

Charged current neutrino and antineutrino induced associated particle production from nucleons

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(Received 28 July 2025; accepted 12 September 2025; published 17 October 2025)

In this work, we study the charged-current (anti)neutrino induced associated particle ($K\Lambda$) production ($\Delta S = 0$) from free nucleons in the energy region of a few GeV, relevant to the (anti)neutrino oscillation experiments with accelerator and atmospheric neutrinos. We employ a model based on effective Lagrangians to evaluate the contributions from the nonresonant and the resonant diagrams. The nonresonant background terms are calculated using a microscopic model derived from the SU(3) chiral Lagrangians. For the resonant contributions, we consider the low-lying spin- $\frac{1}{2}$ resonances, such as $S_{11}(1650)$, $P_{11}(1710)$, $P_{11}(1880)$, and $S_{11}(1895)$, and spin- $\frac{3}{2}$ resonances, such as $P_{13}(1720)$ and $P_{13}(1900)$, which have finite branching ratios to the $K\Lambda$ channel. These resonant contributions are modeled using an effective phenomenological Lagrangian approach, with strong couplings determined from the experimental branching ratios and the decay widths to the $K\Lambda$ channel. To fix the parameters of the vector current interaction, the model is first used to reproduce satisfactorily the MAMI@Mainz experimental data on the real photon induced scattering off the nucleon resulting an eta meson in the final state and with the CLAS@JLab data for the $K\Lambda$ production in the final state. The partial conservation of the axial vector current hypothesis and the generalized Goldberger-Treiman relation are used to fix the parameters of the axial vector interaction. The model is then applied to study the weak production of $K\Lambda$ induced by the neutrinos and antineutrinos and predicts the numerical values for the Q^2 distribution ($\frac{d\sigma}{dQ^2}$), the kinetic energy distribution for the outgoing kaon ($\frac{d\sigma}{dp_K}$), and the total scattering cross sections (σ) with and without a cut on the center of mass energy W . The results presented in this work are relevant for the present and future accelerator experiments like MicroBooNE, T2K, NOvA, MINERvA, SBND, ICARUS, T2-HyperK, and DUNE as well as for the atmospheric neutrino experiments.

DOI: [10.1103/6c5c-sl4t](https://doi.org/10.1103/6c5c-sl4t)

I. INTRODUCTION

The discovery that the neutrinos have nonzero masses through the observation of the neutrino oscillation phenomenon is one of the rare clues pointing toward the new physics beyond the standard model. Consequently, the study of the phenomenology of the neutrino oscillation has led to the measurement of the various oscillation parameters like the three mixing angles including the measurement of very small but nonzero value of the angle θ_{13} . However, no definite measurement of the CP -violating phase angle δ has been reported, and the question of the neutrino mass hierarchy involving the measurements of the three mass square differences, i.e., Δm_{12}^2 , Δm_{23}^2 , and Δm_{13}^2 ,

is still not settled due to the lack of consensus on the values of these parameters obtained from the various experiments. To have more precise measurements of the mixing angles and the CP -violating phase angle as well as to settle the issue of the neutrino mass hierarchy, various current and future neutrino experiments have been planned in laboratories around the world, which will also explore new physics beyond the standard model. Most of these experiments use neutrino beams in the energy region of a few GeV and the target material containing moderate to heavy nuclei. In this energy region, reliable knowledge of the nuclear medium effects on the neutrino-nuclear cross sections as well as a precise understanding of the basic neutrino-nucleon cross sections from the free nucleon targets are essential inputs. Whether the nuclear medium effects are experimentally constrained or theoretically derived from the first principles, the independent understanding of the elementary level neutrino-nucleon amplitudes for the various physical processes in the energy region of a few GeV is essential. The study of the various neutrino-nucleon processes is not only vital for the analysis

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of the neutrino oscillation experiments but also for gaining deeper insights into the hadronic interactions in the weak sector, where additional contribution from the axial vector current exists alongside the vector current contributions. Moreover, in the weak sector, the vector and axial vector form factors appearing in the matrix elements of the hadronic current are determined assuming various symmetries like the time reversal invariance, SU(3) symmetry, conserved vector current (CVC) hypothesis, partial conservation of the axial vector current (PCAC) hypothesis, etc. Therefore, a better understanding of the neutrino-nucleon cross section is important in order to test these symmetry properties [1–4]. For this, we need experimental measurements of neutrino-nucleon cross sections, which have been recently highlighted in a white paper by the Snowmass Collaboration [5].

In the few GeV energy region of (anti)neutrino, the contribution to the cross section comes from various reaction channels such as quasielastic, inelastic, and deep inelastic scattering processes which have different energy dependence [6]. The inelastic scattering on the nucleons involves a wide range of processes triggered when (anti)neutrinos interact with the nucleon target. Among these, the single pion production is the most prominent inelastic reaction. However, nucleons illuminated by neutrinos can also radiate photons and, with increasing energy transfer, give rise to multiple pions, eta, kaons, and other mesons along with a baryon with or without strangeness. In general, the strange mesons like kaons are produced either through the $\Delta S = 0$ reaction along with a hyperon in the final state or through the $\Delta S = \pm 1$ reaction along with a nucleon in the final state. The Cabibbo-suppressed (anti)neutrino-nucleon interactions that lead to the production of kaons through the $\Delta S = \pm 1$ processes require lower threshold energy but have smaller cross sections and have been discussed earlier by us and others [7,8]. In this work, we focus on the associated particle production of kaons induced by the $\Delta S = 0$ reactions, which results in final states such as YK , where Y denotes hyperons like Λ , Σ , etc., and K may be K^+ or K^0 , and features larger cross sections, but the threshold energy for their production is high.

The history of studying weak interaction induced associated particle production processes is more than half-century old, which started with the theoretical works of Shrock [9] using Born approximation on the weak charged and neutral current induced $\Delta S = 0$ reactions off the nucleon targets and followed by many others like Mecklenburg [10], Amer [11], and Dewan [12]. In the last two to three decades, many theoretical efforts have been made to study the single pion production, the single kaon production, the single hyperon production, etc. [1], but only limited efforts have been made to study the associated particle production processes, like the works of Adera *et al.* [13] using Born approximation and

Nakamura *et al.* [14] using the dynamical coupled channel (DCC) model. Adera *et al.* [13] have studied the differential cross sections (at $d^4\sigma$ level) in the threshold region using only the nucleon Born terms, where the hadronic current receives contribution from the s, t, and u channel diagrams and the contributions from the resonance excitations have not been considered. No results have been discussed at the level of the total scattering cross section σ and Q^2 distribution. On the other hand, Nakamura *et al.* [14] have used the DCC model to calculate the total scattering cross section for the neutrino induced production of pion, kaon, eta, and two pions in the final state. The hadronic current receives contribution from both the nonresonant Born terms as well as the resonance excitations. We have made a comparison of the total scattering cross section obtained in the present calculations with the available results of Nakamura *et al.* [14] for the kaon production through the associated particle production (KA) process (Fig. 8). We have also compared the present results with the earlier results obtained by Shrock [9]. The process of associated particle production also plays an important role in deepening our understanding of the fundamental symmetries of the Standard Model, the structure of weak hadronic form factors, the strange-quark content of the nucleon, etc. In addition to its theoretical significance, kaon production via the associated particle production processes also constitutes a critical background in proton decay searches, such as $p \rightarrow K\bar{\nu}$. Consequently, achieving a reliable and accurate estimation of the cross sections for these background-contributing processes to the proton decay channels is of paramount importance and has been strongly emphasized in the literature [15,16].

Experimentally, the observations of neutrino induced associated particle production were carried out in earlier decades at ANL [17,18], CERN [19,20], BNL [21,22], and Fermilab [23]. However, these measurements were limited by very low statistics and significant systematic uncertainties. The theoretical estimates significantly underestimate the experimental data, in some cases by almost a factor of 4 [24]. For example, the theoretical results presented by Shrock [9] and Mecklenburg [10], averaged over the ANL flux, show strong disagreement with the experimentally measured value, and there is also a significant difference between the predictions from these two theoretical calculations. Modern neutrino experiments, equipped with high-intensity (anti)neutrino beams, are now in a better position to revisit and refine these studies. Nonetheless, the cross sections used in the commonly adopted neutrino event generators, such as GENIE [25], NEUT [26], etc., account only for the resonant associated kaon production based on the Rein-Sehgal model, which was originally developed for pion production [27]. This stark discrepancy highlights the urgent need for improved theoretical models and

more accurate cross section predictions for the weak interaction induced associated particle production. Such advancements are essential not only for the precision neutrino physics but also for enhancing the sensitivity of rare event searches, including proton decay.

In the current generation of accelerator experiments, there are no experimental data on the specific reaction channels involving strangeness changing ($\Delta S = 1$) and strangeness conserving ($\Delta S = 0$) processes. The recent experimental measurements on the charged current induced kaon production by the MINERvA [28] and MicroBooNE [29] collaborations are focussed on the inclusive kaon production, where a K^+ meson along with a muon are observed in the final state, while the other hadrons remain undetected. The MINERvA Collaboration [30] has also reported the neutral current induced inclusive kaon production. The MINERvA Collaboration [28,30] has reported the results for the kinetic energy distribution of kaons from the plastic scintillator target, which are consistent with the GENIE simulation results especially in the case of neutral current induced K^+ production. The MicroBooNE Collaboration [29] has reported the results for the flux integrated cross section of the charged current neutrino induced inclusive K^+ production from the argon target and compared the results with the different neutrino event generators like GENIE, NEUT, and NuWro. The results are consistent with the predictions from the different neutrino generators, but large uncertainties in the experimental results require more precise data so that the interaction mechanism may be theoretically understood. However, exclusive production of K^+ with or without a hyperon in the final state has not been reported in any of the recent measurements.

In this work, we present a theoretical calculation to obtain the differential and total scattering cross sections for ΛK production for the $\nu_\mu(\bar{\nu}_\mu)$ induced scattering off the nucleon target. The weak hadronic current receives contribution from the nonresonant Born terms as well as from the resonance excitations. The Born terms are obtained using an SU(3) symmetric chiral model. The contribution from the resonance terms is considered from the excitation of both the spin- $\frac{1}{2}$ resonances like $S_{11}(1650)$, $P_{11}(1710)$, $P_{11}(1880)$, and $S_{11}(1895)$ and the spin- $\frac{3}{2}$ resonances like $P_{13}(1720)$ and $P_{13}(1900)$. In the case of spin- $\frac{1}{2}$ resonances, the s -channel hadronic currents for the positive and negative parity resonances with the explicit form of the vector and axial-vector form factors for the isospin- $\frac{1}{2}$ resonances have been used. The vector form factors of the nucleon-resonance ($N - R$) transition are expressed in terms of the electromagnetic $N - R$ form factors using the isospin symmetry, which in turn are determined from the helicity amplitudes of the resonance excitations.

Recently, the treatment of spin- $\frac{3}{2}$ resonances has been discussed in literature using the Rarita-Schwinger formalism [31] as well as Pascalutsa formalism [32]. It is

well known that the Rarita-Schwinger formalism is not unique for describing the spin- $\frac{3}{2}$ field (nor for describing the higher spin fields) and has a problem associated with the lower-spin degrees of freedom. This leads to some ambiguities in describing the propagation of the off-shell spin- $\frac{3}{2}$ field using a propagator specially in the presence of external interactions like the electromagnetic and/or the strong interactions. The problem has been discussed extensively in literature for many years ever since the field theory of higher spins was developed using either the vector-spinor formalism [31] or the multispinor formalism [33]. Consequently, there are various prescriptions for treating the propagator and the effective Lagrangians for the interacting fields of higher spin in a consistent way for describing the interaction of spin- $\frac{3}{2}$ fields [34]. One of the most popular prescriptions given by Pascalutsa and Timmermans [32] has been investigated further in the latest works of Mart [35] and Vrancx *et al.* [36] and many other references cited in their works. We find [37] that the results for the photon induced associated particle production off the nucleon better explain the experimental data when the numerical calculations are done by including the spin- $\frac{3}{2}$ resonance contribution using the formalism developed by Pascalutsa and Timmermans [32]. Therefore, all the (anti)neutrino results presented in this work are obtained using the Pascalutsa and Timmermans [32] formalism when spin- $\frac{3}{2}$ -resonant states are included.

In view of the above discussion, the parameters of the vector current interaction in the case of (anti)neutrino induced production of $K\Lambda$ are determined by fitting the experimental data on the electromagnetic production of $K\Lambda$ and η meson. In the case of photoproduction of η , our model explains the MAMI experimental data [38,39], where spin- $\frac{1}{2}$ resonances and nonresonant background terms are considered. Then, the experimental data from the CLAS Collaboration [40] for the photon induced associated particle production ($K\Lambda$) are explained, where additional resonances, including spin- $\frac{3}{2}$ resonances, are also taken into account. The Q^2 dependence of the vector form factors is fixed by explaining the CLAS electroproduction data [41] for the virtual photon induced eta production. The PCAC hypothesis and the generalized Goldberger-Treiman (GT) relation are used to fix the parameters of the axial-vector current interaction.

Accordingly, in Secs. II A and II B, we present the formalism for the photon and electron induced associated particle production, respectively, and compared them with the experimental results. The formalism for the charged current (anti)neutrino induced associated particle production is presented in Sec. III. The results and discussions are presented in Sec. IV. In Sec. V, we conclude our findings.

