	T_{2500}	νtb	νWj	νZj	νjj
Basic	1.21	0.25	0.05	38,160	11,100	5365	378,000
Cut 1	1.16	0.25	0.046	954	1576	268	17,010
Cut 2	0.61	0.15	0.03	23	68	11	2438
Cut 3	0.39	0.094	0.018	2.67	0.33	0.53	2.04
Cut 4a	0.242	...	...	0.106	0.054	0.183	0.216
Cut 4b	...	0.078	0.017	0.021	0.053	0.125	0.087
Total eff _{Cut-4a}	20%	...	...	2.8×10^{-6}	4.9×10^{-6}	3.4×10^{-5}	5.7×10^{-7}
Total eff _{Cut-4b}	...	31%	35%	5.6×10^{-7}	4.8×10^{-6}	2.3×10^{-5}	2.3×10^{-7}

TABLE VI. Cut flow of the cross sections (in fb) for the signals and SM backgrounds at $\sqrt{s} = 6.48$ TeV in the W_{had} channel. Here, we set a benchmark value of $g^* = 0.1$.

Cuts	Signals			Backgrounds			
	T_{1500}	T_{2000}	T_{2500}	νtb	νWj	νZj	νjj
Basic	2.88	0.81	0.22	52,920	14,280	6980	480,000
Cut 1	2.76	0.78	0.22	1534	2142	138	23,520
Cut 2	1.44	0.46	0.13	45	114	58	377
Cut 3	0.86	0.29	0.084	4.76	0.61	0.77	0.53
Cut 4a	0.55	...	...	0.40	0.36	0.31	0.17
Cut 4b	...	0.24	0.08	0.167	0.248	0.24	0.082
Total eff _{Cut-4a}	19%	...	...	7.6×10^{-6}	2.5×10^{-5}	4.5×10^{-5}	3.5×10^{-7}
Total eff _{Cut-4b}	...	30%	36%	3.2×10^{-6}	1.7×10^{-5}	3.5×10^{-5}	1.7×10^{-7}

and of the invariant mass of the b -tagged and fat jet system (M_{bJ}). (Again, these are presented for $\sqrt{s} = 5.29$ TeV in the W_{had} channel, but they are very similar for $\sqrt{s} = 6.48$ TeV and 9.16 TeV.) According to the behaviors of these distributions, we impose the following cuts to extract the signal from the SM backgrounds.

- (i) Cut 1: The transverse momentum for the fat jet is required to be $p_T^J > 200$ GeV.
- (ii) Cut 2: The fat jet mass is such that $|M_J - m_W| < 15$ GeV.
- (iii) Cut 3: Events are required to have at least one b -tagged jet ($N_b \geq 1$). The leading b -tagged jet must

satisfy $p_T^b > 200$ GeV, and its separation from the lepton in pseudorapidity-azimuth space must satisfy $\Delta R_{e,b} > 2.7$.

- (iv) Cut 4: The aforementioned invariant mass is required to be: $M_{bJ} > 1300$ GeV for $m_T < 1800$ GeV (Cut 4a), and $M_{bJ} > 1500$ GeV for $m_T \geq 1800$ GeV (Cut 4b).

In Tables V–VII, we present the cross sections for the signals and relevant SM backgrounds after imposing the above cuts. Here, one can see that all the SM backgrounds are suppressed very efficiently, while the signals still have relatively good efficiency at the end of the cut flow. The

TABLE VII. Cut flow of the cross sections (in fb) for the signals and SM backgrounds at $\sqrt{s} = 9.16$ TeV in the W_{had} channel. Here, we set a benchmark value of $g^* = 0.1$.

Cuts	Signals			Backgrounds			
	T_{2000}	T_{3000}	T_{4000}	νtb	νWj	νZj	νjj
Basic	3.62	0.58	0.0884	89,000	21,480	10,700	701,400
Cut 1	3.54	0.58	0.088	3204	3651	2354	38,577
Cut 2	2.32	0.4	0.062	196	301	107	982
Cut 3	1.22	0.232	0.036	12.5	1.85	1.07	2.1
Cut 4	1.09	0.226	0.035	0.56	0.47	0.39	0.22
Total eff.	28%	36.5%	38.2%	3.6×10^{-6}	2.2×10^{-5}	3.7×10^{-5}	3.2×10^{-7}

cross section of the total SM background in the W_{had} channel is about 0.28 fb at $\sqrt{s} = 5.29$ TeV, 0.52 fb at $\sqrt{s} = 6.48$ TeV, and 1.65 fb at $\sqrt{s} = 9.16$ TeV, respectively.

3. Discovery and exclusion significance

In order to analyze the sensitivity, we estimate the expected discovery ($\mathcal{Z}_{\text{dis}}$) and exclusion ($\mathcal{Z}_{\text{exc}}$) limits by using the following formulas [133]:

$$\begin{aligned}\mathcal{Z}_{\text{dis}} &= \sqrt{2 \left[(s+b) \ln \left(\frac{(s+b)(1+\delta_{\text{sys}}^2 b)}{b + \delta_{\text{sys}}^2 b(s+b)} \right) - \frac{1}{\delta_{\text{sys}}^2} \ln \left(1 + \frac{\delta_{\text{sys}}^2 s}{1 + \delta_{\text{sys}}^2 b} \right) \right]}, \\ \mathcal{Z}_{\text{exc}} &= \sqrt{2 \left[s - b \ln \left(\frac{b+s+x}{2b} \right) - \frac{1}{\delta_{\text{sys}}^2} \ln \left(\frac{b-s+x}{2b} \right) \right] - (b+s-x) \left(1 + \frac{1}{\delta_{\text{sys}}^2 b} \right)},\end{aligned}\quad (4)$$

with

$$x = \sqrt{(s+b)^2 - 4\delta_{\text{sys}}^2 s b^2 / (1 + \delta_{\text{sys}}^2 b)}. \quad (5)$$

Here, s and b are the numbers of signal and background events, respectively, which can be obtained by multiplying the total signal and SM background cross sections, respectively, by the integrated luminosity while δ is the percentage systematic error on the SM background estimate. In the limit case of $\delta_{\text{sys}} \rightarrow 0$, these expressions can be simplified to

$$\begin{aligned}\mathcal{Z}_{\text{dis}} &= \sqrt{2[(s+b) \ln(1+s/b) - s]}, \\ \mathcal{Z}_{\text{exc}} &= \sqrt{2[s - b \ln(1+s/b)]}.\end{aligned}\quad (6)$$

The integrated luminosity is set at 100 fb^{-1} for all three center-of-mass energies [96].

