

# **Erratum: Hyperfine splittings of heavy quarkonium hybrids** **[Phys. Rev. D **108**, 014025 (2023)]**

Joan Soto<sup>✉</sup> and Sandra Tomàs Valls<sup>✉</sup>



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This erratum has been issued for three primary reasons. First, an extra global sign involving the matrices in (9) and (11) was brought into our attention.<sup>1</sup> Second, we discovered a missing factor of 2 in one of the long-distance potentials.<sup>2</sup> Third, we identified an omitted factor of  $-2$  associated with Eq. (4) in the code used to compute the spectrum.<sup>3</sup> These errors affect the extraction of the unknown short-distance parameters of the potentials presented in the original paper as well as the signs of the long-distance potentials. However, they only affect the spectrum in a mild way.

We also take this opportunity to offer some clarifications. According to the general expression for  $V_{1+11}^{sa}(r)$  in [2], its sign should be opposite to that used in our work. Since we consistently changed the sign of  $V_{1+11}^{sa}(r)$  in Eqs. (4) and (5), this choice did not alter the final results. However, this convention must be taken into account when we compare with other approaches in Fig. 4.

The first sign issue mentioned above is corrected by changing the global sign in the matrices of formulas (9) and (11). The long-distance potential with the factor of 2 corrected reads

$$\frac{V_{1+10}^{sb}(r)}{m_Q} = \mp \frac{2c_F g \Lambda' \pi^2}{m_Q \sqrt{\pi \kappa}} \frac{1}{r^2} \equiv V_{ld}^{sb}(r), \quad (1)$$

which should replace the second formula in (14). Due to the fact that  $g\Lambda'$  is about four times smaller than  $g\Lambda'''$ , this correction has a very small impact on the results.

Additionally, we correct the following typographical errors. Below (9),  $V_2^{--}$  and  $V_2^{0-}$  should read

$$V_2^{--} = \frac{\sqrt{\mathcal{J}-1}}{\sqrt{\mathcal{J}(2\mathcal{J}-1)}}, \quad V_2^{0-} = \frac{\mathcal{J}^2-1}{\sqrt{\mathcal{J}(\mathcal{J}-1)(4\mathcal{J}^2-1)}}. \quad (2)$$

The matrix in (10) should read,

$$\begin{pmatrix} \frac{1}{\mathcal{J}} & \frac{\sqrt{\mathcal{J}^2-1}}{\mathcal{J}} & 0 & 0 \\ \frac{\sqrt{\mathcal{J}^2-1}}{\mathcal{J}} & -\frac{1}{\mathcal{J}} & 0 & 0 \\ 0 & 0 & \frac{1}{\mathcal{J}+1} & \frac{\sqrt{(\mathcal{J}+2)\mathcal{J}}}{\mathcal{J}+1} \\ 0 & 0 & \frac{\sqrt{(\mathcal{J}+2)\mathcal{J}}}{\mathcal{J}+1} & -\frac{1}{\mathcal{J}+1} \end{pmatrix}. \quad (3)$$

Below (11),  $V_2^{0-+0}$  and  $V_2^{0--0}$  should read

$$V_2^{0-+0} = \sqrt{\frac{\mathcal{J}-1}{\mathcal{J}}} \frac{\mathcal{J}+1}{2\mathcal{J}+1}, \quad V_2^{0--0} = \frac{\sqrt{\mathcal{J}^2-1}(\mathcal{J}+2)}{3\mathcal{J}(2\mathcal{J}+1)}. \quad (4)$$

<sup>1</sup>We thank P. Heinrich and A. Vairo for pointing out this sign as well as for their help with further cross-checks.

<sup>2</sup>Reference [1] shows the results with this factor already corrected.

<sup>3</sup>We thank C. Schlosser and M. Wagner for discussions that helped identifying these two last mistakes.

