

Kinematically enhanced interpolating operators for boosted hadronsRui Zhang^{1,*} Anthony V. Grebe,² Daniel C. Hackett,² Michael L. Wagman,² and Yong Zhao¹¹*Physics Division, Argonne National Laboratory, Lemont, Illinois 60439, USA*²*Fermi National Accelerator Laboratory, Batavia, Illinois 60510, USA*

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We propose to use interpolating operators for lattice quantum chromodynamics calculations of highly boosted pions and nucleons with kinematically enhanced ground-state overlap factors at large momentum. Because this kinematic enhancement applies to the signal but not the variance of the correlation function, these interpolating operators can achieve better signal-to-noise ratios at large momentum. We perform proof-of-principle calculations with boosted pions and nucleons using close-to-physical and larger quark masses to explore the utility of our proposal. Results for effective energies and matrix elements, as well as Lanczos ground-state energy estimators, are consistent with theoretical expectations for signal-to-noise improvement at large momenta.

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Boosted hadrons play a significant role in collider physics at the LHC [1] and the forthcoming Electron-Ion Collider (EIC) [2–4]. Additionally, highly boosted light hadrons also frequently appear in other physical processes with large momentum transfer Q^2 , such as the pion and kaon measurements at Jefferson Lab [5,6], and in high-precision searches for unitary violations which rely on the decay of heavy mesons [7–10] and baryons [11]. Therefore, understanding the structure of boosted hadrons is crucial for advancing modern particle and nuclear physics.

Lattice quantum chromodynamics (QCD) is essential for providing nonperturbative first-principles predictions for these experiments within the Standard Model [12]. To serve this goal, measurements of boosted hadrons on the lattice are necessary, especially in extracting their spin and three-dimension structures using near-light-cone approximations [13–26] and in measuring observables with large Q^2 [27–32]. Recent lattice QCD calculations have employed pions boosted to 2.4 GeV [33,34], as well as kaons and nucleons boosted to 3.0 GeV [35–37]. However, a major computational challenge remains the reliable projection of hadronic states onto large momenta, as the signal-to-noise ratios (SNRs) for hadronic observables decrease rapidly at large boost, limiting the ability of lattice QCD calculations to reliably extract ground-state signals. Techniques to improve the precision and reliability of lattice QCD

calculations including highly boosted hadron states are thus extremely desirable.

Attempts to design interpolating operators (interpolators) that yield better signals for highly boosted hadron states have primarily focused on the spatial structure of the quark fields. Early attempts included anisotropic spatial smearing resembling a “platelike” picture of hadrons in a Minkowski boosted frame [38,39], but these operators did not show significant SNR improvement and seemed to worsen excited-state effects. Significant progress was achieved through the proposal of momentum smearing quark propagators [40], which enhances both the SNR and overlap of standard hadron interpolators with boosted states. In this approach, the fermion fields are smeared with a phase factor to simulate a wave packet carrying specific momentum on the lattice. Momentum smearing can lead to order-of-magnitude improvement in the SNR for large boosts, and its use is now standard in lattice QCD calculations of highly boosted hadrons [24,25,41].

Meanwhile, designing optimal spinor structures for highly boosted hadron interpolators has received less attention. Standard interpolators acting on the vacuum create quark (antiquark) Fock states with the quantum numbers and symmetry properties of rest-frame hadrons and then multiply these states by a momentum phase factor. Physically, highly boosted hadrons can be described by a light cone field theory picture where there is an infinite set of Fock components. The leading light cone Fock states for highly boosted pions and nucleons are constructed from the “plus” component of the quark spinors [42,43], while the overlap with standard interpolators includes a subleading light cone wave function.

In this Letter, we construct interpolators associated with the leading light cone Fock states for pions and nucleons.

