

Dispersion relations of deeply virtual Compton scattering: Investigating twist-four kinematic power corrections

Víctor Martínez-Fernández^{1,2,*} and Cédric Mezrag^{1,†}

¹*Irfu, CEA, Université Paris-Saclay, F-91191, Gif-sur-Yvette, France*

²*Center for Frontiers in Nuclear Science, Stony Brook University, Stony Brook, New York 11794, USA*



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In this paper we include kinematic power corrections up to twist-four to the deeply virtual Compton scattering dispersion relation. We demonstrate that, both for (pseudo)scalar and spin-1/2 targets, the formal expression of the n -subtracted leading-twist dispersion relations is preserved. However, the expression of the subtracted constants is modified by the kinematic powers. Importantly, the minimal-subtracted dispersion relation for the helicity-conserving amplitude, previously thought to depend only on the Polyakov-Weiss D -term, now also depends on the double distributions F and K . These results are consistent with the ones obtained previously in the literature. Such a mixing may be critical for the Jefferson Lab kinematic range, as it is not suppressed for typical values of t and Q^2 in the valence region. We therefore expect a strong impact on the attempts to extract pressure forces from deeply virtual Compton scattering data.

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I. INTRODUCTION

The energy-momentum tensors (EMTs) of hadrons are today at the core of an intense research activity. Many theoretical and phenomenological studies have been performed (for instance [1–7]). Lattice and continuum QCD computations have also been performed in the past few years [8–13]. The goal of this activity is to understand how the macroscopic properties of the nucleon, such as its mass and its spin, emerge from the dynamics of QCD.

However, connecting the EMT with experimental data is challenging. Today only an indirect connection, already noticed three decades ago [14], is available through generalized parton distributions (GPDs). Introduced independently in [14–18], GPDs allow one to describe the amplitude of exclusive processes through factorization with a coefficient function computed in perturbation theory [19,20]. The better studied experimental process connected to GPD is certainly deeply virtual Compton scattering (DVCS) [16]. But other exclusive processes are connected to GPDs, such as timelike Compton scattering (TCS) [21] or deeply virtual meson production (DVMP) [22]. However, all these processes face a severe deconvolution

problem [23,24]. Therefore, double DVCS [25–28] (DDVCS) and multiparticle exclusive processes have been advocated to bypass this issue [29–34]. Nevertheless, DVCS remains today the main source of experimental knowledge on GPDs with measurements spanning the last two decades [35–42].

This large experimental campaign has triggered many theoretical developments improving the description of DVCS. One can for instance mention the description of the perturbative coefficient function at next-to-next-to-leading order (NNLO) [43]. More critical for current facilities running in the valence region, a significant effort has been performed to derive kinematic higher power corrections [44–50]. These corrections have been recently extended to DDVCS for a (pseudo)scalar target [51]. They are expected to contribute up to 40% of the DVCS amplitude for some of the kinematic area [38]. As a consequence, they may have a significant impact on the extraction of the pressure and shear forces from experimental data, usually performed through DVCS dispersion relations bounding the real and imaginary part of DVCS amplitude. DVCS dispersion relations were derived two decades ago, and the size of NLO corrections [7,52] have been shown to be of the order of 10% of the leading contribution [7]. Consequently, there is room for kinematic power corrections to be significantly larger than NLO corrections in the strong coupling constant.

In this paper, we revisit the topic of kinematic power correction up to twist-four to DVCS dispersion relations for (pseudo)scalar and spin-1/2 targets. In Sec. II, we introduce our notations and conventions. In Secs. III and IV we

*Contact author: victor.martinezfernandez@cea.fr

†Contact author: cedric.mezrag@cea.fr

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provide a derivation of dispersion relations with higher kinematic-power corrections, for (pseudo)scalar and spin-1/2 targets respectively, adapting the proof of Ref. [7]. In the nucleon case, an expression of the DVCS subtraction constant including kinematic twist-four corrections was derived and expressed as a double integral over GPDs in Ref. [47]. Our new expression relying on a single integral over double distributions [15,18] agrees with this pioneering result. Beyond the DVCS subtraction constant, we also derived the twist-four corrections to higher-subtracted dispersion relations connecting the Mellin moments of the imaginary part of the CFFs with double distributions. In Sec. V, we compute the impact of the kinematic corrections to the Gegenbauer modes of the Polyakov-Weiss D -term [53] and highlight that these corrections may help one to extract these modes from experimental data. Then, in Sec. VI we provide numerical estimates, demonstrating the relevance of power corrections. Finally, we conclude in Sec. VII.

II. ACCESSING THE ENERGY-MOMENTUM TENSOR VIA GENERALIZED PARTON DISTRIBUTIONS

The energy-momentum tensor of a hadron is obtained by projecting the local and gauge-invariant operator $T^{\mu\nu}$ between two hadron states off-diagonal in momentum. The momentum transfer is labeled $\Delta = p' - p$, introducing also $t = \Delta^2$ and the average momentum $\bar{p} = (p + p')/2$. For a spin-0 target, the nonconserved, symmetric tensor decomposition involves three form factors [54,55]:

$$\langle p' | T_a^{\mu\nu}(0) | p \rangle = 2\bar{p}^\mu \bar{p}^\nu A_a(t) + 2(\Delta^\mu \Delta^\nu - \eta^{\mu\nu} \Delta^2) C_a(t) + \eta^{\mu\nu} M^2 \bar{C}_a(t) \quad (1)$$

where the index a labels quark flavors or gluon contributions. For a spin-1/2 hadron, five form factors are required to parametrize the nonconserved, asymmetric EMT:

$$\langle p', s' | T_a^{\mu\nu}(0) | p, s \rangle = \quad (2)$$

$$\begin{aligned} \bar{u}(p', s') \left\{ \frac{\bar{p}^\mu \bar{p}^\nu}{M} A_a(t) + M \eta^{\mu\nu} \bar{C}_a(t) \right. \\ \left. + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu} \Delta^2}{M} C_a(t) + \frac{\bar{p}^{[\mu} i \sigma^{\nu]\rho} \Delta_\rho}{4M} D_a(t) \right. \\ \left. + \frac{\bar{p}^{\{\mu} i \sigma^{\nu\}\rho} \Delta_\rho}{4M} [A_a(t) + B_a(t)] \right\} u(p, s). \end{aligned} \quad (3)$$

One can build a physical interpretation for several of these form factors in terms of “mechanical properties” of hadrons, like internal pressure or shear forces distributions [1,2,56], and their decomposition in terms of quarks and gluons.

The form factors of the EMT cannot be probed directly experimentally. However, some of them are connected with the generalized parton distributions [14–18] through their Mellin moments over x , the average longitudinal light-front momentum fraction of the active parton (we use the conventions of Ref. [57]):

$$\int_{-1}^1 dx x H^q(x, \xi, t) = A_q(t) + 4\xi^2 C_q(t), \quad (4)$$

$$\int_{-1}^1 dx x E^q(x, \xi, t) = B_q(t) - 4\xi^2 C_q(t), \quad (5)$$

where Eq. (5) is relevant for spin-1/2 hadrons. The last kinematic variable, the skewness ξ , is defined in terms of the “plus” light components of the initial- and final-state momenta of the hadron as

$$\xi = -\frac{\Delta^+}{2\bar{p}^+} = \frac{p^+ - p'^+}{p^+ + p'^+}. \quad (6)$$

In light-cone coordinates we can parametrize any four-vector v as

$$v^\mu = v^+ n'^\mu + v^- n^\mu + v_\perp^\mu, \quad (7)$$

with the conditions $n^2 = n'^2 = 0$, $nn' \neq 0$ and $nv_\perp = n'v_\perp = 0$, such that $v^+ = vn/(nn')$ and $v^- = vn'/(nn')$. Taking into account that in DVCS $q'^2 = 0$, we can select

$$n^\mu = q'^\mu, \quad (8)$$

$$n'^\mu = -q^\mu + \left(1 - \frac{t}{Q^2}\right) q'^\mu \quad \text{with} \quad Q^2 = Q^2 + t. \quad (9)$$

This choice, introduced in Ref. [49], is of physical importance as it keeps the leading kinematic contribution (*leading twist*, LT) $U(1)$ -electromagnetic gauge invariant. In fact, the Compton tensor, encoding the partonic distributions by Compton form factors (CFFs), is dominated at LT by the transverse metric and the CFF labeled $\mathcal{H}^{++}$,

$$T_{\text{LT}}^{\mu\nu} = -g_\perp^{\mu\nu} \mathcal{H}^{++} + \dots \quad (10)$$

Therefore, if the in or out photon carry a transverse component, the LT approximation is $U(1)$ violating. For this reason, we span the longitudinal plane by the momenta q, q' . Through GPDs, some of the form factors of the EMT can thus indirectly be probed experimentally.

We also recall that GPDs can be written in terms of double distributions F and K [15,18] (see for instance Refs. [58–61] for the interpretation of DDs in terms of Radon transforms) plus the so-called Polyakov-Weiss D -term [53]:

$$H^q(x, \xi, t) = \int_{\Omega} d\beta d\alpha [F^q(\beta, \alpha, t) + \xi D^q(\alpha, t) \delta(\beta)] \times \delta(x - \beta - \alpha\xi), \quad (11)$$

$$E^q(x, \xi, t) = \int_{\Omega} d\beta d\alpha [K^q(\beta, \alpha, t) - \xi D^q(\alpha, t) \delta(\beta)] \times \delta(x - \beta - \alpha\xi), \quad (12)$$

where $\Omega = \{(\alpha, \beta) | |\alpha| + |\beta| \leq 1\}$. For convenience, we already introduce the so-called magnetic double distribution given as [62–64]

$$N^q(\beta, \alpha, t) = \frac{F^q(\beta, \alpha, t) + K^q(\beta, \alpha, t)}{2}. \quad (13)$$

Note that the first Mellin moment of the D -term yields the form factor $C_q(t)$ in Eq. (4):

$$C_q(t) = \frac{1}{4} \int_{-1}^1 d\alpha \alpha D^q(\alpha, t). \quad (14)$$

It has been argued in the past that $C_q(t)$ could have been extracted using a deeply virtual Compton scattering dispersion relation, bypassing the deconvolution of GPDs which reveals itself at best delicate [23,24]. Indeed, the Compton form factors ($\mathcal{H}$, $\mathcal{E}$...) parametrizing the DVCS amplitude are related to GPDs through the convolution

$$\mathcal{H}^q(\xi, t, Q^2) = \int_{-1}^1 \frac{dx}{\xi} T^q\left(\frac{x}{\xi}; \frac{Q^2}{\mu^2}, \alpha_s, \frac{t}{Q^2}\right) H^q(x, \xi, t, \mu^2), \quad (15)$$

where T^q represents here the DVCS coefficient function, computed in perturbation theory, and $\mathbb{Q}^2 = Q^2 + t$. Indeed, from kinematic higher-twist studies of the DVCS amplitude such as [50,51,65], the natural scale for expansion on twist is $\mathbb{Q}^2$ rather than Q^2 . The difference between both of them is a higher-twist term so one can choose either one. In this work, we select $\mathbb{Q}^2$ as it simplifies expansions and provides a direct comparison with previous literature.

However, as already pointed out in [5,7], a dispersive approach [7,52,66,67] does not preclude facing an ill-posed deconvolution problem. In that regard, kinematic higher-twist corrections provide a new lever arm, adding an explicit t dependence in the coefficient function that comes with a more involved x behavior (typically involving Li_2 functions). However, such corrections will also impact the derivation of the dispersion relations. Consequently, in the following we rederive the dispersion relations, taking into account the first power corrections.

III. POWER CORRECTIONS TO DISPERSION RELATIONS: THE SCALAR CASE

In this section we adapt the proof of Ref. [7] to take into account the t and target-mass corrections.

