

Consistent canonical quantization of gravity: Recovery of classical GR from BRST-invariant coherent states

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We perform canonical quantization of general relativity, as an effective quantum field theory below the Planck scale, within the Becchi-Rouet-Stora-Tyutin (BRST)-invariant framework. We show that the promotion of constraints to dynamical equations of motion for auxiliary fields leads to the healthy Hamiltonian flow. In particular, we show that the classical properties of Einstein’s gravity, such as vanishing Hamiltonian modulo boundary contribution, are realized merely as an expectation value in appropriate physical states. Most importantly, the physicality is shown not to entail trivial time evolution for correlation functions. In the present approach, we quantize the theory once and for all around the Minkowski vacuum and treat other would-be classical backgrounds as BRST-invariant coherent states. This is especially important for cosmological spacetimes as it uncovers features that are not visible in ordinary semiclassical treatment. The Poincaré invariance of the vacuum, essential for our quantization, provides strong motivation for spontaneously broken supersymmetry.

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I. INTRODUCTION

This paper is about the Becchi-Rouet-Stora-Tyutin (BRST)-invariant formulation of canonically quantized gravity as of consistent low-energy effective field theory. The emphasis is made on recovering the essential aspects of classical general relativity (GR) and, in particular, the existence of time evolution, which is sometimes obscured within more “vintage” approaches to quantizing GR. Contrary to semiclassical approaches of quantizing fields on curved classical backgrounds, this approach allows, at least in principle, the resolution of the would-be classical background as a full-fledged quantum state of gravitational degrees of freedom.

Namely, the present framework provides an explicit, consistent realization of the previously suggested approach [1–7] implying that the would-be classical metric backgrounds must be treated as the coherent states of gravity quantized on top of the Minkowski vacuum. The analogous treatments of classical backgrounds in quantum field theories of other spins can be found in [8–13].

Quantum field theory in curved spacetime [14] is concerned with quantizing fields, as well as fluctuations of the metric, in classical background geometries. Although this approximation is adequate in many cases, it has clear limitations. In particular, it breaks down over the timescales on which the quantum backreaction on a would-be classical background metric becomes significant [1,3–6,15–18]. As shown in previous papers, this breakdown has important implications for black hole physics and cosmology.¹ It has also been shown to have ramifications for the initial conditions for inflation [23,24].

The crucial role of analyzing gauge-invariant observables was stressed [25] already in connection with the earlier attempts [26–29] of accounting for quantum backreaction on de Sitter and inflationary spacetimes. In particular, one has to make sure that departures from de Sitter geometry do not represent the artifacts of the choice of gauge for quantum fluctuations.

In this work, we stress the importance of describing the quantum state of the system in its entirety. That is, the background spacetime itself is viewed as the quantum state. Therefore, we need to quantize gravity before positing the nontrivial background. By quantization, we refer to the

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¹For example, for a recent work on astrophysical implications of the so-called black hole “memory burden effect” [16–18], see [19–22].

identification of canonical phase-space degrees of freedom, the symplectic structure, and their realization as an operator algebra. Upon the discussion of the spectrum of the Hamiltonian, there must indubitably be a special state that would serve as the vacuum of the theory. In the framework pursued here, Minkowski spacetime is a perfect candidate for such a state on top of which other physically relevant backgrounds are described as quantum states of the same underlying theory.

This way of reasoning selects a certain gauge from the beginning, in which the canonical quantization was performed. Moreover, it sets the gauges of the background and perturbations in a connected way. There is residual freedom to readjust the gauge for perturbations as needed, e.g., in the path integral, but only for the computation of gauge-invariant quantities. This stems from general principles we are accustomed to in gauge theories, see, e.g., [30].

This raises important questions about the possibility of switching between frames. Within BRST quantization, quantum states corresponding to the gauge field configuration generically change in such a way that give vanishing contribution to S -matrix elements between physical states.

The canonical formulation is not introduced here as an alternative to the BRST-invariant path-integral description but rather to make explicit a complementary aspect of the same framework. Starting from the BRST-invariant Lagrangian path integral, the existence of the corresponding Hamiltonian description is in fact implicit, since in this formulation all fields, including those associated with unphysical degrees of freedom, possess nonvanishing conjugate momenta. Our interest in the canonical formalism is motivated by questions of real-time dynamics for nonasymptotic physical states. In particular, we would like to associate to a given classical field configuration a quantum state satisfying the BRST physicality conditions, and then follow its time evolution at finite times. This is especially relevant for situations such as cosmological backgrounds or black-hole-like states, where one wishes to understand how an initially approximately classical state loses coherence and becomes increasingly quantum in the fully interacting theory. While the path-integral formalism is ideally suited for scattering amplitudes and in-in correlators, the canonical language makes the implementation of physical-state conditions at a given time more transparent, and thereby provides a natural framework for the questions addressed in this work.

The paper is organized as follows. In Sec. II, we kick start the discussion with a brief outline of the role of having a notion of a well-defined vacuum (like Minkowski) for a consistent canonical quantization of GR, emphasizing that this does not preclude the existence of other consistent states corresponding to nontrivial spacetimes. In Sec. III, we follow this up with a qualitative argument connecting the existence of the low-energy S -matrix for multigraviton states with the existence of the BRST-invariant canonical

Hamiltonian formalism. Section IV recounts the perturbative recovery of classical dynamics from the connection between S -matrix and an effective action. Section V presents the main results of this work, performing canonical (BRST-invariant) quantization of GR, analyzing the equations of motion and discussing the recovery of the classical dynamics in appropriate quantum states. Section VI contains the discussion of coordinate reparametrization within the adopted framework. Section VII touches upon the cutoff sensitivity of low-energy observables. We conclude in Sec. VIII and discuss the outlook.

II. ON BACKGROUND INDEPENDENCE OF QUANTIZATION

Here, we would like to set the framework and clarify the role of the background in our quantization. We shall quantize gravity in the theory of GR with a zero cosmological term. This does not imply that we are limited by Minkowski as the only possible background. On the contrary, once quantized on the Minkowski vacuum, the theory has full quantum power to account for all other consistent backgrounds, including the cosmological ones.

For example, a homogeneous inflationary background is viewed as a coherent state of a scalar field and gravity constructed on top of the Minkowski vacuum [3–7].

Temporarily, the potential energy of the slow-rolling scalar field can be approximated by a positive cosmological term and correspondingly the coherent state can be approximated by a pure gravitational de Sitter. The BRST-invariant construction of such a state can be found in [7].

Of course, in practice, it may be easier to study quantum fluctuations around the inflationary state within a conventional semiclassical treatment in which the de Sitter coherent state is treated as a classical background and perturbations are quantized on top of it. Our quantization method captures the dynamics of these semiclassical fluctuations as a Schwinger-Dyson expansion of the background field method. The outline of the procedure on the example of classical field theories can be found in [11] and references therein.

In summary, the quantization of the theory with zero cosmological constant does not impair its ability to provide the full quantum description of other consistent backgrounds.

However, there exist independent selection criteria for rejecting certain backgrounds. These are not directly related to our quantization methods, but rather to other aspects of a consistent formulation of the theory. For instance, an eternal de Sitter state does not allow for the definition of the S -matrix. Correspondingly, such vacua are excluded by formulations of the theory based on the S -matrix (for the discussion, see [31]). This can act as a powerful selection tool for the parameters of the theory (for various implications, see [32]) but are not essential for the present discussion.

III. FROM S-MATRIX TO CANONICAL HAMILTONIAN FORMALISM

The ultimate goal of this manuscript is the clarification of certain perceived puzzles in the canonical quantization of GR as a low-energy effective field theory.

The starting point of the discussion will be the assumption that there exists an S -matrix formulation of gravity. This, in turn, posits the existence of the scattering theory of an arbitrary number of gravitons, which are quantum gravitational degrees of freedom defined around the Minkowski spacetime and are represented by the massless spin-2 field.

In order to establish that the aforementioned ultimate S -matrix formulation has some semblance of GR at low energies, one would need to match the scattering S -matrix obtained from the quantization of the latter within its regime of applicability to the appropriate limit of the former. For this, one should be able to perform a quantization of GR and define the notion of quantum graviton to begin with.

Luckily, such a quantization had been performed and the scattering matrix elements of low-energy gravitons had been obtained [33]. The starting point is the definition of degrees of freedom. The Poincaré symmetry being the guiding principle, the gravitational degrees of freedom are identified as its massless spin-2 representation. This statement, not being the subject of debates, will be taken for granted. The next, equally important step is the identification of the interaction vertices. It is also a matter of consensus that the self-consistent interactions of said quanta are uniquely fixed to the leading order in derivative expansion, matching the weak-field expansion of GR around the Minkowski spacetime $g_{\mu\nu} = \eta_{\mu\nu}$, which is the only Poincaré invariant classical solution to GR.

In other words, the low-energy scattering amplitudes are obtained from an appropriate quantization of the nonlinear theory of gravitons with the classical action

$$S_{\text{EH}} = \int d^4x \mathcal{L}_{\text{EH}} = \frac{1}{2} M_{\text{pl}}^2 \int d^4x \sqrt{-g} R(g_{\mu\nu}), \quad (1)$$

with $g_{\mu\nu} \equiv \eta_{\mu\nu} + M_{\text{pl}}^{-1} h_{\mu\nu}$ serving as the definition of the graviton field $h_{\mu\nu}$. This is the classical formulation of gravity, with the coordinate reparametrization invariance corresponding to the gauge redundancy of the formalism.

There are different ways one could proceed in quantizing this theory based on the treatment of this gauge redundancy. Modern, consistent procedures reduce to the BRST-invariant path-integral formulation of scattering amplitudes in the Lagrangian formalism. The generating functional can be schematically given as

$$Z[T^{\mu\nu}, \dots] = \int [\mathcal{D}h][\mathcal{D}c][\mathcal{D}\bar{c}][\mathcal{D}b] \times e^{-i \int d^4x (\mathcal{L}_{\text{EH}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}} + h_{\mu\nu} T^{\mu\nu} + \dots)}, \quad (2)$$

where $\mathcal{L}_{\text{GF}}$ and $\mathcal{L}_{\text{FP}}$ stand for the gauge-fixing and ghost Lagrangians respectively. As is customary, the auxiliary field b_μ imposing the gauge-fixing condition and the Hermitian Faddeev-Popov ghosts (c^μ and $\bar{c}_\mu$) have been introduced. In Eq. (2), $T^{\mu\nu}$ is the external current for $h_{\mu\nu}$ and ellipses stand for the external currents for the aforementioned auxiliary sector. In this section, we keep the discussion schematic without being specific about the detailed form of various terms, which can be found in Sec. V. This schematic expression is also obscuring the contribution connected to the noncanonical nature of the kinetic term of GR, which is immaterial for tree-level computations and for the point we are trying to make here.