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We study the Parisi-Lepage scaling [44,45] of correlation functions (correlators) built from these interpolators and show that their SNR is enhanced for large boosts P by a kinematic factor proportional to P^2 . Proof-of-principle numerical calculations corroborate these theoretical expectations. We find that, although excited-state effects are larger for these new interpolators than standard ones for small boost values, excited-state effects are comparable or smaller for very large boosts. To quantify SNR improvements in ground-state energy determinations, we use the Lanczos/Rayleigh-Ritz framework for correlator analysis [46–51], which provides ground-state energy estimators with asymptotically constant SNR for boosted hadrons. For the setup considered here, kinematically enhanced interpolators lead to $\mathcal{O}(100)$ -fold SNR improvement for pions with boosts of $P > 2$ GeV and $\mathcal{O}(10)$ -fold SNR improvement for nucleons with boosts of $P > 3$ GeV. Analogous SNR improvements are seen in three-point correlators, suggesting that these new interpolators can significantly improve the precision of lattice QCD studies of boosted hadrons.

Kinematic enhancement: Theory. In the light cone limit, a Dirac spinor can be decomposed into $\psi = \psi_+ + \psi_-$ with $\psi_{\pm} = \frac{1}{\sqrt{2}}\gamma_{\mp}\not{\epsilon}_z\psi$, in which the plus component ψ_+ dominates the dynamics [42,43]. Orienting the hadron momentum as $\vec{P} = |\vec{P}|\hat{e}_z$, the light cone γ matrices are $\gamma_{\pm} \equiv \frac{1}{\sqrt{2}}(\gamma_t \pm \mathbf{i}\gamma_z)$, where $\gamma_{x/y/z/t} \equiv \gamma_{1/2/3/4}$ are Euclidean γ matrices satisfying $\gamma_{\mu}^{\dagger} = \gamma_{\mu}$ and $\{\gamma_{\mu}, \gamma_{\nu}\} = 2\delta_{\mu\nu}$ for $\mu, \nu = \{1, 2, 3, 4\}$; Minkowski versions are related by $\gamma_4 = \gamma_0^M$ and $\gamma_i = \mathbf{i}\gamma_i^M$ for $i = \{1, 2, 3\}$. Thus quark bilinears constructed with only ψ_+ components describe the leading Fock states of mesons with mass M in the expansion of $M/(E + P_z)$. For pseudoscalar mesons like the pion, the leading contribution is $u_+^{\dagger}\gamma_5 d_+ = \sqrt{2}\bar{u}\gamma_+\gamma_5 d$ [52,53]. On the other hand, the traditional pseudoscalar operator $(\bar{u}\gamma_5 d) = (u_+^{\dagger}\gamma_5 d_- + u_-^{\dagger}\gamma_5 d_+)/2$ is associated with subleading Fock states. These light cone physics considerations suggest that $(\bar{u}\gamma_{\mu}\gamma_5 d)$ operators should have better overlap with highly boosted pion ground states.

Axial-vector pion operators $(\bar{u}\gamma_{\mu}\gamma_5 d)$ transform differently from standard pseudoscalar pion operators $(\bar{u}\gamma_5 d)$ under rotations and only $(\bar{u}\gamma_i\gamma_5 d)$ has nonzero overlap with the pion ground state in the rest frame. Conversely, the axial-vector current $(\bar{u}\gamma_z\gamma_5 d)$ has the same quantum numbers as an axial-vector meson and thus can be used to study the spectrum of mesons with axial-vector quantum numbers at $P_z = 0$, including the $a_1(1260)$ resonance. However, rotational symmetry no longer provides useful constraints for hadron states with nonzero $\vec{P}$, which allows $(\bar{u}\gamma_z\gamma_5 d)$ to overlap with the same states as pseudoscalar pion operators. This effect is familiar in the context of calculating the pion decay constant using the

cross-correlation of $(\bar{u}\gamma_{\mu}\gamma_5 d)$ and $(\bar{u}\gamma_5 d)$, which has the spectral representation

$$\begin{aligned} & \sum_{\vec{x}} e^{i\vec{P}\cdot\vec{x}} \langle [\bar{u}\gamma_{\mu}\gamma_5 d](\vec{x}, t) [\bar{d}\gamma_5 u](0) \rangle \\ &= \frac{e^{-E_{\pi}(\vec{P})t}}{2E_{\pi}(\vec{P})} Z_{\pi}(\vec{P}) \mathbf{i}f_{\pi}P_{\mu} + \dots, \end{aligned} \quad (1)$$

