

Selective particle excitations in Rindler spacetime

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The Unruh effect is a well-understood phenomenon in which a quantum field in a vacuum state within Minkowski spacetime appears thermally populated for a uniformly accelerating Rindler observer. In this article, we derive a variant of the Unruh effect involving two distinct accelerating observers and aim to address the following questions: (i) Is it possible to have a particle excitation for a selective subset of momentum modes for the case of massless scalar fields, and (ii) Is it possible to excite only the left-chiral massless fermions while keeping right-chiral fermionic fields in an unexcited state or vice versa? To this end, we consider a Rindler wedge R_1 constructed from a class of accelerating observers and another Rindler wedge R_2 (with $R_2 \subset R_1$) constructed from another class of accelerating observers such that the wedge R_2 is displaced along a null direction with respect to R_1 by a parameter Δ . By first considering a massless scalar field in the R_1 vacuum, we demonstrate that if we choose the displacement Δ along one null direction, the positive momentum modes are thermally populated, whereas negative momentum modes remain unaffected (and vice versa if we choose the displacement along the other null direction). We then consider a massless fermionic field in a vacuum state in R_1 and show that the reduced state in R_2 is such that the left-chiral fermions are excited and are thermal for large frequencies. In contrast, the right-chiral fermions have negligible particle density (and vice versa). We argue that the null-shifted Rindler spacetime considered in this article is a good toy model for understanding a class of evolving horizons. Additionally, based on our work, we hypothesize that massless fermions underwent selective chiral excitations during the radiation-dominated era of cosmology.

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I. INTRODUCTION

Black holes are among the most mysterious astrophysical objects ever discovered due to their unique classical and quantum properties. They can emit particles and radiation due to the quantum effects near the event horizon. This phenomenon is known in the literature as Hawking radiation [1]. The spectrum of particles exhibits a thermal distribution at late times [1]. Astrophysical black holes are formed from the collapse of matter and radiation, creating an event horizon that prevents any object from escaping to the outside. The existence of black holes raises many unresolved questions, such as spacetime singularities and the

information paradox. These issues remain unsolved and require concrete theoretical proofs and justifications.

Rindler spacetime has been used as a valuable toy model to study and understand various aspects of black hole event horizon and Hawking radiation, particularly due to the discovery of the Unruh effect [2]. While Rindler spacetime is interesting in its own right, its significance primarily comes from the fact that it happens to be near the horizon limit of generic (nonextremal) black holes [3,4]. Consequently, extensive research is being carried out in Rindler spacetime to eventually gain insights into black hole horizons in the context of thermodynamics, entanglement entropy, and quantum information [5–9].

One aspect of black holes that is not fully understood is the quantum signatures associated with evolving horizons. Evolving horizons usually occur during the dynamical formation of a black hole through gravitational collapse. In this context, the concept of marginally trapped surface (which is the boundary of the trapped region) becomes relevant. The apparent horizon becomes particularly important, as it marks the outermost marginally trapped surface. Marginally trapped surfaces can be broadly classified based on their causal properties [10,11]: (i) A dynamical horizon is a spacelike hypersurface constructed using a foliation of

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marginally trapped surfaces. The tangent of the hypersurface forming the dynamical horizon is spacelike. The dynamical horizons typically occur during the formation of black holes due to the collapse of a generic distribution of matter. The relevant idea concerning this article is that during the evolution, the apparent horizon makes a series of spacelike jumps in the radially outward direction. (ii) In contrast, the evolving horizons are termed timelike tubes if the tangent is timelike [10]. These scenarios are less generic and occur in the cosmological evolution of our universe or during the gravitational collapse of homogeneous dust models, such as in the Oppenheimer-Snyder collapse model [12]. (iii) Additionally, there is a less-explored scenario in which the evolving horizon is null. This situation occurs during cosmological evolution for a particular equation of state that corresponds to the radiation-dominated era. This scenario, along with a few interesting properties of this case, is discussed in Refs. [13–16]. A unified concept for evolving horizons, called holographic screens, is introduced in Ref. [17], where a dynamical horizon is interpreted as a spacelike future holographic screen.

In the following paragraphs, we outline the findings of this article, motivated by the fact that they can shed light on quantum aspects related to the evolving horizons and also help in the study of entanglement aspects of subsystems. The Rindler spacetimes provide interesting toy models for understanding the quantum signatures of evolving horizons. When one constructs two different Rindler spacetimes using two separate classes of uniformly accelerating observers (say R_1 and R_2 respectively), we can arrange the observers such that R_2 is a subsystem of R_1 ($R_2 \subset R_1$) as illustrated in Fig. 1. For R_2 to qualify as a subsystem of R_1 , it is evident that the bifurcation point of R_2 is spacelike or null separated from R_1 , as depicted in Figs. 1–3.

For the case of dynamical horizons, a good toy model is a nested sequence of spacelike shifted Rindler spacetimes, as argued in Ref. [18]. This is because the apparent horizon in this context makes a series of spacelike jumps in the increasing radial direction. The situation resembles a nested sequence of Rindler wedges. The nested sequence of wedges that arise in the case of dynamical horizons (or future holographic screens) has been employed to demonstrate important results regarding the increase in entanglement entropy in the outer region of a black hole within the AdS/CFT context [19]. A review of relevant literature on entanglement in a quantum field theoretic setting is discussed in [6,20–24].

A limited number of studies have explored spacelike shifted Rindler scenarios, yielding interesting results. In Ref. [25], the authors examine two Rindler spacetimes, R_1 and R_2 , which are shifted along a spatial direction. They showed that if quantum fields in R_1 are in a vacuum state, the fields in R_2 exhibit a thermal spectrum of particles. In Ref. [18], the authors investigate the uniqueness of the particle content in R_2 when starting from a vacuum state

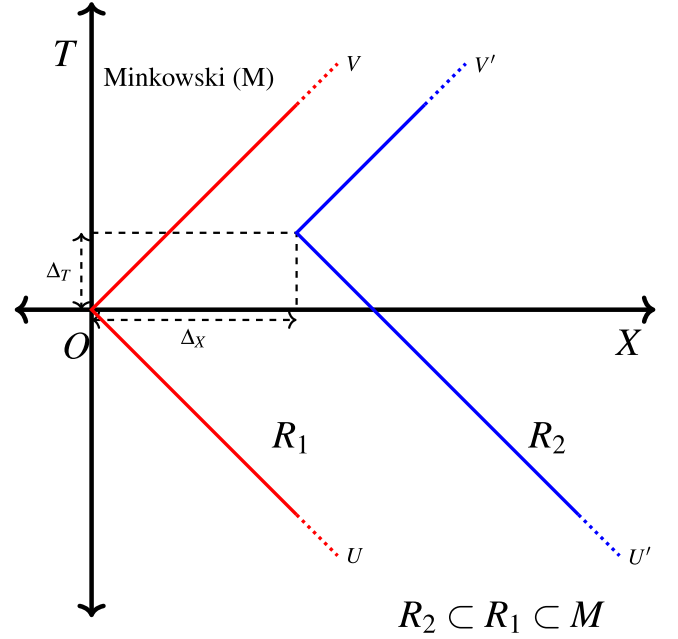


FIG. 1. Spacelike-shifted Rindler spacetimes (R_1) and (R_2).

in Minkowski spacetime and depending on whether we consider R_1 between R_2 and Minkowski spacetime ($R_2 \subset R_1 \subset M$ with M as Minkowski spacetime) or not. A toy model using Rindler spacetimes and its subsystems is difficult to construct for the case of Timelike tubes. On the other hand, the work done in this article yields a good toy model for the third category of evolving horizons, viz, the null case, as outlined in the paragraphs below.

Here are a few interesting questions, some of which are addressed in this article. Consider a quantum field theoretic

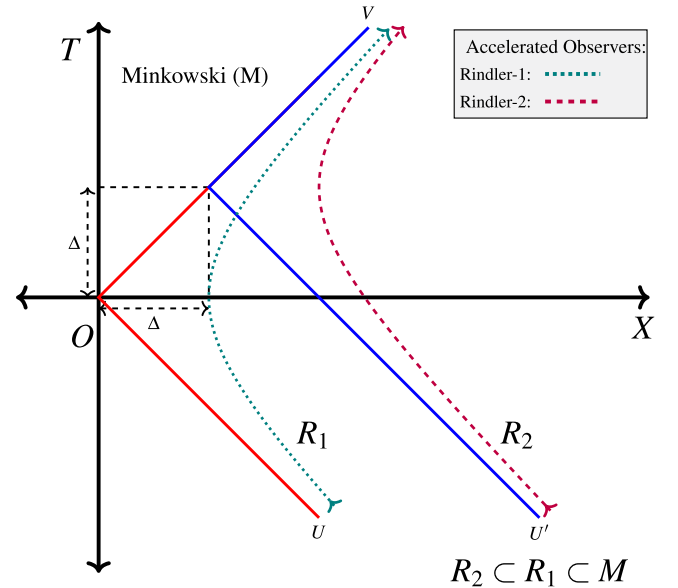


FIG. 2. The Rindler spacetime R_1 and the null-shifted Rindler spacetime R_2 along the V -axis.

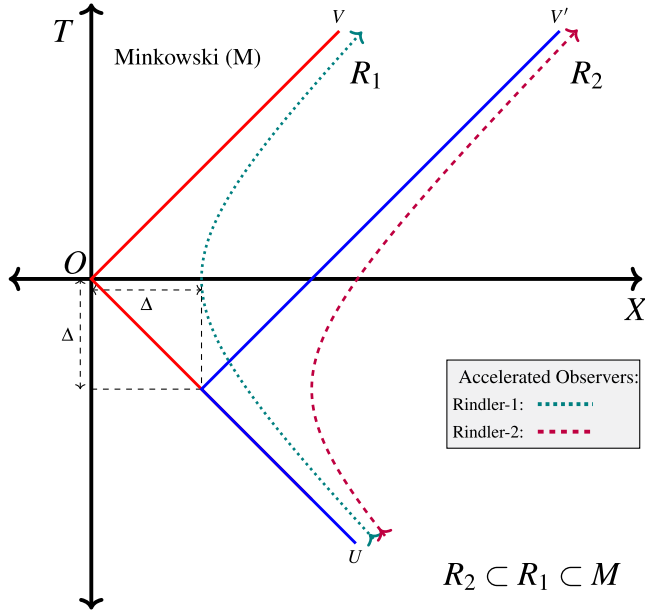


FIG. 3. The Rindler spacetime R_1 and the null-shifted Rindler spacetime R_2 along the U -axis.

system with a spacetime containing a quantum field defined in a vacuum state. Can we construct subsystems of this system demarcated due to horizons to excite a subset of modes? Can Rindler spacetime exhibit a strand of quantum hair? We demonstrate that the answers to the above questions are affirmative. We choose R_1 as a system with a massless scalar field or a massless Dirac field in a vacuum state. Next, we define its subsystem as R_2 , which is shifted along one of the null directions (see Figs. 2 and 3). We then examine the state of the scalar or Dirac fields in R_2 . Our findings show that, depending on the direction of the null shift, either positive or negative momentum modes are in a thermal state, while the other set remains in a no-particle state. We demonstrate that in the case of null-shift along the V -axis, left-chiral fermions are in an excited state, whereas the right-chiral fermions remain with negligible excitation. In contrast, right-chiral (left-chiral) fermions exhibit negligible excitation in Rindler spacetime. Therefore, we can effectively create chiral excitations using Rindler horizons.

The findings of the articles [13,14] examine gravitational situations where apparent horizons evolve along the null direction during the radiation-dominated era. Our findings in this article suggest the possibility of chiral excitations of massless fermions during the radiation-dominated era. It remains to be seen whether this holds or not, and if it is true, are there any important consequences that have not yet been answered? These questions are left for future consideration. We also discuss how the findings in this article present a case where it is possible to observe “quantum hair” in the context of Rindler spacetime.

In the following sections, we will provide a comprehensive overview of how selective excitations manifest in

our scenario. In Sec. II, we will briefly discuss the setup that leads to selective excitations. We will introduce the relevant coordinate systems and their mutual relations. In Sec. III, we will present detailed methods and techniques for understanding how selective thermalization manifests in the case of scalar fields. The first part of Sec. IV will introduce the Dirac fields in our setup and review some existing results, such as the Unruh effect for fermionic fields. In the subsequent parts of Sec. IV, we will outline the theoretical framework underlying our approach on how selective chiral excitations manifest for massless Dirac fields. Finally, in Sec. V, we will conclude and discuss the implications of our results along with the potential future direction for further research on this problem and associated problems.

II. THE PROBLEM STATEMENT AND SETUP

In this article, we investigate the possibility of selective excitations of fields starting from Rindler spacetime R_1 in the relevant vacuum state. We use classes of accelerating observers and arrange them (populating the wedge R_2) so that only a subset of the modes are excited while the other modes have zero particle content. We demonstrate this by analyzing two cases: one involving a free massless scalar field and the other using a free massless Dirac field.

The ideal starting point of the analysis is the quantum field in the Minkowski vacuum since it is well defined. Then the first Rindler wedge R_1 would have been in a thermal state. All the questions posed in this article could have been restated with R_1 in a thermal state rather than a vacuum state. However, we choose the starting point for the questions in this article to be the Rindler wedge R_1 in the vacuum state (the vacuum state is defined subsequently in the later portions of the article) and then examine what the observed particle density is in, say, R_2 . Our reasons for this choice of the starting point are elaborated in the Sec. V.

The spacetimes we consider are Minkowski and Rindler, with coordinates denoted by (X, T) for Minkowski spacetime M and (x_i, t_i) for the various Rindler frames R_i , respectively. The two Rindler frames are chosen so that the bifurcation point of R_1 is at the origin of M as illustrated in Fig. 1. Now, the scenario where $R_2 \subset R_1$ is possible only when the bifurcation point of R_2 is separated from the bifurcation point of R_1 by a spacelike interval or a null interval. The spacelike separation case and some of its interesting implications have been explored in Refs. [18,25]. They demonstrate that starting from a quantum field in vacuum state in R_1 , one can obtain a thermal state in R_2 (at least in the large frequency limit).