As a result, in Fig. 6, we present the 95% CL exclusion limit and 5σ discovery reaches in the plane of $g^* - m_T$ at the three different c.m. energies in the W_{lep} channel. To illustrate the effect of systematic uncertainty on the significance, we select three cases: no systematics ($\delta = 0$) and two typical systematic uncertainties ($\delta = 10\%$ and $\delta = 20\%$). One can see that the higher systematic uncertainty of background can decrease the discovery capability and the exclusion power. With a realistic 10% systematic error, the discoverable (at 5σ level) region is $g^* \in [0.3, 0.5]$ with $m_T \in [1500, 2100]$ GeV at $\sqrt{s} = 5.29$ TeV, $g^* \in [0.27, 0.5]$ with $m_T \in [1500, 2400]$ GeV at $\sqrt{s} = 6.48$ TeV, and $g^* \in [0.18, 0.5]$ with $m_T \in [1500, 3250]$ GeV at $\sqrt{s} = 9.16$ TeV. As for the exclusion (at 95% CL) region, this is $g^* \in [0.17, 0.5]$ with $m_T \in [1500, 2500]$ GeV at $\sqrt{s} = 5.29$ TeV, $g^* \in [0.14, 0.5]$ with $m_T \in [1500, 2900]$ GeV at $\sqrt{s} = 6.48$ TeV, and $g^* \in [0.09, 0.5]$ with $m_T \in [1500, 3700]$ GeV at $\sqrt{s} = 9.16$ TeV.

In Fig. 7, we present the 95% CL exclusion limit and 5σ discovery reach in the plane of $g^* - m_T$ at the three different c.m. energies in the W_{had} channel. One finds that

the sensitivities are better than those for the signal in the W_{lep} channel. In the presence of 10% systematic uncertainty, the discoverable (at 5σ level) region is $g^* \in [0.15, 0.5]$ with $m_T \in [1500, 2520]$ GeV at $\sqrt{s} = 5.29$ TeV, $g^* \in [0.14, 0.5]$ with $m_T \in [1500, 2900]$ GeV at $\sqrt{s} = 6.48$ TeV, and $g^* \in [0.1, 0.5]$ with $m_T \in [1800, 3750]$ GeV at $\sqrt{s} = 9.16$ TeV. As for the exclusion (at 95% CL) region, this is $g^* \in [0.08, 0.5]$ with $m_T \in [1500, 2850]$ GeV at $\sqrt{s} = 5.29$ TeV, $g^* \in [0.07, 0.5]$ with $m_T \in [1500, 3340]$ GeV at $\sqrt{s} = 6.48$ TeV, and $g^* \in [0.06, 0.5]$ with $m_T \in [1800, 4500]$ GeV at $\sqrt{s} = 9.16$ TeV.

So far, we have treated the VLQ- T signals in narrow width approximation (NWA), which is appropriate when the width-to-mass ratio Γ/M of the resonance is per mille level, however, for values at percent level and beyond, the full Breit-Wigner (BW) should be adopted, as demonstrated in [134]. For the high-coupling scenarios where Γ/m becomes substantial, a full off-shell calculation including interference would be necessary, making our NWA results conservative upper bounds. Future studies will incorporate these effects through complete matrix-element calculations. In order to appreciate the difference between the two approaches in our case, we proceed as in Ref. [135] and limit ourselves to sample Γ_T/m_T values below 10%. For large mass values of the VLQ- T , the decay width is approximated as $\Gamma_T \sim (g^*)^2 m_T^3 \times 6.5 \times 10^{-7} \text{ GeV}^{-2}$, and therefore g^* can be chosen to obtain a specific Γ_T/m_T ratio. Thus in Figs. 6 and 7, we also display the contours of Γ_T/m_T for three typical values: 1%, 5%, and 10%. For a fixed value of $\Gamma_T/m_T = 10\%$ and systematic uncertainty of 10% within, e.g., the W_{had} channel (the dominant final state), the signal can be discovered (at 5σ level) for a VLQ- T mass of about 1900, 2000, and 2500 GeV at $\sqrt{s} = 5.29, 6.48$, and 9.16 TeV, respectively. For the case of exclusion (at 95% CL), the lower limit on the VLQ- T mass is about 2250, 2400, and 3000 GeV, at $\sqrt{s} = 5.29, 6.48$, and 9.16 TeV, respectively. Hence, width effects can generally be significant.