where f_{π} is the bare pion decay constant, defined from the ground-state pion-to-vacuum matrix element of the axial-vector current $\langle \Omega | \bar{u}\gamma_{\mu}\gamma_5 d | \pi(\vec{P}) \rangle = \mathbf{i}f_{\pi}P_{\mu}$, and $Z_{\pi}(\vec{P}) \equiv \langle \pi(\vec{P}) | \bar{d}\gamma_5 u | \Omega \rangle$ is the ground-state overlap factor of the pseudoscalar pion operator. Once there is a large boost in the z -direction, the overlap of $(\bar{u}\gamma_z\gamma_5 d)$ with moving pion ground states will be enhanced by P_z as in Eq. (1) and dominate the signal. Some excited-state overlaps may also receive enhancements proportional to P_z ; however, excited states can involve additional polarization and/or relative momentum vectors, which can lead to different relative enhancement of ground-state versus excited-state overlaps.

The overlap between axial-vector and pseudoscalar operators follows from the symmetries of QCD in boosted frames. In the continuum and infinite-volume limits, both pseudoscalar and axial-vector meson operators parallel to the momentum vector, $(\bar{u}\gamma_z\gamma_5 d)$ for $\vec{P} = P_z\hat{e}_z$, transform trivially under the little group of rotations leaving $\vec{P}$ invariant. The symmetries of the finite-volume lattice theory form a discrete subgroup of this little group, and $(\bar{u}\gamma_5 d)$ and $(\bar{u}\gamma_z\gamma_5 d)$ transform in the same irreducible representations (IRs) of the lattice symmetry groups C_{4v} for boosted frames. Explicitly, the pseudoscalar IR subduces as $A_1^- \rightarrow A_2$, and the axial-vector IR subduces as $T_1^+ \rightarrow A_2 \oplus E$ [54–56]. It is precisely the $(\bar{u}\gamma_z\gamma_5 d)$ component of $(\bar{u}\gamma_i\gamma_5 d)$ that transforms in the A_2 IR, confirming that $(\bar{u}\gamma_z\gamma_5 d)$ and $(\bar{u}\gamma_5 d)$ transform identically.

Higher-spin interpolators overlap with boosted lower-spin states only through extra P_{μ} -dependent kinematic factors as in Eq. (1). At large momentum the overlap of $(\bar{u}\gamma_i\gamma_5 d)$ with the pion ground state is related to that of the $(\bar{u}\gamma_5 d)$ operator by $f_{\pi}E_{\pi}/Z_{\pi}$, which provides an enhanced overlap of $\mathcal{O}(E_{\pi})$ with boosted pion states. A similar argument also applies to $(\bar{u}\gamma_z\gamma_5 d)$, where the kinematic enhancement is $\mathcal{O}(P_z)$, except that this operator has zero overlap with the pion ground state in the rest frame. In general, the signal of $(\bar{u}\gamma_{\mu}\gamma_5 d)$ two-point correlators receives a kinematic enhancement of $\mathcal{O}(P_{\mu}^2)$ at large momentum.

On the other hand, the statistical fluctuation of these correlation functions can be estimated through an analysis of variance correlators following the methods of Parisi [44] and Lepage [45]. For a generic pion interpolator $O_{\pi}(\vec{x}, t)$, the variance correlator associated with $C_{\pi}(\vec{P}, t) \equiv \sum_{\vec{x}} O_{\pi}(\vec{x}, t) O_{\pi}^{\dagger}(0) e^{i\vec{P}\cdot\vec{x}}$,

$$\begin{aligned}\text{Var}(C_\pi) &= \langle \text{Re}(C_\pi)^2 \rangle - \langle C_\pi \rangle^2 \\ &= \frac{1}{2} \langle |C_\pi|^2 \rangle + \frac{1}{2} \langle C_\pi^2 \rangle - \langle C_\pi \rangle^2,\end{aligned}\quad (2)$$

involves $C_\pi(\vec{P}, t)^2$, which includes two-pion states with total momentum $2\vec{P}$, as well as

$$|C_\pi|^2 = \sum_{\vec{x}, \vec{y}} e^{i\vec{P} \cdot (\vec{x} - \vec{y})} O_\pi(\vec{y}, t) O_\pi^\dagger(\vec{x}, t) O_\pi^\dagger(0) O_\pi(0), \quad (3)$$

which has total momentum zero and is dominated at late time by states with two $|\vec{P}| = 0$ pions decaying at a rate $\sim 2m_\pi$, leading to exponential SNR decay for boosted pion states.