A. DVCS dispersion relation with power corrections for $\mathcal{H}^j$ amplitude

We start by considering DVCS on a scalar target (such as a pion in a Sullivan process [68–71], or the ^4He nucleus):

$$N(p) + \gamma^{(*)}(q) \rightarrow N(p') + \gamma(q'). \quad (16)$$

For a process of this kind:

$$p^2 = p'^2 = M^2, \quad q^2 = -Q^2, \quad q'^2 = 0, \quad (17)$$

where M is the mass of the hadron and, from now on, Q^2 is positive and t is negative. Then, the Mandelstam variables take the form

$$s = M^2 - Q^2 + 2p \cdot q, \quad (18)$$

$$t = -2p \cdot \Delta, \quad (19)$$

$$u = M^2 - 2p \cdot q - t, \quad (20)$$

$$s + t + u = 2M^2 - Q^2. \quad (21)$$

For fixed negative values of t , the process can be described with just one variable: s , u or an appropriate combination of both of them. Following Ref. [7], one can choose (note the relative minus sign in the definition of ν in Eq. (22) here with respect to Ref. [7])

$$\nu = -\frac{s - u}{s + u}. \quad (22)$$

This choice is particularly convenient as it is the inverse of the DVCS skewness ξ at leading twist accuracy. However, higher power corrections modify this simple relation as we will see below. Reshuffling the expression connecting the Mandelstam variables, one gets

$$\begin{aligned} s + u &= 2M^2 - Q^2 + |t| = -Q^2 \left[1 - \frac{|t| + 2M^2}{Q^2} \right] \\ &= -(Q^2 + t) \left[1 - \frac{2M^2}{Q^2 + t} \right] \\ &= -Q^2 \left[1 - \frac{2M^2}{Q^2} \right], \end{aligned} \quad (23)$$

where $s + u < 0$ can be kept as long as

$$M^2/(Q^2 + t) < 1/2 \quad \text{and} \quad |t| < Q^2. \quad (24)$$

In such a case, there exists a region of the kinematics domain for which both s and u are negative, hence closed for particle production. As we will demonstrate in the following, this region corresponds to $-1 < \nu < 1$ [see Eq. (33)]. The amplitude is thus real and analytic on a segment of the real axis allowing us to define analytic continuation in the entire complex plane through the Schwartz principle (see Ref. [7] for details).

Note also that

$$s - u = -Q^2 + t + 4pq = 4\bar{p}q = 4\bar{p}q', \quad (25)$$

with $\bar{p} = (p + p')/2$, and

$$-\Delta q' = -qq' = \frac{1}{2} \left[\underbrace{(q - q')^2}_t - q^2 - \underbrace{q'^2}_0 \right] = \frac{Q^2 + t}{2}. \quad (26)$$

Taking into account the choice of longitudinal plane given in Eqs. (8) and (9), the skewness reads

$$\xi = -\frac{\Delta n}{2\bar{p}n} = -\frac{\Delta q'}{2\bar{p}q'} = \frac{t + Q^2}{s - u} = \frac{Q^2}{s - u}. \quad (27)$$

Using this expression for ξ together with the hard scale $Q^2 = -2qq' = Q^2 + t$ and Eq. (23), ν is given by

$$\nu = \frac{1/\xi}{1 - 2M^2/Q^2} \xrightarrow[\text{limit}]{\text{Bjorken}} \frac{1}{\xi}. \quad (28)$$

Since for $M^2/Q^2 \geq 1/2$ there is at least one channel open for particle production, we will consider $M^2/Q^2 < 1/2$ in this work. Note, however, that this is a sufficient but not necessary condition.

Because $\nu \geq 1$ implies $s \geq 0$, the amplitude will be analytically continued to the upper half of the complex plane of s : $s \rightarrow s + i\eta$ while $u \rightarrow u - i\eta$ with $\eta \in \mathbb{R}^+$. Then, $\nu \rightarrow \nu + i\eta$. For the case of $\nu \leq -1$, we have $u \geq 0$, then $s \rightarrow s - i\eta$ while $u \rightarrow u + i\eta$, and $\nu \rightarrow \nu - i\eta$. This implies that the amplitude is recovered for $\nu > 0$ by approaching the real axis from above, while for $\nu < 0$ it is approached from below. For $\nu_0 \in \mathbb{R}^+$:

$$\mathcal{F}(\nu_0) = \lim_{\eta \rightarrow 0^+} \mathcal{F}(\nu_0 + i\eta), \quad (29)$$

$$\mathcal{F}(-\nu_0) = \lim_{\eta \rightarrow 0^+} \mathcal{F}(-\nu_0 - i\eta), \quad (30)$$

and by Schwartz's reflection principle:

$$\mathcal{F}(\nu_0 - i\eta) = \mathcal{F}^*(\nu_0 + i\eta), \quad (31)$$

$$\mathcal{F}(-\nu_0 + i\eta) = \mathcal{F}^*(-\nu_0 - i\eta). \quad (32)$$

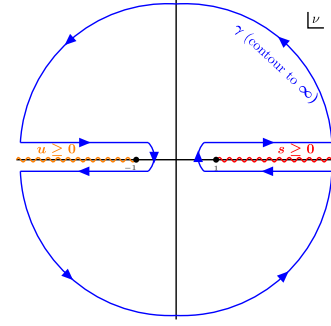


FIG. 1. Complex plane for variable ν where the region where analyticity is not granted, i.e., the physical domain $\nu \in (-\infty, -1] \cup [1, \infty)$, has been highlighted in red for the positive- s region and in orange for the positive- u segment. Note that they are exchanged with respect to Ref. [7]. The contour γ runs over and its interior is within the analytic domain of ν so that $\oint_{\gamma} d\nu' \frac{\mathcal{F}(\nu')}{\nu' - \nu} = 0$ for ν in the physical region, accordingly to Cauchy's integral theorem.

Because ν is inversely proportional to ξ , cf. Eq. (28), $\mathcal{F}(\nu \pm i\eta)$ is equivalent to $\tilde{\mathcal{F}}(\xi \mp i\eta)$ as $\eta \rightarrow 0^+$.

Finally, $\mathcal{F}(\nu)$ is granted to be analytic for

$$\nu \in \{\mathbb{C} - (-\infty, -1] \cup [1, +\infty)\}, \quad (33)$$

as illustrated in Fig. 1, while $\tilde{\mathcal{F}}(\xi)$ for

$$\xi \in \{\mathbb{C} - [-1 - \Lambda, 1 + \Lambda]\}, \quad \Lambda = \frac{2M^2}{Q^2}, \quad (34)$$

as shown in Fig. 2. The factor Λ comes from

$$|\nu| < 1 \Rightarrow |\xi| > \frac{1}{1 - 2M^2/Q^2} = 1 + \frac{2M^2}{Q^2} + O\left(\frac{M^4}{Q^4}\right) > 1. \quad (35)$$

This expansion is possible thanks to the condition $M^2/Q^2 < 1/2$. Note the difference with respect to a

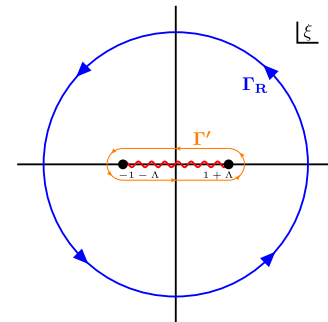


FIG. 2. Complex plane for variable ξ where the region where analyticity is not granted, i.e., $\xi \in [-1 - \Lambda, 1 + \Lambda]$, has been highlighted in red. Note that this interval is larger than the physical domain which corresponds to $|\xi| < 1$.

standard mass correction which comes in powers of $\xi M/Q$, cf. [50,51]. There, the skewness dependence guarantees that the mass corrections are tamed for large nuclei. Indeed, up to t/Q^2 terms and binding energy corrections ($M \approx AM^{\text{proton}}$, with A the mass number), the skewness takes the form

$$\xi \approx \frac{x_B}{2 - x_B} \approx \frac{x_B^{\text{proton}}}{2A - x_B^{\text{proton}}}, \quad (36)$$

thus the mass correction reads

$$\frac{\xi M}{Q} = \frac{x_B^{\text{proton}}}{2 - x_B^{\text{proton}}} \frac{M^{\text{proton}}}{Q} \xrightarrow{A \rightarrow \infty} \frac{x_B^{\text{proton}} M^{\text{proton}}}{2Q}. \quad (37)$$

This implies that the mass corrections, even for large nuclei, ought to be of the same order as for the proton. However, the condition for granted analyticity ($M^2/Q^2 < 1/2$) is not regulated by the skewness, making it dependent of the target mass.

From the considerations above, $\mathcal{F}(\nu)$ can be written around $\nu = 0$ as a entire series of radius of convergence $|\nu| < 1$ in the complex plane as

$$\mathcal{F}(\nu)|_{|\nu|<1} = \sum_{j=0}^{\infty} f_j \nu^j \quad (38)$$

where the f_j are guaranteed to be real because of the Schwartz principle. Consequently, within the domain (34), $\tilde{\mathcal{F}}(\xi)$ can be expanded such that

$$\tilde{\mathcal{F}}(\xi)|_{|\xi|>1+\Lambda} = \sum_{j=0}^{\infty} f_j \frac{1}{\xi^j}, \quad f_j \in \mathbb{R}. \quad (39)$$

Going back to standard CFF notations, and regardless of the incoming (A) or outgoing (B) photon polarization, the CFF $\mathcal{H}^{AB}$ can thus be expanded as

$$\mathcal{H}^{AB}(\xi)|_{|\xi|>1+\Lambda} = \sum_{j=0}^{\infty} h_j^{AB} \frac{1}{\xi^j}, \quad h_j^{AB} \in \mathbb{R}. \quad (40)$$

At this point, the demonstration follows the one of Ref. [7]. Briefly we introduce the n -subtracted integral $\mathcal{I}_n^{AB}(\xi)$ over the contour Γ_R of Fig. 2:

$$\begin{aligned} \mathcal{I}_n^{AB}(\xi)|_{|\xi| \in \text{branch cut}} &= \oint_{\Gamma_R} d\xi' \frac{\mathcal{H}^{AB}(\xi')}{\xi' - \xi} \left(\frac{\xi'}{\xi}\right)^n \\ &= 2\pi i \sum_{j=0}^n h_j^{AB} \frac{1}{\xi^j} \end{aligned} \quad (41)$$

which truncates the series of Eq. (40) for ξ in the physical region. This result, obtained with the residue theorem, has

to match that of the same integration with respect to the closed curve Γ' because both Γ_R and Γ' are inside the domain (34) and, therefore, are homotopic.

Integrating over Γ' while keeping ξ on the branch cut, one is left with

$$\begin{aligned} \mathcal{I}_n^{AB}(\xi) &= \int_{-(1+\Lambda)}^{1+\Lambda} d\xi' \frac{\mathcal{H}^{AB}(\xi' - i0)}{\xi' - \xi - i0} \left(\frac{\xi' - i0}{\xi}\right)^n - (\text{c.c.}) \\ &= \text{PV} \int_{-(1+\Lambda)}^{1+\Lambda} d\xi' \frac{\mathcal{H}^{AB}(\xi' - i0)}{\xi' - \xi} \left(\frac{\xi' - i0}{\xi}\right)^n \\ &\quad + i\pi \mathcal{H}^{AB}(\xi - i0) \left(1 - \frac{i0}{\xi}\right)^n - (\text{c.c.}) \\ &= \text{PV} \int_{-(1+\Lambda)}^{1+\Lambda} d\xi' \frac{\mathcal{H}^{AB}(\xi' - i0) - \mathcal{H}^{AB}(\xi' + i0)}{\xi' - \xi} \left(\frac{\xi'}{\xi}\right)^n \\ &\quad + i\pi [\mathcal{H}^{AB}(\xi - i0) + \mathcal{H}^{AB}(\xi + i0)], \end{aligned} \quad (42)$$

where (c.c.) stands for “complex conjugate” and PV represents Cauchy’s principal value. From the first to the second line we used the Sokhotski-Plemelj formula. Considering $\xi \in (0, 1)$, the singular behavior around zero skewness is avoided and we can safely take $i0\mathcal{H}^{AB}(\xi' \pm i0) \rightarrow 0$ when going from the second to the third line. Remember that $\mathcal{H}^{AB}(\xi) \sim \xi^{-\alpha}$ as $\xi \rightarrow 0$, then the formula above is only valid for $n > \alpha - 1$.

One finally gets for physical values of ξ :

$$\begin{aligned} \sum_{j=0}^n h_j^{AB} \frac{1}{\xi^j} &= \text{Re}(\mathcal{H}^{AB}(\xi)) \\ &\quad + \frac{1}{\pi} \text{PV} \int_{-(1+\Lambda)}^{1+\Lambda} d\xi' \frac{\text{Im}(\mathcal{H}^{AB}(\xi'))}{\xi' - \xi} \left(\frac{\xi'}{\xi}\right)^n. \end{aligned} \quad (43)$$

This expression coincides with the one obtained in Ref. [7] up to the term Λ in the integration limits. However, according to the factorization theorem, the Compton form factor of an exclusive process such as DVCS is given by the convolution of a coefficient function T with a generalized parton distribution [see Eq. (15)]. The imaginary part of the CFF $\mathcal{H}$ is generated solely from the so-called DGLAP kinematic region of the GPDs, i.e., $|x| \geq |\xi|$. For $|\xi| \geq 1$, the convolution of Eq. (15) does not probe the DGLAP region and thus, the imaginary part of the CFF vanishes.

In the Efremov-Radyushkin-Brodsky-Lepage (ERBL) region ($|x| < |\xi|$), no imaginary part is generated because of the different signs of the momentum fractions $x + \xi$ and $x - \xi$. In that situation, no on-shell intermediate state can be generated in the u or s channels. Indeed, the imaginary part of the amplitude, according to Cutkosky’s rules, is generated by on-shell intermediate states (poles or cuts), in the s and u

channels, respectively (we recall that t is chosen spacelike from the start).