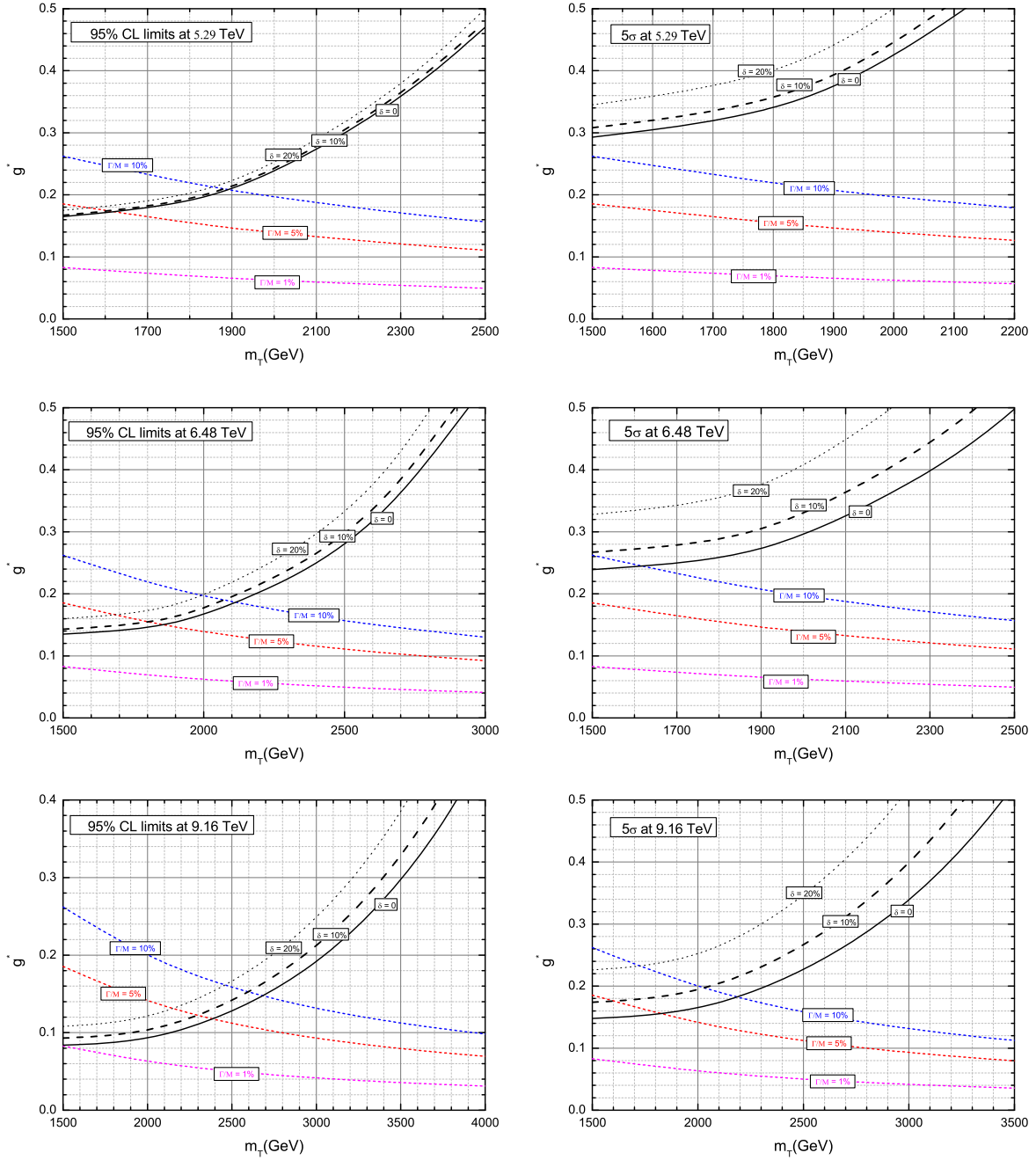


FIG. 6. 95% CL exclusion limit (left panel) and 5σ discovery reach (right panel) contour plots for the signal in $g^* - m_T$ in the W_{lep} channel at the three different c.m. energies. Short-dashed lines denote the contours of Γ_T/m_T .

Very recently, the authors of Ref. [122] have presented a comprehensive review of the most up-to-date exclusion limits on VLQs derived from ATLAS and CMS data at the LHC, wherein single VLQ- T production constrains the mixing parameter κ to values below 0.26 (0.42) for $m_T \sim 1.5(2.0)$ TeV. To enable more intuitive comparison of VLQ- T search capabilities across different collider scenarios, we have supplemented Table VIII with two contour plots in Fig. 8, visually presenting the

exclusion limits and discovery reaches in the $g^* - m_T$ parameter space. Our analysis for the μp collider assumes a conservative 10% systematic uncertainty, while other studies employ different assumptions (see respective references for details). This comprehensive comparison demonstrates that a future muon-proton collider would provide complementary sensitivity for singlet VLQ- T searches, particularly in the high-mass region.

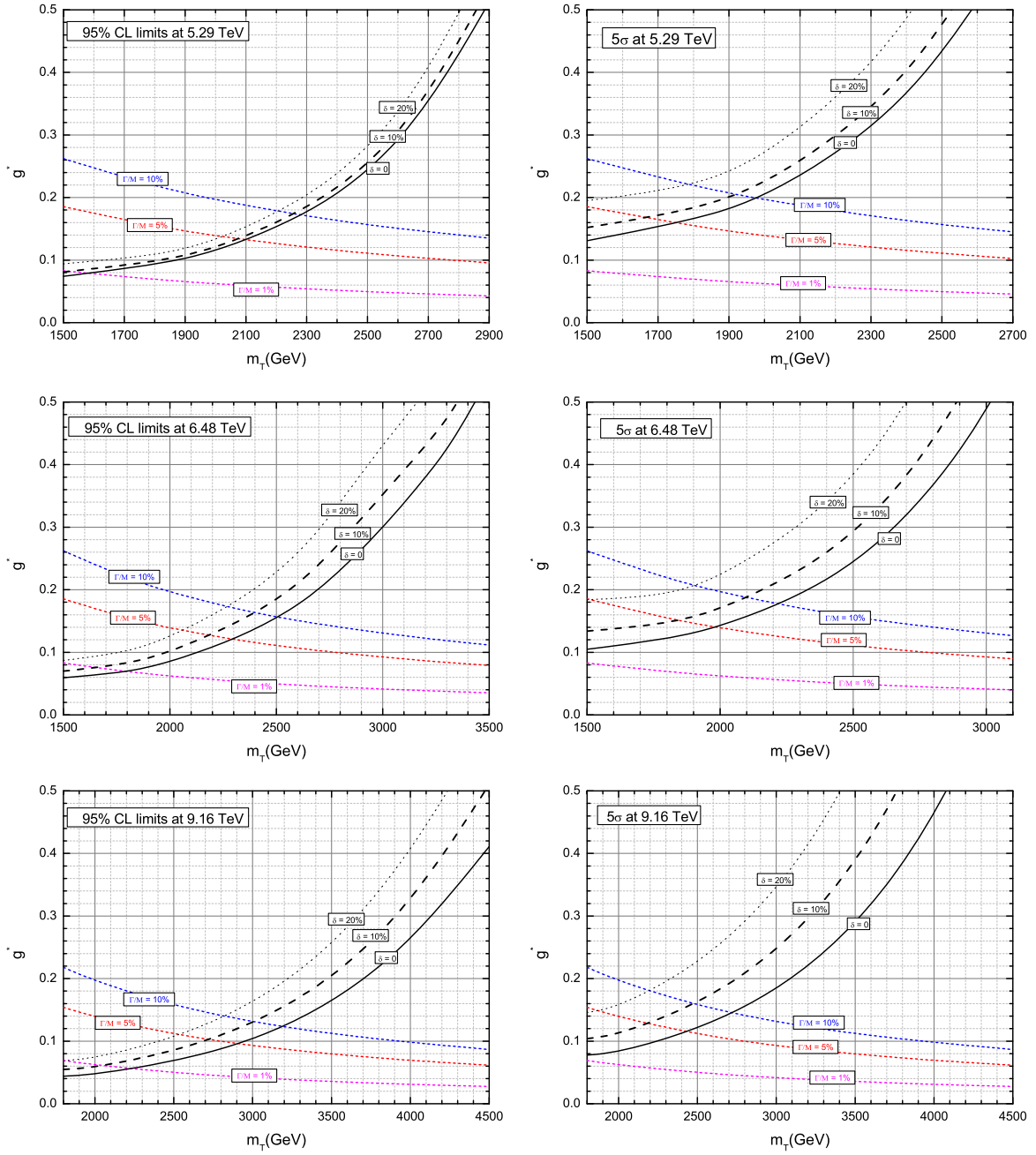


FIG. 7. 95% CL exclusion limit (left panel) and 5σ discovery reach (right panel) contour plots for the signal in $g^* - m_T$ in the W_{had} channel at the three different c.m. energies. Short-dashed lines denote the contours of Γ_T/m_T .

