

Next-to-leading order QCD corrections to electromagnetic production and decay of fully charm tetraquarks

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We investigate the electromagnetic properties of the fully charm tetraquark states, particularly incorporating contributions from internal gluon radiation. The paper first presents analytical expressions for the next-to-leading-order QCD corrections to the decay amplitudes of fully charm tetraquarks into two photons. It is found that the QCD corrections are significant for the $J^{PC} = 0^{++}$ and $J^{PC} = 2^{++}$ fully charm tetraquark decay process. Subsequently, by considering photon-photon fusion in ultraperipheral high-energy collisions of protons and nuclei and in electron-positron collision, we provide theoretical predictions for the production cross sections of fully charm tetraquark states. The results presented in this work regarding the electromagnetic production and decay of fully charm tetraquarks shall be tested in current and future experiments.

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I. INTRODUCTION

It is well known that the standard quark model postulated by Gell-Mann and Zweig provided an excellent explanation for the hadron spectrum in the past century, such as the light meson nonet, light baryon octet, and decuplet, all of which are successful examples. However, in 2003, with the discovery of exotic hadronic states such as $X(3872)$ [1,2], the situation changed. Up to now, a new-generation quark model that can neatly accommodate all these new hadronic states and explain all the production and decay properties has not yet been established.

In 2020, the LHCb Collaboration first reported a peak around 6900 MeV in the J/ψ -pair invariant mass distribution [3], which is considered the first candidate for a fully charm tetraquark state. Three years later, the 6900 MeV peak was confirmed by both the ATLAS and CMS collaborations [4,5]. In the same year, a possible peak with mass 6267 ± 43 MeV and width 121 ± 72 MeV was observed in the Belle experiment [6]. Very recently, the 6900 MeV peak and two other peaks at 6600 MeV and 7100 MeV in the J/ψ -pair channel and their spin-parity quantum numbers were determined by the CMS Collaboration using a data sample

corresponding to an integrated luminosity of 135 fb^{-1} at a center-of-mass energy of 13 TeV [7]. Unlike the previous channel, the 6900 MeV peak has also been observed recently in the $J/\psi - \psi(2S)$ invariant mass distribution by the ATLAS Collaboration [8].

Correctly understanding the structure and properties of these exotic hadron states presents a significant theoretical challenge, as it involves the nonperturbative interactions of QCD that are difficult to handle. The pioneering idea of the possibility of four charm quark states near the J/ψ -pair threshold dates back to the 1970s–1980s [9,10]. In the new century, these exotic peaks in the J/ψ pair spectrum have been studied using the tetraquark picture from the quark potential model [11–21], diquark-antidiquark model [22–24], gluonic tetraquark hybrid model [25–27], tetraquarks in QCD sum rules [28–39], tetraquarks in lattice QCD calculations [40–42], tetraquark-molecule mixing model [43], tetraquarks in Bethe-Salpeter equations [44–46], and dynamically generated resonance poles [47–51]. The extension to other fully heavy tetraquark systems is studied in Refs. [52–55]. The production of fully charm tetraquarks is studied in hadron-hadron colliders [23,56–62]. Therein, the first complete next-to-leading order QCD calculation for the fully charm tetraquark states' hadron production is given in Ref. [58]. The quark and gluon to fully charm tetraquark fragmentation functions are studied in Refs. [63–65]. The decay properties of fully charm tetraquarks are studied in Refs. [66–73]. A short review paper on the fully charm tetraquarks can be found in Ref. [74].

In this paper, we investigate the interaction strength between photons and fully charm tetraquark states.

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Although their couplings are certainly weaker compared to processes involving gluons, such interactions can provide new observables that will contribute to a deeper understanding of exotic states in the J/ψ pair spectrum. On the one hand, the decay of fully charm tetraquark states into two photons offers a new decay channel distinct from J/ψ pair hadronic final states. Using photon final states allows for a clearer view of the internal structural features of fully charm tetraquark states. On the other hand, ultraperipheral collisions (UPCs), which are reactions where two ions interact via their cloud of virtual photons, provide a novel production mechanism and offer new experimental detection opportunities. Related studies of exotic hadrons in UPCs can also be found in Refs. [75,76].

Since fully charm tetraquark states are composed entirely of heavy quarks, we can employ the nonrelativistic QCD (NRQCD) effective theory to separate the high- and low-energy scale interactions in their production and decay processes, thereby enabling factorized calculations for these processes [77]. The rest of this paper is organized as follows. In Sec. II, we present the formula for the electromagnetic decays of fully charm tetraquarks. In Sec. III, the next-to-leading order calculation of short-distance coefficients is obtained. In Sec. IV, the $\gamma + \gamma \rightarrow T_{4c}$ photoproduction processes are studied. The cross sections of UPCs for different heavy-ion systems and electron-positron collisions are also given. We will discuss the phenomenological numerical results in Sec. V. The conclusion is given at the end.