Following this argument, one recovers the same result as Ref. [7] for physical values of ξ :

$$\sum_{j=0}^n h_j^{AB} \frac{1}{\xi^j} = \text{Re}(\mathcal{H}^{AB}(\xi)) + \frac{1}{\pi} \text{PV} \int_{-1}^1 d\xi' \frac{\text{Im}(\mathcal{H}^{AB}(\xi'))}{\xi' - \xi} \left(\frac{\xi'}{\xi}\right)^n. \quad (44)$$

B. Subtraction constant with higher power corrections

The formal expression of the dispersion relation given in terms of the real and imaginary parts of the CFF is left unchanged compared to the leading twist case. However, the way the h_j^{AB} are connected with the GPDs is significantly impacted, as the expression of the coefficient function is strongly modified. We explore here the impact of these corrections and their dependence for the $\mathcal{H}^{++}$ Compton form factor, of great importance for being the only one participating of the LT amplitude. The connection between $\mathcal{H}^{++}$ and the GPDs can be written in a more compact way, separating terms by their number of derivatives ∂_ξ :

$$\begin{aligned} \mathcal{H}^{++} = & \int_{-1}^1 \frac{dx}{\xi} \left\{ T_0^{++} \left(\frac{x}{\xi}, \frac{t}{Q^2} \right) H \right. \\ & + \frac{t}{Q^2} \xi \partial_\xi \left(T_1^{++} \left(\frac{x}{\xi}, \frac{t}{Q^2} \right) H \right) \\ & + \frac{-2\xi^2 \tilde{p}_1^2}{Q^2} \xi^2 \partial_\xi^2 \left(T_1^{++} \left(\frac{x}{\xi}, \frac{t}{Q^2} \right) H \right) \Big\} \\ & + O(\text{tw}-6, \alpha_s \cdot \text{tw}-4), \end{aligned} \quad (45)$$

where

$$\begin{aligned} T_0^{++} \left(\frac{x}{\xi}, \frac{t}{Q^2} \right) = & C_{\text{LT}}^{(+)} \left(\frac{x}{\xi} \right) + \frac{t}{Q^2} \tilde{\mathbb{P}}_{(\text{iii})}^{(+)} \left(\frac{x}{\xi} \right) \\ & - \frac{t}{Q^2} \frac{\mathcal{L}^{(+)} \left(\frac{x}{\xi} \right) + C_0^{(+)} \left(\frac{x}{\xi} \right)}{2}, \end{aligned} \quad (46)$$

$$T_1^{++} \left(\frac{x}{\xi} \right) = \frac{\mathcal{L}^{(+)} \left(\frac{x}{\xi} \right) - \tilde{\mathbb{P}}_{(\text{iii})}^{(+)} \left(\frac{x}{\xi} \right)}{2}. \quad (47)$$

The reader is invited to refer to Appendix A for complete definitions of all previous functions. Here we just highlight that $C_{\text{LT}}^{(+)}$ is the leading-twist coefficient function (at arbitrary precision in α_s), while $\mathcal{L}^{(+)}$ and $\tilde{\mathbb{P}}_{(\text{iii})}^{(+)}$ arise from kinematic higher-twist corrections. Considering implicitly the t dependence, one can inject the DDs representation

(11) term by term in the previous equation. Doing so allows us to analytically continue the GPDs for $|\xi| > 1$ as in this region the Radon transform (11) admits non-vanishing values for $x < |\xi|$ instead of $|x| < 1$. This is a consequence of the support of the double distributions $|\beta| + |\alpha| \leq 1$. We treat the terms one by one following the decomposition:

$$\mathcal{H}^{++} = \mathcal{H}_0^{++} + \mathcal{H}_1^{++} + \mathcal{H}_2^{++} \quad (48)$$

starting by analytically continuing $\mathcal{H}_0^{++}$ for $|\xi| \geq 1$ with¹

$$\begin{aligned} \mathcal{H}_0^{++} = & \int_{-\xi}^{\xi} dx \frac{1}{\xi} T_0^{++}(x/\xi, t/Q^2) H \\ = & \int_{\Omega} d\beta d\alpha \int_{-\xi}^{\xi} \frac{dx}{\xi} T_0^{++} \left(\frac{x}{\xi}, \frac{t}{Q^2} \right) \\ & \times \delta(x - \beta - \alpha\xi) [F(\beta, \alpha) + \xi D(\alpha) \delta(\beta)] \\ = & \int_{\Omega} d\beta d\alpha \frac{1}{\xi} T_0^{++} \left(\frac{\beta}{\xi} + \alpha, \frac{t}{Q^2} \right) F(\beta, \alpha) \\ & + \int_{-1}^1 d\alpha T_0^{++} \left(\alpha, \frac{t}{Q^2} \right) D(\alpha) \\ = & \sum_{n=0}^{\infty} \frac{1}{n!} \int_{\Omega} d\beta d\alpha \frac{\beta^n}{\xi^{n+1}} T_0^{++(n)}(\alpha, t/Q^2) F(\beta, \alpha) \\ & + \int_{-1}^1 d\alpha T_0^{++}(\alpha, t/Q^2) D(\alpha), \end{aligned} \quad (49)$$

where we use the notation $f^{(n)}(y) = \frac{\partial^n f(y)}{\partial y^n} \big|_{y=y}$. From the second to the third equality, we integrate with respect to $x \in (-\xi, \xi)$ with the $\delta(x - \beta - \alpha\xi)$. From the third to the fourth equality, we expand the coefficient function in powers of $1/\xi$ which is only possible in the unphysical domain of ξ . This is precisely what we want to do in order to identify the different coefficients h_j^{++} from series (40), only valid when the amplitude is factorized for $|\xi| > 1$, and be able to read out the subtraction constant (h_0^{++}). This is the strategy followed in Ref. [7] and here.

¹Following our argument before Eq. (44), the natural bound for $|\xi|$ is $|\xi| > 1 + \Lambda$. However, once we factorize the amplitude into a GPD and a coefficient function, the analytic properties of the coefficient function dictate the ones of the amplitudes. Thus, the natural bound obtained at the level of the amplitude can be relaxed once we assume the factorization regime holds. As we mentioned before, s and u negative is a sufficient condition to guarantee that the amplitude is analytic, not a necessary one.

The term on ∂_ξ is given by (again for $|\xi| > 1$)

$$\begin{aligned}
 \mathcal{H}_1^{++} &= \frac{t}{\mathbb{Q}^2} \int_{-\xi}^{\xi} dx \partial_\xi [T_1^{++}(x/\xi)H] \\
 &= \frac{t}{\mathbb{Q}^2} \partial_\xi \left[\int_{\Omega} d\beta d\alpha T_1^{++} \left(\frac{\beta}{\xi} + \alpha \right) F(\beta, \alpha) + \xi \int_{-1}^1 d\alpha T_1^{++}(\alpha) D(\alpha) \right] \\
 &= \frac{t}{\mathbb{Q}^2} \left[\sum_{n=0}^{\infty} \frac{-n}{n!} \int_{\Omega} d\beta d\alpha \frac{\beta^n}{\xi^{n+1}} T_1^{++(n)}(\alpha) F(\beta, \alpha) + \int_{-1}^1 d\alpha T_1^{++}(\alpha) D(\alpha) \right] \\
 &= \frac{t}{\mathbb{Q}^2} \left[\sum_{n=0}^{\infty} \frac{-1}{n!} \int_{\Omega} d\beta d\alpha \frac{\beta^{n+1}}{\xi^{n+2}} T_1^{++(n+1)}(\alpha) F(\beta, \alpha) + \int_{-1}^1 d\alpha T_1^{++}(\alpha) D(\alpha) \right]
 \end{aligned} \tag{50}$$

while the term with ∂_ξ^2 in the region $|\xi| > 1$ is

$$\begin{aligned}
 \mathcal{H}_2^{++} &= \frac{-2\xi^3 \bar{p}_\perp^2}{\mathbb{Q}^2} \int_{-\xi}^{\xi} dx \partial_\xi^2 [T_1^{++}(x/\xi)H] \\
 &= \frac{-2\xi^2 \bar{p}_\perp^2}{\mathbb{Q}^2} \partial_\xi^2 \left[\sum_{n=0}^{\infty} \frac{1}{n!} \int_{\Omega} d\beta d\alpha \frac{\beta^n}{\xi^n} T_1^{++(n)}(\alpha) F(\beta, \alpha) + \xi \int_{-1}^1 d\alpha T_1^{++}(\alpha) D(\alpha) \right] \\
 &= \frac{-2\xi^3 \bar{p}_\perp^2}{\mathbb{Q}^2} \sum_{n=0}^{\infty} \frac{n(n+1)}{n!} \int_{\Omega} d\beta d\alpha \frac{\beta^n T_1^{++(n)}(\alpha)}{\xi^{n+2}} F(\beta, \alpha) \\
 &= \frac{-2\xi^3 \bar{p}_\perp^2}{\mathbb{Q}^2} \sum_{n=0}^{\infty} \frac{n+2}{n!} \int_{\Omega} d\beta d\alpha \frac{\beta^{n+1} T_1^{++(n+1)}(\alpha)}{\xi^{n+3}} F(\beta, \alpha) \\
 &= \frac{-2\bar{p}_\perp^2}{\mathbb{Q}^2} \sum_{n=0}^{\infty} \frac{n+2}{n!} \int_{\Omega} d\beta d\alpha \frac{\beta^{n+1} T_1^{++(n+1)}(\alpha)}{\xi^n} F(\beta, \alpha).
 \end{aligned} \tag{51}$$

Taking into account that $\bar{p}_\perp^2$ depends on ξ :

$$\bar{p}_\perp^2 = M^2 - \frac{t}{4} \left(1 - \frac{1}{\xi^2} \right), \tag{52}$$

as well as the above series $\mathcal{H}_i^{++}$, we get for $|\xi| > 1$:

$$\begin{aligned}
 \sum_{j=0}^{\infty} h_j^{++} \frac{1}{\xi^j} &= \int_{-1}^1 d\alpha \left[T_0^{++}(\alpha, t/\mathbb{Q}^2) + \frac{t}{\mathbb{Q}^2} T_1^{++}(\alpha) \right] D(\alpha) - \frac{4M^2 - t}{\mathbb{Q}^2} \int_{\Omega} d\beta d\alpha \beta F(\beta, \alpha) T_1^{++(1)}(\alpha) \\
 &\quad + \frac{1}{\xi} \int_{\Omega} d\beta d\alpha F(\beta, \alpha) \left[T_0^{++}(\alpha, t/\mathbb{Q}^2) - \frac{6M^2 - 3t/2}{\mathbb{Q}^2} \beta^2 T_1^{++(2)}(\alpha) \right] \\
 &\quad + \sum_{n=2}^{\infty} \frac{1}{\xi^n} \int_{\Omega} d\beta d\alpha F(\beta, \alpha) \left[\beta^{n-1} \left\{ \frac{T_0^{++(n-1)}(\alpha, t/\mathbb{Q}^2)}{(n-1)!} - \frac{t}{\mathbb{Q}^2} \frac{n+2}{2 \cdot (n-2)!} T_1^{++(n-1)}(\alpha) \right\} \right. \\
 &\quad \left. - \beta^{n+1} \frac{2M^2 - t/2}{\mathbb{Q}^2} \frac{n+2}{n!} T_1^{++(n+1)}(\alpha) \right].
 \end{aligned} \tag{53}$$

The double distribution $F(\beta, \alpha)$ is even in α . Taking into account that $\int_{\Omega} d\beta d\alpha$ is done for a symmetric interval in both β and α , the terms multiplying this DD and that are odd in α vanish upon integration.² As a consequence, the above expression simplifies to

²Since $T_i^{++}(\alpha)$, $i \in \{0, 1\}$ are superpositions of functions that are odd in α , then an even (odd) number of derivatives with respect to α renders an odd (even) function with respect to that variable. Note also that $D(\alpha)$ is odd in α , as opposed to $F(\beta, \alpha)$.