The generating functional (2) can be, and has been, used to compute the in-out correlation functions and scattering amplitudes for low-energy gravitons. Having the low-energy S -matrix from the aforementioned Lagrangian path-integral formalism is very good, but in order to connect it with real-time physical observables, we need to identify the Hamiltonian. The usual quantization procedure begins with the latter and, upon integrating out conjugate momenta, results in the former. However, since the canonical quantization raises more eyebrows these days than the Lagrangian path-integral formalism (2), we can retrace our steps backward. From (2), we proceed by integrating in the canonical conjugate momenta for the spatial metric degrees of freedom h_{ij} and ghosts. We also need to keep in mind that b_μ is related to the canonical conjugate of $h_{0\mu}$ via a linear transformation, as we will see explicitly in Sec. VB, that has a unit Jacobian. Not surprisingly, following this procedure, we recover the classical Hamiltonian of GR upon reintroducing the missing conjugate momenta by means of the Gaussian integral, which is due to the fact that the Hamiltonian is at most quadratic in canonical momenta (as is well known and is reiterated in Sec. VB). The resulting schematic form of the generating functional is

$$Z[J^{\mu\nu}, \dots] = \int [\mathcal{D}h][\mathcal{D}\Pi][\mathcal{D}c][\mathcal{D}\Pi_c][\mathcal{D}\bar{c}][\mathcal{D}\Pi_{\bar{c}}] \times e^{-i \int d^4x (\dot{h}_{\mu\nu} \Pi^{\mu\nu} + \dot{c}^\mu \Pi_\mu^c + \dot{\bar{c}}_\mu \Pi_{\bar{c}}^\mu - H + h_{\mu\nu} J^{\mu\nu} + \dots)}. \quad (3)$$

After all is said and done, we end up with the path integral in phase space where all degrees of freedom have the canonical conjugate momenta. We can use the result to identify the Hamiltonian of the system in the field and conjugate momentum eigenstate bases, which we can then use to build the Hamiltonian operator. This should be highly indicative that the canonical quantization framework in phase space should have merit and therefore be able to account for the time dependence of physical quantities. In other words, the existence of the low-energy scattering S -matrix of gravitons is in one-to-one correspondence with

the existence of the well-defined Hamiltonian quantization within an appropriate regime of validity.

We would like to underline that not all canonical quantization attempts are equal. What we have outlined in this section is how to recover the BRST-invariant canonical formalism of [34] from the Lagrangian path integral (2). This should not come as a surprise, as the BRST symmetry is inherent to the latter as well. However, some of the canonical quantization attempts entail such a peculiar treatment of the constraints of the GR that they would be impossible to recover from (2).

IV. RECOVERY OF CLASSICAL GR FROM S-MATRIX

In this section, we recount the connection between the S -matrix and effective action. In particular, the classical effective action in the background gravitational field created by a set of classical sources is given by the expectation value of the S -matrix operators over coherent states describing these sources. Schematically, this is given by the general equation:

$$\langle coh|\hat{S}|coh\rangle = \Gamma_{\text{eff}}, \quad (4)$$

where $\hat{S}$ is the total S -matrix operator implying summation over all $1/M_{\text{pl}}$ contributions and $|coh\rangle$ stands for corresponding coherent states. Since we are mainly interested in the recovery of classical gravity from the quantum theory of the graviton, we shall present the derivation in $1/M_{\text{pl}}$ expansion taking into account the contributions into $\hat{S}$ coming from tree-level graviton exchanges. The corresponding discussions can be found, e.g., in lectures [35].

Let us start with linearized Einstein gravity. The Lagrangian is

$$\mathcal{L} = \frac{1}{2} h^{\mu\nu} \mathcal{E}_{\mu\nu} - \frac{1}{M_{\text{pl}}} h_{\mu\nu} T^{\mu\nu}, \quad (5)$$

where

$$\begin{aligned} \mathcal{E}_{\mu\nu} \equiv & \square h_{\mu\nu} - \eta_{\mu\nu} h - \partial_\mu \partial^\beta h_{\nu\beta} - \partial_\nu \partial^\beta h_{\mu\beta} \\ & + \eta_{\mu\nu} \partial^\alpha \partial^\beta h_{\alpha\beta} + \partial_\mu \partial_\nu h \end{aligned} \quad (6)$$

is the linearized Einstein tensor and $T_{\mu\nu}$ is a classical energy-momentum source. In quantum theory, this source should be understood as the expectation value of an operator $\hat{T}_{\mu\nu}$ over a certain coherent state, $|coh\rangle$, such that $\langle coh|\hat{T}_{\mu\nu}|coh\rangle = T_{\mu\nu}$. The operator $\hat{T}_{\mu\nu}$ is composed of field operators that source gravity.

As is well known, the above Lagrangian describes a unique linear ghost-free theory of a massless spin-2 field. It matches the theory obtained “top-down” by linearization of the Einstein-Hilbert action. In this sense, Einstein’s theory

passes an immediate test of uniqueness and of quantum consistency at the linear level.

The corresponding equations of motion have the form

$$\mathcal{E}_{\mu\nu} = \frac{1}{M_{\text{pl}}} T_{\mu\nu}. \quad (7)$$

In de Donder gauge ($\partial^\mu h_{\mu\nu} = \frac{1}{2} \partial_\nu h$) this equation takes the form

$$\square h_{\mu\nu} = \frac{1}{M_{\text{pl}}} \left(T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T \right) \quad (8)$$

and is solved by

$$h_{\mu\nu}^{(1)}(x) = \frac{1}{M_{\text{pl}}} \int d^4 x_1 \Delta_{\mu\nu,\alpha\beta}(x - x_1) T^{\alpha\beta}(x_1), \quad (9)$$

where

$$\begin{aligned} \Delta_{\mu\nu,\alpha\beta}(x - x_1) = & \int \frac{d^4 p}{(2\pi)^4} e^{-ip(x-x_1)} \\ & \times \frac{\frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}) - \frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta}}{p^2} \end{aligned} \quad (10)$$

is the Green’s function evaluated with retarded pole prescription. The superscript on $h_{\mu\nu}$ indicates the order in $1/M_{\text{pl}}$.

In order to monitor the gravitational field, we can introduce a probe energy-momentum tensor denoted by $\tau_{\mu\nu}$. For simplicity, we can assume that $\tau_{\mu\nu}$ is arbitrarily soft ($\tau_{\mu\nu} \ll T_{\mu\nu}$), so that the higher-order effects in it can be safely ignored. However, this is not essential, since these effects can be consistently taken into account whenever needed. In classical theory $\tau_{\mu\nu}$ is composed out of some classical fields.

To the linear order in $\tau_{\mu\nu}$, its effective action in the classical gravitational field is given by

$$A_{\text{eff}} = \frac{1}{M_{\text{pl}}} \int d^4 x h_{\mu\nu}(x) \tau^{\mu\nu}(x). \quad (11)$$

After taking the variational derivative of the full action with respect to degrees of freedom composing $\tau_{\mu\nu}$, we obtain their equation of motion in the background gravitational field.

This procedure is trivially lifted to a quantum theory in which the fields composing the probe are promoted to quantum operators, and so is their energy-momentum tensor $\tau_{\mu\nu} \rightarrow \hat{\tau}_{\mu\nu}$. Correspondingly, the action (11) is promoted into a quantum effective action in a background classical field,

$$A_{\text{eff}}(q) = \frac{1}{M_{\text{pl}}} \int d^4 x h_{\mu\nu}(x) \hat{\tau}^{\mu\nu}(x). \quad (12)$$

The actions (11) and (12) can be reconstructed perturbatively as a series in $1/M_{\text{pl}}$. To the order $1/M_{\text{pl}}^2$, the action of the classical probe $\tau_{\mu\nu}$ moving in the classical metric $h_{\mu\nu}(x)$ created by the source $T_{\mu\nu}$ is

$$\begin{aligned} A_{\text{eff}}^{(2)} &= \frac{1}{M_{\text{pl}}} \int d^4x h_{\mu\nu}^{(1)}(x) \tau^{\mu\nu}(x) \\ &= \frac{1}{M_{\text{pl}}^2} \int d^4x d^4x_1 \tau^{\mu\nu}(x) \Delta_{\mu\nu,\alpha\beta}^{\text{F}}(x-x_1) T^{\alpha\beta}(x_1), \end{aligned} \quad (13)$$

where

$$\begin{aligned} \Delta_{\mu\nu,\alpha\beta}^{\text{F}}(x-x_1) &= \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-x_1)} \\ &\quad \times \frac{\frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}) - \frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta}}{p^2 + i\epsilon} \end{aligned} \quad (14)$$

is the Feynman propagator.

This expression is equal to the expectation value of the quantum $\hat{S}$ -matrix operator of one-graviton exchange,

$$\hat{S}^{(2)} = \frac{1}{M_{\text{pl}}^2} \int d^4x d^4x_1 \hat{\tau}^{\mu\nu} \langle \hat{h}_{\mu\nu}(x) \hat{h}_{\alpha\beta}(x_1) \rangle \hat{T}^{\alpha\beta}(x_1), \quad (15)$$

taken over the coherent state $|coh\rangle = |coh\rangle_{\tau} \times |coh\rangle_T$ of quantum degrees of freedom composing the operators $\hat{\tau}_{\mu\nu}$ and $\hat{T}_{\mu\nu}$. That is, the coherent states satisfy $\langle coh | \hat{\tau}_{\mu\nu} | coh \rangle = \tau_{\mu\nu}$ and $\langle coh | \hat{T}_{\mu\nu} | coh \rangle = T_{\mu\nu}$. For simplicity, we assume that the sources $\hat{\tau}_{\mu\nu}$ and $\hat{T}_{\mu\nu}$ are composed of independent degrees of freedom, so that the coherent state factorizes into two independent ones. However, this is not essential. In the above expression, the graviton operators are fully contracted, giving the Feynman propagator,

$$\langle \hat{h}_{\mu\nu}(x) \hat{h}_{\alpha\beta}(x_1) \rangle = \Delta_{\mu\nu,\alpha\beta}^{\text{F}}(x-x_1). \quad (16)$$

It is then obvious that we have the relation

$$\langle coh | \hat{S}^{(2)} | coh \rangle = A_{\text{eff}}^{(2)}. \quad (17)$$

In other words, the quantum S -matrix operator generates the classical gravitational action for the sources.

This connection holds for all orders in $1/M_{\text{pl}}$ expansion [36]. That is, the contribution of the higher-order nonlinearities to the classical metric is in one-to-one correspondence with quantum S -matrix elements generated by tree-graviton exchanges among the classicalized sources to the same order in $1/M_{\text{pl}}$.

For example, consider an effect of an addition of a cubic-order graviton vertex to the Lagrangian, which we shall schematically denote by $\frac{1}{M_{\text{pl}}}(\partial^2 h(x) h(x) h(x))$, accounting

for the presence of two derivatives and $1/M_{\text{pl}}$. The tensorial structure, which is rather lengthy, is not shown explicitly.