What happens to the results in [18,25] if the shift is not along the x -axis but along the configuration depicted in Fig. 1? We argue below that we have fundamentally two distinct possibilities.

For the first case, suppose we consider wedge R_1 to be in vacuum. If we let the bifurcation point of the wedge R_2 tend to the null ray $T - X = 0$ (or $T + X = 0$) but not exactly on the null ray, we then argue that both the left-moving modes and right-moving modes have a thermal distribution. No selective particle excitation can take place. The justification is as follows: We let the bifurcation point of R_2 be spacelike separated from the bifurcation point of R_1 (this can be as close to the null ray as desired). We can now select an inertial frame T', X' such that in the new frame, the Rindler wedge R_2 lies along the X' axis. It is straightforward to demonstrate that R_1 remains in vacuum in the new inertial frame as well, since both the wedge and the accelerating observers in the wedge, R_1 , lie on an invariant hyperbolae in Minkowski spacetime. So the Rindler vacuum state $|0\rangle_{R_1}$ is invariant under Lorentz boosts. The particle density when R_1 is in a vacuum and R_2 has a bifurcation point on the X -axis is already computed in [18,25]. Based on these results, it is easy to see that both the left-moving and right-moving modes exhibit a thermal distribution of particles.

For the second case, if we now let the bifurcation point of the wedge R_2 to be exactly on null ray $T - X = 0$ (or $T + X = 0$), then we show in this article that only the left moving modes (or the right moving modes) have particles in thermal distribution whereas the right moving modes (left moving modes) have zero particle content. Selective particle excitation, therefore, occurs, and the null shifted case is fundamentally different from the results of the spacelike shifted case and hence the former cannot be derived as a limit of the latter.

A. Rindler coordinates

We consider two specific cases in this article: *case 1* involves a shift of Rindler-2 (R_2) frame along the “ V -axis,” and *case 2* involves a shift along the “ U -axis,” both are in the null directions (see Figs. 2 and 3 respectively).

1. Case 1: Null-shift in V -axis

We denote the coordinates of the Rindler-1 (R_1) frame as (x_1, t_1) and those of Rindler-2 (R_2) as (x_2, t_2) . The relationship between R_1 coordinates and the Minkowski (M) coordinates, as illustrated in Fig. 2 can be expressed as follows:

$$T = \frac{e^{g_1 x_1}}{g_1} \sinh(g_1 t_1), \quad (1)$$

$$X = \frac{e^{g_1 x_1}}{g_1} \cosh(g_1 t_1). \quad (2)$$

In terms of the Light-cone coordinates, this can be expressed as follows:

$$U_M = T - X = -\frac{e^{-g_1 t_1}}{g_1}, \quad (3)$$

$$V_M = T + X = \frac{e^{g_1 t_1}}{g_1}. \quad (4)$$

Likewise, we define the R_2 frame with the coordinates (x_2, t_2) , such that it is null-shifted in the V -axis. It is important to note that the V -axis is represented by $U = 0$, indicating that $T = X$. The relationship between R_2 and that of the Minkowski (M) coordinates is given as follows:

$$T = \frac{e^{g_2 x_2}}{g_2} \sinh(g_2 t_2) + \Delta, \quad (5)$$

$$X = \frac{e^{g_2 x_2}}{g_2} \cosh(g_2 t_2) + \Delta, \quad (6)$$

where Δ represents the shift in the coordinate axis. Now, when expressed in light-cone coordinates, we have

$$U_M = T - X = -\frac{e^{-g_2 t_2}}{g_2}, \quad (7)$$

$$V_M = T + X = \frac{e^{g_2 t_2}}{g_2} + 2\Delta. \quad (8)$$

Similarly, the relationship between R_i coordinates and the corresponding light-cone coordinates of R_i are denoted as follows:

$$u_i = t_i - x_i, \quad (9)$$

$$v_i = t_i + x_i, \quad (10)$$

where the subscript index “ i ” takes values “1” or “2” and labels the Rindler frames R_1 or R_2 respectively. The general form of the metric in any of the Rindler frames takes the following form:

$$ds^2 = e^{2g_i x_i} (dt_i^2 - dx_i^2), \quad (11)$$

here also, the subscript index $i \in (0, 1)$, labels a specific Rindler frame under consideration. To understand and interpret the results of our article in terms of uniformly accelerating observers, we identify a canonical trajectory of a uniformly accelerating observer in R_1 represented by a “teal dotted” hyperbola in Fig. 2. Similarly, we plot another canonical trajectory of uniformly accelerating observer for the frame R_2 represented by a “purple dashed” hyperbola. The accelerating observers in R_1 have access to the information from accelerating observers in R_2 throughout the entire trajectory. In contrast, accelerating observers in frame R_2 will see the accelerating observers in R_1 disappear behind the horizon, limiting their access to observers in R_1 to only a restricted part of their trajectory.

Note: “Accelerating observers” refers to uniformly accelerating observers unless specified otherwise.

2. Case 2: Null-shift in U -axis

For case 2, as seen in Fig. 3, we have considered a null-shift of R_2 along the U -axis, which implies $V = 0$. The relationship between the R_2 frame with coordinates (x_2, t_2) and the Minkowski coordinates are given as

$$T = \frac{e^{g_2 x_2}}{g_2} \sinh(g_2 t_2) - \Delta, \quad (12)$$

$$X = \frac{e^{g_2 x_2}}{g_2} \cosh(g_2 t_2) + \Delta. \quad (13)$$

When expressed in light-cone coordinates, we obtain the following:

$$U_M = T - X = -\frac{e^{-g_2 u_2}}{g_2} - 2\Delta, \quad (14)$$

$$V_M = T + X = \frac{e^{g_2 v_2}}{g_2}. \quad (15)$$

As previously mentioned, the form of the Rindler metric remains the same as in Eq. (11). To carry out an analysis using accelerating observers, we now identify a canonical trajectory of an accelerating observer in R_1 represented by a “teal dotted” hyperbola in Fig. 3. Similarly; we plot another canonical trajectory for an accelerating observer in the frame R_2 denoted by a “purple dashed” hyperbola. Here, the accelerating observers in R_2 have access to the information from accelerating observers in R_1 during the entire trajectory. In contrast, accelerating observers in frame R_1 can only obtain information from observers in R_2 after they have crossed the horizon in R_2 . Note: for the rest of the analysis, we take the acceleration parameter for different Rindler frames as $g_1 = g_2 = g$; this simplifies calculations.

III. SELECTIVE EXCITATION FOR MASSLESS SCALAR FIELDS

This section examines a more straightforward scenario involving a free massless scalar field, specifically in a vacuum state in the frame R_1 . We calculate the particle content in R_2 , where R_2 is shifted from R_1 along the V -axis (see Fig. 2). It will be shown that the particle content of the left-moving modes becomes excited, which aligns with the thermal spectrum for large frequencies. In contrast, the right-moving modes remain with no particle content. Based on symmetry considerations, we will similarly predict the particle content when R_2 is shifted along the U -axis.

A. Mode expansion in Rindler spacetime

Consider a real free massless scalar field in $(1+1)$ dimensional Rindler spacetime. The equations of motion are given by the Klein-Gordon equation:

$$\square \hat{\phi}(x^\mu) = 0. \quad (16)$$

The mode expansion of the field operator $\hat{\phi}(x^\mu)$ is similar across all the Rindler frames and can be expressed as follows:

$$\hat{\phi}(x_i, t_i) = \int_0^\infty dp [\hat{c}_i(p) h_i(p) + \hat{c}_i^\dagger(p) h_i^*(p) + \hat{d}_i(p) j_i(p) + \hat{d}_i^\dagger(p) j_i^*(p)], \quad (17)$$

where h and j are the right-moving and left-moving modes, the subscript index “ i ” labels the Rindler frame. In a given Rindler frame, the modes can be represented by plane waves as follows:

$$h_i(p) = \frac{e^{i(-\omega_p t_i + p x_i)}}{\sqrt{4\pi\omega_p}} \quad (18)$$

and

$$j_i(p) = \frac{e^{i(-\omega_p t_i - p x_i)}}{\sqrt{4\pi\omega_p}}. \quad (19)$$

The plane wave solutions can be normalized using the Klein-Gordon inner product, forming a complete basis for describing the field operator. The analysis presented below on a free massless scalar field uses Wald’s [26] representation of the Klein-Gordon inner product and takes the form

$$\langle \phi_1, \phi_2 \rangle = -i \int_\Sigma \sqrt{-\sigma} d\Sigma^\mu (\phi_1^* \partial_\mu \phi_2 - \phi_2 \partial_\mu \phi_1^*), \quad (20)$$

where Σ is an appropriate constant time Cauchy hypersurface for the particular Klein-Gordon equation. Furthermore, we observe that for a massless scalar field, the on-shell condition implies $\omega_p = |p|$, such that $\omega_p > 0$. Now for notational convenience relabeling ω_p as “ p ,” we can infer from the above discussion that “ p ” is non-negative. Consequently, for R_1 the Eq. (17) can be rewritten using Eqs. (18) and (19) as follows:

$$\hat{\phi}(x_1, t_1) = \int_0^\infty dp \left[\hat{c}_1(p) \frac{e^{-ip(t_1 - x_1)}}{\sqrt{4\pi p}} + \hat{c}_1^\dagger(p) \frac{e^{ip(t_1 - x_1)}}{\sqrt{4\pi p}} + \hat{d}_1(p) \frac{e^{-ip(t_1 + x_1)}}{\sqrt{4\pi p}} + \hat{d}_1^\dagger(p) \frac{e^{ip(t_1 + x_1)}}{\sqrt{4\pi p}} \right]. \quad (21)$$

The coefficients $\hat{c}_1^\dagger(p)$, $\hat{d}_1^\dagger(p)$ and $\hat{c}_1(p)$, $\hat{d}_1(p)$ are the usual creation and annihilation operators of the field,

obeying the standard commutation relations

$$\begin{aligned} [\hat{c}_1(p), \hat{c}_1^\dagger(p')] &= \delta(p-p') & [\hat{d}_1(p), \hat{d}_1^\dagger(p')] &= \delta(p-p') \\ [\hat{c}_1(p), \hat{c}_1(p')] &= 0 & \text{and } [\hat{d}_1(p), \hat{d}_1(p')] &= 0 \\ [\hat{c}_1^\dagger(p), \hat{c}_1^\dagger(p')] &= 0 & [\hat{d}_1^\dagger(p), \hat{d}_1^\dagger(p')] &= 0 \end{aligned} \quad (22)$$

The Fock space is constructed from the above nonvanishing commutation relations. Thus we define the Rindler-1 (R_1) vacuum state $|0\rangle_{R_1}$, as the state annihilated by all the annihilation operators [27], i.e.,

$$\hat{c}_1(p)|0\rangle_{R_1} = 0 \quad \text{and} \quad \hat{d}_1(p)|0\rangle_{R_1} = 0, \quad \forall p > 0. \quad (23)$$

Therefore, the Rindler-1 vacuum state $|0\rangle_{R_1}$ is devoid of both the right-moving and left-moving excitations. The operator $\hat{c}_1^\dagger(p)$ creates a right-moving excited state with momentum “ p ” out of the vacuum, while the operator $\hat{d}_1^\dagger(p)$ creates a left-moving excited state with momentum “ p ” out of the vacuum state.

Similarly, from Eqs. (17) to (19), we obtain the mode expansion of the field operator for R_2 as follows:

$$\begin{aligned} \hat{\phi}(x_2, t_2) &= \int_0^\infty dp \left[\hat{c}_2(p) \frac{e^{-ip(t_2-x_2)}}{\sqrt{4\pi p}} + \hat{c}_2^\dagger(p) \frac{e^{ip(t_2-x_2)}}{\sqrt{4\pi p}} \right. \\ &\quad \left. + \hat{d}_2(p) \frac{e^{-ip(t_2+x_2)}}{\sqrt{4\pi p}} + \hat{d}_2^\dagger(p) \frac{e^{ip(t_2+x_2)}}{\sqrt{4\pi p}} \right]. \end{aligned} \quad (24)$$

As before, the coefficients $\hat{c}_2^\dagger(p)$, $\hat{d}_2^\dagger(p)$ and $\hat{c}_2(p)$, $\hat{d}_2(p)$ are the usual creation and annihilation operators, obeying the standard commutation relations as in Eq. (23). The Fock space constructed from the nonvanishing commutation relations contains the Rindler-2 (R_2) vacuum state $|0\rangle_{R_2}$, which is defined as the state annihilated by all the annihilation operators [27], i.e.,

$$\hat{c}_2(p)|0\rangle_{R_2} = 0 \quad \text{and} \quad \hat{d}_2(p)|0\rangle_{R_2} = 0, \quad \forall p > 0. \quad (25)$$

Also, the operators $\hat{c}_2^\dagger(p)$ and $\hat{d}_1^\dagger(p)$ are used to define excited states from the vacuum state.

The field operator $\hat{\phi}(x_i, t_i)$, when expressed in Rindler light-cone coordinates [using Eqs. (9) and (10)], results in the following:

$$\hat{\phi}(x_i, t_i) = \hat{\phi}(u_i) + \hat{\phi}(v_i). \quad (26)$$

It is important to note that Eq. (26) shows that the left-moving $[\hat{\phi}(u_i)]$ and right-moving $[\hat{\phi}(v_i)]$ sectors of the field are decoupled.