II. PHOTOPRODUCTION AND ELECTROPRODUCTION OF $K\Lambda$ AND $N\eta$

A. Photoproduction

The differential cross section for the photoproduction of associated kaons and η off the proton target, i.e.,

$$\gamma(q) + p(p) \longrightarrow \Lambda(p') + K^+(p_K), \quad (1)$$

$$\gamma(q) + p(p) \longrightarrow p(p') + \eta(p_\eta), \quad (2)$$

is written as [42]

$$\left. \frac{d\sigma}{d\Omega} \right|_{CM} = \frac{1}{64\pi^2 s} \frac{|\vec{p}'|}{|\vec{p}|} \sum_r \sum_{\text{spin}} |\mathcal{M}^r|^2, \quad (3)$$

where the quantities in the parentheses of Eqs. (1) and (2) represent the four momenta of the corresponding particles. The center of mass (c.m.) energy $\sqrt{s}$ is expressed as $s = W^2 = (q + p)^2 = M^2 + 2ME_\gamma$, with E_γ being the energy of the incoming photon in the laboratory frame. $\sum_r \sum |\mathcal{M}^r|^2$ is the square of the transition matrix element $\mathcal{M}^r$, for the photon polarization state r , averaged and summed over the initial and final spin states, where $\mathcal{M}^r$ is written in terms of the real photon polarization vector ϵ_μ^r , as

$$\mathcal{M}^r = e\epsilon_\mu^r(q)J^\mu, \quad (4)$$

with e being the electromagnetic coupling constant and J^μ being the matrix element of the electromagnetic current taken between the hadronic initial and final states. The hadronic matrix element receives contribution from the

nonresonant Born terms, kaon resonances being exchanged in the t -channel diagrams, and the terms corresponding to the resonance excitations and their subsequent decay to $K\Lambda$ or $N\eta$ mode. The hadronic currents for the nonresonant Born terms are obtained using the nonlinear sigma model and the total hadronic current J^μ is obtained by adding the currents corresponding to the nonresonant and resonance terms coherently. For the detailed description of the formalism, readers are referred to Ref. [1]. The detailed formalism for the photon and electron induced η production, along with the relevant Feynman diagrams, hadronic currents, etc., are discussed in Refs. [42,43].

The hadronic currents for the various non-resonant terms contributing to the $K\Lambda$ production, which are shown diagrammatically in Fig. 1, are expressed as [44]:

$$J^\mu|_s = ieA_s F_s(s) \bar{u}(p') \not{p}' \gamma_5 \frac{\not{p} + \not{q} + M}{s - M^2} \times \left(\gamma^\mu e_p + i \frac{\kappa_p}{2M} \sigma^{\mu\nu} q_\nu \right) u(p), \quad (5)$$

$$J^\mu|_t = ieA_t F_t(t) \bar{u}(p') [(\not{p}' - \not{p}) \cdot \gamma_5] u(p) \frac{(2p_K^\mu - q^\mu)}{t - M_K^2}, \quad (6)$$

$$J^\mu|_{u\Lambda} = ieA_u^\Lambda F_u^\Lambda(u) \bar{u}(p') \left(\gamma^\mu e_\Lambda + i \frac{\kappa_\Lambda}{2M_\Lambda} \sigma^{\mu\nu} q_\nu \right) \times \frac{\not{p}' - \not{q} + M_\Lambda}{u - M_\Lambda^2} \not{p}' \gamma_5 u(p), \quad (7)$$

$$J^\mu|_{u\Sigma^0} = ieA_u^{\Sigma^0} F_u^{\Sigma^0}(u) \bar{u}(p') \left(\gamma^\mu e_{\Sigma^0} + i \frac{\kappa_{\Sigma^0}}{2M_{\Sigma^0}} \sigma^{\mu\nu} q_\nu \right) \times \frac{\not{p}' - \not{q} + M_{\Sigma^0}}{u - M_{\Sigma^0}^2} \not{p}' \gamma_5 u(p), \quad (8)$$

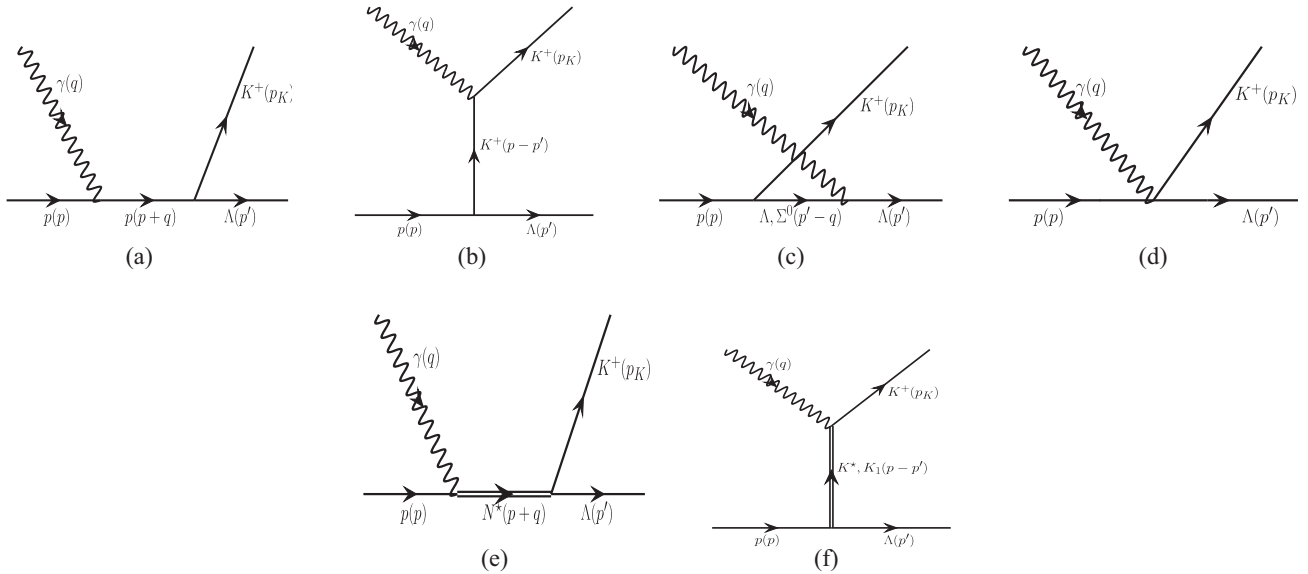


FIG. 1. Feynman diagram for the various channels possible for the process $\gamma(q) + p(p) \rightarrow K^+(p_K) + \Lambda(p')$. (a) s -channel, (b) t -channel, (c) u -channel, and (d) contact term constitute the nonresonant terms; (e) nucleon resonances in the s -channel; and (f) kaon resonances in the t -channel. The quantities in the bracket represent four momenta of the corresponding particles.

TABLE I. Properties of the spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ resonances available in the PDG [47], with Breit-Wigner mass M_R , the total decay width Γ_R , isospin I , spin J , parity P , the branching ratio full range available from the PDG (used in the present calculations) into different meson-baryon channels like $N\pi$, $N\eta$, $K\Lambda$, and $N\pi\pi$ and the strong coupling constants $g_{RN\pi}$ and $g_{RK\Lambda}$.

Resonance $\rightarrow$ Parameters $\downarrow$	$S_{11}(1535)$	$S_{11}(1650)$	$P_{11}(1710)$	$P_{13}(1720)$	$P_{11}(1880)$	$S_{11}(1895)$	$P_{13}(1900)$
M_R (GeV)	1.510 ± 0.01	1.655 ± 0.015	1.700 ± 0.02	1.680 ± 0.02	1.860 ± 0.04	1.910 ± 0.02	1.920 ± 0.02
Γ_R (GeV)	0.130 ± 0.02	0.135 ± 0.035	0.120 ± 0.04	0.150 ± 0.05	0.230 ± 0.05	0.110 ± 0.03	0.130 ± 0.03
$I(J^P)$	$\frac{1}{2}(\frac{1}{2}^-)$	$\frac{1}{2}(\frac{1}{2}^-)$	$\frac{1}{2}(\frac{1}{2}^+)$	$\frac{1}{2}(\frac{3}{2}^+)$	$\frac{1}{2}(\frac{1}{2}^+)$	$\frac{1}{2}(\frac{1}{2}^-)$	$\frac{1}{2}(\frac{3}{2}^+)$
BR (%)	$N\pi$	32–52 (43)	50–70 (60)	5–20 (16)	8–14 (11)	3–31 (34)	2–18 (23)
	$N\eta$	30–55 (40)	15–35 (25)	10–50 (20)	1–5 (3)	1–55 (20)	5–45 (30)
	$K\Lambda$	...	5–15 (10)	5–25 (15)	4–19 (2)	1–3 (2)	3–23 (13)
	$N\pi\pi$	4–31 (17)	0–58 (5)	14–48 (49)	> 50 (84)	> 32 (44)	17–74 (34)
$ g_{RN\pi} $	0.1019	0.0915	0.0418	0.1105	0.0466	0.0229	0.0644
$ g_{RK\Lambda} $	...	0.1362	0.1973	1.29	0.0604	0.08556	0.5684

$$J^\mu|_{CT} = -ieA_{CT}F_{CT}\bar{u}(p')\gamma^\mu\gamma_5u(p), \quad (9)$$

where CT stands for the contact term and s , t , and u are the Mandelstam variables defined as

$$s = (p + q)^2, \quad t = (p - p')^2, \quad u = (p' - q)^2. \quad (10)$$

$A_i' s(i = s, t, u, CT)$ are the coupling strengths of s , t , and u channels and the contact term, respectively, and are obtained as

$$A_s = A_t = A_u^\Lambda = A_{CT} = -\left(\frac{D + 3F}{2\sqrt{3}f_K}\right);$$

$$A_u^{\Sigma^0} = \left(\frac{D - F}{2f_K}\right). \quad (11)$$

All these couplings of the nonresonant terms are generated by the chiral symmetry and are fixed by the low-energy electroweak phenomenology consistent with the experimental data. D and F are the axial-vector couplings of the baryon octet, and $f_K = 105$ MeV [45] is the kaon decay constant.

To take into account the hadronic structure of the nucleons, the form factors $F_s(s)$, $F_t(t)$, $F_u(u)$, and $F_{CT}(s, t, u)$, are introduced at the strong vertices. We use a dipole form for the hadronic form factor [44],

$$F_x(x) = \frac{\Lambda_B^4}{\Lambda_B^4 + (x - M^2)^2}, \quad x = s, t, u, \quad (12)$$

where Λ_B is the cutoff parameter taken to be the same for the s -, t - and u - channel nonresonant Born terms and x represents the Mandelstam variables s , t , and u . The value of Λ_B is fitted to the experimental data, and the best-fitted value is $\Lambda_B = 0.54$ GeV. For the strong form factor of the contact term $F_{CT}(s, t, u)$, we have used the prescription of Davidson and Workman [46], who parametrize F_{CT} in terms of F_s and F_t as

$$F_{CT}(s, t, u) = F_s(s) + F_t(t) - F_s(s) \times F_t(t). \quad (13)$$

One of the most important property of the electromagnetic current is the gauge invariance which corresponds to the current conservation. In Ref. [44], we have discussed in detail the implementation of the gauge invariance in the case of the photoproduction of $K\Lambda$ for the Born terms, while for the spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ nucleon resonances, the hadronic current is gauge invariant by construction.

We have taken into account those resonances, which have mass $M_R < 2$ GeV and a significant branching ratio to the $K\Lambda$ decay mode reported in the particle data group (PDG) [47]. Specifically, we have considered four spin- $\frac{1}{2}$ resonances, viz., $S_{11}(1650)$, $P_{11}(1710)$, $P_{11}(1880)$, and $S_{11}(1895)$, and two spin- $\frac{3}{2}$ resonances, viz., $P_{13}(1720)$ and $P_{13}(1900)$. The general properties of these resonances like mass, decay width, spin, etc., are given in Table I.

The most general form of the hadronic current for the s -channel processes where a resonance state R is produced and decays to a Λ baryon and a K meson in the final state is written as [2]

$$j^\mu|_s = F_s^*(s) \frac{g_{RK\Lambda}}{f_K} \bar{u}(p') \not{p}_K \gamma_5 \Gamma_s \left(\frac{\not{p} + \not{q} + M_R}{s - M_R^2 + iM_R\Gamma_R} \right) \Gamma_{\frac{1}{2}^\pm}^\mu u(p), \quad (14)$$

where Γ_R and M_R , respectively, are the decay width and mass of the resonance. $\Gamma_s = 1(\gamma_5)$ stands for the positive (negative) parity resonances, and $g_{RK\Lambda}$ is the strong coupling strength of the $RK\Lambda$ vertex, which has been determined using the partial decay width of the resonance to the $K\Lambda$ mode where the central values of the full width tabulated in Table I are used in the numerical calculations. The values of the strong coupling constant of the different resonances are also tabulated in Table I. The vertex functions $\Gamma_{\frac{1}{2}^+}^\mu$ and $\Gamma_{\frac{1}{2}^-}^\mu$ for the positive and negative parity resonances are defined as

$$\Gamma_{\frac{1}{2}^{\pm}}^{\mu} = V_{\frac{1}{2}}^{\mu} \Gamma_s, \quad (15)$$

where $V_{\frac{1}{2}}^{\mu}$ represents the vector current parameterized in terms of $F_2^{R^+, R^0}$, as

$$V_{\frac{1}{2}}^{\mu} = \left[\frac{F_2^{R^+, R^0}}{2M} i\sigma^{\mu\alpha} q_{\alpha} \right]. \quad (16)$$

The coupling $F_2^{R^+, R^0}$ is derived from the helicity amplitudes extracted from the real photon scattering experiments. The explicit relation between the coupling $F_2^{R^+, R^0}$ and the helicity amplitude $A_{\frac{1}{2}}^{p,n}$ is given by [42]

$$A_{\frac{1}{2}}^{p,n} = \sqrt{\frac{2\pi\alpha (M_R \mp M)^2}{M (M_R^2 - M^2)}} \left[\frac{M_R \pm M}{2M} F_2^{R^+, R^0} \right], \quad (17)$$

where the upper (lower) sign stands for the positive (negative) parity resonance. R^+ and R^0 correspond, respectively, to the charged and neutral states of the isospin- $\frac{1}{2}$ resonances. The values of the helicity amplitude $A_{\frac{1}{2}}^{p,n}$ for the different resonances are quoted in Table II.

In analogy with the nonresonant terms, we have considered the following form factor at the strong vertex for the

various resonances, in order to take into account their hadronic structure,

$$F_x^*(x) = \frac{\Lambda_R^4}{\Lambda_R^4 + (x - M_R^2)^2}, \quad x = s, t \quad (18)$$

where Λ_R is the cutoff parameter whose value is fitted to the experimental data and the best fit for Λ_R for spin- $\frac{1}{2}$ resonances is obtained as $\Lambda_R = 1$ GeV.