Variation of this vertex with respect to $h_{\mu\nu}$ corrects Eq. (7) by a bilinear term,

$$\mathcal{E}_{\mu\nu} = \frac{1}{M_{\text{pl}}} T_{\mu\nu} + \frac{1}{M_{\text{pl}}} (\partial^2 h^2)_{\mu\nu}. \quad (18)$$

The leading-order effect of this term is an order $1/M_{\text{pl}}^3$ correction to $h_{\mu\nu}(x)$ which can be taken into account iteratively, and gives

$$h_{\mu\nu}^{(3)}(x) = \frac{1}{M_{\text{pl}}} \int d^4x_1 \Delta_{\mu\nu,\alpha\beta}(x-x_1) (\partial^2 (h^{(1)}(x_1))^2)^{\alpha\beta}, \quad (19)$$

where the $h^{(1)}(x_1)$ is a solution (9). Clearly, (19) is of order $1/M_{\text{pl}}^3$ and the corresponding correction to the effective action,

$$A_{\text{eff}}^{(4)} = \frac{1}{M_{\text{pl}}} \int d^4x h_{\mu\nu}^{(3)}(x) \tau^{\mu\nu}(x), \quad (20)$$

is of order $1/M_{\text{pl}}^4$. Again, this effective action is matched by the expectation value of the $\hat{S}$ -matrix operator to the same order,

$$\begin{aligned} \hat{S}^{(4)} &= \sum \frac{1}{M_{\text{pl}}^4} \int d^4x d^4x_1 d^4x_2 d^4x_3 \hat{\tau}^{\mu\nu} \hat{h}_{\mu\nu}(x) \hat{h}_{\alpha\beta}(x_1) \\ &\quad \times \hat{T}^{\alpha\beta}(x_1) \hat{h}_{\gamma\delta}(x_2) \hat{T}^{\gamma\delta}(x_2) (\partial^2 (h(x_3))^3), \end{aligned} \quad (21)$$

where summation is taken over all possible combinations in which all graviton operators are singly contracted. The expectation value is taken over the same coherent states that classicalize the sources. This gives

$$\langle coh | \hat{S}^{(4)} | coh \rangle = A_{\text{eff}}^{(4)}. \quad (22)$$

Of course, already at the level of cubic interactions, we get contributions at arbitrary higher orders in $1/M_{\text{pl}}$. The example of a $1/M_{\text{pl}}^4$ and $1/M_{\text{pl}}^6$ contributions are given in Fig. 1.

With the increasing order of nonlinearities in the Lagrangian, the number of contributions is increasing drastically, but order by order, the same relation holds. This is understandable, since with gravitons appearing as virtual states among the classical sources, the classical contribution in each order in $1/M_{\text{pl}}$ must be accounted for by the tree-level exchanges.

For example, denoting a k -graviton interaction vertex at spacetime point x by $V(h(x)^k)$ lets us consider a generic $\hat{S}$ -operator which is order n in $1/M_{\text{pl}}$, first order in the

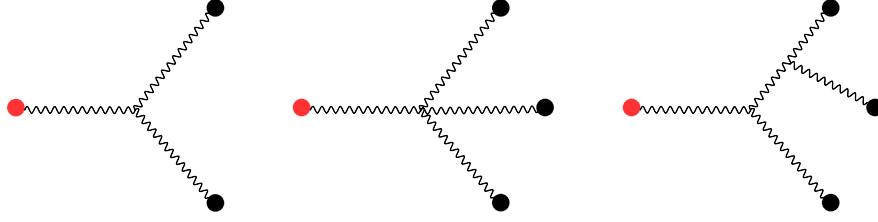


FIG. 1. Diagrammatic examples of order $1/M_{\text{pl}}^4$ and $1/M_{\text{pl}}^6$ contributions to the effective action of the probe $\tau_{\mu\nu}$ denoted by the red dot. The source $T_{\mu\nu}$ is denoted by the black dots.

probe $\tau_{\mu\nu}$, and l th order in the source $T_{\mu\nu}$ and with all sources interacting with gravitons linearly. This operator has a form

$$\begin{aligned} \hat{S}^{(n)} = & \frac{1}{M_{\text{pl}}^n} \sum \int d^4x d^4x_1 \dots d^4x_l d^4y_1 \dots d^4y_q \\ & \times \hat{\tau}^{\mu\nu}(x) \hat{h}_{\mu\nu}(x) \hat{T}^{\alpha\beta}(x_1) \hat{h}_{\alpha\beta}(x_1) \dots \hat{T}^{\gamma\delta}(x_l) \hat{h}_{\gamma\delta}(x_l) \\ & \times V(h(y_1)^{k_1}) \dots V(h(y_q)^{k_q}). \end{aligned} \quad (23)$$

The summation goes over all possible contractions as well as over various insertions of vertices that satisfy $n = l + 1 + \sum_{i=1}^q (k_i - 2)$. Again, all gravitons are singly contracted. After classicalizing the sources by taking the expectation values over proper coherent states, this expression maps to a corresponding effective action of the probe. The mapping is straightforwardly generalized to the case of nonlinear interactions between the sources and gravitons as long as all the graviton operators are singly contracted.

We thus arrive at a general schematic relation,

$$\langle \text{coh} | \hat{S}^{(n)} | \text{coh} \rangle = A_{\text{eff}}^{(n)}, \quad (24)$$

that holds in n th order in $1/M_{\text{pl}}$ expansion.

Strictly speaking, the connection must hold beyond the weak field regime. The only caveat is that beyond the weak field, the $1/M_{\text{pl}}$ expansion breaks down and the series must be resummed.

The above result is a particular manifestation of the generic relation between the effective action and the S -matrix, which holds for an arbitrary quantum field $\hat{\phi}$.

V. QUANTIZATION OF GR

To proceed, we need to choose the framework. We employ the BRST-invariant formulation of GR, contrary to a more commonly sought after Wheeler-DeWitt quantization.² Its initial development goes back to [34,39], where

²Some potential differences between Wheeler-DeWitt and BRST quantizations have been discussed in the literature (see, e.g., [37,38]). The full scope of the distinction between the two frameworks shall become transparent from the analysis below.

the recovery of Einstein's equations together with various other aspects were discussed.

In this framework, the gauge-fixed Lagrangian density takes the following form:

$$\mathcal{L} = \mathcal{L}_{\text{EH}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}}, \quad (25)$$

$$\mathcal{L}_{\text{EH}} = \sqrt{-g} M_{\text{pl}}^2 R, \quad (26)$$

$$\mathcal{L}_{\text{GF}} = M_{\text{pl}} b_\nu \partial_\mu (\sqrt{-g} g^{\mu\nu}) - \frac{1}{2} \alpha \eta^{\mu\nu} b_\mu b_\nu, \quad (27)$$

$$\mathcal{L}_{\text{FP}} = i \partial_\mu \bar{c}_\nu (\sqrt{-g} g^{\mu\sigma} \partial_\sigma c^\nu - \partial_\sigma (\sqrt{-g} g^{\sigma\nu}) c^\mu). \quad (28)$$

Here $\eta_{\mu\nu}$ is the Minkowski metric and α is a free parameter which plays the same role as ξ in R_ξ gauge of quantum electrodynamics. In fact, we could have simply chosen $\alpha = 0$. As it is customary, we have employed the auxiliary field b_μ to impose the de Donder gauge and have introduced the Hermitian ghost fields c^μ and $\bar{c}_\mu$ accordingly.

The reason the introduction of gauge-fixing auxiliary fields facilitates the canonical quantization is that, in the gauge-invariant formulation, the temporal components of the metric lack the conjugate momenta. In the presence of b_μ , on the other hand, the temporal metric components acquire canonical counterparts.

In the Lagrangian formalism it is convenient to work with $\tilde{g}^{\mu\nu} \equiv \sqrt{-g} g^{\mu\nu}$, as it was done in [34]. In terms of this variable, the Einstein-Hilbert Lagrangian can be written in the following convenient form:

$$\begin{aligned} \mathcal{L}_{\text{EH}} = & M_{\text{pl}}^2 \left(\tilde{g}^{\rho\sigma} \tilde{g}_{\lambda\mu} \tilde{g}_{\kappa\nu} - 2 \delta_\kappa^\sigma \delta_\lambda^\rho \tilde{g}_{\mu\nu} - \frac{1}{2} \tilde{g}^{\rho\sigma} \tilde{g}_{\mu\kappa} \tilde{g}_{\lambda\nu} \right) \partial_\rho \tilde{g}^{\mu\kappa} \partial_\sigma \tilde{g}^{\lambda\nu} \\ & + 2 M_{\text{pl}}^2 \partial_\mu \left(\frac{1}{2} \tilde{g}^{\mu\nu} \tilde{g}_{\alpha\beta} \partial_\nu \tilde{g}^{\alpha\beta} + \partial_\nu \tilde{g}^{\mu\nu} \right). \end{aligned} \quad (29)$$

However, it is easy to see that the theory has been written in such a way that there are time derivatives of the temporal components of the metric even in the absence of the gauge-fixing sector. This makes the formulation of the canonical formalism somewhat cumbersome. Therefore, in order to find a more convenient parametrization for the temporal components, we switch to the Arnowitt-Deser-Misner (ADM) formalism.

A. ADM formalism

The variables of ADM decomposition [40] are lapse N , shift N_j , and the spatial metric γ_{ij} , which are introduced as

$$g_{\mu\nu} = \begin{pmatrix} -N^2 + N_k N^k & N_j \\ N_i & \gamma_{ij} \end{pmatrix},$$

$$g^{\mu\nu} = \begin{pmatrix} -N^{-2} & N^{-2} N^j \\ N^{-2} N^i & \gamma^{ij} - N^{-2} N^i N^j \end{pmatrix} \quad (30)$$

(with $\gamma_{ik}\gamma^{kj} = \delta_i^j$ and $N^i = \gamma^{ij}N_j$). In these variables, the Einstein-Hilbert part reduces to

$$\begin{aligned} \mathcal{L}_{\text{EH}} &= \sqrt{-g} M_{\text{pl}}^2 R \\ &= M_{\text{pl}}^2 (N \sqrt{\gamma} (K_{ij} K^{ij} - K^2 + {}^{(3)}R) \\ &\quad - 2\partial_i(\sqrt{\gamma} K) + 2\partial_i(\sqrt{\gamma} (KN^i - \gamma^{ij}\partial_j N))), \end{aligned} \quad (31)$$

where

$$K_{ij} = \frac{1}{2N} (\nabla_i N_j + \nabla_j N_i - \partial_i \gamma_{ij}). \quad (32)$$

As is customary, the total derivatives have been isolated in such a way that N and N_j are free of derivatives, everywhere except on the boundary. After the adjustment of the boundary term following the Gibbons-Hawking-York procedure, the only surviving boundary contribution is precisely what the doctor prescribed to recover the ADM mass for the asymptotically Minkowski geometry. As it plays no role in the bulk dynamics, we will ignore it for the time being and reintroduce it when relevant,

$$\mathcal{L}_{\text{EH}} = M_{\text{pl}}^2 N \sqrt{\gamma} (K_{ij} K^{ij} - K^2 + {}^{(3)}R). \quad (33)$$

Next, we proceed with the identification of the convenient canonical degrees of freedom.