II. NRQCD FACTORIZATION FORMULA FOR FULLY CHARM TETRAQUARK ELECTROMAGNETIC DECAYS

The NRQCD factorization formula for $T_{4c} \rightarrow \gamma\gamma$ at the lowest order in velocity can be expressed as [66]

$$\begin{aligned} \Gamma(T_{4c}^{0^{++}} \rightarrow \gamma\gamma) &= \frac{m_H}{128m_c^4} (c_1 \langle \mathcal{O}_{6\otimes 6}^{(0)} \rangle + c_2 \langle \mathcal{O}_{3\otimes 3}^{(0)} \rangle \\ &\quad + c_{\text{mix}} \langle \mathcal{O}_{\text{mix}}^{(0)} \rangle), \\ \Gamma(T_{4c}^{2^{++}} \rightarrow \gamma\gamma) &= \frac{m_H}{128m_c^4} c_3 \langle \mathcal{O}_{3\otimes 3}^{(2)} \rangle, \end{aligned} \quad (1)$$

where the superscript in T_{4c} denotes the spin-parity quantum number J^{PC} of the T_{4c} and m_c and m_H are the masses of the charm quark and T_{4c} , respectively. The coefficients $\{c_i\}$ are the short distance coefficients (SDCs), which can be perturbatively calculated order by order. And the long distance matrix elements (LDMEs) are defined as

$$\begin{aligned} \langle \mathcal{O}_{6\otimes 6}^{(0)} \rangle &= |\langle 0 | \mathcal{O}_{6\otimes 6}^{(0)} | T_{4c} \rangle|^2, \\ \langle \mathcal{O}_{3\otimes 3}^{(J)} \rangle &= |\langle 0 | \mathcal{O}_{3\otimes 3}^{(J)} | T_{4c} \rangle|^2, \\ \langle \mathcal{O}_{\text{mix}}^{(0)} \rangle &= \text{Re}[\langle 0 | \mathcal{O}_{3\otimes 3}^{(0)} | T_{4c} \rangle \langle T_{4c} | \mathcal{O}_{6\otimes 6}^{(0)\dagger} | 0 \rangle], \end{aligned} \quad (2)$$

where $J = 0, 2$. The above operators are constructed in the diquark-antidiquark configurations. In this assignment, the S-wave spin-singlet diquark is a color singlet, while the S-wave spin-triplet diquark is a color triplet [78–80]. Explicitly, these local operators read

$$\begin{aligned} \mathcal{O}_{6\otimes 6}^{(0)} &= [\psi_a^T(i\sigma^2)\psi_b][\chi_c^\dagger(i\sigma^2)\chi_d^*]C_{6\otimes 6}^{ab;cd}, \\ \mathcal{O}_{3\otimes 3}^{(0)} &= -\frac{1}{\sqrt{3}}[\psi_a^T(i\sigma^2)\sigma^i\psi_b][\chi_c^\dagger(i\sigma^2)\chi_d^*]C_{3\otimes 3}^{ab;cd}, \\ \mathcal{O}_{3\otimes 3}^{(2)} &= [\psi_a^T(i\sigma^2)\sigma^m\psi_b][\chi_c^\dagger\sigma^n(i\sigma^2)\chi_d^*]C_{3\otimes 3}^{ab;cd} \\ &\quad \times \epsilon_H^{ij*}\Gamma^{ij;mn}, \end{aligned} \quad (3)$$

where ψ and χ are two-dimensional Pauli spinor fields, σ^i is the Pauli matrix, and ϵ_H^{ij} is the polarization tensor of the T_{4c} . And the symmetric traceless color tensor is

$$\Gamma^{ij;mn} = \delta^{im}\delta^{jn} + \delta^{in}\delta^{jm} - \frac{2}{3}\delta^{ij}\delta^{mn}. \quad (4)$$

The color projection tensors are

$$\begin{aligned} C_{6\otimes 6}^{ab;cd} &= \frac{1}{2\sqrt{6}}(\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}), \\ C_{3\otimes 3}^{ab;cd} &= \frac{1}{2\sqrt{3}}(\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc}). \end{aligned} \quad (5)$$

III. NLO QCD CORRECTIONS TO SDCs

With the factorization formula, the nonperturbative long-distance effects are absorbed into the LDMEs. Then, the remaining SDCs c_i can be perturbatively expanded in terms of the strong coupling constant α_s ,

$$c_i = \sum_{j=0}^{\infty} \left(\frac{\alpha_s}{\pi} \right)^j c_{i,j}. \quad (6)$$

To calculate the coefficient $c_{i,j}$, we first generate the amplitudes involving four free charm quarks $c\bar{c}c\bar{c}$, then project the four free charm quarks onto the targeted state with certain total angular momentum and parity. We use the spin operator Π_0 and Π_1 to obtain the spin-0 and spin-1 diquarks, respectively. And the color projectors $C_{6\otimes 6}^{ab;cd}$ and $C_{3\otimes 3}^{ab;cd}$ are used to extract the color-sextet and color-antitriplet of the diquark, respectively. For the spin-singlet diquark contribution, the state of $c(p_1)\bar{c}(p_2)c(p_3)\bar{c}(p_4)$ is projected onto $J^{PC} = 0^{++}$ by the replacements

$$\begin{aligned} \bar{u}(p_1)X_1v(p_3)\bar{u}(p_2)X_2v(p_4) &\rightarrow -\text{Tr}[\Pi_0 X_2 \Pi_0 X_1^C]C_{6\otimes 6}^{ab;cd}, \\ \bar{u}(p_1)X_1v(p_4)\bar{u}(p_2)X_2v(p_3) &\rightarrow \text{Tr}[\Pi_0 X_2 \Pi_0^C X_1^C]C_{6\otimes 6}^{ab;cd}, \end{aligned} \quad (7)$$

where the subscript C stands for the charge conjugate, and the spin-singlet projector is [81,82]

$$\Pi_0 = \frac{(\not{P} + m_c)\gamma_5}{\sqrt{2}}. \quad (8)$$

Here, P is the momentum of the T_{4c} , and the on-shell condition $P^2 = m_H^2$ is satisfied. In the lowest order of velocity, the relative velocity between quarks is zero; then, we have