B. Particle content of the right-moving modes

To analyze the particle content of the right-moving modes, we start by expressing the field in terms of Rindler light-cone coordinates (u_i , v_i). Since the right-moving and left-moving modes are independent, they form complete bases separately [28]. The general form of the Bogoliubov transformation, which relates the modes of R_2 to those of R_1 , is expressed as follows:

$$h_2(q) = \int_0^\infty dk (\alpha_{21u}(q, k) h_1(k) + \beta_{21u}(q, k) h_1^*(k)), \quad (27)$$

$$h_1(k) = \int_0^\infty dq (\alpha_{21u}^*(q, k) h_2(q) - \beta_{21u}(q, k) h_2^*(q)). \quad (28)$$

Please note that the subscript “ u ” in the Bogoliubov transformations indicates that it is associated with the right-moving modes, as seen in expressions such as $\alpha_{21u}(q, k)$. Based on the Bogoliubov transformation relations [see Eqs. (27) and (28)], we can express how the creation and annihilation operators of R_2 are related to those of R_1 [29] as follows:

$$\hat{c}_2(p) = \int_0^\infty dk (\alpha_{21u}^*(p, k) \hat{c}_1(k) - \beta_{21u}^*(p, k) \hat{c}_1^\dagger(k)), \quad (29)$$

$$\hat{c}_2^\dagger(p) = \int_0^\infty dk (\alpha_{21u}(p, k) \hat{c}_1^\dagger(k) - \beta_{21u}(p, k) \hat{c}_1(k)). \quad (30)$$

Notably, the values of the above Bogoliubov coefficients will determine the particle content of the right-moving field sector. Returning to our analysis, we observe from Eqs. (3) and (7) that $u_1 = u_1(u_2)$ and since the left-moving and right-moving modes do not interact with each other [see Eq. (26)], we can conclude that $\hat{\phi}(u_1) = \hat{\phi}(u_2)$. Now, consider the standard mode expansion of the field operator in the domain of spacetime where the coordinate frames of R_1 and R_2 overlap [29] and based on the equality of the right-moving field sectors [30], we can express the following:

$$\begin{aligned} \phi(u_i) &= \int_0^\infty dk \left[\hat{c}_1(k) \frac{e^{-iku_1}}{\sqrt{4\pi k}} + \hat{c}_1^\dagger(k) \frac{e^{iku_1}}{\sqrt{4\pi k}} \right] \\ &= \int_0^\infty dp \left[\hat{c}_2(p) \frac{e^{-ipu_2}}{\sqrt{4\pi p}} + \hat{c}_2^\dagger(p) \frac{e^{ipu_2}}{\sqrt{4\pi p}} \right]. \end{aligned} \quad (31)$$

Here, as mentioned initially, we have used the relations Eqs. (9) and (10) to write the mode expansions in light-cone coordinates of R_1 and R_2 . The relation between the null coordinates of R_1 and R_2 can be deduced from Eqs. (3) and (7),

$$\frac{e^{-gu_1}}{g} = \frac{e^{-gu_2}}{g}, \quad (32)$$

which implies $u_1 = u_1(u_2) = u_2$. Since, in Eq. (31), $\hat{\phi}(u_1(u_2)) = \hat{\phi}(u_2)$ are a function of u_2 , we can apply the Fourier transform to obtain the left-hand side (lhs) and right-hand side (rhs) in terms of Ω , which can be expressed as

$$\begin{aligned} & \int_{-\infty}^{\infty} \frac{du_2}{\sqrt{2\pi}} e^{i\Omega u_2} \int_0^{\infty} \frac{dk}{\sqrt{2\pi}} \left[\hat{c}_1(k) \frac{e^{-iku_1}}{\sqrt{2k}} + \hat{c}_1^\dagger(k) \frac{e^{iku_1}}{\sqrt{2k}} \right] \\ &= \int_{-\infty}^{\infty} \frac{du_2}{\sqrt{2\pi}} e^{i\Omega u_2} \int_0^{\infty} \frac{dp}{\sqrt{2\pi}} \left[\hat{c}_2(p) \frac{e^{-ipu_2}}{\sqrt{2p}} + \hat{c}_2^\dagger(p) \frac{e^{ipu_2}}{\sqrt{2p}} \right]. \end{aligned} \quad (33)$$

A further simplification gives

$$\begin{aligned} \hat{c}_2(\Omega) &= \int_0^{\infty} dk \left[\hat{c}_1(k) \sqrt{\frac{\Omega}{k}} \int_{-\infty}^{\infty} \frac{du_2}{2\pi} e^{i(\Omega u_2 - k u_1)} \right. \\ &\quad \left. + \hat{c}_1^\dagger(k) \sqrt{\frac{\Omega}{k}} \int_{-\infty}^{\infty} \frac{du_2}{2\pi} e^{i(\Omega u_2 + k u_1)} \right]. \end{aligned} \quad (34)$$

The above Eq. (34) can be compared with Eq. (29) to get the expressions for the Bogoliubov coefficients for the right-moving modes. It is expressed as follows:

$$\alpha_{21u}^*(k, \Omega) = \sqrt{\frac{\Omega}{k}} \int_{-\infty}^{\infty} \frac{du_2}{2\pi} e^{i(\Omega u_2 - k u_1)}, \quad (35)$$

and,

$$\beta_{21u}^*(k, \Omega) = -\sqrt{\frac{\Omega}{k}} \int_{-\infty}^{\infty} \frac{du_2}{2\pi} e^{i(\Omega u_2 + k u_1)}. \quad (36)$$

Using $u_1 = u_2$ [see Eq. (32)] to evaluate the integrals in Eqs. (35) and (36), we obtain the results for the Bogoliubov coefficients as follows:

$$\alpha_{21u}^*(k, \Omega) = \sqrt{\frac{\Omega}{k}} \delta(\Omega - k), \quad \forall \Omega = k, \quad (37)$$

and

$$\beta_{21u}^*(k, \Omega) = -\sqrt{\frac{\Omega}{k}} \delta(\Omega + k) = 0, \quad \forall \Omega > 0. \quad (38)$$

It is important to note that both Ω and k are positive quantities. This leads to the conclusion that $\beta_{21u}^*(k, \Omega) = 0$, ($\forall \Omega > 0$ & $\Omega = k$). The fact that $\beta_{21u}(k, \Omega) = 0$, implies that the right-moving modes of R_2 (h_2) are made of purely positive frequency functions. Thus, there is no mixing of modes manifesting in the context of right-moving modes of R_2 , where mixing of modes is considered as a crucial criterion for having nontrivial ground states and, thus, nonzero particle distribution [31]. From Eqs. (29), (37),

and (38), we can write $\hat{c}_2(p) \propto \hat{c}_1(p)$. Therefore, beginning from the vacuum state in R_1 , i.e., $\hat{c}_1(p)|0\rangle_{R_1} = 0$, we get,

$$\hat{c}_2(p)|0\rangle_{R_1} \propto \hat{c}_1(p)|0\rangle_{R_1} = 0, \quad (39)$$

thus, the right-moving modes remain in a vacuum state.

C. Particle content of the left-moving modes

The discussion regarding the particle content of the left-moving modes follows the same procedure outlined in the previous section. Unlike the right-moving modes that remain in the vacuum state, we show that the left-moving modes display a thermal particle distribution. To show this, we consider the Bogoliubov transformation between the left-moving modes of R_2 and R_1 , defined as follows:

$$j_2(q) = \int_0^{\infty} dk (\alpha_{21v}(q, k) j_1(k) + \beta_{21v}(q, k) j_1^*(k)), \quad (40)$$

$$j_1(k) = \int_0^{\infty} dq (\alpha_{21v}^*(q, k) j_2(q) - \beta_{21v}^*(q, k) j_2^*(q)), \quad (41)$$

where $j_2(q)$ and $j_1(k)$ are the positive frequency modes in R_2 and R_1 respectively. From the above, the annihilation and creation operators of R_2 can be expressed as a linear combination of the creation and annihilation operators of R_1 , given by

$$\hat{d}_2(p) = \int_0^{\infty} dk (\alpha_{21u}^*(p, k) \hat{d}_1(k) - \beta_{21u}^*(p, k) \hat{d}_1^\dagger(k)), \quad (42)$$

$$\hat{d}_2^\dagger(p) = \int_0^{\infty} dk (\alpha_{21u}(p, k) \hat{d}_1^\dagger(k) - \beta_{21u}(p, k) \hat{d}_1(k)). \quad (43)$$

Consider the standard mode expansion of the field operator in the common spacetime domain of R_1 and R_2 . It can be expressed in the Rindler light-cone coordinates [using Eqs. (9) and (10)], to get decoupled left-moving and right-moving sectors [see Eq. (26)]. Since, given that $v_1 = v_1(v_2)$ and the independence of the left and right-moving sectors, we can conclude that $\widehat{\phi}(v_1) = \widehat{\phi}(v_2)$. Thus, we can write

$$\begin{aligned} \phi(v_i) &= \int_0^{\infty} \frac{dk}{\sqrt{2\pi}} \left[\hat{d}_1(k) \frac{e^{-ikv_1}}{\sqrt{2\omega_k}} + \hat{d}_1^\dagger(k) \frac{e^{ikv_1}}{\sqrt{2\omega_k}} \right] \\ &= \int_0^{\infty} \frac{dp}{\sqrt{2\pi}} \left[\hat{d}_2(p) \frac{e^{-ipv_2}}{\sqrt{2\omega_p}} + \hat{d}_2^\dagger(p) \frac{e^{ipv_2}}{\sqrt{2\omega_p}} \right]. \end{aligned} \quad (44)$$

From Eqs. (4) and (8), we compute the relation between the light-cone coordinates of R_1 and R_2 as

$$\frac{e^{gv_1}}{g} = \frac{e^{gv_2}}{g} + 2\Delta. \quad (45)$$

Let us take the magnitude of the shift, denoted as Δ , to be equal to $\lambda/(2g)$, where λ is a positive parameter. Therefore, from Eq. (45), we can conclude the relationship between the light-cone coordinates v_1 and v_2 as follows:

$$v_1 = \frac{\ln(e^{gv_2} + \lambda)}{g}. \quad (46)$$

From the above mapping, it is evident that both the lhs and rhs of Eq. (44) are functions of v_2 . So, applying the Fourier transform on both sides to get lhs and rhs as a function of Ω , we obtain:

$$\begin{aligned} & \int_{-\infty}^{\infty} \frac{dv_2}{\sqrt{2\pi}} e^{i\Omega v_2} \int_0^{\infty} \frac{dk}{\sqrt{2\pi}} \left[\hat{d}_1(k) \frac{e^{-ikv_1}}{\sqrt{2k}} + \hat{d}_1^\dagger(k) \frac{e^{ikv_1}}{\sqrt{2k}} \right] \\ &= \int_{-\infty}^{\infty} \frac{dv_2}{\sqrt{2\pi}} e^{i\Omega v_2} \int_0^{\infty} \frac{dp}{\sqrt{2\pi}} \left[\hat{d}_2(p) \frac{e^{-ipv_2}}{\sqrt{2p}} + \hat{d}_2^\dagger(p) \frac{e^{ipv_2}}{\sqrt{2p}} \right]. \end{aligned} \quad (47)$$

Further simplification of Eq. (47) expresses $\hat{d}_2(\Omega)$ in terms of the annihilation and creation operators of R_1 as follows:

$$\begin{aligned} \hat{d}_2(\Omega) = & \int_0^{\infty} dk \left[\hat{d}_1(k) \sqrt{\frac{\Omega}{k}} \int_{-\infty}^{\infty} \frac{dv_2}{2\pi} e^{i(\Omega v_2 - kv_1)} \right. \\ & \left. + \hat{d}_1^\dagger(k) \sqrt{\frac{\Omega}{k}} \int_{-\infty}^{\infty} \frac{dv_2}{2\pi} e^{i(\Omega v_2 + kv_1)} \right]. \end{aligned} \quad (48)$$

By comparing the above with the general expression of the R_2 annihilation operator in terms of operators of R_1 [see, Eqs. (29) and (30)], the Bogoliubov coefficients $\alpha_{21v}^*(k, \Omega)$ and $\beta_{21v}^*(k, \Omega)$ can be written as follows:

$$\alpha_{21v}^*(k, \Omega) = \sqrt{\frac{\Omega}{k}} \int_{-\infty}^{\infty} \frac{dv_2}{2\pi} e^{i(\Omega v_2 - kv_1)}, \quad (49)$$

$$\beta_{21v}^*(k, \Omega) = -\sqrt{\frac{\Omega}{k}} \int_{-\infty}^{\infty} \frac{dv_2}{2\pi} e^{i(\Omega v_2 + kv_1)}. \quad (50)$$

Using Eq. (46), we can evaluate Eqs. (49) and (50) to obtain nonzero Bogoliubov coefficients. Specifically, we obtain $\beta_{21v}^*(k, \Omega) \neq 0$ (which will be shown in the next subsection), we can confirm that the left-moving positive/negative frequency modes in R_2 can be expanded in terms of the left-moving positive and negative frequency modes of R_1 [31]. Additionally, since the positive frequency modes ($j_2(q) \propto e^{-i\omega_q t_2}$) are proportional to the combination of the positive and negative frequency modes of R_1 due to the nonzero $\beta_{21v}(k, \Omega)$, the R_1 vacuum state will not be a vacuum state for R_2 . In the following subsections, we will first evaluate $\beta_{21v}^*(k, \Omega)$, and then compute the mean number density by multiplying $[\beta_{21v}^*(k, \Omega)\beta_{21v}(k, \Omega)]$. We will explicitly demonstrate that, for large values of

frequencies (Ω), the computed number density distribution aligns with the Planckian distribution.