B. Canonical variables

For convenience, we choose the canonical fields to be γ_{ij} and $A^\mu \equiv \sqrt{-g} g^{0\mu}$. The latter are related to ADM variables by

$$N = -\frac{\sqrt{\gamma}}{A^0} \quad \text{and} \quad N^j = -\frac{A^j}{A^0}. \quad (34)$$

This choice follows from the examination of the Lagrangian for ghosts (27). The classical expression for the conjugate momentum of γ_{ij} , in the absence of the ghost sector, is given by

$$\Pi^{ij} = -M_{\text{pl}}^2 \sqrt{\gamma} (K^{ij} - \gamma^{ij} K). \quad (35)$$

One of the advantages of rewriting the ghost Lagrangian of [34] into (27) by integration by parts is to maintain this classical relation. Moreover, had we chosen N and N_j as

canonical variables instead of A^μ , we would have gotten an additional contribution to (35) from the ghost sector. The conjugate momentum of A^μ is readily given by

$$\Pi_\nu = M_{\text{pl}} b_\nu - i(\partial_\mu \bar{c}_\nu) c^\mu. \quad (36)$$

As for the ghosts, we have

$$\Pi_c^\nu = i(A^\sigma \partial_\sigma c^\nu - \partial_\sigma (\sqrt{-g} g^{\sigma\nu}) c^0), \quad (37)$$

$$\Pi_\nu^c = -iA^\mu \partial_\mu \bar{c}_\nu, \quad (38)$$

where we defined the conjugate momenta using the left differentiation. This way, the Hamiltonian is obtained as

$$H = \int d^3x (\dot{A}^\mu \Pi_\mu + \dot{c}^\mu \Pi_\mu^c + \dot{\bar{c}}_\mu \Pi_\mu^c + \dot{\gamma}_{ij} \Pi^{ij} - \mathcal{L}). \quad (39)$$

It must be noted that the time derivative of temporal components is undetermined from the canonical variables, due to the fact that they only enter in Π_c^ν together with $\dot{c}$. However, as it is customary in such cases, the undetermined time derivative drops out from the Hamiltonian. In particular, upon collecting relevant terms

$$H \supset \int d^3x \dot{A}^\nu (\Pi_\nu - M_{\text{pl}} b_\nu + i\partial_\mu \bar{c}_\nu c^\mu), \quad (40)$$

which vanishes in the light of (36).

Although $\dot{A}^\nu$ drops out from the Hamiltonian, it is nevertheless determined from Hamilton's equation for A^μ , in a complete analogy with the Coulomb potential of electrodynamics. It is worthwhile to mention that the relevant equation follows from the gauge-fixing terms of the Lagrangian.

Moving forward, upon simplification, the ghost and gauge-fixing contributions to the Hamiltonian take the form

$$\begin{aligned} H_{\text{FP+GF}} &= \int d^3x \left[\frac{i}{A^0} \{ \Pi_c^\nu + iA^j \partial_j \bar{c}_\nu \} \{ \Pi_c^\nu - iA^k \partial_k c^\nu \} \right. \\ &\quad \left. - i\partial_i \bar{c}_\nu \sqrt{-g} g^{ij} \partial_j c^\nu - \Pi_0 \partial_k A^k - \Pi_i \partial_j (\sqrt{-g} g^{ij}) \right. \\ &\quad \left. + \frac{1}{2} \alpha \eta^{\mu\nu} b_\mu b_\nu \right], \end{aligned} \quad (41)$$

where we keep in mind that $\sqrt{-g} g^{ij} = \frac{1}{A^0} (-\gamma \gamma^{ij} + A^i A^j)$. Also, we have kept the last term written in terms of b_ν , despite the fact that it is not a canonical degree of freedom, for notational convenience. Furthermore, this is an expendable term as we can always focus on $\alpha = 0$ case.

The Einstein-Hilbert part of the Hamiltonian is well known and, up to the boundary term, reduces to

$$H_{\text{EH}} = \int d^3x \left[-\frac{1}{A^0} \mathcal{H} + \frac{A^i}{A^0} \mathcal{P}_i \right], \quad (42)$$

where $\mathcal{H}$ and $\mathcal{P}^i$ stand for the so-called Hamiltonian and momentum constraints respectively

$$\mathcal{H} \equiv \frac{1}{2M_{\text{pl}}^2} (\gamma_{ik}\gamma_{j\ell} + \gamma_{i\ell}\gamma_{jk} - \gamma_{ij}\gamma_{k\ell}) \Pi^{ij}\Pi^{kl} - M_{\text{pl}}^2 \gamma R^{(3)}, \quad (43)$$

$$\mathcal{P}_i \equiv -2\gamma_{ik}\partial_j \Pi^{kj} - (2\partial_k \gamma_{ji} - \partial_i \gamma_{jk}) \Pi^{jk}. \quad (44)$$

Once again, we would like to emphasize that the boundary term, responsible for reproducing the ADM mass for the asymptotically flat geometry, must be added; upon relevance, we will comment about its necessity. So far, the discussion has been classical; that is why we have not yet paid attention to the order of bosonic operators in the Hamiltonian. The next step is to promote fields to operators, assign the canonical (anti)commutation relations and define the vacuum. It is worth emphasizing that the latter condition will be indicative of how the field operators defined so far differ from the operators corresponding to the elementary quantum degrees of freedom. This difference will be merely a constant shift, which does not affect the canonical (anti)commutation relations. These points will be further clarified as we proceed with quantization.

C. Canonical quantization

Within the adopted framework, all components of the metric possess nonzero canonical conjugates. Therefore, we proceed in a usual fashion by assigning the following equal-time (anti)commutation relations:

$$[\hat{\gamma}_{ij}(x), \hat{\Pi}^{k\ell}(y)] = \frac{i}{2} (\delta_i^k \delta_j^\ell + \delta_i^\ell \delta_j^k) \delta^{(3)}(x-y), \quad (45)$$

$$[\hat{A}^\mu(x), \hat{\Pi}_\nu(y)] = i\delta_\nu^\mu \delta^{(3)}(x-y), \quad (46)$$

$$\{\hat{c}^\mu(x), \hat{\Pi}_\nu^c(y)\} = i\delta_\nu^\mu \delta^{(3)}(x-y), \quad (47)$$

$$\{\hat{\bar{c}}_\nu(x), \hat{\Pi}_\mu^c(y)\} = i\delta_\mu^\nu \delta^{(3)}(x-y). \quad (48)$$

Here, hats indicate the promotion of fields to operators, while [...] and {...} indicate the commutation and anti-commutation, respectively. Notice that, unlike QED and the linearized gravity, b_μ is not strictly speaking the conjugate momentum of the temporal degrees of freedom A^μ due to the additional ghost contribution to (36). However, it is determined in terms of Π_μ and Π_μ^c , using (38).

Let us emphasize that when promoting the Hamiltonian to the operator

$$\hat{H} = \hat{H}_{\text{EH}} + \hat{H}_{\text{FP+GF}}, \quad (49)$$

we need to be cautious with ordering noncommuting fields. For starters, it needs to be Hermitian, a task which is rather straightforward to achieve. We would like to note that noncommuting operators enter the Hamiltonian at most in quartic order. In fact, the first term of (43) is the only source

of the term containing four noncommuting quantities. Notice that this is the virtue of our approach to quantize A^μ rather than the corresponding ADM variables N and N_j . In particular, using the shift vector would introduce the inverse metric in terms that involve the conjugate momentum of the spatial metric. For the rest of this work, we will not pay close attention to the ordering of non-commuting operators at all times, as it is not relevant for our discussion.

At the quantum level, the Hamiltonian contains products of noncommuting operators of schematic form $\gamma\gamma\Pi\Pi$. A complete quantization therefore requires an explicit operator ordering, in particular to make the Hamiltonian Hermitian, or, more precisely, self-adjoint. In the present work we do not attempt to solve this problem in full generality. Rather, our aim is to show that, assuming a consistent BRST-invariant quantization, the classical limit reproduces the conventional time evolution of Einstein gravity. The corresponding consistency requirement is that the Hermitian-ordered BRST charge remain nilpotent and compatible with the Hermitian-ordered Hamiltonian, as discussed further in Sec. V E.

This brings us to the next important point, concerning the vacuum of the theory. If the BRST quantization procedure we have adopted has any merit to it, then the lowest energy eigenstate $|\Omega\rangle$ of the Hamiltonian (which incorporates the boundary contribution resulting in the ADM mass) must correspond to the Minkowski spacetime,³ i.e.

$$\langle\Omega|\hat{\gamma}_{ij}|\Omega\rangle = \delta_{ij}, \quad (50)$$

$$\langle\Omega|\hat{A}^0|\Omega\rangle = -1, \quad (51)$$

$$\langle\Omega|\hat{A}^j|\Omega\rangle = 0, \quad (52)$$

$$\langle\Omega|\hat{b}_\mu|\Omega\rangle = 0. \quad (53)$$

It must be stressed that the last condition must hold in any physical state, not merely in the vacuum. The reason for this is the same as in QED; it follows from the BRST transformation properties of the antighost field. Namely, we have

$$\hat{b}_\nu = i\{\hat{Q}, \hat{\bar{c}}_\nu\}, \quad (54)$$

implying $\langle\psi|\hat{b}_\nu|\psi\rangle = 0$ in any state satisfying $\hat{Q}|\psi\rangle = 0$. Here, $\hat{Q}$ is the Noether charge of the BRST symmetry, and the physical Hilbert space is defined as a set of all zero-BRST charge eigenstates with vanishing ghost number. As it straightforwardly follows from (50) and (51), quantum

³In the classical limit, this is achieved via positivity theorem [41]. For the full quantum theory, the same can be achieved using supergravity, which in turn is motivated by the vacuum structure of GR [32].

fields corresponding to the elementary excitations around the vacuum are defined as

$$\hat{h}_{ij} \equiv \hat{\gamma}_{ij} - \delta_{ij}, \quad (55)$$

$$\hat{a}^0 \equiv \hat{A}^0 + 1. \quad (56)$$

In other words, the creation-annihilation operators creating particles in the vacuum should be defined by the decomposition of $\hat{h}_{ij}$, $\hat{a}^0$, and $\hat{A}^j$ (together with ghost fields). Therefore, we could have simply started with these shifted fields when introducing equal-time commutation relations. Obviously, this would not have changed anything, as both sets of fields satisfy the same commutation relations.

It is straightforward to demonstrate that the canonical formulation at hand leads to the quantum Einstein's equations for the operators supplemented with the gauge-fixing and ghost terms. In order to show without a doubt that the classical dynamics can be recovered from these equations, one needs to construct BRST-invariant states corresponding to classical geometries, with the prime candidates being coherent states.

The explicit construction of such states is challenging; however, we can analyze the expectation value of the above-mentioned equation of motion in a physical state to see if the physicality of the state automatically entails the inconsistency with classical dynamics.

In fact, we will argue (in Sec. V E) that the BRST-construction recovers classical nonlinear equations of motion for 1-point functions supplemented with additional quantum terms that can be treated perturbatively in $\hbar$ for certain backgrounds.