For these $|\vec{P}| = 0$ two-pion states, the P_μ^2 factors arising for kinematically enhanced interpolators are simply equal to m_π^2 when they are nonzero. At large t the variance of C_π is not kinematically enhanced, so $\text{SNR}(C_\pi) \equiv \langle C_\pi \rangle / \sqrt{\text{Var}(C_\pi)}$ for $(\bar{u}\gamma_\mu\gamma_5 d)$ receives an $\mathcal{O}(P_\mu^2/m_\pi^2)$ enhancement compared to $(\bar{u}\gamma_5 d)$ at large boost, as analyzed in Supplemental Material [57]. Empirically, the precision of the traditional interpolator $(\bar{u}\gamma_5 d)$ is better than $(\bar{u}\gamma_i\gamma_5 d)$ by an $\mathcal{O}(1)$ factor for static pions (in our test, this factor is around 2), which can be easily compensated by the kinematic enhancement. Note that the enhancement should not depend significantly on the lattice spacing, because the improved interpolators have the same dimension as the traditional ones. Recent applications of these interpolators to $a \approx 0.06$ fm lattice is consistent with these expectations [58]. Current state-of-the-art continuum-extrapolated parton structure calculations with near-physical pion masses have employed boosts of up to $P_z \sim 2$ GeV [34,59], corresponding to SNR enhancement factors for two-point correlators as large as $E_\pi^2/m_\pi^2 \sim 200$.

Some of these interpolators have been explored in a generalized eigenvalue problem setup previously [54,60,61] for static or slightly boosted meson systems. More recently, $(\bar{u}\gamma_i\gamma_5 d)$ has been used along with $(\bar{u}\gamma_5 d)$ at larger boosts to calculate the pion distribution amplitude (DA) [62], but its SNR benefits were not revealed. One reason is that the enhancement factor for pion DA is linear in E_π/m_π , which was not large enough ($\lesssim 5$) in those calculations to provide a noteworthy enhancement. Moreover, as we show below, $(\bar{u}\gamma_i\gamma_5 d)$ has worse SNR than $(\bar{u}\gamma_5 d)$ for a static pion, which offsets its enhancement at small boosts.

An identical strategy can be used to construct nucleon interpolators using the quark-field plus components. The standard interpolator for a static nucleon is

$$N_\Gamma = \epsilon_{abc} (d_a^T C \Gamma u_b) \mathcal{P}_+ u_c, \quad (4)$$

with $\Gamma = \gamma_5$ and C the charge conjugate operator, $\mathcal{P}_\pm \equiv (1 \pm \gamma_t)/2$ to project the positive-parity sector in the rest frame, and ϵ_{abc} the Levi-Civita symbol with a, b, c as color indices. Here, the diquark $(d_a^T C \gamma_5 u_b)$ has spin 0. Conversely,

the leading Fock component built from quark-field plus components has a spin-1 diquark [42,43,63]. Both $\Gamma = \gamma_5\gamma_\mu$ and $\Gamma = \gamma_\mu$ correspond to the same baryon leading-twist wave function in the light cone limit [64],

$$\begin{aligned}\langle 0 | N_{\gamma_5\gamma_\mu} | N(\vec{P}) \rangle &= \alpha P_\mu \mathcal{P}_+ u(\vec{P}) + \beta \gamma_\mu \mathcal{P}_+ u(\vec{P}), \\ \langle 0 | N_{\gamma_\mu} | N(\vec{P}) \rangle &= \alpha' P_\mu \gamma_5 \mathcal{P}_+ u(\vec{P}) + \beta' \gamma_\mu \gamma_5 \mathcal{P}_+ u(\vec{P}),\end{aligned}\quad (5)$$