$$p_1 = p_2 = p_3 = p_4 = \frac{P}{4}. \quad (9)$$

The spin-triplet diquark contribution can be obtained by the replacements

$$\begin{aligned} \bar{u}(p_1)X_1v(p_3)\bar{u}(p_2)X_2v(p_4) &\rightarrow -\text{Tr}[\Pi_{1\mu}X_2\Pi_{1\nu}X_1^C]C_{3\otimes\bar{3}}^{cd;ef}J_{0,2}^{\mu\nu}, \\ \bar{u}(p_1)X_1v(p_4)\bar{u}(p_2)X_2v(p_3) &\rightarrow \text{Tr}[\Pi_{1\mu}X_2\Pi_{1\nu}X_1^C]C_{3\otimes\bar{3}}^{cd;ef}J_{0,2}^{\mu\nu}, \end{aligned} \quad (10)$$

where the spin-triplet projector is

$$\Pi_{1,\mu} = \frac{(\not{P} + m_c)\gamma_\mu}{\sqrt{2}}. \quad (11)$$

And the covariant projectors $J_{0,2}^{\mu\nu}$ in D dimensions are

$$\begin{aligned} J_0^{\mu\nu} &= \frac{1}{\sqrt{D-1}}\eta^{\mu\nu}, \\ J_2^{\mu\nu} &= \epsilon_{H,\alpha\beta} \left[\frac{1}{2}\eta^{\mu\alpha}\eta^{\nu\beta} + \frac{1}{2}\eta^{\mu\beta}\eta^{\nu\alpha} - \frac{1}{D-1}\eta^{\mu\nu}\eta^{\alpha\beta} \right], \end{aligned} \quad (12)$$

where

$$\eta^{\mu\nu} = -g^{\mu\nu} + \frac{P^\mu P^\nu}{m_H^2}. \quad (13)$$

Since there are no real emission processes in the next-to-leading-order (NLO) QCD corrections, the calculation remains within the framework of a two-body final state. After performing the trivial two-body phase space integration and the averaging of spin, the SDCs can be represented as

$$c_i = \frac{|\mathcal{M}_i|^2}{32\pi m_H(2J+1)}, \quad (14)$$

where J is the spin of T_{4c} .

To calculate the NLO corrections of c_i , we generate the tree-level and one-loop amplitudes of $T_{4c} \rightarrow \gamma\gamma$ by using the package FeynArts [83]. We find that there are 40 tree and 856 one-loop Feynman diagrams contributing to $T_{4c} \rightarrow \gamma\gamma$. The typical Feynman diagrams are plotted in Fig. 1. Since the tree-level amplitudes are easy to calculate, here we describe the computing procedure of the one-loop amplitudes. After

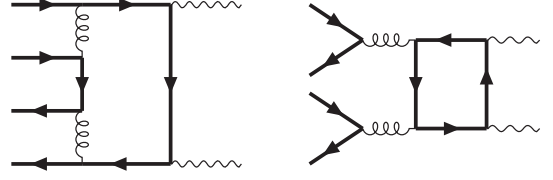


FIG. 1. Typical one-loop Feynman diagrams for the $T_{4c} \rightarrow \gamma\gamma$. The thick black lines stand for the massive charm quarks.

substituting the fermion chains with Eqs. (7) and (10), the package FeynCalc [84,85] is used to simplify the Dirac matrices. Then, the one-loop bare amplitudes can be projected into several Lorentz structures $\{F_i\}$,

$$\mathcal{M}^{\text{bare}} = \sum_i d_i F_i. \quad (15)$$

In the one-loop amplitudes, the corresponding coefficients $\{d_i\}$ are the linear combinations of one-loop scalar integrals, which can be reduced to a set of basis integrals called master integrals (MIs) due to the identities from integration by parts [86,87] with the package Kira [88]. Then the analytical results of one-loop MIs can be easily obtained by package FeynHelpers [89] based on Package-X [90].

The one-loop amplitudes computed above contain UV divergences, which will be canceled with the diagrams from counterterms, arising from the renormalization of the α_s , charm quark masses, and quark-field strength. The charm quark mass and field are renormalized in the on-shell scheme, while the strong coupling α_s is renormalized in the $\overline{\text{MS}}$ scheme,

$$\begin{aligned} m_{c,\text{bare}} &= Z_m m_c, \\ \psi_{c,\text{bare}} &= \sqrt{Z_2} \psi_c, \\ \alpha_s^{\text{bare}} C^\epsilon &= \mu^{2\epsilon} \alpha_s Z_{\alpha_s}, \end{aligned} \quad (16)$$

where $C_\epsilon = (4\pi e^{-\gamma_E})^\epsilon$ within D -dimension $D = 4 - 2\epsilon$. And the renormalization constants are

$$\begin{aligned} Z_{\alpha_s} &= 1 + \left(\frac{\alpha_s}{4\pi}\right) \frac{2n_l - 31}{3\epsilon} + \mathcal{O}(\alpha_s^2), \\ Z_m &= 1 + \left(\frac{\alpha_s}{4\pi}\right) D_\epsilon \left(-\frac{4}{\epsilon} - \frac{16}{3} - \frac{32\epsilon}{3}\right) + \mathcal{O}(\alpha_s^2), \\ Z_2 &= 1 + \left(\frac{\alpha_s}{4\pi}\right) D_\epsilon \left(-\frac{4}{\epsilon} - \frac{16}{3} - \frac{32\epsilon}{3}\right) + \mathcal{O}(\alpha_s^2), \end{aligned} \quad (17)$$

where $C_A = 3$ and $C_F = 4/3$ have been substituted, and we have

$$D_\epsilon \equiv \frac{\Gamma(1+\epsilon)}{e^{-\gamma_E\epsilon}} \left(\frac{\mu^2}{m_c^2}\right)^\epsilon. \quad (18)$$