To determine the value of the strong $RK\Lambda$ coupling, we start by writing the most general form of $RK\Lambda$ interaction Lagrangian [42]:

$$\mathcal{L}_{RK\Lambda}(x) = \frac{g_{RK\Lambda}}{f_K} \bar{\Psi}_R(x) \Gamma_s^{\mu} \partial_{\mu} K(x) \Psi(x), \quad (19)$$

where $\Psi(x)$ is the nucleon field, $\Psi_R(x)$ is the resonance field, and $K(x)$ is the kaon field. The interaction vertex $\Gamma_s^{\mu} = \gamma^{\mu} \gamma^5$ (γ^{μ}) stands for the positive (negative) parity resonance states.

Using the above Lagrangian, one obtains the expression for the decay width in the resonance rest frame as [1]

$$\Gamma_{R \rightarrow K\Lambda} = \frac{\mathcal{C}}{4\pi} \left(\frac{g_{RK\Lambda}}{f_K} \right)^2 (M_R \pm M)^2 \frac{E_N \mp M}{M_R} |\vec{p}_K^{\text{cm}}|, \quad (20)$$

TABLE II. Parametrization of the helicity amplitude for $S_{11}(1535)$, $S_{11}(1650)$, $P_{11}(1710)$, $P_{13}(1720)$, $P_{11}(1880)$, $S_{11}(1895)$, and $P_{13}(1900)$ resonances on the proton and neutron targets. $\mathcal{A}_{\alpha}(0)$ is given in units of 10^{-3} GeV^{-2} , and the coefficients a_1 and b_1 are given in units of GeV^{-2} .

Resonance	Helicity amplitude	Proton target			Neutron target		
		$\mathcal{A}_{\alpha}(0)$	a_1	b_1	$\mathcal{A}_{\alpha}(0)$	a_1	b_1
$S_{11}(1535)$	$A_{\frac{1}{2}}$	95.0	0.85	0.85	-78.0	1.75	1.75
	$S_{\frac{1}{2}}$	-2.0	1.9	0.81	32.5	0.4	1.0
$S_{11}(1650)$	$A_{\frac{1}{2}}$	33.3	0.45	0.72	26.0	0.1	2.5
	$S_{\frac{1}{2}}$	2.5	1.88	0.96	3.8	0.4	0.71
$P_{11}(1710)$	$A_{\frac{1}{2}}$	55.0	1.0	1.05	-45.0	-0.02	0.95
	$S_{\frac{1}{2}}$	4.4	2.18	0.88	-31.5	0.35	0.85
$P_{13}(1720)$	$A_{\frac{1}{2}}$	100.0	1.89	1.55	-2.9	1.7	1.55
	$A_{\frac{3}{2}}$	-11.0	10.0	1.55	-31.0	3.0	1.55
	$S_{\frac{1}{2}}$	-53.0	2.46	1.55	0	0	0
$P_{11}(1880)$	$A_{\frac{1}{2}}$	-60.0	0.4	1.0	-45.0	-0.02	0.95
	$S_{\frac{1}{2}}$	0.4	0.75	0.5	-31.5	0.35	0.85
$S_{11}(1895)$	$A_{\frac{1}{2}}$	-15.0	1.45	0.6	26.0	0.1	2.5
	$S_{\frac{1}{2}}$	-3.5	0.88	0.6	3.8	0.4	0.71
$P_{13}(1900)$	$A_{\frac{1}{2}}$	8.0	1.89	1.55	-2.9	12.7	1.55
	$A_{\frac{3}{2}}$	-98.0	1.0	1.55	-31.0	3.0	1.55
	$S_{\frac{1}{2}}$	-10.0	0.46	1.0	0	0	0

where the upper (lower) sign represents the positive (negative) parity resonance; $\mathcal{C} = 1$ for the kaon production processes; $|\vec{p}_K^{\text{cm}}|$ is the outgoing kaon momentum measured in the resonance rest frame and is given by

$$|\vec{p}_K^{\text{cm}}| = \frac{\sqrt{(M_R^2 - M_K^2 - M^2)^2 - 4M_K^2 M^2}}{2M_R}; \quad (21)$$

and E_N , the outgoing Λ baryon energy, is

$$E_N = \frac{M_R^2 + M^2 - M_K^2}{2M_R}. \quad (22)$$

1. Spin- $\frac{3}{2}$ resonances

The general form of the hadronic current for the s -channel processes where a positive parity resonance state is produced and decays to a lambda (Λ) and a kaon (K) in the final state, is written as [2]

$$j^\mu|_s = F_s^*(s) \frac{g_{RK\Lambda}}{M_R M_K} \bar{u}(p') \epsilon_{\sigma\nu\alpha\beta} p_R^\sigma \gamma_5 \gamma^\alpha p_K^\beta \left(\frac{\mathcal{P}^{\nu\delta}(p_R)}{s - M_R^2 + iM_R \Gamma_R} \right) \Gamma_{\frac{3}{2}}^{\delta\mu} u(p), \quad p_R = p + q, \quad (23)$$

where Γ_R and M_R , respectively, are the decay width and mass of the resonance. $F_s^*(s)$ is defined in Eq. (18), with the value of the cutoff parameter Λ_R determined for spin- $\frac{3}{2}$ resonances to be $\Lambda_R = 1.62$ GeV. $g_{RK\Lambda}$ is the strong coupling strength of the $RK\Lambda$ vertex, which has been determined using the partial decay width of the resonance to $K\Lambda$ mode. The values of the strong coupling constant of the different resonances are also tabulated in Table I.

The spin- $\frac{3}{2}$ projection operator $\mathcal{P}_{\alpha\beta}(p_R)$, in the prescription of Pascalutsa [32], is given by

$$\begin{aligned} \mathcal{P}_{\alpha\beta}(p_R) = & -(\not{p}_R + M_R) \\ & \times \left(g_{\alpha\beta} - \frac{1}{3} \gamma_\alpha \gamma_\beta - \frac{1}{3p_R^2} (\not{p}_R \gamma_\alpha \not{p}_{R\beta} + p_{R\alpha} \gamma_\beta \not{p}_R) \right). \end{aligned} \quad (24)$$

The vertex function $\Gamma_{\frac{3}{2}}^{\mu\alpha}$ for the positive parity resonances is defined as

$$\Gamma_{\frac{3}{2}}^{\mu\alpha} = V_{\frac{3}{2}}^{\mu\alpha}, \quad (25)$$

where $V_{\frac{3}{2}}^\mu$ represents the vector current and is parametrized in terms of the vector couplings $C_{3,4}^{R^+, R^0}$, as

$$V_{\frac{3}{2}}^{\mu\alpha} = \left[(\gamma^\mu q^\alpha - \not{q} g^{\alpha\mu}) \frac{C_3^{R^+, R^0}}{M} + (q \cdot p' g^{\alpha\mu} - q^\alpha p'^\mu) \frac{C_4^{R^+, R^0}}{M^2} \right]. \quad (26)$$

The couplings $C_{3,4}^{R^+, R^0}$ are derived from the helicity amplitudes extracted from the real photon scattering experiments. The explicit relation between the coupling $C_{3,4}^{R^+, R^0}$ and the helicity amplitude $A_{\frac{1}{2}}^{p,n}$ is given by [42]

$$\begin{aligned} A_{\frac{1}{2}}^{p,n} = & \sqrt{\frac{\pi\alpha}{3M} \frac{M_R - M}{M_R + M}} \left[\frac{C_3^{R^+, R^0}}{M} (M + M_R) \right. \\ & \left. + \frac{C_4^{R^+, R^0}}{2M^2} (M^2 - M_R^2) \right], \end{aligned} \quad (27)$$

$$\begin{aligned} A_{\frac{1}{2}}^{p,n} = & \sqrt{\frac{\pi\alpha}{M} \frac{M_R - M}{M_R + M}} \left[\frac{C_3^{R^+, R^0}}{M_R} (M + M_R) \right. \\ & \left. + \frac{C_4^{R^+, R^0}}{2M^2} (M_R^2 - M^2) \right], \end{aligned} \quad (28)$$

where R^+ and R^0 correspond, respectively, to the charged and neutral states of the isospin- $\frac{1}{2}$ resonances. The value of the helicity amplitude $A_{\frac{1}{2}}^{p,n}$ for the different resonances are quoted in Table II.

To determine the value of the strong $RK\Lambda$ coupling for spin- $\frac{3}{2}$ resonances, we start by writing the general form of $RK\Lambda$ Lagrangian given by Pascalutsa [32], as

$$\mathcal{L}(x) = \frac{g_{RK\Lambda}}{M_R M_K} \epsilon_{\mu\nu\alpha\beta} \partial^\mu \bar{\Psi}_R^\nu(x) \gamma_5 \gamma^\alpha \psi(x) \partial^\beta K(x), \quad (29)$$

where $\psi(x)$ is the nucleon field, $\Psi_R(x)$ is the spin- $\frac{3}{2}$ resonance field, and $K(x)$ is the kaon field.

Using the above Lagrangian, one obtains the expression for the decay width in the resonance rest frame as

$$\Gamma_{R \rightarrow K\Lambda} = \frac{\mathcal{C}}{12\pi} \left(\frac{g_{RK\Lambda}}{m_K} \right)^2 \frac{E_N + M}{M_R} |\vec{p}_K^{\text{cm}}|^3, \quad (30)$$

where $\mathcal{C} = 1$ for kaon production processes and $|\vec{p}_K^{\text{cm}}|$ and E_N , respectively, are given in Eqs. (21) and (22), respectively.

TABLE III. Properties of kaon resonances included in the present model, with mass M_R , spin J , isospin I , parity P , the total decay width Γ , and the vector G_K^v and tensor G_K^t couplings for the kaon resonances. It is to be noted that the couplings G_K^v and G_K^t contain both the electromagnetic as well as the strong coupling strengths.

Resonances	M_R (GeV)	J	I	P	Γ (GeV)	G_K^v	G_K^t
$K^*(892)$	0.89166 ± 0.00026	1	$\frac{1}{2}$	—	0.0508 ± 0.0009	−0.18	0.02
$K_1(1270)$	1.272 ± 0.007	1	$\frac{1}{2}$	+	0.090 ± 0.020	0.28	−0.28

2. Kaon resonances

In the present work, we have considered two kaon resonances in the t channel: a vector meson $K^*(892)$ and an axial vector meson $K_1(1270)$. For details, see Ref. [44].

The hadronic current for the K^* exchange in the t channel is obtained as

$$J_\mu|_{K^*} = ieF_t^*(t)\bar{u}(p')\epsilon_{\mu\nu\rho\sigma}q^\rho(p' - p)^\sigma \left(\frac{-g^{\nu\alpha} + (p - p')^\nu(p - p')^\alpha/M_{K^*}^2}{t - M_{K^*}^2 + iM_{K^*}\Gamma_{K^*}} \right) \times \left[G_{K^*}^v\gamma_\alpha + \frac{G_{K^*}^t}{M + M_\Lambda}(\not{p}' - \not{p})\gamma_\alpha \right] u(p), \quad (31)$$

with $G_{K^*}^v = \kappa_{KK^*}g_{K^*\Lambda p}^v/\mu$ and $G_{K^*}^t = \kappa_{KK^*}g_{K^*\Lambda p}^t/\mu$, where $g_{K^*\Lambda p}^v$ and $g_{K^*\Lambda p}^t$ are the vector and the tensor couplings, respectively, at the strong $K^*\Lambda p$ vertex, κ_{KK^*} is the coupling strength of the γKK^* vertex, and μ is an arbitrary mass factor which is introduced to make the Lagrangian dimensionless. μ is chosen to be 1 GeV. M_{K^*} and Γ_{K^*} are the mass and width of the K^* resonance, respectively.

Because of the lack of the experimental data on the K^* and K_1 resonances, the values of $G_{K^*}^v$ and $G_{K^*}^t$ cannot be determined phenomenologically and are treated as free parameters to be fitted to the experimental data of the $K\Lambda$ production and are quoted in Table III.

The hadronic current for the axial vector kaon K_1 exchange in the t channel is obtained as

$$J_\mu|_{K_1} = ieF_t^*(t)\bar{u}(p')[g_{\alpha\mu}q \cdot (p - p') - q_\alpha(p - p')_\mu] \left(\frac{-g^{\alpha\rho} + (p - p')^\alpha(p - p')^\rho/M_{K_1}^2}{t - M_{K_1}^2 + iM_{K_1}\Gamma_{K_1}} \right) \times \left[G_{K_1}^v\gamma_\rho\gamma_5 + \frac{G_{K_1}^t}{M + M_\Lambda}(\not{p}' - \not{p})\gamma_\rho\gamma_5 \right] u(p), \quad (32)$$

with $G_{K_1}^v = \kappa_{KK_1}g_{K_1\Lambda p}^v/\mu$ and $G_{K_1}^t = \kappa_{KK_1}g_{K_1\Lambda p}^t/\mu$, where κ_{KK_1} is the coupling strength of the electromagnetic γKK_1 vertex and $g_{K_1\Lambda p}^v$ and $g_{K_1\Lambda p}^t$ are the vector and the tensor couplings, respectively, at the strong $K_1\Lambda p$ vertex. M_{K_1} and Γ_{K_1} are the mass and width of the K_1 resonance, respectively. The values of $G_{K_1}^v$ and $G_{K_1}^t$ are treated as free parameters to be fitted to the experimental data of the $K\Lambda$ production and are quoted in Table III.

In Eqs. (31) and (32), $F_t^*(t)$ is the strong form factor with Λ_R taken to be the same as used in the case of Born terms, i.e., $\Lambda_R = \Lambda_B = 0.54$ GeV.

3. Cross section

To obtain the η production as well as the associated particle production cross sections, in the numerical calculations, we consider the following:

- (i) The nonresonant Born terms are obtained using the SU(3) symmetric chiral Lagrangians based on the nonlinear sigma model.
- (ii) For all the spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ resonances, the coupling constant at the strong $R \rightarrow N\eta$ or $R \rightarrow K\Lambda$ vertex is determined using the partial decay width of the resonance into $N\eta$ or $K\Lambda$ decay mode.
- (iii) In the case of η production, for the spin- $\frac{1}{2}$ resonances, the couplings at the electromagnetic γNR vertex for both the proton and neutron targets are taken from the PDG [47] and varied within the errors bars to fit the η photoproduction data [42,43].
- (iv) In the case of $K\Lambda$ production, for the spin- $\frac{1}{2}$ resonances, we have used the same values of the electromagnetic couplings as fitted for the η photoproduction case, which has been discussed by us in Refs. [42,43].