where $u(\vec{P})$ is the nucleon's Dirac spinor and $\alpha, \alpha', \beta, \beta'$ are scalar functions of $\vec{P}$. The term proportional to P_μ is not present in $\langle 0 | N_{\gamma_5} | N(\vec{P}) \rangle = Z(\vec{P}) \mathcal{P}_+ u(\vec{P})$ with scalar $Z(\vec{P})$. Compared to the static nucleon, this term generates an enhancement $\propto P_\mu^2$ at large boost for the $N_{\gamma_5\gamma_\mu}$ and N_{γ_μ} two-point correlators that we estimate to be $\mathcal{O}(P_\mu^2/M_N^2)$ based on analogous arguments to those in Supplemental Material [57]. Both $N_{\gamma_5\gamma_\mu}$ and N_{γ_μ} overlap with spin- $\frac{3}{2}$ baryons, such as the $\Delta(1232)$ [65], but the overlap is found below to be numerically small at large momentum.

In the rest frame, N_Γ can also be projected by $\mathcal{P}_-$ to isolate the negative-parity sector containing the $N^*(1535)$ resonance. Boosted states are no longer eigenstates of the parity, thus any projection will contain states boosted from both parity sectors. Alternatives to the parity projector $\mathcal{P}_+$ in Eq. (4) are considered in Supplemental Material [57], where we also found that the γ_t term in the traditional projector $\mathcal{P}_\pm$ projects the quark with free spinor indices to its u_+ components and thus enhances the correlator at large boost compared to $\mathcal{P} = 1$. In the remaining part of this work, we will show results only with $\mathcal{P}_+$ projection.

Parisi-Lepage analysis analogous to the pion case above shows that the boosted nucleon variance is dominated by states with three zero-momentum pions with energies $\sim 3m_\pi$ that do not receive kinematic enhancements, and therefore the $N_{\gamma_5\gamma_\mu}$ and N_{γ_μ} correlator SNR should receive $\mathcal{O}(P_\mu^2/M_N^2)$ enhancement.

Lattice QCD verification. We test these new interpolators numerically on a gauge ensemble produced by the MILC Collaboration [66] with $2+1+1$ flavors of highly improved staggered quarks tuned to reproduce the physical pion mass and the one-loop Symanzik improved gauge action [67]. The lattice has a volume $L^3 \times T = 32^3 \times 48$ and lattice spacing $a \approx 0.15$ fm. We apply two steps of hypercubic smearing [68] to the gauge fields and then use a Wilson-clover action [69] for the valence quarks with $c_{\text{SW}} = 1$ and $\kappa = 0.12635$, tuned to produce ≈ 190 MeV pions. We perform measurements with 64 source locations on 334 configurations. To increase the signal at large momentum, we use momentum smearing of $k \approx 1.55$ GeV for pion momentum from $P_z = 0$ to $P_z = 2.32$ GeV [40].

Figure 1 confirms significant SNR improvement in the effective masses $E_{\text{eff}}(t, P) \equiv 1/a \ln [C(t, P)/C(t + a, P)]$ of $(\bar{u}\gamma_\mu\gamma_5 d)$ interpolators at large momenta. It is clear that

