

Black hole interior operators and dilatation symmetry in planar black branes

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Planar anti-de Sitter (AdS) black branes have a scaling symmetry that maps a brane solution at one temperature to a solution at another. It is natural to expect that boundary representations of bulk field modes should inherit this symmetry, i.e., their correlators should transform covariantly under boundary dilatations. We derive a covariance condition that any boundary representation of interior modes in a planar AdS black brane should satisfy. We then show that Papadodimas-Raju mirror operators satisfy this condition. Thus the Papadodimas-Raju reconstruction of the bulk interior, although state-dependent, inherits the scaling symmetry of planar AdS black holes.

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I. INTRODUCTION

Planar AdS black branes possess a scaling symmetry: the simultaneous rescaling

$$r \rightarrow r/\lambda, \quad t \rightarrow \lambda t, \quad \vec{x} \rightarrow \lambda \vec{x}$$

leaves the metric invariant up to a rescaling of the horizon radius $r_h \rightarrow r_h/\lambda$, and hence of the Hawking temperature $T \rightarrow T/\lambda$ [1]. Since this rescaling acts on the boundary coordinates as $(t, \vec{x}) \rightarrow (\lambda t, \lambda \vec{x})$, it corresponds to a dilatation of the boundary conformal field theory (CFT) that lives on $\mathbb{R}^{d,1}$. A well-known consequence is that physical observables—transport coefficients and quasinormal mode frequencies—depend only on scaling-covariant dimensionless ratios [2–8]. This symmetry is absent for spherical black holes, whose boundary duals are thermal states that do not enjoy dilatation invariance.

It is natural to ask whether this symmetry extends to bulk reconstruction. For exterior reconstruction, the HKLL map [9–14] provides a state-independent construction of bulk fields outside the horizon, and covariance of exterior operators under dilatations follows in a relatively straightforward manner (see [15–18] for explicit reconstructions of exterior operators). The interior is more subtle. Both the mirror operator construction of Papadodimas and Raju (PR) [19–22] and the more recent nonisometric encoding proposal [23–28] are state-dependent, and it is therefore a

nontrivial question whether interior operators can be consistently defined so as to transform covariantly under boundary dilatations at all.

In this paper we address this question. We first show that demanding covariance of boundary correlators under dilatations constrains interior operators to satisfy a precise transformation law, either in the strong form

$$U_\lambda \tilde{\mathcal{O}}_{\omega, \vec{k}}^{(\Psi_T)} U_\lambda^\dagger = \lambda^{\Delta-d-1} \tilde{\mathcal{O}}_{\omega/\lambda, \vec{k}/\lambda}^{(\Psi_{T/\lambda})} \text{ on } \mathcal{H}_{\text{code}}(T/\lambda),$$

or a weaker projected version thereof. We then verify that the PR mirror operators satisfy the stronger condition. This shows that the PR interior reconstruction is fully consistent with the scaling symmetry of planar AdS black branes. The nonisometric proposal does not provide a concrete prescription that can be checked against our criterion. We argue that in this case, there should be an additional scaling condition purely in the bulk.

II. DILATATION SYMMETRY AND ITS ACTION ON THE CODE SUBSPACE

In this section, we spell out the action of dilatations at three levels: on Fourier modes of primary operators, on black brane microstates, and on the small algebra and its associated code subspace.

The dual to a $d+2$ -dimensional planar AdS black brane is a CFT on $\mathbb{R}^{d,1}$. This CFT has a dilatation operator D and corresponding unitary $U_\lambda = e^{i\alpha D}$ with $\lambda = e^\alpha$. Its action on the boundary coordinates is given by

$$(t, \vec{x}) \mapsto (\lambda t, \lambda \vec{x}), \quad (1)$$

and on a primary operator of dimension Δ

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$$U_\lambda \mathcal{O}(t, \vec{x}) U_\lambda^\dagger = \lambda^\Delta \mathcal{O}(\lambda t, \lambda \vec{x}). \quad (2)$$

The Hamiltonian H satisfies $[D, H] = -iH$, so

$$U_\lambda H U_\lambda^\dagger = \lambda H. \quad (3)$$

a. Action on Fourier modes: The Fourier mode of a primary of dimension Δ is given by

$$\mathcal{O}_{\omega, \vec{k}} = \int d^d x dt e^{i\omega t - i\vec{k} \cdot \vec{x}} \mathcal{O}(t, \vec{x})$$