1. Bogoliubov coefficients

As discussed, showing $\beta_{21v}^*(k, \Omega) \neq 0$ is enough to validate the existence of a nontrivial Bogoliubov transformation between modes. In this subsection, we will evaluate $\beta_{21v}^*(k, \Omega)$, represented by the integral in Eq. (50), and show that it is indeed nonzero. Since the integral [see Eq. (50)] is evaluated with respect to v_2 , we will express v_1 in terms of v_2 using the relationship given by Eq. (46). Thus we get,

$$\beta_{21v}^*(k, \Omega) = -\sqrt{\frac{\Omega}{k}} \int_{-\infty}^{\infty} \frac{dv_2}{2\pi} e^{i\left(\Omega v_2 + k \frac{\log(e^{gv_2} + \lambda)}{g}\right)}. \quad (51)$$

Taking the acceleration parameter g and shift parameter λ as constants, and then substituting $e^{gv_2} = h$, such that $dv_2 = dh/(gh)$. Consequently, the integral part of Eq. (51), denoted by “ $I(\Omega)$,” can be expressed in terms of h as follows:

$$I(\Omega) = \int_0^{\infty} \frac{dh}{g} \frac{h^{\frac{\Omega}{h}} (h + \lambda)^{\frac{ik}{h}}}{h}, \quad (52)$$

It is important to note that the new limits of integration range from 0 to ∞ . By evaluating the above integral $I(\Omega)$ and then combining it with the other constants, we obtain,

$$\beta_{21v}^*(k, \Omega) = \frac{-1}{2\pi g} \sqrt{\frac{\Omega}{k}} \frac{\lambda^{\frac{i(k+\Omega)}{g}} \Gamma\left(\frac{i\Omega}{g}\right) \Gamma\left(-\frac{i(k+\Omega)}{g}\right)}{\Gamma\left(-\frac{ik}{g}\right)}. \quad (53)$$

The corresponding complex conjugate can be written as follows:

$$\beta_{21v}(k, \Omega) = \frac{-1}{2\pi g} \sqrt{\frac{\Omega}{k}} \frac{\lambda^{\frac{-i(k+\Omega)}{g}} \Gamma\left(\frac{-i\Omega}{g}\right) \Gamma\left(\frac{i(k+\Omega)}{g}\right)}{\Gamma\left(\frac{ik}{g}\right)}. \quad (54)$$

Using these Bogoliubov coefficients, we can compute the mean particle number density for the R_1 vacuum state $|0\rangle_{R_1}$ in the R_2 frame. It is important to note that, from the expression for the Bogoliubov coefficients expressed above, the mean particle number density does not depend on the shift parameter λ .

2. Particle spectrum

To understand the particle content of left-moving modes, we compute the mean number density of the R_2 particle number operator (defined for the left-moving sector) for a specific frequency Ω with respect to the R_1 vacuum state $|0\rangle_{R_1}$. We know, the standard number operator for a specific

mode “ Ω ” is defined as $\hat{N}(\Omega) \equiv \hat{d}_2^\dagger(\Omega) \hat{d}_2(\Omega)$ and for states with multiples modes (labeled with Ω), the expectation value of the total number operator is defined as [32],

$$\hat{N} = \int \frac{d\Omega}{2\pi} \hat{N}(\Omega) = \int \frac{d\Omega}{2\pi} \hat{d}_2^\dagger(\Omega) \hat{d}_2(\Omega). \quad (55)$$

For our analysis, we focus on the expectation value of the R_2 particle number operator for a specific frequency Ω with respect to the R_1 vacuum state, which can be expressed as follows:

$$\langle 0|_{R_1} \hat{N}(\Omega) |0\rangle_{R_1} = \langle 0|_{R_1} \hat{d}_2^\dagger(\Omega) \hat{d}_2(\Omega) |0\rangle_{R_1}. \quad (56)$$

The above expression can be interpreted as the mean number of particles with the frequency Ω found in the R_2 frame. To evaluate Eq. (56), the R_2 creation/annihilation operators are expressed in terms of the R_1 creation and annihilation operators [see Eqs. (42) and (43)]. We then apply the commutation relations satisfied by these operators: $[\hat{d}_1(k), \hat{d}_1^\dagger(p)] = \delta(k-p)$. Thus, we obtain the following:

$$\langle 0|_{R_1} \hat{N}(\Omega) |0\rangle_{R_1} = \int_0^\infty dk |\beta_{21v}(\Omega, k)|^2. \quad (57)$$

Using Eq. (53) and its complex conjugate Eq. (54), we can express the integrand of the above as follows:

$$|\beta_{21v}(k, \Omega)|^2 = \left[\frac{1}{e^{\frac{2\pi\Omega}{g}} - 1} \right] \frac{\sinh\left(\frac{\pi k}{g}\right) \text{csch}\left(\frac{\pi(k+\Omega)}{g}\right)}{2\pi g(k+\Omega) e^{\frac{-\pi\Omega}{g}}}. \quad (58)$$

Thus, the expectation value integral Eq. (57) can be written as,

$$\begin{aligned} \langle 0|_{R_1} \hat{N}(\Omega) |0\rangle_{R_1} &= \int_0^\infty dk F(k, \Omega), \\ &= \int_0^\infty dk \frac{\sinh\left(\frac{\pi k}{g}\right) \text{csch}\left(\frac{\pi(k+\Omega)}{g}\right)}{2\pi g(k+\Omega) e^{\frac{-\pi\Omega}{g}}}, \end{aligned} \quad (59)$$

where the integrand function can be represented by $F(k, \Omega)$. We observe that the integrand function $F(k, \Omega)$ tends to zero in the $k \rightarrow 0$ limit, indicating that it is proportional to k . Consequently, the integral does not exhibit any infrared divergence. In the limit $k \rightarrow \infty$, the integrand tends to zero again and is proportional to $1/k$. Therefore, we expect the integral to have logarithmic divergence. We will investigate this below. In order to solve the integral Eq. (59), we take the approximated form for the integrand function $F(k, \Omega)$, such that the approximated function denoted by $\tilde{F}(k, \Omega)$, coincides with $F(k, \Omega)$ for large k . This is because the main contribution to the integrand comes from large k (see Fig. 4). From the

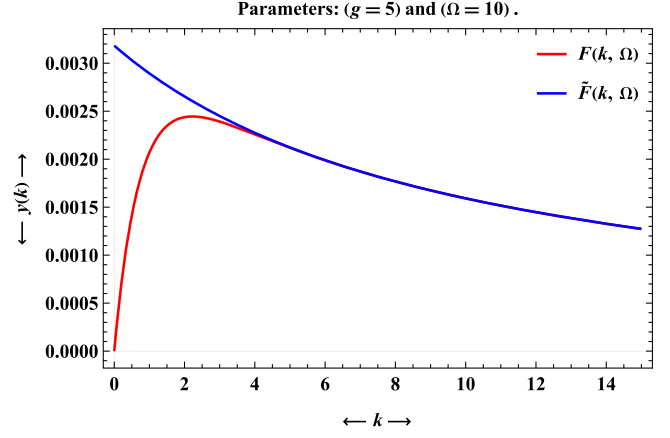


FIG. 4. For large k the original integrand function $F(k, \Omega)$ and the approximated function $\tilde{F}(k, \Omega)$ coincides.

expression of $F(k, \Omega)$, and using the fact that $\sinh(x) \approx e^x/2$, for large values of x , we can obtain the approximate form for $F(k, \Omega)$ as follows:

$$F(k, \Omega) \approx \tilde{F}(k, \Omega) = \frac{\frac{e^{\frac{\pi k}{g}}}{2} e^{\frac{\pi\Omega}{g}}}{2\pi g(k+\Omega) \frac{e^{\frac{\pi(k+\Omega)}{g}}}{2}}. \quad (60)$$

On simplification, we get,

$$\tilde{F}(k, \Omega) = \frac{1}{2\pi g(k+\Omega)}. \quad (61)$$

From Fig. 4, we can see that $\tilde{F}(k, \Omega)$ bounds $F(k, \Omega)$ and it perfectly matches with $F(k, \Omega)$ for large values of k . Thus, we can express the expectation value integral using the approximated integrand as follows:

$$\langle 0|_{R_1} \hat{N}(\Omega) |0\rangle_{R_1} \approx \int_0^\infty dk \tilde{F}(k, \Omega). \quad (62)$$

Therefore, evaluating the mean value of the R_2 particle number operator for a specific frequency “ Ω ” with respect to the R_1 vacuum state, we obtain the following:

$$\langle 0|_{R_1} \hat{N}(\Omega) |0\rangle_{R_1} = \frac{1}{e^{\frac{2\pi\Omega}{g}} - 1} \int_0^\infty dk \frac{1}{2\pi g(k+\Omega)}. \quad (63)$$

The integral in Eq. (63) diverges logarithmically, which leads to a divergent term with units of spatial volume. We assume that this volume divergence can be addressed by incorporating a volume term in the denominator. To achieve this, we define the mean number density of the particles with frequency “ Ω ” (as discussed in Ref. [29]) as follows:

$$\hat{n}(\Omega) = \frac{\langle \hat{N}(\Omega) \rangle}{V}. \quad (64)$$

Specifically, for our case, we obtain:

$$\hat{n}(\Omega) = \frac{\langle 0|_{R_1} \hat{N}(\Omega) |0\rangle_{R_1}}{V}. \quad (65)$$

Consequently, from Eqs. (63) and (65), we obtain the mean particle number density for a specific frequency “ Ω ” as follows:

$$\hat{n}(\Omega) = \frac{1}{[e^{\frac{2\pi\Omega}{g}} - 1]}. \quad (66)$$

Notably, the particle number density obeys a Planckian distribution, as expected for the case of scalar fields.

Thus, the above analysis demonstrates that modes can be selectively excited in unique scenarios, such as in our case, where we have considered null-shifted wedges (see Figs. 2 and 3). In particular, only the left-moving modes are excited in the case under consideration (see Fig. 2). We claim that this is a case of informative excitation since the particle content reveals the causal positioning of the superset (R_1) that yields the given selectively excited state. We discuss this in more detail in the Sec. V.

IV. CHIRAL EXCITATION FOR MASSLESS DIRAC FIELDS

We will now explore the excitations of fermionic fields, which, due to spin connections, are more directly connected to spacetime points than massless scalar fields. Since the Rindler spacetime works very well as a toy model for the black hole scenarios involving horizons, we set up Dirac fields in Rindler spacetime to capture the effect of null-shifted horizons on Dirac fields. The subsections below will investigate Dirac fields in Minkowski and Rindler spacetime backgrounds. We follow the same notations as in the previous sections, where the Minkowski coordinates are labeled using (T, X) and the Rindler using (t, x) and specifically for the R_1 frame, it is (t_1, x_1) , and for R_2 it is (t_2, x_2) . Furthermore, since we consider only massless fields for our analysis, we obtain $\omega_k > 0$ from the on-shell conditions $\omega_k = |k|$. For notational convenience, we have relabeled ω_k as “ k ,” where $k > 0$ indicates that it is non-negative. These notations will be used in the mode expansions of the field operators in the following sections.

A. Dirac fields in Minkowski spacetime

The Dirac equation for massless spin 1/2 particle in $(1+1)$ dimensional Minkowski spacetime can be expressed as

$$i\gamma^\mu \partial_\mu \Psi = 0. \quad (67)$$

We will choose the chiral representation of the gamma matrices for two-dimensional Minkowski spacetime as given in [33,34],

$$\gamma^0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \gamma^1 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \quad (68)$$

Such that the above gamma matrices obey the Clifford algebra,

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2\eta^{\mu\nu}. \quad (69)$$

Furthermore, the conditions $(\gamma^0)^2 = I$ and $(\gamma^1)^2 = -I$ are additional constraints imposed due to the particular choice of chiral representation (see Appendix B.3 of Ref. [35]). We will consider the chirality operator for $(1+1)$ dimensions (see Ref. [36]) as

$$\gamma^5 := \gamma^0 \gamma^1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \quad (70)$$

Now, returning to the Dirac equation, substituting Eq. (68) in Eq. (67), we can obtain the expanded form of the Dirac equation as

$$[i\gamma^0 \partial_0 + i\gamma^1 \partial_1] \Psi = 0. \quad (71)$$

The spinor solution Ψ of the above equation can be expressed as a two-component column matrix as follows:

$$\Psi = \begin{bmatrix} \psi_{(+)} \\ \psi_{(-)} \end{bmatrix}. \quad (72)$$

Here, we can observe that the components of the spinor wavefunction are eigenstates of the chirality operator γ^5 , i.e.,

$$\begin{aligned} \gamma^5 \psi_{(+)} &= +\psi_{(+)}, \\ \gamma^5 \psi_{(-)} &= -\psi_{(-)}, \end{aligned} \quad (73)$$

where, “+” and “−” are the eigenvalues. The $\psi_{(+)}$ and $\psi_{(-)}$ denote the right-chiral and left-chiral spinor wave functions, respectively. Now, substituting Eq. (72) in Eq. (71) leads to two decoupled differential equations, which can be expressed as follows:

$$\frac{\partial \psi_{(-)}}{\partial T} - \frac{\partial \psi_{(-)}}{\partial X} = 0, \quad (74)$$

and,

$$\frac{\partial \psi_{(+)}}{\partial T} + \frac{\partial \psi_{(+)}}{\partial X} = 0. \quad (75)$$

The above equations can be solved separately and combined to get Ψ . The mode expansion of the Dirac field Ψ can be expressed as follows:

$$\Psi = \frac{1}{\sqrt{2\pi}} \int_0^\infty dk \left(\hat{a}(k) \begin{bmatrix} e^{ik(T-X)} \\ 0 \end{bmatrix} + \hat{c}^\dagger(k) \begin{bmatrix} e^{-ik(T-X)} \\ 0 \end{bmatrix} + \hat{b}(k) \begin{bmatrix} 0 \\ e^{-ik(T+X)} \end{bmatrix} + \hat{d}^\dagger(k) \begin{bmatrix} 0 \\ e^{ik(T+X)} \end{bmatrix} \right). \quad (76)$$

Expressing the above in Minkowski lightcone coordinates $[(U = T - X) \text{ and } (V = T + X)]$, we obtain,

$$\Psi = \psi_{(+)}^U + \psi_{(-)}^V, \quad (77)$$

where $\psi_{(+)}^U$ and $\psi_{(-)}^V$ are the decoupled right-chiral and left-chiral sectors of the field, respectively. For the rest of the calculations in this article, we consider the simplest case of Dirac fields, i.e., the real representation of the Dirac field commonly known as the Majorana representation. In the case of Majorana fields, we have to impose the constraint that the field Ψ has to be real, i.e.,

$$\Psi = \Psi^*, \quad (78)$$

where the “*” denotes complex conjugate. It has to be noted that a Majorana field describes particles with properties similar to Dirac particles, except that particles and antiparticles are identical [37]. Now, with these considerations, the Eq. (76) can be expressed as follows:

$$\Psi = \frac{1}{\sqrt{2\pi}} \int_0^\infty dk \left(\hat{a}(k) \begin{bmatrix} e^{ik(T-X)} \\ 0 \end{bmatrix} + \hat{a}^\dagger(k) \begin{bmatrix} e^{-ik(T-X)} \\ 0 \end{bmatrix} + \hat{b}(k) \begin{bmatrix} 0 \\ e^{-ik(T+X)} \end{bmatrix} + \hat{b}^\dagger(k) \begin{bmatrix} 0 \\ e^{ik(T+X)} \end{bmatrix} \right). \quad (79)$$