$$\begin{aligned}
\sum_{\substack{j=0, \\ j \text{ even}}}^{\infty} h_j^{++} \frac{1}{\xi^j} &= \int_{-1}^1 d\alpha \left[T_0^{++}(\alpha, t/\mathbb{Q}^2) + \frac{t}{\mathbb{Q}^2} T_1^{++}(\alpha) \right] D(\alpha) - \frac{4M^2 - t}{\mathbb{Q}^2} \int_{\Omega} d\beta d\alpha \beta F(\beta, \alpha) T_1^{++(1)}(\alpha) \\
&+ \sum_{\substack{n=2, \\ n \text{ even}}}^{\infty} \frac{1}{\xi^n} \int_{\Omega} d\beta d\alpha F(\beta, \alpha) \left[\beta^{n-1} \left\{ \frac{T_0^{++(n-1)}(\alpha, t/\mathbb{Q}^2)}{(n-1)!} - \frac{t}{\mathbb{Q}^2} \frac{n+2}{2 \cdot (n-2)!} T_1^{++(n-1)}(\alpha) \right\} \right. \\
&\left. - \beta^{n+1} \frac{2M^2 - t/2}{\mathbb{Q}^2} \frac{n+2}{n!} T_1^{++(n+1)}(\alpha) \right], \quad (54)
\end{aligned}$$

from where the first line, after restoring t dependence, provides the subtraction constant of the dispersion relation:

$$\begin{aligned}
h_0^{++}(t) &= \int_{-1}^1 d\alpha T_2^{++} \left(\alpha, \frac{t}{\mathbb{Q}^2} \right) D(\alpha, t) \\
&- 4 \frac{M^2 - t/4}{\mathbb{Q}^2} \int_{\Omega} d\beta d\alpha F(\beta, \alpha, t) \beta T_1^{++(1)}(\alpha), \quad (55)
\end{aligned}$$

where

$$T_2^{++} \left(\alpha, \frac{t}{\mathbb{Q}^2} \right) = T_0^{++}(\alpha, t/\mathbb{Q}^2) + \frac{t}{\mathbb{Q}^2} T_1^{++}(\alpha), \quad (56)$$

and the coefficients for even $n \geq 2$

$$\begin{aligned}
h_n^{++} &= \int_{\Omega} d\beta d\alpha F(\beta, \alpha) \left[\beta^{n-1} \left\{ \frac{T_0^{++(n-1)}(\alpha, t/\mathbb{Q}^2)}{(n-1)!} \right. \right. \\
&- \frac{t}{\mathbb{Q}^2} \frac{n+2}{2 \cdot (n-2)!} T_1^{++(n-1)}(\alpha) \left. \right\} \\
&\left. - \beta^{n+1} \frac{M^2 - t/4}{\mathbb{Q}^2} \frac{2(n+2)}{n!} T_1^{++(n+1)}(\alpha) \right]. \quad (57)
\end{aligned}$$

For odd n , $h_n^{++} = 0$. That only even n contributes to the CFFs is a consequence of the time-reversal symmetry of the theory which leads to $\mathcal{H}^{++}(\xi) = \mathcal{H}^{++}(-\xi)$.

Owing to the Schwartz's reflection principle, h_j^{++} s must be real numbers. This implies that only the real part of the coefficient functions should contribute to the above integrals. In fact, the integration with respect to α is restricted to the interval $\alpha \in (-1, 1)$. Taking into account that $\alpha = x/\xi$, we conclude that x falls in the ERBL region ($|x| < |\xi|$) while the imaginary parts of T_0^{++} and T_1^{++} are found in the DGLAP domain ($|x| > |\xi|$). As a consequence, the imaginary parts of these coefficients do not contribute to the h_j^{++} factors. Note also that the argument of logarithms and dilogarithms in T_1^{++} is $y = (1 \pm \alpha)/2 \in (0, 1)$ for $\alpha \in (-1, 1)$, so $\ln|y| = \ln y$ and branch cuts are not crossed for either $\ln y$ or $\text{Li}_2(y)$.

Equation (55) deserves comments. The kernel relating h_0^{++} and D is, as expected, modified by the kinematic power corrections, adding an explicit dependence in $t/\mathbb{Q}^2$ and a convolution with a Li_2 function. However, the

unexpected output relies in the new mixing with the DD F . This mixing is not suppressed at small values of t , as it comes with an explicit mass dependence. And in fact, the prefactor $4M^2/\mathbb{Q}^2$ is not small, especially for JLab kinematics, thus this term cannot be considered negligible and needs to be taken into account. It breaks the simple³ relation between the subtraction constant $-h_0^{++}$ and a convolution with D . Worse, this term is “unprotected,” in the sense that it is hadron dependent, compared to standard kinematic twist expansion of the type $\xi^2 M^2/\mathbb{Q}^2$. It triggers that, for ^4He , this mass term is expected to be by far the dominant contribution, most probably precluding the extraction of the D -term as it was envisioned in the literature [72]. This is, provided that the dispersion relation holds for ^4He despite the breaking of Eq. (24), which is only a sufficient but not strictly necessary condition. On the other hand, the case of the pion is expected to be much better, which may allow a study of the D -term through the Sullivan process [68–71].

IV. POWER CORRECTIONS TO DISPERSION RELATIONS: THE SPIN-1/2 CASE

In this section, we generalize the previous discussion from (pseudo)scalar to spin-1/2 targets. As the derivation in Sec. III A is independent of the spin of the target, Eq. (44) still holds for the nucleon amplitudes. We will thus focus on the expression of h_0^{++} in terms of DDs.

A. Coefficient function for spin-1/2 targets

Kinematic higher-twist corrections are associated to the twist decomposition of the parton operators describing the hadronic structure and, therefore, being affected by features such as spin: different spin renders different GPD parametrization, cf. Appendix A 3. In Ref. [65], authors present a calculation of the Compton tensor, $T^{\mu\nu}$, in DVCS for a spin-1/2 target up to kinematic twist-six. We are interested in the transverse-helicity conserving amplitude $\mathcal{A}^{++}$ from the vector part of the Compton tensor (which is a

³As strongly emphasized in [7], if the h_0^{++} is at leading twist provided by the convolution with a D -term, the deconvolution problem remains very challenging, with the possibility to reconstruct only a single Gegenbauer mode for now.

combination of the corresponding vector $\mathcal{H}^{++}, \mathcal{E}^{++}$ Compton form factors),

$$T^{\mu\nu} = -g_{\perp}^{\mu\nu} \mathcal{A}^{++} + (\text{terms} \sim \mathcal{A}^{+-}, \mathcal{A}^{0+}) + (\text{axial part}), \quad (58)$$

which in Ref. [65] is given in terms of two invariant amplitudes $V_0^{(1)}$ and $V_0^{(2)}$. We would like to match that expression to the usual CFFs $\mathcal{H}^{++}$ and $\mathcal{E}^{++}$ [57] which is

$$\mathcal{A}^{++} = \frac{v \cdot q'}{q \cdot q'} V_0^{(1)} + \frac{v \cdot \bar{p}}{M^2} V_0^{(2)} = h \mathcal{H}^{++} + e \mathcal{E}^{++}. \quad (59)$$

The amplitudes $V_0^{(1)}$ and $V_0^{(2)}$ read

$$V_0^{(1)} = -\left(1 + \frac{t}{4(qq')}\right) (G^{(+)} \otimes T_0) - \frac{t}{2(qq')} (G^{(+)} \otimes T_{10}) - \frac{1}{2} D_{\xi}^2 \frac{|\bar{p}_{\perp}|^2}{(qq')} (G^{(+)} \otimes T_2) + O(\text{tw}-6), \quad (60)$$

$$V_0^{(2)} = -\left(1 + \frac{t}{4(qq')}\right) \left(\frac{E^{(+)}}{2} \odot T_0\right) - \frac{t}{2(qq')} \left(\frac{E^{(+)}}{2} \odot T_{10}\right) - \frac{1}{2} D_{\xi} \frac{|\bar{p}_{\perp}|^2}{(qq')} D_{\xi} \left(\frac{E^{(+)}}{2} \odot T_2\right) - \frac{M^2}{qq'} D_{\xi} (G^{(+)} \otimes T_2) + O(\text{tw}-6), \quad (61)$$

where $D_{\xi}^n = (-2\xi^2 \partial_{\xi})^n$ and the hard kernels read as

$$\begin{aligned} T_0(z) &= \frac{1}{1-z} = 2C_0(2(z-i0)-1), \\ T_{10}(z) &= \frac{1}{z} \ln(1-z) = -\tilde{\mathbb{P}}_{(\text{iii})}(2(z-i0)-1), \\ T_2(z) &= \frac{1}{1-z} (\text{Li}_2(z) - \text{Li}_2(1)) - \frac{1}{2z} \ln(1-z) \\ &= \frac{\tilde{\mathbb{P}}_{(\text{iii})}(2(z-i0)-1) - \mathcal{L}(2(z-i0)-1)}{2}. \end{aligned} \quad (62)$$

Here, $z = \frac{x+\xi}{2\xi} + i0$, thus $2(z-i0)-1 = \frac{x}{\xi}$. The “magnetic” GPD was defined⁴ as in Ref. [65] (see also Refs. [62–64]):

$$G^{(+)} = \frac{1}{2} (H^{(+)} + E^{(+)}), \quad (63)$$

with $F^{(+)}(x) = F(x) - F(-x)$, and the symbols $\otimes, \odot$ stand for convolutions between hard coefficient kernels and GPDs with different normalizations [65]:

$$G^{(+)} \otimes T = \int_{-1}^1 dx G^{(+)}(x, \xi, t) T\left(\frac{x+\xi}{2\xi} + i0\right), \quad (64)$$

$$E^{(+)} \odot T = \frac{1}{2\xi} \int_{-1}^1 dx E^{(+)}(x, \xi, t) T\left(\frac{x+\xi}{2\xi} + i0\right). \quad (65)$$

⁴In Ref. [65] the odd in x “magnetic” GPD was represented as M so that the mapping to the notation in this manuscript is $M \rightarrow G^{(+)}$.

Meanwhile, vectors and spinor bilinears in Eq. (59) are given by

$$\begin{aligned} v^{\mu} &= \bar{u}(p') \gamma^{\mu} u(p), \quad \bar{p}^{\mu} = \frac{p+p'}{2} = \frac{(n'^{\mu} - \frac{t}{Q^2} n^{\mu})}{2\xi} + \bar{p}_{\perp}^{\mu}, \\ h &= \frac{vn}{2\bar{p}^+}, \quad e = \frac{\bar{u}(p') i\sigma^{\alpha\beta} n_{\alpha} \Delta_{\beta} u(p)}{4M\bar{p}^+}, \end{aligned} \quad (66)$$

with $\sigma^{\alpha\beta} = i[\gamma^{\alpha}, \gamma^{\beta}]/2$. Using the definition of the skewness $\xi = -\Delta \cdot n / (2\bar{p} \cdot n)$ and $n = q'$, we find

$$\frac{v \cdot q'}{q \cdot q'} = \frac{v \cdot n}{-2\bar{p}^+ \xi} = -\frac{1}{\xi} h. \quad (67)$$

With the Dirac equation

$$v \cdot \bar{p} = \bar{u}(p') \not{p} u(p) = M \bar{u}(p') u(p), \quad (68)$$

and Gordon’s identity,

$$v^{\alpha} = \bar{u}(p') \gamma^{\alpha} u(p) = \bar{u}(p') \left[\frac{\bar{p}^{\alpha}}{M} + \frac{i\sigma^{\alpha\beta} \Delta_{\beta}}{2M} \right] u(p), \quad (69)$$

we find

$$\begin{aligned} vn &= \frac{Q^2}{4\xi M} \frac{v\bar{p}}{M} + 2\bar{p}^+ e \Rightarrow \frac{vn}{2\bar{p}^+} = h = \underbrace{\frac{Q^2}{8\xi\bar{p}^+}}_{1/2} \frac{v\bar{p}}{M^2} + e \\ &\Rightarrow h - e = \frac{v\bar{p}}{2M^2}. \end{aligned} \quad (70)$$

Finally,

$$\mathcal{A}^{++} = h \left(2V_0^{(2)} - \frac{1}{\xi} V_0^{(1)} \right) + e(-2V_0^{(2)}), \quad (71)$$

where we identify

$$\mathcal{H}^{++} = 2V_0^{(2)} - \frac{1}{\xi} V_0^{(1)}, \quad \mathcal{E}^{++} = -2V_0^{(2)}. \quad (72)$$

$V_0^{(1)}$ is given as a convolution of some kernel with GPDs. As these kernels are analytic in the domain (34), $V_0^{(1)}$ can be expanded in powers of $1/\xi$. As the CFFs are even in ξ , it

follows that $V_0^{(1)}/\xi$ can be expressed as a series in even powers of the inverse of the skewness within the domain (34). Therefore, only $V_0^{(2)}$ contributes to the subtraction constant of both $\mathcal{H}^{++}$ and $\mathcal{E}^{++}$. Thus,

$$h_0^{++} + e_0^{++} = 0. \quad (73)$$

Now, we want to write down the CFFs in a similar way as we did for the spin-0 target. In order to do so we need to translate the terms with total derivatives $D_\xi^n = (-2\xi^2 \partial_\xi)^n$ to terms with $\xi^n \partial_\xi^n$.