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TABLE I. Corrected version of Table I in this paper.

| $\text{sign}(V_{ld}^{sa}) \text{ sign}(V_{ld}^{sb})$ |          | $+-$    | $++$    | $--$    | $+-$    |
|------------------------------------------------------|----------|---------|---------|---------|---------|
| $\chi^2/\text{d.o.f.}$                               |          | 0.6471  | 0.5818  | 0.8968  | 0.7418  |
| $A$ (GeV)                                            |          | -0.0858 | -0.0700 | -0.0257 | -0.0448 |
| $B$ (GeV <sup>3</sup> )                              |          | 0.0098  | 0.0117  | -0.0100 | -0.0123 |
| $(s/d)_1$ mass (GeV)                                 | $1^{--}$ | 4.0107  | 4.0107  | 4.0107  | 4.0107  |
|                                                      | $0^{-+}$ | 3.8862  | 3.8917  | 3.9041  | 3.8985  |
|                                                      | $1^{-+}$ | 3.9513  | 3.9526  | 3.9582  | 3.9556  |
|                                                      | $2^{-+}$ | 4.0593  | 4.0617  | 4.0602  | 4.0621  |
| $p_1$ mass (GeV)                                     | $1^{++}$ | 4.1450  | 4.1450  | 4.1450  | 4.1450  |
|                                                      | $0^{+-}$ | 4.1018  | 4.1012  | 4.1046  | 4.1051  |
|                                                      | $1^{+-}$ | 4.1140  | 4.1089  | 4.0920  | 4.1014  |
|                                                      | $2^{+-}$ | 4.1510  | 4.1478  | 4.1396  | 4.1439  |
| $(p/f)_2$ mass (GeV)                                 | $2^{++}$ | 4.2316  | 4.2316  | 4.2316  | 4.2316  |
|                                                      | $1^{+-}$ | 4.2032  | 4.1906  | 4.1662  | 4.1767  |
|                                                      | $2^{+-}$ | 4.2260  | 4.2248  | 4.2301  | 4.2301  |
|                                                      | $3^{+-}$ | 4.2466  | 4.2584  | 4.2791  | 4.2695  |
| $p_0$ mass (GeV)                                     | $0^{++}$ | 4.4864  | 4.4864  | 4.4864  | 4.4864  |
|                                                      | $1^{+-}$ | 4.4627  | 4.4773  | 4.4592  | 4.4461  |

Due to the missing factor of  $-2$  in the original code and the other reasons above, in Sec. IV the best fit now corresponds to a positive  $V_{ld}^{sa}(r)$  and  $V_{ld}^{sb}(r)$ , as shown in Table I and illustrated in Fig. 1, which should replace the Fig. 1 in the original manuscript. The former implies  $g\Lambda''' < 0$ , and the later selects the negative sign in the  $\mp$  of (1). Based on the ranges for  $g\Lambda'$  and  $g\Lambda'''$  provided in [3], the fit exhibits a mild preference for smaller absolute values of these parameters, yielding a  $\chi^2/\text{d.o.f.} = 0.582$  (now the  $\chi^2/\text{d.o.f.} = 0.806$  if we set the long-distance potentials to zero). The optimal values that we adopt henceforth are  $g\Lambda' = -0.059$  GeV and  $g\Lambda''' = -0.230$  GeV, which led to  $A = -0.0700$  GeV and  $B = 0.0117$  GeV<sup>3</sup>. When studying the dependence on  $r_0$ , the best  $\chi^2/\text{d.o.f.}$  corresponds to the default value  $r_0 = 3.96$  GeV<sup>-1</sup>. We also take this opportunity to improve the method used to estimate the uncertainties in A and B. We realized that, due to the proximity of a number of energy levels, the linearization that was carried out before in order to estimate the errors was not a good approximation. Assuming instead that the lattice data errors are fully correlated and performing fits to the average, maximum, and minimum values, we obtain now  $A = -0.070 \pm 0.010$  GeV and  $B = 0.0117 \pm 0.0003$  GeV<sup>3</sup>. The corresponding parameters for bottomonium in Sec. V are  $A' = -0.017 \pm 0.002$  GeV and  $B' = 0.0028 \pm 0.0001$  GeV<sup>3</sup>. The last paragraph of Sec. V should be removed, since now the best fit corresponds to the default value of  $r_0$ . Tables I–V here present the