In Minkowski light-cone coordinates, the respective right-chiral and left-chiral sectors can be written as

$$\psi_{(+)}^U = \frac{1}{\sqrt{2\pi}} \int_0^\infty dk \left(\hat{a}(k) \begin{bmatrix} e^{-ikU} \\ 0 \end{bmatrix} + \hat{a}^\dagger(k) \begin{bmatrix} e^{ikU} \\ 0 \end{bmatrix} \right), \quad (80)$$

and,

$$\psi_{(-)}^V = \frac{1}{\sqrt{2\pi}} \int_0^\infty dk \left(\hat{b}(k) \begin{bmatrix} 0 \\ e^{-ikV} \end{bmatrix} + \hat{b}^\dagger(k) \begin{bmatrix} 0 \\ e^{ikV} \end{bmatrix} \right), \quad (81)$$

where $1/\sqrt{2\pi}$ is the normalization factor determined from the inner product defined on the constant time hypersurface. It is important to note that k is always positive, and the operators $\hat{a}_1^\dagger(k)$, $\hat{b}_1^\dagger(k)$ and $\hat{a}_1(k)$, $\hat{b}_1(k)$ are the usual creation and annihilation operators of the Dirac field (Majorana), and they obey the standard anticommutation relations

$$\begin{aligned} \{\hat{a}(k), \hat{a}^\dagger(k')\} &= \delta(k - k'), & \{\hat{b}(k), \hat{b}^\dagger(k')\} &= \delta(k - k'), \\ \{\hat{a}(k), \hat{a}(k')\} &= 0, & \text{and } \{\hat{b}(k), \hat{b}(k')\} &= 0, \\ \{\hat{a}^\dagger(k), \hat{a}^\dagger(k')\} &= 0, & \{\hat{b}^\dagger(k), \hat{b}^\dagger(k')\} &= 0. \end{aligned} \quad (82)$$

As discussed in the analysis of scalar fields, the Fock space is constructed from the above nonvanishing anticommutation relations. Thus we define the Minkowski vacuum state $|0\rangle_M$, as the state annihilated by all the annihilation operators [27], i.e.,

$$\hat{a}(k)|0\rangle_M = 0 \quad \text{and} \quad \hat{b}(k)|0\rangle_M = 0, \quad \forall k > 0. \quad (83)$$

Here, the excited states can be obtained from the vacuum state using the creation operators. The following section focuses on Dirac fields (Majorana) in the Rindler spacetime.

B. Dirac fields in Rindler spacetime

Dirac fields in the Rindler metric can be studied using the vierbein formalism [38,39]. Thus, we can express the local inertial frame metric η_{AB} as

$$\eta_{AB} = e_A^\mu e_B^\nu g_{\mu\nu}, \quad (84)$$

where the Zweibeins e_A^μ are used to project vectors from a coordinated frame (Global) to a noncoordinated frame (local Lorentz) [33]. For the case of the Rindler metric, we can express the Zweibeins as,

$$e_A^\mu = \begin{bmatrix} e^{-gx} & 0 \\ 0 & e^{-gx} \end{bmatrix}. \quad (85)$$

Another important thing required to express the Dirac equation in Rindler spacetime is the spacetime-dependent gamma matrix, $\tilde{\gamma}^\mu$. It can be computed from the zweibeins and the normal flat spacetime gamma matrix as follows:

$$\tilde{\gamma}^\mu = e_A^\mu \gamma^A. \quad (86)$$

Also, we note that the Dirac spinors obey the following inner product [40]

$$\langle \phi | \psi \rangle = \int_\Sigma \bar{\phi} \tilde{\gamma}^\mu n_\mu \psi d\Sigma, \quad (87)$$

where $\bar{\phi} = \phi^\dagger \gamma^0$, n is the future directed normal to the hypersurface Σ and $d\Sigma$ is the invariant volume element on Σ . Following a similar analysis as in Refs. [33,41], we can express the Dirac equation in a general (1 + 1) dimensional spacetime as follows:

$$\left[i\gamma^A e_A^\mu \partial_\mu + \frac{1}{2} \gamma^A \frac{1}{\sqrt{-\bar{g}}} \partial_\mu (\sqrt{-\bar{g}} e_A^\mu) \right] \Psi = 0, \quad (88)$$

where $\bar{g}$ is the determinant of the metric. Substituting the parameters of the Rindler metric on Eq. (88), we obtain as follows:

$$\left[i e^{-gx} \gamma^0 \partial_0 + i e^{-gx} \gamma^1 \partial_1 + \frac{1}{2} i g e^{-gx} \gamma^1 \partial_1 \right] \Psi = 0. \quad (89)$$

The above equation is identical to the Dirac equation for Minkowski spacetime [see Eq. (67)], under a conformal transformation, where the field $\Psi \rightarrow e^{-gx/2} \Psi$. Now, to solve the above equation for the positive frequency modes $\Psi_{\mathbf{P}}(k)$, we introduce a trial solution (see Refs. [33,42]),

$$\Psi_{\mathbf{P}}(k) = e^{-i\omega_k t} \begin{bmatrix} f_{(+)}(x) \\ f_{(-)}(x) \end{bmatrix}, \quad (90)$$

where $f_{(+)}(x)$ and $f_{(-)}(x)$ are the space-dependent part of the solution and the notation “(+)” and “(−)” denotes the right and left chirality respectively. Substituting this in Eq. (89), we obtain,

$$2i \frac{d f_{(-)}(x)}{dx} - (2\omega_k - ig) f_{(-)}(x) = 0, \quad (91)$$

and

$$2i \frac{d f_{(+)}(x)}{dx} + (2\omega_k + ig) f_{(+)}(x) = 0. \quad (92)$$

The above differential equations are solved to get the spatial part. It is then combined with the temporal part to get the total solution and is then normalized using the inner product defined by Eq. (87) (see Ref. [33]). Since $f_{(+)}(x)$ and $f_{(-)}(x)$ are decoupled, we can express the complete positive mode solution as

$$\Psi_{\mathbf{P}}(k) = \frac{e^{-\frac{gx}{2}}}{\sqrt{2\pi}} \left(\hat{c}(k) \begin{bmatrix} e^{-ik(t-x)} \\ 0 \end{bmatrix} + \hat{d}(k) \begin{bmatrix} 0 \\ e^{-ik(t+x)} \end{bmatrix} \right), \quad (93)$$

where $1/\sqrt{2\pi}$ is the normalization constant. Following the same procedure, we can obtain the mode expansion for the negative mode solutions as follows:

$$\Psi_{\mathbf{N}}(k) = \frac{e^{-\frac{gx}{2}}}{\sqrt{2\pi}} \left(\hat{c}^\dagger(k) \begin{bmatrix} e^{ik(t-x)} \\ 0 \end{bmatrix} + \hat{d}^\dagger(k) \begin{bmatrix} 0 \\ e^{ik(t+x)} \end{bmatrix} \right). \quad (94)$$

Combining Eqs. (93) and (94), i.e., $\Psi = \Psi_{\mathbf{P}} + \Psi_{\mathbf{N}}$ and expressing them as right-chiral and left-chiral sectors of the field [as similar to Eq. (77)], we obtain

$$\begin{aligned} \Psi = & \frac{e^{-\frac{gx}{2}}}{\sqrt{2\pi}} \int_0^\infty dk \left(\hat{c}(k) \begin{bmatrix} e^{-ik(t-x)} \\ 0 \end{bmatrix} + \hat{c}^\dagger(k) \begin{bmatrix} e^{ik(t-x)} \\ 0 \end{bmatrix} \right) \\ & + \frac{e^{-\frac{gx}{2}}}{\sqrt{2\pi}} \int_0^\infty dk \left(\hat{d}(k) \begin{bmatrix} 0 \\ e^{-ik(t+x)} \end{bmatrix} + \hat{d}^\dagger(k) \begin{bmatrix} 0 \\ e^{ik(t+x)} \end{bmatrix} \right). \end{aligned} \quad (95)$$

In Rindler light-cone coordinates $[(u = t - x) \text{ and } (v = t + x)]$, we can write the above as

$$\Psi = \psi_{(+)} + \psi_{(-)}, \quad (96)$$

where as usual the (+) and (−) denotes the right/left chirality respectively. It is important to note that the mode expansion mentioned above applies to a Dirac field operator in a general Rindler spacetime setting. We note that for the case of a massless scalar field, one can separate out the left-moving and right-moving sectors, whereas, for the fermionic case, one can similarly separate out the right chiral and left chiral sectors. When performing calculations in a specific Rindler frame, the variables “ t ” and “ x ” will be replaced with the frame’s coordinates (t_i, x_i) , where the label “ i ” denotes the corresponding Rindler frame. Thus, the Eq. (95) can be rewritten as:

$$\begin{aligned} \Psi = & \frac{e^{-\frac{gx_i}{2}}}{\sqrt{2\pi}} \int_0^\infty dk \left(\hat{c}_i(k) \begin{bmatrix} e^{-ik(t_i-x_i)} \\ 0 \end{bmatrix} + \hat{c}_i^\dagger(k) \begin{bmatrix} e^{ik(t_i-x_i)} \\ 0 \end{bmatrix} \right) \\ & + \frac{e^{-\frac{gx_i}{2}}}{\sqrt{2\pi}} \int_0^\infty dk \left(\hat{d}_i(k) \begin{bmatrix} 0 \\ e^{-ik(t_i+x_i)} \end{bmatrix} + \hat{d}_i^\dagger(k) \begin{bmatrix} 0 \\ e^{ik(t_i+x_i)} \end{bmatrix} \right). \end{aligned} \quad (97)$$

Consequently, we can define R_1 vacuum state $(|0\rangle_{R_1})$ and R_2 vacuum state $(|0\rangle_{R_2})$ from the corresponding set of annihilation operators of R_1 and R_2 as the state which is annihilated by all the annihilation operators defined in the corresponding Hilbert spaces. It can be shown as

$$\hat{c}_i(k)|0\rangle_{R_i} = 0 \quad \text{and} \quad \hat{d}_i(k)|0\rangle_{R_i} = 0, \quad \forall k > 0, \quad (98)$$

where “ $i = 1$ ” and “ $i = 2$ ” denote the R_1 and R_2 spacetimes respectively. In the following sections, we will choose different Rindler frames for our analysis of the particle spectrum and investigate the phenomenon of selective excitations.

C. Thermal spectrum of fermions in Rindler spacetime from Minkowski vacuum state

In our analysis, we first briefly derive the well-known result for the thermal spectrum of fermions in R_1 spacetime from the Minkowski vacuum state (see Fig. 2). As discussed in the section on scalar fields, we adopt the ansatz that the modes in R_1 are related to the modes in Minkowski space through a Bogoliubov transformation of

the following form:

$$h_{(+/-)1}(q) = \int_0^\infty dk (\alpha_{10}^{(+/-)}(q, k) f_{(+/-)M}(k) + \beta_{10}^{(+/-)}(q, k) f_{(+/-)M}^*(k)), \quad (99)$$

$$f_{(+/-)M}(k) = \int_0^\infty dq (\alpha_{10}^{(+/-)*}(q, k) h_{(+/-)1}(q) - \beta_{10}^{(+/-)}(q, k) h_{(+/-)1}^*(q)), \quad (100)$$

where $f_{(+/-)M}$ are the Minkowski modes and $h_{(+/-)1}$ are the R_1 modes respectively. It is important to note that the subscript “ $(+)$ ” and “ $(-)$ ” in the Bogoliubov transformations [e.g., $\alpha_{10}^{(+/-)}(q, k)$] denotes the right/left-chiral cases respectively and the subscript “ $_{10}$ ” indicates R_1 and Minkowski spacetimes. Based on the above ansatz [see Eqs. (99) and (100)], the creation and annihilation operators of R_1 can be expressed in terms of those of Minkowski as follows:

$$\hat{c}_1(p) = \int_0^\infty dk (\alpha_{10}^{(+/-)*}(p, k) \hat{a}_M(k) - \beta_{10}^{(+/-)*}(p, k) \hat{a}_M^\dagger(k)), \quad (101)$$

$$\hat{c}_1^\dagger(p) = \int_0^\infty dk (\alpha_{10}^{(+/-)}(p, k) \hat{a}_M^\dagger(k) - \beta_{10}^{(+/-)}(p, k) \hat{a}_M(k)). \quad (102)$$

Returning to the analysis of the Dirac fields, we consider the standard mode expansion of the Dirac field operator in terms of the right-chiral and left-chiral sectors, given by Eqs. (79) and (95) in Minkowski and R_1 spacetime, respectively. Here, the right-chiral and left-chiral modes are decoupled. Now, since the massless Dirac field considered is conformally invariant, at any point in the common spacetime domain of R_1 and Minkowski coordinates, we can equate the right-chiral modes of both R_1 and Minkowski spacetime [30], i.e., $\psi_{(+)} = \psi_{(+)}^U$. Thus, we obtain

$$\begin{aligned} & \int_0^\infty dk \frac{e^{-\frac{gx_1}{2}}}{\sqrt{2\pi}} \left(\hat{c}_1(k) \begin{bmatrix} e^{-ik(t_1-x_1)} \\ 0 \end{bmatrix} + \hat{c}_1^\dagger(k) \begin{bmatrix} e^{ik(t_1-x_1)} \\ 0 \end{bmatrix} \right) \\ &= \int_0^\infty dp \frac{1}{\sqrt{2\pi}} \left(\hat{a}_M(p) \begin{bmatrix} e^{-ip(T-X)} \\ 0 \end{bmatrix} + \hat{a}_M^\dagger(p) \begin{bmatrix} e^{ip(T-X)} \\ 0 \end{bmatrix} \right). \end{aligned} \quad (103)$$