(v) In the case of $K\Lambda$ production, for the spin- $\frac{3}{2}$ resonances, the electromagnetic couplings at the γNR vertex for the proton target are taken from the PDG [47] and varied within the errors bars to fit the $K\Lambda$ photoproduction data [44], while for the neutron target, we have used the central values of these couplings from the PDG.

In the left panel of Fig. 2, we have shown the results for σ vs W for $\gamma p \rightarrow \eta p$ (solid line) and $\gamma n \rightarrow \eta n$ (dashed line) processes using the full model in the region of W from η production threshold to $W = 1.9$ GeV. We have compared our theoretical results with the experimental data obtained by McNicoll *et al.* [38] for the MAMI Crystal Ball Collaboration on the proton target and the quasifree neutron data from Werthmüller *et al.* [39] for the MAMI A2 Collaboration. It may be observed from the figure that in the case of η production, our results, with a very few free parameters, viz., Λ_B and Λ_R , are in a very good agreement with the available experimental data for the proton in the range of $W \leq 1.65$ GeV and for the neutron target in the entire range of W considered in the present work. However, in the case of η production from the proton target, our results underestimate the cross sections in the range of $W > 1.65$ GeV, which may be due to the contribution from the higher resonances that need further studies. In the right panel of Fig. 2, we have presented the results for the total scattering cross section σ as a function of center of mass energy W for the photon induced $K\Lambda$ production. The theoretical calculations are presented for the full model, which receives contribution from the nonresonant Born terms, kaon resonance exchange diagrams, and the $S_{11}(1650)$, $P_{11}(1710)$, $P_{13}(1720)$, $P_{11}(1880)$, $S_{11}(1895)$,

and $P_{13}(1900)$ resonance excitations. The results shown by the dotted line also include the contributions from K^* and K_1 resonances, whose contributions are found out to be almost negligible. The theoretical calculations are compared with the experimental data from the CLAS experiment [40]. It may be observed from the figure that our results are in good agreement with the experimental data.

B. Electroproduction

The electron induced $K\Lambda$ and η productions off the nucleon target are given by the reactions

$$e^-(k) + p(p) \rightarrow e^-(k') + \Lambda(p') + K^+(p_K), \quad (33)$$

$$e^-(k) + p(p) \rightarrow e^-(k') + p(p') + \eta(p_\eta), \quad (34)$$

where the 4-momentum for each particle is indicated in the parentheses. The 4-momentum of the virtual photon exchanged in the electroproduction is given by $q = k - k'$.

The five-fold differential cross section for the electroproduction can be expressed as [48–50]

$$\frac{d\sigma}{d\Omega_l dE_l d\Omega_{qp_K}} = \Gamma \frac{d\sigma_v}{d\Omega_{qp_K}}, \quad (35)$$

with the flux of the virtual photon Γ given by

$$\Gamma = \frac{\alpha}{2\pi^2} \frac{E_l}{E_e} \frac{K}{Q^2} \frac{1}{1 - \varepsilon}. \quad (36)$$

In the above equation, $K = (W^2 - M^2)/2M$ denotes the photon equivalent energy; i.e., the laboratory energy

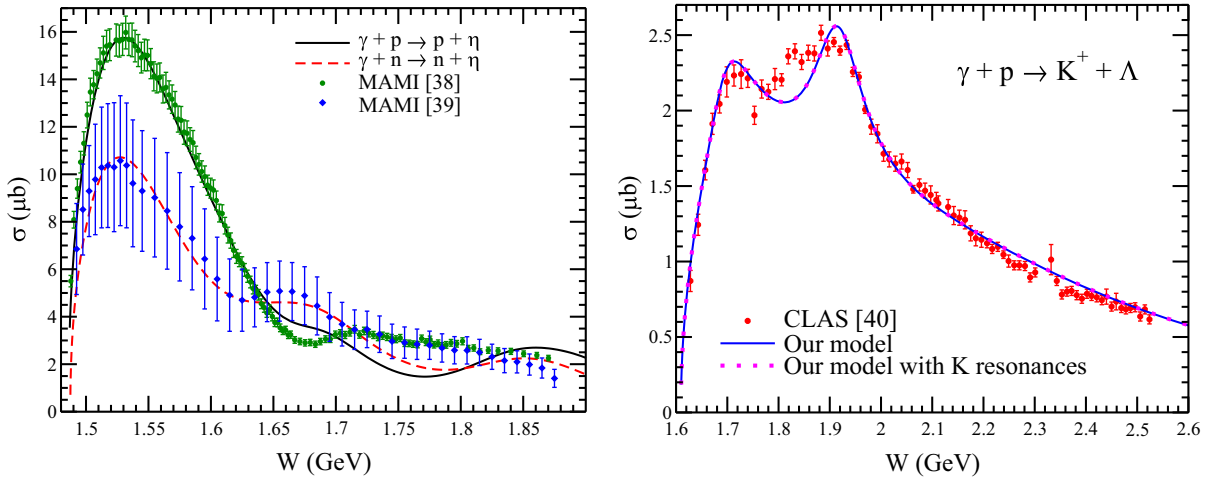


FIG. 2. Left panel: total cross section σ vs W for $\gamma p \rightarrow \eta p$ (solid line) and $\gamma n \rightarrow \eta n$ (dashed line) processes using the full model, obtained using the formalism discussed in Refs. [42,43]. The experimental points for the proton target (solid circle) are obtained from MAMI Crystal Ball Collaboration [38], and for the neutron target (solid diamond), we have used the quasifree neutron data from the MAMI A2 Collaboration [39]. Right panel: comparison of σ vs W for the process $\gamma p \rightarrow \Lambda K$ by including all the possible Feynman diagrams depicted in Fig. 1, without K-meson intermediate state (dotted line), and with the K meson, i.e., result of our full calculation (solid line). Solid circles are the data from the CLAS experiment [40].

necessary for a real photon to excite a hadronic system with c.m. energy W , and ε is the transverse polarization parameter of the virtual photon and is given as

$$\varepsilon = \left(1 + 2 \frac{|\vec{q}|^2}{Q^2} \tan^2 \frac{\theta_l}{2}\right)^{-1}, \quad (37)$$

with $Q^2 = -q^2 = -(k - k')^2$.

The detailed discussion of the electron induced η production has been discussed by us in Refs. [42,43].

The hadronic currents corresponding to the nucleon Born terms for the electroproduction of associated kaons are obtained using the nonlinear sigma model and are written as

$$J^\mu|_s = ieA_s F_s(s) \bar{u}(p') \not{p} \gamma_5 \frac{\not{p} + \not{q} + M}{s - M^2} \mathcal{O}_N^\mu u(p), \quad (38)$$

$$J^\mu|_t = ieA_t F_t(t) F_K(Q^2) \bar{u}(p') [(\not{p} - \not{p}') \cdot \gamma_5] u(p) \frac{(2p_k^\mu - q^\mu)}{t - M_k^2}, \quad (39)$$

$$J^\mu|_{u\Lambda} = ieA_u^\Lambda F_u^\Lambda(u) \bar{u}(p') \mathcal{O}_\Lambda^\mu \frac{\not{p}' - \not{q} + M_\Lambda}{u - M_\Lambda^2} \not{p}_k \gamma_5 u(p), \quad (40)$$

$$J^\mu|_{u\Sigma^0} = ieA_u^{\Sigma^0} F_u^{\Sigma^0}(u) \bar{u}(p') \mathcal{O}_{\Sigma^0}^\mu \frac{\not{p}' - \not{q} + M_{\Sigma^0}}{u - M_{\Sigma^0}^2} \not{p}_k \gamma_5 u(p), \quad (41)$$

$$J^\mu|_{CT} = -ieA_{CT} F_{CT} F_C(Q^2) \bar{u}(p') \gamma^\mu \gamma_5 u(p), \quad (42)$$

where the vertex operator $\mathcal{O}_{N,\Lambda,\Sigma^0}^\mu$ corresponding, respectively, to γNN , $\gamma \Lambda \Lambda$, $\gamma \Lambda \Sigma^0$ vertices, is expressed in terms of the Q^2 dependent form factors as

$$\mathcal{O}_{N,\Lambda,\Sigma^0}^\mu = F_1^{N,\Lambda,\Sigma^0}(Q^2) \gamma^\mu + F_2^{N,\Lambda,\Sigma^0}(Q^2) i\sigma^{\mu\nu} \frac{q_\nu}{2M_{N,\Lambda,\Sigma^0}}. \quad (43)$$

The Dirac and Pauli form factors of the nucleon and hyperon, viz., $F_1^{N,\Lambda,\Sigma^0}(Q^2)$ and $F_2^{N,\Lambda,\Sigma^0}(Q^2)$, respectively, are expressed in terms of the Sachs electric $[G_E^{p,n}(Q^2)]$ and magnetic $[G_M^{p,n}(Q^2)]$ form factors of the nucleons, for which various parametrizations are available in the literature. In the present work, we have taken the parametrization of these form factors from Bradford *et al.* [51] also known as BBBA05 parametrization. For details, see Ref. [42].

The form factors $F_K(Q^2)$ and $F_C(Q^2)$ are introduced in the t -channel and contact term diagrams, respectively, to take into account the hadronic structure at the electromagnetic vertices. The implementation of gauge invariance determines these form factors in terms of the electromagnetic form factors of nucleons and hyperons as

$$F_K(Q^2) = F_1^P(Q^2) - F_1^\Lambda(Q^2) + \sqrt{3} F_1^{\Sigma^0}(Q^2) \frac{D - F}{D + 3F} \times \left(\frac{u - M_{\Sigma^0} M_\Lambda + M M_{\Sigma^0} - M M_\Lambda}{u - M_{\Sigma^0}^2} + \left[\frac{M_{\Sigma^0} - M_\Lambda}{M + M_\Lambda} \right] \left[\frac{u - M^2}{u - M_{\Sigma^0}^2} \right] \right), \quad (44)$$

$$F_C(Q^2) = F_1^P(Q^2) - F_1^\Lambda(Q^2) + \sqrt{3} F_1^{\Sigma^0}(Q^2) \frac{D - F}{D + 3F} \times \left(\frac{u - M_{\Sigma^0} M_\Lambda + M M_{\Sigma^0} - M M_\Lambda}{u - M_{\Sigma^0}^2} \right). \quad (45)$$

The general expression of the hadronic current for the resonance excitation in the s channel is written as

$$j^\mu|_s = F_s^*(s) \frac{g_{RK\Lambda}}{f_K} \bar{u}(p') \not{p}_K \gamma_5 \Gamma_s \left(\frac{\not{p} + \not{q} + M_R}{s - M_R^2 + iM_R \Gamma_R} \right) \times \Gamma_{\frac{1}{2}\pm}^\mu u(p). \quad (46)$$

The vertex function $\Gamma_{\frac{1}{2}\pm}^\mu$ for the positive and negative parity resonances is given in Eq. (15), where the vector current $V_{\frac{1}{2}}^\mu$ in the case of electroproduction processes is expressed in terms of the form factors $F_{1,2}^{R^+,R^0}(Q^2)$ as

$$V_{\frac{1}{2}}^\mu = \frac{F_1^R(Q^2)}{(2M)^2} (q q^\mu + Q^2 \gamma^\mu) + \frac{F_2^R(Q^2)}{2M} i\sigma^{\mu\nu} q_\nu, \quad (47)$$

$$R = R^+, R^0.$$

The electromagnetic $N - R$ transition form factors for the charged $[F_{1,2}^{R^+}(Q^2)]$ and neutral $[F_{1,2}^{R^0}(Q^2)]$ states are then related to the helicity amplitudes given by the relations [1]

$$A_{\frac{1}{2}} = \sqrt{\frac{2\pi\alpha}{K_R}} \left\langle R, J_Z = \frac{1}{2} \left| \epsilon_\mu^+ J_i^\mu \right| N, J_Z = \frac{-1}{2} \right\rangle \zeta$$

$$S_{\frac{1}{2}} = -\sqrt{\frac{2\pi\alpha}{K_R}} \frac{|\vec{q}|}{\sqrt{Q^2}} \left\langle R, J_Z = \frac{1}{2} \left| \epsilon_\mu^0 J_i^\mu \right| N, J_Z = \frac{-1}{2} \right\rangle \zeta, \quad (48)$$

where in the resonance rest frame

$$K_R = \frac{M_R^2 - M^2}{2M_R}, \quad |\vec{q}|^2 = \frac{(M_R^2 - M^2 - Q^2)^2}{4M_R^2} + Q^2,$$

$$\epsilon_\pm^\mu = \mp \frac{1}{\sqrt{2}} (0, 1, \pm i, 0), \quad \epsilon_0^\mu = \frac{1}{\sqrt{Q^2}} (|\vec{q}|, 0, 0, q^0). \quad (49)$$

The parameter ζ is model dependent, which is related to the sign of $R \rightarrow N\pi$ and for the present calculation is taken as $\zeta = 1$.

Using Eq. (49) in Eq. (48), the helicity amplitudes $A_{\frac{1}{2}}(Q^2)$ and $S_{\frac{1}{2}}(Q^2)$ in terms of the electromagnetic form factors $F_1^{R^+,R^0}(Q^2)$ and $F_2^{R^+,R^0}(Q^2)$ are obtained as [42]

$$\begin{aligned} A_{\frac{1}{2}}^{p,n}(Q^2) &= \sqrt{\frac{2\pi\alpha(M_R \mp M)^2 + Q^2}{M}} \left(\frac{Q^2}{4M^2} F_1^{R^+,R^0}(Q^2) + \frac{M_R \pm M}{2M} F_2^{R^+,R^0}(Q^2) \right) \\ S_{\frac{1}{2}}^{p,n}(Q^2) &= \mp \sqrt{\frac{\pi\alpha(M_R \pm M)^2 + Q^2}{M}} \frac{(M_R \mp M)^2 + Q^2}{4M_R M} \left(\frac{M_R \pm M}{2M} F_1^{R^+,R^0}(Q^2) - F_2^{R^+,R^0}(Q^2) \right), \end{aligned} \quad (50)$$

where upper (lower) sign corresponds to positive (negative) parity resonances.