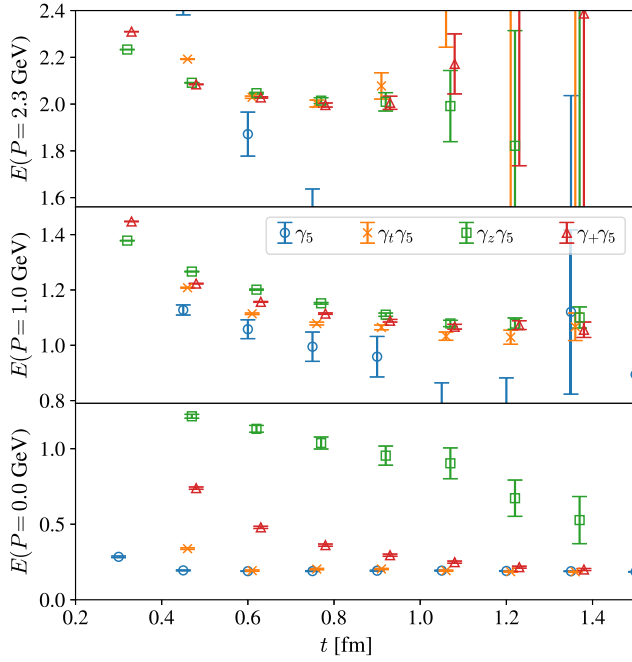


FIG. 1. Comparison of the pion effective mass at various boosts with four interpolators. At large momentum, all three kinematically enhanced interpolators ($\bar{u}\Gamma d$) with $\Gamma = \{\gamma_t\gamma_5, \gamma_z\gamma_5, \gamma_+\gamma_5\}$ show substantial precision improvements compared with the traditional ($\bar{u}\gamma_5 d$) interpolators.

($\bar{u}\gamma_z\gamma_5 d$) only overlaps with heavier states at $P_z = 0$ but becomes dominated by pion states at large momenta. At the largest momentum, $P_z \sim 2.32$ GeV, ($\bar{u}\gamma_5 d$) correlators become very noisy for $t \gtrsim 0.6$ fm and decrease to unphysical values, which is a clear sign of noise dominance; however, ($\bar{u}\gamma_\mu\gamma_5 d$) interpolators can achieve reliable signals out to larger $t \sim 0.9$ fm.

As shown in Fig. 1, at small momentum, ($\bar{u}\gamma_\mu\gamma_5 d$) correlators converge slower than ($\bar{u}\gamma_5 d$), indicating larger excited-state effects. At large momentum, ($\bar{u}\gamma_\mu\gamma_5 d$) converges faster. In a naive implementation, the operator ($\bar{u}\gamma_+\gamma_5 d$) works best for large momentum, but ($\bar{u}\gamma_t\gamma_5 d$) is more suitable to scan a large range of momenta. A strategy for removing excited-state contamination from ($\bar{u}\gamma_z\gamma_5 d$) correlation functions by forming differences with correlation functions built from a transverse component, e.g., ($\bar{u}\gamma_x\gamma_5 d$), is presented in Supplemental Material [57].

We extract the ground-state energy $E_\pi(P_z)$ from the two-point correlators with the Lanczos/Rayleigh-Ritz method [46–51] using nested bootstrap median estimators and spurious-state filtering [50]. To avoid the effects from extra terms arising from Wick’s theorem at $t = 0$, as explained in Supplemental Material [57], we start with $t = 2a$ data. The Lanczos analysis then converges within a few iterations; results after 22 iterations, incorporating $t \in [2, N_t - 3]a$, are shown in Fig. 2. The results deviate from the continuous dispersion relation up to 15% at very large momentum due to discretization effects, as has been shown in Ref. [34]. For

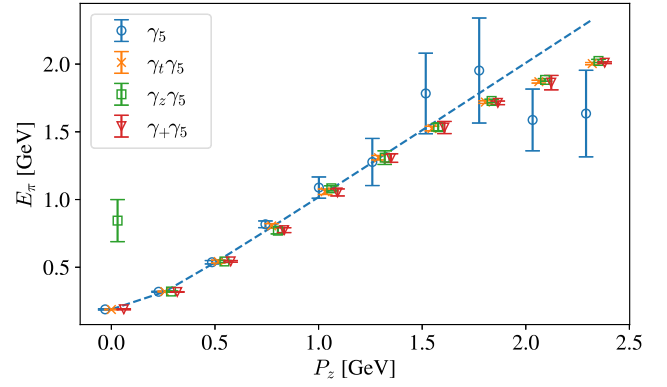


FIG. 2. The ground-state energy of the pion extracted using the Lanczos/Rayleigh-Ritz method at various boosts for the interpolators ($\bar{u}\Gamma d$) with $\Gamma = \{\gamma_5, \gamma_t\gamma_5, \gamma_z\gamma_5, \gamma_+\gamma_5\}$. The dashed line represents the continuum dispersion relation. The precision is substantially improved at large momentum when using the kinematically enhanced ($\bar{u}\gamma_\mu\gamma_5 d$). Note that, for systems at rest, ($\bar{u}\gamma_z\gamma_5 d$) has zero overlap with the pion.