B. Expected discovery and exclusion reaches for VLQ- Y

As explained, the MC analysis for the VLQ- Y case is identical to the previous one. Thus, similarly to what previously done, in Fig. 9, we present the 95% CL exclusion limit and 5σ discovery reach in the plane of $\kappa_Y - m_Y$ at the usual three different c.m. energies, albeit limitedly to the fat jet final state (the dominant

one, thereby neglecting the subdominant leptonic signal). In the presence of 10% systematic uncertainty, the discoverable (at 5σ level) region is $\kappa_Y \in [0.12, 0.5]$ with $m_Y \in [1700, 2700]$ GeV at $\sqrt{s} = 5.29$ TeV, $\kappa_Y \in [0.10, 0.5]$ with $m_Y \in [1700, 3100]$ GeV at $\sqrt{s} = 6.48$ TeV, and $\kappa_Y \in [0.08, 0.5]$ with $m_Y \in [1700, 4100]$ GeV at $\sqrt{s} = 9.16$ TeV. Instead, a VLQ- Y can be excluded (at 95% CL) for $\kappa_Y \in [0.07, 0.5]$ with $m_Y \in [1700, 3020]$ GeV

TABLE VIII. Some results of searching for the singlet VLQ- T at different high-energy colliders. Here, the symbol “...” stands for no relevant results in the reference. The results in this work correspond to a mild systematic uncertainty of 10% at a μp collider with an integrated luminosity of 100 fb^{-1} .

Channel	Dataset	Excluding capability		Discovery capability		References
		g^*	m_T/TeV	g^*	m_T/TeV	
$T \rightarrow tZ$	LHC @14 TeV, 3 ab^{-1}	[0.06, 0.25]	[0.9, 1.5]	[0.10, 0.42]	[0.9, 1.5]	[139]
$T \rightarrow th$	LHC @14 TeV, 3 ab^{-1}	[0.16, 0.5]	[1.0, 1.6]	[0.24, 0.72]	[1.0, 1.6]	[140]
$T \rightarrow bW^+$	LHC @14 TeV, 3 ab^{-1}	[0.19, 0.5]	[1.3, 2.4]	[0.31, 0.5]	[1.3, 1.9]	[136]
$T \rightarrow bW^+$	$e\gamma$ collider @2 TeV, 1 ab^{-1}	[0.13, 0.5]	[0.8, 1.6]	...	...	[141]
$T \rightarrow tZ$	$e\gamma$ collider @3 TeV, 3 ab^{-1}	[0.15, 0.23]	[1.3, 2.0]	[0.23, 0.5]	[1.3, 2.0]	[137]
$T \rightarrow th$	$e\gamma$ collider @3 TeV, 3 ab^{-1}	[0.14, 0.50]	[1.3, 2.0]	[0.27, 0.5]	[1.3, 2.0]	[142]
$T \rightarrow bW^+$	e^+e^- collider @3 TeV, 5 ab^{-1}	[0.15, 0.40]	[1.5, 2.6]	[0.24, 0.44]	[1.5, 2.4]	[138]
$T \rightarrow tZ$	e^+e^- collider @3 TeV, 5 ab^{-1}	[0.19, 0.40]	[1.3, 2.5]	[0.31, 0.5]	[1.3, 2.3]	[143]
$T \rightarrow bW \rightarrow b\ell\nu$	μp collider @4.58 TeV	[0.16, 0.48]	[1.5, 2.5]	[0.31, 0.5]	[1.5, 2.1]	This work
	μp collider @6.48 TeV	[0.14, 0.5]	[1.5, 2.9]	[0.24, 0.5]	[1.5, 2.4]	
	μp collider @9.16 TeV	[0.09, 0.4]	[1.5, 3.7]	[0.18, 0.5]	[1.5, 3.2]	
$T \rightarrow bW \rightarrow bJ$	μp collider @4.58 TeV	[0.08, 0.5]	[1.5, 2.85]	[0.14, 0.5]	[1.5, 2.52]	
	μp collider @6.48 TeV	[0.07, 0.5]	[1.5, 3.34]	[0.13, 0.5]	[1.5, 2.9]	
	μp collider @9.16 TeV	[0.06, 0.5]	[2.0, 4.5]	[0.1, 0.5]	[1.8, 3.75]	

at $\sqrt{s}=5.29 \text{ TeV}$, $\kappa_Y \in [0.06, 0.5]$ with $m_Y \in [1700, 3540] \text{ GeV}$ at $\sqrt{s} = 6.48 \text{ TeV}$, and $\kappa_Y \in [0.04, 0.5]$ with $m_Y \in [1700, 4800] \text{ GeV}$ at $\sqrt{s} = 9.16 \text{ TeV}$. Furthermore, for a fixed value of $\Gamma_Y/m_Y = 10\%$, the VLQ- Y can be discovered (at 5σ level) with a mass of about 2300, 2500, and 3000 GeV at $\sqrt{s} = 5.29, 6.48$, and 9.16 TeV , respectively. Instead, the 95% CL excluded region for the VLQ- T mass is below about 2600, 2900, and 3600 GeV at $\sqrt{s} = 5.29, 6.48$, and 9.16 TeV , respectively.