For that purpose, and denoting by $\mathbb{T}$ the hard coefficient functions of Eqs. (62), we employ the following relations:

$$D_\xi^2 \left(\frac{|\bar{p}_\perp|^2}{qq'} G^{(+)} \otimes \mathbb{T} \right) = 4 \frac{t}{Q^2} G^{(+)} \otimes \mathbb{T} + \frac{2\xi^2 \bar{p}_\perp^2 - t}{Q^2} 8\xi \partial_\xi (G^{(+)} \otimes \mathbb{T}) + \frac{8\xi^2 \bar{p}_\perp^2}{Q^2} \xi^2 \partial_\xi^2 (G^{(+)} \otimes \mathbb{T}), \quad (74)$$

$$D_\xi \left[\frac{|\bar{p}_\perp|^2}{qq'} D_\xi \left(\frac{E^{(+)}}{2} \odot \mathbb{T} \right) \right] = \frac{2\xi^2 \bar{p}_\perp^2 - t}{Q^2} 8\xi \partial_\xi \left(\frac{E^{(+)}}{2} \odot \mathbb{T} \right) + \frac{8\xi^2 \bar{p}_\perp^2}{Q^2} \xi^2 \partial_\xi^2 \left(\frac{E^{(+)}}{2} \odot \mathbb{T} \right). \quad (75)$$

Note also that in Ref. [65], the notation $H, E, \tilde{H}, \tilde{E}$ refers in fact to half of the C-even part of the GPDs as introduced in [57]; this is $H, E, \tilde{H}, \tilde{E} \mapsto H^{(+)}/2, E^{(+)}/2, \tilde{H}^{(+)}/2, \tilde{E}^{(+)}/2$.

Collecting the above results,

$$\begin{aligned} \mathcal{H}^{++} = & - \left(1 - \frac{t}{2Q^2} \right) \left[E^{(+)} \odot \mathbb{T}_0 - \frac{1}{\xi} G^{(+)} \otimes \mathbb{T}_0 \right] + \frac{t}{Q^2} \left[E^{(+)} \odot \mathbb{T}_{10} - \frac{1}{\xi} G^{(+)} \otimes \mathbb{T}_{10} \right] \\ & - \frac{1}{2} \left[\frac{2\xi^2 \bar{p}_\perp^2 - t}{Q^2} 8\xi \left(\partial_\xi [E^{(+)} \odot \mathbb{T}_2] - \frac{1}{\xi} \partial_\xi [G^{(+)} \otimes \mathbb{T}_2] \right) + \frac{8\xi^2 \bar{p}_\perp^2}{Q^2} \xi^2 \left(\partial_\xi^2 [E^{(+)} \odot \mathbb{T}_2] - \frac{1}{\xi} \partial_\xi^2 [G^{(+)} \otimes \mathbb{T}_2] \right) \right] \\ & + \frac{1}{\xi} \frac{2t}{Q^2} G^{(+)} \otimes \mathbb{T}_2 - \frac{8M^2}{Q^2} \xi^2 \partial_\xi (G^{(+)} \otimes \mathbb{T}_2). \end{aligned} \quad (76)$$

Making use of the previously introduced expressions for $\mathbb{T}_0, \mathbb{T}_{10}$, and $\mathbb{T}_2$, we find the following relations to the convolutions encountered in the spin-0 case. Starting with the term free of derivatives:

$$E^{(+)} \odot \mathbb{T}_0 - \frac{1}{\xi} G^{(+)} \otimes \mathbb{T}_0 = - \int_{-1}^1 dx \frac{1}{\xi} C_0(x/\xi) H^{(+)}(x, \xi, t), \quad (77)$$

and

$$E^{(+)} \odot \mathbb{T}_{10} - \frac{1}{\xi} G^{(+)} \otimes \mathbb{T}_{10} = \int_{-1}^1 dx \frac{1}{2\xi} \tilde{\mathbb{P}}_{(\text{iii})}(x/\xi) H^{(+)}(x, \xi, t). \quad (78)$$

The first derivative term yields:

$$\begin{aligned} \partial_\xi (E^{(+)} \odot \mathbb{T}_2) - \frac{1}{\xi} \partial_\xi (G^{(+)} \otimes \mathbb{T}_2) = & - \frac{1}{2} \left(\frac{1}{\xi} E^{(+)} \otimes \mathbb{T}_2 + \partial_\xi [H^{(+)} \otimes \mathbb{T}_2] \right) \\ = & - \frac{1}{4} \int_{-1}^1 \frac{dx}{\xi} \left(\frac{1}{\xi} [\tilde{\mathbb{P}}_{(\text{iii})}(x/\xi) - \mathcal{L}(x/\xi)] E^{(+)}(x, \xi, t) \right. \\ & \left. + \partial_\xi \{ [\tilde{\mathbb{P}}_{(\text{iii})}(x/\xi) - \mathcal{L}(x/\xi)] H^{(+)}(x, \xi, t) \} \right), \end{aligned} \quad (79)$$

while the second derivative term gives

$$\begin{aligned}
 & \partial_\xi^2(E^{(+)} \odot T_2) - \frac{1}{\xi} \partial_\xi^2(G^{(+)} \otimes T_2) \\
 &= \frac{1}{\xi^3} \left(E^{(+)} \otimes T_2 - \xi \partial_\xi [E^{(+)} \otimes T_2] - \frac{\xi^2}{2} \partial_\xi^2 [H^{(+)} \otimes T_2] \right) \\
 &= \frac{1}{2\xi^2} \int_{-1}^1 \frac{dx}{\xi} \left([\tilde{\mathbb{P}}_{(\text{iii})}(x/\xi) - \mathcal{L}(x/\xi)] E^{(+)}(x, \xi, t) \right. \\
 &\quad \left. - \xi \partial_\xi \{ [\tilde{\mathbb{P}}_{(\text{iii})}(x/\xi) - \mathcal{L}(x/\xi)] E^{(+)}(x, \xi, t) \} \right. \\
 &\quad \left. - \frac{\xi^2}{2} \partial_\xi^2 \{ [\tilde{\mathbb{P}}_{(\text{iii})}(x/\xi) - \mathcal{L}(x/\xi)] H^{(+)}(x, \xi, t) \} \right). \quad (80)
 \end{aligned}$$

As a consequence, the CFF $\mathcal{H}^{++}$ for the spin-1/2 case can be written in the compact form

$$\mathcal{H}^{++} = \mathbb{F}_0[H^{(+)}] + \Delta\mathcal{H}^{++} + O(\text{tw} - 6), \quad (81)$$

where $\mathbb{F}_0[H^{(+)}]$ is the convolution of the spin-0 case (45) and

$$\begin{aligned}
 \Delta\mathcal{H}^{++} &= \int_{-1}^1 \frac{dx}{\xi} \left(\frac{-t}{2Q^2} \right) \{ (\tilde{\mathbb{P}}_{(\text{iii})} - \mathcal{L}) E^{(+)} \\
 &\quad - \xi \partial_\xi [(\tilde{\mathbb{P}}_{(\text{iii})} - \mathcal{L}) E^{(+)}] \\
 &\quad + \xi^3 \partial_\xi^3 [(\tilde{\mathbb{P}}_{(\text{iii})} - \mathcal{L}) (H^{(+)} + E^{(+)})] \}. \quad (82)
 \end{aligned}$$

Following the same procedure and after some algebra,

$$\mathcal{E}^{++} = \mathbb{F}_0[E^{(+)}] + \Delta\mathcal{E}^{++} + O(\text{tw} - 6), \quad (83)$$

where $\mathbb{F}_0[E^{(+)}]$ corresponds to the convolution introduced in Eq. (45) but changing the GPD $H^{(+)}$ by $E^{(+)}$, and $\Delta\mathcal{E}^{++}$ is

$$\begin{aligned}
 \Delta\mathcal{E}^{++} &= \int_{-1}^1 dx \frac{1}{\xi} \left(\frac{t}{2Q^2} \right) \{ (\tilde{\mathbb{P}}_{(\text{iii})} - \mathcal{L}) E^{(+)} \\
 &\quad - \xi \partial_\xi [(\tilde{\mathbb{P}}_{(\text{iii})} - \mathcal{L}) E^{(+)}] \\
 &\quad + 2\xi^3 \partial_\xi^3 [(\tilde{\mathbb{P}}_{(\text{iii})} - \mathcal{L}) G^{(+)}] \\
 &\quad - 2\xi \partial_\xi [(\tilde{\mathbb{P}}_{(\text{iii})} - \mathcal{L}) G^{(+)}] \} \\
 &\quad + \int_{-1}^1 \frac{dx}{\xi} \frac{4\xi^3 \bar{p}_1^2}{Q^2} \partial_\xi [(\tilde{\mathbb{P}}_{(\text{iii})} - \mathcal{L}) G^{(+)}]. \quad (84)
 \end{aligned}$$

Note that beyond LT there is no 1-to-1 relation between GPDs H , E , and CFFs $\mathcal{H}^{++}$, $\mathcal{E}^{++}$. However, considering the combination given by the magnetic GPD (63) a 1-to-1 relation still holds:

$$\begin{aligned}
 \mathcal{G}^{++} &= \frac{1}{2} (\mathcal{H}^{++} + \mathcal{E}^{++}) \\
 &= \mathbb{F}_0[G^{(+)}] - 4 \int_{-1}^1 \frac{dx}{\xi} \frac{M^2 - t/4}{Q^2} \xi^3 \partial_\xi \left(\frac{\mathcal{L} - \tilde{\mathbb{P}}_{(\text{iii})}}{2} G^{(+)} \right). \quad (85)
 \end{aligned}$$

B. Subtraction constant at twist-four for spin-1/2 targets

Taking into account that the CFF $\mathcal{H}^{++}$ of a spin-1/2 hadron can be decomposed into the convolution of a spin-0 particle, $\mathbb{F}_0[\mathcal{H}^{(+)}]$ (45), and an addendum $\Delta\mathcal{H}^{++}$ (82), we can profit from the previous calculation and focus on the latter term. In the notation of the preceding section,

$$\Delta\mathcal{H}^{++} = \sum_{i=0}^2 \Delta\mathcal{H}_i^{++}, \quad (86)$$

where

$$\begin{aligned}
 \Delta\mathcal{H}_0^{++} &= \frac{t}{Q^2} \int_{-1}^1 dx \frac{1}{\xi} T_1^{++} E \\
 &= \frac{t}{Q^2} \left[\sum_{n=0}^{\infty} \frac{1}{n!} \int_{\Omega} d\beta d\alpha \frac{\beta^n}{\xi^{n+1}} T_1^{++(n)}(\alpha) K(\beta, \alpha, t) \right. \\
 &\quad \left. - \int_{-1}^1 d\alpha T_1^{++}(\alpha) D(\alpha) \right], \quad (87)
 \end{aligned}$$

$$\begin{aligned}
 \Delta\mathcal{H}_1^{++} &= -\frac{t}{Q^2} \int_{-1}^1 dx \partial_\xi (T_1^{++} E) \\
 &= \frac{t}{Q^2} \left[\sum_{n=0}^{\infty} \frac{1}{n!} \int_{\Omega} d\beta d\alpha \frac{\beta^{n+1}}{\xi^{n+2}} T_1^{++(n+1)}(\alpha) K(\beta, \alpha, t) \right. \\
 &\quad \left. + \int_{-1}^1 d\alpha T_1^{++}(\alpha) D(\alpha) \right], \quad (88)
 \end{aligned}$$

and

$$\begin{aligned}
 \Delta\mathcal{H}_2^{++} &= \int_{-1}^1 dx \xi^2 \partial_\xi (2T_1^{++} G) \\
 &= \frac{-t}{Q^2} \sum_{n=0}^{\infty} \frac{1}{n!} \int_{\Omega} d\beta d\alpha \frac{\beta^{n+1}}{\xi^n} 2T_1^{++(n+1)}(\alpha) N(\beta, \alpha, t). \quad (89)
 \end{aligned}$$

The zeroth order in the expansion of $\Delta\mathcal{H}^{++}(\xi, t)$ in the analytical unphysical domain of the skewness is then

$$\begin{aligned}
 \Delta\mathcal{H}^{++}|_{1/\xi^0} &= -\frac{t}{Q^2} \int_{\Omega} d\beta d\alpha \beta T_1^{++(1)}(\alpha) \\
 &\quad \times [F(\beta, \alpha, t) + K(\beta, \alpha, t)]. \quad (90)
 \end{aligned}$$

Adding this result to the corresponding term $\mathbb{F}_0[H^{(+)}]|_{1/\xi^0}$ which takes the form of Eq. (55), we find the subtraction constant for the dispersion relation associated to the spin-1/2 hadron in DVCS:

$$h_0^{++}(t) = \int_{-1}^1 d\alpha \left[T_0^{++}(\alpha, t/\mathbb{Q}^2) + \frac{t}{\mathbb{Q}^2} T_1^{++}(\alpha) \right] D(\alpha, t) - 4 \frac{M^2}{\mathbb{Q}^2} \int_{\Omega} d\beta d\alpha \left[F(\beta, \alpha, t) + \frac{t}{4M^2} K(\beta, \alpha, t) \right] \beta T_1^{++(1)}(\alpha). \quad (91)$$

Taking $K \mapsto -F$ recovers the subtraction constant of the spin-0 case.

