

Holographic Krylov complexity in $\mathcal{N}=4$ SYM theory

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We propose and calculate a holographic Krylov complexity in $\mathcal{N}=4$ SYM theory via the proper momentum for motion in AdS_5 sliced by AdS_3 . The motion in an AdS_3 subgroup corresponds to the Krylov complexity of the $Sl(2)$ subsector. The general motion corresponds to the Krylov complexity of the $\mathcal{N}=4$ SYM.

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I. INTRODUCTION

The notion of complexity in quantum mechanical systems is recent and one that has received a lot of interest. The geometrical description of Nielsen [1] was followed by extensions via holography to quantum field theories, for the subsequent conjectures of “complexity = volume” [2], “complexity = action” [3] or, more recently, a one-dimensional set of conjectures dubbed “complexity = anything” [4], and it has then led to understanding in quantum field theory of the notion of circuit complexity [5]. But these notions of complexity are not algorithmic. A more recent notion of complexity that *is* algorithmic, at least in quantum mechanics ($0+1$ dimensional systems) is Krylov complexity, initially defined as a measure of quantum chaos [6] (for reviews of Krylov complexity, see Refs. [7,8], and for a review of all notions of quantum complexity, see Ref. [9]), and then defined in a more general way, equivalently for states and operators, as “spread complexity” in [10].

The holographic description of Krylov complexity has been more difficult to define. In one line of research, for the quantum mechanical double-scaled SYK model (DSSYK), the holographic complexity was defined from the worm-hole length in the dual JT gravity [11,12] (see also the review [8]). In another line of research, the time derivative of the Krylov complexity $\dot{C}(t)$ was shown to be related to

the *proper momentum* $P_{\bar{p}}(t)$ for a motion in an AdS_3 gravity dual (so, for a CFT_2), a relation made precise in [13], as

$$\dot{C}(t) = -\frac{P_{\bar{p}}}{\epsilon}, \quad (1.1)$$

based on various earlier, but less precise relations. Such a less precise relation was proven from the “complexity = volume” conjecture for the Nielsen complexity in [14] and previous work by the same authors. Note also the extension of Krylov complexity to supersymmetric theories, and to semiclassical strings in a gravity dual defined in [15].

In this paper we seek to extend the range of the proposal in (1.1) to the original case of holographic duality, $\mathcal{N}=4$ SYM vs $\text{AdS}_5 \times S^5$. We will further extend this in two companion papers to appear shortly, one where we will extend this to the case of nonconformal confining theories, with only asymptotically AdS , but the space ending in the IR, based on the Anabalon-Ross solution [16] (related also to the generalization to the nonsingular version of the Coulomb branch of $\mathcal{N}=4$ SYM in [17]), itself generalizing [18], and another where we will extend this to the case of AdS_3 and AdS_2 solutions, but with an extra direction corresponding to a quiver in the field theory [19–23] and [24–26], respectively.

In this $\mathcal{N}=4$ SYM case, we will first note that there is an AdS_3 subspace construction, dual to the $Sl(2)$ subset of $\mathcal{N}=4$ SYM, where previous constructions apply, and then generalize to the case of a slicing of AdS_5 by AdS_3 , and propose a holographic construction that predicts a field theory Krylov complexity.

The paper is organized as follows. In Sec. II we review Krylov complexity and better define our proposal for its holographic form. In Sec. III we consider the AdS_3

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subspace of AdS_5 and its dual $Sl(2)$ subsector of $\mathcal{N} = 4$ SYM. In Sec. IV we consider the general case, of slicing AdS_5 by AdS_3 slices, calculating the motion starting at zero velocity in the UV, and the resulting proper momentum. In Sec. V we conclude.

II. KRYLOV COMPLEXITY AND HOLOGRAPHIC PROPOSAL

For a quantum mechanical system, the Krylov complexity is defined as the average number (or discrete “length,” the equivalent of the number of gates in a circuit for a quantum system) n in a basis of the Hilbert space, the Krylov basis $|n\rangle$, to go from a reference state $|0\rangle$ to a target space $|\psi(t)\rangle$. Using the Lanczos algorithm, from $|0\rangle$ one defines the Krylov basis $|n\rangle$, in which the (Hermitian) Hamiltonian has the most general tridiagonal form,

$$\hat{H}|n\rangle = a_n|n\rangle + b_n|n-1\rangle + b_{n+1}|n+1\rangle, \quad (2.1)$$

or the time-dependent target space obeys the (Schrödinger) equation

$$i\partial_t\psi_n(t) = a_n\psi_n(t) + b_n\psi_{n-1}(t) + b_{n+1}\psi_{n+1}(t), \quad (2.2)$$

where $\psi_n(t) \equiv \langle n|\psi(t)\rangle$. The Krylov complexity is then

$$C(t) \equiv \sum_n n |\psi_n(t)|^2 = \sum_n n p_n(t) = \langle n \rangle(t). \quad (2.3)$$

In [13], the calculation of the Krylov (or spread) complexity of states excited by primary operators $\mathcal{O}$ of dimension Δ and energy $E_0 = \Delta/\epsilon$ (with regulator $e^{-\epsilon H}$) in a generic CFT_2 was done, in the cases of the CFT on the cylinder $\mathbb{R}_t \times S_L^1$, so for a CFT on a finite size L , and the CFT on the plane $\mathbb{R}_t \times \mathbb{R}_x$, so for a CFT on an infinite line, obtaining

$$\dot{C}_L(t) = \frac{E_0}{\epsilon} \frac{L}{2\pi} \sin \frac{2\pi t}{L} \Rightarrow \quad (2.4)$$

$$\dot{C}_\infty(t) = \frac{E_0}{\epsilon} t \text{ (for } L \rightarrow \infty), \quad (2.5)$$

respectively, and also at finite temperature $T = 1/\beta$, obtaining

$$\dot{C}_T(t) = \frac{E_0}{\epsilon} \frac{\beta}{2\pi} \sinh \frac{2\pi t}{\beta}, \quad (2.6)$$

consistent in the $T \rightarrow 0$ limit with the $C_\infty(t)$ case. These calculations were matched by the calculations in global AdS_3 (with $\mathbb{R}_t \times S^1$ boundary) and Poincaré AdS_3 (with $\mathbb{R}_t \times \mathbb{R}_x$ boundary),