The results of SDCs at leading order (LO) read [66]

$$\begin{aligned} c_{1,0} &= \frac{128\pi^3 e_c^4 \alpha_s^2}{3m_H m_c^4}, \\ c_{2,0} &= \frac{2304\pi^3 e_c^4 \alpha_s^2}{m_H m_c^4}, \\ c_{\text{mix},0} &= \frac{256\sqrt{6}\pi^3 e_c^4 \alpha_s^2}{m_H m_c^4}, \\ c_{3,0} &= \frac{16384\pi^3 e_c^4 \alpha_s^2}{15m_H m_c^4}, \end{aligned} \quad (19)$$

and the NLO corrections can be expressed as

$$c_{i,1} = c_{i,0} \left(\frac{\alpha_s}{\pi} \right) X_{i,1}, \quad (20)$$

where $X_{i,1}$ is the dimensionless ratio. Note that the amplitude square in $c_{\text{mix},1}$ is defined as

$$2\text{Re}[\mathcal{M}_{1,1}^{\text{ren}} \mathcal{M}_{2,0}^\dagger + \mathcal{M}_{2,1}^{\text{ren}} \mathcal{M}_{1,0}^\dagger]. \quad (21)$$

The analytic results for spin-0 tetraquark electromagnetic decay are as follows:

$$\begin{aligned} X_{1,1} &= \frac{(31-2n_l)}{6} \log\left(\frac{\mu^2}{m_c^2}\right) + \frac{5}{32} \log^2(7-4\sqrt{3}) - \frac{151}{48} \log^2(2\sqrt{2}+3) + \frac{559}{18\sqrt{2}} \log(2\sqrt{2}+3) + \frac{673}{72} \text{Li}_2\left(-\frac{1}{3}\right) \\ &\quad - \frac{527}{144} \text{Li}_2\left(\frac{1}{3}\right) + \frac{91}{96} \left(-4\text{Li}_2\left(\frac{1}{5}\right) - 2\text{Li}_2\left(\frac{3}{5}\right) + 2\text{Li}_2\left(-\frac{1}{5}\right) + 4\text{Li}_2\left(-\frac{3}{5}\right) + 3\log^2(3) - 2\log(5)\log(3) \right) \\ &\quad + \frac{207-16n_l}{64} \pi^2 - \frac{1595-36n_l}{54} \log(2) - \frac{21-8n_l}{18} \\ &\approx (5.16667 - 0.333333n_l) \log\left(\frac{\mu^2}{m_c^2}\right) + 31.6078 - 1.56086n_l, \\ X_{2,1} &= \frac{(31-2n_l)}{6} \log\left(\frac{\mu^2}{m_c^2}\right) - \frac{11}{288} \log^2(7-4\sqrt{3}) - \frac{5}{16} \log^2(2\sqrt{2}+3) + \frac{139\log^2(3)}{288} + \frac{673}{648} \text{Li}_2\left(-\frac{1}{3}\right) \\ &\quad - \frac{95}{1296} \text{Li}_2\left(\frac{1}{3}\right) + \frac{7}{144} \left(2\text{Li}_2\left(\frac{1}{5}\right) + \text{Li}_2\left(\frac{3}{5}\right) - \text{Li}_2\left(-\frac{1}{5}\right) - 2\text{Li}_2\left(-\frac{3}{5}\right) + \log(3)\log(5) \right) \\ &\quad + \frac{31-16n_l}{576} \pi^2 - \frac{401}{162\sqrt{2}} \log(2\sqrt{2}+3) - \frac{67-36n_l}{54} \log(2) + \frac{65-24n_l}{54} \\ &\approx (5.16667 - 0.333333n_l) \log\left(\frac{\mu^2}{m_c^2}\right) - 3.00931 - 0.256502n_l, \\ X_{\text{mix},1} &= \frac{(31-2n_l)}{6} \log\left(\frac{\mu^2}{m_c^2}\right) + \frac{17}{288} \log^2(7-4\sqrt{3}) - \frac{83}{48} \log^2(2\sqrt{2}+3) + \frac{479\log^2(3)}{288} + \frac{3365}{648} \text{Li}_2\left(-\frac{1}{3}\right) \\ &\quad - \frac{2419}{1296} \text{Li}_2\left(\frac{1}{3}\right) + \frac{133}{144} \left(-2\text{Li}_2\left(\frac{1}{5}\right) - \text{Li}_2\left(\frac{3}{5}\right) + \text{Li}_2\left(-\frac{1}{5}\right) + 2\text{Li}_2\left(-\frac{3}{5}\right) - \log(3)\log(5) \right) \\ &\quad + \frac{947-80n_l}{576} \pi^2 + \frac{2315}{162\sqrt{2}} \log(2\sqrt{2}+3) - \frac{277-12n_l}{18} \log(2) + \frac{1}{54}, \\ &\approx (5.16667 - 0.333333n_l) \log\left(\frac{\mu^2}{m_c^2}\right) + 14.2992 - 0.908680n_l. \end{aligned} \quad (22)$$