Very recently, the authors of Ref. [63] investigated the expected limits for the VLQ- Y state via single production of Y followed by decay channel $Y \rightarrow Wb$ at the LHC with

$\sqrt{s} = 14 \text{ TeV}$ and the future high-energy pp colliders. Considering an integrated luminosity of 300 (3000) fb^{-1} at the 14 TeV LHC with a systematic uncertainty $\delta = 10\%$, the VLQ- Y can be discovered (at 5σ level) over the region $\kappa_Y \in [0.11, 0.5]$ with $m_Y \in [1500, 3200] \text{ GeV}$ ($\kappa_Y \in [0.1, 0.5]$ with $m_Y \in [1500, 3350] \text{ GeV}$), and excluded (at 95% CL) over the region $\kappa_Y \in [0.06, 0.5]$ with $m_Y \in [1500, 3800] \text{ GeV}$ ($\kappa_Y \in [0.05, 0.5]$ with $m_Y \in [1500, 3970] \text{ GeV}$). For a fixed value of $\Gamma_Y/m_Y = 10\%$, the VLQ- Y can be discovered (excluded) with a mass about 2200 (2600) GeV at the High-Luminosity LHC (HL-LHC). Besides, the authors of Ref. [64] studied single production

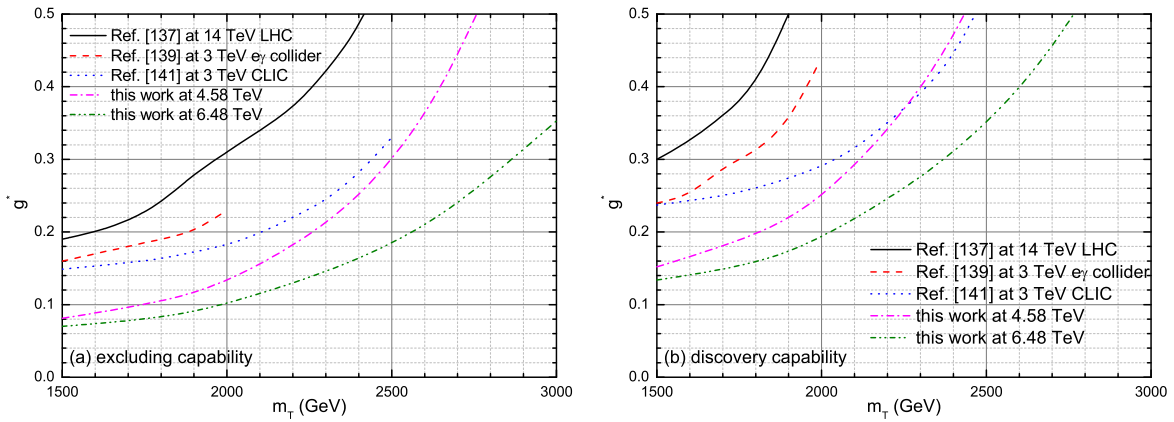


FIG. 8. (a) Exclusion limits and (b) discovery reaches for the singlet VLQ- T in the $g^* - m_T$ plane, comparing sensitivities across future high-energy colliders. Results include projections for LHC (14 TeV, 3 ab^{-1}), $e\gamma$ (3 TeV, 3 ab^{-1}), e^+e^- (3 TeV, 5 ab^{-1}), and μp (4.58 and 9.16 TeV, 100 fb^{-1}) colliders from Refs. [136–138].

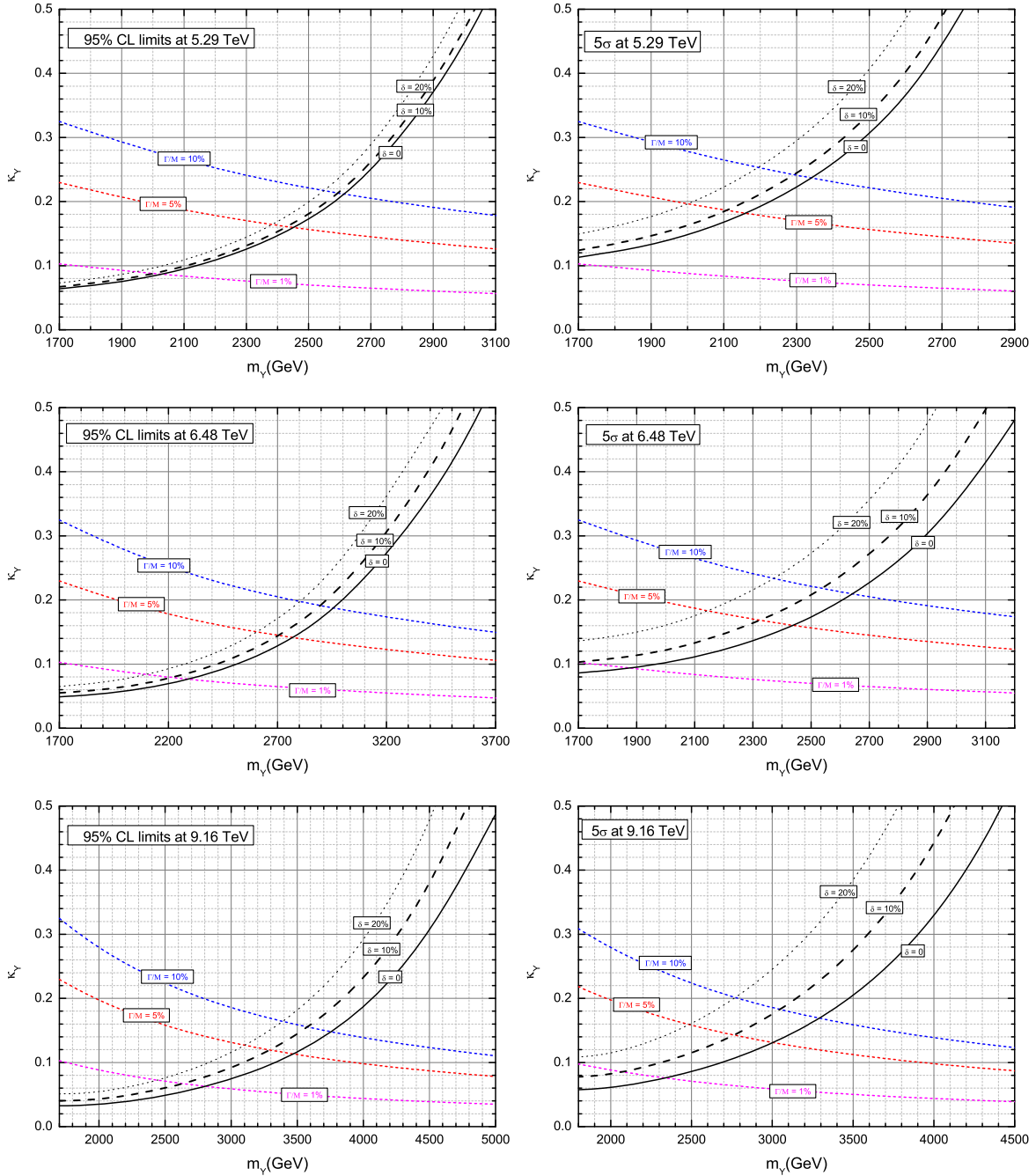


FIG. 9. 95% CL exclusion limit (left panel) and 5σ discovery reach (right panel) contour plots for the signal in $\kappa - m_Y$ at the three different center-of-mass energies. Short-dashed lines denote the contours of Γ_Y/m_Y .