Following (2), this transforms as

$$U_\lambda \mathcal{O}_{\omega, \vec{k}} U_\lambda^\dagger = \lambda^{\Delta-d-1} \mathcal{O}_{\omega/\lambda, \vec{k}/\lambda} \quad (4)$$

b. Action on black hole: First, let us recall the scaling symmetry on the bulk side. The metric of a planar AdS_{d+2} Schwarzschild black brane can be written as

$$ds^2 = -r^2 f(r) dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 d\vec{x}^2, \quad f(r) = 1 - \left(\frac{r_h}{r}\right)^{d+1}. \quad (5)$$

The metric is invariant under the scaling transformation

$$r \rightarrow r/\lambda, \quad t \rightarrow \lambda t, \quad \vec{x} \rightarrow \lambda \vec{x}, \quad r_h \rightarrow r_h/\lambda. \quad (6)$$

So the metric is mapped to the same functional form, but with horizon radius

$$r_h \rightarrow r_h/\lambda, \quad (7)$$

As the Hawking temperature is proportional to the horizon radius,

$$T = \frac{(d+1)r_h}{4\pi}. \quad (8)$$

this implies a change of temperature

$$T \rightarrow T/\lambda. \quad (9)$$

c. Action on black-brane equilibrium states. To discuss normalizable microstates and entropy, we temporarily introduce an infrared regulator by placing the CFT in a spatial box of linear size L . Let $\mathcal{H}_L$ denote the corresponding Hilbert space. A black-brane equilibrium microstate at temperature T is taken from a narrow band $\mathcal{B}_{T,L}$ centered on the thermodynamic energy $E_0(T, L)$:

$$|\Psi_{T,L}\rangle = \sum_{E \in \mathcal{B}_{T,L}} \sum_a C_{E,a} |E, a; L\rangle, \quad \sum_{E,a} |C_{E,a}|^2 = 1. \quad (10)$$

Here a labels the degeneracy at fixed energy. We do not assume that the individual coefficients $C_{E,a}$ have a Boltzmann form.

The property required below is that $|\Psi_{T,L}\rangle$ is an equilibrium state for the chosen small set of observables. Namely, for every normalized $A \in \mathcal{A}_{\text{small}}(T, L)$,

$$\langle \Psi_{T,L} | A | \Psi_{T,L} \rangle = \langle A \rangle_{\beta,L} + \delta_A, \quad \delta_A = O(1/N) + O(e^{-S(T,L)/2}), \quad (11)$$

where $\langle A \rangle_{\beta,L}$ is the thermal expectation value at $\beta = 1/T$. The estimate is understood for observables whose thermal correlators have been normalized to be $O(1)$; it expresses the working accuracy of bulk reconstruction and the typical state-to-state fluctuations within the band.

On the infinite plane the dilation is implemented by U_λ . With the finite-volume regulator it should be regarded as a unitary identification

$$U_{\lambda,L} : \mathcal{H}_L \longrightarrow \mathcal{H}_{\lambda L}, \quad U_{\lambda,L} H_L U_{\lambda,L}^\dagger = \lambda H_{\lambda L}. \quad (12)$$

Thus an energy eigenstate is carried to an eigenstate of energy E/λ in $\mathcal{H}_{\lambda L}$. Up to a unitary change of basis within degenerate energy eigenspaces,

$$U_{\lambda,L} |E, a; L\rangle = \sum_b V_{ba}(E) |E/\lambda, b; \lambda L\rangle. \quad (13)$$

Define

$$|\Psi_{T/\lambda, \lambda L}\rangle := U_{\lambda,L} |\Psi_{T,L}\rangle. \quad (14)$$

For

$$A' = U_{\lambda,L} A U_{\lambda,L}^\dagger \in \mathcal{A}_{\text{small}}(T/\lambda, \lambda L), \quad (15)$$

unitarity gives

$$\langle \Psi_{T/\lambda, \lambda L} | A' | \Psi_{T/\lambda, \lambda L} \rangle = \langle \Psi_{T,L} | A | \Psi_{T,L} \rangle. \quad (16)$$