The analytic result for spin-2 tetraquark electromagnetic decay is

$$\begin{aligned} X_{3,1} &= \frac{(31-2n_l)}{6} \log\left(\frac{\mu^2}{m_c^2}\right) + \frac{479}{144} \text{Li}_2\left(-\frac{1}{3}\right) - \frac{2431}{1152} \text{Li}_2\left(\frac{1}{3}\right) + \frac{21}{128} \log^2(7-4\sqrt{3}) - \frac{615}{256} \log^2(2\sqrt{2}+3) \\ &\quad + \frac{467}{768} \log^2(3) - \frac{2665\pi^2}{4608} - \frac{49}{16\sqrt{3}} \log(4\sqrt{3}+7) + \frac{575}{96\sqrt{2}} \log(2\sqrt{2}+3) - \frac{22357}{2592} \log(2) + \frac{463-168n_l}{432} \\ &\approx (5.16667 - 0.333333n_l) \log\left(\frac{\mu^2}{m_c^2}\right) - 15.1997 - 0.388889n_l, \end{aligned} \quad (23)$$

where n_l is the number of the light quark flavor.

IV. APPLICATION TO THE PHOTOPRODUCTION OF FULLY CHARM TETRAQUARKS

We have obtained explicit NLO QCD corrections to the electromagnetic decays of fully charm S-wave tetraquarks in the preceding section. Next, we employ them to obtain the NLO QCD corrections to the photoproduction of fully charm tetraquarks. The parton-level amplitude in the $\gamma + \gamma \rightarrow T_{4c}$ photoproduction process is conveniently obtained from the parton-level amplitude in the $T_{4c} \rightarrow \gamma + \gamma$ decay process through the crossing symmetry.

The NRQCD factorization formula for the $\gamma + \gamma \rightarrow T_{4c}$ photoproduction process at the lowest order in velocity can be written as

$$\begin{aligned} \sigma(\gamma\gamma \rightarrow T_{4c}^{0++}) &= \frac{\pi^2 \delta(1-z)}{32m_H^2 m_c^4} (c_1 \langle 0 | O_{6\otimes\bar{6},6\otimes\bar{6}}^0 | 0 \rangle \\ &\quad + c_2 \langle 0 | O_{\bar{3}\otimes 3,\bar{3}\otimes 3}^0 | 0 \rangle \\ &\quad + c_{\text{mix}} \langle 0 | O_{\bar{3}\otimes 3,6\otimes\bar{6}}^0 | 0 \rangle), \\ \sigma(\gamma\gamma \rightarrow T_{4c}^{2++}) &= \frac{5\pi^2 \delta(1-z)}{32m_H^2 m_c^4} c_3 \langle 0 | O_{\bar{3}\otimes 3,\bar{3}\otimes 3}^2 | 0 \rangle, \end{aligned} \quad (24)$$

where $z = m_H^2/\hat{s}$ with $\hat{s}$ as the center-of-mass energy of two photons. O_{g_1,g_2}^J with $g_i = 6 \otimes \bar{6}$ or $\bar{3} \otimes 3$ are the production operators for tetraquarks with spin quantum number J ,

$$O_{g_1,g_2}^J = O_{g_1}^{(J)} \sum_X |T_{4c}^J + X\rangle \langle T_{4c}^J + X | O_{g_2}^{(J)\dagger}, \quad (25)$$

where the NRQCD operators $O_{g_1}^{(J)}$ have been defined in the previous section. By employing the vacuum saturation effect, a connection can be established between the NRQCD production LDMEs and the decay LDMEs.

Then, we can apply our formulae to the production via photon fusion in high-energy ultraperipheral collisions of protons and nuclei. The UPC cross section via photon fusion can be written as

$$\sigma(\text{AB} \xrightarrow{\gamma\gamma} \text{A} \text{B} T_{4c}) = \int \frac{dE_{\gamma_1}}{E_{\gamma_1}} \frac{dE_{\gamma_2}}{E_{\gamma_2}} \frac{d^2\sigma_{\gamma_1\gamma_2}^{(\text{AB})}}{dE_{\gamma_1} dE_{\gamma_2}}, \quad (26)$$

where E_{γ_i} is the photon energy and the values of maximum energy $E_{\gamma_i}^{\text{max}}$ for different UPCs at the LHC can be found in Ref. [91]. In general, the photon number densities in UPCs are dependent on both the photon energy and the impact parameters from hadrons A and B. If we ignore the hadronic-nonoverlap effects, the two-photon cross section distribution becomes [91,92]

$$\frac{d^2\sigma_{\gamma_1\gamma_2}^{(\text{AB})}}{dE_{\gamma_1} dE_{\gamma_2}} = n_{\gamma/\text{A}}(\bar{x}_1) n_{\gamma/\text{B}}(\bar{x}_2) \sigma(\gamma\gamma \rightarrow T_{4c}), \quad (27)$$

where $\bar{x}_i = x_i m_N b_{\text{min}}$ and $x_i = E_{\gamma_i}/E_{\text{beam}}$. $m_N = 0.93$ GeV is the nucleon mass. The minimum of impact

parameter b_{min} is set to be the nuclear radius as $b_{\text{min}} = R_A \approx 1.2A^{1/3}$ fm with the ion mass number A . The effective photon distribution function is

$$\begin{aligned} n_{\gamma/\text{A}}(\chi) &= \frac{2Z^2\alpha}{\pi} \left[(1 - \gamma_L^{-2}) \frac{\chi^2}{2} (K_0^2(\chi) - K_1^2(\chi)) \right. \\ &\quad \left. + \chi K_0(\chi) K_1(\chi) \right], \end{aligned} \quad (28)$$

where Z is the charge number and γ_L is the Lorentz boost factor for ion beams. K_0 and K_1 are modified Bessel functions.