$P_z = 0$, the ground state identified from ($\bar{u}\gamma_z\gamma_5 d$) correlators is significantly heavier than the pion mass, as expected due to its T_1^+ quantum numbers. For nonzero momenta, all correlators provide statistically consistent ground-state energy estimates. As the momentum increases, the kinematically enhanced interpolators clearly show growing SNR improvement compared to the traditional ($\bar{u}\gamma_5 d$) interpolator. The improvement is consistent with $\mathcal{O}(P_\mu^2/m_\pi^2)$ scaling for all but the smallest momenta and reaches factors of ~ 30 – 50 for $P_z > 2$ GeV. Supplemental Material [57] provides another way to estimate the enhancement as a function of P_z/m_π , where we have also included data with a heavier pion mass $m_\pi \approx 400$ MeV to show the scaling. It demonstrates an improvement factor of up to ~ 50 that is consistent with the Lanczos analysis.

We measure the nucleon two-point correlators on the same lattice with 16 sources and 202 configurations. To reach higher momentum, we choose $P = \frac{2\pi n}{L} \times (1, 1, 1)$ for $n \in [0, 7]$, i.e., up to 3.1 GeV. The momentum smearing is optimized for the largest momentum $P = 3.1$ GeV. Figure 3 shows the effective mass for static and boosted correlators with the five different N_Γ ’s where $\Gamma \in \{\gamma_5, \gamma_5\gamma_t, \gamma_5\gamma_z, \gamma_t, \gamma_z\}$.¹ All kinematically enhanced interpolators show similar SNR improvements compared to N_{γ_5} at large P . Among them, $N_{\gamma_5\gamma_t}$ has the least excited-state contamination where differences are visible.

Unlike the pion case where the pseudoscalar interpolator is optimal in the rest frame, the SNR for the nucleon interpolator $N_{\gamma_5\gamma_t}$ is equally good as N_{γ_5} at $P = 0$. At large momentum, the enhancement factor is of similar size as

¹For a general boost with momentum $\vec{P}$ not aligned with the z -axis, the spatial γ matrix becomes $\vec{\gamma} \cdot \hat{P}$. In our case, where $\vec{P} \propto (1, 1, 1)$, the projector is $\Gamma = (\gamma_x + \gamma_y + \gamma_z)/\sqrt{3}$, which we denote as the z -component for simplicity.

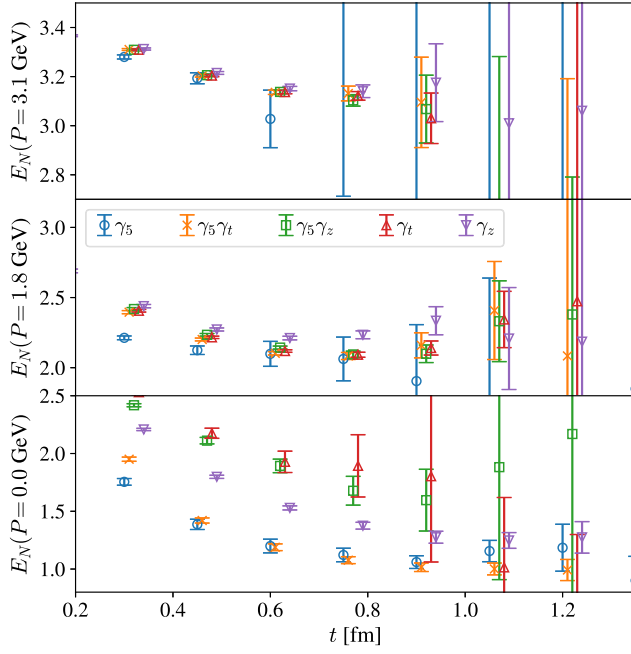


FIG. 3. The analog of Fig. 1 for the nucleon, where the interpolating operators are $(d^T C \Gamma u) \mathcal{P}_+ u$ with $\Gamma = \{\gamma_5, \gamma_5 \gamma_t, \gamma_5 \gamma_z, \gamma_t, \gamma_z\}$. At large momentum, the last four operators all yield higher precision than the choice of $\Gamma = \gamma_5$.