Thermal correlators transform in the same way, with $\beta' = \lambda\beta$. Hence the deviations δ_A are transported with the same parametric suppression: the dilation introduces no new error. It therefore maps the class of equilibrium microstates at (T, L) onto the class at $(T/\lambda, \lambda L)$. We suppress the regulator label L below.

d. Action on code subspace/small algebra: Given a black brane microstate $|\Psi_T\rangle$ at temperature T , the small algebra is defined as

$$\mathcal{A}_{\text{small}}(T; \omega_*, k_*, K, E_*) := \text{span} \left\{ \mathbf{1}, \mathcal{O}_{\omega_1, \vec{k}_1} \cdots \mathcal{O}_{\omega_n, \vec{k}_n} : \right. \\ \left. n \leq K, |\omega_i| < \omega_*, |\vec{k}_i| < k_*, \sum_i |\omega_i| \leq E_* \right\}. \quad (17)$$

We take this set to be closed under Hermitian conjugation. The cutoffs K and E_* are chosen so that its dimension is much smaller than that of the black-brane Hilbert space. The associated state-dependent code subspace is

$$\mathcal{H}_{\text{code}}(T) = \mathcal{A}_{\text{small}}(T)|\Psi_T\rangle. \quad (18)$$

The exterior modes used to generate this space are state-independent, whereas $\mathcal{H}_{\text{code}}(T)$ itself depends on the reference state $|\Psi_T\rangle$.

From (4), the dilatation U_λ maps $\mathcal{O}_{\omega,\vec{k}} \mapsto \lambda^{\Delta-d-1} \mathcal{O}_{\omega/\lambda,\vec{k}/\lambda}$. The image of the algebra $\mathcal{A}(T, \omega_*, k_*)$ under conjugation by U_λ is therefore

$$\begin{aligned} U_\lambda \mathcal{A}_{\text{small}}(T; \omega_*, k_*, K, E_*) U_\lambda^\dagger \\ = \mathcal{A}_{\text{small}}(T/\lambda; \omega_*/\lambda, k_*/\lambda, K, E_*/\lambda). \end{aligned} \quad (19)$$

The ratio

$$q := \left(\frac{\omega}{T}, \frac{\vec{k}}{T} \right) \quad (20)$$

remains constant under scaling. Thus each q defines an orbit of dilatation.

Now that we have established how exterior operators transform under dilatations, we proceed to ask what the analogous transformation law must be for operators representing fields in the black brane interior.

III. CORRELATOR COVARIANCE FOR INTERIOR OPERATORS

For correlators of operators representing the exterior, covariance under dilatations is immediate from (4). Operators inside the horizon, however, do not admit a state-independent prescription. We assume only that a boundary representation $\tilde{\mathcal{O}}_{\omega,\vec{k}}^{(\Psi_T)}$ of interior modes exists: a family of CFT operators, possibly state-dependent, whose correlators within the code subspace $\mathcal{H}_{\text{code}}(T)$ reproduce the bulk EFT correlators of the corresponding interior mode on the black-brane background at temperature T , to the standard $O(1/N)$ accuracy of bulk reconstruction. This defining property fixes every matrix element of $\tilde{\mathcal{O}}_{\omega,\vec{k}}^{(\Psi_T)}$ between code-subspace states, and hence fixes the projected operator $P_{\text{code}}(T) \tilde{\mathcal{O}}_{\omega,\vec{k}}^{(\Psi_T)} P_{\text{code}}(T)$ uniquely for any admissible construction, where $P_{\text{code}}(T)$ denotes the orthogonal projector onto $\mathcal{H}_{\text{code}}(T)$. In what follows we denote exterior representations by $\mathcal{O}$ and interior ones by $\tilde{\mathcal{O}}$ [29].