In a similar way, one can expect the fully charm tetraquark production via photon fusion in electron-positron collisions. The production cross section via two-photon fusion in electron-positron collisions can be written as [93]

$$\begin{aligned} \sigma(e^-e^+ \xrightarrow{\gamma\gamma} e^-e^+ T_{4c}) &= (2J+1) \frac{8\alpha^2 \Gamma_{\gamma\gamma}}{m_T^3} \left[-\frac{1}{3} \left(\ln \frac{s}{m_R^2} \right)^3 \right. \\ &\quad \left. + f\left(\frac{m_T^2}{s}\right) \left(\ln \frac{m_V^2 s}{m_e^2 m_T^2} - 1 \right)^2 \right], \end{aligned} \quad (29)$$

where α is the electromagnetic coupling and s is the collision center of mass energy. m_V is usually chosen as the ρ meson mass. The auxiliary function is defined as

$$f(z) = \left(1 + \frac{z}{2}\right)^2 \log\left(\frac{1}{z}\right) - \frac{(1-z)(3+z)}{2}. \quad (30)$$

V. DISCUSSION AND NUMERICAL RESULTS

With the analytical expressions of NLO corrections at hand, we can obtain the numerical results of SDCs up to NLO corrections. First, we take the input parameters from [5,93]

$$\begin{aligned} m_H &= 4m_c = 6847_{-28}^{+44+48} \text{ MeV}, \quad m_b = 5 \text{ GeV}, \\ m_w &= 91.1876 \text{ GeV}, \quad \alpha_s(m_w) = 0.1181, \quad n_l = 3, \end{aligned} \quad (31)$$

where the fully charm tetraquark mass is chosen from the latest CMS measurement. Then, the package RunDec [94] is used to obtain α_s at other scales. The strong coupling α_s at $2m_c$, $4m_c$, and $8m_c$ is evaluated to be

$$\begin{aligned} \alpha_s(2m_c) &= 0.242997, \quad \alpha_s(4m_c) = 0.195959, \\ \alpha_s(8m_c) &= 0.166230. \end{aligned} \quad (32)$$

To perform the decay width, we still need the LDMEs as in Eq. (1). In this work, we take the LDMEs in Ref. [58], in which we have

$$\langle O_{\bar{3}\otimes 3}^{(2)} \rangle \mathcal{B}(T_{4c}) = 4.44_{-1.60-1.14-0.00}^{+1.60+2.58+1.24} \times 10^{-5} \text{ GeV}^9, \quad (33)$$

TABLE I. The numerical values of $\mathcal{R}_{\gamma\gamma}(T_{4c}^{0++})$ and $\mathcal{R}_{\gamma\gamma}(T_{4c}^{2++})$ with different scales.

	(10^{-8} MeV)	$\mu = 2m_c$	$\mu = 4m_c$	$\mu = 8m_c$
$\mathcal{R}_{\gamma\gamma}(T_{6\otimes 6,4c}^{0++})$	LO	0.39	0.25	0.18
	NLO	1.37	0.86	0.60
$\mathcal{R}_{\gamma\gamma}(T_{3\otimes 3,4c}^{0++})$	LO	20.89	13.59	9.78
	NLO	24.12	20.17	16.79
$\mathcal{R}_{\gamma\gamma}(T_{\text{mix},4c}^{0++})$	LO	5.69	3.70	2.66
	NLO	13.32	9.03	6.73
$\mathcal{R}_{\gamma\gamma}(T_{4c}^{0++})$	LO	26.97	17.54	12.62
	NLO	38.81	30.06	24.12
$\mathcal{R}_{\gamma\gamma}(T_{4c}^{2++})$	LO	9.90	6.44	4.63
	NLO	1.79	4.51	4.87

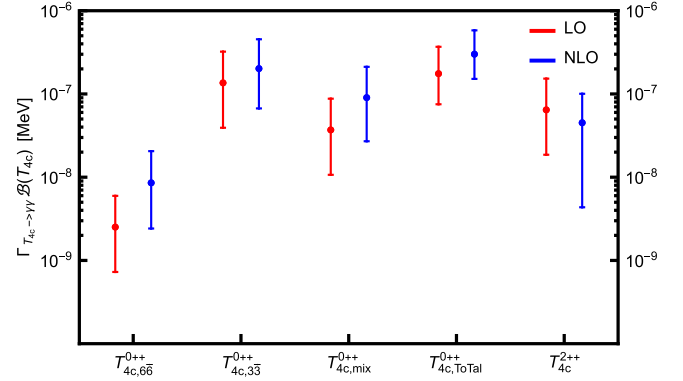
where $\mathcal{B}(T_{4c})$ stands for the branch ratio of $T_{4c} \rightarrow 2J/\psi$. The first column in uncertainties is from the existing LHCb measurements; the second column is from the scale uncertainty with and the third column is from the parton distribution function sets. It should be noted that, due to the difference between the production and decay processes, the LDMEs used in the decay calculation are related to those extracted from production by a factor of $1/(2J+1) = 1/5$ from the Vacuum Saturation Approximation [77]. In the absence of experimental data for 0^{++} fully charm tetraquark, we assume that

$$\begin{aligned} \langle \mathcal{O}_{6\otimes 6}^{(0)} \rangle &= \langle \mathcal{O}_{3\otimes 3}^{(0)} \rangle = \langle \mathcal{O}_{\text{mix}}^{(0)} \rangle \\ &= 4.44_{-1.60-1.14-0.00}^{+1.60+2.58+1.24} \times \frac{10^{-5}}{\mathcal{B}(T_{4c})} \text{ GeV}^9. \end{aligned} \quad (34)$$