Before we delve into a discussion of how classical dynamics is recovered within the adopted framework, it is imperative to discuss the question of the gauge choice. For this, we only need the part of the Hamiltonian. This is an important point for understanding the introduction of classical backgrounds in different coordinates.

D. Gauge freedom

The relevant equation follows from the gauge-fixing sector and, in the Lagrangian formalism, descends from varying the action with respect to b_ν . In the canonical operator framework, on the other hand, it comes from Hamilton's equation for temporal degrees of freedom

$$\dot{\hat{A}}^\mu = i[\hat{H}, \hat{A}^\mu]. \quad (57)$$

This is both the gauge-fixing equation and the equation defining $\dot{\hat{A}}^\mu$. Luckily, we have the time derivative of only the antighost field in (36). As such, we can straightforwardly deduce

$$\begin{aligned} [\hat{A}^\mu(x), \hat{\Pi}_\nu(y)] &= i\delta_\nu^\mu \delta^{(3)}(x-y) \Leftrightarrow [\hat{A}^\mu(x), M_{\text{pl}} \hat{b}_\nu(y)] \\ &= i\delta_\nu^\mu \delta^{(3)}(x-y). \end{aligned} \quad (58)$$

Rewriting the relevant part of the Hamiltonian in terms of Π_ν , we have

$$\begin{aligned} \hat{H} \supset \int d^3x &\left(-\hat{\Pi}_0 \partial_j \hat{A}^j - \hat{\Pi}_i \partial_j \left(\frac{\hat{\gamma} \hat{\gamma}^{ij}}{\hat{A}^0} \right) \right. \\ &\left. - \frac{1}{2\hat{A}^0} \hat{A}^i (\partial_i \hat{\Pi}_j + \partial_j \hat{\Pi}_i) \hat{A}^j + \frac{1}{2} \alpha \eta^{\mu\nu} \hat{b}_\mu \hat{b}_\nu \right). \end{aligned} \quad (59)$$

Here we have ordered the third term to ensure Hermiticity. Also, $1/\hat{A}^0$ needs to be understood as a Taylor series expansion in $\hat{a}^0 \equiv 1 + \hat{A}^0$.

As a result, (57) takes the following form in three-dimensional decomposition

$$\dot{\hat{A}}^0 = -\partial_j \hat{A}^j + \frac{\alpha}{M_{\text{pl}}} \hat{b}_0, \quad (60)$$

$$\dot{\hat{A}}^i = \partial_j \left(\frac{1}{\hat{A}^0} [\hat{\gamma} \hat{\gamma}^{ij} - \hat{A}^i \hat{A}^j] \right) - \frac{\alpha}{M_{\text{pl}}} \hat{b}_i. \quad (61)$$

As we have already stated, $\hat{b}_\mu$ has vanishing matrix elements between physical states. This property is a direct consequence of (54). We are interested in the expectation value of the metric since it serves as a classical proxy for the quantum state. Therefore, we have

$$\partial_i \langle \psi | \hat{A}^0 | \psi \rangle = -\partial_j \langle \psi | \hat{A}^j | \psi \rangle, \quad (62)$$

$$\partial_i \langle \psi | \hat{A}^i | \psi \rangle = \partial_j \langle \psi | \left(\frac{1}{\hat{A}^0} [\hat{\gamma} \hat{\gamma}^{ij} - \hat{A}^i \hat{A}^j] \right) | \psi \rangle, \quad (63)$$

where $|\psi\rangle$ is a physical state satisfying $\hat{Q}|\psi\rangle = 0$. As we can see, (62) is very similar to the QED gauge-fixing condition. Linearity simplifies the story since it means that the 1-point expectation value satisfies the classical constraint. The other three gauge-fixing conditions (63) are not as pure, and it seems that they may include quantum corrections for certain states. However, the absence of quantum corrections may be possible to demonstrate using perturbative expansion in $\hbar$, utilizing the background field method and realizing that the correlation functions at coincidence should have vanishing gradients. Although this is not guaranteed if the state in question corresponds to an inhomogeneous background. This is an interesting point, the investigation of which we postpone to future work.

Here, we would like to focus on states corresponding to classical configurations, i.e. states with macroscopic occupancy with nonvanishing 1-point function in $\hbar \rightarrow 0$ limit. For such states, the leading and most relevant part of the gauge-fixing condition (63) reduces to

$$\partial_i \langle \hat{A}^i \rangle = \partial_j \left(\frac{1}{\langle \hat{A}^0 \rangle} [\langle \hat{\gamma} \rangle \langle \hat{\gamma}^{ij} \rangle - \langle \hat{A}^i \rangle \langle \hat{A}^j \rangle] \right) + \mathcal{O}(\hbar). \quad (64)$$

It is important to keep in mind that for well-defined physical states one can choose the coordinate frames which satisfy $\langle \hat{A}^0 \rangle < 0$.

As expected (62) and (64) are equivalent to

$$\partial_\mu (\sqrt{-g_{\text{cl}}} g_{\text{cl}}^{\mu\nu}) = 0 \quad (65)$$

at the classical level, where $g_{\mu\nu}^{\text{cl}}$ is the 1-point expectation value of the metric. This equation could have been derived by bypassing the Hamiltonian formalism, as in [34], by promoting the Euler-Lagrange equation for b_μ to an operator equation and evaluating the expectation value in the BRST-invariant state.

E. Reproducing classical dynamics

The above naturally brings us to the discussion of how BRST formalism recovers the classical dynamics, which is an important consistency check of the approach. Let us begin with the aspects of GR that are traditionally considered to cause issues upon quantization. The relevant equations consist of Hamiltonian and momentum constraints

$$\mathcal{H} = 0, \quad (66)$$

$$\mathcal{P}_i = 0, \quad (67)$$

explicitly given by (43) and (44) respectively, together with dynamical equations for the spatial metric and its conjugate momentum. Interestingly, the satisfaction of constraints entails the vanishing of the Einstein-Hilbert Hamiltonian density away from the boundary, given by the integrand of (42). Notably, the theory admits time-dependent configurations that satisfy constraints and have vanishing bulk Hamiltonian density. In classical theory, there is no contradiction in this observation.

The question arising in quantum theory concerns the implementation of constraints. Namely, if one chooses to define the Hilbert space of physical states by

$$\hat{\mathcal{H}}|phys\rangle = 0, \quad (68)$$

$$\hat{\mathcal{P}}_i|phys\rangle = 0, \quad (69)$$

then they are annihilated by $\hat{H}_{\text{EH}}$ of (42), in the absence of the boundary term. As a result, one is led to the conclusion that the subsequent quantum gravity is incapable of generating a nontrivial Hamiltonian flow in physical Hilbert space [42]. Consequently, the quantization at hand reproduces one of the properties of its classical counterparts by yielding a vanishing bulk contribution to the Hamiltonian from physical states.

However, even upon the introduction of the appropriate boundary terms, the main challenge is the reproduction of the time-dependent expectation values of metric degrees of freedom (away from the boundary) in physical states.

A successful quantization must reproduce both of the above-mentioned properties of the classical theory. Namely, there must exist a notion of vanishing Hamiltonian (up to the boundary contribution) in the classical limit, while permitting a time-dependent one-point function of the metric degrees of freedom. The only possibility for achieving these within the BRST formalism is by satisfying the following:

- (i) $\lim_{\hbar \rightarrow 0} \langle phys | \hat{H} | phys \rangle = 0$, where $\hat{H}$ is given by (49) and lacks the boundary term that yields the ADM mass. In order for (49) to satisfy this condition, we need to show that the auxiliary sector gives a vanishing contribution in the classical limit.
- (ii) $\hat{H}|phys\rangle \neq 0$, for the states with nontrivial bulk dynamics.

Now, the preceding argument should not be perceived as a statement about the irrelevance of the aforementioned boundary condition that yields the ADM mass. In fact, this contribution becomes irreplaceable for describing the configurations that are eternal in the classical limit. A good example of such a configuration is a black hole. Classically, a black hole is an eternal state in the sense that its characteristics, such as the mass or the angular momentum, do not change in time. However, in quantum theory, it evolves via Hawking evaporation, and this evolution inevitably triggers a departure from the classical description. There exist excellent arguments indicating that after a certain critical time, referred to as “quantum break-time,” the departure becomes order-one, invalidating the semi-classical picture [3,16,17]. However, the magnitude of the deviation is not central to the present discussion. The key point is that, no matter what, the difference between classical and quantum evolution is nonzero. Correspondingly, the question arises how the ADM mass-fixing boundary Hamiltonian accounts for it.

Within the presented picture of BRST quantization, the above connection is guaranteed by the fundamental principles of formulation. That is, the would-be classical dynamics is recovered by the time evolution of a proper coherent state describing the object. At the same time, the departure of the true quantum evolution from the classical one is accounted for by an inevitable loss of coherence.

For instance, a classical gravitational wave, which can be initially approximated by a coherent state of gravitons, shall depart from classical evolution due to a quantum rescattering of its constituents. Of course, the corresponding quantum break time can be extremely long, but this does not change the essence of the issue.

In other words, in the BRST approach, the main consistency test is the recovery of classical approximation within a certain time scale, as the existence of quantum corrections to such evolution is built into the very framework.

Let us now demonstrate the recovery of the classical dynamics from BRST quantization more explicitly. Within this framework, the fate of constraints stems from Hamilton's equations for the canonical conjugate to the temporal degrees of freedom

$$\partial_t \hat{\Pi}_\mu = i[\hat{H}, \hat{\Pi}_\mu]. \quad (70)$$

The resulting expression for the time and space components of this equation takes the form

$$\begin{aligned} \partial_t \hat{\Pi}_0 = & -\frac{1}{(\hat{A}^0)^2} \hat{\mathcal{H}} + \frac{\hat{A}^i}{(\hat{A}^0)^2} \hat{\mathcal{P}}_i + \frac{1}{(\hat{A}^0)^2} \partial_j \hat{\Pi}_i (-\gamma \gamma^{ij} + \hat{A}^i \hat{A}^j) \\ & + \frac{i}{(\hat{A}^0)^2} [\hat{\Pi}_\nu^c \hat{\Pi}_\nu^c + i \hat{A}^k (\partial_k \hat{c}_\nu \cdot \hat{\Pi}_\nu^c - \hat{\Pi}_\nu^c \cdot \partial_k \hat{c}^\nu) \\ & + \gamma \gamma^{ij} \partial_i \hat{c}_\nu \cdot \partial_j \hat{c}^\nu], \end{aligned} \quad (71)$$

$$\begin{aligned} \partial_t \hat{\Pi}_j = & -\frac{1}{\hat{A}^0} \hat{\mathcal{P}}_j - \partial_j \hat{\Pi}_0 - \frac{1}{\hat{A}^0} [(\partial_j \hat{\Pi}_k + \partial_k \hat{\Pi}_j) \hat{A}^k \\ & + \hat{A}^k (\partial_j \hat{\Pi}_k + \partial_k \hat{\Pi}_j)] \\ & + \frac{1}{\hat{A}^0} (\partial_j \hat{c}_\nu \cdot \Pi_\nu^c - \hat{\Pi}_\nu^c \cdot \partial_j \hat{c}^\nu). \end{aligned} \quad (72)$$

In the current operatorial form, these expressions represent the dynamical equations rather than constraints. The reason is that within the presented quantization scheme, all the metric components are treated as the dynamical degrees of freedom. However, as it is well known, upon acting on physical states, some of the equations generate constraints. In fact, similar to the recovery of Gauss's constraint in QED, the

quantum generalization of Hamiltonian and momentum constraints are recovered as matrix elements of the aforementioned equations between the physical states.