For the Cauchy hypersurface $t_1 = T = 0$, the above equation becomes,

$$\begin{aligned} & \int_0^\infty dk \frac{e^{-\frac{gx_1}{2}}}{\sqrt{2\pi}} \left(\hat{c}_1(k) \begin{bmatrix} e^{ikx_1} \\ 0 \end{bmatrix} + \hat{c}_1^\dagger(k) \begin{bmatrix} e^{-ikx_1} \\ 0 \end{bmatrix} \right) \\ &= \int_0^\infty dp \frac{1}{\sqrt{2\pi}} \left(\hat{a}_M(p) \begin{bmatrix} e^{ipX} \\ 0 \end{bmatrix} + \hat{a}_M^\dagger(p) \begin{bmatrix} e^{-ipX} \\ 0 \end{bmatrix} \right). \end{aligned} \quad (104)$$

It is clear from Eq. (2) that $X = e^{gx_1}/g$, and for $t_1 = 0$, the lhs and rhs of Eq. (104) are functions of x_1 . Therefore, applying the Fourier transform on both sides to get everything in terms of Ω , we obtain as follows:

$$\begin{aligned} & \int_0^\infty dk \int_{-\infty}^\infty \frac{dx_1}{2\pi} (\hat{c}_1(k) e^{i(k+\Omega)x_1} + \hat{c}_1^\dagger(k) e^{i(\Omega-k)x_1}) \\ &= \int_0^\infty dp \left[\left(\int_{-\infty}^\infty \frac{dx_1}{2\pi} e^{\frac{gx_1}{2}} e^{i\Omega x_1} e^{ip \frac{e^{gx_1}}{g}} \right) \hat{a}_M(p) \right. \\ & \quad \left. + \left(\int_{-\infty}^\infty \frac{dx_1}{2\pi} e^{\frac{gx_1}{2}} e^{i\Omega x_1} e^{-ip \frac{e^{gx_1}}{g}} \right) \hat{a}_M^\dagger(p) \right]. \end{aligned} \quad (105)$$

For $\Omega > 0$, i.e., positive, we can evaluate the lhs to be $c_1^\dagger(\Omega)$ and then compare Eq. (105) with Eq. (102), to get,

$$\alpha_{10}^{(+)}(\Omega, p) = \int_{-\infty}^\infty \frac{dx_1}{2\pi} e^{\frac{gx_1}{2}} e^{i\Omega x_1} e^{-ip \frac{e^{gx_1}}{g}}, \quad (106)$$

and,

$$\beta_{10}^{(+)}(\Omega, p) = - \int_{-\infty}^\infty \frac{dx_1}{2\pi} e^{\frac{gx_1}{2}} e^{i\Omega x_1} e^{ip \frac{e^{gx_1}}{g}}. \quad (107)$$

Evaluating the above integral, we get expressions for the Bogoliubov coefficients; we see that $\beta_{10}^{(+)}(\Omega, p) \neq 0$, which implies the existence of a nontrivial Bogoliubov transformation between the modes. Since we are interested in the particle content, we compute the mean number density of R_1 particle number operator for a specific frequency Ω with respect to the Minkowski vacuum state $|0\rangle_M$ [see Eqs. (57) and (65)]. Thus, we have to evaluate $\langle 0 |_M \hat{N}(\Omega) | 0 \rangle_M = \int_0^\infty dk |\beta_{10}^{(+)}(\Omega, k)|^2$. Now, in order to get the above, we have to compute $\beta_{10}^{(+)}(\Omega, p)$ from Eq. (107). We substitute $e^{gx_1} = h$, such that $dx_1 = dh/(gh)$, and express Eq. (107) in terms of h and then apply new limits to obtain

$$\beta_{10}^{(+)}(\Omega, p) = \frac{-e^{\frac{i\pi}{4}} e^{-\frac{\pi\Omega}{2g}} \left(\frac{p}{g}\right)^{-\frac{1}{2} - \frac{i\Omega}{g}} \Gamma\left(\frac{1}{2} + \frac{i\Omega}{g}\right)}{2\pi g}. \quad (108)$$

From Eq. (108), we can evaluate the expectation of the particle number operator (for a specific frequency Ω) as follows:

$$\begin{aligned}
\langle 0|_M \hat{N}(\Omega)|0\rangle_M &= \int_0^\infty dp |\beta_{10}^{(+)}(\Omega, p)|^2, \\
&= \int_0^\infty dp \left(\frac{e^{-\frac{\pi\Omega}{g}} \left(\frac{p}{g}\right)^{-1} \left|\Gamma\left(\frac{1}{2} + \frac{i\Omega}{g}\right)\right|^2}{4\pi^2 g^2} \right), \\
\therefore \langle 0|_M \hat{N}(\Omega)|0\rangle_M &= \int_0^\infty \frac{d\Omega}{2\pi} \frac{1}{[e^{\frac{2\pi\Omega}{g}} + 1]} \underbrace{\int_0^\infty dp \frac{1}{p} \frac{1}{2\pi g}}_{\delta(0) \rightarrow \text{divergent factor}},
\end{aligned}$$

The above integral is divergent in nature; to deal with this divergence, we define the mean number density of the particles with frequency Ω [see Eq. (65)] [29], thus we obtain

$$\hat{n}(\Omega) = \frac{1}{[e^{\frac{2\pi\Omega}{g}} + 1]}. \quad (109)$$

The above result affirms the Unruh effect for the case of Dirac fields [43,44]. The subsequent sections provide a detailed discussion of the primary focus of this work, i.e., the selective excitation of the Dirac fields.

D. Negligible excitations of right-chiral modes

In the following subsections, we investigate the phenomenon of selective excitations of left/right chiral modes of Dirac (Majorana) fields. We analyze how the vacuum state of a Dirac (Majorana) field in R_1 is perceived in the R_2 frame and evaluate the corresponding particle density spectrum for different chiral particles. Specifically, we will examine the scenario described by Fig. 2, where we have a null-shifted R_2 frame in the V -axis. Now, following the similar line of analysis adopted as in Sec. III for the case of scalar fields, we first consider the ansatz that the modes of R_2 represented by $h_{(+/-)2}$ are related to the modes of R_1 represented by $h_{(+/-)1}$, through a Bogoliubov transformation of the following form,

$$\begin{aligned}
h_{(+/-)2}(q) &= \int_0^\infty dk (\alpha_{21}^{(+/-)}(q, k) h_{(+/-)1}(k) \\
&\quad + \beta_{21}^{(+/-)}(q, k) h_{(+/-)1}^*(k)), \quad (110)
\end{aligned}$$

$$\begin{aligned}
h_{(+/-)1}(k) &= \int_0^\infty dq (\alpha_{21}^{*(+/-)}(q, k) h_{(+/-)2}(q) \\
&\quad + \beta_{21}^{(+/-)}(q, k) h_{(+/-)2}^*(q)). \quad (111)
\end{aligned}$$

Note, as mentioned earlier, the subscript “ $(+/-)$ ” in the Bogoliubov transformations [e.g., $\alpha_{21}^{(+/-)}(q, k)$] implies the right/left chirality and the “ $_{21}$ ” refers to R_2 and R_1 spacetimes.

Returning to our analysis of the selective excitations for the case of right-chiral Dirac particles, based on the above ansatz, the creation/annihilation operators of R_2 can be expressed in terms of R_1 (for the right-chiral case) as follows:

$$\begin{aligned}
\hat{c}_2(p) &= \int_0^\infty dk (\alpha_{21}^{*(+)}(p, k) \hat{c}_1(k) \\
&\quad - \beta_{21}^{*(+)}(p, k) \hat{c}_1^\dagger(k)), \quad (112)
\end{aligned}$$

$$\begin{aligned}
\hat{c}_2^\dagger(p) &= \int_0^\infty dk (\alpha_{21}^{(+)}(p, k) \hat{c}_1^\dagger(k) \\
&\quad - \beta_{21}^{(+)}(p, k) \hat{c}_1(k)). \quad (113)
\end{aligned}$$

In the overlapping region of the coordinates for R_1 and R_2 spacetimes, we can perform the standard mode expansion as in Eq. (97), such that we observe the right-chiral and left-chiral sectors are decoupled and can be treated separately. Since our Dirac field is conformally invariant and decoupled into the right/left sector, we can conclude the equivalence of the right-chiral sectors of R_1 and R_2 . Thus, we obtain

$$\begin{aligned}
&\int_0^\infty dk \frac{e^{-\frac{gk_1}{2}}}{\sqrt{2\pi}} \left(\hat{c}_1(k) \begin{bmatrix} e^{-ik(t_1-x_1)} \\ 0 \end{bmatrix} + \hat{c}_1^\dagger(k) \begin{bmatrix} e^{ik(t_1-x_1)} \\ 0 \end{bmatrix} \right) \\
&= \int_0^\infty dp \frac{e^{-\frac{gp_2}{2}}}{\sqrt{2\pi}} \left(\hat{c}_2(p) \begin{bmatrix} e^{-ip(t_2-x_2)} \\ 0 \end{bmatrix} + \hat{c}_2^\dagger(p) \begin{bmatrix} e^{ip(t_2-x_2)} \\ 0 \end{bmatrix} \right). \quad (114)
\end{aligned}$$

Expressing in Rindler null coordinates [see Eqs. (9) and (10)], the above becomes,

$$\begin{aligned}
&\int_0^\infty dk \frac{e^{-\frac{g}{4}(v_1-u_1)}}{\sqrt{2\pi}} \left(\hat{c}_1(k) \begin{bmatrix} e^{-iku_1} \\ 0 \end{bmatrix} + \hat{c}_1^\dagger(k) \begin{bmatrix} e^{iku_1} \\ 0 \end{bmatrix} \right) \\
&= \int_0^\infty dp \frac{e^{-\frac{g}{4}(v_2-u_2)}}{\sqrt{2\pi}} \left(\hat{c}_2(p) \begin{bmatrix} e^{-ipu_2} \\ 0 \end{bmatrix} + \hat{c}_2^\dagger(p) \begin{bmatrix} e^{ipu_2} \\ 0 \end{bmatrix} \right). \quad (115)
\end{aligned}$$

We multiply both lhs and rhs of Eq. (115) with

$$\int_{-\infty}^\infty \frac{du_2}{2\pi} e^{\frac{g}{4}(v_2-u_2)} [e^{i\Omega u_2} \quad 0], \quad (116)$$

such that the rhs of Eq. (115) becomes,

$$\begin{aligned}
&\int_0^\infty dp \left(\hat{c}_2(p) s \int_{-\infty}^\infty \frac{du_2}{2\pi} e^{i(\Omega-p)u_2} \right. \\
&\quad \left. + \hat{c}_2^\dagger(p) \int_{-\infty}^\infty \frac{du_2}{2\pi} e^{i(\Omega+p)u_2} \right). \quad (117)
\end{aligned}$$

Similarly, the lhs becomes,

$$\int_0^\infty dk \left(\hat{c}_1(k) \int_{-\infty}^\infty \frac{du_2}{2\pi} e^{\frac{g}{4}(v_2-u_2-v_1+u_1)} e^{i(\Omega u_2 - k u_1)} + \hat{c}_1^\dagger(k) \int_{-\infty}^\infty \frac{du_2}{2\pi} e^{\frac{g}{4}(v_2-u_2-v_1+u_1)} e^{i(\Omega u_2 + k u_1)} \right). \quad (118)$$

Using the fact $p > 0$ and $\Omega > 0$, we can further simplify the rhs to obtain the following:

$$\int_0^\infty dp (\hat{c}_2(p) \delta(\Omega - p) + \hat{c}_2^\dagger(p) \delta(\Omega + p)) = \hat{c}_2(\Omega), \quad (119)$$

where the R_2 annihilation operator is related to the creation and annihilation operators of R_1 . Now, comparing Eq. (118) with Eq. (112), we can write down the expressions for the Bogoliubov coefficients as follows:

$$\alpha^{*(+)}(\Omega, k) = \int_{-\infty}^\infty \frac{du_2}{2\pi} e^{\frac{g}{4}(v_2-u_2-v_1+u_1)} e^{i(\Omega u_2 - k u_1)}, \quad (120)$$

and

$$\beta_{21}^{*(+)}(\Omega, k) = - \int_{-\infty}^\infty \frac{du_2}{2\pi} e^{\frac{g}{4}(v_2-u_2-v_1+u_1)} e^{i(\Omega u_2 + k u_1)}. \quad (121)$$

To evaluate the integral for $\beta_{21}^{*(+)}(\Omega, k)$, we consider a constant time hypersurface $t_2 = 0$ on the common patch of R_1 and R_2 coordinates and from the fact $u_1 = u_2$, for the case described by the Fig. 2, we get $v_2 = -u_2$ from Eqs. (9) and (10). Using the above facts and the Eq. (46), we can rewrite the integral in Eq. (121) as follows:

$$\beta_{21}^{*(+)}(\Omega, k) = - \int_{-\infty}^\infty \frac{du_2}{2\pi} e^{\frac{g}{4}(u_2 + \ln[e^{-gu_2} + \lambda])} e^{i(\Omega + k)u_2}. \quad (122)$$

Now, substituting $h = e^{-gu_2}$, such that $du_2 = -dh/(gh)$ and applying the new limits, the above integration can be expressed as,

$$\beta_{21}^{*(+)}(\Omega, k) = \frac{-1}{2\pi g} \int_0^\infty dh \left[\frac{h}{h + \lambda} \right]^{\frac{1}{4}} h^{\frac{-i(\Omega + k)}{g}}. \quad (123)$$

Solving the above, we obtain,

$$\beta_{21}^{*(+)}(\Omega, k) = \frac{-\lambda^{\frac{-i(k+\Omega)}{g}} \Gamma\left(\frac{i(k+\Omega)}{g}\right) \Gamma\left(\frac{1}{4} - \frac{i(k+\Omega)}{g}\right)}{2\pi g \Gamma\left(\frac{1}{4}\right)}. \quad (124)$$