$$\begin{aligned} ds_{3,\text{gl.}}^2 &= d\rho^2 + \frac{4\pi^2}{L^2} (-\cosh^2 \rho dt^2 + \sinh^2 \rho d\phi^2), \\ ds_{3,\text{P.}}^2 &= d\rho^2 + e^{2\rho} (-dt^2 + dx^2), \end{aligned} \quad (2.7)$$

respectively, where we have used the coordinates ρ such that $ds^2 = d\rho^2$ when the boundary coordinates are constant. The canonical momentum P_ρ in these coordinates was referred to as the *proper momentum*. By starting in the UV, i.e., at very large ρ , with $z = e^{-\rho} = \epsilon$, and calculating the geodesic of a massive particle starting at zero velocity, one finds in both cases that

$$\dot{C}(t) = -\frac{P_\rho(t)}{\epsilon}, \quad (2.8)$$

as advertised in (1.1), with $\bar{\rho} = \rho$ here. Similarly, at finite temperature T , for the AdS_3 black hole of temperature $T = 1/\beta$,

$$ds_{3,\text{BH}}^2 = d\rho^2 + \frac{4\pi^2}{\beta^2} (-\sinh^2 \rho dt^2 + \cosh^2 \rho dx^2), \quad (2.9)$$

which has a horizon at $\rho = 0$, that gives a periodicity of Euclidean time $\Delta t_E = \beta = 1/T$, one finds the same relation to be satisfied.

The isometry of AdS_3 is $SO(2, 2) \simeq SO(2, 1) \times SO(2, 1) \simeq Sl(2; \mathbb{R}) \times Sl(2; \mathbb{R})$, with the Lorentz group being $SO(2, 1) \simeq Sl(2; \mathbb{R})$, and then AdS_3 , which is the coset manifold $SO(2, 2)/SO(2, 1) \simeq Sl(2; \mathbb{R})$, and is isometric to the noncompact group manifold $Sl(2; \mathbb{R})$, where the two $Sl(2; \mathbb{R})$ symmetries are left and right actions on the group elements.

This $Sl(2; \mathbb{R})$ symmetry structure is at the core of the solvability of Krylov complexity in this case, dual to CFT_2 , and the sinusoidal form in the finite size L case is due to the finite size of the Hilbert space, if discretized by ϵ down to a spin chain form.

We are proposing that even in a non- AdS_3 situation, the formula (1.1) continues to hold, if we identify correctly the *proper distance* coordinate $\bar{\rho}$, such that $ds^2 = d\bar{\rho}^2$ on the motion of the massive particle falling from large $\bar{\rho}$, and then $P_{\bar{\rho}}$ is the canonical momentum of this coordinate.

Specifically, then, we write on the timelike geodesic parametrized as $(r(t), \rho(t))$, that

$$ds^2 = A(t)dr(t)^2 + B(t)d\rho(t)^2 \equiv d\bar{\rho}(t)^2, \quad (2.10)$$

so that

$$\frac{d\rho^2}{dr^2} = \frac{\dot{\rho}^2}{\dot{r}^2}, \quad (2.11)$$

and the proper momentum is

$$P_{\bar{\rho}} = P_r \frac{dr}{d\bar{\rho}} + P_\rho \frac{d\rho}{d\bar{\rho}}. \quad (2.12)$$

III. AdS₃ SUBSPACE VS $Sl(2)$ SECTOR

The first thing to understand is that there is an AdS₃ subspace inside AdS₅, and it is dual to the so-called $Sl(2)$ sector, and thus the complexity within this sector is given by the same calculation as the generic CFT₂ calculation of [13].

A. AdS₃ subspace of AdS₅ $\times$ S⁵ and its Krylov complexity

The relevant AdS₃ subspace is obtained by writing the AdS₅ metric as

$$\begin{aligned} ds^2 &= R^2[-dt^2 \cosh^2 \rho + d\rho^2 + \sinh^2 \rho d\Omega_3^2] \\ &= R^2[-dt^2 \cosh^2 \rho + d\rho^2 + \sinh^2 \rho (\cos^2 \theta d\phi^2 + d\theta^2 \\ &\quad + \sin^2 \theta d\psi^2)], \end{aligned} \quad (3.1)$$

and the AdS₃ subspace in which the $Sl(2)$ subsector lives (on its boundary) is the Equator $\theta = 0$, with metric

$$ds_3^2 = R^2[-dt^2 \cosh^2 \rho + d\rho^2 + \sinh^2 \rho d\phi^2]. \quad (3.2)$$

Note that we have an AdS₃ *only* at $\theta = 0$. This subspace is what contained the Gubser-Klebanov-Polyakov (GKP) spinning folded string [27]. Another (equivalent) form is given in [28] as in the embedding (hyperboloid) coordinates $X_{-1}, X_0, X_1, X_2, X_3, X_4$ for AdS₅, one fixes X_3, X_4 to constants, obtaining AdS₃.

At the boundary $\rho \rightarrow \infty$, we have

$$ds_2^2 \simeq \frac{R^2 e^{2\rho}}{4} (-dt^2 + d\phi^2), \quad (3.3)$$

so the $\mathbb{R}_t \times S^1$ of the cylinder (CFT₂ on a finite line), in $w = t_E + i\phi$ complex Euclideanized coordinates, is defined only at $\theta = 0$. The corresponding plane (for CFT₂ on the infinite line) in $z = e^w = e^{t_E + i\phi} = x_2 + ix_1$ complex coordinates is also only defined for $\theta = 0$ in the S^3 (at its equator).

The Krylov complexity for this AdS₃ subspace is then exactly (2.4) in global coordinates, corresponding to the CFT₂ on the cylinder, in w coordinates, and (2.5) in Poincaré coordinates, corresponding to the CFT₂ on the plane, in z coordinates.

B. $Sl(2)$ subsector of $\mathcal{N} = 4$ SYM and its Krylov complexity

Corresponding to the AdS₃ subspace above, we have the $Sl(2)$ subspace of $\mathcal{N} = 4$ SYM, defined by Beisert in [29].

The generators of the relevant $sl(2)$ subalgebra of the $so(4, 2)$ algebra of $\mathcal{N} = 4$ SYM (a CFT₄) are $sl(2) \times sl(2)$ Wick rotated Lorentz generators within CFT₄, for $SO(3, 1) \rightarrow SO(2, 2) \simeq SO(2, 1) \times SO(2, 1) \simeq Sl(2) \times Sl(2)$, called L^a_b and $\tilde{L}^a_b$, also four-dimensional momenta $P^a, \tilde{P}^a$, four-dimensional boosts $K^a, \tilde{K}^a$ and D (dilatations in CFT₄, associated with the *overall* radial direction of AdS₅). Here $a, \tilde{a} = 1, 2$.