We could follow the same steps for the CFF $\mathcal{E}^{++}$ and obtain

$$e_0^{++} = -h_0^{++}, \quad (92)$$

which holds up to twist-six at least, as discussed in Eq. (73). This has been cross-checked by computing the corresponding $g_0^{++} = (h_0^{++} + e_0^{++})/2 = 0$ from Eq. (85).

The remnant coefficients for the series in powers of $1/\xi$ of the CFF $\mathcal{H}^{++}$ are, for even $n \geq 2$,

$$\begin{aligned} h_n^{++} = & \int_{\Omega} d\beta d\alpha F(\beta, \alpha, t) \left[\beta^{n-1} \left\{ \frac{T_0^{++(n-1)}(\alpha, t/\mathbb{Q}^2)}{(n-1)!} - \frac{t}{\mathbb{Q}^2} \frac{n+2}{2 \cdot (n-2)!} T_1^{++(n-1)}(\alpha) \right\} \right. \\ & \left. - \beta^{n+1} \left\{ \frac{M^2 - t/4}{\mathbb{Q}^2} \frac{1}{(n-1)!} + \frac{M^2}{\mathbb{Q}^2} \frac{2}{n!} \right\} 2T_1^{++(n+1)}(\alpha) \right] \\ & + \int_{\Omega} d\beta d\alpha K(\beta, \alpha, t) \frac{t}{\mathbb{Q}^2} \left[\beta^{n-1} \frac{n}{(n-1)!} T_1^{++(n-1)}(\alpha) - \beta^{n+1} \frac{1}{n!} T_1^{++(n+1)}(\alpha) \right], \end{aligned} \quad (93)$$

and for odd n , $h_n^{++} = 0$.

These coefficients together with those for the magnetic CFF can be used to compute those of $\mathcal{E}^{++}$. For $\mathcal{G}^{++}$ and even $n \geq 2$,

$$\begin{aligned} g_n^{++} = & \int_{\Omega} d\beta d\alpha N(\beta, \alpha, t) 2 \left[\beta^{n-1} \left\{ \frac{T_0^{++(n-1)}(\alpha, t/\mathbb{Q}^2)}{(n-1)!} - \frac{t}{\mathbb{Q}^2} \frac{n+2}{2 \cdot (n-2)!} T_1^{++(n-1)}(\alpha) \right\} \right. \\ & \left. - \beta^{n+1} \frac{M^2 - t/4}{\mathbb{Q}^2} \frac{2n}{n!} T_1^{++(n+1)}(\alpha) \right], \end{aligned} \quad (94)$$

and for odd n , $g_n^{++} = 0$ as usual.

The dispersion relation connecting $D(\alpha)$ to h_0^{++} suffers the same issue as in the scalar case, with an additional complication: it also mixes GPDs H and E through DDs F and K . Indeed, on top of GPD H as in the scalar case, GPD E also contributes to the dispersion relation. This triggers a new challenge, as E is poorly known today, adding uncertainties on the extraction of D .

C. Comparison with BMMP results

In this section, we compare the results of Eq. (91) with the pioneering one obtained by Braun *et al.* [47]. First, let us recall the expression for the subtraction constant obtained in [47]:

$$\begin{aligned} h_0^{++} = & 2 \int_0^1 \frac{du}{1-u} \left(1 - \frac{t}{Q^2} (1 - 2 \ln u) \right) \varphi_D(u) \\ & - 4 \int_0^1 d\xi \xi \int_{\xi}^1 \frac{dx}{x} t_2(x) \\ & \times \left[\frac{4M^2}{Q^2} H^{(+)}(\xi/x, \xi, t) + \frac{t}{Q^2} E^{(+)}(\xi/x, \xi, t) \right], \end{aligned} \quad (95)$$

where

$$t_2(x) = \frac{\ln(\frac{1+x}{2x})}{1-x} - \frac{1}{2(1+x)}, \quad (96)$$

and

$$\varphi_D(u) = D(2u - 1). \quad (97)$$

The first line of Eq. (95) can be related to our result by making the change of variable $u \rightarrow \frac{1+\alpha}{2}$ and antisymmetrizing the coefficient function. The second line is more challenging. Since the structure in terms of powers of M^2/Q^2 and t/Q^2 is the same as in our case⁵ we will only focus on $H^{(+)}$. Since

⁵We remind the reader that the difference between choosing Q^2 or $\mathbb{Q}^2$ is a twist-six effect.

$$\begin{aligned}
H^{(+)}(x, \xi, t) &= \int_{\Omega} d\beta d\alpha \delta(x - \beta - \alpha\xi) \\
&\quad \times (F(\beta, \alpha, t) - F(-\beta, \alpha, t)) \\
&= \int_{\Omega} d\beta d\alpha \delta(x - \beta - \alpha\xi) F^{(+)}(\beta, \alpha, t), \quad (98)
\end{aligned}$$

we can inject this DD representation into the $H^{(+)}$ integral of (95):

$$\begin{aligned}
I_2 &= 4 \int_0^1 d\xi \xi \int_{\xi}^1 \frac{dx}{x} t_2(x) H^{(+)}(\xi/x, \xi, t) \\
&= 4 \int_{\Omega} d\beta d\alpha F^{(+)}(\beta, \alpha, t) \\
&\quad \times \int_0^1 d\xi \xi \int_{\xi}^1 \frac{dx}{x} t_2(x) \delta\left(\frac{\xi}{x} - \beta - \alpha\xi\right). \quad (99)
\end{aligned}$$

We then perform the change of variable $y = \xi/x$ and integrate the Dirac delta over y :

$$\begin{aligned}
I_2 &= 4 \int_{\Omega} d\beta d\alpha F^{(+)}(\beta, \alpha) \int_0^1 d\xi \frac{\xi}{\beta + \alpha\xi} t_2\left(\frac{\xi}{\beta + \alpha\xi}\right) \\
&\quad \times \Theta(\xi \leq \beta + \xi\alpha \leq 1) \quad (100)
\end{aligned}$$

where Θ is nonzero only for $\xi \leq \beta + \xi\alpha \leq 1$. Because of the DD support Ω , the upper bound does not provide additional constraint. Since $\xi \geq 0$, the lower bound can be understood as $0 \leq \xi \leq \beta/(1 - \alpha)$ and $\beta \geq 0$. Reinjecting this condition in Eq. (100), we obtain

$$I_2 = 4 \int_{\Omega} d\beta d\alpha F^{(+)}(\beta, \alpha) \Theta(\beta) \int_0^{\frac{\beta}{1-\alpha}} \frac{d\xi \xi}{\beta + \alpha\xi} t_2\left(\frac{\xi}{\beta + \alpha\xi}\right). \quad (101)$$

Now performing the change of variable $\xi = \beta z$, we are left with

$$\begin{aligned}
I_2 &= 4 \int_{\Omega} d\beta d\alpha F^{(+)}(\beta, \alpha) \Theta(\beta) \beta \int_0^{\frac{1}{1-\alpha}} \frac{dz z}{1 + \alpha z} t_2\left(\frac{z}{1 + \alpha z}\right) \\
&= \int_{\Omega} d\beta d\alpha \beta F(\beta, \alpha) 4 \int_0^{\frac{1}{1-\alpha}} dz \frac{z}{1 + \alpha z} t_2\left(\frac{z}{1 + \alpha z}\right), \quad (102)
\end{aligned}$$

where in the second line we have exploited the definition of $F^{(+)}$ to remove the Θ function and come back to F . Introducing now $G(\alpha)$,

$$G(\alpha) = \int_0^{\frac{1}{1-\alpha}} dz \frac{z}{1 + \alpha z} t_2\left(\frac{z}{1 + \alpha z}\right), \quad (103)$$

and symmetrizing it,

$$G^{(-)}(\alpha) = G(\alpha) + G(-\alpha), \quad (104)$$

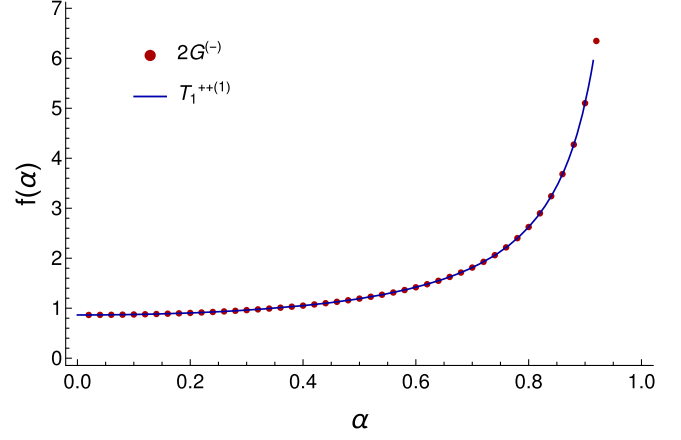


FIG. 3. Comparison between the present result $T_1^{++(1)}$ and the first result $2G^{(-)}$ obtained in Ref. [47].

we are left with

$$I_2 = \int_{\Omega} d\beta d\alpha \beta F(\beta, \alpha) 2G^{(-)}(\alpha), \quad (105)$$

an analytic structure in agreement with our result in Eq. (91). The remaining task is to prove that

$$2G^{(-)}(\alpha) = T_1^{++(1)}(\alpha). \quad (106)$$

We provide here a numerical comparison on Fig. 3. This equality is verified with a maximal relative error of 10^{-13} reached when $\alpha \rightarrow 1$, i.e., when approaching the divergence. We thus conclude that our new derivation is in agreement with the one obtained previously in Ref. [47].

Finally, let us mention that our sufficient kinematic condition, $Q^2 \geq 2M^2 - t$, is not mentioned in the proof of Ref. [47]. Note that in our case, this condition is only sufficient, not necessary, to guarantee that on a segment of the real axis, the amplitude is real. Relaxing it would require one to analyze the possible production threshold in the Mandelstam plane.

V. IMPACT ON THE DECONVOLUTION PROBLEM

In this section, we assume that h_0^{++} is known experimentally and that F and K have already been extracted. The question we ask is whether the modifications of the coefficient function are sufficient to allow one to deconvolute two (or more) Gegenbauer modes of the D -term.

For both the spin-0 and 1/2 cases, the integral containing the D -term is

$$\begin{aligned}
\mathcal{D}(t) &= \int_{-1}^1 d\alpha T_2^{++}(\alpha, t/Q^2) D(\alpha, t) \\
&= \int_{-1}^1 d\alpha \left[T_0^{++}(\alpha, t/Q^2) + \frac{t}{Q^2} T_1^{++}(\alpha) \right] D(\alpha, t). \quad (107)
\end{aligned}$$

With Eqs. (A1), (A9), (A10), and (46), we can express the above term in the square brackets as

$$T_2^{++}(\alpha, t/\mathbb{Q}^2) = C_{\text{LT}}^{(+)}(\alpha) + \frac{t}{2\mathbb{Q}^2} [\tilde{\mathbb{P}}_{(\text{iii})}^{(+)}(\alpha) - C_0^{(+)}(\alpha)] \quad (108)$$

$$\stackrel{\text{LO}}{=} \left(1 - \frac{t}{2\mathbb{Q}^2}\right) C_0^{(+)}(\alpha) + \frac{t}{2\mathbb{Q}^2} \tilde{\mathbb{P}}_{(\text{iii})}^{(+)}(\alpha). \quad (109)$$

To solve the integral (107) with the LO kernel (109), we choose the traditional Gegenbauer parametrization of the D -term that for quarks reads

$$D(\alpha, t) = (1 - \alpha^2) \sum_{n=0}^{\infty} d_{2n+1}(t) \mathcal{C}_{2n+1}^{(3/2)}(\alpha). \quad (110)$$

Here, $\mathcal{C}_{2n+1}^{(3/2)}(\alpha)$ is a Gegenbauer polynomial of degree $2n+1$ in α .