of VLQ- Y at the HL-LHC with $\sqrt{s} = 14$ TeV via the fully hadronic mode $Y \rightarrow bW \rightarrow bj\bar{j}$. For the integrated luminosity projection of 3000 fb^{-1} and $\kappa_Y = 0.5(0.3)$, the lower limits for m_Y were obtained as 2350 (1550) GeV for exclusion (at 95% CL), and 1900 (1250) GeV for discovery (at 5σ level). Thus, we conclude again that our study can drive complementary searches for a possible doublet VLQ- Y at a future muon-proton collider.

IV. CONCLUSIONS

In this paper, we have studied the potential of a future μp collider to search for heavy VLQs of type T and Y via the single production mode $\mu g \rightarrow \nu_\mu bT/Y$ and subsequent decay $T/Y \rightarrow bW$. To be as model-independent as possible, a simplified framework with only two free parameters was applied: the VLQs mass $m_{T/Y}$ and the EW coupling constant g^*/κ_Y . Specifically, we have presented a search

strategy at such a possible future machine with the three typical c.m. energies $\sqrt{s} = 5.29, 6.48$, and 9.16 TeV, for VLQ- T and VLQ- Y signals with two final states: one electrons plus one b -tagged jet and missing energy in the W_{lep} channel, and one fat jet plus one b -tagged jet and missing energy in the W_{had} channel. After performing a detector level simulation for the signal and relevant SM backgrounds, the 5σ discovery prospects and 95% CL exclusion limits over the relevant parameter space were obtained.

From the numerical results, all tested against existing experimental results from the LHC from searches for VLQ, we have obtained the following results.

- (1) Due to a larger cross section for the final state including a fat W -jet with respect to the leptonic W decay channel, the sensitivities in the W_{had} channel are better than those in the W_{lep} channel, respectively.
- (2) Considering a systematic uncertainty of 10% with an integrated luminosity of 100 fb^{-1} , for the VLQ- T , the discoverable (at 5σ level) region is $g^* \in [0.15, 0.5]$ with $m_T \in [1500, 2520]$ GeV at $\sqrt{s} = 5.29$ TeV, gradually changing to $g^* \in [0.13, 0.5]$ with $m_T \in [1500, 2900]$ GeV at $\sqrt{s} = 6.48$ TeV, and to $g^* \in [0.1, 0.5]$ with $m_T \in [1800, 3750]$ GeV at $\sqrt{s} = 9.16$ TeV. Conversely, the 95% CL exclusion limit is attained over the parameter space region $g^* \in [0.08, 0.5]$ with $m_T \in [1500, 2750]$ GeV at $\sqrt{s} = 5.29$ TeV, $g^* \in [0.07, 0.5]$ with $m_T \in [1500, 3340]$ GeV at $\sqrt{s} = 6.48$ TeV, and $g^* \in [0.06, 0.5]$ with $m_T \in [1800, 4500]$ GeV at $\sqrt{s} = 9.16$ TeV.
- (3) For the above systematic uncertainty and integrated luminosity, in the case of the VLQ- Y , the discoverable (at 5σ level) region is $\kappa_Y \in [0.12, 0.5]$ with $m_Y \in [1700, 2700]$ GeV at $\sqrt{s} = 5.29$ TeV, $\kappa_Y \in [0.10, 0.5]$ with $m_Y \in [1700, 3100]$ GeV at $\sqrt{s} = 6.48$ TeV, and $\kappa_Y \in [0.08, 0.5]$ with $m_Y \in [1700, 4100]$ GeV at $\sqrt{s} = 9.16$ TeV. Conversely, the 95% CL exclusion limit is attained over the parameter space of $\kappa_Y \in [0.07, 0.5]$ with $m_Y \in [1700, 3020]$ GeV at $\sqrt{s} = 5.29$ TeV, $\kappa_Y \in [0.06, 0.5]$ with $m_Y \in [1700, 3540]$ GeV at $\sqrt{s} = 6.48$ TeV, and $\kappa_Y \in [0.04, 0.5]$ with $m_Y \in [1700, 4800]$ GeV at $\sqrt{s} = 9.16$ TeV.
- (4) As previous literature has emphasized the role of the T and Y width in the case of their single production mode at the LHC, for a fixed value of $\Gamma_{T/Y}/m_{T/Y} = 10\%$, the VLQ- $T(Y)$ can be discovered (at 5σ level) with a mass about 1900 (2300), 2000 (2500), and 2500 (3000) GeV at $\sqrt{s} = 5.29, 6.48$ and 9.16 TeV, respectively. Conversely, the 95% CL exclusion limit is attained for VLQ- T mass about 2250 (2600), 2400 (2900), and 3000

(3600) GeV, at $\sqrt{s} = 5.29, 6.48$ and 9.16 TeV, respectively.

Finally, by comparing our results to existing literature on the VLQ- T and VLQ- Y states at a variety of present and future colliders, we have concluded that a future μp machine can be competitive in the search for such possible new states of Nature.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX: EVENT COUNTS AND SELECTION EFFICIENCIES

The cumulative efficiency after a series of selection cuts is defined as the ratio of surviving events (E_K) to the initial generated events (E_{in})

$$\epsilon = \frac{E_K}{E_{\text{in}}} \quad (\text{A1})$$

1. Analysis at $\sqrt{s} = 5.29$ TeV: Leptonic W channel

The Monte Carlo samples for this channel include:

- (i) 10^5 events for each signal benchmark point: T_{1500} , T_{2000} , and T_{2500}
- (ii) 5×10^5 events for each SM background process: $\nu_\mu tb$ and $\nu_\mu Wj$

Figure 10 presents the preselection kinematic distributions for both signal and background processes, showing:

- (i) Transverse momentum of the lepton (p_T^ℓ)
- (ii) Transverse momenta of the leading and subleading b -jets ($p_T^{b_1}, p_T^{b_2}$)
- (iii) Angular separation between the leading b -jet and lepton ($\Delta R_{\ell, b_1}$)
- (iv) Missing transverse energy (E_T)
- (v) Invariant mass of the b -jet and electron system (M_{be})

Table IX presents the event counts and cumulative efficiencies for the leptonic W channel (W_{lep}) at $\sqrt{s} = 5.29$ TeV.