Then, the $sl(2)$ generators for the symmetry of the $Sl(2)$ subsector are

$$\begin{aligned} J'_+ &= P_1 + iP_2, \\ J'_- &= K_1 + iK_2, \\ J'_3 &= \frac{1}{2}(D + \delta D + L^1_1 + \tilde{L}^1_1), \end{aligned} \quad (3.4)$$

where δD is the quantum correction to D , satisfying

$$[J'_+, J'_-] = -2J'_3, \quad [J'_3, J'_\pm] = \pm J'_\pm. \quad (3.5)$$

As we can see, this $Sl(2)$ is associated, in AdS₅, with the *overall* radial direction ρ , the Euclideanized time direction 2, and a spatial direction 1.

The generators $J'_\pm, J'_3$ are represented in terms of oscillators as

$$J'_- = a, \quad J'_+ = a^\dagger + a^\dagger a^\dagger a, \quad J'_3 = \frac{1}{2} + a^\dagger a. \quad (3.6)$$

Then, the $Sl(2)$ sector of $\mathcal{N} = 4$ SYM is defined by combinations of the objects (complex scalar and complex derivative)

$$Z = \Phi_{5+i6} = \Phi_5 + i\Phi_6, \quad D_+ = D_{1+i2} = D_1 + iD_2, \quad (3.7)$$

where D_+ can be defined either on the cylinder w or on the plane z , as above, and the identification with the oscillator states is given by

$$|n\rangle = \frac{1}{n!} (a^\dagger)^n |0\rangle = \frac{1}{n!} D_+^n |0\rangle, \quad (3.8)$$

which corresponds in the $\mathcal{N} = 4$ SYM CFT to operators of given spin $n = S$ and given dimension, for instance

$$\mathcal{O}_S \sim \text{Tr}[D_+^S Z]. \quad (3.9)$$

This is an operator corresponding in a spin chain to only parallel spins.

More generally, we can construct a *spin chain* by using the general operator

$$\mathcal{O}_{m_1, \dots, m_L} = \text{Tr}[D_+^{m_1} Z D_+^{m_2} Z \dots D_+^{m_L} Z], \quad (3.10)$$

with the $D_+^{m_k}$ identified with spins inserted at lattice site k , and with spin S , dilatation charge Δ and R-charge L given by

$$S = \sum_k m_k, \quad L = \sum_k 1, \quad \Delta_{g=0} = S + L. \quad (3.11)$$

The one-loop Hamiltonian found by Beisert in [29] is in fact the XXX spin 1/2 Heisenberg Hamiltonian, which is nearest-neighbor in site (k) space.

Thus, defining, within the subspace of operators $\mathcal{O}_{m_1, \dots, m_L}$, the operators of given S and L , ordered by site (increasing m_k , then increasing k) insertion, it is also nearest-neighbor in the Krylov ($|n\rangle$) sense, and so can be identified with the Krylov basis, so the corresponding Heisenberg XXX_{1/2} Hamiltonian with the Krylov basis Hamiltonian (2.1). This is so, since both the subsector Hamiltonian and the Krylov Hamiltonian are for quantum mechanical ($0+1$ dimensional) finite dimensional systems.

One could then just compute the complexity from the Hamiltonian in this Krylov basis (which we leave for further work) but, given that for generic AdS₃/CFT₂ this was done in [13] (in Supplemental Material A), there is no need to do it here as well.

Note that for only two D_+ insertions in a chain of length L , the operators of momentum n that are *eigenstates of the Hamiltonian* (but therefore *not Krylov basis states*: diagonal vs tridiagonal form) are

$$\begin{aligned} \mathcal{O}_n^L &= 2 \cos \frac{\pi n}{L+1} \text{Tr}(a_1^\dagger)^2 |L\rangle \\ &+ \sum_{p=2}^L \cos \frac{\pi n(2p-1)}{L+1} a_1^\dagger a_p^\dagger |L\rangle, \end{aligned} \quad (3.12)$$

which are the 2-magnon Berenstein-Maldacena-Nastase (BMN) [30] operators, of energy and correction to the dilatation operator of

$$E = 8 \sin^2 \frac{\pi n}{L+1}, \quad \delta D = \frac{g_{\text{YM}}^2 N}{\pi^2} \sin^2 \frac{\pi n}{L+1}. \quad (3.13)$$

On the other hand, in the opposite case of large spin S , so large Δ , but with fixed J , the gravity dual corresponding to the operator that is an eigenstate of the Hamiltonian is the folded spinning string at the center of AdS₅, but restricted to move in AdS₃, of GKP [27] with behavior

$$\Delta - S = \frac{\sqrt{\lambda}}{\pi} \ln \frac{S}{\sqrt{\lambda}} + \dots, \quad \lambda = g_{\text{YM}}^2 N. \quad (3.14)$$

In conclusion, the generic $SI(2)$ subsector state moves in AdS₃ only (it is not necessarily a large string, but maybe just a particle), and stays inside it, consistent with what we have found for the complexity.

IV. MOTION IN AdS₅ SLICED BY AdS₃ AND HOLOGRAPHIC KRYLOV COMPLEXITY

We now consider a more general case, where in the gravity dual of AdS₅ (we ignore the S^5 , considering that there is no motion there) we have a motion that still respects the $SI(2)$ symmetry at every given point, but the motion itself is not $SI(2)$ symmetric. The result of the gravity dual calculation will be then considered as a prediction, if we consider that the matching we had in the AdS₃ vs $SI(2)$ subsector should extend to the full $SI(2)$ invariant theory.

A. Motion in AdS₅ sliced by AdS₃

That means that we must find a slicing of AdS₅ by AdS₃ slices, which in the more general case of AdS _{$d+2$} sliced by AdS _{d} is given by

$$ds_{\text{AdS}_{d+2}}^2 = \sinh^2 r d\psi^2 + dr^2 + \cosh^2 r ds_{\text{AdS}_d}^2. \quad (4.1)$$

1. Global coordinates for AdS₃

In the case of AdS₅ vs AdS₃ and global coordinates, we have

$$\begin{aligned} ds^2 &= dr^2 + \sinh^2 r d\psi^2 \\ &+ \cosh^2 r (d\rho^2 - \cosh^2 \rho dt^2 + \sinh^2 \rho d\phi^2), \end{aligned} \quad (4.2)$$

so, with the ansatz for timelike geodesics with $r = r(t)$, $\rho = \rho(t)$, the Lagrangian for a massive particle is

$$S = -m \int dt \sqrt{\cosh^2 r (\cosh^2 \rho - \dot{\rho}^2) - \dot{r}^2}. \quad (4.3)$$

In this case, the proper distance will be found by replacing the solution with $r(t)$, $\rho(t)$ in

$$ds^2 = dr^2 + \cosh^2 r d\rho^2 \quad (4.4)$$

and rewriting this as a $d\bar{\rho}^2$.

