

Phenomenology of D_{s1} mesons radiative transitions

A. E. Bondar^{*} and A. I. Milstein[†]

*Budker Institute of Nuclear Physics of SB RAS, 630090 Novosibirsk, Russia
and Novosibirsk State University, 630090 Novosibirsk, Russia*



(Received 6 May 2025; accepted 6 June 2025; published 20 June 2025)

We discuss radiative transitions $D_{s1} \rightarrow D_s \gamma$ and $D_{s1} \rightarrow D_s^* \gamma$ of p -wave mesons. Since D_{s1} mesons have no certain C -parity, and the masses of quarks in these mesons differ significantly, then due to the spin-orbit interaction, each of the p -wave mesons with the total angular momentum $J = 1$ is a superposition of states with the total spin of quarks $S_{\text{tot}} = 0$ and $S_{\text{tot}} = 1$. We explain why the partial width of $D_{s1}(2460) \rightarrow D_s \gamma$ radiative transition is significantly larger than the corresponding value for $D_{s1}(2536) \rightarrow D_s \gamma$. We also predict the corresponding partial widths of B_{s1} and B_{c1} .

DOI: 10.1103/64y1-qyvt

I. INTRODUCTION

Currently, various phenomenological approaches are used to study spectroscopy and radiative transitions of hadronic states containing heavy quarks. In particular, one can try to use a nonrelativistic quark model accounting for the first relativistic corrections. The corrections are especially important if the photon radiation amplitude obtained in the leading nonrelativistic approximation is suppressed for one reason or another. In this case, when calculating the radiation amplitudes, in addition to relativistic corrections it is also necessary to take into account the retardation effect in the photon wave function. A similar problem arises in both atomic and nuclear physics. An example is a single-photon transition $2s \rightarrow 1s$ in the hydrogen atom [1] and bremsstrahlung during scattering of two identical nonrelativistic nuclei [2]. This is precisely the situation that arises when studying radiative transitions of p -wave states, for example, $c\bar{s}$ systems with the total momentum $J = 1$. The nonrelativistic electric dipole moment of this system is

$$\mathbf{d} = \left[\frac{e_{\bar{s}}}{m_s} - \frac{e_c}{m_c} \right] M_R \mathbf{r} \approx \frac{e_{\bar{s}}}{3m_s} M_R \mathbf{r}, \quad (1)$$

where m_s and $e_{\bar{s}} = e/3$ are the mass and charge of $\bar{s}$ quark, m_c and $e_c = 2e/3$ are the mass and charge of c quark, e is the proton charge, $\mathbf{r} = \mathbf{r}_{\bar{s}} - \mathbf{r}_c$, and $M_R = m_c m_s / (m_c + m_s)$ is the reduced mass. In the center-of-mass frame,

$\mathbf{r}_{\bar{s}} = M_R \mathbf{r} / m_s$ and $\mathbf{r}_c = -M_R \mathbf{r} / m_c$. For the estimate, the values $m_s = 0.5$ GeV and $m_c = 1.7$ GeV are used. A strong compensation in the magnitude of electric dipole moment is seen.