This equality corresponds to mix angle $\theta = \pi/4$ in [69]. For simplicity, here we define

$$\mathcal{R}_{\gamma\gamma}(T_{4c}) \equiv \Gamma(T_{4c} \rightarrow \gamma\gamma) \mathcal{B}(T_{4c} \rightarrow 2J/\psi). \quad (35)$$

In Table I, we show the numerical results of $\mathcal{R}_{\gamma\gamma}(T_{4c})$ up to NLO with different renormalization scale μ . All other inputs, including the LDMEs and m_c , are set to their central values. Here, we have $\mathcal{R}_{\gamma\gamma}(T_{4c}^{0++}) = \mathcal{R}_{\gamma\gamma}(T_{6\otimes 6,4c}^{0++}) + \mathcal{R}_{\gamma\gamma}(T_{3\otimes 3,4c}^{0++}) + \mathcal{R}_{\gamma\gamma}(T_{\text{mix},4c}^{0++})$. Among the three components of T_{4c}^{0++} , $\mathcal{R}_{\gamma\gamma}(T_{6\otimes 6,4c}^{0++})$ are quite smaller than others, and $\mathcal{R}_{\gamma\gamma}(T_{3\otimes 3,4c}^{0++})$ gives the dominant contribution. Then, $\mathcal{R}_{\gamma\gamma}(T_{4c}^{0++})$ of T_{4c}^{0++} is much larger than T_{4c}^{2++} . At LO, $\mathcal{R}_{\gamma\gamma}(T_{4c}^{0++})$ is about three times that of $\mathcal{R}_{\gamma\gamma}(T_{4c}^{2++})$ at the typical scale $\mu = 4m_c$. And one can see that the QCD corrections are quite large. The NLO QCD corrections increase the LO value of $\mathcal{R}_{\gamma\gamma}(T_{4c}^{0++})$ by approximately 70% at $\mu = 4m_c$. In contrast, the correction for the 2^{++} state is negative, decreasing the LO prediction by about 30% at $\mu = 4m_c$. After considering the all uncertainties including m_c and LDMEs, up to NLO, we have

FIG. 2. The numerical values of $\mathcal{R}_{\gamma\gamma}(T_{4c}^{0++})$ and $\mathcal{R}_{\gamma\gamma}(T_{4c}^{2++})$. The error bars denote the uncertainties.

$$\begin{aligned} \mathcal{R}_{\gamma\gamma}(T_{6\otimes 6,4c}^{0++}) &= 0.86_{-0.61}^{+1.20} \times 10^{-8} \text{ MeV}, \\ \mathcal{R}_{\gamma\gamma}(T_{3\otimes 3,4c}^{0++}) &= 20.17_{-13.47}^{+25.22} \times 10^{-8} \text{ MeV}, \\ \mathcal{R}_{\gamma\gamma}(T_{\text{Mix},4c}^{0++}) &= 9.03_{-6.33}^{+12.14} \times 10^{-8} \text{ MeV}, \\ \mathcal{R}_{\gamma\gamma}(T_{4c}^{0++}) &= 30.00_{-14.89}^{+28.01} \times 10^{-8} \text{ MeV}, \\ \mathcal{R}_{\gamma\gamma}(T_{4c}^{2++}) &= 4.51_{-4.07}^{+5.54} \times 10^{-8} \text{ MeV}. \end{aligned} \quad (36)$$

To show these properties explicitly, we plot the results in Fig. 2. We also consider the dependence of the mixing angle θ in Ref. [69], in which the $\mathcal{R}_{\gamma\gamma}(T_{4c}^{0++})$ can be expressed as $\mathcal{R}_{\gamma\gamma}^\theta(T_{4c}^{0++}) = 2\sin^2(\theta)\mathcal{R}_{\gamma\gamma}(T_{6\otimes 6,4c}^{0++}) + 2\cos^2(\theta)\mathcal{R}_{\gamma\gamma}(T_{3\otimes 3,4c}^{0++}) + 2\sin(\theta)\cos(\theta)\mathcal{R}_{\gamma\gamma}(T_{\text{mix},4c}^{0++})$. The numerical results for $\mathcal{R}_{\gamma\gamma}^\theta(T_{4c}^{0++})$ as a function of θ are shown in Fig. 3, together with the $\mathcal{R}_{\gamma\gamma}(T_{4c}^{2++})$ value for comparison.

For the photon fusion in UPCs of proton-proton (P-P) and nucleus-nucleus (A-A), the electromagnetic field of the charged particles accelerated at high energies can be identified as a flux of quasireal photons in the equivalent

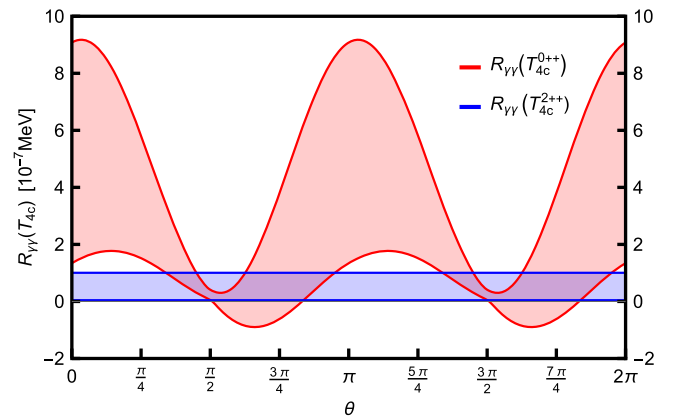
FIG. 3. The numerical values of $\mathcal{R}_{\gamma\gamma}^\theta(T_{4c}^{0++})$ and $\mathcal{R}_{\gamma\gamma}(T_{4c}^{2++})$ with the different mix angle θ . The error bars denote the uncertainties.