The canonical momenta associated with the variables $r(t)$, $\rho(t)$ are

$$\begin{aligned} P_\rho &= \frac{m \cosh^2 r \dot{\rho}}{\sqrt{\cosh^2 r (\cosh^2 \rho - \dot{\rho}^2) - \dot{r}^2}}, \\ P_r &= \frac{m \dot{r}}{\sqrt{\cosh^2 r (\cosh^2 \rho - \dot{\rho}^2) - \dot{r}^2}}, \end{aligned} \quad (4.5)$$

and the resulting conserved (time-independent) Hamiltonian is

$$H = P_\rho \dot{\rho} + P_r \dot{r} - L = m \frac{\cosh^2 r \cosh^2 \rho}{\sqrt{\cosh^2 r (\cosh^2 \rho - \dot{\rho}^2) - \dot{r}^2}}, \quad (4.6)$$

which means that, from the constancy of H , we can solve for $\dot{r}$ as

$$\begin{aligned} \dot{r}^2 &= \cosh^2 r (\cosh^2 \rho - \dot{\rho}^2) - \frac{m^2 \cosh^4 r \cosh^4 \rho}{H^2} \\ &= \cosh^2 r \left[\cosh^2 \rho \left(1 - \frac{m^2}{H^2} \cosh^2 r \cosh^2 \rho \right) - \dot{\rho}^2 \right]. \end{aligned} \quad (4.7)$$

On the other hand, the equations of motion of the action (4.3), for ρ and r , respectively, are

$$\begin{aligned} \frac{d}{dt} \left[\frac{m \cosh^2 r \dot{\rho}}{\sqrt{\cosh^2 r (\cosh^2 \rho - \dot{\rho}^2) - \dot{r}^2}} \right] &= -m \frac{\cosh \rho \sinh \rho \cosh^2 r}{\sqrt{\cosh^2 r (\cosh^2 \rho - \dot{\rho}^2) - \dot{r}^2}}, \\ \frac{d}{dt} \left[\frac{m \dot{r}}{\sqrt{\cosh^2 r (\cosh^2 \rho - \dot{\rho}^2) - \dot{r}^2}} \right] &= -m \frac{\cosh r \sinh r \cosh^2 \rho}{\sqrt{\cosh^2 r (\cosh^2 \rho - \dot{\rho}^2) - \dot{r}^2}}. \end{aligned} \quad (4.8)$$

In the case of pure AdS₃ (in our case, at constant r), the initial boundary condition for the massive geodesic was that it starts near the boundary (at $\rho_0 \rightarrow \infty$), with zero velocity, $\dot{\rho}_0 = 0$.

In the case of general motion in AdS₅, with $r(t), \rho(t)$, we will assume that (since we want to consider AdS₃ as a basis for the calculation), together with the boundary condition for r that it also starts near the boundary, at large r , $r_0 \rightarrow \infty$.

Note that, for $r = r_0 \rightarrow \infty, \rho = \rho_0 \rightarrow \infty$, the AdS₅ (boundary) metric becomes

$$\begin{aligned} ds^2 &\simeq \frac{e^{2r_0}}{4} \left[d\psi^2 + \frac{e^{2\rho_0}}{4} (-dt^2 + d\phi^2) \right] \\ &= \frac{e^{2r_0+2\rho_0}}{16} [4e^{-2\rho_0} d\psi^2 - dt^2 + d\phi^2], \end{aligned} \quad (4.9)$$

so the circle $d\psi^2$ becomes of zero length, and can be dropped, so the boundary metric is just $\mathbb{R}_t \times S^1$, just the boundary of global AdS₃, consistent with a modification of the $Sl(2)$ sector described by similar operators.

The simplest and most interesting case is when we also start at zero velocity in r , so $\dot{r}_0 = 0$, in which case from (4.7), we get

$$H = m \cosh r_0 \cosh \rho_0. \quad (4.10)$$

Then, eliminating m/H , we get

$$\dot{r} = \cosh r \sqrt{\cosh^2 \rho \left(1 - \frac{\cosh^2 \rho \cosh^2 r}{\cosh^2 \rho_0 \cosh^2 r_0} \right) - \dot{\rho}^2}, \quad (4.11)$$

but we will not use this for now.

Substituting

$$m \frac{\cosh^2 r}{\sqrt{\cosh^2 r (\cosh^2 \rho - \dot{\rho}^2) - \dot{r}^2}} = \frac{H}{\cosh^2 \rho} \quad (4.12)$$

in the ρ equation of motion, and thus getting rid of $\dot{r}$, we obtain the constant Hamiltonian H on both sides of the equation, so it cancels out, and we remain with the simple equation

$$\frac{d}{dt} \left[\frac{\dot{\rho}}{\cosh^2 \rho} \right] = -\tanh \rho. \quad (4.13)$$

At general ρ , we write the equation as

$$\begin{aligned} \frac{d^2}{dt^2} \tanh \rho + \tanh \rho &= 0 \Rightarrow \tanh \rho \\ &= A' \cos t + B' \sin t = A \cos(t - t_0), \end{aligned} \quad (4.14)$$

or (imposing that at $\rho = \rho_0$ we have $\dot{\rho}_0 = 0$),

$$\tanh \rho = \tanh \rho_0 \cos(t - t_0). \quad (4.15)$$

Note that at large ρ , so for small times (t near t_0 , so ρ near $\rho_0 \rightarrow \infty$), we get

$$\rho \simeq -\frac{1}{2} \log \left[e^{-2\rho_0} + \frac{(t - t_0)^2}{4} \right]. \quad (4.16)$$