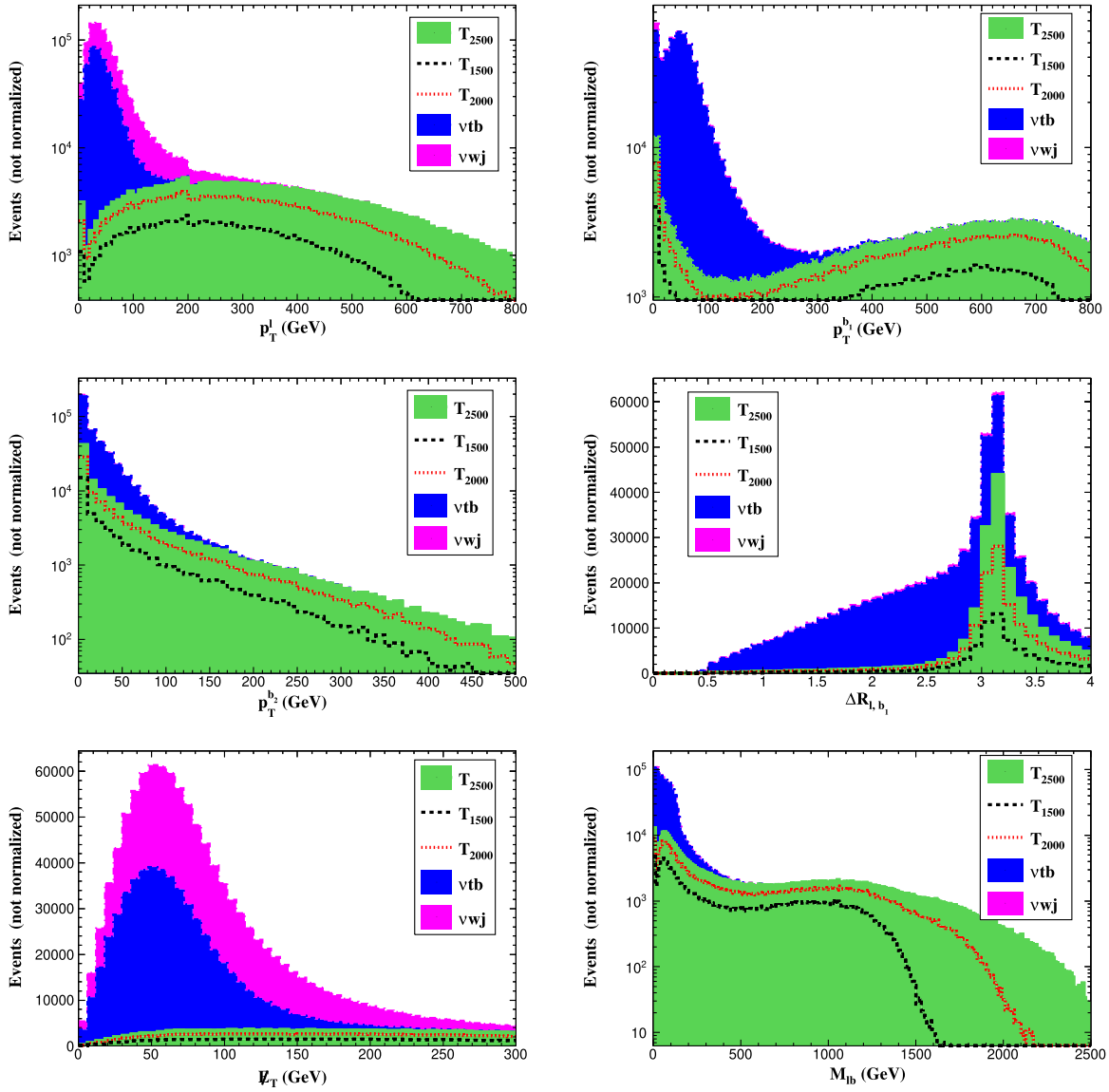


FIG. 10. Kinematic distributions prior to selection cuts for p_T^l , $p_T^{b_{1,2}}$, $\Delta R_{l,b_1}$, E_T , and M_{be} at $\sqrt{s} = 5.29$ TeV in the W_{lep} channel. The green band corresponds to $m_T = 2500$ GeV, while the dashed ($m_T = 1500$ GeV) and dotted ($m_T = 2000$ GeV) lines show independent predictions overlaid for comparison. All distributions display raw Monte Carlo event counts without cross section or luminosity normalization.

TABLE IX. Event counts and cumulative efficiencies after relevant selection cuts for the W_{lep} channel at $\sqrt{s} = 5.29$ TeV. Values in parentheses show efficiencies.

Selection	Signal			Background	
	T_{1500}	T_{2000}	T_{2500}	νtb	νWj
No cuts	100,000	100,000	100,000	500,000	500, 000
Basic selection	87,612 (0.88)	88,042 (0.88)	88,305 (0.88)	282,463 (0.56)	312,960 (0.63)
Cut 1	29,585 (0.30)	32,835 (0.33)	34,167 (0.34)	1033 (0.002)	13,908 (0.028)
Cut 2	20,702 (0.21)	23,674 (0.24)	24,910 (0.25)	196 (3.9×10^{-4})	57 (1.1×10^{-4})
Cut 3	14,385 (0.14)	18,174 (0.18)	20,308 (0.20)	33 (6.6×10^{-5})	18 (3.6×10^{-5})
Cut 4	5903 (0.06)	14,268 (0.14)	18,325 (0.18)	3 (6×10^{-6})	5 (1.0×10^{-5})

2. Analysis at $\sqrt{s} = 5.29$ TeV: Hadronic W channel

Figure 11 displays the preselection distributions for the hadronic channel, featuring:

- (i) Transverse momentum of the fat jet (p_T^J)
- (ii) Transverse momentum of the leading b -jet ($p_T^{b_1}$)
- (iii) Invariant mass of the fat jet (M_J)
- (iv) Combined invariant mass of the b -jet and fat jet (M_{bJ})

Table X presents the complete event counts and selection efficiencies for signal and background processes in the W_{had} channel at $\sqrt{s} = 5.29$ TeV. The analysis includes a

comprehensive simulation of the inclusive charged-current deep inelastic scattering (DIS) background process, $\mu p \rightarrow \nu_\mu j b \bar{b}$, which has an initial production cross section of 439 fb after basic kinematic cuts. Upon applying the full analysis selection criteria, this background is significantly suppressed, yielding final cross sections of 0.01 fb (Cut-4a) and 0.005 fb (Cut-4b). In comparison, the total cross section of all SM backgrounds amounts to approximately 0.56 fb (Cut-4a) and 0.28 fb (Cut-4b). Given its negligible contribution relative to the total background, the DIS process can safely be disregarded.