In order to extract the classical Hamiltonian and the momentum constraints as the leading effects, we need to make certain assumptions about physical states. This would not be necessary if we were able to perform an explicit construction of BRST-invariant (nonasymptotic) states.

We begin by noticing that, since one-point functions for ghost fields should vanish in physical states, upon bracketing (71) and (72) with such states, the leading-order ghost contribution should be in the form of their two-point functions at coincidence. It is important to keep in mind that some of such corrections will vanish for kinematic reasons, while the others will be divergent and would have to be absorbed in counterterms, with possible finite left-overs. Either way, all such contributions must be dropped in the classical limit.

In order to simplify the rest of the terms, we would like to use properties of $\hat{b}_\mu$ utilizing (36) with the promotion to operators with proper ordering,

$$\hat{\Pi}_\nu = M_{\text{pl}} \hat{b}_\nu + \frac{1}{2\hat{A}^0} [\hat{\Pi}_\nu^c, \hat{c}^0] + i \frac{\hat{A}^j}{\hat{A}^0} \partial_j \hat{c}_\nu \cdot \hat{c}^0 - i \partial_j \hat{c}_\nu \cdot \hat{c}^j. \quad (73)$$

Correspondingly, in the classical limit, the ghost contributions must be dropped from the expectation value of this quantity. If we further employ the fact that $\langle \text{phys} | \hat{b}_\mu | \text{phys} \rangle = 0$, we find the vanishing expectation value for $\hat{\Pi}_\mu$. Therefore, the expectation values of (71) and (72) reduce to

$$-\left\langle \frac{1}{(\hat{A}^0)^2} \hat{\mathcal{H}} \right\rangle + \left\langle \frac{\hat{A}^i}{(\hat{A}^0)^2} \hat{\mathcal{P}}_i \right\rangle + \left\langle \frac{1}{(\hat{A}^0)^2} \partial_j \hat{\Pi}_i (-\gamma \gamma^{ij} + \hat{A}^i \hat{A}^j) \right\rangle + \mathcal{O}(\hbar) = 0, \quad (74)$$

$$\left\langle \frac{1}{\hat{A}^0} \hat{\mathcal{P}}_j \right\rangle + \left\langle \frac{1}{\hat{A}^0} [(\partial_j \hat{\Pi}_k + \partial_k \hat{\Pi}_j) \hat{A}^k + \hat{A}^k (\partial_j \hat{\Pi}_k + \partial_k \hat{\Pi}_j)] \right\rangle + \mathcal{O}(\hbar) = 0. \quad (75)$$

Moreover, since we are interested in states that correspond to classical configurations built over the Minkowski vacuum, we make further assumptions about the correlation functions of canonical degrees of freedom $\{\hat{h}_{ij}, \hat{\Pi}_{ij}, \hat{a}^0, \hat{A}^j, \hat{\Pi}_\mu\}$ [defined around (55) and (56)] that describe the excitations on top of the vacuum. Namely, we posit that the quantum state in question corresponds to the state of the system with a nonvanishing one-point function for these operators. At the same time, their higher-order correlation functions are assumed to be dominated by disconnected contributions. This assumption can be justified perturbatively in M_{pl}^{-1} , following the construction of [7]. In other words, we are assuming that there exist BRST-invariant coherent states for which the composite operators satisfy

$$\langle f(\hat{h}, \hat{\Pi}, \dots) \rangle = f(\langle \hat{h} \rangle, \langle \hat{\Pi} \rangle, \dots) (1 + \mathcal{O}(\hbar)). \quad (76)$$

Then, the equations further reduce to

$$-\frac{1}{\langle \hat{A}^0 \rangle^2} \mathcal{H}(\langle \hat{\gamma}_{k\ell} \rangle, \langle \hat{\Pi}_{mn} \rangle) + \frac{\langle \hat{A}^i \rangle}{\langle \hat{A}^0 \rangle^2} \mathcal{P}_i(\langle \hat{\gamma}_{k\ell} \rangle, \langle \hat{\Pi}_{mn} \rangle) + \mathcal{O}(\hbar) = 0, \quad (77)$$

$$\frac{1}{\langle \hat{A}^0 \rangle} \mathcal{P}_j(\langle \hat{\gamma}_{k\ell} \rangle, \langle \hat{\Pi}_{mn} \rangle) + \mathcal{O}(\hbar) = 0. \quad (78)$$

Under these assumptions, one can easily convince oneself that the expectation value of the Hamiltonian (49) vanishes in the classical limit. In other words, we have the expression

$$\langle \hat{H} \rangle = M_{\text{pl}}^2 \int d^3x \left[-\frac{1}{\langle \hat{A}^0 \rangle} \mathcal{H}(\langle \hat{\gamma}_{k\ell} \rangle, \langle \hat{\Pi}_{mn} \rangle) \{1 + \mathcal{O}(\hbar)\} + \frac{\langle \hat{A}^i \rangle}{\langle \hat{A}^0 \rangle} \mathcal{P}_i(\langle \hat{\gamma}_{k\ell} \rangle, \langle \hat{\Pi}_{mn} \rangle) \{1 + \mathcal{O}(\hbar)\} + \mathcal{O}(\hbar) \right], \quad (79)$$

which vanishes in $\hbar \rightarrow 0$ limit, in light of (77) and (78).

Having demonstrated the recovery of one of the classical properties of GR, it is important to appreciate that this does not directly lead to

$$\hat{H}|phys\rangle \stackrel{?}{=} 0. \quad (80)$$

Obviously, this condition would be much stronger than what we have demonstrated for the expectation value. Within the adopted framework, the constraints (68) and (69) are replaced by a single constraint based on the BRST charge

$$\hat{Q}|phys\rangle = 0. \quad (81)$$

Now, it is important to acknowledge that this constraint does bear a resemblance to (68) and (69), just as its QED counterpart contains the operator of Gauss's law within it, see, e.g., [7]. In particular, even though

$$\hat{Q}_{\text{QED}} = \int d^3x [\hat{c}(g\hat{p} - \partial_j \hat{E}_j) + \hat{B}\hat{\Pi}_c + \partial_j(\hat{c} \hat{E}_j)], \quad (82)$$

the constraint (81) does not entail

$$(g\hat{p} - \partial_j \hat{E}_j)|phys\rangle \stackrel{?}{=} 0, \quad (83)$$

because of the auxiliary sector. In fact, for the BRST quantization, the constraint equation (83) takes the form

$$(g\hat{p} - \partial_j \hat{E}_j)|phys\rangle = -i\{\hat{Q}_{\text{QED}}, \hat{\Pi}_c\}|phys\rangle, \quad (84)$$

and differs from (83) due to the nontrivial contribution from the right-hand side. The outlined difference is crucial, as it tells us that the BRST quantization has a larger Hilbert space than a naive constraint quantization.

Gauss's law is recovered between arbitrary physical states,

$$\begin{aligned} \langle phys' | (g\hat{p} - \partial_j \hat{E}_j) | phys \rangle &= -i \langle phys' | \{\hat{Q}_{\text{QED}}, \hat{\Pi}_c\} | phys \rangle \\ &= 0, \end{aligned} \quad (85)$$

even for different $|phys\rangle$ and $|phys'\rangle$. This equation shows the difference between BRST and the aforementioned naive quantization. The latter puts all the constraints on the state appearing from one side, where in the BRST quantization it is “shared” between ket- and bra- states.

This feature is well known to be exhibited in the Gupta-Bleuler quantization. The gauge-fixing condition is partially set on the Hilbert space. The positive-frequency part is set on the ket states, while the negative-frequency part is imposed on the bra states. This is naturally realized in the BRST quantization, since the right-hand side of (85) can be related to the gauge condition using

$$i\{\hat{Q}_{\text{QED}}, \hat{\Pi}_c\} = \hat{B}, \quad (86)$$

where $\hat{B}$ is equal to the gauge-fixing condition based on Hamilton's equation, and it vanishes only between physical states.

Let us reiterate that, unlike the naive constraint quantization, Gauss's law is realized dynamically, in the BRST framework. The constraint appears from a dynamical equation of motion and is fulfilled only between two physical states. All of the above suggest that the gravitational Hamiltonian $\hat{H}$ does not necessarily annihilate the physical states and is capable of generating the time evolution.

The dynamics follows from the spatial Einstein's equations, which take the form

$$\begin{aligned} \partial_i \hat{\Pi}^{mn} &= \frac{2}{M_{\text{pl}}^2} \frac{1}{\hat{A}^0} \left(\hat{\Pi}^{mk} \hat{\Pi}_k^n - \frac{1}{2} \hat{\Pi}^{mn} \hat{\Pi} \right) + \partial_k \left(\frac{\hat{A}^m}{\hat{A}^0} \right) \hat{\Pi}^{nk} + \partial_k \left(\frac{\hat{A}^n}{\hat{A}^0} \right) \hat{\Pi}^{mk} - \partial_i \left(\frac{\hat{A}^i}{\hat{A}^0} \hat{\Pi}^{mn} \right) \\ &\quad - \frac{M_{\text{pl}}^2}{\hat{A}^0} \hat{\gamma}(\hat{\gamma}^{mn(3)} \hat{R} + {}^{(3)}\hat{R}^{mn}) - M_{\text{pl}}^2 \hat{\gamma}(\nabla^m \nabla^n - \gamma^{mn} \nabla^2) \frac{1}{\hat{A}^0} \\ &\quad + \frac{1}{2} \frac{\hat{\gamma}}{\hat{A}^0} \hat{\gamma}^{mi} \hat{\gamma}^{nj} (\partial_i \hat{\Pi}_j + \partial_j \hat{\Pi}_i - \partial_i \hat{c}_\nu \partial_j \hat{c}^\nu - \partial_j \hat{c}_\nu \partial_i \hat{c}^\nu) + \frac{\hat{\gamma}}{\hat{A}^0} \hat{\gamma}^{mn} (\partial_k \hat{\Pi}^k - \partial_k \hat{c}_\nu \partial^k \hat{c}^\nu). \end{aligned} \quad (87)$$

At this point, it is straightforward to see that upon computing the expectation value of this equation in a physical state and following the same line of reasoning as in recovering the classical constraint equations, we will arrive at the classical equations for $\langle \hat{\Pi}^{mn} \rangle$ which will coincide with the corresponding Einstein's equations. In particular, for a generic physical state for which the right-hand side (87) has a nonzero disconnected contribution at $\hbar^0$ order, we obtain

$$\lim_{\hbar \rightarrow 0} \partial_t \langle \hat{\Pi}^{mn} \rangle \neq 0. \quad (88)$$

We have thus demonstrated that BRST quantization of gravity correctly reproduces the classical dynamics (for further clarification, see [43]). Upon reintroduction of quantum terms, the corresponding correction to the dynamics can be straightforwardly obtained within this framework. However, this goes beyond the scope of the current work and will be considered elsewhere.