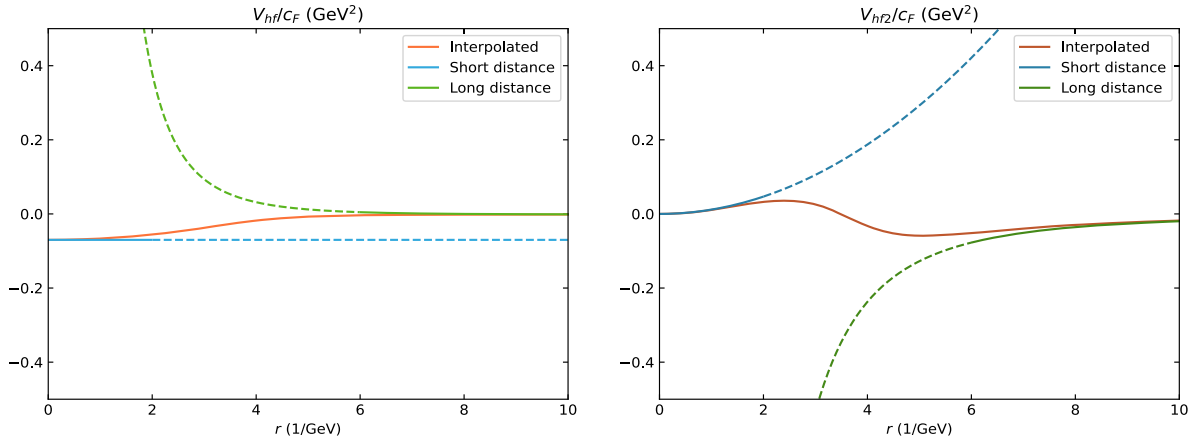


FIG. 1. Corrected version of Fig. 1 in this paper.

TABLE II. Corrected version of Table II in this paper.

| $r_0$ (GeV <sup>-1</sup> ) |          | 2.5     | 3       | 3.5     | 3.964   | 5       |
|----------------------------|----------|---------|---------|---------|---------|---------|
| $\chi^2/\text{d.o.f.}$     |          | 1.4686  | 0.7943  | 0.6148  | 0.5818  | 0.6266  |
| $A$ (GeV)                  |          | -0.1691 | -0.1133 | -0.0846 | -0.0700 | -0.0546 |
| $B$ (GeV <sup>3</sup> )    |          | 0.1723  | 0.0618  | 0.0253  | 0.0117  | 0.0020  |
| $(s/d)_1$ mass (GeV)       | $1^{--}$ | 4.0107  | 4.0107  | 4.0107  | 4.0107  | 4.0107  |
|                            | $0^{-+}$ | 3.8756  | 3.8821  | 3.8879  | 3.8917  | 3.8973  |
|                            | $1^{-+}$ | 3.9470  | 3.9490  | 3.9511  | 3.9526  | 3.9550  |
|                            | $2^{-+}$ | 4.0329  | 4.0513  | 4.0588  | 4.0617  | 4.0628  |
| $p_1$ mass (GeV)           | $1^{++}$ | 4.1450  | 4.1450  | 4.1450  | 4.1450  | 4.1450  |
|                            | $0^{+-}$ | 4.1036  | 4.1008  | 4.1001  | 4.1012  | 4.1040  |
|                            | $1^{+-}$ | 4.1255  | 4.1189  | 4.1129  | 4.1089  | 4.1032  |
|                            | $2^{+-}$ | 4.1362  | 4.1468  | 4.1486  | 4.1478  | 4.1450  |
| $(p/f)_2$ mass (GeV)       | $2^{++}$ | 4.2316  | 4.2316  | 4.2316  | 4.2316  | 4.2316  |
|                            | $1^{+-}$ | 4.2253  | 4.2092  | 4.1983  | 4.1906  | 4.1798  |
|                            | $2^{+-}$ | 4.2205  | 4.2189  | 4.2221  | 4.2248  | 4.2273  |
|                            | $3^{+-}$ | 4.2241  | 4.2403  | 4.2512  | 4.2584  | 4.2681  |
| $p_0$ mass (GeV)           | $0^{++}$ | 4.4864  | 4.4864  | 4.4864  | 4.4864  | 4.4864  |
|                            | $1^{+-}$ | 4.4638  | 4.4865  | 4.4816  | 4.4773  | 4.4754  |

