

Saturation of Pauli blocking in near-extremal charged Nariai black holes

Chiang-Mei Chen^{1,2,3,*}, Chun-Chih Huang^{1,†} and Sang Pyo Kim^{4,2,3,5,‡}

¹*Department of Physics, National Central University, Chungli 32001, Taiwan*

²*Center for High Energy and High Field Physics (CHiP),
National Central University, Chungli 32001, Taiwan*

³*Asia Pacific Center for Theoretical Physics, Pohang 37673, Korea*

⁴*Department of Physics, School of Science and Technology,
Kunsan National University, Kunsan 54150, Korea*

⁵*Center for Relativistic Laser Science, Institute for Basic Science, Gwangju 61005, Korea*



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We solve the Dirac equation for a massive charged fermion in the near-extremal charged Nariai black hole with the near-horizon geometry $dS_2 \times S^2$. At the one-loop level, contrary to the catastrophic emission of charged spinless bosons in C.-M. Chen *et al.*, [Phys. Rev. D **110**, 085020 (2024)], we show that the emission of fermions in a narrow timelike inner region between the cosmological horizon and black hole horizon saturates the bound from the Pauli blocking and does not give an amplification (quantum superradiance). Using the reciprocal relation, we find the Schwinger emission of fermions from $AdS_2 \times S^2$, and compare the Schwinger emission of fermions and bosons from near-extremal Nariai black holes.

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I. INTRODUCTION

Extremal or near-extremal black holes have enhanced symmetry [1,2]. The near-extremal Reissner-Nordström (RN) black holes in an asymptotically flat space or anti-de Sitter (AdS) space have the near-horizon geometry of $AdS_2 \times S^2$, whereas RN black holes in a de Sitter (dS) space with three horizons have two coincident limits: one is the coincidence of inner (Cauchy) and outer (event) horizon of black hole with the near-horizon geometry of $AdS_2 \times S^2$ and the other is the coincidence of black hole horizon and cosmological horizon, the so-called Nariai limit, with the geometry $dS_2 \times S^2$.

The low temperature quantum theory of charged black holes with AdS_2 horizons provides an effective theory of the Einstein-Maxwell theory [3]. Recently the enhanced symmetries have been used to study quantum corrections of the near-extremal black holes and their entropies [4–11]. It is argued that two-dimensional Jackiw-Teitelboim gravity is an appropriate effective field theory that dominates the near-extremal black holes at low temperature, and

furthermore, this description may go away from the horizon [12,13].

The authors have studied Schwinger emission of charged scalars (spin-0 bosons) from near-extremal charged black holes [14], in which the extremely low Hawking temperature suppresses Hawking radiation of charged particles as well as charge neutral particles while the electric field on the horizon provides a channel for charge emission via Schwinger mechanism [15]. In fact, the enhanced symmetry allows one to solve the Klein-Gordon equation for massive charged scalars in near-extremal RN black holes [16–19] and in near-extremal Kerr-Newman black holes [14,20–22], both in terms of hypergeometric functions. Recently we have found the Schwinger effect of spontaneous pair production of massive spinors (spin-1/2 fermions) by a uniform electric field in the (A)dS₂ [23].

In this paper, we will study the spontaneous production of spin-1/2 massive charged fermions from near-extremal charged Nariai black holes. RN black holes in a dS space have three horizons: the inner (Cauchy) horizon, the outer (event) horizon, and the cosmological horizon. Near-extremal charged Nariai black holes have the black hole horizon close to the cosmological horizon, in which the black hole horizon emits Hawking radiation with Hawking temperature and the cosmological horizon emits with Gibbons-Hawking temperature. Though they are extremely small with slightly higher Hawking temperature than the Gibbons-Hawking temperature, the electric field that threads out from the black hole forces the black hole horizon to emit charges with the same sign as the black hole

*Contact author: cmchen@phy.ncu.edu.tw

†Contact author: makedate0809@gmail.com

‡Contact author: sangkim@kunsan.ac.kr

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and the cosmological horizon to emit charges of the opposite sign (for the scalar case, see Refs. [14,19]). We solve the Dirac equation in the near-extremal charged Nariai black holes, find the Bogoliubov coefficients and the mean number of spontaneously produced fermions in the inner and outer regions. We then show that two narrow horizons produce fermion pairs still obeying the Pauli exclusion principle and saturating the Pauli blocking when two horizons coincide.

The organization of this paper is as follows: in Sec. II, we solve the Dirac equation for spin-1/2 massive charged fermion in the spacelike outer region of near-extremal Nariai black holes, find the Bogoliubov transformation and relation, and compute the mean number of fermion pair production. In Sec. III, we find the Bogoliubov transformation and relation, and then compute the mean number of fermion pair production in the timelike inner region of near-extremal Nariai black holes. In Sec. IV, we compare the fermion production in an exact Nariai black hole and in a uniform electric field in dS_2 space, and apply the reciprocal relation¹ of mean numbers to find the fermion production in AdS_2 space. In Sec. V, we compare the fermion emission with catastrophic scalar emission from near-extremal Nariai black holes.

II. OUTER REGION OF NEAR-EXTREMAL NARIAI BLACK HOLES

A. Dirac equation

The Dirac equation for a massive charged spinor field, Ψ , in a charged black hole is given by

$$\gamma^a e_a^\mu (\partial_\mu + \Gamma_\mu - iqA_\mu) \Psi + m\Psi = 0, \quad (1)$$

where m and q are the mass and charge of the spinor field, respectively, and A_μ is the gauge potential of black hole. The tetrad e_a^μ , one-form Γ , and connection one-form ω_{ab} are defined as

$$\begin{aligned} \eta_{ab} &= e_a^\mu e_b^\nu g_{\mu\nu}, \quad \Gamma = \Gamma_\mu dx^\mu = \frac{1}{4} \gamma^a \gamma^b \omega_{ab}, \\ d\vartheta^a &= -\omega^a_b \wedge \vartheta^b, \end{aligned} \quad (2)$$

where the gamma matrices satisfy $\gamma^a \gamma^b + \gamma^b \gamma^a = 2\eta^{ab}$. Below, Greek letters will denote coordinate indices, and Latin letters denote frame indices.