We conclude this section by stressing a crucial conceptual point. In the BRST framework, the gravitational constraints are not imposed as strong operator equations annihilating physical states in the Wheeler-DeWitt sense. Instead, physical states are defined cohomologically: they are BRST-closed modulo BRST-exact states. Accordingly, Eqs. (68) and (69) should not be interpreted here as strong operator identities on the physical Hilbert space. Their content is recovered only weakly, i.e. in matrix elements or expectation values on BRST-physical states. This is the sense in which the physical-state conditions used here differ from the Wheeler-DeWitt prescription. Thus, in the present quantization scheme, the nonlinear classical constraint algebra is replaced by the BRST algebra on the extended state space. The relevant quantum consistency condition is therefore the nilpotence of the BRST charge and its compatibility with the Hamiltonian, rather than the direct strong-operator realization of the original constraint algebra. In $3+1$ dimensions, where no genuine gravitational anomaly is expected, this strongly suggests that BRST symmetry can be maintained at the quantum level. Nevertheless, a complete proof would require an explicit Hermitian ordering of the BRST charge and Hamiltonian and a verification of nilpotence in that ordering.

VI. REPARAMETRIZATION OF COORDINATES

Let us now discuss the question of reference frames within the canonical quantization of gravity at hand. In particular, any physical state corresponding to a certain classical geometry must satisfy the gauge-fixing condition introduced upon quantization. As a result, the configurations corresponding to different frames are still allowed; however, not all classical slices of a geometry are reproducible by the 1-point expectation value in a physical state. In fact, even if we construct a BRST-invariant state for which the 1-point function reproduces a classical geometry in a certain frame at a given moment of time, the classical

evolution away from the original slicing of this 1-point function will be governed by the gauge-fixing condition (65) (for a related discussion in the context of electrodynamics, see [44]). It goes without saying that the states corresponding to different frames are physically equivalent, as it can be demonstrated by the equivalence of the corresponding S -matrix elements. Moreover, as it is well known, the correlation functions of gauge-invariant quantities are independent of the gauge-fixing condition. However, we are often interested in nongauge-invariant correlation functions, especially in cosmology. Furthermore, the standard gauge-invariant variables in cosmological perturbation theory are invariant to the linear order.

We begin demonstrating some of these points by considering the vacuum state corresponding to Minkowski geometry. In the following sections, we will also see the implications for the FLRW universe.

A. From Minkowski to Rindler

The Minkowski spacetime $g_{\mu\nu} = \eta_{\mu\nu}$ obviously satisfies de Donder gauge condition (65). An accelerated observer views the same spacetime in the Rindler coordinates x_r^μ , which are related to Minkowski coordinates x^μ by

$$t_r = \frac{1}{a} \operatorname{arctanh}\left(\frac{t}{x}\right), \quad x_r = \sqrt{x^2 - t^2}, \quad (89)$$

where a stands for the acceleration in the positive x direction and the observer is at rest at $x = 1/a$ at $t = 0$. In these coordinates, the spacetime interval takes the form

$$ds^2 = -(ax_r)^2 dt_r^2 + dx_r^2 + dy_r^2 + dz_r^2. \quad (90)$$

Due to the covariance of Einstein's equations, classically, the Rindler metric

$$g_{\mu\nu} = \operatorname{diag}(-(ax)^2, 1, 1, 1) \quad (91)$$

is as good a solution as the Minkowski space $g_{\mu\nu} = \eta_{\mu\nu}$. However, it is straightforward to see that this metric does not satisfy (65). This implies that if we construct a physical coherent state in which the expectation value of the metric at some moment of time t is given by (91), the equality will not be maintained at later times even in the classical limit.

In order to ensure that $\langle \hat{g}_{\mu\nu} \rangle$ maintains a given classical form, we need to adjust the gauge-fixing term from the beginning accordingly. For example, had we imposed $\partial_\mu(g^{\mu\nu}) = 0$ instead of (65), the Rindler metric (91) would have been a proper background at all times, at least classically. Incidentally, so would the Minkowski space itself.

The corresponding modification of the gauge-fixing term (27) would have been

$$\mathcal{L}'_{\text{GF}} = M_{\text{pl}} b_\nu \partial_\mu g^{\mu\nu} - \frac{1}{2} \alpha \eta^{\mu\nu} b_\mu b_\nu, \quad (92)$$

with appropriately adjusted ghost terms. Which, in turn, would force us to reconsider the parametrization of the temporal degrees of freedom of the metric. As we are about to see, this particular gauge-fixing is relevant to the FLRW background as well.

B. FLRW cosmology

Although the explicit construction of fully nonlinear BRST-invariant states is challenging, we can make certain statements about the consistency of the gauge-fixing sector with backgrounds of cosmological significance.

We shall now discuss the implications of our quantization framework for cosmology. First, as any other would-be classical state, in this framework, a cosmological background represents an expectation value of the metric operator over a coherent state. The metric operator is obtained by BRST quantization of the metric field once and for all on a special state representing the vacuum of the theory (in the present case Minkowski). The coherent state describing the cosmological background is also constructed on top of this vacuum [7].

In this language, the entire cosmology is captured by the quantum evolution of the state. This, in particular, encodes the information about all possible higher-order correlators.

In this description, the standard treatment of quantum perturbations on a classical cosmological background represents an approximation, in which the true quantum state is split into a coherent part describing the background and a perturbation. However, this approximation is expected to “wear out” in time due to loss of coherence. For certain backgrounds, this can lead to a complete quantum-break [3–7].

In the standard background field treatment, in principle, this breakdown can be captured by the backreaction on a would-be classical background and should manifest itself in the relative growth of higher-order correlators [11].

However, we would like to separate the issues of breakdown of perturbative treatment on large timescales from the questions of differences in gauge-fixings in the two approaches. In our approach, the procedure must begin with the gauge-fixing sector in place before introducing the nontrivial background state for the degrees of freedom in question.

The next step is the construction of the cosmological background state. This requires a source. The role of the source can be played by an additional massive scalar field with the following Lagrangian:

$$\Delta\mathcal{L} = \sqrt{-g} \left(-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right). \quad (93)$$

Upon canonical quantization, the vacuum of the theory would still correspond to Minkowski spacetime with properties

$$\langle \Omega | \hat{g}_{\mu\nu} | \Omega \rangle = \eta_{\mu\nu}, \quad \langle \Omega | \hat{\phi} | \Omega \rangle = 0. \quad (94)$$

However, there should also exist states in the physical Hilbert space for which $\langle \hat{\phi} \rangle \neq 0$. And such states are expected to source the nontrivial state of gravitational degrees of freedom with $\langle \hat{g}_{\mu\nu} \rangle \neq \eta_{\mu\nu}$.

Let us consider the state that classically corresponds to spatially flat cosmology with the spacetime interval

$$ds^2 = -dt^2 + a^2(t) dx^2, \quad (95)$$

with $a(t)$ being the scale factor; e.g., during quasi-de Sitter evolution, like cosmic inflation, $a(t) \simeq e^{Ht}$ with high accuracy.

In quantum theory, on the other hand, we are interested in quantum (presumably coherent) states, in which the expectation value of the metric operator $\langle \hat{g}_{\mu\nu} \rangle$ reproduces the desired classical geometry. Again, it is important to appreciate that one has to quantize the theory before constructing the above state.

The BRST quantization begins with the introduction of the gauge-fixing sector. We begin by considering the de Donder gauge. Let us assume that there exists a BRST-invariant state $|g_{cl}\rangle$, such that $\langle g_{cl} | \hat{g}_{\mu\nu} | g_{cl} \rangle = g_{\mu\nu}^{cl}$ reproduces the aforementioned FLRW metric in the classical limit. As we have already argued above, the de Donder gauge implies (65) up to $\mathcal{O}(\hbar)$ corrections. It is straightforward to see that this condition is not satisfied by (95), implying that we will be deviating from the corresponding slicing dynamically. If we would like to maintain classical FLRW background geometry, an appropriate gauge-fixing condition is required. For example, the choice we have considered for the Rindler background (92) also works for FLRW slicing (95).

With the proper gauge-fixing at hand, we can proceed with computations of correlators, order by order in $\hbar$ expansion. This procedure must correctly match the standard background field method for accounting for the quantum backreaction in the specific gauge (92).

C. A class of pure-gauge states

Although, in general, a construction of BRST-invariant states in quantum gravity can be highly involved, for illustrative purposes, we shall produce a simple class of manifestly BRST-invariant states. These are the states built by $\hat{b}_\nu$ which are invariant under BRST transformations, in complete analogy with QED [44]. Despite the modification of the 1-point expectation value and correspondingly the background, the utilization of $\hat{b}_\nu$ is incapable of generating a physically distinct state, because of (54).

Let us consider the following coherent state:

$$|f\rangle = e^{-i \int d^3x f_\nu(x) \hat{b}_\nu} |\Omega\rangle, \quad (96)$$

where $f_c^\nu(x)$ are the c-number functions of spacial coordinates, while $|\Omega\rangle$ stands for the Minkowski vacuum, i.e. $\langle\Omega|\hat{g}_{\mu\nu}|\Omega\rangle = \eta_{\mu\nu}$. Despite similarities, it must be mentioned that this state differs from a similar one constructed in QED. Namely, $\hat{b}_\nu$ is not *per se*, a conjugate momentum for the temporal degree of freedom $\hat{A}^\nu$, because of the participation of ghosts in (36). However, it is easy to see that this difference does not generate a ghost number, nor any other nontrivial ghost configuration.

It is a direct consequence of (54) and the nilpotence of the BRST charge ($\hat{Q}^2 = 0$) that (96) is physically equivalent to the Minkowski vacuum, i.e.

$$|f\rangle = |\Omega\rangle + \hat{Q}|\psi\rangle, \quad (97)$$

with some, not necessarily a physical, state $|\psi\rangle$. As usual, this implies that this state has the same S -matrix elements with physical states as $|\Omega\rangle$. Similarly, we could replace $|\Omega\rangle$ in (96) with any other physical state, and the result of the exponential operator would be equivalent.

Yet, the expectation value of the metric is modified from its Minkowskian value

$$\langle f|A^\mu|f\rangle = -\delta_0^\mu + f_c^\mu, \quad \langle f|g_{ij}|f\rangle = \delta_{ij}. \quad (98)$$

The above indicates that the coherent state in question corresponds to a Minkowski space in different coordinates.

It is important to appreciate that this class of pure gauge states is quite limited and by no means guarantees that a particular slicing of the Minkowski spacetime can be realized within it. For example, it may naively seem that the quantum state corresponding to the Rindler space (91) may be consistently captured within (96). However, the coordinate singularity of that frame complicates the construction. Recalling that in the de Donder gauge $A^\mu = \sqrt{-g}g^{0\mu}$, we can easily find f_c^μ that would correspond to (91). In this gauge, however, the metric would depart from (91) classically at later times.