TABLE III. Corrected version of Table III in this paper.

|                         |          |        | Fit error | Total error |
|-------------------------|----------|--------|-----------|-------------|
| $A$ (GeV)               |          | -0.070 | 0.010     |             |
| $B$ (GeV <sup>3</sup> ) |          | 0.0117 | 0.0003    |             |
| $(s/d)_1$ mass (GeV)    | $1^{--}$ | 4.011  |           | 0.030       |
|                         | $0^{-+}$ | 3.892  | 0.021     | 0.036       |
|                         | $1^{-+}$ | 3.953  | 0.010     | 0.032       |
|                         | $2^{-+}$ | 4.062  | 0.009     | 0.031       |
| $p_1$ mass (GeV)        | $1^{++}$ | 4.145  |           | 0.030       |
|                         | $0^{+-}$ | 4.101  | 0.009     | 0.031       |
|                         | $1^{+-}$ | 4.109  | 0.010     | 0.032       |
|                         | $2^{+-}$ | 4.148  | 0.002     | 0.030       |
| $(p/f)_2$ mass (GeV)    | $2^{++}$ | 4.232  |           | 0.030       |
|                         | $1^{+-}$ | 4.191  | 0.007     | 0.031       |
|                         | $2^{+-}$ | 4.225  | 0.002     | 0.030       |
|                         | $3^{+-}$ | 4.258  | 0.005     | 0.030       |
| $p_0$ mass (GeV)        | $0^{++}$ | 4.486  |           | 0.030       |
|                         | $1^{+-}$ | 4.477  | 0.004     | 0.030       |

TABLE IV. Corrected version of Table IV in this paper.

|            |          | Mass (GeV) | $A$ and $B$ error | Total error |
|------------|----------|------------|-------------------|-------------|
| $d_2$      | $2^{--}$ | 4.334      |                   | 0.030       |
|            | $1^{-+}$ | 4.325      | 0.002             | 0.030       |
|            | $2^{-+}$ | 4.325      | 0.002             | 0.030       |
|            | $3^{-+}$ | 4.334      | 0.0001            | 0.030       |
| $2(s/d)_1$ | $1^{--}$ | 4.355      |                   | 0.030       |
|            | $0^{-+}$ | 4.316      | 0.010             | 0.032       |
|            | $1^{-+}$ | 4.338      | 0.006             | 0.031       |
|            | $2^{-+}$ | 4.374      | 0.004             | 0.030       |

TABLE V. Corrected version of Table V in this paper.

|            |          | Mass (GeV) | $A$ and $B$ error | Total error |
|------------|----------|------------|-------------------|-------------|
| $(s/d)_1$  | $1^{--}$ | 10.690     |                   | 0.003       |
|            | $0^{-+}$ | 10.680     | 0.0021            | 0.004       |
|            | $1^{-+}$ | 10.685     | 0.0010            | 0.003       |
|            | $2^{-+}$ | 10.695     | 0.0007            | 0.003       |
| $p_1$      | $1^{++}$ | 10.761     |                   | 0.003       |
|            | $0^{+-}$ | 10.755     | 0.0011            | 0.003       |
|            | $1^{+-}$ | 10.758     | 0.0007            | 0.003       |
|            | $2^{+-}$ | 10.764     | 0.0002            | 0.003       |
| $(p/f)_2$  | $2^{++}$ | 10.819     |                   | 0.003       |
|            | $1^{+-}$ | 10.814     | 0.0011            | 0.003       |
|            | $2^{+-}$ | 10.817     | 0.0003            | 0.003       |
|            | $3^{+-}$ | 10.822     | 0.0005            | 0.003       |
| $d_2$      | $2^{--}$ | 10.870     |                   | 0.003       |
|            | $1^{-+}$ | 10.868     | 0.0005            | 0.003       |
|            | $2^{-+}$ | 10.869     | 0.0003            | 0.003       |
|            | $3^{-+}$ | 10.871     | 0.0001            | 0.003       |
| $2(s/d)_1$ | $1^{--}$ | 10.885     |                   | 0.003       |
|            | $0^{-+}$ | 10.880     | 0.0012            | 0.003       |
|            | $1^{-+}$ | 10.883     | 0.0006            | 0.003       |
|            | $2^{-+}$ | 10.888     | 0.0004            | 0.003       |
| $2p_1$     | $1^{++}$ | 10.970     |                   | 0.003       |
|            | $0^{+-}$ | 10.967     | 0.0008            | 0.003       |
|            | $1^{+-}$ | 10.968     | 0.0005            | 0.003       |
|            | $2^{+-}$ | 10.971     | 0.0002            | 0.003       |
| $2(p/f)_2$ | $2^{++}$ | 11.005     |                   | 0.003       |
|            | $1^{+-}$ | 11.002     | 0.0007            | 0.003       |
|            | $2^{+-}$ | 11.004     | 0.0002            | 0.003       |
|            | $3^{+-}$ | 11.007     | 0.0003            | 0.003       |
| $p_0$      | $0^{++}$ | 11.012     |                   | 0.003       |
|            | $1^{+-}$ | 11.012     | 0.0001            | 0.003       |