To find $r(t)$, we first write the conservation of H/m as

$$\frac{H^2}{m^2} \frac{\dot{r}^2}{\cosh^2 r} = \frac{H^2}{m^2} \cosh^2 \rho - \cosh^2 r \cosh^4 \rho - \frac{H^2}{m^2} \dot{\rho}^2, \quad (4.17)$$

then, in the $\dot{r}_0 = 0$ case, we replace H/m with $\cosh r_0 \cosh \rho_0$ on the right-hand side, and then substitute the solution for $\rho(t)$ and so also for $\dot{\rho}(t)$ on the right-hand side, to obtain

$$\begin{aligned} \frac{H^2}{m^2} \frac{\dot{r}^2}{\cosh^2 r} &= \frac{1}{(1 - \tanh^2 \rho_0 \cos^2(t - t_0))^2} \\ &\times [\cosh^2 r_0 - \cosh^2 r], \end{aligned} \quad (4.18)$$

and then taking the square root and separating the dr and dt integrals gives

$$\begin{aligned} \frac{H}{m} \int_{r_0}^r \frac{dr}{\cosh r \sqrt{\cosh^2 r_0 - \cosh^2 r}} \\ = \int_{t_0}^t \frac{dt}{1 - \tanh^2 \rho_0 \cos^2(t - t_0)}, \end{aligned} \quad (4.19)$$

and doing the integrals on both sides,

$$\begin{aligned} \frac{H}{m \cosh r_0} \left(\arcsin \frac{\tanh r}{\tanh r_0} \right) \Big|_{r_0}^r \\ = -\cosh \rho_0 \arctan \left[\frac{\cot(t - t_0)}{\cosh \rho_0} \right] \Big|_{t_0}^t \Rightarrow \\ r(t) = \tanh^{-1} \left\{ \tanh r_0 \sin \left[-\arctan \left[\frac{\cot(t - t_0)}{\cosh \rho_0} \right] + \frac{3\pi}{4} \right] \right\}. \end{aligned} \quad (4.20)$$

At this point it seems from (4.18) that $r = r_0 = \text{constant}$ is a solution, yet, the equation of motion for r , which can be rewritten, by substituting H on both sides of the equation, and cancelling this constant value (as for the equation of motion for ρ) gives

$$\frac{d}{dt} \left[\frac{\dot{r}}{\cosh^2 r \cosh^2 \rho} \right] = -\tanh r, \quad (4.21)$$

which clearly does not admit the $r = r_0$ solution (the left-hand side is zero, but the right-hand side is not). If $\dot{r}_0 = 0$, then we find $\ddot{r}_0 = -\cosh^2 r_0 \tanh r_0 \cosh^2 \rho_0$, very large. But there is no need to go to this; we can already see in the solution in (4.20) that $r = r_0$ is not possible: in fact, the solution is unique, without any free constants.

We can also choose the more general boundary condition for r , namely a nonzero velocity, $\dot{r}_0 \neq 0$, in which case we just keep H/m as it is, and obtain instead of (4.18) that

$$\begin{aligned} \frac{H^2}{m^2} \frac{\dot{r}^2}{\cosh^2 r} &= \frac{1}{(1 - \tanh^2 \rho_0 \cos^2(t - t_0))^2} \\ &\times \left[\frac{H^2}{m^2 \cosh^2 \rho_0} - \cosh^2 r \right], \end{aligned} \quad (4.22)$$

so that

$$\begin{aligned} \frac{H}{m} \int_{r_0}^r \frac{dr}{\cosh r \sqrt{\frac{H^2}{m^2 \cosh^2 \rho_0} - \cosh^2 r}} \\ = \int_{t_0}^t \frac{dt}{1 - \tanh^2 \rho_0 \cos^2(t - t_0)}, \end{aligned} \quad (4.23)$$

which integrates to

$$\cosh \rho_0 \left(\arcsin \frac{m \cosh \rho_0 \tanh r}{\sqrt{H^2 - m^2 \cosh^2 \rho_0}} \right) \Big|_{r_0}^r = -\cosh \rho_0 \arctan \left[\sqrt{1 - \tanh^2 \rho_0} \cot(t - t_0) \right] \Big|_{t_0}^t \quad (4.24)$$

or

$$\begin{aligned} \left(\arcsin \frac{m \cosh \rho_0 \tanh r}{\sqrt{H^2 - m^2 \cosh^2 \rho_0}} \right) \Big|_{r_0}^r &= -\arctan \left[\frac{\cot(t - t_0)}{\cosh \rho_0} \right] \Big|_{t_0}^t \Rightarrow \\ r(t) &= \tanh^{-1} \left\{ \frac{\sqrt{H^2 - m^2 \cosh^2 \rho_0}}{m \cosh \rho_0} \sin \left[-\arctan \left(\frac{\cot(t - t_0)}{\cosh \rho_0} \right) + \frac{\pi}{2} \right] \right. \\ &\quad \left. + \arcsin \frac{m \cosh \rho_0 \tanh r_0}{\sqrt{H^2 - m^2 \cosh^2 \rho_0}} \right\}. \end{aligned} \quad (4.25)$$

An example of evolution of $r(t)$ is in Fig. 1.

2. Poincaré coordinates for AdS₃

The slicing of AdS₅ by AdS₃ is independent on the metric of AdS₃, so we can use the Poincaré patch of AdS₃, described, like in [13], in coordinates

$$ds_3^2 = d\rho^2 + e^{2\rho}(-dt^2 + dx^2), \quad (4.26)$$

and corresponding to the CFT₂ on an infinite line. Indeed, the boundary of AdS₅ at $r = r_0 \rightarrow \infty$ and $\rho = \rho_0 \rightarrow \infty$ is now

$$ds_5^2 = \frac{e^{2r_0+2\rho_0}}{4} [e^{-2\rho_0} d\psi^2 - dt^2 + dx^2], \quad (4.27)$$

so just two-dimensional Minkowski space, $\mathbb{R}_t \times \mathbb{R}_x$, corresponding to a CFT₂ on an infinite line, meaning that we

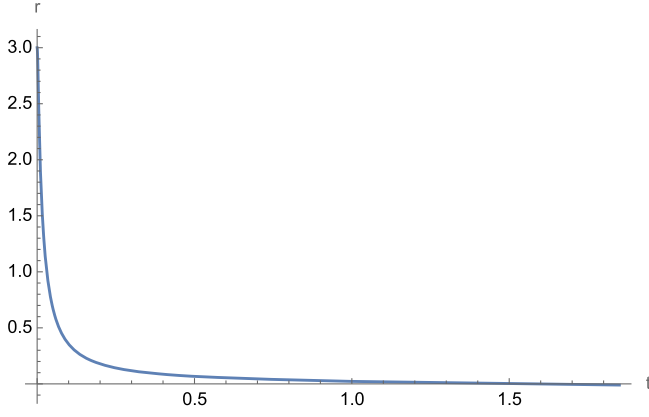


FIG. 1. The evolution of $r(t)$ for AdS_3 in global coordinates that follows from (4.22). The initial conditions are set as $\dot{r}_0 = 0.1$, $\rho_0 = 4$, $r_0 = 3$, together with $t_0 = 0$.

are calculating the complexity for a CFT_2 on an infinite line.