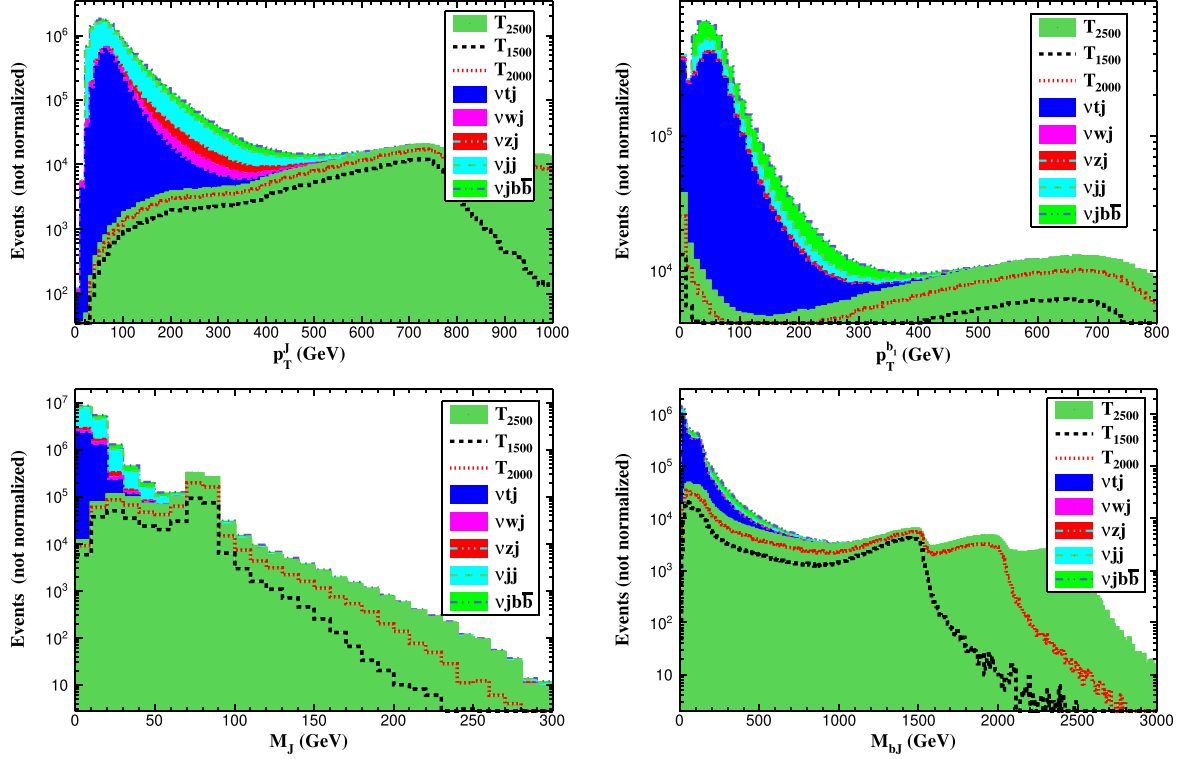


FIG. 11. Preselection kinematic distributions for p_T^J , $p_T^{b_1}$, M_J , and M_{bJ} in the W_{had} channel at $\sqrt{s} = 5.29$ TeV. The green band corresponds to $m_T = 2500$ GeV, while the dashed ($m_T = 1500$ GeV) and dotted ($m_T = 2000$ GeV) lines show independent predictions overlaid for comparison. Signal benchmarks and background contributions are shown with raw Monte Carlo statistics.

TABLE X. Event counts and efficiencies for the W_{had} channel at $\sqrt{s} = 5.29$ TeV.

Selection	Signal and background processes			
	T_{1500}	T_{2000}	T_{2500}	νtb
No cuts	379,619	378,324	375,503	3,605,809
Basic selection	379,608 (1.00)	378,314 (1.00)	375,494 (1.00)	3,590,580 (1.00)
Cut 1	363,168 (0.96)	368,979 (0.98)	369,027 (0.98)	87,660 (0.024)
Cut 2	187,079 (0.49)	213,026 (0.56)	223,324 (0.59)	2256 (6.2×10^{-4})
Cut 3	119,531 (0.31)	137,441 (0.36)	144,219 (0.38)	262 (7.3×10^{-5})
Cut 4a	75,887 (0.20)	...	...	10 (2.8×10^{-6})
Cut 4b	...	7619 (0.02)	8838 (0.02)	2 (5.6×10^{-7})

Selection	Background processes (continued)			
	νWj	νZj	νjj	$\nu j b \bar{b}$
No cuts	611,400	906,941	8,734,540	2,243,809
Basic selection	607,125 (0.99)	904,826 (1.00)	8,096,377 (0.93)	2,198,247 (0.98)
Cut 1	85,794 (0.14)	162,572 (0.18)	444,173 (0.05)	293,552 (0.13)
Cut 2	3799 (6.2×10^{-3})	5850 (6.4×10^{-3})	17,902 (2.1×10^{-3})	15,326 (6.8×10^{-3})
Cut 3	18 (2.9×10^{-5})	89 (9.8×10^{-5})	52 (6×10^{-6})	403 (1.8×10^{-4})
Cut 4a	3 (4.9×10^{-6})	31 (3.4×10^{-5})	5 (5.7×10^{-7})	55 (2.4×10^{-5})
Cut 4b	3 (4.9×10^{-6})	21 (2.3×10^{-5})	2 (2.3×10^{-7})	24 (1.1×10^{-5})

Note: Table divided for clarity. Values in parentheses show efficiencies. “...” indicates nonapplicable entries.

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