Its complex conjugate can be expressed as

$$\beta_{21}^{(+)}(\Omega, k) = \frac{-\lambda^{\frac{i(k+\Omega)}{g}} \Gamma\left(\frac{-i(k+\Omega)}{g}\right) \Gamma\left(\frac{1}{4} + \frac{i(k+\Omega)}{g}\right)}{2\pi g \Gamma\left(\frac{1}{4}\right)}. \quad (125)$$

Now, in order to understand the particle content of the right-chiral sector of the Dirac field, we evaluate the expectation value of the R_2 particle number operator with respect to the R_1 vacuum state. Thus, we obtain,

$$\langle 0|_{R_1} \hat{N}(\Omega) |0\rangle_{R_1} = \int_0^\infty dk \beta_{21}^{*(+)}(\Omega, k) \beta_{21}^{(+)}(\Omega, k). \quad (126)$$

Substituting the value of $\beta_{21}^{*(+)}(\Omega, k)$ and its complex conjugate $\beta_{21}^{(+)}(\Omega, k)$ in Eq. (126) the integrand of the above equation [denoted by $G(\Omega, g, k)$], can be written as,

$$G(\Omega, g, k) = \frac{\left| \Gamma\left(-\frac{i(k+\Omega)}{g}\right) \right|^2 \left| \Gamma\left(\frac{1}{4} - \frac{i(k+\Omega)}{g}\right) \right|^2}{4\pi^2 g^2 \Gamma\left(\frac{1}{4}\right)^2}. \quad (127)$$

From the identity $|\Gamma(x + iy)| \leq |\Gamma(x)|$ (see Ref. [45]), we can rewrite the above as (evaluating an upper bound for the above integral),

$$G(\Omega, g, k) \approx \frac{\left| \Gamma\left(-\frac{i(k+\Omega)}{g}\right) \right|^2 \left| \Gamma\left(\frac{1}{4}\right) \right|^2}{4\pi^2 g^2 \Gamma\left(\frac{1}{4}\right)^2}. \quad (128)$$

We now prove using a couple of justifications that using the above expression as the integrand in Eq. (126) yields a finite number of particles and hence a negligible number density when divided by the infinite volume. Thus, the particle number integral can be written as follows:

$$\langle \hat{N}(\Omega) \rangle_{R_1} \approx \int_0^\infty dk \frac{1}{4\pi g(k + \Omega)} \frac{1}{\sinh\left(\frac{\pi(k+\Omega)}{g}\right)}. \quad (129)$$

Now, since k is always positive, we know $k + \Omega$ is always greater than Ω , also we take $\sinh(x) \approx e^x/2$ for large values of x . Consequently, the integral in Eq. (129) can be approximated by,

$$\begin{aligned} \int_0^\infty dk \frac{e^{\frac{-\pi(k+\Omega)}{g}}}{2\pi g \Omega} &= \langle \hat{N}_{\text{upp}}(\Omega) \rangle \\ &\geq \int_0^\infty dk \frac{1}{4\pi g(k + \Omega) \sinh\left(\frac{\pi(k+\Omega)}{g}\right)}, \end{aligned} \quad (130)$$

where the integral in the lhs " $\langle \hat{N}_{\text{upp}}(\Omega) \rangle$ " provides an upper bound for our required particle number integral. By evaluating the lhs of Eq. (130), we get the upper bound as,

$$\langle \hat{N}_{\text{upp}}(\Omega) \rangle = \frac{e^{\frac{-\pi\Omega}{g}}}{2\pi^2 \Omega}, \quad (131)$$

which is finite. We note that the above expression $\langle \hat{N}_{\text{upp}}(\Omega) \rangle$ is the expectation value of the particle number operator with frequency Ω in R_2 with respect to the R_1 vacuum state, and we got it as finite. Now, evaluating the mean particle number density by dividing the above by an infinite volume term ($V \rightarrow \infty$), we get the $\hat{n}_{\text{upp}}(\Omega) \rightarrow 0$; therefore, it is negligible. Now, since $\langle \hat{N}_{\text{upp}}(\Omega) \rangle$ is always greater than $\langle \hat{N}(\Omega) \rangle$, and since from Eq. (131), $\langle \hat{N}_{\text{upp}}(\Omega) \rangle$, we can claim the integral $\langle \hat{N}(\Omega) \rangle$ is also finite. Consequently, this suggests that the mean number density of the right-chiral particles with frequency Ω given by $\hat{n}(\Omega)$ is negligible.

E. Excitations of left-chiral modes

In this section, we investigate the phenomenon of selective excitation for left-chiral particles, extending the analysis performed for right-chiral particles (see Sec. IV D). We follow similar calculations outlined in the previous section. In the overlapping patch of coordinates of R_1 and R_2 spacetimes, we consider the standard mode expansion of the Dirac field (Majorana) operator, decomposed into left-chiral and right-chiral sectors. From the ansatz, where the modes of R_2 are related to the modes of R_1 through a Bogoliubov transformation given by Eqs. (110) and (111), we obtain the relation between the creation/annihilation operators of R_2 and R_1 for the left-chiral particles as follows:

$$\hat{c}_2(p) = \int_0^\infty dk (\alpha_{21}^{*(-)}(p, k) \hat{c}_1(k) - \beta_{21}^{*(-)}(p, k) \hat{c}_1^\dagger(k)), \quad (132)$$

$$\hat{c}_2^\dagger(p) = \int_0^\infty dk (\alpha_{21}^{(-)}(p, k) \hat{c}_1^\dagger(k) - \beta_{21}^{(-)}(p, k) \hat{c}_1(k)). \quad (133)$$

Given that the left-chiral and right-chiral sectors are decoupled, and the Dirac field under consideration is conformally invariant, the left-chiral sector of R_1 is equivalent to that of R_2 . Thus, we obtain,

$$\begin{aligned} & \int_0^\infty dk \frac{e^{-\frac{gk}{2}}}{\sqrt{2\pi}} \left(\hat{c}_1(k) \begin{bmatrix} 0 \\ e^{-ik(t_1+x_1)} \end{bmatrix} + \hat{c}_1^\dagger(k) \begin{bmatrix} 0 \\ e^{ik(t_1+x_1)} \end{bmatrix} \right) \\ &= \int_0^\infty dp \frac{e^{-\frac{gpx_2}{2}}}{\sqrt{2\pi}} \left(\hat{c}_2(p) \begin{bmatrix} 0 \\ e^{-ip(t_2+x_2)} \end{bmatrix} + \hat{c}_2^\dagger(p) \begin{bmatrix} 0 \\ e^{ip(t_2+x_2)} \end{bmatrix} \right). \end{aligned} \quad (134)$$

Expressing the above in terms of Rindler null coordinates, we get,

$$\begin{aligned} & \int_0^\infty dk \frac{e^{-\frac{g}{4}(v_1-u_1)}}{\sqrt{2\pi}} \left(\hat{c}_1(k) \begin{bmatrix} 0 \\ e^{-ikv_1} \end{bmatrix} + \hat{c}_1^\dagger(k) \begin{bmatrix} 0 \\ e^{ikv_1} \end{bmatrix} \right) \\ &= \int_0^\infty dp \frac{e^{-\frac{g}{4}(v_2-u_2)}}{\sqrt{2\pi}} \left(\hat{c}_2(p) \begin{bmatrix} 0 \\ e^{-ipv_2} \end{bmatrix} + \hat{c}_2^\dagger(p) \begin{bmatrix} 0 \\ e^{ipv_2} \end{bmatrix} \right). \end{aligned} \quad (135)$$

Now, to get the expressions for the Bogoliubov coefficients, we multiply both the lhs and rhs of Eq. (135) with,

$$\int_{-\infty}^\infty \frac{dv_2}{2\pi} e^{\frac{g}{4}(v_2-u_2)} [0 \quad e^{i\Omega v_2}]. \quad (136)$$

Thus, the rhs becomes,

$$\begin{aligned} & \int_0^\infty dp \left(\hat{c}_2(p) \int_{-\infty}^\infty \frac{dv_2}{2\pi} e^{i(\Omega-p)v_2} \right. \\ & \left. + \hat{c}_2^\dagger(p) \int_{-\infty}^\infty \frac{dv_2}{2\pi} e^{i(\Omega+p)v_2} \right). \end{aligned} \quad (137)$$

Similarly, the lhs becomes,

$$\begin{aligned} & \int_0^\infty dk \left(\hat{c}_1(k) \int_{-\infty}^\infty \frac{dv_2}{2\pi} e^{\frac{g}{4}(v_2-u_2-v_1+u_1)} e^{i(\Omega v_2-kv_1)} \right. \\ & \left. + \hat{c}_1^\dagger(k) \int_{-\infty}^\infty \frac{dv_2}{2\pi} e^{\frac{g}{4}(v_2-u_2-v_1+u_1)} e^{i(\Omega v_2+kv_1)} \right), \end{aligned} \quad (138)$$

The rhs can be simplified further to obtain,

$$\int_0^\infty dp (\hat{c}_2(p) \delta(\Omega-p) + \hat{c}_2^\dagger(p) \delta(\Omega+p)) = \hat{c}_2(\Omega), \quad (139)$$

where $\hat{c}_2(\Omega)$ is the annihilation operator for the left-chiral Dirac field modes in R_2 . Comparing the Eq. (138), i.e., the lhs with the Eq. (132), we can express the Bogoliubov coefficients as follows:

$$\alpha_{21}^{*(-)}(\Omega, k) = \int_{-\infty}^\infty \frac{dv_2}{2\pi} e^{\frac{g}{4}(v_2-u_2-v_1+u_1)} e^{i(\Omega v_2-kv_1)}, \quad (140)$$

and

$$\beta_{21}^{*(-)}(\Omega, k) = - \int_{-\infty}^\infty \frac{dv_2}{2\pi} e^{\frac{g}{4}(v_2-u_2-v_1+u_1)} e^{i(\Omega v_2+kv_1)}. \quad (141)$$

Now, in order to study the selective excitations of left-chiral modes and the corresponding particle content in R_2 , we must evaluate the Bogoliubov coefficient $\beta_{21}^{*(-)}(\Omega, k)$. For that, we will consider a constant time hypersurface $t_2 = 0$ on the common patch of R_1 and R_2 coordinates, such that from Eqs. (9) and (10) we get $u_2 = -v_2$. Also, as per the scenario described by the Fig. 2 and from Eq (32), we get $u_1 = u_2$. Using the above facts, and Eq. (46), the integral for

the Bogoliubov coefficient $\beta_{21v}^*(\Omega, k)$ can be simplified as follows:

$$\beta_{21}^{*(-)}(\Omega, k) = - \int_{-\infty}^{\infty} \frac{dv_2}{2\pi} e^{\frac{g}{4}(v_2 - \frac{\ln[e^{g v_2} + \lambda]}{g})} e^{i(\Omega v_2 + k \frac{\ln[e^{g v_2} + \lambda]}{g})}. \quad (142)$$

On substituting $h = e^{g v_2}$, such that $dv_2 = \frac{dh}{gh}$ and applying the new limits for the above integration, we obtain,

$$\beta_{21}^{*(-)}(\Omega, k) = \frac{-1}{2\pi g} \int_0^{\infty} dh \left[\frac{h}{h + \lambda} \right]^{\frac{1}{4}} h^{\frac{i\Omega}{g} - 1} (h + \lambda)^{\frac{ik}{g}}. \quad (143)$$

This integral can be evaluated to get,

$$\beta_{21}^{*(-)}(\Omega, k) = \frac{-\lambda^{\frac{i(k+\Omega)}{g}} \Gamma\left(\frac{i\Omega}{g} + \frac{1}{4}\right) \Gamma\left(-\frac{i(k+\Omega)}{g}\right)}{2\pi g \Gamma\left(\frac{1}{4} - \frac{ik}{g}\right)}. \quad (144)$$

Its complex conjugate can be expressed as

$$\beta_{21}^{(-)}(\Omega, k) = \frac{-\lambda^{\frac{-i(k+\Omega)}{g}} \Gamma\left(-\frac{i\Omega}{g} + \frac{1}{4}\right) \Gamma\left(\frac{i(k+\Omega)}{g}\right)}{2\pi g \Gamma\left(\frac{1}{4} + \frac{ik}{g}\right)}. \quad (145)$$

In order to obtain the particle content of the left-chiral sector of the Dirac field, we evaluate the expectation value of the R_2 particle number operator with respect to the R_1 vacuum state. Thus, we obtain,

$$\langle 0|_{R_1} \hat{N}(\Omega) |0\rangle_{R_1} = \int_0^{\infty} dk \beta_{21}^{*(-)}(\Omega, k) \beta_{21}^{(-)}(\Omega, k). \quad (146)$$

Substituting the value of $\beta_{21}^{*(-)}(\Omega, k)$ and $\beta_{21}^{(-)}(\Omega, k)$ in the above expression, we get particle number integral, where the integrand $|\beta_{21}^{(-)}(\Omega, k)|^2$ can be expressed as follows:

$$|\beta_{21}^{(-)}(\Omega, k)|^2 = \frac{\left| \Gamma\left(-\frac{i\Omega}{g} + \frac{1}{4}\right) \right|^2 \left| \Gamma\left(\frac{i(k+\Omega)}{g}\right) \right|^2}{4\pi^2 g^2 \left| \Gamma\left(\frac{1}{4} + \frac{ik}{g}\right) \right|^2}. \quad (147)$$

Using properties of gamma functions (the reflection formula and the asymptotic expansion formula) [45,46], we can approximate the integrand and simplify it. After evaluating the particle number integral [see the Appendix for more details], we get,

$$\langle \hat{N}(\Omega) \rangle \approx \text{sech}\left(\frac{2\pi\Omega}{g}\right) \int_0^{\infty} dk \left[\frac{1}{4\pi g \sqrt{\Omega} \sqrt{k}} \right]. \quad (148)$$

In the above expression, the integral exhibits divergence, and from dimensional analysis, we can conclude that this divergent part has spatial units. Although the integrand results from an asymptotic form of the Gamma functions, the formula is extended to lower values of k , where the

integrand remains finite. Therefore, divergence occurs only for larger values of k . Thus, to deal with this divergence, we define the mean number density of particles with frequency Ω [see Eq. (65)] [29]. Thus, we obtain

$$\hat{n}(\Omega) = \lim_{V \rightarrow \infty} \frac{\langle 0|_{R_1} \hat{N}(\Omega) |0\rangle_{R_1}}{V}, \quad (149)$$

where Eq. (148) is divided by the infinite spatial volume of R_2 . Here, we assume that dividing by the infinite volume cancels the divergent term, resulting in a finite particle number density distribution. Thus, from Eqs. (148) and (149), we can evaluate the mean number density distribution for large Ω for the case of left-chiral particles as follows,

$$n(\Omega) \sim \frac{1}{e^{\frac{2\pi\Omega}{g}}}. \quad (150)$$

It is important to note that at large frequencies, the expression [see Eq. (150)] resembles the standard thermal distribution of fermions, which is given as $1/(e^{\frac{2\pi\Omega}{g}} + 1)$, where we can ignore the “1” in the denominator due to the dominance of the exponential term. Thus, we can interpret the particle number density distribution for the left-chiral modes given by Eq. (150) as a thermallike distribution for fermions, with the temperature defined as $g/(2\pi)$, at least for high frequencies. Therefore, we can observe that the particle number density distribution and the corresponding chiral excitations for the left-chiral modes are significantly stronger compared to those observed in the case of right-chiral modes, which yields negligible particle density.