Therefore, in order for the classical dynamics not to take us away from the Rindler coordinates, it is better to work with (92), in which case $A^\mu = g^{0\mu}$. The resulting coherent state corresponding to (91) would be

$$|R\rangle = e^{-i \int d^3x [1 + (ax)^{-2}] \hat{b}_0} |\Omega\rangle. \quad (99)$$

Notice that the state becomes singular at the horizon, which invalidates the construction. In other words, it does not seem possible to represent (91) as a coherent state [of the form (96)] built over the Minkowski space.

VII. ON CUTOFF SENSITIVITY AND SUPERSYMMETRY

We would like to comment on the issue of the cutoff sensitivity of the low-energy theory. First of all, let us

remark that the presented quantization on the Minkowski vacuum can be performed without any reference to the cutoff, the value of which depends on the details of the theory. While the Planck mass sets an upper bound on this scale, with the extended field content, the cutoff can be much lower. In general, the cutoff of a low-energy effective theory propagating N_{sp} particle species is lowered by the black hole physics to the “species scale,” $M_{\text{pl}}/\sqrt{N_{\text{sp}}}$ [45–49].

Although the canonical quantization itself is not sensitive to the cutoff, the computation of higher-order corrections inevitably raises such questions. The corresponding cutoff-sensitive contributions must be evaluated within the UV-complete theory. At the level of low-energy effective theory, integrating out UV physics generates an infinite series of higher-dimensional operators suppressed by the cutoff. Their impact on low-energy observables, such as planetary orbits, depends on the scale of the process relative to the cutoff length and is a matter of phenomenology. For reasonable values of the cutoff, such corrections are negligible for most macroscopic systems of astrophysical interest, so that the low-energy dynamics is dominated by Einstein gravity.

An important cutoff-sensitive quantity is the vacuum energy, Λ . At the level of the effective action, its contributions arise from diagrams whose resummation generates the $\sqrt{-g}\Lambda$ term, as required by general covariance. As with other UV-sensitive quantities, its precise value can only be determined in the UV-complete theory. The point of the present discussion is therefore not to claim that our framework, by itself, proves the vanishing of the cosmological constant. In the effective-field-theory sense, vacuum-energy contributions can be absorbed into the bare cosmological constant in the usual way. Our concern here is not with that renormalization procedure itself, but with the role played by the vacuum state in the canonical formulation.

From this viewpoint, the canonical BRST-invariant framework developed in this work requires the existence of a well-defined vacuum state in the Hilbert space. More precisely, in the canonical picture we would like to have a Hilbert space containing at least one vacuum state, understood as the lowest-energy eigenstate of the Hamiltonian with the corresponding ADM energy. The same Hilbert space should then also contain many other states corresponding, in an appropriate semiclassical sense, to various classical solutions of Einstein’s equations. This should be understood in close analogy with ordinary interacting quantum field theory, where the existence of a distinguished vacuum does not preclude the presence of non-trivial approximately classical backgrounds, but instead provides the reference state on top of which such configurations are described as nonstationary quantum states.

In the same sense, cosmological or other time-dependent geometries are not excluded in our framework; rather, they

are interpreted as BRST-invariant states built on top of the vacuum instead of vacuum states themselves. Accordingly, time-dependent backgrounds, or backgrounds exhibiting particle production, should not in general be identified with Hamiltonian eigenstates. This makes the case of vanishing net cosmological constant conceptually the most straightforward in the present framework, since Minkowski space can then play the role of the vacuum state and the corresponding BRST quantization can be implemented most explicitly. By contrast, when the net vacuum energy is positive, the identification of the vacuum becomes less clear, since de Sitter spacetime is time dependent in global description and exhibits particle production, making its interpretation as a Hamiltonian eigenstate problematic.

For negative net vacuum energy, anti-de Sitter (AdS) space is in some respects a better candidate for a vacuum state, since it is static and does not exhibit particle production in the same way. However, it raises different difficulties once one considers generic perturbations and the effective low-energy dynamics. In particular, AdS space is known to exhibit a nonlinear instability under a broad class of small perturbations, which may drive the system toward singular evolution of big-crunch type. This makes it more difficult to discuss approximate unitarity within the low-energy effective theory, since energy can cascade into ultraviolet modes.

In this sense, the special role of Minkowski space in our construction is not that it is derived here as the only conceivable possibility, but that it furnishes the vacuum structure for which the canonical quantization program can be carried out most explicitly and whose physical implications we can analyze self-consistently.

This viewpoint is compatible with the condition $\Lambda = 0$ that is also suggested by the requirement of a consistent S -matrix (see [31] and references therein). This is not a coincidence as the Minkowski vacuum is required both as the S -matrix vacuum that admits a globally defined time, as well as the vacuum for consistent quantization of the asymptotic S -matrix states. In the present context, however, we use this connection only as additional motivation for focusing on the Minkowski-vacuum case, rather than as a stand-alone proof from our low-energy framework itself.

Notice that, although none of the above constraints exclude the existence of AdS states, they do exclude the transitions into such states from our Minkowski vacuum. In other words, the AdS vacua consistent with our quantization must not jeopardize the stability of the Minkowski one. In this sense, the two must belong to different superselection sectors.

We remark that the impossibility of decay of the Minkowski vacuum into an AdS one is also supported by the S -matrix considerations as it is impossible to consistently describe such a decay as a unitary S -matrix process [50,51].

The prominent role of the Minkowski vacuum is intrinsically intertwined with the question of supersymmetry.

First, the key feature of the valid Minkowski vacuum is its stability. This can be guaranteed by the Poincaré invariance. Notice that this requirement is exact and must go beyond the perturbative level. That is, the decay of the Minkowski vacuum via the nonperturbative tunnelling processes is also excluded. In particular, this implies [50] that a consistent theory must exclude the existence of a neighboring AdS vacuum with the energy splitting exceeding the Coleman-De Luccia [52] tunnelling threshold. The absence of tunnelling gives a first indication of the potential role of supersymmetry. Namely, supersymmetry plays a prominent role in guaranteeing the exact Poincaré invariance of the Minkowski vacuum.

However, there is more to it, as supersymmetry appears to be a necessity rather than merely a stabilizing tool. Even in the absence of any scalar potential or a cosmological term at the perturbative level, supersymmetry appears to be necessary for excluding the violation of Poincaré symmetry by the gravitational instantons [32]. As shown in the latter work, even in pure gravity, the Poincaré symmetry of Minkowski vacuum is inevitably violated (by generating a continuum of the eternal de Sitter states) by Eguchi-Hanson instantons [53,54] which are the intrinsic feature of the topological structure of such a vacuum. Supersymmetry restores the Poincaré symmetry thanks to the fermionic zero modes deposited in Eguchi-Hanson instantons by the spin-3/2 gravitino. Simultaneously, the gravitino gets a mass, thereby signalling the super-Higgs effect.

In light of the above arguments, our quantization framework provides a strong motivation for supersymmetry.

VIII. CONCLUSION

In this work, we have performed the canonical quantization of GR in the BRST-invariant framework. We demonstrated the recovery of dynamical classical properties of GR in the appropriate limit. This is of central importance, since a quantization of GR within effective field theory is usually believed to be problematic because of the constraint structure.

We showed that the BRST-invariant approach [34] is fully consistent with the existence of the Hamiltonian time flow in the physical Hilbert space. We pinpoint the place where the over-constraining of the physical states takes place in other approaches and how they differ from the well-known and well-tested procedures of handling constraints in gauge theories.

Let us stress the scope of our results. The purpose of this work is to show that, assuming a consistent BRST-invariant quantization exists, the BRST physicality condition is compatible with the emergence of the standard classical time evolution of Einstein gravity. We have not provided a complete construction of the Hermitian-ordered quantum theory, nor an explicit proof that the corresponding BRST charge remains nilpotent after operator ordering. These are

necessary ingredients for a full demonstration of the consistency of the framework and remain for future work.

Our framework sheds a very different light on the role of the background in quantum gravity. The distinguishing feature is that the theory is quantized with the gauge-fixing in place before we select a quantum state that corresponds to a nontrivial (non-Minkowski) spacetime. This solidifies the idea that in quantum gravity, the curved classical backgrounds must be treated as coherent states constructed on top of the Minkowski vacuum [1,3–7,31]. In particular, the BRST-invariant construction of a de Sitter state can be found in [7].

Naturally, the Minkowski spacetime is given a well-deserved special treatment as the proclaimed vacuum of quantum gravity. In this picture, the characteristic “democracy” of GR, according to which any consistent classical metric can be treated as a valid vacuum for quantum perturbations, is the emergent property of coherent states in the proper limit. In this way, the fundamental gauge-fixing term is universally consistent for all physical spacetimes.

This picture has a number of implications, in particular for cosmology. The coherent state description of a cosmological space-time in full quantum theory can uncover new properties that are invisible in the standard semiclassical treatment. The example is provided by the quantum break time of de Sitter, which shows that the eternal de Sitter is not a valid vacuum of quantum gravity [3–7,15,55].

The prominent status of Minkowski as the vacuum resonates with the formulation of quantum gravity via an S -matrix theory in which in and out states are given by a complete set of asymptotic states defined on Minkowski.

Although the presented framework is independent of the UV completion of the theory, the above-discussed conceptual features of our framework are validated by the string theory in very general terms.

In particular, the string theory makes manifest both the defining role of the S -matrix and the coherent state interpretation of cosmological backgrounds [31]. For example, the de Sitter-like inflationary backgrounds in string theory are obtained via D -branes [56,57], which are the only known string-theoretic sources with negative pressure. However, D -branes [58] represent the string solitons and thus the coherent states of stringy degrees of freedom. This makes the coherent state view of the inflationary cosmological background [3–7,15,55] organic for string theory.

Last but not least, our quantization framework, due to the crucial role of the Minkowski vacuum, provides strong motivation for supersymmetry. In particular, as shown in [32], without supersymmetry, the Poincaré invariance would be inevitably broken at a nonperturbative level by

the Eguchi-Hanson instantons (due to generation of the de Sitter “vacua”). These instantons are fundamentally linked with the topological structure of the Minkowski vacuum in gravity, and their existence is insensitive to UV properties of the theory. Therefore, they must be removed within a low-energy effective field theory. This is accomplished by the fermionic zero modes of spin-3/2 gravitino, thereby justifying supersymmetry. It is important to stress that supersymmetry must be spontaneously broken by the consistency of the instanton-generated gravitino mass with the Poincaré symmetry. That is, the low-energy effective theory below the gravitino mass is Einstein’s gravity with an exact Poincaré-invariant vacuum.

Supersymmetry is also instrumental for eliminating other processes of violation of the Poincaré symmetry, such as via tunnelling, which is essential for having a consistent theory with Minkowski vacuum [50,51].

Thus, via the requirement of exactness of the Poincaré symmetry of the Minkowski vacuum, our framework supports a spontaneously broken supersymmetry.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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