¹The reciprocity of Schwinger effect of either scalar or spinor between the dS and AdS space, $\mathcal{N}_{dS}(E, \mathcal{R}) \mathcal{N}_{AdS}(E, \mathcal{R}) = 1$ with $\mathcal{R}$ denoting either the dS or AdS scalar curvature, is the inverse proportionality and should not be confused with the same terminology used in other contexts, such as the Lorentz reciprocity, Onsager reciprocal relation, etc.

For the RN-dS black holes,

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2 d\Omega_2^2, \quad A_{[1]} = \frac{Q}{r} dt, \quad (3)$$

where the lapse function can have maximally three positive horizons and a negative root, as follows:

$$\begin{aligned} F(r) &= 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{r^2}{L^2} \\ &= -\frac{(r-r_-)(r-r_+)(r-r_c)(r+r_-+r_++r_c)}{L^2 r^2}. \end{aligned} \quad (4)$$

The Nariai limit corresponds $r_+ = r_c \equiv r_n$ which can be parametrized by charge Q_n as

$$\begin{aligned} r_n^2 &= \frac{L^2}{6} \left(1 + \sqrt{1 - 12Q_n^2/L^2} \right), \\ M_n &= \frac{r_n}{3} \left(2 - \sqrt{1 - 12Q_n^2/L^2} \right). \end{aligned} \quad (5)$$

We can further consider the near Nariai limit (near-extremal Nariai black hole) with a slight derivation as

$$\begin{aligned} r_+ &= r_n - \epsilon B, \quad r_c = r_n + \epsilon B, \\ M &= M_n - \epsilon^2 B^2 \frac{2r_n}{L^2}, \quad Q^2 = Q_n^2 - \epsilon^2 B^2 \frac{M_n}{r_n}. \end{aligned} \quad (6)$$

We first consider the Dirac equation in the spacelike outer region as $r = r_n + \epsilon\tau$, $t = \epsilon^{-1}r_{ds}^2\rho$ of the near-extremal Nariai black hole, which, after taking $\epsilon \rightarrow 0$, is described by the geometry of $dS_2 \times S^2$ [19], for $\tau \in (B, \infty)$ as follows:

$$\begin{aligned} ds^2 &= r_{ds}^2 \left(-\frac{d\tau^2}{f(\tau)} + f(\tau) d\rho^2 \right) + r_n^2 (d\theta^2 + \sin^2\theta d\varphi^2), \\ f(\tau) &= \tau^2 - B^2, \end{aligned} \quad (7)$$

and the gauge (vector) potential is given by

$$A = \frac{r_{ds}^2 Q_n}{r_n^2} \tau d\rho = \phi(\tau) d\rho, \quad (8)$$

where

$$r_{ds}^2 = \frac{L^2}{6} \left(\frac{1}{\sqrt{1 - 12Q_n^2/L^2}} + 1 \right). \quad (9)$$

In the orthonormal frame $\vartheta^a = e^a_\mu dx^\mu$, we define the tetrad as

$$e^0_\tau = \frac{r_{ds}}{\sqrt{f}}, \quad e^1_\rho = r_{ds} \sqrt{f}, \quad e^2_\theta = r_n, \quad e^3_\varphi = r_n \sin\theta, \quad (10)$$

and then write the term $\gamma^a e_a^\mu \Gamma_\mu$ as

$$\gamma^a e_a{}^\mu \Gamma_\mu = \gamma^1 e_1{}^\rho \Gamma_\rho + \gamma^3 e_3{}^\varphi \Gamma_\varphi = \frac{\gamma^0}{4r_{\text{ds}}\sqrt{f}} \frac{df}{d\tau} + \frac{\gamma^2}{2r_n} \cot\theta. \quad (11)$$

Setting the spinor field as

$$\Psi = f^{-1/4}(\sin\theta)^{-1/2}\Phi, \quad (12)$$

the Dirac equation can be written as

$$\left(\gamma^0 \frac{\sqrt{f}}{r_{\text{ds}}} \partial_\tau + \frac{\gamma^1}{r_{\text{ds}}\sqrt{f}} (\partial_\rho - iq\phi) + \frac{\gamma^2}{r_n} \partial_\theta + \frac{\gamma^3}{r_n \sin\theta} \partial_\varphi + m \right) \Phi = 0. \quad (13)$$

B. Exact solution

We expand the spinor field (12) in terms of the orthonormal spherical spinors

$$\begin{aligned} \Phi_{v,n}^+(\theta, \varphi) &= \begin{bmatrix} i\mathbb{Y}_{j\mp 1/2}^n(\theta, \varphi) \\ 0 \end{bmatrix}, \\ \Phi_{v,n}^-(\theta, \varphi) &= \begin{bmatrix} 0 \\ \mathbb{Y}_{j\pm 1/2}^n(\theta, \varphi) \end{bmatrix}, \end{aligned} \quad (14)$$

where each entry is a two-component spherical spinor defined in terms of the spherical harmonics. The parameter

$$v = \mp \left(j + \frac{1}{2} \right) \quad \text{for } j = l \pm \frac{1}{2}, \quad (15)$$

is determined by the quantum number of the angular momentum j and its projection n , where $-l \leq n \leq l$ and l is an integer. Moreover, the spherical spinors satisfy the following properties:

$$\begin{aligned} \gamma^0 \Phi_{v,n}^\pm &= \mp i \Phi_{v,n}^\pm, & \gamma^1 \Phi_{v,n}^\pm &= \pm i \Phi_{v,n}^\mp, \\ \left(\gamma^2 \partial_\theta + \gamma^3 \frac{\partial_\varphi}{\sin\theta} \right) \Phi_{v,n}^\pm &= i v \Phi_{v,n}^\mp. \end{aligned} \quad (16)$$

Now, we assume that the solution of the spinor field Φ is given by

$$\begin{aligned} \Phi(\tau, \rho, \theta, \varphi) &= \xi_+(\tau) \exp(ik\rho) \begin{bmatrix} i\mathbb{Y}_{j-1/2}^n(\theta, \varphi) \\ 0 \end{bmatrix} \\ &+ \xi_-(\tau) \exp(ik\rho) \begin{bmatrix} 0 \\ \mathbb{Y}_{j+1/2}^n(\theta, \varphi) \end{bmatrix}. \end{aligned} \quad (17)$$

Hereafter we will consider only $v = -(j + 1/2)$.