Covariance of correlators with interior insertions is then not an independent assumption; it follows from the bulk scaling map. The transformation (6) maps the black-brane solution at temperature T exactly onto the solution at T/λ , acting on the *entire* spacetime, interior included. Bulk EFT correlation functions on the two backgrounds are therefore

identified by this map, with an interior mode of boundary quantum numbers $(\omega, \vec{k})$ on the T background carried to the mode $(\omega/\lambda, \vec{k}/\lambda)$ on the T/λ background. The labels and normalization of the interior partner mode are fixed by the same boundary normalization as the corresponding exterior mode. In particular, writing

$$a := \Delta - d - 1, \quad (21)$$

the bulk scaling map carries the normalized partner mode according to

$$\tilde{\mathcal{O}}_{\omega,\vec{k}}^{\text{bulk},(T)} \mapsto \lambda^a \tilde{\mathcal{O}}_{\omega/\lambda,\vec{k}/\lambda}^{\text{bulk},(T/\lambda)}. \quad (22)$$

Note that this is a statement about the bulk mode whose boundary representation is being reconstructed, rather than an assumption about that boundary representation. The associated weight is the same $\lambda^{\Delta-d-1}$ as for exterior modes in (4): in the two-sided description an interior free-field mode is a linear combination of left and right boundary modes whose coefficients depend only on the dimensionless ratios $(\omega/T, \vec{k}/T)$, each boundary mode carrying weight $\lambda^{\Delta-d-1}$ under the diagonal dilatation, and a single-sided representation must reproduce the same correlators. For example, in the two-sided description the interior field has the schematic mode expansion

$$\begin{aligned} \phi_{\text{int}}^{(T)} \sim \int d\omega d^d k \left[f^{(1)} \left(\frac{\omega}{T}, \frac{\vec{k}}{T}, \frac{r}{T} \right) \mathcal{O}_{\omega,\vec{k}} \right. \\ \left. + f^{(2)} \left(\frac{\omega}{T}, \frac{\vec{k}}{T}, \frac{r}{T} \right) \tilde{\mathcal{O}}_{\omega,\vec{k}} + \text{H.c.} \right]. \end{aligned} \quad (23)$$

The coefficient functions depend only on scaling-invariant arguments, so the exterior and partner modes carry the same Fourier-mode weight.

Meanwhile, Sec. II established that U_λ transports the microstate, the small algebra, and hence the code subspace covariantly: $|\Psi_T\rangle \mapsto |\Psi_{T/\lambda}\rangle$, $\mathcal{A}(T, \omega_*) \mapsto \mathcal{A}(T/\lambda, \omega_*/\lambda)$, and $\mathcal{H}_{\text{code}}(T) \mapsto \mathcal{H}_{\text{code}}(T/\lambda)$. Combining these statements, the code-subspace correlators that an admissible representation at T/λ must reproduce are precisely the U_λ -transforms of those at T . For correlators with a single interior insertion this reads

$$\begin{aligned} \langle \Psi_{T/\lambda} | (A'_L)^\dagger \tilde{\mathcal{O}}_{\omega/\lambda,\vec{k}/\lambda}^{(\Psi_{T/\lambda})} A'_R | \Psi_{T/\lambda} \rangle \\ = \lambda^{-(\Delta-d-1)} \langle \Psi_T | (A_L)^\dagger \tilde{\mathcal{O}}_{\omega,\vec{k}}^{(\Psi_T)} A_R | \Psi_T \rangle, \quad A'_{L,R} := U_\lambda A_{L,R} U_\lambda^\dagger, \end{aligned} \quad (24)$$

for all $A_L, A_R \in \mathcal{A}(T, \omega_*)$. Equation (24) is thus a necessary condition on any boundary representation of interior modes, state-dependent or not. We now recast it as an operator statement. Define the difference operator

$$D_{\omega, \vec{k}} := U_\lambda \tilde{\mathcal{O}}_{\omega, \vec{k}}^{(\Psi_T)} U_\lambda^\dagger - \lambda^{\Delta-d-1} \tilde{\mathcal{O}}_{\omega/\lambda, \vec{k}/\lambda}^{(\Psi_{T/\lambda})}. \quad (25)$$

Using $U_\lambda |\Psi_T\rangle = |\Psi_{T/\lambda}\rangle$, the correlator condition becomes

$$\langle \Psi_{T/\lambda} | (A'_L)^\dagger D_{\omega, \vec{k}} A'_R | \Psi_{T/\lambda} \rangle = 0 \quad (26)$$

for all A'_L, A'_R in the transformed small operator set.