TABLE II. Photon fusion cross sections of T_{4c}^{0++} and T_{4c}^{2++} in various UPCs processes ($AB \xrightarrow{\gamma\gamma} AB T_{4c}$) with $\theta = \pi/4$ and $\mathcal{B}(T_{4c}) = 0.5$.

System	E_{beam}^i (TeV)	$\sigma(T_{4c}^{0++})$	$\sigma(T_{4c}^{2++})$
P-P	7.0 + 7.0 TeV	$0.13^{+0.12}_{-0.07}$ fb	$0.10^{+0.12}_{-0.09}$ fb
O-O	3.5 + 3.5 TeV	$0.16^{+0.15}_{-0.08}$ pb	$0.12^{+0.15}_{-0.11}$ pb
Ca-Ca	3.5 + 3.5 TeV	$5.10^{+4.77}_{-2.53}$ pb	$3.83^{+4.71}_{-3.46}$ pb
Ar-Ar	3.15 + 3.15 TeV	$3.10^{+2.90}_{-1.54}$ pb	$2.33^{+2.87}_{-2.11}$ pb
Kr-Kr	3.23 + 3.23 TeV	$0.04^{+0.04}_{-0.02}$ nb	$0.03^{+0.04}_{-0.03}$ nb
Xe-Xe	2.93 + 2.93 TeV	$0.17^{+0.16}_{-0.09}$ nb	$0.13^{+0.16}_{-0.12}$ nb
Pb-Pb	2.76 + 2.76 TeV	$0.76^{+0.71}_{-0.38}$ nb	$0.57^{+0.70}_{-0.51}$ nb

TABLE III. Photon fusion cross sections of T_{4c}^{0++} and T_{4c}^{2++} for different collision center of mass energy $\sqrt{s}$ in electron-positron collision processes ($e^-e^+ \xrightarrow{\gamma\gamma} e^-e^+ T_{4c}$) at SCTF, Belle, and CEPC with $\theta = \pi/4$ and $\mathcal{B}(T_{4c}) = 0.5$.

Experiments	$\sqrt{s}$ (GeV)	$\sigma(T_{4c}^{0++})$	$\sigma(T_{4c}^{2++})$
SCTF	7	$0.18^{+0.17}_{-0.09}$ ab	$0.14^{+0.17}_{-0.13}$ ab
Belle	10.6	$5.25^{+4.90}_{-2.61}$ ab	$3.95^{+4.85}_{-3.56}$ ab
CEPC	91.2	$0.11^{+0.11}_{-0.06}$ fb	$0.08^{+0.10}_{-0.08}$ fb
CEPC	172	$0.17^{+0.16}_{-0.08}$ fb	$0.13^{+0.16}_{-0.12}$ fb
CEPC	240	$0.20^{+0.19}_{-0.10}$ fb	$0.15^{+0.19}_{-0.14}$ fb

photon approximation. In this procedure, we use $\theta = \pi/4$ and assume that $\mathcal{B}(T_{4c}) = 0.5$. We then give the photon fusion cross section in various UPC processes in Table II. The UPC cross section of fully charm tetraquarks is not small. The cross section via photon fusion in proton-proton collision is around 0.1 pb, while the related cross section becomes around μb in lead-lead collision because of the electromagnetic enhancing factor, i.e., the square of its electric charge Z^2 . We also show our results of photon fusion cross sections of T_{4c}^{0++} and T_{4c}^{2++} for different collision center-of-mass energy $\sqrt{s}$ in electron-positron collision processes ($e^-e^+ \xrightarrow{\gamma\gamma} e^-e^+ T_{4c}$) in Table III. From the table, the cross sections of fully charm tetraquarks are small. Nonetheless, these physical findings offer valuable guidance for the feasibility studies of future planned experimental facilities such as STCF and CEPC regarding fully charm tetraquark states [95–97].

VI. CONCLUSION

In this paper, we investigate the interaction strength between fully charm tetraquark states and two photons. We obtain the next-to-leading-order corrections to the decay widths of fully charm tetraquark states decaying into two photons. The clean two-photon channel will improve the signal recognition. Our final results, however, show that detecting this decay channel poses significant challenges in present experiments given that the partial decay width for the two-photon channel is on the order of 10^{-8} MeV, while the total width of fully charm tetraquark is around 100 MeV. Nevertheless, we believe that future experiments at the LHC may make its observation possible.

On the other hand, the production cross sections via photon fusion in various ultraperipheral collisions and electron-positron collision processes are also investigated. Our results show that the photon fusion cross sections of T_{4c}^{0++} and T_{4c}^{2++} in various UPCs and electron-positron collision processes may be detected. Based on the cross section results for the fully charm tetraquarks, there are approximately three signal events when using a data sample corresponding to an integrated luminosity of 5 nb^{-1} in lead-lead collision. These results can be tested experimentally at the current LHC/Belle experiments and future STCF/CEPC experiments and also provide a relatively clean approach for extracting the nonperturbative LDMEs of fully charm tetraquark states. Through studies of electromagnetic properties of fully charm tetraquark states in current proton-proton or nucleus-nucleus collisions, a deeper understanding of their internal structure is anticipated.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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