Then the Dirac equation can be separated into a pair of coupled equations

$$\left(\sqrt{f} \frac{d}{d\tau} \pm imr_{\text{ds}} \right) \xi_\pm + \left(\frac{ik - iq\phi}{\sqrt{f}} \mp v \frac{r_{\text{ds}}}{r_n} \right) \xi_\mp = 0. \quad (18)$$

To solve Eq. (18), we first introduce new functions

$$T_\pm(\tau) = \xi_\pm(\tau) \pm \xi_\mp(\tau) \Rightarrow \xi_\pm(\tau) = \frac{T_+(\tau) \pm T_-(\tau)}{2}, \quad (19)$$

and then obtain

$$\left(\sqrt{f} \frac{d}{d\tau} \pm \frac{ik - iq\phi}{\sqrt{f}} \right) T_\pm + \left(imr_{\text{ds}} \pm v \frac{r_{\text{ds}}}{r_n} \right) T_\mp = 0. \quad (20)$$

Note that the function f have been combined in Eq. (20).

To further simplify Eq. (20), we introduce a new variable and apply a transformation to the functions $T_\pm$ as

$$\begin{aligned} z &= \frac{\tau + B}{2B}, & T_\pm(z) &= \Sigma^{\mp 1} \tilde{T}_\pm(z), \\ \Sigma &= z^{-i(\tilde{\kappa} + \kappa)/2} (z - 1)^{i(\tilde{\kappa} - \kappa)/2}, \end{aligned} \quad (21)$$

where $\tilde{\kappa} = k/B$ and $\kappa = r_{\text{ds}}^2(qQ_n/r_n^2) = r_{\text{ds}}^2 E_n$ is the dimensionless electric field on the horizon. Substituting these into Eq. (20), we obtain

$$\Sigma^{\mp 2} \sqrt{z(z-1)} \frac{d\tilde{T}_\pm}{dz} + \left(imr_{\text{ds}} \pm v \frac{r_{\text{ds}}}{r_n} \right) \tilde{T}_\mp = 0. \quad (22)$$

Finally, after decoupling $\tilde{T}_\pm$, we find the solutions of Eq. (22) in terms of the hypergeometric functions

$$\begin{aligned} \tilde{T}_\pm &= C_\pm F \left(\pm i(\mu + \kappa), \mp i(\mu - \kappa); \frac{1}{2} \mp i(\tilde{\kappa} - \kappa); 1 - z \right) \\ &+ D_\pm \Sigma^{\pm 2} \sqrt{z(z-1)} F \left(1 \mp i(\mu + \kappa), 1 \pm i(\mu - \kappa); \frac{3}{2} \right. \\ &\left. \pm i(\tilde{\kappa} - \kappa); 1 - z \right), \end{aligned} \quad (23)$$

where $\mu^2 = \kappa^2 + m^2 r_{\text{ds}}^2 + v^2 r_{\text{ds}}^2 / r_n^2$. The relations of coefficients $C_\pm$ and $D_\pm$ can be determined by Eq. (22), as follows:

$$D_\pm = -C_\mp \frac{imr_{\text{ds}} \pm v r_{\text{ds}} / r_n}{1/2 \pm i(\tilde{\kappa} - \kappa)}. \quad (24)$$

C. Asymptotic and near-horizon behaviors

Now we find the Bogoliubov relation between the modes at $\tau \rightarrow B$ and $\tau \rightarrow \infty$ in the near-horizon region. To do so, we identify the ingoing mode at the horizon $\tau \rightarrow B$ and the outgoing mode at the future infinity $\tau \rightarrow \infty$. In the region of

$\tau \rightarrow B$ ($z \rightarrow 1$), the function $T_{\pm}(z)$ has the asymptotic form

$$\lim_{z \rightarrow 1} T_{\pm}(z) = C_{\pm}(z-1)^{\mp i(\tilde{\kappa}-\kappa)/2}. \quad (25)$$

Then the spinor field Φ can be decomposed into two modes, as follows:

$$\Phi_B^{(\pm)} = \frac{C_{\pm}}{2} \exp(ik\rho)(z-1)^{\mp i(\tilde{\kappa}-\kappa)/2} \begin{bmatrix} i\mathbb{Y}_{j-1/2}^n \\ \pm \mathbb{Y}_{j+1/2}^n \end{bmatrix}, \quad (26)$$

where the upper (lower) sign corresponds to the positive (negative) frequency modes. Note that we need to consider the magnitudes of $\tilde{\kappa}$ and κ , which will be discussed later. In the other region $\tau \rightarrow \infty$ ($z \rightarrow \infty$), the function $T_{\pm}(z)$ can be written as

$$\begin{aligned} \lim_{z \rightarrow \infty} T_+(z) &= \left(C_+ \Xi_+ + C_- \frac{imr_{\text{ds}} + vr_{\text{ds}}/r_n}{i(\mu + \kappa)} \Omega_- \right) z^{-i\mu} \\ &+ \left(C_+ \Omega_+ - C_- \frac{imr_{\text{ds}} + vr_{\text{ds}}/r_n}{i(\mu - \kappa)} \Xi_- \right) z^{i\mu}, \end{aligned} \quad (27)$$

$$\begin{aligned} \lim_{z \rightarrow \infty} T_-(z) &= \left(C_- \Omega_- + C_+ \frac{imr_{\text{ds}} - vr_{\text{ds}}/r_n}{i(\mu - \kappa)} \Xi_+ \right) z^{-i\mu} \\ &+ \left(C_- \Xi_- - C_+ \frac{imr_{\text{ds}} - vr_{\text{ds}}/r_n}{i(\mu + \kappa)} \Omega_+ \right) z^{i\mu}, \end{aligned} \quad (28)$$

where

$$\begin{aligned} \Xi_{\pm} &= \frac{\Gamma(1/2 \mp i(\tilde{\kappa} - \kappa)) \Gamma(\mp 2i\mu)}{\Gamma(1/2 \mp i(\tilde{\kappa} + \mu)) \Gamma(\mp i(\mu - \kappa))}, \\ \Omega_{\pm} &= \frac{\Gamma(1/2 \mp i(\tilde{\kappa} - \kappa)) \Gamma(\pm 2i\mu)}{\Gamma(1/2 \mp i(\tilde{\kappa} - \mu)) \Gamma(\pm i(\mu + \kappa))}. \end{aligned} \quad (29)$$