Because the set is closed under Hermitian conjugation, vectors of the form $A'_R |\Psi_{T/\lambda}\rangle$ span $\mathcal{H}_{\text{code}}(T/\lambda)$ and the corresponding bras span its dual. Therefore

$$\langle \phi | D_{\omega, \vec{k}} | \chi \rangle = 0, \quad |\phi\rangle, |\chi\rangle \in \mathcal{H}_{\text{code}}(T/\lambda), \quad (27)$$

or equivalently

$$P_{\text{code}}(T/\lambda) D_{\omega, \vec{k}} P_{\text{code}}(T/\lambda) = 0. \quad (28)$$

Substituting the definition of $D_{\omega, \vec{k}}$, we find that the interior operator must transform covariantly after projection to the code subspace:

$$P_{\text{code}}(T/\lambda) U_\lambda \tilde{\mathcal{O}}_{\omega, \vec{k}}^{(\Psi_T)} U_\lambda^\dagger P_{\text{code}}(T/\lambda) = \lambda^{\Delta-d-1} P_{\text{code}}(T/\lambda) \tilde{\mathcal{O}}_{\omega/\lambda, \vec{k}/\lambda}^{(\Psi_{T/\lambda})} P_{\text{code}}(T/\lambda). \quad (29)$$

In addition, if both

$$U_\lambda \tilde{\mathcal{O}}_{\omega, \vec{k}}^{(\Psi_T)} U_\lambda^\dagger \quad \text{and} \quad \tilde{\mathcal{O}}_{\omega/\lambda, \vec{k}/\lambda}^{(\Psi_{T/\lambda})}$$

preserve $\mathcal{H}_{\text{code}}(T/\lambda)$, then

$$U_\lambda \tilde{\mathcal{O}}_{\omega, \vec{k}}^{(\Psi_T)} U_\lambda^\dagger = \lambda^{\Delta-d-1} \tilde{\mathcal{O}}_{\omega/\lambda, \vec{k}/\lambda}^{(\Psi_{T/\lambda})} \quad \text{on } \mathcal{H}_{\text{code}}(T/\lambda) \quad (30)$$

Equation (29) is the central constraint of our analysis. Given the spanning property of the code subspace, it is in fact equivalent to the correlator condition (24): the content of this section is to convert the covariance of code-subspace correlators, derived above from the bulk scaling map and the defining accuracy of bulk reconstruction, into an operator criterion against which concrete constructions can be tested. Any boundary representation of interior bulk fields, regardless of the specific construction, must satisfy (29) to $O(1/N)$; since the defining property fixes the projected operator uniquely, (29) states that this unique projection transforms covariantly. If the interior operators additionally preserve the code subspace—as the PR mirrors do up to edge effects at the cutoff ω_* , which itself scales covariantly by (19)—the stronger condition (30) holds. In the next section we verify that the PR construction satisfies (30) directly, without invoking (24) at any point.

For the truncated PR operator set it is useful to distinguish an inner code subspace from the full cutoff space. For example, let $\mathcal{H}_{\text{code}}^{\text{in}}(T)$ be generated by monomials of degree at most $K-1$ and total excitation energy at most $E_* - \omega_*$. Acting with one additional exterior or mirror mode then maps

$$\mathcal{H}_{\text{code}}^{\text{in}}(T) \longrightarrow \mathcal{H}_{\text{code}}(T). \quad (31)$$

Statements that an operator “preserves the code subspace” are understood in this nested sense when a sharp truncation is used.

IV. ACTION ON MIRROR OPERATORS

In this section, we show that the PR mirror operators satisfy (30). We give two proofs. The first uses the explicit definition of the mirror operator directly, while the second uses the Tomita-Takesaki modular theory and shows that the result is a consequence of the algebraic properties of the construction rather than specific details of the state.

The PR definition of mirror operator is as follows. Given the small algebra $\mathcal{A}$ and the state $|\Psi_T\rangle$, the code subspace/little Hilbert space is $\mathcal{H}_{\text{code}} = \mathcal{A}|\Psi_T\rangle$. The mirror operator $\tilde{\mathcal{O}}_{\omega, \vec{k}}$ is defined by its action on $\mathcal{H}_{\text{code}}$: for all $A \in \mathcal{A}$,

$$\tilde{\mathcal{O}}_{\omega, \vec{k}}(A|\Psi\rangle) = A e^{-\beta\omega/2} \mathcal{O}_{\omega, \vec{k}}^\dagger |\Psi\rangle. \quad (32)$$