E_N^2/M_N^2 without the $\mathcal{O}(1)$ loss that appears in the pion case. Lanczos analysis of the nucleon correlators with the same spurious-state filtering as above gives a ground-state energy estimator with SNR enhanced by factors of 3–10 for the kinematically enhanced interpolators in comparison with N_{γ_5} .

The kinematic enhancement applies exactly the same way in the three-point correlators. For an illustration, we measure the bare unpolarized quark quasi-parton-distribution-function matrix element [17,70] $h_B^U(z, P_z)$ of the nucleon for up quark connected diagrams in the Coulomb gauge,

$$C_{3\text{pt}}(\vec{P}, t, t_{\text{sep}}) = \sum_{\vec{x}, \vec{w}, \vec{y}} e^{i(\vec{x}-\vec{y}) \cdot \vec{P}} \text{Tr} \left[\mathcal{P}_+ \langle N_{\Gamma}(t_{\text{sep}}, \vec{x}) \right. \\ \left. \times \bar{u}(t, \vec{w} + z\hat{z}) \gamma_t u(t, \vec{w}) \bar{N}_{\Gamma}(0, \vec{y}) \rangle \right], \quad (6)$$

which has the same kinematic enhancement as the two-point correlators. Thus we take the ratio

$$R^U(t, t_{\text{sep}}) = \frac{C_{3\text{pt}}(t, t_{\text{sep}})}{C_{2\text{pt}}(t_{\text{sep}})} = h_B^U(z, P_z) + \dots, \quad (7)$$

where the omitted terms are contamination from excited states. We show the results at $z = a$ and $P = 3.1$ GeV in Fig. 4. The results are consistent among the three interpolators, and we clearly observe a significant improvement with

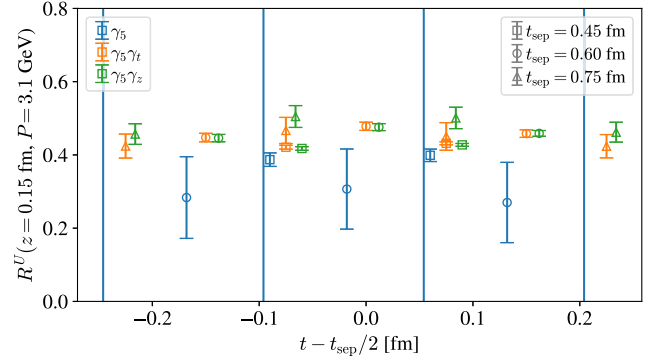


FIG. 4. A ratio of three-point to two-point functions using three nucleon interpolators from Fig. 3 shows improved precision with kinematically enhanced choices $\Gamma = \{\gamma_5 \gamma_t, \gamma_5 \gamma_z\}$ over the standard $\Gamma = \gamma_5$.

the new interpolators at the level of correlators, especially for larger t_{sep} where the ground states are dominating.

Conclusion. In this work, we propose interpolating operators for lattice QCD calculations of highly boosted pions and nucleons with kinematically enhanced ground-state overlap factors at large momentum. The signal of lattice correlators is enhanced quadratically in the Lorentz boost factor, while the noise is insensitive to the momentum, resulting in a kinematically enhanced SNR. Compared to the traditional interpolators, we find an improvement in the SNR by up to ~ 50 for pions with $P_\mu^2/m_\pi^2 \approx 150$ and up to ~ 10 for nucleons with $P_\mu^2/M_N^2 \approx 10$, which correspond to increases of statistics by $\mathcal{O}(2000)$ and $\mathcal{O}(100)$, respectively. Using these interpolators will tremendously reduce the cost of measuring boosted hadron spectra and matrix elements, significantly improving the precision of lattice calculations of form factors at large Q^2 and partonic observables. Such high-precision, high-momentum lattice calculations of form factors and parton distributions are necessary inputs for analyzing collider experiments, including the LHC and the upcoming EIC. Moreover, they could potentially be extended to processes such as $\pi\pi$ scattering [71,72] and heavy meson decays to energetic final states, which are essential to resolve CP matrix elements for high-precision unitarity violation searches.

Our calculation is performed using the GLU [73] and QUDA [74] software packages.

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Data availability. The data are not publicly available. The data are available from the authors upon reasonable request.

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