Then the spinor field Φ_B can be decomposed into the positive frequency mode $\Phi_{\infty}^{(+)}$ and the negative frequency mode $\Phi_{\infty}^{(-)}$, as follows:

$$\begin{aligned} \Phi_{\infty}^{(+)} &= \frac{C_+ \Xi_+}{2} \exp(ik\rho) z^{-i\mu} \sqrt{\frac{2\mu}{\mu - \kappa}} \begin{bmatrix} i\Lambda_1 \mathbb{Y}_{j-1/2}^n \\ \Lambda_4 \mathbb{Y}_{j+1/2}^n \end{bmatrix} \\ &+ \frac{C_- \Omega_-}{2} \exp(ik\rho) z^{-i\mu} \sqrt{\frac{2\mu}{\mu + \kappa}} \begin{bmatrix} i\Lambda_3 \mathbb{Y}_{j-1/2}^n \\ -\Lambda_2 \mathbb{Y}_{j+1/2}^n \end{bmatrix}, \end{aligned} \quad (30)$$

$$\begin{aligned} \Phi_{\infty}^{(-)} &= \frac{C_+ \Omega_+}{2} \exp(ik\rho) z^{i\mu} \sqrt{\frac{2\mu}{\mu + \kappa}} \begin{bmatrix} i\Lambda_2^* \mathbb{Y}_{j-1/2}^n \\ \Lambda_3^* \mathbb{Y}_{j+1/2}^n \end{bmatrix} \\ &+ \frac{C_- \Xi_-}{2} \exp(ik\rho) z^{i\mu} \sqrt{\frac{2\mu}{\mu - \kappa}} \begin{bmatrix} i\Lambda_4^* \mathbb{Y}_{j-1/2}^n \\ -\Lambda_1^* \mathbb{Y}_{j+1/2}^n \end{bmatrix}, \end{aligned} \quad (31)$$

where

$$\begin{aligned} \Lambda_1 &= \sqrt{\frac{\mu - \kappa}{2\mu}} \left(1 + \frac{imr_{\text{ds}} - vr_{\text{ds}}/r_n}{i(\mu - \kappa)} \right), \\ \Lambda_2 &= \sqrt{\frac{\mu + \kappa}{2\mu}} \left(1 - \frac{imr_{\text{ds}} + vr_{\text{ds}}/r_n}{i(\mu + \kappa)} \right), \\ \Lambda_3 &= \sqrt{\frac{\mu + \kappa}{2\mu}} \left(1 + \frac{imr_{\text{ds}} + vr_{\text{ds}}/r_n}{i(\mu + \kappa)} \right), \\ \Lambda_4 &= \sqrt{\frac{\mu - \kappa}{2\mu}} \left(1 - \frac{imr_{\text{ds}} - vr_{\text{ds}}/r_n}{i(\mu - \kappa)} \right). \end{aligned} \quad (32)$$

Note that a necessary condition for pair production is that the parameter μ should be real; otherwise, the particle concept for spinors cannot be defined.

In the region of $B \ll 1$, the positive and negative modes depend on the relative magnitude of $\tilde{\kappa}$ and κ , and we will assume that $\tilde{\kappa} \gg \kappa$, so $\Phi_B^{(+)}$ and $\Phi_B^{(-)}$ represent the positive and negative frequency modes, respectively. To find the Bogoliubov coefficients, we impose the boundary condition that a positive frequency mode outgoes from the horizon:

$$\Phi_B^{(-)} = 0 \Rightarrow C_- = 0. \quad (33)$$

Then, from the Bogoliubov transformation

$$\Phi_B^{(+)} = \alpha \Phi_{\infty}^{(+)} + \beta^* \Phi_{\infty}^{(-)}, \quad (34)$$

we obtain the corresponding Bogoliubov coefficients

$$\alpha = \sqrt{\frac{2\mu}{\mu - \kappa}} \Xi_+ \Rightarrow |\alpha|^2 = \frac{\sinh(\pi\mu - \pi\kappa) \cosh(\pi\tilde{\kappa} + \pi\mu)}{\cosh(\pi\tilde{\kappa} - \pi\kappa) \sinh(2\pi\mu)}, \quad (35)$$

$$\beta^* = \sqrt{\frac{2\mu}{\mu + \kappa}} \Omega_+ \Rightarrow |\beta|^2 = \frac{\sinh(\pi\mu + \pi\kappa) \cosh(\pi\tilde{\kappa} - \pi\mu)}{\cosh(\pi\tilde{\kappa} - \pi\kappa) \sinh(2\pi\mu)}. \quad (36)$$

They satisfy the Bogoliubov relation $|\alpha|^2 + |\beta|^2 = 1$ for fermions.

For $\tilde{\kappa} > \kappa$, introducing the Unruh and effective temperatures² similarly as the scalar case [19], as follows:

²These are not temperatures associated to black holes. The Unruh temperature corresponds to the acceleration of produced particles. However, the effective temperatures appearing in the mean number should include corrections by the gravitational curvature effects.

$$T_U = \frac{\kappa}{2\pi\bar{m}r_{\text{ds}}^2}, \quad T_{\text{eff}} = \sqrt{T_U^2 + \frac{1}{(2\pi r_{\text{ds}})^2}} + T_U, \\ \bar{T}_{\text{eff}} = \sqrt{T_U^2 + \frac{1}{(2\pi r_{\text{ds}})^2}} - T_U, \quad (37)$$

where $\bar{m}$ is the effective mass in the dS space

$$\bar{m}^2 = m^2 + \frac{(j + 1/2)^2}{r_n^2}, \quad (38)$$

we express the mean number in terms of thermodynamic variables as

$$|\beta|^2 = \frac{(1 - e^{-\bar{m}/\bar{T}_{\text{eff}}})(1 + e^{\bar{m}/T_{\text{eff}}}e^{-(k-q\Phi_H)/T_H})}{(e^{\bar{m}/T_{\text{eff}}} - e^{-\bar{m}/\bar{T}_{\text{eff}}})(1 + e^{-(k-q\Phi_H)/T_H})}, \quad (39)$$

where the Hawking temperature and chemical potential are

$$T_H = \frac{B}{2\pi}, \quad \Phi_H = B \frac{Qr_{\text{ds}}^2}{r_n^2}. \quad (40)$$

The temperature T_H measured in the black hole coordinates is $T_H = B/2\pi \times \epsilon/r_{\text{ds}}^2$. Note that the effective mass for scalars have $m^2 - 1/(4r_{\text{ds}}^2)$ and $l(l+1)$ instead of m^2 and $(j+1/2)^2$ for fermions.