The calculation in Poincaré AdS_3 follows the same one in the global case. With the $r = r(t)$, $\rho = \rho(t)$ ansatz, we have the massive particle action

$$S = -m \int dt \sqrt{\cosh^2 r (e^{2\rho} - \dot{\rho}^2) - \dot{r}^2}, \quad (4.28)$$

giving the canonical momenta

$$\begin{aligned} P_\rho &= \frac{m \cosh^2 r \dot{\rho}}{\sqrt{\cosh^2 r (e^{2\rho} - \dot{\rho}^2) - \dot{r}^2}}, \\ P_r &= \frac{m \dot{r}}{\sqrt{\cosh^2 r (e^{2\rho} - \dot{\rho}^2) - \dot{r}^2}}, \end{aligned} \quad (4.29)$$

and the conserved Hamiltonian is

$$H = P_\rho \dot{\rho} + P_r \dot{r} - L = m \frac{\cosh^2 r e^{2\rho}}{\sqrt{\cosh^2 r (e^{2\rho} - \dot{\rho}^2) - \dot{r}^2}}, \quad (4.30)$$

so that, solving for $\dot{r}$, we obtain

$$\begin{aligned} \dot{r}^2 &= \cosh^2 r (e^{2\rho} - \dot{\rho}^2) - \frac{m^2 \cosh^4 r e^{4\rho}}{H^2} \\ &= \cosh^2 r \left[e^{2\rho} \left(1 - \frac{m^2}{H^2} \cosh^2 r e^{2\rho} \right) - \dot{\rho}^2 \right]. \end{aligned} \quad (4.31)$$

The equations of motion for ρ and r coming from the massive particle action are

$$\begin{aligned} \frac{d}{dt} \left[\frac{m \cosh^2 r \dot{\rho}}{\sqrt{\cosh^2 r (e^{2\rho} - \dot{\rho}^2) - \dot{r}^2}} \right] &= -m \frac{e^{2\rho} \cosh^2 r}{\sqrt{\cosh^2 r (e^{2\rho} - \dot{\rho}^2) - \dot{r}^2}}, \\ \frac{d}{dt} \left[\frac{m \dot{r}}{\sqrt{\cosh^2 r (e^{2\rho} - \dot{\rho}^2) - \dot{r}^2}} \right] &= -m \frac{\cosh r \sinh r (e^{2\rho} - \dot{\rho}^2)}{\sqrt{\cosh^2 r (e^{2\rho} - \dot{\rho}^2) - \dot{r}^2}}. \end{aligned} \quad (4.32)$$

Substituting the constant Hamiltonian on both sides of the first equation, it cancels, and we obtain simply (using the same boundary conditions)

$$\frac{d^2}{dt^2} e^{-2\rho} = +2 \Rightarrow \rho(t) = -\frac{1}{2} \log [e^{-2\rho_0} + (t - t_0)^2]. \quad (4.33)$$

To find $r(t)$, we first write the conservation of H/m as

$$\frac{H^2}{m^2} \frac{\dot{r}^2}{\cosh^2 r} = \frac{H^2}{m^2} e^{2\rho} - \cosh^2 r e^{4\rho} - \frac{H^2}{m^2} \dot{\rho}^2, \quad (4.34)$$

then replace H/m with $\cosh r_0 e^{\rho_0}$ on the right-hand side, and then substitute the solution for $\rho(t)$ and so also for $\dot{\rho}(t)$ on the right-hand side, to obtain

$$\frac{H^2}{m^2} \frac{\dot{r}^2}{\cosh^2 r} = \frac{e^{4\rho_0}}{(1 + e^{2\rho_0}(t - t_0)^2)^2} [\cosh^2 r_0 - \cosh^2 r], \quad (4.35)$$

so taking the square root and separating the dr and dt integrals gives

$$\frac{H}{m} \int_{r_0}^r \frac{dr}{\cosh r \sqrt{\cosh^2 r_0 - \cosh^2 r}} = \int_{t_0}^t \frac{dt}{e^{-2\rho_0} + (t - t_0)^2}, \quad (4.36)$$

or, doing the integrals on both sides,

$$\begin{aligned} \frac{H}{m \cosh r_0} \left(\arcsin \frac{\tanh r}{\tanh r_0} \right) \Big|_{r_0}^r &= e^{\rho_0} \arctan [e^{\rho_0}(t - t_0)] \Big|_{t_0}^t \Rightarrow \\ r(t) &= \tanh^{-1} \left\{ \tanh r_0 \sin \left[\arctan [e^{\rho_0}(t - t_0)] + \frac{\pi}{4} \right] \right\}. \end{aligned} \quad (4.37)$$

Again for the boundary condition $\dot{r}_0 = 0$ we seem to have the solution $r = r_0 = \text{constant}$, but it is actually not a solution of the equation for r , which is (substituting H on both sides of the equation, and then cancelling it)

$$\frac{d}{dt} \left[\frac{\dot{r} e^{-2\rho}}{\cosh^2 r} \right] = -\tanh r (1 - \dot{\rho}^2 e^{-2\rho}), \quad (4.38)$$

so for $\dot{r}_0 = 0$ we would obtain the initial acceleration $\ddot{r}_0 = -\cosh^2 r_0 \tanh r_0 e^{2\rho_0}$, very large.