A. First integral

The first part of the convolution involves

$$J = \left(1 - \frac{t}{2\mathbb{Q}^2}\right) \times \sum_{n=0}^{\infty} d_{2n+1}(t) \int_{-1}^1 d\alpha C_0^{(+)}(\alpha) (1 - \alpha^2) \mathcal{C}_{2n+1}^{(3/2)}(\alpha), \quad (111)$$

which is the same integral as in the pure LO-LT case. Thus the result is already well known and given as

$$J = 4 \left(1 - \frac{t}{2\mathbb{Q}^2}\right) \sum_{n=0}^{\infty} d_{2n+1}(t). \quad (112)$$

B. Second integral

The second part of Eq. (107) is given by

$$\begin{aligned} & \int_{-1}^1 d\alpha \tilde{\mathbb{P}}_{(\text{iii})}^{(+)}(\alpha) D(\alpha, t) \\ &= \int_{-1}^1 d\alpha \left[-\frac{2}{1+\alpha} \ln\left(\frac{1-\alpha}{2}\right) + \frac{2}{1-\alpha} \ln\left(\frac{1+\alpha}{2}\right) \right] D(\alpha, t) \\ &= \int_{-1}^1 d\alpha \frac{4}{1-\alpha} \ln\left(\frac{1+\alpha}{2}\right) D(\alpha, t) \\ &= 4 \sum_{n=0}^{\infty} d_{2n+1}(t) \int_{-1}^1 d\alpha (1+\alpha) \ln\left(\frac{1+\alpha}{2}\right) \mathcal{C}_{2n+1}^{(3/2)}(\alpha). \end{aligned} \quad (113)$$

The integral with respect to α on the right-hand side,

$$I_N = \int_{-1}^1 d\alpha (1+\alpha) \ln\left(\frac{1+\alpha}{2}\right) \mathcal{C}_N^{(3/2)}(\alpha), \quad (114)$$

can be solved for any degree N of the Gegenbauer polynomial by making use of property (B6):

$$\begin{aligned} \sum_{N=0}^{\infty} \tau^N I_N &= \int_{-1}^1 d\alpha (1+\alpha) \ln\left(\frac{1+\alpha}{2}\right) \frac{1}{(1-2\tau\alpha+\tau^2)^{3/2}} \\ &= \frac{1}{\tau^2} \left[-\sqrt{1-2\tau\alpha+\tau^2} + \frac{1+\tau+\tau^2-\tau\alpha}{\sqrt{1-2\tau\alpha+\tau^2}} \ln\left(\frac{1+\alpha}{2}\right) \right. \\ &\quad \left. + (1+\tau) \left\{ \ln\left(1 + \frac{\sqrt{1-2\tau\alpha+\tau^2}}{1+\tau}\right) - \ln\left(1 - \frac{\sqrt{1-2\tau\alpha+\tau^2}}{1+\tau}\right) \right\} \right] \Big|_{\alpha \rightarrow -1}^{\alpha \rightarrow +1} \\ &= \frac{1}{\tau^2} \left[2\tau + (1+\tau) \left\{ \ln 2 - \ln 0^+ + \ln\left(\frac{2}{1+\tau}\right) - \ln 2 - \ln\left(\frac{2\tau}{1+\tau}\right) + \mathbf{L} \right\} \right], \end{aligned} \quad (115)$$

where $\ln 0^+ = \lim_{\varepsilon \rightarrow 0^+} \ln \varepsilon$ and

$$\mathbf{L} = \lim_{\alpha \rightarrow -1^+} \ln\left(1 - \frac{\sqrt{1-2\tau\alpha+\tau^2}}{1+\tau}\right). \quad (116)$$

Introducing $\alpha = -1 + \varepsilon$, $\varepsilon > 0$, then

$$\begin{aligned} \mathbf{L} &= \lim_{\varepsilon \rightarrow 0^+} \ln\left(\frac{1+\tau-\sqrt{1+2\tau+\tau^2-2\tau\varepsilon}}{1+\tau}\right) = -\ln(1+\tau) + \lim_{\varepsilon \rightarrow 0^+} \ln\left(1+\tau-\sqrt{(1+\tau)^2-2\tau\varepsilon}\right) \\ &= -\ln(1+\tau) + \lim_{\varepsilon \rightarrow 0^+} \ln\left(1+\tau-(1+\tau)\sqrt{1-\frac{2\tau\varepsilon}{(1+\tau)^2}}\right). \end{aligned} \quad (117)$$

The function $2\tau/(1+\tau)^2$ is monotonic increasing for $\tau \in [0, 1]$ taking values in $[0, 1/2]$ with the maximum located at $\tau = 1$. Then, we can safely consider $2\tau\epsilon/(1+\tau)^2 < 1$ and expand the above square root in Taylor series:

$$\begin{aligned} \mathbf{L} &= -\ln(1+\tau) + \lim_{\epsilon \rightarrow 0^+} \ln\left(\frac{\tau\epsilon}{1+\tau}\right) \\ &= -2\ln(1+\tau) + \ln\tau + \ln 0^+. \end{aligned} \quad (118)$$

Going back to Eq. (115),

$$\begin{aligned} \sum_{N=0}^{\infty} \tau^N I_N &= \frac{2}{\tau^2} [\tau - (1+\tau) \ln(1+\tau)] \\ &= \frac{2}{\tau^2} \left[\tau - (1+\tau) \sum_{n=0}^{\infty} (-1)^n \frac{\tau^{n+1}}{n+1} \right] \\ &= \frac{2}{\tau^2} \left[\tau - (1+\tau) \left(\tau + \sum_{n=1}^{\infty} (-1)^n \frac{\tau^{n+1}}{n+1} \right) \right] \\ &= -2 \left[1 - \sum_{n=0}^{\infty} (-1)^n \frac{\tau^n}{n+2} - \sum_{n=0}^{\infty} (-1)^n \frac{\tau^{n+1}}{n+2} \right] \\ &= -1 - 2 \sum_{n=1}^{\infty} \tau^n \frac{(-1)^n}{(n+1)(n+2)} \\ &= \sum_{n=0}^{\infty} \tau^n \frac{(-1)^{n+1} 2}{(n+2)(n+1)}. \end{aligned} \quad (119)$$

As the equality holds for all $|\tau| \leq 1$, we conclude

$$\begin{aligned} I_N &= \int_{-1}^1 d\alpha (1+\alpha) \ln\left(\frac{1+\alpha}{2}\right) \mathcal{C}_N^{(3/2)}(\alpha) \\ &= \frac{(-1)^{N+1} 2}{(N+2)(N+1)}, \end{aligned} \quad (120)$$

from where it follows that

$$\int_{-1}^1 d\alpha \tilde{\mathbb{P}}_{(iii)}^{(+)}(\alpha) D(\alpha, t) = \sum_{n=0}^{\infty} d_{2n+1}(t) \frac{4}{(2n+3)(n+1)}. \quad (121)$$

C. Full integral with the D -term

With the results from Eqs. (112) and (121), we can finally write

$$\begin{aligned} \mathcal{D}(t) &\stackrel{\text{LO}}{=} 4 \sum_{n=0}^{\infty} d_{2n+1}(t) - \frac{2t}{\mathbb{Q}^2} \sum_{n=0}^{\infty} d_{2n+1}(t) \frac{(2+n)(1+2n)}{(1+n)(3+2n)} \\ &\stackrel{\text{LO}}{=} 4 \sum_{n=0}^{\infty} d_{2n+1}(t) \left[1 - \frac{t}{2\mathbb{Q}^2} \left(1 - \frac{1}{(2n+3)(n+1)} \right) \right]. \end{aligned} \quad (122)$$

Contrary to the pure LT case, there is an n dependence introduced in the description of the subtraction constant in terms of Gegenbauer modes of order n . However, these coefficient converge quadratically to 1, and thus we cannot expect to distinguish the behavior beyond the very first modes. The $t/\mathbb{Q}^2$ dependence becomes degenerate.

D. Shadow contributions to the D -term

At LO in α_s we have found for DVCS that the integral containing the D -term can be analytically computed by a Gegenbauer parametrization and it takes the form of Eq. (122). Taking into account that $d_{2n+1}(t)$ is shorthand for $d_{2n+1}(t; \mu^2)$ with μ^2 the energy scale, then a term that produces a vanishing $\mathcal{D}(t; \mu^2)$ at a certain energy scale μ^2 and a certain ratio $t/\mathbb{Q}^2$ is referred to as a *shadow D -term* [7].

Assuming dominance by the first two Gegenbauer modes [$d_n(t) = 0, \forall n > 3$], and omitting the dependence on μ^2 , a shadow D -term is given by

$$\mathcal{D}^{\text{sh}}(t) = 0 = d_1^{\text{sh}}(t) \left[4 - \frac{4}{3} \frac{t}{\mathbb{Q}^2} \right] + d_3^{\text{sh}}(t) \left[4 - \frac{9}{5} \frac{t}{\mathbb{Q}^2} \right]. \quad (123)$$

At LO and LT, a shadow D -term is manifest through the condition $d_1^{\text{sh}}(t) = -d_3^{\text{sh}}(t)$. For a nonzero but fixed ratio $|t|/\mathbb{Q}^2 < 1$, we find

$$d_1^{\text{sh}}(t) = -d_3^{\text{sh}}(t) \frac{1 - \frac{9}{20} \frac{t}{\mathbb{Q}^2}}{1 - \frac{1}{3} \frac{t}{\mathbb{Q}^2}}. \quad (124)$$

In order to determine the impact of the kinematic higher-twist corrections on the deconvolution problem, we consider the difference:

$$\begin{aligned} \frac{1 - \frac{9}{20} \frac{t}{\mathbb{Q}^2}}{1 - \frac{1}{3} \frac{t}{\mathbb{Q}^2}} - 1 &= \left(1 - \frac{9}{20} \frac{t}{\mathbb{Q}^2} \right) \left(1 + \frac{1}{3} \frac{t}{\mathbb{Q}^2} + O(|t|^2/\mathbb{Q}^4) \right) - 1 \\ &= -\frac{7}{60} \frac{t}{\mathbb{Q}^2} + O(|t|^2/\mathbb{Q}^4) \\ &\simeq 0.12 \left| \frac{t}{\mathbb{Q}^2} \right| + O(|t|^2/\mathbb{Q}^4). \end{aligned} \quad (125)$$

This difference represents the modification on the LO + LT shadow D -term [$d_1^{\text{sh}}(t) = -d_3^{\text{sh}}(t)$] due to the kinematic power corrections. We find that said modification is of order $\sim 10\%$ of a twist-four, rendering the effect of the $t/\mathbb{Q}^2$ corrections on the deconvolution problem probably not better than the one obtained through evolution [7]. An improvement on the extraction of the D -term by including these effects should not be expected, at least if the extraction is performed from the dispersion relation associated to $\mathcal{H}^{++}$ as in Ref. [7]. This motivates the study of the dispersion relation of the other CFFs ($\mathcal{H}^{+-}$, $\mathcal{H}^{0+}$) as for those there is no LT component that could obscure the

kinematic corrections. From Eq. (124), we deduce that their effect on purely higher-twist components should be of the order of

$$\frac{9/20}{1/3} = 27/20 = 1.35 \Rightarrow d_1^{\text{sh}}(t) \sim 1.35 d_3^{\text{sh}}(t). \quad (126)$$

This is an estimated 35% difference between the first two Gegenbauer modes allowing us to study the deconvolution problem and the relation between the different modes. The computation of the dispersion relations associated to the CFFs $\mathcal{H}^{+-}$ and $\mathcal{H}^{0+}$ and the subsequent extraction of the D -term will be taken care of in a future publication.

VI. NUMERICAL EVALUATION

In this section, we will assess the impact of kinematic higher-twist corrections on the description of the subtraction constant of DVCS. To do so, we will combine the experimental extraction of d_1 [see Eq. (110)] obtained in Ref. [7], with the Goloskokov-Kroll (GK) GPD model [73–76]. Note that this choice is not fully consistent, in the sense that the GK model has been extracted assuming explicitly that the D -term vanishes. Moreover, since the extractions performed both for d_1 and the GK model were performed assuming only leading-twist contributions, one should expect that some higher-twist effects might taint the values extracted from data. Nevertheless, this procedure will provide an order of magnitude of the impact of kinematic-power corrections. For this analysis, we used the numerical implementation of the GK model found in PARTONS [77] and use the Apfel++ library [78–81] to perform the leading order evolution from 4 GeV^2 to 2 GeV^2 .

A. Model of the D -term

The D -term extracted in Ref. [7] was based on a Gegenbauer decomposition, following Eq. (110), and thus only the $d_{2n+1}(t)$ coefficients have to be extracted. Note that the t dependence is modeled by

$$d_{2n+1}(t) = \frac{d_{2n+1}(0)}{\left(1 - \frac{t}{\Lambda_{2n+1}^2}\right)^3}, \quad (127)$$

where the power-law behavior is fixed and not extracted. In practice, because of the deconvolution problem of the D -term already mentioned, only $d_1(t)$ is extracted with a large uncertainty. We assume all higher Gegenbauer modes to be zero. Reference [7] reports $d_1(0) = -0.7 \pm 1.3$. In the following we will stick to the central value of -0.7 in order to obtain an order of magnitude of the next-to-leading power corrections impact. For completeness, we highlight that $\Lambda_1 = 0.8 \text{ GeV}$.