corrected versions of those originally published in this paper. The overall comparison of results remains unchanged, as only minor shifts are observed in Figs. 2 and 3, which should replace Figs. 2 and 3 in the original manuscript. The previous discussion and conclusions remain correct, the only change necessary being the update of our  $\chi^2$  to  $\chi^2/\text{d.o.f.} = 0.582$ , a slightly lower value than before.

Additionally, we now include in a new figure, Fig. 4, a comparison between our potentials, those from [4], and the recent lattice computation of the potentials in [5].<sup>4</sup> The potentials in [4] relate to  $V_{1^{+11}}^{sa}(r)$  and  $V_{1^{+10}}^{sb}(r)$  in Ref. [2] by

$$V_{1^{+11}}^{sa}(r) = V_{SK}(r), \quad (5)$$

$$V_{1^{+10}}^{sb}(r) = V_{SK}(r) + V_{SKb}(r)r^2. \quad (6)$$

Recall that our  $V_{1^{+11}}^{sa}$  has the opposite sign to the one in Ref. [2].

<sup>4</sup>We thank J. Tarrús Castellà for discussions regarding this comparison.

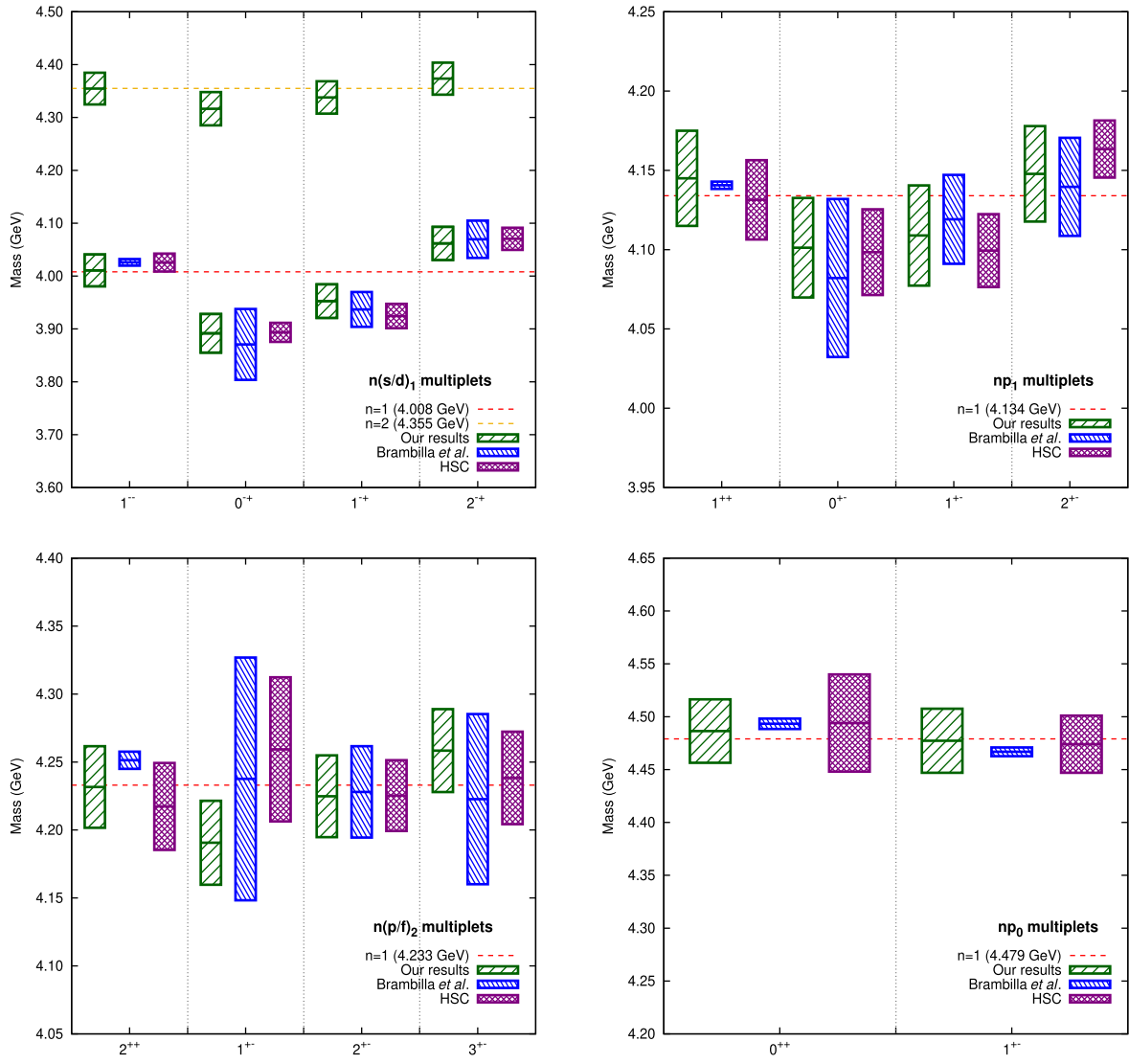


FIG. 2. Corrected version of Fig. 2 in this paper.