V. CONCLUSIONS AND DISCUSSION

In this article, we demonstrate that for the case of massless scalar fields, the left-moving modes are thermally excited, while the right-moving modes have zero particle content. In contrast, in the case of massless fermionic fields, the right-chiral modes exhibit negligible particle number density, whereas the left-chiral modes are excited and have a nonzero particle number density. The thermal characteristics of the spectrum for all modes remain unclear at this point, although it is shown to be thermal for high-frequency modes.

While this work is set in a more elementary framework, several important questions arise. For instance, is selective excitation possible if we extend the analysis to include massive fields? In the case of massive particles, the distinction between left-moving and right-moving modes becomes frame-dependent. Are there scenarios where chiral asymmetric excitation can occur, even for massive Dirac fields? Additionally, can the results of this article be applied to four-dimensional spacetime? These are some of the more pertinent questions that stem from the findings discussed in the paper.

As pointed out earlier, the questions in the article could have been posed starting from R_1 in a thermal state (the quantum field in Minkowski spacetime chosen to be in vacuum) and then examining what is the observed particle density in, say, R_2 . We present our reasoning and shortcomings as to why we had to proceed with R_1 in a vacuum state, even though the Rindler vacuum state has an issue with being nonrenormalizable. First, we currently lack techniques to evaluate the particle distribution of particles in R_2 starting from a thermal particle distribution in R_1 for the null-shifted case. In [18], a similar analysis was done where we considered a thermal distribution of particles in R_1 and then evaluated particle content in R_2 , where R_2 is spacelike shifted from R_1 . Unfortunately, the technique developed in [18] is not adaptable to the null-shifted cases. A search for a better technique is left for future considerations.

Second, in this article, we are interested in how the vacuum of the Rindler wedge R_1 relates to the vacuum of its subregions of the type R_2 . If we define more null shifted wedges beyond R_2 and want to evaluate the particle density, the expression of R_1 vacuum in terms of the excited states or R_2 is needed as an intermediate step. Our article provides this information. Also, the Rindler vacuum, as demonstrated in past literature [47], is similar to the Boulware vacuum in the context of black holes. The expectation value of the local stress tensor with respect to the Rindler vacuum diverges on the horizon, but in this paper, we are more interested in proving and comparing the inequivalence of the Rindler vacuum and the vacuum of the uniformly accelerating observer, which perceives the shifted null wedge as the horizon boundary. The issue of renormalizability, or its absence, in either the Rindler vacuum or the vacuum of the null-shifted wedge does not influence the analysis carried out in our article.

Several challenging questions remain to be explored based on the work done in this article. Specifically, how can one solve for the entanglement entropy across a nested sequence of Rindler observers that are shifted from each other by spacelike or null intervals using information-theoretic techniques or the methods of algebraic quantum field theory? [6,21,48,49]. A standard method for deriving the particle content in Rindler spacetime from a Minkowski vacuum state is the Euclidean path integral formulation, as proposed in the renowned paper by Gibbons and Hawking [50]. How can this technique be extended to determine the particle density across a sequence of Rindler spacetimes that are shifted along spacelike or null intervals?

As pointed out earlier in this article, the shifted Rindler spacetimes, or the nested sequence of Rindler spacetimes, may provide important insights into capturing quantum signatures from evolving horizons. During the radiation-dominated era of cosmological evolution, the marginally trapped region develops along a lightlike direction [13,14]. This evolution can be represented by a series of wedges

shifted along a null direction. Now, suppose one considers a massless scalar field or a Dirac field in this spacetime, could it be possible that only left-chiral or right-chiral particles get excited during this cosmological era? This might lead to an asymmetry in particle density regarding chirality for massless or low-mass fermions, such as neutrinos. These questions will be addressed in future discussions.

The findings in the article imply the possibility of Rindler spacetime exhibiting “quantum hair.” It is possible to infer, based on the particle content, whether the scenario in question corresponds to Fig. 3 or Fig. 4. We explain this below. Suppose we find that a Rindler wedge R_2 has a particle density such that the left-moving massless particles are thermally populated while the right-moving massless particles are absent. Suppose now we ask a question: is there a larger spacetime A (where $R_2 \subset A$) with the quantum field in a pure vacuum state, such that its reduced state in R_2 yields the observed particle density? The answer is yes. The Rindler wedges of the form R_1 will induce the observed particle density. Now the wedges R_1 and R_2 share a null ray $U = 0$. So the location of R_1 is such that $R_2 \subset R_1$ and along a null ray, though the displacement along the shared null ray is arbitrary. The observed particle density, where one set of modes is thermally populated while the other is absent, is therefore informative. With the information of the selective particle density, we can locate the position of the wedge R_1 (up to a “displacement” since R_1 can be located anywhere on the common null ray such that $R_2 \subset R_1$). This indicates that informative excitations are possible within Rindler spacetime, suggesting that Rindler spacetimes can have “quantum hairs.” The implications of these results for the information loss problem are yet to be explored.

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX: MEAN PARTICLE NUMBER DENSITY DISTRIBUTION FOR THE LEFT-CHIRAL MODES

To find the particle content for the left-chiral modes, we first compute the expectation value of the particle number operator (as discussed in Sec. IV E),

$$\langle 0|_{R_1} \hat{N}(\Omega) |0\rangle_{R_1} = \int_0^\infty dk \beta_{21}^{*(-)}(\Omega, k) \beta_{21}^{(-)}(\Omega, k). \quad (\text{A1})$$

Now, substituting the values of $\beta_{21}^{*(-)}(\Omega, k)$ and $\beta_{21}^{(-)}(\Omega, k)$ in the above expression, we get particle number integral,

where the integrand $|\beta_{21}^{(-)}(\Omega, k)|^2$ can be expressed as follows:

$$|\beta_{21}^{(-)}(\Omega, k)|^2 = \frac{\left| \Gamma\left(\frac{-i\Omega}{g} + \frac{1}{4}\right) \right|^2 \left| \Gamma\left(\frac{i(k+\Omega)}{g}\right) \right|^2}{4\pi^2 g^2 \left| \Gamma\left(\frac{1}{4} + \frac{ik}{g}\right) \right|^2}. \quad (\text{A2})$$

To simplify the integrand, we apply the well-known results for gamma functions, specifically the reflection formula [46] given by

$$\Gamma(1-z) = \frac{\pi}{\Gamma(z) \sin \pi z}, \quad (\text{A3})$$

and the asymptotic expansion for the gamma functions [45] given by

$$|\Gamma(x+iy)| \sim \sqrt{2\pi} |y|^{x-\frac{1}{2}} e^{-\frac{\pi|y|}{2}}. \quad (\text{A4})$$

First, consider the numerator, the function $\Gamma\left(\frac{1}{4} - \frac{i\Omega}{g}\right)$ can be replaced by,

$$\Gamma\left(\frac{1}{4} - \frac{i\Omega}{g}\right) = \frac{\pi}{\sin\left(\pi\left(\frac{3}{4} + \frac{i\Omega}{g}\right)\right) \Gamma\left(\frac{3}{4} + \frac{i\Omega}{g}\right)}. \quad (\text{A5})$$

Therefore, we get,

$$\left| \Gamma\left(\frac{1}{4} - \frac{i\Omega}{g}\right) \right|^2 = \frac{2\pi^2 \text{sech}\left(\frac{2\pi\Omega}{g}\right)}{\Gamma\left(\frac{3}{4} - \frac{i\Omega}{g}\right) \Gamma\left(\frac{3}{4} + \frac{i\Omega}{g}\right)}. \quad (\text{A6})$$

Now using the Eq. (A4) (asymptotic expansion of gamma function), the $\Gamma\left(\frac{3}{4} - \frac{i\Omega}{g}\right)$ can be written as

$$\Gamma\left(\frac{3}{4} - \frac{i\Omega}{g}\right) \sim \sqrt{2\pi} \left(\frac{\Omega}{g}\right)^{\frac{3}{4}-\frac{1}{2}} e^{-\frac{\pi\Omega}{2g}}. \quad (\text{A7})$$

Consequently, the denominator of Eq. (A6) becomes,

$$\left| \Gamma\left(\frac{3}{4} - \frac{i\Omega}{g}\right) \right|^2 \sim 2\pi \left(\frac{\Omega}{g}\right)^{\frac{1}{2}} e^{-\frac{\pi\Omega}{g}}. \quad (\text{A8})$$

Similarly, Eq. (A6) can be simplified as

$$\left| \Gamma\left(\frac{1}{4} - \frac{i\Omega}{g}\right) \right|^2 = \frac{2\pi^2 \sqrt{g} \text{sech}\left(\frac{2\pi\Omega}{g}\right)}{2\pi \sqrt{\Omega} e^{-\frac{\pi\Omega}{g}}}. \quad (\text{A9})$$

The other term in the numerator, $|\Gamma(\frac{i(k+\Omega)}{g})|^2$, when simplified results in,

$$\left| \Gamma\left(\frac{i(k+\Omega)}{g}\right) \right|^2 = \frac{\pi g \text{csch}\left(\frac{\pi(k+\Omega)}{g}\right)}{(k+\Omega)}. \quad (\text{A10})$$

Now, considering the denominator, using the asymptotic expression, we obtain,

$$\left| \Gamma\left(\frac{1}{4} + \frac{ik}{g}\right) \right| \sim \sqrt{2\pi} \left(\frac{k}{g}\right)^{\frac{1}{4}-\frac{1}{2}} e^{-\frac{\pi k}{2g}}, \quad (\text{A11})$$

which results in,

$$\left| \Gamma\left(\frac{1}{4} + \frac{ik}{g}\right) \right|^2 \sim 2\pi \left(\frac{k}{g}\right)^{-\frac{1}{2}} e^{-\frac{\pi k}{g}}. \quad (\text{A12})$$

Now substituting Eqs. (A9), (A10), and (A12) in Eq. (A2), the simplified integrand expression takes the form:

$$|\beta_{21v}(\Omega, k)|^2 = \frac{\text{sech}\left(\frac{2\pi\Omega}{g}\right)}{e^{-\frac{\pi\Omega}{g}}} \frac{\sqrt{k} \text{csch}\left(\frac{\pi(k+\Omega)}{g}\right)}{8\pi g \sqrt{\Omega} (k+\Omega) e^{-\frac{\pi k}{g}}}. \quad (\text{A13})$$

Therefore, the particle number integral can now be expressed as follows:

$$\langle \hat{N}(\Omega) \rangle = F(\Omega) \int_0^\infty dk \left[\frac{\sqrt{k} \text{csch}\left(\frac{\pi(k+\Omega)}{g}\right)}{8\pi g \sqrt{\Omega} (k+\Omega) e^{-\frac{\pi k}{g}}} \right], \quad (\text{A14})$$

where the function $F(\Omega)$ denotes the particle spectrum in terms of Ω . Now, to evaluate the integral, we take the asymptotic form of the function at large k , i.e., for $k \rightarrow \infty$, where, we approximate the function $\text{csch}\left(\frac{\pi(k+\Omega)}{g}\right) \approx 2e^{-\frac{\pi(k+\Omega)}{g}}$ and $(k+\Omega) \approx k$. Thus we get,

$$\langle \hat{N}(\Omega) \rangle \approx F(\Omega) e^{-\frac{\pi\Omega}{g}} \int_0^\infty dk \left[\frac{\sqrt{k} 2e^{-\frac{\pi(k+\Omega)}{g}}}{8\pi g \sqrt{\Omega} k e^{-\frac{\pi k}{g}}} \right], \quad (\text{A15})$$

and on further simplification, we obtain,

$$\langle \hat{N}(\Omega) \rangle \approx \text{sech}\left(\frac{2\pi\Omega}{g}\right) \int_0^\infty dk \left[\frac{1}{4\pi g \sqrt{\Omega} \sqrt{k}} \right]. \quad (\text{A16})$$

Expressing the above in exponential terms, we get,

$$\langle \hat{N}(\Omega) \rangle \approx \left[\frac{2e^{\frac{2\pi\Omega}{g}}}{e^{\frac{4\pi\Omega}{g}} + 1} \right] \int_0^\infty dk \left[\frac{1}{4\pi g \sqrt{\Omega} \sqrt{k}} \right]. \quad (\text{A17})$$

The above integral diverges, so to address the divergence, we evaluate the mean particle number density by dividing the above by an infinite volume of R_2 . Thus, we get the required particle number density spectrum as

$$\hat{n}(\Omega) \approx \frac{e^{\frac{2\pi\Omega}{g}}}{e^{\frac{4\pi\Omega}{g}} + 1}. \quad (\text{A18})$$

This number density distribution, when considered for large values of Ω , can be expressed as

$$\hat{n}(\Omega) \sim \frac{1}{e^{\frac{2\pi\Omega}{g}}}. \quad (\text{A19})$$

Thus, we get a thermallike particle number density distribution for the left-chiral modes of the Dirac field.

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