For the more general boundary condition, $\dot{r}_0 \neq 0$, we leave H/m as it is, and obtain

$$\frac{H^2}{m^2} \frac{\dot{r}^2}{\cosh^2 r} = \frac{e^{4\rho_0}}{(1 + e^{2\rho_0}(t - t_0))^2} \left[\frac{H^2}{m^2} e^{-2\rho_0} - \cosh^2 r \right], \quad (4.39)$$

so that

$$e^{\rho_0} \left(\arcsin \frac{m e^{\rho_0} \tanh r}{\sqrt{H^2 - m^2 e^{2\rho_0}}} \right) \Big|_{r_0}^r = e^{\rho_0} \arctan [e^{\rho_0}(t - t_0)] \Big|_{t_0}^t \Rightarrow$$

$$r(t) = \tanh^{-1} \left\{ \frac{\sqrt{H^2 - m^2 e^{2\rho_0}}}{m e^{\rho_0}} \sin \left[\arctan (e^{\rho_0}(t - t_0)) + \arcsin \frac{m e^{\rho_0} \tanh r_0}{\sqrt{H^2 - m^2 e^{2\rho_0}}} \right] \right\}. \quad (4.41)$$

An example of evolution of $r(t)$ is in Fig. 2.

B. Proper momentum as derivative of Krylov complexity in $\mathcal{N} = 4$ SYM

We now calculate the proper momentum $P_{\bar{\rho}}$ which, by the proposed formula, gives the time derivative of the Krylov complexity, $\dot{C}(t)$.

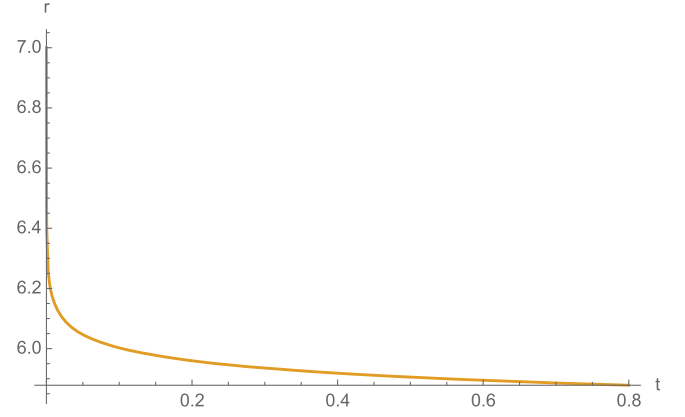


FIG. 2. We evaluate $r(t)$ for AdS_3 in Poincaré coordinates following (4.39). The initial conditions are set to be $\dot{r}_0 = 0.01$, $\rho_0 = 8$, $r_0 = 7$.

$$\frac{H}{m} \int_{r_0}^r \frac{dr}{\cosh r \sqrt{\frac{H^2}{m^2} e^{-2\rho_0} - \cosh^2 r}} = \int_{t_0}^t \frac{dt}{e^{-2\rho_0} + (t - t_0)^2}, \quad (4.40)$$

and so

1. Global coordinates for AdS_3

In global coordinates, we define the proper distance by

$$d\bar{\rho}^2 = dr^2 + \cosh^2 r d\rho^2, \quad (4.42)$$

and since the canonical momenta are

$$P_{\rho} = \frac{\partial L}{\partial \dot{\rho}} = \frac{H \dot{\rho}}{\cosh^2 \rho}, \quad P_r = \frac{\partial L}{\partial \dot{r}} = \frac{H \dot{r}}{\cosh^2 \rho \cosh^2 r}, \quad (4.43)$$

the proper momentum is

$$\begin{aligned} P_{\bar{\rho}} &= \frac{P_r}{\sqrt{1 + \cosh^2 r \rho'^2(r)}} + \frac{P_{\rho}}{\sqrt{r'^2 + \cosh^2 r}} \\ &= \frac{H \dot{r}}{\cosh^2 \rho \cosh^2 r} \frac{1}{\sqrt{1 + \cosh^2 r \rho'^2(r)}} + \frac{H \dot{\rho} \rho'(r)}{\cosh^2 \rho \sqrt{1 + \cosh^2 r \rho'^2(r)}} \\ &= \frac{H}{\cosh^2 \rho \cosh^2 r} \dot{\rho} \sqrt{\cosh^2 r + \frac{\dot{r}^2}{\dot{\rho}^2}} \\ &= \frac{H}{\cosh \rho \cosh r} \sqrt{1 - \frac{m^2 \cosh^2 \rho \cosh^2 r}{H^2}}, \end{aligned} \quad (4.44)$$

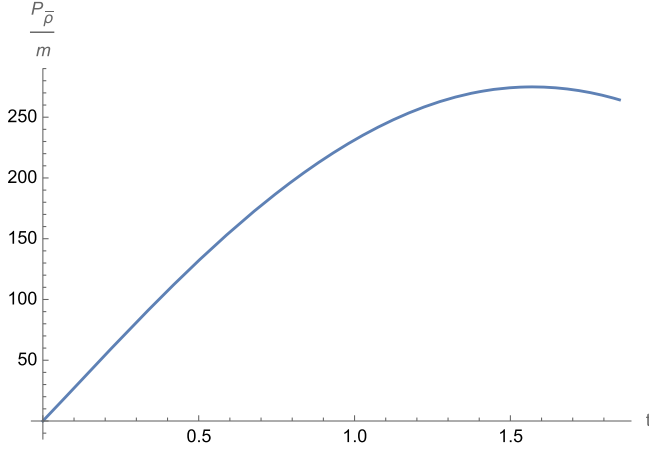


FIG. 3. The evolution of the proper momentum $P_{\bar{\rho}}(t)$ for AdS_3 in global coordinates has been carried out following (4.44). The initial conditions are set to be $\dot{r}_0 = 0.1$, $\rho_0 = 4$, $r_0 = 3$, $t_0 = 0$.

where we have substituted $\dot{\rho}, \dot{r}$ in terms of just r, ρ from their solutions.

If $\dot{r}_0 = 0$, we substitute H/m in terms of r_0, ρ_0 and obtain the simpler form

$$P_{\bar{\rho}} = \frac{H}{\cosh \rho \cosh r} \sqrt{1 - \frac{\cosh^2 \rho \cosh^2 r}{\cosh^2 \rho_0 \cosh^2 r_0}}. \quad (4.45)$$

An example of evolution of the proper momentum in this global case is in Fig. 3.

Note that if we would also have $r = r_0$, that would mean

$$P_{\bar{\rho}} = \frac{H}{\cosh \rho \cosh r_0} \sqrt{1 - \frac{\cosh^2 \rho}{\cosh^2 \rho_0}} = m \tanh \rho_0 \sin(t - t_0), \quad (4.46)$$

which is indeed the result for AdS_3 in global coordinates but, as we saw, this is not a good solution.