The emission formula (36) in the extremal case, i.e., ($B \rightarrow 0$, $T_H \rightarrow 0$, and $\tilde{\kappa} \rightarrow \infty$), is

$$|\beta|^2 = \frac{e^{2\pi\kappa} - e^{-2\pi\mu}}{2 \sinh(2\pi\mu)}. \quad (41)$$

A few comments are in order. First, in the Nariai limit of an uncharged black hole ($Q = 0$) that is the Nariai limit of a Schwarzschild black hole in dS space, the fermion production reduces to the Fermi-Dirac distribution with $T_C = 1/2\pi r_{\text{ds}}$ and cannot saturate the Pauli blocking since $T_C \geq 0$. Second, the emission of fermions from an extremal charged Nariai black hole can saturate the Pauli blocking ($|\beta|^2 = 1$) provided that $\kappa \rightarrow \infty$ which corresponds to $r_{\text{ds}} \rightarrow \infty$ and the Nariai black hole gets close to the ultracold limit below. Third, in the ultracold limit, where the three horizons are degenerate, $r_- = r_+ = r_c$, so that $r_n = L/\sqrt{6}$, $M_n = \sqrt{2/27}L$ and $Q_n = L/\sqrt{12}$, the near horizon dS_2 geometry reduces to two-dimensional Minkowski spacetime as $r_{\text{ds}} \rightarrow \infty$. Accordingly the parameters μ, κ become infinite, which leads to $|\beta|^2 = e^{-2\pi(\mu-\kappa)}$. Moreover, in this limit we have $\mu - \kappa = (m^2 + v^2/r_n^2)/(2qQ/r_n^2) = (m^2 + v^2/r_n^2)/2qE_n$, which gives the Schwinger formula for charge with transverse momenta v^2/r_n^2 [15]; therefore, the pair production is exponentially suppressed when $m^2 \gg qQ/L^2$. One may compare this exponential suppression of the Schwinger effect with the

Festina-Lente bound $m^2 \gg qM_P/L$ with M_P denoting the Planck mass that protects Nariai black holes from evaporating into a big crunch [24]. It corresponds to the Planckian scale electrostatic potential at the cosmological horizon. However, the slow (adiabatic) emission of charges from Nariai black holes violates the weak gravity conjecture (for instance, Ref. [25]).

III. INNER REGION OF NEAR-EXTREMAL NARIAI BLACK HOLES

A. Dirac equation

The timelike inner region, $r = r_n + \epsilon\rho$, $t = \epsilon^{-1}r_{\text{ds}}^2\tau$ with $-B < \rho < B$, of the near-extremal Nariai black hole, after taking $\epsilon \rightarrow 0$, is still described by the geometry of $\text{dS}_2 \times S^2$, as follows:

$$ds^2 = r_{\text{ds}}^2 \left(-f(\rho)d\tau^2 + \frac{d\rho^2}{f(\rho)} \right) + r_n^2 (d\theta^2 + \sin^2\theta d\varphi^2), \\ f(\rho) = B^2 - \rho^2, \quad (42)$$

and the gauge (Coulomb) potential is given by

$$A = -\frac{r_{\text{ds}}^2 Q_n}{r_n^2} \rho d\tau = \phi(\rho) d\tau. \quad (43)$$

Similarly to Eq. (10) and replacing $e^0_\tau \rightarrow e^1_\rho$ and $e^1_\rho \rightarrow e^0_\tau$, we define the tetrad as

$$e^0_\tau = r_{\text{ds}}\sqrt{f}, \quad e^1_\rho = \frac{r_{\text{ds}}}{\sqrt{f}}, \quad e^2_\theta = r_n, \quad e^3_\varphi = r_n \sin\theta, \quad (44)$$

and write

$$\gamma^a e_a^\mu \Gamma_\mu = \gamma^0 e_0^\tau \Gamma_\tau + \gamma^1 e_1^\rho \Gamma_\rho = \frac{\gamma^1}{4r_{\text{ds}}\sqrt{f}} \frac{df}{d\rho} + \frac{\gamma^2}{2r_n} \cot\theta. \quad (45)$$

Using the spinor field (12), the Dirac equation now takes the form

$$\left(\frac{\gamma^0}{r_{\text{ds}}\sqrt{f}} (\partial_\tau - iq\phi) + \gamma^1 \frac{\sqrt{f}}{r_{\text{ds}}} \partial_\rho + \frac{\gamma^2}{r_n} \partial_\theta \right. \\ \left. + \frac{\gamma^3}{r_n \sin\theta} \partial_\varphi + m \right) \Phi = 0. \quad (46)$$

B. Exact solution

Now, we assume the positive frequency solution of the spinor field Φ , as follows:

$$\Phi(\tau, \rho, \theta, \varphi) = \xi_+(\rho) \exp(-i\omega\tau) \begin{bmatrix} i\mathbb{Y}_{j-1/2}^n(\theta, \varphi) \\ 0 \end{bmatrix} + \xi_-(\rho) \exp(-i\omega\tau) \begin{bmatrix} 0 \\ \mathbb{Y}_{j+1/2}^n(\theta, \varphi) \end{bmatrix}, \quad (47)$$

and only consider $v = -(j + 1/2)$ below. Then the Dirac equation equals two coupled equations:

$$\left(\sqrt{f} \frac{d}{d\rho} \pm v \frac{r_{\text{ds}}}{r_n} \right) \xi_{\pm} - \left(\frac{i\omega + iq\phi}{\sqrt{f}} \pm imr_{\text{ds}} \right) \xi_{\mp} = 0. \quad (48)$$

As in the spacelike outer region, a linear combination

$$R_{\pm}(\rho) = \xi_+(\rho) \pm \xi_-(\rho) \Rightarrow \xi_{\pm}(\rho) = \frac{R_{\pm}(\rho) \pm R_{\mp}(\rho)}{2}, \quad (49)$$

leads to

$$\left(\sqrt{f} \frac{d}{d\rho} \mp \frac{i\omega + iq\phi}{\sqrt{f}} \right) R_{\pm} + \left(v \frac{r_{\text{ds}}}{r_n} \pm imr_{\text{ds}} \right) R_{\mp} = 0. \quad (50)$$