Now we check that mirror operators transform covariantly under dilatations:

$$U_\lambda [A \cdot e^{-\beta\omega/2} \mathcal{O}_{\omega, \vec{k}}^\dagger |\Psi_T\rangle] = e^{-\beta\omega/2} A' \cdot \lambda^{\Delta-d-1} \mathcal{O}_{\omega/\lambda, \vec{k}/\lambda}^\dagger |\Psi_{T/\lambda}\rangle$$

where $A' = U_\lambda A U_\lambda^\dagger$ and $|\Psi_{T/\lambda}\rangle = U_\lambda |\Psi_T\rangle$.

So

$$(U_\lambda \tilde{\mathcal{O}}_{\omega, \vec{k}} U_\lambda^\dagger)(A' |\Psi_{T/\lambda}\rangle) = A' \cdot \lambda^{\Delta-d-1} e^{-\beta\omega/2} \mathcal{O}_{\omega/\lambda, \vec{k}/\lambda}^\dagger |\Psi_{T/\lambda}\rangle.$$

Now compare with what the correct mirror operator formula demands at temperature $T' = T/\lambda$ ($\beta' = \lambda\beta$) and, $\omega' = \omega/\lambda$ and momentum $\vec{k}' = \vec{k}/\lambda$:

$$\begin{aligned}\tilde{\mathcal{O}}_{\omega/\lambda, \vec{k}/\lambda}^{(\Psi_{T/\lambda})}(A'|\Psi_{T/\lambda}) &= A' \cdot e^{-\beta'\omega'/2} \mathcal{O}_{\omega', \vec{k}'}^\dagger |\Psi_{T/\lambda}\rangle \\ &= A' \cdot e^{-(\beta\lambda)(\omega/(2\lambda))} \mathcal{O}_{\omega/\lambda, \vec{k}/\lambda}^\dagger |\Psi_{T/\lambda}\rangle = A' \cdot e^{-\beta\omega/2} \mathcal{O}_{\omega/\lambda, \vec{k}/\lambda}^\dagger |\Psi_{T/\lambda}\rangle.\end{aligned}$$

This gives us

$$U_\lambda \tilde{\mathcal{O}}_{\omega, \vec{k}} U_\lambda^\dagger = \lambda^{\Delta-d-1} \tilde{\mathcal{O}}_{\omega/\lambda, \vec{k}/\lambda}^{(\Psi_{T/\lambda})} \quad \text{on } \mathcal{H}_{\text{code}}(T/\lambda). \quad (33)$$

Thus the transformed mirror has the same defining action as the mirror associated with $|\Psi_{T/\lambda}\rangle$, on the transformed code subspace.

We now give a complementary algebraic explanation of the covariance property using Tomita-Takesaki theory. The relation between PR mirror operators and Tomita-Takesaki modular conjugation is detailed in [19]. We use this relation here to give a second proof of covariance. For an adjoint-closed algebra $\mathfrak{A}$ with a cyclic and separating state $|\Psi\rangle$, the identification

$$\tilde{\mathcal{O}}_{\omega, \vec{k}}^{(\Psi)} = J_\Psi \mathcal{O}_{\omega, \vec{k}} J_\Psi \quad (34)$$

is an exact statement on the Hilbert space generated by $\mathfrak{A}|\Psi\rangle$. Here J_Ψ is the modular conjugation associated to the state $|\Psi\rangle$ and the algebra $\mathcal{A}$.

In the PR application, the small operator set is truncated and only approximately closed. Accordingly, its modular description, and the equalities derived from it below, are understood as statements about the action of the operators within the code subspace, to the working large- N accuracy and away from cutoff-edge effects. They do not specify an extension of the mirror operators to the full CFT Hilbert space. The direct proof above does not require these additional modular-theory assumptions.