The Q^2 dependence of the helicity amplitudes [Eq. (50)] is generally parametrized as [52]

$$\mathcal{A}_\alpha(Q^2) = \mathcal{A}_\alpha(0)(1 + \alpha Q^2)e^{-\beta Q^2}, \quad (51)$$

where $\mathcal{A}_\alpha(Q^2)$ are the helicity amplitudes; $A_{\frac{1}{2}}(Q^2)$ and $S_{\frac{1}{2}}(Q^2)$ and parameters $\mathcal{A}_\alpha(0)$ are generally determined

by a fit to the photoproduction data of the corresponding resonance. The values of these parameters for the different nucleon resonances are tabulated in Table II.

1. Spin- $\frac{3}{2}$ resonances

The general expression of the hadronic current for the resonance excitation in the s channel is written as [1]

$$j^\mu|_s = F_s^*(s) \frac{g_{RK\Lambda}}{M_R M_K} \bar{u}(p') \epsilon^{\sigma\nu\alpha\beta} p_{R\sigma} \gamma_5 \gamma_\alpha p_{K\beta} \left(\frac{\mathcal{P}_{\nu\delta}(p_R)}{s - M_R^2 + iM_R \Gamma_R} \right) \Gamma_{\frac{3}{2}}^{\delta\mu}(Q^2) u(p), \quad p_R = p + q. \quad (52)$$

The vertex function $\Gamma_{\frac{3}{2}}^{\delta\mu}$ for the positive parity resonances is given in Eq. (25), where the vector current $V_{\frac{3}{2}}^{\delta\mu}$ in the case of electroproduction processes is expressed in terms of the form factors $C_{3,4,5}^{R^+,R^0}(Q^2)$ as

$$\begin{aligned} V_{\frac{3}{2}}^{\delta\mu} &= (\gamma^\mu q^\delta - q^\mu g^{\delta\mu}) \frac{C_3^R(Q^2)}{M} + (q \cdot p' g^{\delta\mu} - q^\delta p'^\mu) \frac{C_4^R(Q^2)}{M^2} \\ &+ (q^\delta q^\mu - q^2 g^{\delta\mu}) \frac{C_5^R(Q^2)}{M^2}, \quad R = R^+, R^0. \end{aligned} \quad (53)$$

The electromagnetic $N - R$ transition form factors for the charged [$C_{3,4,5}^{R^+}(Q^2)$] and neutral [$C_{3,4,5}^{R^0}(Q^2)$] states are then related to the helicity amplitudes [$A_{\frac{1}{2}}(Q^2)$, $A_{\frac{3}{2}}(Q^2)$, and $S_{\frac{1}{2}}(Q^2)$], where the explicit relations between the helicity

amplitudes $A_{\frac{1}{2}}$ and $S_{\frac{1}{2}}$ and the electromagnetic current are given in Eq. (48), and $A_{\frac{3}{2}}$ is expressed in terms of the electromagnetic current by the following expression:

$$A_{\frac{3}{2}} = \sqrt{\frac{2\pi\alpha}{K_R}} \langle R, J_z^R = +\frac{3}{2} | \epsilon_\mu^+ J_i^\mu | N, J_z^N = +\frac{1}{2} \rangle \zeta. \quad (54)$$

The different variables appearing in the above expression are given in Eq. (49).

Using Eq. (49) in Eqs. (48) and (54), the helicity amplitudes $A_{\frac{1}{2}}(Q^2)$, $A_{\frac{3}{2}}(Q^2)$, and $S_{\frac{1}{2}}(Q^2)$ in terms of the electromagnetic form factors $C_3^{R^+,R^0}(Q^2)$, $C_4^{R^+,R^0}(Q^2)$, and $C_5^{R^+,R^0}(Q^2)$ are obtained as [42]

$$A_{\frac{1}{2}}(Q^2) = \sqrt{\frac{\pi\alpha[(M_R - M)^2 + Q^2]}{3M}} \left[\frac{C_3^R(Q^2)}{M} (M + M_R) + \frac{C_4^R(Q^2)}{2M^2} (M^2 - M_R^2 + Q^2) - C_5^R(Q^2) \frac{Q^2}{M^2} \right], \quad (55)$$

$$A_{\frac{3}{2}}(Q^2) = \sqrt{\frac{\pi\alpha[(M_R - M)^2 + Q^2]}{M}} \left[\frac{C_3^R(Q^2)}{MM_R} [M(M + M_R) + Q^2] + \frac{C_4^R(Q^2)}{2M^2} [M_R^2 - M^2 - Q^2] + C_5^R(Q^2) \frac{Q^2}{M^2} \right], \quad (56)$$

$$S_{\frac{1}{2}}(Q^2) = -\sqrt{\frac{\pi\alpha}{6M} \frac{[(M_R + M)^2 + Q^2]}{M_R^2 - M^2}} [(M_R - M)^2 + Q^2] \left[-\frac{C_3^R(Q^2)}{MM_R} + \frac{C_4^R(Q^2)}{M^2} + C_5^R(Q^2) \frac{[M^2 - M_R^2 + Q^2]}{2M^2 M_R^2} \right]. \quad (57)$$

The parametrization of the Q^2 dependence of the helicity amplitudes [Eq. (55)] is given in Eq. (51), with $\mathcal{A}_\alpha(Q^2)$ being the helicity amplitudes $A_{\frac{1}{2}}(Q^2)$, $A_{\frac{3}{2}}(Q^2)$, and $S_{\frac{1}{2}}(Q^2)$. The values of the parameters appearing in Eq. (51) for the different spin- $\frac{3}{2}$ resonances are tabulated in Table II.

2. Kaon resonances

In the case of associated particle production induced by the electrons, the expressions for the hadronic current of the kaon exchange diagrams remain the same as given in Eqs. (31) and (32), except that these terms are multiplied by an additional form factor, which accounts for the electromagnetic structure of the kaons. In the literature, generally a monopole form factor of the type

$$F_{K^*, K_1}(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\Lambda_{K^*, K_1}^2}\right)} \quad (58)$$

is taken into account with $\Lambda_{K^*} = 0.95$ GeV [53] and $\Lambda_{K_1} = 0.55$ GeV [53].

3. Cross section

To obtain the electron induced η production as well as the $K\Lambda$ production cross sections, in the numerical calculations, we consider the following:

- (i) In the nonresonant Born terms, the nucleon and hyperon vector form factors are expressed in terms of the Sachs electric and magnetic form factors of the nucleons, for which the BBBA05 parametrization is used.
- (ii) For the spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ resonances, we have used the same values of the coupling constant at the strong $R \rightarrow N\eta$ or $R \rightarrow K\Lambda$ vertex as determined in the case of the photoproduction of $K\Lambda$ or $N\eta$.
- (iii) In the case of η production, for the spin- $\frac{1}{2}$ resonances, the Q^2 dependence of the helicity amplitudes, which are used to determine the electromagnetic $N - R$ transition form factors, is fitted to explain the electroproduction data for the η production available from the CLAS experiment [42,43].
- (iv) In the case of $K\Lambda$ production, for the spin- $\frac{1}{2}$ resonances, we have used the same Q^2 dependence of the electromagnetic form factors as fitted for the η electroproduction case [42,43].
- (v) In the case of $K\Lambda$ production, for the spin- $\frac{3}{2}$ resonances, we have fitted the Q^2 dependence of the helicity amplitudes, which are used to determine the electromagnetic form factors.

In the left panel of Fig. 3, we present the results for the total cross section σ for $\gamma^* p \rightarrow \eta p$ process by integrating

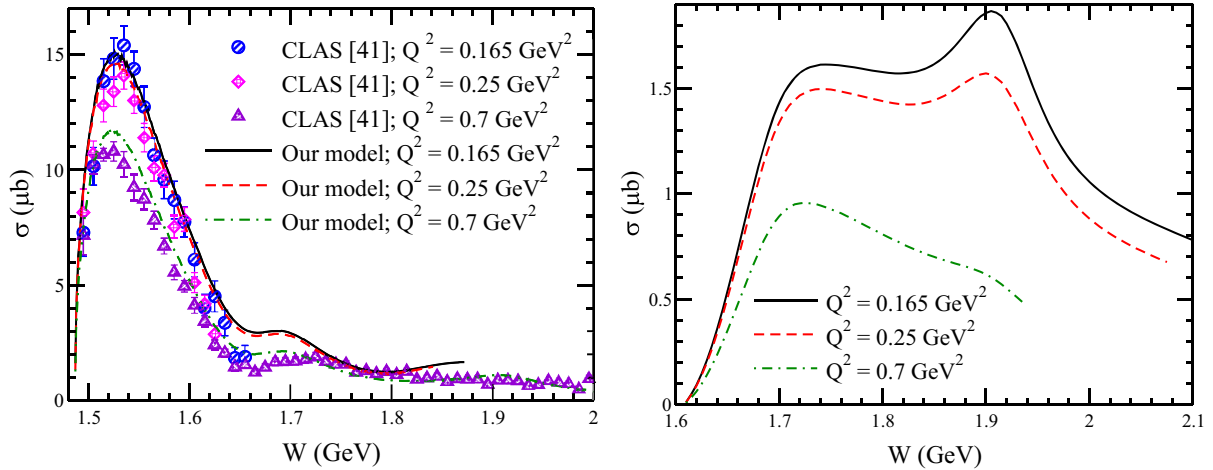


FIG. 3. Left panel: integrated cross section σ vs W at different Q^2 for $\gamma^* p \rightarrow \eta p$ process. The solid line is the result at $Q^2 = 0.165$ GeV², the dashed line is the result at $Q^2 = 0.25$ GeV² and the dot-dashed line is the result at $Q^2 = 0.7$ GeV². These results are presented using the full model which receives contribution from the nonresonant Born terms as well as from the nucleon resonance excitations [43]. The experimental points are the CLAS 2007 data [41]. Right panel: integrated cross section σ vs W at different Q^2 for $\gamma^* p \rightarrow K\Lambda$ process. The solid line is the result at $Q^2 = 0.165$ GeV², the dashed line is the result at $Q^2 = 0.25$ GeV², and the dot-dashed line is the result at $Q^2 = 0.7$ GeV². These result are presented using the full model which receives contribution from the nonresonant Born terms, kaon resonance exchanges, and the nucleon resonance excitations.

the angular distribution ($\frac{d\sigma}{d\Omega_{qp\eta}}$) given in Eq. (35) over the polar and azimuthal angles as a function of c.m. energy W at different values of Q^2 , viz., $Q^2 = 0.165 \text{ GeV}^2$ (solid line), 0.25 GeV^2 (dashed line), and 0.7 GeV^2 (dot-dashed line). The theoretical calculations are presented for the full model, which receives contribution from the nonresonant Born terms as well as from the $S_{11}(1535)$, $S_{11}(1650)$, $P_{11}(1710)$, $P_{11}(1880)$, and $S_{11}(1895)$ resonance excitations. We have compared our theoretical results with the experimental data available from the CLAS experiment [41] and found very good agreement between them at all values of Q^2 . This has been discussed in Ref. [43] in more detail. In the right panel of Fig. 3, we have shown the results for the total cross section σ for $\gamma^* p \rightarrow \Lambda K$ at the three different values of Q^2 , viz., $Q^2 = 0.165 \text{ GeV}^2$ (solid line), 0.25 GeV^2 (dashed line), and 0.7 GeV^2 (dot-dashed line).

III. WEAK PRODUCTION OF ΛK

The weak charged current associated particle production processes induced by the (anti)neutrinos are given by the reactions

$$\nu_\mu(k) + n(p) \longrightarrow \mu^-(k') + \Lambda(p') + K^+(p_K), \quad (59)$$

$$\bar{\nu}_\mu(k) + p(p) \longrightarrow \mu^+(k') + \Lambda(p') + K^0(p_K), \quad (60)$$

where the quantities in the parentheses represent the 4-momentum of the corresponding particles.

The differential scattering cross section $\frac{d\sigma}{dQ^2}$ for the reactions shown in Eqs. (59) and (60) is expressed as

$$\frac{d\sigma}{dQ^2} = \int_{W_{\min}}^{W_{\max}} dW \int_0^{2\pi} d\phi_{qpK} \times \int_{E_K^{\min}}^{E_K^{\max}} dE_K \frac{1}{(2\pi)^4} \frac{1}{64E_\nu^2 M^2} \frac{W}{|\vec{q}|} \overline{\sum} \sum |\mathcal{M}|^2, \quad (61)$$

where $W_{\min}(=M_\Lambda + M_K)$ and $W_{\max}$ are, respectively, the minimum and maximum c.m. energies; $E_K^{\min}$ and $E_K^{\max}$ are, respectively, the minimum and maximum energy of the outgoing kaon; and ϕ_{qpK} is the azimuthal angle between the scattering plane and the reaction plane as shown in Fig. 4. $\overline{\sum} \sum |\mathcal{M}|^2$ is the square of the transition amplitude averaged (summed) over the spins of the initial (final) states, and the transition matrix element is written in terms of the leptonic and the hadronic currents as

$$\mathcal{M} = \cos \theta_c \frac{G_F}{\sqrt{2}} l_\mu J^\mu, \quad (62)$$

where $\theta_c(=13.1^\circ)$ is the Cabibbo angle and $G_F(=1.166 \times 10^{-5} \text{ GeV}^{-2})$ is the Fermi coupling constant. The leptonic current l_μ is given by

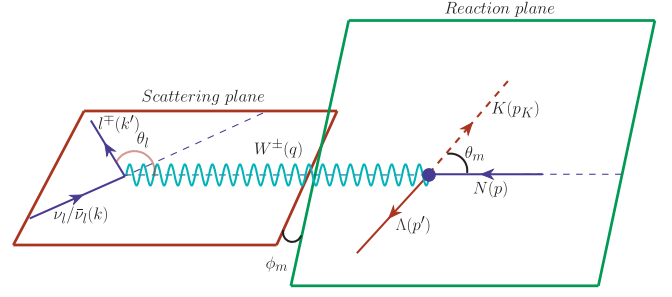


FIG. 4. (Anti)neutrino scattering and reaction planes.