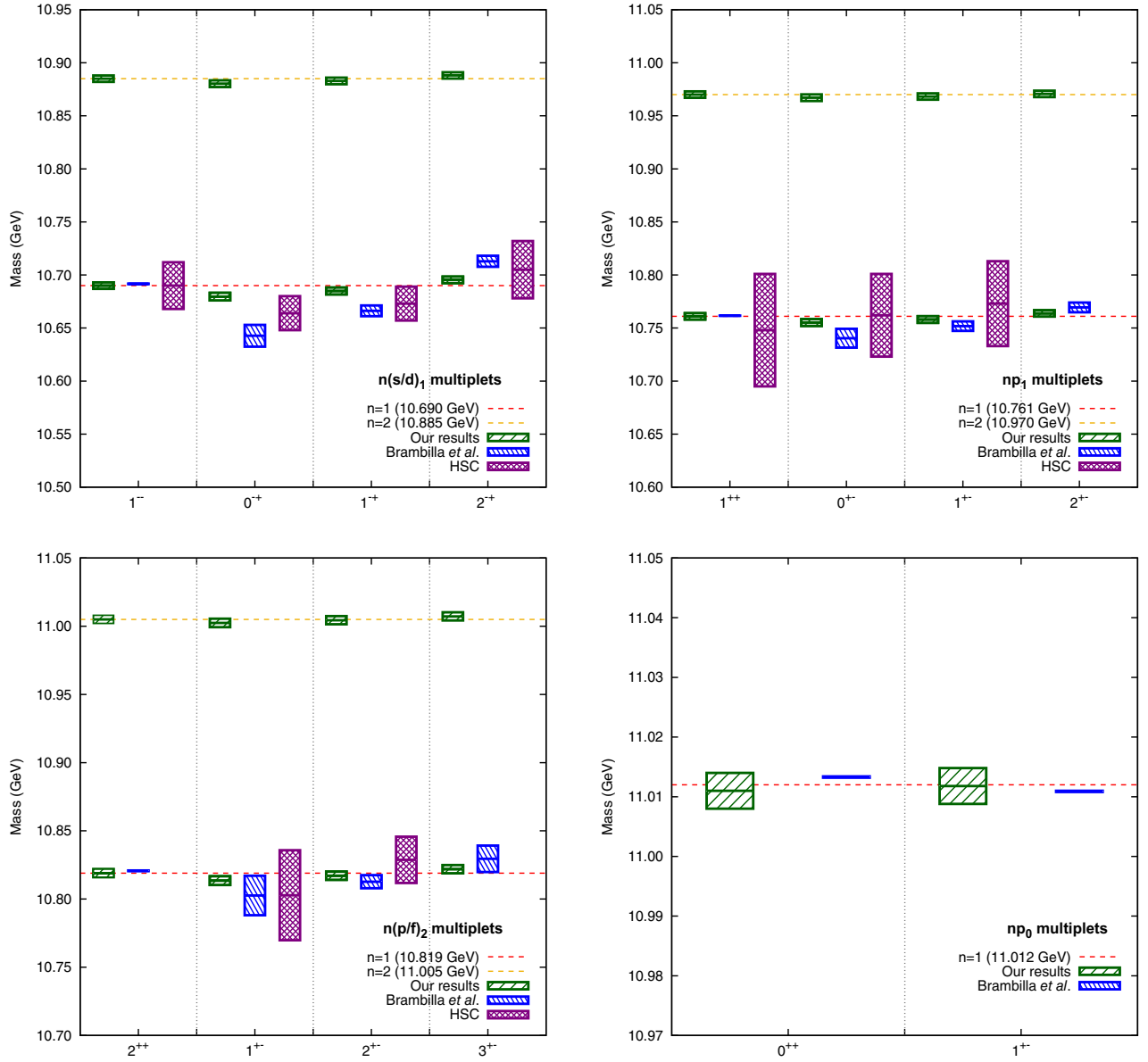
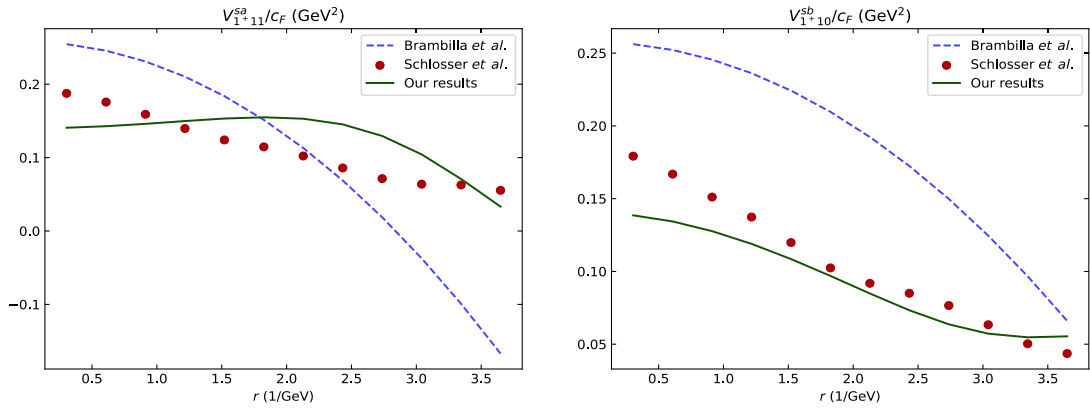


FIG. 3. Corrected version of Fig. 3 in this paper.

FIG. 4. Comparison of the  $V_{1+11}^{sa}(r)/c_F$  (left) and  $V_{1+10}^{sb}(r)/c_F$  (right) potentials in Ref. [2] with our model (solid green), with lattice results from [5] (red dots), and with results from Table II of [4] (dashed blue).

*Data availability.* No data were created or analyzed in this study.

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- [5] C. Schlosser and M. Wagner, [Phys. Rev. D \*\*111\*\*, 074504 \(2025\)](#).