The Tomita operator S_Ψ is the antilinear operator defined by

$$S_\Psi(A|\Psi) = A^\dagger|\Psi\rangle, \quad A \in \mathfrak{A}. \quad (35)$$

First, we show that the Tomita operator transforms covariantly. We conjugate both sides by U_λ . On the left-hand side,

$$\begin{aligned}U_\lambda S_\Psi U_\lambda^\dagger (U_\lambda A|\Psi) &= U_\lambda S_\Psi (A|\Psi) = U_\lambda A^\dagger|\Psi\rangle \\ &= (U_\lambda A^\dagger U_\lambda^\dagger) |\Psi_{T/\lambda}\rangle = (A')^\dagger |\Psi_{T/\lambda}\rangle, \quad (36)\end{aligned}$$

where $A' := U_\lambda A U_\lambda^\dagger \in \mathcal{A}(T/\lambda)$ and we used $U_\lambda |\Psi_T\rangle = |\Psi_{T/\lambda}\rangle$. Since vectors of the form $A'|\Psi_{T/\lambda}\rangle$ span $\mathcal{H}_{\text{code}}(T/\lambda)$, and since the right-hand side equals $S_{\Psi_{T/\lambda}}(A'|\Psi_{T/\lambda})$, we conclude

$$U_\lambda S_\Psi U_\lambda^\dagger = S_{\Psi_{T/\lambda}}. \quad (37)$$

Next, we show that the modular conjugation transforms covariantly. The polar decomposition of the Tomita

operator is $S_\Psi = J_\Psi \Delta_\Psi^{1/2}$, where J_Ψ is antiunitary and $\Delta_\Psi^{1/2}$ is positive. Since U_λ is unitary, conjugation by U_λ preserves this decomposition. Applying $U_\lambda(\cdot)U_\lambda^\dagger$ to both sides of $S_{\Psi_T} = J_{\Psi_T} \Delta_{\Psi_T}^{1/2}$ and using (37) gives

$$S_{\Psi_{T/\lambda}} = (U_\lambda J_{\Psi_T} U_\lambda^\dagger) (U_\lambda \Delta_{\Psi_T}^{1/2} U_\lambda^\dagger). \quad (38)$$

By uniqueness of the polar decomposition, we can identify the two factors separately:

$$J_{\Psi_{T/\lambda}} = U_\lambda J_{\Psi_T} U_\lambda^\dagger, \quad \Delta_{\Psi_{T/\lambda}}^{1/2} = U_\lambda \Delta_{\Psi_T}^{1/2} U_\lambda^\dagger. \quad (39)$$

Finally, using (4) and (39) we have

$$\begin{aligned}U_\lambda \tilde{\mathcal{O}}_{\omega, \vec{k}}^{(\Psi_T)} U_\lambda^\dagger &= U_\lambda J_{\Psi_T} \mathcal{O}_{\omega, \vec{k}} J_{\Psi_T} U_\lambda^\dagger \\ &= J_{\Psi_{T/\lambda}} (U_\lambda \mathcal{O}_{\omega, \vec{k}} U_\lambda^\dagger) J_{\Psi_{T/\lambda}} \\ &= J_{\Psi_{T/\lambda}} (\lambda^{\Delta-d-1} \mathcal{O}_{\omega/\lambda, \vec{k}/\lambda}) J_{\Psi_{T/\lambda}} \\ &= \lambda^{\Delta-d-1} \tilde{\mathcal{O}}_{\omega/\lambda, \vec{k}/\lambda}^{(\Psi_{T/\lambda})}.\end{aligned}$$

This agrees with (33). The action of dilatations on the interior reconstruction data can therefore be summarized as a map

$$\begin{aligned}R_\lambda: (\mathcal{A}(T, \omega_*, k_*), |\Psi_T\rangle, \tilde{\mathcal{A}}^{(\Psi_T)}(T, \omega_*, k_*)) \\ \mapsto (\mathcal{A}(T/\lambda, \omega_*/\lambda, k_*/\lambda), |\Psi_{T/\lambda}\rangle, \tilde{\mathcal{A}}^{(\Psi_{T/\lambda})}(T/\lambda, \omega_*/\lambda, k_*/\lambda)).\end{aligned}$$

Here $\tilde{\mathcal{A}}^{(\Psi_T)}$ denotes the mirror algebra associated with the state $|\Psi_T\rangle$. The corresponding generators transform as

$$\mathcal{O}_{\omega, \vec{k}} \mapsto \lambda^{\Delta-d-1} \mathcal{O}_{\omega/\lambda, \vec{k}/\lambda}, \quad \tilde{\mathcal{O}}_{\omega, \vec{k}}^{(\Psi_T)} \mapsto \lambda^{\Delta-d-1} \tilde{\mathcal{O}}_{\omega/\lambda, \vec{k}/\lambda}^{(\Psi_{T/\lambda})}.$$

Thus the entire reconstruction triple transforms covariantly under the planar black brane scaling symmetry.