Again, by introducing a new variable and a factor into the functions $R_{\pm}$, as follows:

$$z = \frac{\rho + B}{2B}, \quad R_{\pm}(z) = \Sigma^{\mp 1} \tilde{R}_{\pm}(z), \quad \Sigma = z^{-i(\tilde{\kappa} + \kappa)/2} (1 - z)^{i(\tilde{\kappa} - \kappa)/2}, \quad (51)$$

where

$$\tilde{\kappa} = \frac{\omega}{B}, \quad \kappa = qQ_n \frac{r_{\text{ds}}^2}{r_n^2}, \quad (52)$$

we obtain

$$\Sigma^{\mp 2} \sqrt{z(1-z)} \frac{d\tilde{R}_{\pm}}{dz} + \left(v \frac{r_{\text{ds}}}{r_n} \pm imr_{\text{ds}} \right) \tilde{R}_{\mp} = 0. \quad (53)$$

Finally, the solutions of Eq. (53) are given by the hypergeometric functions

$$\begin{aligned} \tilde{R}_{\pm} = & C_{\pm} F \left(\pm i(\mu + \kappa), \mp i(\mu - \kappa); \frac{1}{2} \pm i(\tilde{\kappa} + \kappa); z \right) \\ & + D_{\pm} \Sigma^{\pm 2} \sqrt{z(1-z)} F \left(1 \mp i(\mu + \kappa), 1 \pm i(\mu - \kappa); \frac{3}{2} \right. \\ & \left. \mp i(\tilde{\kappa} + \kappa); z \right), \end{aligned} \quad (54)$$

where

$$\mu^2 = \kappa^2 + m^2 r_{\text{ds}}^2 + v^2 \frac{r_{\text{ds}}^2}{r_n^2}, \quad D_{\pm} = -C_{\mp} \frac{vr_{\text{ds}}/r_n \pm imr_{\text{ds}}}{1/2 \mp i(\tilde{\kappa} + \kappa)}. \quad (55)$$

C. Near-horizon behaviors

As $z \rightarrow 0$ ($\rho \rightarrow -B$), the function $R_{\pm}(z)$ has the asymptotic form

$$\lim_{z \rightarrow 0} R_{\pm}(z) = C_{\pm} z^{\pm i(\tilde{\kappa} + \kappa)/2}. \quad (56)$$

Then, the spinor field Φ can be decomposed into the outgoing (ingoing) modes $\Phi_{-B}^{(\pm)}$ as

$$\Phi_{-B}^{(\pm)} = \frac{C_{\pm}}{2} \exp(-i\omega\tau) z^{\pm i(\tilde{\kappa} + \kappa)/2} \begin{bmatrix} i\mathbb{Y}_{j-1/2}^n \\ \pm \mathbb{Y}_{j+1/2}^n \end{bmatrix}. \quad (57)$$

As $z \rightarrow 1$ ($\rho \rightarrow B$), the function $R_{\pm}(z)$ can be written as

$$\lim_{z \rightarrow 1} R_{\pm}(z) = (C_{\pm} \Xi_{\pm} - C_{\mp} \Omega_{\pm}) (1 - z)^{\mp i(\tilde{\kappa} - \kappa)/2}, \quad (58)$$

where

$$\begin{aligned} \Xi_{\pm} &= \frac{\Gamma(1/2 \pm i(\tilde{\kappa} + \kappa)) \Gamma(1/2 \pm i(\tilde{\kappa} - \kappa))}{\Gamma(1/2 \pm i(\tilde{\kappa} + \mu)) \Gamma(1/2 \pm i(\tilde{\kappa} - \mu))}, \\ \Omega_{\pm} &= \left(v \frac{r_{\text{ds}}}{r_n} \pm imr_{\text{ds}} \right) \frac{\Gamma(1/2 \mp i(\tilde{\kappa} + \kappa)) \Gamma(1/2 \pm i(\tilde{\kappa} - \kappa))}{\Gamma(1 \mp i(\mu + \kappa)) \Gamma(1 \pm i(\mu - \kappa))}. \end{aligned} \quad (59)$$

Then the spinor field Φ can be decomposed into two modes:

$$\begin{aligned} \Phi_B^{(+)} &= \left(\frac{C_+ \Xi_+}{2} - \frac{C_- \Omega_+}{2} \right) \exp(-i\omega\tau) (1 - z)^{-i(\tilde{\kappa} - \kappa)/2} \\ &\times \begin{bmatrix} i\mathbb{Y}_{j-1/2}^n \\ \mathbb{Y}_{j+1/2}^n \end{bmatrix}, \end{aligned} \quad (60)$$

$$\begin{aligned} \Phi_B^{(-)} &= \left(\frac{C_- \Xi_-}{2} - \frac{C_+ \Omega_-}{2} \right) \exp(-i\omega\tau) (1 - z)^{i(\tilde{\kappa} - \kappa)/2} \\ &\times \begin{bmatrix} i\mathbb{Y}_{j-1/2}^n \\ -\mathbb{Y}_{j+1/2}^n \end{bmatrix}. \end{aligned} \quad (61)$$

In the near-extremal limit $B \rightarrow 0$, we assume that $\tilde{\kappa} \gg \kappa$ and impose the boundary condition

$$\Phi_B^{(-)} = 0 \Rightarrow C_- = C_+ \frac{\Omega_-}{\Xi_-}. \quad (62)$$

By applying the Bogoliubov transformation,

$$\Phi_{-B}^{(+)} + \alpha \Phi_{-B}^{(-)} = \beta^* \Phi_B^{(+)}, \quad (63)$$

we obtain the corresponding Bogoliubov coefficients:

$$\alpha = \frac{\Omega_-}{\Xi_-} \Rightarrow |\alpha|^2 = \frac{\sinh(\pi\mu + \pi\kappa) \sinh(\pi\mu - \pi\kappa)}{\cosh(\pi\tilde{\kappa} + \pi\mu) \cosh(\pi\tilde{\kappa} - \pi\mu)}, \quad (64)$$

$$\beta^* = \frac{1}{\Xi_-} \Rightarrow |\beta|^2 = \frac{\cosh(\pi\tilde{\kappa} + \pi\kappa) \cosh(\pi\tilde{\kappa} - \pi\kappa)}{\cosh(\pi\tilde{\kappa} + \pi\mu) \cosh(\pi\tilde{\kappa} - \pi\mu)}. \quad (65)$$

They satisfy the Bogoliubov relation. Remarkably, as $\tilde{\kappa} \rightarrow \infty$ in the exact Nariai limit, the mean number approaches the unity that saturates the bound from the Pauli blocking due to the exclusion principle. That is, as two spherical shells of the cosmological and black hole horizons collapse, the mean number $|\beta|^2 = 1$, while $|\alpha|^2 = 0$.