$$l_\mu = \bar{u}(k') \gamma_\mu (1 \mp \gamma_5) u(k), \quad (63)$$

where the $-(+)$ sign stands for the neutrino (antineutrino) induced reaction and J^μ is the hadronic current for $W^\pm + N \rightarrow \Lambda + K$ interaction. The total hadronic current J^μ is written as the sum of the contributions from the nonresonant Born terms (J_{NR}^μ) and the resonance contributions from the spin- $\frac{1}{2}$ resonances ($J_{R_{\frac{1}{2}}}^\mu$) and spin- $\frac{3}{2}$ resonance ($J_{R_{\frac{3}{2}}}^\mu$), i.e.,

$$J^\mu = J_{\text{NR}}^\mu + J_{R_{\frac{1}{2}}}^\mu + J_{R_{\frac{3}{2}}}^\mu. \quad (64)$$

A. Nonresonant contribution

The nonresonant terms are calculated using the nonlinear sigma model [1,44], and the expressions for the hadronic currents corresponding to the diagrams shown in Fig. 5 are obtained as

$$\begin{aligned} j^\mu|_s &= iA_{SY} \bar{u}(p') \not{p}_K \gamma_5 \frac{\not{p} + \not{q} + M}{(p+q)^2 - M^2} [V^\mu - A^\mu] u(p) \\ j^\mu|_{PF} &= iA_{TY} f_{PF}(Q^2) (M + M_\Lambda) \bar{u}(p') \gamma_5 u(p) \frac{2p_K^\mu - q^\mu}{(p-p')^2 - m_K^2} \\ j^\mu|_u &= iA_{UY} \bar{u}(p') [V^\mu - A^\mu] \frac{\not{p} - \not{p}_K + M_\Sigma}{(p-p_K)^2 - M_\Sigma^2} \not{p}_K \gamma_5 u(p) \\ j^\mu|_{CT} &= iA_{CT} \bar{u}(p') [\gamma^\mu f_\rho((q-p_K)^2) + B_{CT} f_{CT}(Q^2) \gamma^\mu \gamma_5] u(p) \\ j^\mu|_{PP} &= iA_\pi f_\rho((q-p_K)^2) \bar{u}(p') [\not{q} + \not{p}_K] u(p) \frac{q^\mu}{q^2 - m_\pi^2}, \end{aligned} \quad (65)$$

where

$$V^\mu = f_1^{BB'}(Q^2) \gamma^\mu + i \frac{f_2^{BB'}(Q^2)}{M + M'} \sigma^{\mu\nu} q_\nu \quad (66)$$

$$A^\mu = g_1^{BB'}(Q^2) \gamma^\mu \gamma_5 + g_3^{BB'}(Q^2) \frac{2q^\mu}{M + M'} \gamma_5 \quad (67)$$

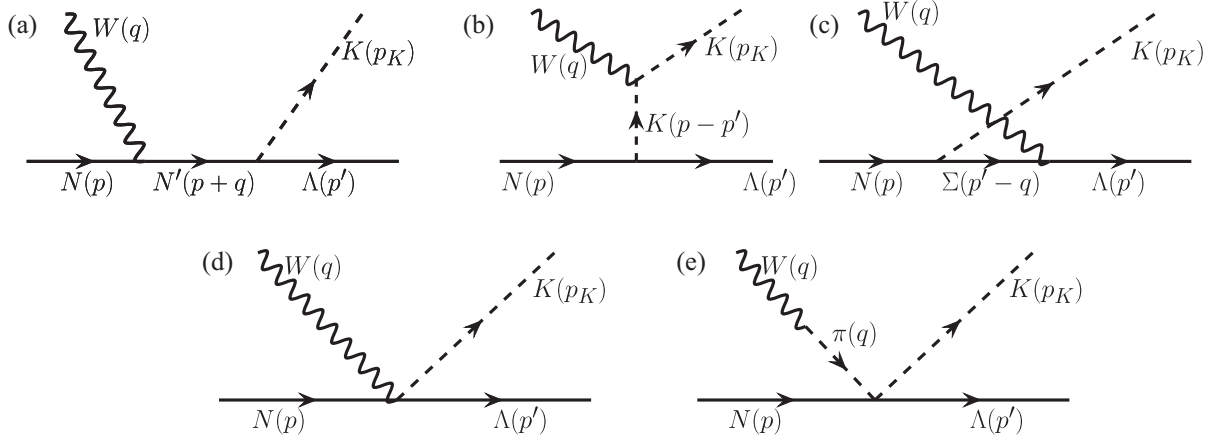


FIG. 5. Feynman diagrams corresponding to the nonresonant background terms of the (anti)neutrino induced $\Delta S = 0$ associated particle production processes. (a) s -channel, (b) t -channel or pion in flight (PF) term, (c) u -channel, (d) contact term (CT), and (e) pion pole (PP) term constitute the nonresonant Born diagrams. The wiggly line represents the intermediate vector meson W , the solid line represents the baryons like nucleon and hyperons, and the dashed line represents the meson like the pion and kaon.

are the vector (V^μ) and axial-vector (A^μ) transition currents for $BB' = NN'$ for the s -channel diagram and $BB' = YY'$ for the u -channel diagram, with $N, N' = n, p$ and $Y, Y' = \Lambda, \Sigma^\pm$. The constant factors appearing in Eq. (65) have been tabulated in Table IV. The vector and axial-vector form factors $f_{1,2}^{BB'}(Q^2)$ and $g_{1,3}^{BB'}(Q^2)$ are determined assuming the Cabibbo theory and the various symmetry properties of the weak hadronic current (for details see Ref. [1]). The isovector vector form factors $f_{1,2}^{BB'}(Q^2)$ are expressed in terms of the Dirac [$F_1^{p,n}(Q^2)$] and Pauli [$F_2^{p,n}(Q^2)$] form factors, discussed in Sec. II B, for the proton and the neutron, using the relationships

$$f_{1,2}^{NN'}(Q^2) = F_{1,2}^p(Q^2) - F_{1,2}^n(Q^2), \quad (68)$$

$$f_{1,2}^{YY'}(Q^2) = -\sqrt{\frac{3}{2}} F_{1,2}^n(Q^2). \quad (69)$$

These electromagnetic form factors may be rewritten in terms of the electric [$G_E^N(Q^2)$] and magnetic [$G_M^N(Q^2)$] Sachs form factors.

The axial-vector form factor $g_1^{BB'}(Q^2)$ is parametrized as

$$g_1^{BB'}(Q^2) = g_1^{BB'}(0) \left[1 + \frac{Q^2}{M_A^2} \right]^{-2}, \quad (70)$$

where $g_1^{NN'}(0) = D + F = 1.267$ and $g_1^{YY'}(0) = -\sqrt{\frac{3}{2}}D$ are the axial-vector charges for the $N - N'$ and $Y - Y'$ transitions, respectively, and M_A is the axial dipole mass, which in the numerical calculations is taken as the world average value, i.e., $M_A = 1.026$ GeV [54].

On the other hand, the pseudoscalar form factor $g_3^{BB'}(Q^2)$ is expressed in terms of $g_1^{BB'}(Q^2)$ using the PCAC hypothesis and Goldberger-Treiman relation as

$$g_3^{BB'}(Q^2) = \frac{(M + M')^2 g_1^{BB'}(Q^2)}{2(m_\pi^2 + Q^2)}, \quad (71)$$

with m_π being the pion mass.

The form factors $f_{CT}(Q^2)$, $f_{PF}(Q^2)$ and $f_\rho((q - p_K)^2)$ are introduced in the contact, pion pole, and pion in flight terms to take into account the hadronic structure. It may be observed from the Feynman diagrams (Fig. 5) that the pion in the flight term is purely vector in nature while the pion pole diagram is possible only with axial-vector current. In the case of contact term, the term associated with B_{CT} represents the vector part of the weak hadronic current, while the term with γ^μ is associated with the axial-vector part. CVC hypothesis imposes the following condition on the form factors $f_{CT}(Q^2)$ and $f_{PF}(Q^2)$,

TABLE IV. Constant factors appearing in the hadronic current in Eq. (65).

A_{CT}	B_{CT}	A_{SY}	A_{UY}		A_{TY}	A_π
			$Y' = \Sigma$	$Y' = \Lambda$		
$-\frac{\sqrt{3}}{2f_K}$	$\frac{-1}{3}(D + 3F)$	$\frac{D+3F}{2\sqrt{3}f_K}$	$\frac{\sqrt{2}}{3f_K}(D - F)$	0	$\frac{D+3F}{2\sqrt{3}f_K}$	$-\frac{\sqrt{3}}{4f_K}$

$$f_{CT}(Q^2) = f_1^{NN'}(Q^2) - 2F_1^n(Q^2) \left(\frac{D-F}{D+3F} \right) \times \left(\frac{u - M_\Sigma M_\Lambda + MM_\Sigma - MM_\Lambda}{M_\Sigma^2 - u} \right), \quad (72)$$

$$f_{PF}(Q^2) = 2F_1^n(Q^2) \left(\frac{D-F}{D+3F} \right) \times \left(\frac{(M+M_\Sigma)(u-M_\Lambda^2)}{(M+M_\Lambda)(M_\Sigma^2-u)} \right) - f_1^{NN'}(Q^2), \quad (73)$$

where $u = (p - p_K)^2$, $f_1^{NN'}(Q^2) = F_1^p(Q^2) - F_1^n(Q^2)$ is the vector form factor with $F_1^p(Q^2)$ and $F_1^n(Q^2)$ being the nucleon electromagnetic form factors. The form factor $f_\rho(Q^2)$ corresponding to the axial-vector current is given by [1,55]

$$f_\rho(Q^2) = \frac{1}{1 + Q^2/m_\rho^2}; \quad \text{with } m_\rho = 0.776 \text{ GeV}. \quad (74)$$

B. Resonance contribution

1. Spin- $\frac{1}{2}$ resonance excitations

Next, we discuss the positive and negative parity resonance excitation mechanisms for the weak interaction induced $K\Lambda$ production. The general expression of the hadronic current for the s -channel resonance excitation and their subsequent decay to $K\Lambda$ mode are given in Eq. (46), where the vertex factor $\Gamma_{\frac{1}{2}\pm}^\mu$ is now written as

$$\Gamma_{\frac{1}{2}+}^\mu = V_{\frac{1}{2}}^\mu - A_{\frac{1}{2}}^\mu, \quad (75)$$

for the positive parity resonance, and as

$$\Gamma_{\frac{1}{2}-}^\mu = (V_{\frac{1}{2}}^\mu - A_{\frac{1}{2}}^\mu)\gamma_5, \quad (76)$$

for the negative parity resonance. The vector and axial-vector vertex factors for the weak charged current interaction processes are given by

$$V_{\frac{1}{2}}^\mu = \frac{f_1^{CC}(Q^2)}{(2M)^2} (Q^2 \gamma^\mu + \not{q} q^\mu) + \frac{f_2^{CC}(Q^2)}{2M} i\sigma^{\mu\alpha} q_\alpha, \quad (77)$$

$$A_{\frac{1}{2}}^\mu = \left[g_1^{CC}(Q^2) \gamma^\mu + \frac{g_3^{CC}(Q^2)}{M} q^\mu \right] \gamma_5, \quad (78)$$

where $f_i^{CC}(Q^2)$ ($i = 1, 2$) are the isovector $N - R$ transition form factors which, in turn, are expressed in terms of the charged $[F_i^{R+}(Q^2)]$ and neutral $[F_i^{R0}(Q^2)]$ electromagnetic $N - R$ transition form factors as

$$f_i^{CC}(Q^2) = F_i^{R+}(Q^2) - F_i^{R0}(Q^2), \quad i = 1, 2. \quad (79)$$

Further, these form factors are related to the helicity amplitudes as discussed in Sec. II B.

The axial-vector current consists of two form factors, viz., $g_1^{CC}(Q^2)$ and $g_3^{CC}(Q^2)$, which are determined assuming the PCAC hypothesis and the pion pole dominance of the divergence of the axial-vector current through the generalized GT relation for $N - R$ transition [1].

The axial-vector coupling g_1^{CC} at $Q^2 = 0$ is obtained as [42]

$$g_1^{CC}(0) = 2g_{RN\pi}, \quad (80)$$

with $g_{RN\pi}$ being the coupling strength for $R \rightarrow N\pi$ decay, which has been determined by the partial decay width of the resonance and tabulated in Table I. Since no information about the Q^2 dependence of the axial-vector form factor for the $N - R$ transition is known experimentally, a dipole form is assumed,

$$g_1^{CC}(Q^2) = \frac{g_1^{CC}(0)}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}, \quad (81)$$

with $M_A = 1.026$ GeV, and the pseudoscalar form factor $g_3^{CC}(Q^2)$ is given by

$$g_3^{CC}(Q^2) = \frac{(MM_R \pm M^2)}{m_\pi^2 + Q^2} g_1^{CC}(Q^2), \quad (82)$$

where the $+$ ($-$) sign is for positive (negative) parity resonances. However, the contribution of $g_3^{CC}(Q^2)$ being directly proportional to the lepton mass is almost negligible for the charged current $\nu_\mu(\bar{\nu}_\mu)$ induced processes.

2. Spin- $\frac{3}{2}$ resonance excitations

In the case of positive parity spin- $\frac{3}{2}$ resonance excitation and its subsequent decay to the $K\Lambda$ channel, the expression of the hadronic current for the s -channel diagram is given in Eq. (52), where the vertex factor $\Gamma_{\frac{3}{2}}^{\mu\alpha}$ is now written as

$$\Gamma_{\frac{3}{2}}^{\mu\alpha} = V_{\frac{3}{2}}^{\mu\alpha} - A_{\frac{3}{2}}^{\mu\alpha}. \quad (83)$$

The vector and axial-vector vertex factors for the weak charged current interaction processes are given by

$$V_{\frac{3}{2}}^{\alpha\mu} = (\gamma^\mu q^\alpha - \not{q} g^{\alpha\mu}) \frac{C_3^V(Q^2)}{M} + (q \cdot p' g^{\alpha\mu} - q^\alpha p'^\mu) \frac{C_4^V(Q^2)}{M^2} + (q^\alpha q^\mu - q^2 g^{\alpha\mu}) \frac{C_5^V(Q^2)}{M^2}, \quad (84)$$

$$A_{\frac{3}{2}}^{\alpha\mu} = - \left[\frac{C_3^A(Q^2)}{M} (g^{\mu\alpha} \not{q} - q^\alpha \not{\gamma}^\mu) + \frac{C_4^A(Q^2)}{M^2} (g^{\mu\alpha} \not{q} \cdot \not{p}' - q^\alpha \not{p}'^\mu) + C_5^A(Q^2) g^{\mu\alpha} + \frac{C_6^A(Q^2)}{M^2} q^\alpha \not{q}^\mu \right] \gamma_5, \quad (85)$$

where $C_i^V(Q^2)$ and $C_i^A(Q^2)$ are the vector and axial-vector form factors.