2. Poincaré coordinates for AdS_3

In the Poincaré case, we still have the same proper distance definition,

$$d\bar{\rho}^2 = dr^2 + \cosh^2 r d\rho^2, \quad (4.47)$$

and the canonical momenta are now

$$P_{\rho} = \frac{\partial L}{\partial \dot{\rho}} = \frac{H \dot{\rho}}{e^{2\rho}}, \quad P_r = \frac{\partial L}{\partial \dot{r}} = \frac{H \dot{r}}{e^{2\rho} \cosh^2 r}, \quad (4.48)$$

so the proper momentum is

$$\begin{aligned} P_{\bar{\rho}} &= \frac{P_r}{\sqrt{1 + \cosh^2 r \rho'^2(r)}} + \frac{P_{\rho}}{\sqrt{r'^2 + \cosh^2 r}} \\ &= \frac{H \dot{r}}{e^{2\rho} \cosh^2 r} \frac{1}{\sqrt{1 + \cosh^2 r \rho'^2(r)}} + \frac{H \dot{\rho} \rho'(r)}{e^{2\rho} \sqrt{1 + \cosh^2 r \rho'^2(r)}} \\ &= \frac{H}{e^{2\rho} \cosh^2 r} \dot{\rho} \sqrt{\cosh^2 r + \frac{\dot{r}^2}{\dot{\rho}^2}} \\ &= \frac{H}{e^{\rho} \cosh r} \sqrt{1 - \frac{m^2}{H^2} e^{2\rho} \cosh^2 r}, \end{aligned} \quad (4.49)$$

where we have substituted $\dot{\rho}, \dot{r}$ in terms of ρ, r .

If $\dot{r}_0 = 0$, we substitute H/m in terms of r_0, ρ_0 and obtain

$$P_{\bar{\rho}} = \frac{H}{e^{\rho} \cosh r} \sqrt{1 - \frac{e^{2\rho} \cosh^2 r}{e^{2\rho_0} \cosh^2 r_0}}. \quad (4.50)$$

An example of momentum dependence is in Fig. 4.

If we would have also $r = r_0$, then we would obtain

$$P_{\bar{\rho}} = \frac{H}{\cosh r_0} e^{-\rho} \sqrt{1 - e^{2(\rho - \rho_0)}} = m e^{\rho_0} (t - t_0), \quad (4.51)$$

which is indeed the result for AdS_3 in Poincaré coordinates but, as we saw, this is not a good solution.

3. Field theory interpretation

As we saw, the dual to the AdS_3 subspace is the $Sl(2)$ sector of $\mathcal{N} = 4$ SYM, and the complexity is considered within it, in that case. The subsector is obtained by acting with $D_+ = D_1 + iD_2$ on $Z = \Phi_5 + i\Phi_6$, where $x_1 + ix_2 = z = e^w = e^{t_E + i\phi}$, and it has $Sl(2)$ symmetry.

In our case, however, there is a sort of instantaneous (fixed time) $Sl(2)$ symmetry, due to the AdS_3 slicing, and

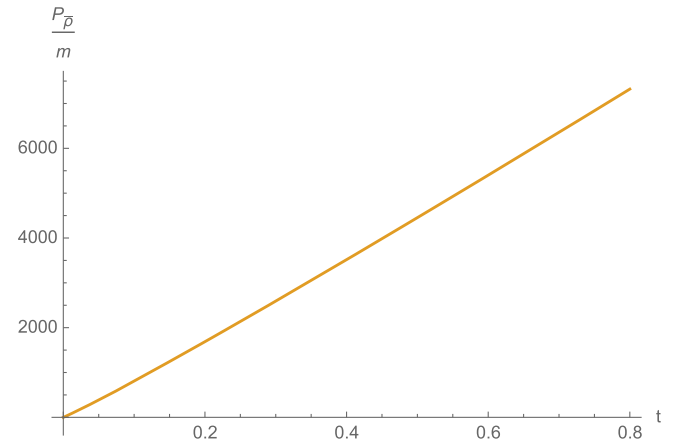


FIG. 4. The proper momentum for AdS_3 in Poincaré coordinates has been estimated using (4.49). The initial conditions are set to be $\dot{r}_0 = 0.01$, $\rho_0 = 8$, $r_0 = 7$.

resulting instantaneous AdS_3 symmetry. Moreover, as we saw, the boundary of AdS_5 on our $(r(t), \rho(t))$ motion is still $\mathbb{R}_t \times S^1$, like for AdS_3 .

One way to describe this is that ϕ , which was a coordinate along an equator of the S^3 in the $\mathcal{N} = 4$ SYM on the $\mathbb{R}_t \times S^3$ cylinder, now becomes a coordinate along an *instantaneous equator* defined by the motion along the S^3 induced by the motion in AdS_5 , described by $(\rho(t), r(t))$, so we have a sort of $\phi(t)$, and corresponding $\partial\phi \equiv \partial/\partial\phi(t)$ derivative.

Therefore we can say that the $SI(2)$ sector now varies with time, but stays essentially $SI(2)$. The details of this construction we will leave for further work.

In support of this picture, we saw that the proper momentum, giving the derivative of the Krylov complexity, has essentially the same behavior as in the pure AdS_3 case: in global coordinates, corresponding to CFT_2 on a circle, it has a sinusoidal-like behavior, due to the finiteness of the Hilbert space, while in Poincaré coordinates, corresponding to CFT_2 on the infinite line, it has a linearlike behavior, increasing without bound due to the nonfiniteness of the Hilbert space.

V. CONCLUSIONS

In this paper, we considered the holographic complexity of $\mathcal{N} = 4$ SYM, extending the formula $\dot{C}(t) = -P_{\bar{\rho}}/\epsilon$, where $P_{\bar{\rho}}$ is the proper momentum, defined when the evolution from the UV of the gravity dual is written like $ds^2 = d\bar{\rho}^2$ on the motion.

We took advantage of the fact that AdS_5 has a sub-manifold AdS_3 situated at $\theta = 0$, dual to the $SI(2)$ sector of

$\mathcal{N} = 4$ SYM, and of the fact that we can slice AdS_5 by AdS_3 slices. The AdS_3 case was exactly proven previously, so we just imported it here, and this allowed us to predict the behavior in the general case, as dual to a modified $SI(2)$ sector.

There are many issues left for further work. One was the actual calculation of complexity in the $SI(2)$ sector, without using the general CFT_2 methods, but rather in a quantum mechanical spin chain system (as opposed to a CFT_2). Another was the better definition of the deformed $SI(2)$ sector giving the holographic result. And, of course, to extend the methods defined here to other cases.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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