IV. FERMION PRODUCTION IN (A)dS₂ × S²

We have shown that as the near-extremality disappears ($B \rightarrow 0$) and Nariai black holes become an exact limit, the fermion production saturates the bound from the Pauli blocking between the black hole horizon and cosmological horizon, i.e., $|\beta|^2 \rightarrow 1$. In the exact Nariai limit, the outer region reduces to dS₂ × S².

With a rescaling $\tau = r_{\text{ds}} e^{t/r_{\text{ds}}}$, $\rho = r_{\text{ds}}^{-2} x$, the exact Nariai metric (7) ($B = 0$) can be written as

$$ds^2 = -d\tau^2 + e^{2t/r_{\text{ds}}} dx^2 + r_n^2 d\Omega_2^2, \quad (66)$$

which is dS₂ × S² in the planar coordinates. In other words, the outer region of a Nariai black hole corresponds to the dS space in the planar coordinates. One may thus expect that in the exact Nariai limit ($\tilde{\kappa} = \infty$), the mean number

$$|\beta|^2 = \frac{\sinh(\pi\mu + \pi\kappa)}{e^{\pi\mu - \pi\kappa} \sinh(2\pi\mu)} \quad (67)$$

has the same form as that of fermions in the planar coordinates of dS₂ [23].

On the other hand, the extremal RN black hole in dS space has the near-horizon geometry [17]

$$ds^2 = -\frac{\rho^2}{r_{\text{ads}}^2} d\tau^2 + \frac{r_{\text{ads}}^2}{\rho^2} d\rho^2 + r_n^2 d\Omega_2^2, \quad (68)$$

which, after rescaling the coordinate $\rho = r_{\text{ads}} e^{x/r_{\text{ads}}}$, can be written as the product AdS₂ × S² in the planar coordinates

$$ds^2 = -e^{2x/r_{\text{ads}}} d\tau^2 + dx^2 + r_n^2 d\Omega_2^2. \quad (69)$$

According to the reciprocal relation $\mathcal{N}_{\text{ds}}(\mathcal{R}) \mathcal{N}_{\text{ads}}(\mathcal{R}) = 1$ [23], the mean number of spontaneously produced fermion

(spinor) pairs becomes

$$\mathcal{N}_{\text{eRN}}^{(\text{sp})} = \frac{\sinh(2\pi\mu(r_{\text{ads}}))}{e^{\pi\kappa - \pi\mu(r_{\text{ads}})} \sinh(\pi\kappa + \pi\mu(r_{\text{ads}}))}, \quad (70)$$

where

$$\mu(r_{\text{ads}}) = \sqrt{\kappa_{\text{ads}}^2 - \tilde{m}^2 r_{\text{ads}}^2}, \quad \kappa_{\text{ads}} = qQ_n \frac{r_{\text{ads}}^2}{r_n^2}. \quad (71)$$

The result (70) agrees with the mean number for fermion production by a uniform electric field in AdS₂.

It should be noted that the spontaneous production of fermion pairs is blocked by the Pauli exclusion principle both in extremal charged Nariai and RN black holes in an asymptotic dS space. Assuming the reciprocal relation between the near-extremal Nariai black hole and the near-extremal RN black hole, the mean number for fermion production in near-extremal RN black hole would become

$$\mathcal{N}_{\text{neRN}}^{(\text{sp})} = \frac{\cosh(\pi\tilde{\kappa}_{\text{ads}} + \pi\mu(r_{\text{ads}})) \cosh(\pi\tilde{\kappa}_{\text{ads}} - \pi\mu(r_{\text{ads}}))}{\cosh(\pi\tilde{\kappa}_{\text{ads}} + \pi\kappa_{\text{ads}}) \cosh(\pi\tilde{\kappa}_{\text{ads}} - \pi\kappa_{\text{ads}})}, \quad (72)$$

where $\tilde{\kappa}_{\text{ads}} = \omega/B$.

The spontaneous fermion production can be compared with the spinless boson production. In the inner region of a near-extremal Nariai black hole, the mean number of bosons for $\tilde{\kappa} \gg 1$,

$$\mathcal{N}_{\text{in}}^{(\text{sc})} = \frac{e^{2\pi\tilde{\kappa}}}{\cosh(\pi\mu + \pi\kappa) \cosh(\pi\mu - \pi\kappa)}, \quad (73)$$

exponentially explodes. The catastrophic boson production is a strong contrast to fermion production with saturation of unity. The mean numbers of bosons from the Nariai black hole and extremal RN black hole have the same forms as in dS₂ and AdS₂ [26] and also satisfy the reciprocal relation [23].