An interesting aside about covariant transformation of operators. If we take an operator string built from small-algebra operators and mirror operators,

$$\mathcal{X}_{\{\omega_i, \vec{k}_i\}} = X_{\omega_1, \vec{k}_1} \overrightarrow{X_{\omega_2, \vec{k}_2}} \cdots X_{\omega_n, \vec{k}_n} \overrightarrow{},$$

where each $X_{\omega_i, \vec{k}_i} \overrightarrow{}$ is either a small-algebra mode $\mathcal{O}_{\omega_i, \vec{k}_i} \overrightarrow{}$ or a mirror mode $\tilde{\mathcal{O}}_{\omega_i, \vec{k}_i} \overrightarrow{}$ with conformal weight Δ_i .

As discussed prior, it follows that

$$\langle \Psi_{T/\lambda} | \mathcal{X}_{\{\omega_i/\lambda, \vec{k}_i/\lambda\}} | \Psi_{T/\lambda} \rangle = \lambda^{-\sum_i (\Delta_i - d - 1)} \langle \Psi_T | \mathcal{X}_{\{\omega_i, \vec{k}_i\}} | \Psi_T \rangle.$$

Defining $\hat{X}_{\omega, \vec{k}}^{(T)} = \beta^{\Delta-d-1} X_{\omega, \vec{k}}^{(T)}$ for either an exterior or a mirror mode, the corresponding dimensionless correlators

depend only on $q_i = (\omega_i/T, \vec{k}_i/T)$ and are invariant along a dilatation orbit.

Clearly the PR reconstruction for planar black branes, though state dependent, inherits the scaling symmetry of AdS black branes.

V. DISCUSSION

In this paper we used the bulk scaling map, together with the defining accuracy of a boundary reconstruction, to establish covariance of code-subspace correlators containing one interior insertion. We then showed that this correlator statement is equivalent to a projected operator criterion on the boundary representation of the interior mode. Accordingly, every admissible representation that reproduces the scale-related bulk-EFT matrix elements must satisfy the projected criterion to the same accuracy.

For the PR construction we proved the stronger transformation law on the code subspace directly from the defining action of the mirror operators, and independently from the general correlator argument. Thus the state dependence of the PR reconstruction is compatible with the planar black-brane scaling symmetry.

Another case of interest is the nonisometric picture, which addresses the apparent overcounting of interior states by noting that exponentially many distinct bulk configurations are virtually indistinguishable to simple boundary observers. This picture suggests that the encoding of the black hole interior Hilbert space $\mathcal{H}_{\text{interior}}$ into $\mathcal{H}_{\text{CFT}}$ is a composition of two maps. One first quotients by a space of null states, which are invisible to simple probes. The quotient Hilbert space $\mathcal{H}_{\text{code}}$ is then mapped to $\mathcal{H}_{\text{CFT}}$ via an approximately isometric embedding. From the boundary point of view, the covariance condition (29) should still hold, since it is a necessary condition on any CFT

representation of interior operators, regardless of whether the underlying encoding is isometric or nonisometric. However, there is a further natural condition at the level of the bulk description: the bulk scaling S_λ induced by (6) should preserve the null subspace itself. This is because null states are defined by indistinguishability under simple probes. As simple probes transform covariantly, the partition of $\mathcal{H}_{\text{interior}}$ into detectable and null sectors should itself be covariant—null states at temperature T should be mapped to null states at temperature T/λ . Since null states are expected to be high-complexity states, S_λ should map high(low) complexity states to high(low) complexity states, with any complexity threshold formulated in terms of scaling-covariant dimensionless variables.

This additional requirement would not be visible directly in the boundary CFT, since the CFT only sees the encoded code subspace and not the null-state quotient itself; rather, it would be a structural property of the bulk scaling map S_λ in the nonisometric framework. Tensor network or qubit models of planar black branes would provide a concrete setting in which to test whether the covariance of the null-state partition under bulk scaling can be made precise, and whether complexity thresholds respect the dilatation symmetry.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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