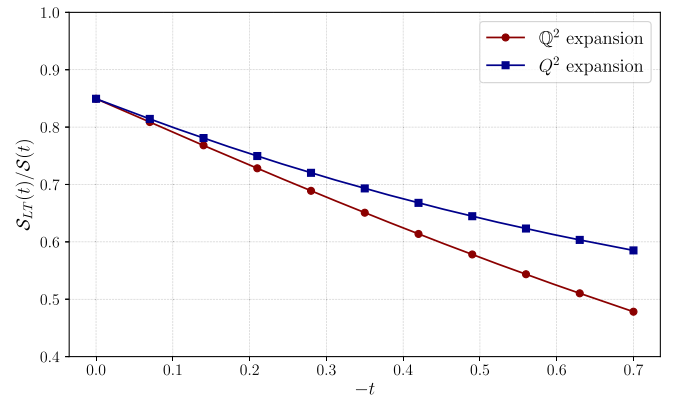


FIG. 4. Ratio S_{LT}/S as a function of t for $Q^2 = 2 \text{ GeV}^2$. In red, S is expanded as a power of $1/Q^2$, while in blue it is in terms of a power of $1/Q^2$. The difference is expected to be twist-six but it is already visible for $|t|$ as small as 0.3 GeV^2 . The dots and squares are the actual computed points; the line is only an interpolation used as a guide.

B. Ratio computation

The DVCS subtraction constant S is obtained by summing the different flavor contributions, weighted by their electric charge:

$$S(t, Q^2) = \sum_q e_q^2 h_0^{+,+,q}. \quad (128)$$

Using Eq. (122), one can readily get the pure leading-twist contribution:

$$S_{LT}(t, Q^2) \approx 4 \sum_q e_q^2 d_1^q(t, Q^2) \approx 4 d_1(t, Q^2) \sum_q e_q^2, \quad (129)$$

where we have used the fact that a flavor-degenerate value of d_1 was extracted in Ref. [7]. Note however that, for computing the part corresponding to the DD F and K , we have kept the flavor dependence modeled by Goloskokov and Kroll.

The ratio of the pure leading-twist subtraction constant to the total one is shown on Fig. 4. This figure highlights the important impact of the higher-twist corrections, going from 15% to 50% on the kinematic range selected. We also illustrate the impact of choosing Q^2 or Q^2 as the typical power-expansion scale. Above $|t| > 0.3 \text{ GeV}^2$ the difference becomes noticeable highlighting the potential need for a twist-six correction.

VII. CONCLUSION

In this work, we have generalized the dispersion relations of DVCS to include higher-twist kinematic power corrections, both for spin-0 and spin-1/2 targets. The results are twofold. First, we prove that the expression for the n -subtracted dispersion relations is the same as at leading twist [see Eq. (44)]. This follows from the fact that the

imaginary part of the CFF is generated solely by the DGLAP region of the GPD—a result that was, at least to us, unexpected. The second important point is the modification of the coefficient function; the latter being itself connected to GPDs and DDs. These modifications are such that an additional term is introduced and comes with a dependence in the full double distributions F and K , and cannot be considered suppressed, especially for JLab kinematics. It calls into question the common thought that DVCS dispersion relations allow one to bypass the extraction of GPDs to get access to the pressure and shear forces of quarks within the nucleon. This issue is discussed in a preceding article [82]. Note that our results for the one-time subtraction constant for the spin-1/2 target (91) expressed in terms of double distribution is in agreement with a previous proof obtained in Ref. [47] in terms of GPDs and recalled here in Eq. (95). We also derived the kinematic twist-four corrections to higher subtracted relations in Eqs. (57) and (93) for spin-0 and spin-1/2, respectively. Finally, Eq. (122) provides the first calculation of the higher-twist convolution with the Polyakov-Weiss D -term.

That being written, assuming that the additional term is taken into account by some procedure, we also investigated the impact of the kinematic corrections to the deconvolution problem of the D -term. To do so, we stay to leading order in α_s , two Gegenbauer modes, and studied how the shadow D -term is impacted by the t/Q^2 corrections. If the coefficient becomes indeed dependent of the mode n considered, this dependence is suppressed quadratically in n , which precludes any help for deconvoluting modes beyond the first few ones. And if one is restricted to the helicity conserving amplitudes, then this mode dependence is a small perturbation of the mode independent, leading-twist part. So that, if the situation may improve with respect to the pure LT, we do not expect a significant effect compared to the one provided by evolution (which is itself already small).

This leads to the considerations of future work. On the one hand, with a large contribution from the twist-four nucleon mass correction at JLab kinematics, we wonder what happens at kinematic twist-six. On the other hand, we believe that studying the photon helicity flip amplitude, $\mathcal{H}^{+-}$, would be of great interest. Indeed, at leading order, there is no pure leading-twist contribution, which may allow one to disentangle between the two first Gegenbauer modes. However, one would also need to assess the impact of gluon “transversity” GPDs, that contribute to $\mathcal{H}^{+-}$ at NLO.

With all these points in mind, we believe that more theoretical and phenomenological work is necessary before we can extract reliable distributions of pressure and shear forces within the nucleon from experimental data.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: COEFFICIENT FUNCTIONS

1. Leading-twist coefficient function

We start by providing the leading-twist coefficient function for the helicity conserving amplitude taken from Ref. [83] up to NLO in α_s :

$$C_0(x/\xi) = \frac{-1}{x/\xi - 1 + i0}, \quad (\text{A1})$$

$$C_1(x/\xi) = \frac{\alpha_s C_F}{4\pi} \frac{1}{x/\xi + 1 - i0} \left[9 - 3 \frac{x+\xi}{x-\xi} \ln \left(\frac{x+\xi}{2\xi} - i0 \right) - \ln^2 \left(\frac{x+\xi}{2\xi} - i0 \right) \right], \quad (\text{A2})$$

$$C_{\text{coll}}(x/\xi) = \frac{\alpha_s C_F}{4\pi} \frac{1}{x/\xi + 1 - i0} \left[-3 - 2 \ln \left(\frac{x+\xi}{2\xi} - i0 \right) \right]. \quad (\text{A3})$$

Defining the antisymmetric coefficients

$$C_0^{(+)}(x/\xi) = C_0(x/\xi) - (x/\xi \rightarrow -x/\xi), \quad (\text{A4})$$

$$C_1^{(+)}(x/\xi) = C_1(x/\xi) - (x/\xi \rightarrow -x/\xi), \quad (\text{A5})$$

$$C_{\text{coll}}^{(+)}(x/\xi) = C_{\text{coll}}(x/\xi) - (x/\xi \rightarrow -x/\xi), \quad (\text{A6})$$

they combine to give the full LT coefficient function

$$C_{\text{LT}}^{(+)}\left(\frac{x}{\xi}\right) = \left[C_0^{(+)}\left(\frac{x}{\xi}\right) + C_1^{(+)}\left(\frac{x}{\xi}\right) + \ln\left(\frac{Q^2}{\mu_F^2}\right) C_{\text{coll}}^{(+)}\left(\frac{x}{\xi}\right) \right] + O(\alpha_s^2). \quad (\text{A7})$$

2. Twist-four scalar coefficient function for $\mathcal{H}^{++}$

To kinematic twist-four accuracy, the CFF $\mathcal{H}^{++}$ reads⁶ [50,51,65]

$$\begin{aligned} \mathcal{H}^{++} = & \int_{-1}^1 dx \frac{1}{\xi} \left\{ \left(1 - \frac{t}{2Q^2} \right) C_0^{(+)}(x/\xi) H \right. \\ & + \frac{t}{Q^2} \left[\tilde{\mathbb{P}}_{(\text{iii})}^{(+)}(x/\xi) - \frac{\mathcal{L}^{(+)}(x/\xi)}{2} \right] H \\ & - \frac{t}{2Q^2} \xi \partial_\xi [(\tilde{\mathbb{P}}_{(\text{iii})}^{(+)}(x/\xi) - \mathcal{L}^{(+)}(x/\xi)) H] \\ & + \frac{\xi^2 \bar{p}_1^2}{Q^2} \xi^2 \partial_\xi^2 [(\tilde{\mathbb{P}}_{(\text{iii})}^{(+)}(x/\xi) - \mathcal{L}^{(+)}(x/\xi)) H] \left. \right\} \\ & + O(\alpha_s, \text{tw} - 6, \alpha_s \cdot \text{tw} - 4) \end{aligned} \quad (\text{A8})$$

where we truncated C_{LT} to order $(\alpha_s)^0$ following Eq. (A7). Here, we introduced the notation

$$\tilde{\mathbb{P}}_{(\text{iii})}^{(+)}(x/\xi) = \tilde{\mathbb{P}}_{(\text{iii})}(x/\xi) - (x/\xi \rightarrow -x/\xi), \quad (\text{A9})$$

$$\mathcal{L}^{(+)}(x/\xi) = \mathcal{L}(x/\xi) - (x/\xi \rightarrow -x/\xi), \quad (\text{A10})$$

upon

$$\tilde{\mathbb{P}}_{(\text{iii})}(x/\xi) = \frac{-2}{x/\xi + 1} \ln \left(\frac{x/\xi - 1 + i0}{-2 + i0} \right), \quad (\text{A11})$$

$$\mathcal{L}(x/\xi) = \frac{4}{x/\xi - 1} \left[\text{Li}_2 \left(\frac{x/\xi + 1}{2 - i0} \right) - \text{Li}_2(1) \right]. \quad (\text{A12})$$

3. Twist-4 spin-1/2 coefficient function for $\mathcal{H}^{++}$ and for $\mathcal{E}^{++}$

The hard coefficient functions of a spin-1/2 particle at LO and up to kinematic twist-four accuracy were detailed in Eq. (62); see main text. All in all, these kernels provide an alternative but equivalent formulation to the spin-0 basis of functions for computing the kinematic corrections. However, one must notice that the final convolutions giving rise to the $\mathcal{H}^{++}$ of spin-0 and spin-1/2 targets are not fully equivalent, viz. Eq. (81). This can be understood by the works of Belitsky and Müller [44] and Braun and Manashov [84] where it is manifest that the kinematic twist corrections stem from higher-order diagrams involving gluon exchanges

between the active quark in the scattering process and the spectator structure of the hadron. In fact, it is only at leading twist that all those exchanges can be fully resummed into the well-known Wilson links that guarantee the gauge-invariant properties of GPDs.

Conversely, NLO corrections in perturbation theory consist of loop diagrams involving self-energy and quark-photon vertex corrections that carry information on the spin and nature of said active quark and not on the characteristics of the hadron from where this parton has been originated. Consequently, at NLO-LT accuracy, there is no difference in the hard kernel C_{LT} between targets of different spins.

APPENDIX B: GEGENBAUER POLYNOMIALS

The Gegenbauer polynomials $\mathcal{C}_n^{(\lambda)}(x)$ are a specific case of Jacobi polynomials of degree n , such that they are orthogonal for a certain weight of the type $(1 - x^2)^{\lambda - \frac{1}{2}}$. For instance, considering $\lambda = \frac{3}{2}$, one gets the first terms as

$$\mathcal{C}_1^{(3/2)}(\alpha) = 3\alpha, \quad (\text{B1})$$

$$\mathcal{C}_3^{(3/2)}(\alpha) = \frac{5}{2}\alpha(7\alpha^2 - 3), \quad (\text{B2})$$

and an orthogonality condition provided by

$$\begin{aligned} & \int_{-1}^1 d\alpha (1 - \alpha^2) \mathcal{C}_n^{(3/2)}(\alpha) \mathcal{C}_m^{(3/2)}(\alpha) \\ &= \delta_{n,m} \frac{\pi \Gamma(n+3)}{n! 4(n+3/2) [\Gamma(3/2)]^2} \\ &= \delta_{n,m} \frac{2(n+3)(n+2)(n+1)}{2n+3}. \end{aligned} \quad (\text{B3})$$

These polynomials satisfy two properties that will be of use later on:

$$\mathcal{C}_N^{(\lambda)}(-\alpha) = (-1)^N \mathcal{C}_N^{(\lambda)}(\alpha), \quad (\text{B4})$$

$$\begin{aligned} 2(n+\lambda) \mathcal{C}_n^{(\lambda)}(\alpha) &= \frac{d}{d\alpha} [\mathcal{C}_{n+1}^{(\lambda)}(\alpha) - \mathcal{C}_{n-1}^{(\lambda)}(\alpha)] \\ &= 2\lambda [\mathcal{C}_n^{(\lambda+1)}(\alpha) - \mathcal{C}_{n-2}^{(\lambda+1)}(\alpha)], \end{aligned} \quad (\text{B5})$$

$$\sum_{N=0}^{\infty} \mathcal{C}_N^{(\lambda)}(\alpha) \tau^N = \frac{1}{(1 - 2\alpha\tau + \tau^2)^\lambda}, \quad |\tau| \leq 1. \quad (\text{B6})$$

⁶ $\mathcal{A}_{[51]}^{++} = \mathcal{H}^{++}$ at zeroth-order in α_s .

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