V. COMPARISON OF EMISSION OF SPINORS AND SCALARS

The mean number (65) of fermions emitted in the inner region may be written in terms of the Hawking temperature for near-extremal Nariai black holes, the Unruh temperature for charge acceleration, and the Gibbons-Hawking temperature for dS₂ space as

$$N_{\text{in}}^{(\text{sp})} = \frac{\cosh\left(\frac{\omega}{2T_H} + \frac{\tilde{m}T_U}{2T_C^2}\right) \cosh\left(\frac{\omega}{2T_H} - \frac{\tilde{m}T_U}{2T_C^2}\right)}{\cosh\left(\frac{\omega}{2T_H} + \frac{\tilde{m}\sqrt{T_C^2 + T_U^2}}{2T_C^2}\right) \cosh\left(\frac{\omega}{2T_H} - \frac{\tilde{m}\sqrt{T_C^2 + T_U^2}}{2T_C^2}\right)}. \quad (74)$$

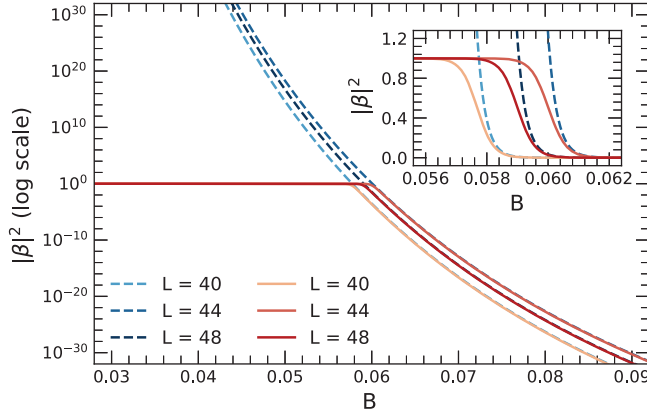


FIG. 1. Comparison of the emission of fermions and scalars in near-extremal Nariai black holes. Dotted lines for scalars and solid lines for fermions. Figures are drawn for $m = q = 1$, $Q_n = 10$, $\omega = 2$ and the s-wave $l = 0$.

The leading terms up to $e^{-\omega/T_H}$ are approximately given by

$$N_{\text{in}}^{(\text{sp})} \approx 1 - 4e^{-\frac{\omega}{T_H}} \sinh\left(\frac{\tilde{m}T_{\text{eff}}}{2T_C^2}\right) \sinh\left(\frac{\tilde{m}\tilde{T}_{\text{eff}}}{2T_C^2}\right). \quad (75)$$

Thus, the spinor emission rapidly reaches the limit ($|\beta|^2 = 1$) due to the Pauli blocking when $B(T_H)$ approaches zero, as shown in Fig. 1.

It is interesting to compare the spinor emission with that of the scalar emission [19] in the region of $B \ll 1$, which may be written as

$$N_{\text{in}}^{(\text{sc})} = \frac{e^{-\frac{q\Phi_H}{T_H}} \sinh\left(\frac{\omega+q\Phi_H}{2T_H}\right) \sinh\left(\frac{\omega-q\Phi_H}{2T_H}\right)}{e^{-\frac{\tilde{m}T_U}{T_C^2}} \cosh\left(\frac{\tilde{m}T_{\text{eff}}}{2T_C^2}\right) \cosh\left(\frac{\tilde{m}\tilde{T}_{\text{eff}}}{2T_C^2}\right)}. \quad (76)$$

Indeed, there is a catastrophic emission of the form $e^{(\omega-q\Phi_H)/T_H}$ provided that $\omega > q\Phi_H$, which is the opposite of superradiance [27], which is shown in Fig. 1.

VI. CONCLUSION

In the in-out formalism [28] (for review, see Ref. [29]), the pair production of scalars and spinors in background fields is determined by the Bogoliubov relations between the in-vacuum and the out-vacuum that obey the spin statistics. The Bogoliubov coefficients satisfy the relations

$$|\alpha|^2 \mp |\beta|^2 = 1, \quad (77)$$

where the upper (lower) sign is the boson (fermion). Thus, the mean number for boson production can have, in principle, an amplification greater than 1 while the mean number of fermion production cannot be greater than 1. Near-extremal Nariai black holes have a timelike region between two horizons while near-extremal RN black holes

have a spacelike region between two horizons. It has been shown that the emission of charged bosons from (near-) extremal black holes exhibits the characteristic of the Schwinger formula $|\beta|^2 = e^{-\pi m^2/qE}$ in a constant electric field in Minkowski spacetime, which is not greater than 1 and whose leading Boltzmann factor is given by nonzero instanton action [16–19], even the rotation of black holes also does not change this feature [14,20–22]. This no-bosonic amplification seems contradictory to our intuition. However, we have shown in Ref. [19] that near-extremal Nariai black holes emit charged bosons catastrophically, similar to two narrow spherical shells with a high potential difference.

In this paper, we have studied the spontaneous production of spin-1/2 fermions from near-extremal Nariai black holes. The scattering of fermions contrasts with that of bosons in the black hole background. Bosons can have superradiance modes, in which the reflected flux becomes greater than the incident flux due to pair production [30]. Hence it is interesting to study the emission of charged fermions from Nariai black holes. We have found that the mean number of fermions in the near-extremal Nariai black holes still satisfies the Bogoliubov relation but saturates the bound $|\beta|^2 = 1$ from Pauli blocking due to the exclusion principle. The extremal charged Nariai black hole saturates the bound as it approaches the ultracold limit in which black hole has the maximal charge. However, the extremal Nariai limit of Schwarzschild black hole in dS space emits fermions in accordance with the Fermi-Dirac distribution, which cannot saturate the bound.

From the view of quantum field theory, spontaneous pair or particle production is the result of complex one-loop effective action, as explicitly shown by Heisenberg-Euler and Schwinger QED action for charged bosons or fermions in a constant electromagnetic field [15,31]. Our formulation of Schwinger effect in near-extremal Nariai or RN black holes is related to the imaginary part of one-loop effective action, since $2\Im\mathcal{L}^{(1)} = \pm \sum \mathcal{D} \ln(1 \pm |\beta|^2)$, where $\mathcal{D}$ denotes the density of states per unit volume available for given quantum number and the upper (lower) sign is for bosons (fermions).

We have assumed here that black holes are large enough to neglect the backreaction of emitted charges and that the Schwinger emission processes are adiabatic. The backreaction may be found by calculating the loss rates of black hole charge Q and mass M due to the Schwinger emission of charge q with m : $dQ/dt \propto -q|\beta|^2$ and $dM/dt \propto -m|\beta|^2$, which explain quantitatively the evolution of charged black holes [32,33]. A systematic approach to the backreaction of the emission would be the energy-momentum tensor $T_{\mu\nu}^{(1)} = (-2/\sqrt{-g}) \times \delta\mathcal{L}^{(1)}/\delta g^{\mu\nu}$ at one loop. Another approach to the backreaction is the probability for emission processes $P = e^{\Delta S}$ in terms of the entropy change ΔS before and after the emission [34].

The quantitative description and comparison of both approaches requires further study that is beyond the scope of this paper.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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