The isovector $C_i^V(Q^2)$ ($i = 3, 4, 5$) form factors are written in terms of the electromagnetic $C_i^{R+}(Q^2)$ and $C_i^{R0}(Q^2)$ form factors through a simple relation [2] as

$$C_i^V(Q^2) = C_i^{R+}(Q^2) - C_i^{R0}(Q^2), \quad i = 3, 4, 5. \quad (86)$$

Further, the electromagnetic form factors are related to the helicity amplitudes as discussed in Sec. II B.

The form factors $C_i^A(Q^2)$ ($i = 3, 4, 5, 6$) corresponding to the axial current have not been studied in the case of higher resonances. The earlier calculations have used PCAC to determine $C_5^A(Q^2)$ and $C_6^A(Q^2)$ and taken other form factors to be zero [56]. In view of this, we have also taken a simple model for the determination of the axial form factors based on PCAC and GT relation and write $C_6^A(Q^2)$ in terms of $C_5^A(Q^2)$ as

$$C_6^A(Q^2) = C_5^A(Q^2) \frac{M^2}{Q^2 + m_\pi^2}. \quad (87)$$

For $C_5^A(Q^2)$, a dipole form has been assumed,

$$C_5^A(Q^2) = \frac{C_5^A(0)}{(1 + Q^2/M_A^2)^2}, \quad (88)$$

with $C_5^A(0) = -2g_{RN\pi}$ [1] for the isospin- $\frac{1}{2}$ resonances considered in the present work, with $g_{RN\pi}$ being the coupling for $R \rightarrow N\pi$ decay for each resonance R . M_A^R is taken as 1.026 GeV. $C_3^A(Q^2)$ as well as $C_4^A(Q^2)$ are taken as zero.

3. Spin-1 kaon resonances

We have taken the same form of the hadronic current for the spin 1 kaon resonances being exchanged in the t -channel as that in the case of electroproduction, discussed in Sec. II B 2.

IV. RESULT AND DISCUSSION

In this section, we present the numerical results for the (anti)neutrino induced $K\Lambda$ production from the nucleon target, using the following inputs for evaluating the contributions from the various terms:

- (i) In the case of Born terms:
 - (a) The nucleon and hyperon weak vector form factors are expressed in terms of the electromagnetic form factors of the nucleons, which in

turn are expressed in terms of the Sachs electric and magnetic form factors of the nucleons, for which the BBBA05 parametrization is used.

- (b) The nucleon and hyperon axial vector form factors are obtained assuming the PCAC hypothesis and the generalized Goldberger-Treiman relation.
- (ii) In the spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ resonances:
 - (a) The value of the strong coupling constant at the $R \rightarrow K\Lambda$ vertex, determined from the photo-production of $K\Lambda$, is used.
 - (b) The weak vector $N - R$ transition form factors are expressed in terms of the electromagnetic $N - R$ transition form factors, as discussed in Sec. III.
 - (c) For the determination of the axial vector $N - R$ transition form factors, the PCAC hypothesis along with the generalized Goldberger-Treiman relation is used. For the Q^2 dependence of the axial vector $N - R$ transition form factor, a dipole form factor is used with $M_A = 1.026$ GeV.

In Fig. 6, we have presented the results for the total cross section σ for $\nu_\mu n \rightarrow \mu^- \Lambda K^+$ (left panel) and $\bar{\nu}_\mu n \rightarrow \mu^+ \Lambda K^0$ (right panel) processes as a function of incoming (anti)neutrino energy. These results have been presented when no cut on Q^2 and W is applied, for the full model (solid line) as well as for the individual contributions from several spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ resonances, viz., $S_{11}(1650)$ (solid line with solid circles), $P_{11}(1710)$ (solid line with solid squares), $P_{13}(1720)$ (solid line with solid triangles), $P_{11}(1880)$ (dashed line), $S_{11}(1895)$ (double dash-dotted line), and $P_{13}(1900)$ (double dot-dashed line) resonances, considered in this work. For comparison, the results for the background contribution are also presented and depicted by the dotted line. We find the contributions of $P_{11}(1710)$ and $P_{13}(1720)$ resonances as well as the background terms to be quite significant for both the neutrino as well as the antineutrino induced processes. Quantitatively, in the case of neutrino induced $K\Lambda$ production channel, the $P_{11}(1710)$ resonance has a contribution of about 55% at $E_{\nu_\mu} = 1.5$ GeV, which decreases with the increase in neutrino energy and becomes 38% at $E_{\nu_\mu} = 2.5$ GeV. The $P_{13}(1720)$ resonance has a contribution of about 30% in the energy range of $E_{\nu_\mu} = 1.5$ –2.5 GeV. While the nonresonant background terms have a contribution of about 17% at $E_{\nu_\mu} = 1.5$ GeV, which increases with the increase in neutrino energy and becomes 28% at $E_{\nu_\mu} = 2.5$ GeV. On the other hand, in the case of antineutrino induced reaction, the $P_{11}(1710)$ resonance has a contribution of about 40% at $E_{\bar{\nu}_\mu} = 1.5$ GeV, which decreases with the increase in antineutrino energy and becomes 34% at $E_{\bar{\nu}_\mu} = 2.5$ GeV. The $P_{13}(1720)$ resonance has a contribution of about 22% at $E_{\bar{\nu}_\mu} = 1.5$ GeV, which

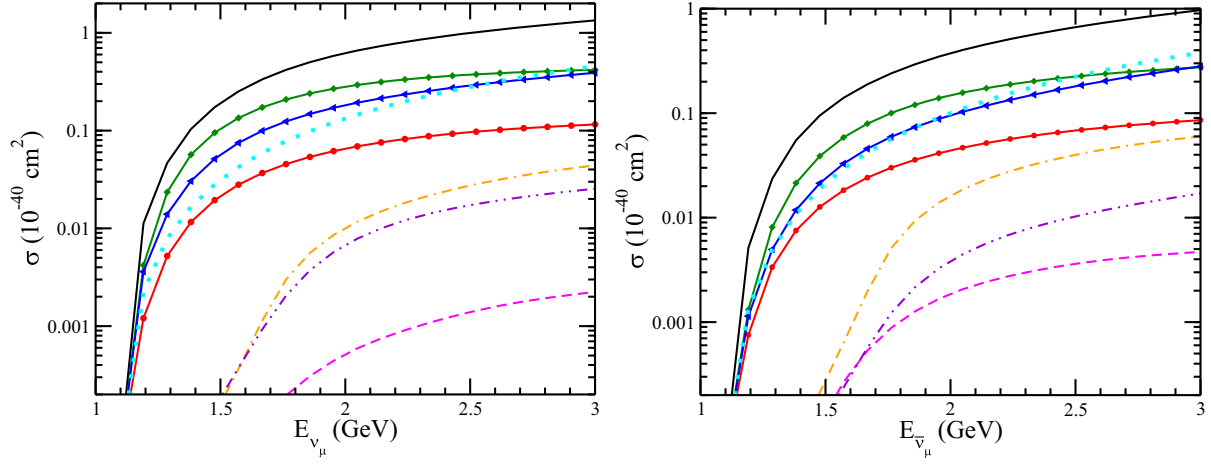


FIG. 6. Cross section for $\nu_\mu + n \rightarrow \mu^- + \Lambda + K^+$ (left panel) and $\bar{\nu}_\mu + p \rightarrow \mu^+ + \Lambda + K^0$ (right panel) processes. Solid (dotted) line represents the results of the full model (only nonresonant Born terms); the solid line with the circle, diamond, and left triangle, respectively, represents the individual contribution from $S_{11}(1650)$, $P_{11}(1710)$, and $P_{13}(1720)$ resonances. The dashed line, double-dash-dotted line, and double-dot dashed line, respectively, represents the individual contribution from $P_{11}(1880)$, $S_{11}(1895)$, and $P_{13}(1900)$ resonances.

increases with the increase in antineutrino energy and becomes 27% at $E_{\bar{\nu}_\mu} = 2.5$ GeV, whereas the nonresonant background terms have a contribution of about 20% at $E_{\bar{\nu}_\mu} = 1.5$ GeV, which increases with the increase in antineutrino energy and becomes 34% at $E_{\bar{\nu}_\mu} = 2.5$ GeV.

In the few GeV energy region, the understanding of the transition from the hadronic regime, where hadrons are the effective degrees of freedom, to the deep inelastic regime, governed by the perturbative QCD dynamics of quarks and gluons, is of significant importance. The intermediate region is commonly referred to as the shallow inelastic scattering (SIS) region. However, defining the exact kinematic boundaries between SIS and deep inelastic scattering (DIS) remains a challenge due to the limited theoretical understanding and a lack of high-statistics experimental data. Because of this scarcity of experimental data in the SIS region, studies of neutrino-nucleon and neutrino-nucleus scattering are primarily guided by the theoretical models. Athar and Morfin [57] have recently highlighted this issue, proposing the kinematic constraints of $W \geq 2$ GeV and $Q^2 \geq 1$ GeV² as cuts to distinguish contributions from the resonance and DIS regions. Nevertheless, defining a boundary between SIS and DIS is not straightforward as different experiments apply varying cuts on W , making comparisons across the datasets difficult. Following Ref. [57], we have considered the region of $W < 2$ GeV and all values of Q^2 , as well as the region of $W > 2$ GeV and $Q^2 < 1$ GeV² as the inelastic region, and obtained the results for the $K\Lambda$ production, while the region of $W \geq 2$ GeV and $Q^2 \geq 1$ GeV² has been excluded considering it to be purely the DIS regime. However, the present choice on the numerical values of cuts on W and Q^2 is motivated by the values used in the literature [57,58], and

more study is required to understand the transition region from resonance to DIS. In view of this, we have presented the results for the total cross section σ for $\nu_\mu n \rightarrow \mu^- \Lambda K^+$ (left panel) and $\bar{\nu}_\mu n \rightarrow \mu^+ \Lambda K^0$ (right panel) processes as a function of incoming (anti)neutrino energy in Fig. 7, by considering four different cases:

- (i) without applying any cuts on W and Q^2 (shown by the solid line),
- (ii) when an upper cut of 2 GeV on W is applied, i.e., $W < 2$ GeV but without any cut on Q^2 (shown by the dashed line),
- (iii) when the cuts on W and Q^2 are applied, i.e., $W > 2$ GeV and $Q^2 < 1$ GeV² (shown by the dot-dashed line),
- (iv) when a cut on $W < 2$ GeV is applied without any cut on Q^2 , and also included in this result are the contributions from $W > 2$ GeV having $Q^2 < 1$ GeV² (shown by the double-dash dotted line).

It may be observed from Fig. 7 that excluding the region of $W > 2$ GeV reduces the cross section for $E_\nu > 2$ GeV, whereas, when the cross sections are evaluated by considering the contributions from $W < 2$ GeV with no cut on Q^2 and $W > 2$ GeV and $Q^2 < 1$ GeV², we find the results to be very close to the ones obtained by applying no constraints on W and Q^2 ; i.e., the results shown by double-dash dotted line vs the solid line are close by for the (anti) neutrino energies considered in this work.

In Fig. 8, we compare our results of the total scattering cross section with the old theoretical calculations and experimental results available in the energy region of a few GeV. The experimental data are from the BNL Collaboration [21], and the theoretical results are from Shrock [9], obtained using the Born approximation

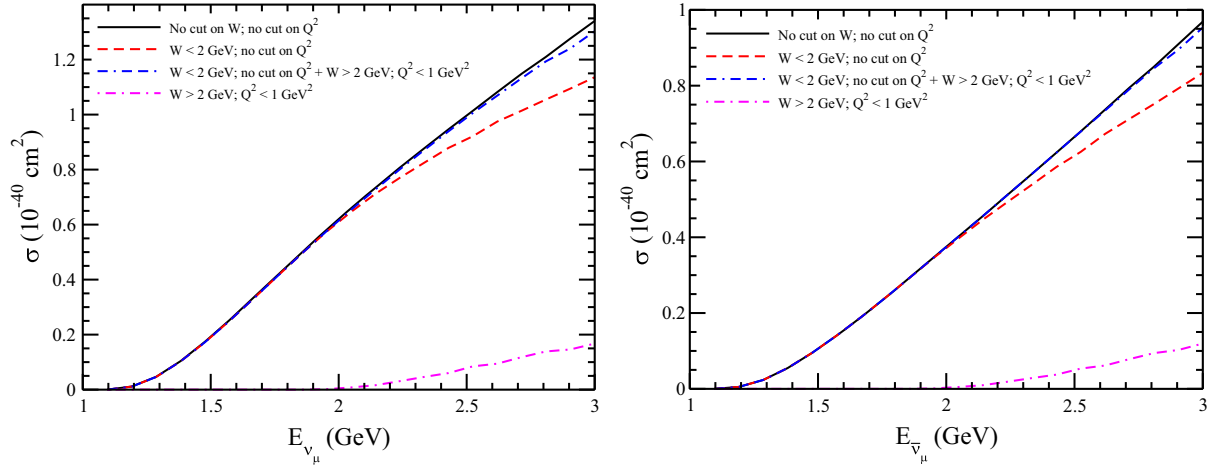


FIG. 7. Cross section for $\nu_\mu + n \rightarrow \mu^- + \Lambda + K^+$ (left panel) and $\bar{\nu}_\mu + p \rightarrow \mu^+ + \Lambda + K^0$ (right panel) processes. Solid line represents the results of the full model when no cuts on W and Q^2 are included in the numerical calculations. The dashed line represents the results of the full model when a cut of $W < 2$ GeV and no cut on Q^2 is applied. The dot-dashed line represents the results of the full model when a cut of $W > 2$ GeV and $Q^2 < 1$ GeV² is applied. The double-dashed line represents the results of the full model when a cut of $W < 2$ GeV and no cut on Q^2 is applied along with the contribution of the cross section when $W > 2$ GeV and $Q^2 < 1$ GeV² cuts are applied.

and Nakamura *et al.* [14] obtained in the DCC model. Moreover, Shrock [9] and Mecklenburg [10] have also reported the results for the Argonne neutrino flux-averaged cross sections ($\langle\sigma\rangle$) as 3.4×10^{-42} and 39.1×10^{-42} cm², respectively, to be compared with the experimentally derived data of the ANL experiment [17], i.e., $\langle\sigma\rangle = 20 \times 10^{-42}$ cm².

In Fig. 9, we present the results for the Q^2 distribution, i.e., $\frac{d\sigma}{dQ^2}$ vs Q^2 at two representative (anti)neutrino energies viz. $E_\nu = 1.5$ GeV (lower curves) and 2.5 GeV (upper curves) for $\nu_\mu n \rightarrow \mu^- \Lambda K^+$ (left panel) and $\bar{\nu}_\mu p \rightarrow \mu^+ \Lambda K^0$ (right panel) processes. These results are presented using the full contribution to the scattering amplitude, i.e.,

resonant as well as nonresonant terms. These results are presented for the two different cases: (i) when no cut on W is applied and (ii) when a cut of 2 GeV on W is applied, i.e., $W < 2$ GeV. It may be observed from the figure that the effect of the W cut is quite important in the peak region of Q^2 , which decreases with the increase in Q^2 and becomes almost negligible at $Q^2 \geq 1$ GeV². The Q^2 distribution is more sharply peaked for the $K\Lambda$ production induced by the antineutrinos than for the neutrinos because of the different sign of the interference terms of the vector and axial vector form factors. The neutrino induced reactions occur with a constructive interference term, which makes the cross section more pronounced in the low Q^2 region, while

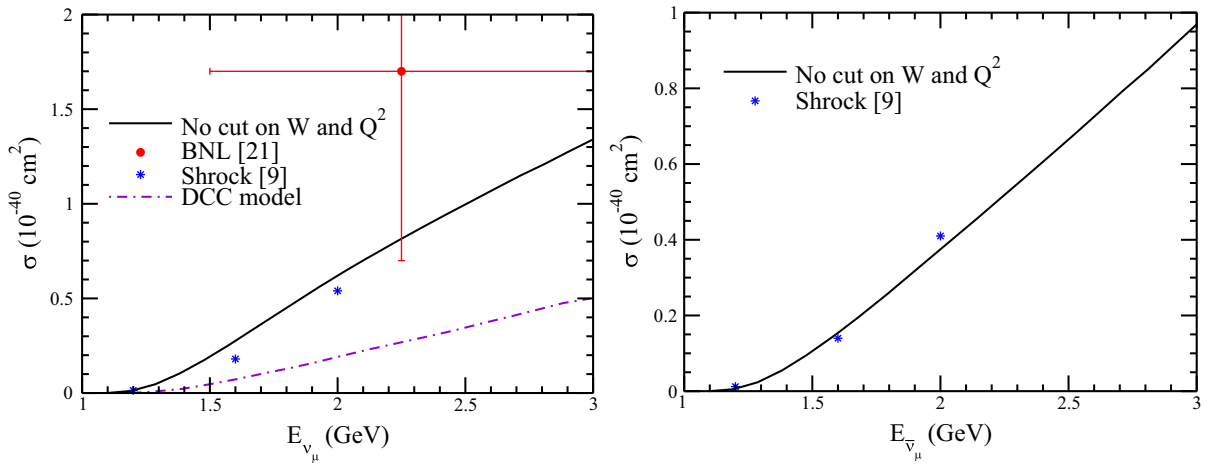


FIG. 8. Cross section for $\nu_\mu + n \rightarrow \mu^- + \Lambda + K^+$ (left panel) and $\bar{\nu}_\mu + p \rightarrow \mu^+ + \Lambda + K^0$ (right panel) processes. The solid line represents the results of the full model when no cuts on W and Q^2 are included in the numerical calculations. The dot-dashed line represents the results of the DCC model [14], while the star represents the results tabulated in Table II of the work by Shrock [9]. The solid circle is the experimental results of the BNL experiment [21].

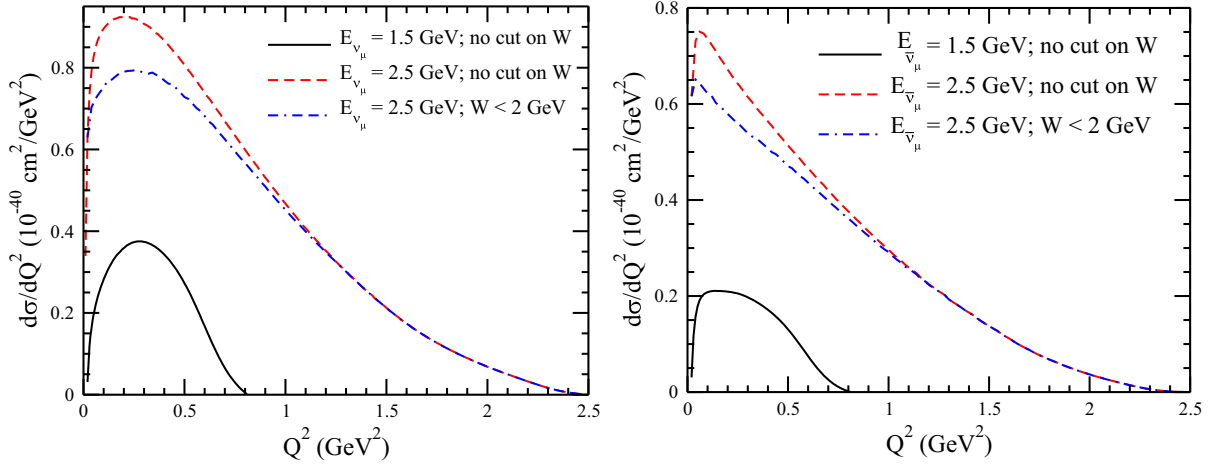


FIG. 9. $\frac{d\sigma}{dQ^2}$ vs Q^2 for $\nu_\mu + n \rightarrow \mu^- + \Lambda + K^+$ (left panel) and $\bar{\nu}_\mu + p \rightarrow \mu^+ + \Lambda + K^0$ (right panel) processes at two different (anti)neutrino energies, viz., 1.5 (lower curves) and 2.5 GeV (upper curves). The solid line is the result when no cut on W is applied for (anti)neutrino energy of 1.5 GeV, and the dashed line (double-dash dotted line) is the result when no cut on W (a cut of $W < 2$ GeV) is applied for (anti)neutrino energy of 2.5 GeV.

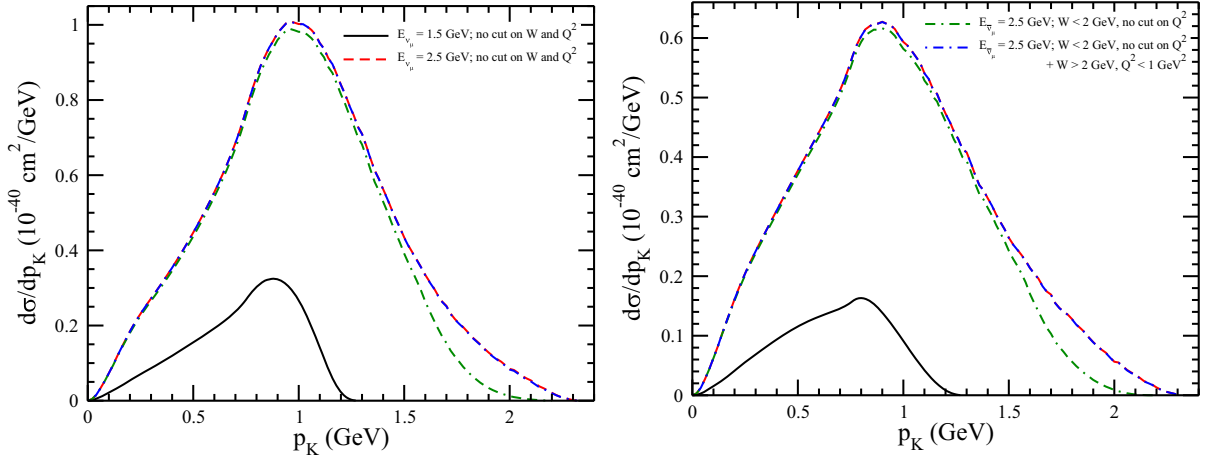


FIG. 10. $\frac{d\sigma}{dp_K}$ vs p_K for $\nu_\mu + n \rightarrow \mu^- + \Lambda + K^+$ (left panel) and $\bar{\nu}_\mu + p \rightarrow \mu^+ + \Lambda + K^0$ (right panel) processes. The solid line is the result when no cut on W is applied for (anti)neutrino energy of 1.5 GeV, and the dashed line (double-dash dotted line) is the result when no cut on W (a cut of $W < 2$ GeV) is applied for (anti)neutrino energy of 2.5 GeV. The dot-dashed line represents the results at an (anti)neutrino energy of 2.5 GeV, when the cuts of $W < 2$ GeV and $W \geq 2$ GeV with $Q^2 < 1$ GeV² are applied while evaluating the numerical calculations.

the antineutrino induced reactions occur with a destructive interference making the cross section more sharply peaked in the low Q^2 region.

In Fig. 10, we present the results for the kaon momentum distribution, i.e., $\frac{d\sigma}{dp_K}$ vs p_K at two representative (anti)neutrino energies, viz., $E_\nu = 1.5$ GeV (lower curves) and 2.5 GeV (upper curves) for $\nu_\mu n \rightarrow \mu^- \Lambda K^+$ (left panel) and $\bar{\nu}_\mu n \rightarrow \mu^+ \Lambda K^0$ (right panel) processes. These results are presented using the full contribution to the scattering amplitude, i.e., resonant as well as nonresonant terms. These results are presented for the three different cases, first when no cut on W and Q^2 is applied; the second when a cut of 2 GeV on W is applied, i.e., $W < 2$ GeV without

applying any cut on Q^2 ; and the third one by considering the case of $W < 2$ GeV and no cut on Q^2 along with the contribution when $W > 2$ GeV but $Q^2 < 1$ GeV². It may be observed that the results obtained by considering the combined effect of $W < 2$ GeV with no cut on Q^2 and $W > 2$ GeV with $Q^2 < 1$ GeV² are almost comparable to the results obtained using no cut on W and Q^2 .

V. CONCLUSION

We have studied the charged current $\nu_\mu(\bar{\nu}_\mu)$ induced KA production off the nucleon target in the few GeV energy region, which is relevant for the ongoing and proposed accelerator and atmospheric based neutrino experiments.

The results are presented for the total scattering cross section and compared with the available theoretical and experimental works in the literature. The results are also presented for the Q^2 distribution and the kaon momentum distribution.

The calculations have been performed using a model based on the effective Lagrangians to evaluate the contribution from the nonresonant and resonant diagrams. The contribution from the nonresonant background terms has been obtained using a microscopic model based on the SU(3) chiral Lagrangians. The contribution from the resonant diagrams due to the low-lying spin- $\frac{1}{2}$ resonances like $S_{11}(1650)$, $P_{11}(1710)$, $P_{11}(1880)$, and $S_{11}(1895)$ and spin- $\frac{3}{2}$ resonances like $P_{13}(1720)$ and $P_{13}(1900)$, for which there is finite branching ratios to the $K\Lambda$ channel, has been considered using an effective phenomenological Lagrangian with its parameters determined from the experimental values of their branching ratios and decay widths to the $K\Lambda$ channel. The vector form factors in the nucleon-resonance transition are expressed in terms of the electromagnetic $N - R$ form factors using the isospin symmetry, which in turn are determined in terms of the helicity amplitudes of the resonances. The PCAC hypothesis and generalized Goldberger-Treiman relation are used to fix the parameters of the axial-vector current interaction. First, the model has been used to study the photo- and electro-production of $K\Lambda$ and η meson, to fix the vector part of the weak hadronic current.

We find that:

- (i) The present model explains the electromagnetic induced ΛK and η production data quite well. This is used to fix the various parameters of the vector current interaction in the (anti)neutrino induced associated particle production.
- (ii) The results of the total scattering cross section for (anti)neutrino induced ΛK production off the nucleon target has contributions from the resonant and nonresonant background terms, which are both important. The resonance contribution is dominated by the $P_{11}(1710)$ and $P_{13}(1720)$ resonances.
- (iii) The effect of c.m. energy cut of $W < 2$ GeV reduces the cross section for $E_\nu > 2$ GeV, which is

about 8% at $E_\nu = 2.5$ GeV and becomes 15% at $E_\nu = 3.5$ GeV, for both the neutrino and antineutrino induced $K\Lambda$ production. However, when the contribution of $W > 2$ GeV with $Q^2 < 1$ GeV² is taken into account, the numerical results are very close to the results obtained without any cut on W and Q^2 , for (anti)neutrino energies up to 3 GeV.

- (iv) $\frac{d\sigma}{dQ^2}$ peaks around $Q^2 = 0.2$ GeV² both for (anti)neutrino energies of 1.5 and 2.5 GeV. The Q^2 distribution for the antineutrino induced $K\Lambda$ production is more sharply peaked as compared to the neutrino induced reaction.
- (v) The kaon momentum distribution $\frac{d\sigma}{dp_K}$ peaks around $p_K = (0.88) \pm 1$ GeV at $E_\nu = (1.5)2.5$ GeV for the neutrino induced $K\Lambda$ production. The nature of kaon momentum distribution in the antineutrino induced reaction is almost similar to that observed in the neutrino case, except the fact that the peak shifts slightly to lower values of momentum in the case of antineutrino induced reaction.

To conclude, this is the first work which discusses Q^2 distribution and the kaon momentum distribution and presents an updated calculation of the total scattering cross section for ν_μ and $\bar{\nu}_\mu$ induced associated particle production of $K\Lambda$ from the free nucleon target. The energy region considered in this work may be quite useful in the experimental analysis of atmospheric and accelerator neutrino experiments like T2K, MicroBooNE, ICARUS, NOvA, MINERvA, SBND, and DUNE, where the average energy is expected to be around 2.5 GeV.

ACKNOWLEDGMENTS

A. F. and M. S. A. are thankful to the Department of Science and Technology (DST), Government of India, for providing financial assistance under Grant No. SR/MF/PS-01/2016-AMU.

DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

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