The nonrelativistic dipole moment (1) is independent of quark spins $S_{\bar{s}}$ and S_c and cannot lead to radiative transitions between the states with $S_{\text{tot}} = 1$ and $S_{\text{tot}} = 0$ of the total spin $S_{\text{tot}} = S_{\bar{s}} + S_c$. Meanwhile, there are two p -wave mesons consisting of c and $\bar{s}$ quarks, $D_{s1}(2460)$ and $D_{s1}(2536)$, with the total momentum $J = 1$ and close masses. Each of these mesons is a superposition of states with $S_{\text{tot}} = 1$ and $S_{\text{tot}} = 0$ [3–6]. This is because the fine structure operator of the system depends not only on S_{tot} but also on the operator $\Sigma = S_{\bar{s}} - S_c$ having nonzero matrix elements between the states with different values of the operator S_{tot} . Therefore, the total spin is not conserved, unlike the total angular momentum J , corresponding to the operator $J = L + S_{\text{tot}}$, where L is the angular momentum operator of the system. In addition, the operator responsible for radiative transitions between states with $L = 1$ and $L = 0$ also contains contributions proportional to S_{tot} and Σ . This, as it is shown in our paper, explains the unusual ratio of widths of D_{s1} -meson radiative transitions [7]. The conclusions of Ref. [7] are based on the analysis of experimental data of BABAR Collaboration [8]. As noted in Ref. [7], the width of transition $D_{s1}(2536) \rightarrow D_s \gamma$ is less than 8 KeV, and the width of transition $D_{s1}(2460) \rightarrow D_s \gamma$ is less than 2.3 KeV, which is strikingly different from the generally accepted expectations presented in Table I. Note that the estimation for the width of $D_{s1}(2460) \rightarrow D_s \gamma$ transition in Ref. [7] was based on the account of the nonrelativistic dipole moment contribution solely. We show that this assumption is not valid due to the importance of relativistic corrections. Taking these corrections into account within a potential model, we obtain the formulas for the probabilities of $D_{s1} \rightarrow D_s \gamma$, $D_{s1} \rightarrow D_s^* \gamma$,

^{*}Contact author: A.E.Bondar@inp.nsk.su

[†]Contact author: A.I.Milstein@inp.nsk.su

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](#). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

TABLE I. Theoretical predictions for radiative decay widths $D_{s1} \rightarrow D_s^{(*)}\gamma$ in KeV.

Publication	[13]	[14]	[15]	[16]	[17]	[18]	This work
$\Gamma(D_{s1}(2536) \rightarrow D_s\gamma)$	15	25.2–31.1	61.2	54.5	18.18–18.85	1.6 ± 2.3	12
$\Gamma(D_{s1}(2536) \rightarrow D_s^*\gamma)$	5.6	14.6–22.8	9.21	8.9	2.96–3.02	10.4 ± 1	29
$\Gamma(D_{s1}(2460) \rightarrow D_s\gamma)$	6.2	10.3–17.2	13.2	12.8	3.53–3.61		297
$\Gamma(D_{s1}(2460) \rightarrow D_s^*\gamma)$	5.5	14.0–25.1	17.4	15.5	4.74–4.79		104

$B_{s1} \rightarrow B_s\gamma$, $B_{s1} \rightarrow B_s^*\gamma$, $B_{c1} \rightarrow B_c\gamma$, and $B_{c1} \rightarrow B_c^*\gamma$ transitions.

Our work is organized as follows. We first derive the spin-dependent corrections to the operator responsible for radiative transitions, make qualitative predictions for the corresponding probabilities, and compare them with available experimental data. The conclusion formulates the main results of our work.

II. FINE STRUCTURE OF $c\bar{s}$ SYSTEM IN THE RADIATION FIELD

To obtain a radiative-transition operator that takes into account relativistic corrections and the retardation effect, we consider the Hamiltonian of the $c\bar{s}$ system in the radiation field. This Hamiltonian contains several terms, which we discuss separately. The vector potential $\mathbf{A}(\mathbf{r}, t)$ of the electromagnetic wave in the final state, the corresponding electric field $\mathcal{E}(\mathbf{r}, t)$, and the magnetic field $\mathcal{H}(\mathbf{r}, t)$ have the forms

$$\begin{aligned} \mathbf{A}(\mathbf{r}, t) &= \mathbf{A}_W e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})}, & \mathcal{H}(\mathbf{r}, t) &= -i[\mathbf{k} \times \mathbf{A}(\mathbf{r}, t)], \\ \mathcal{E}(\mathbf{r}, t) &= -i\omega \mathbf{A}(\mathbf{r}, t), \end{aligned} \quad (2)$$

where $\mathbf{A}_W$ is a constant vector perpendicular to the momentum $\mathbf{k}$ of the photon, and the system of units $\hbar = c = 1$ is used.

We start with the Hamiltonian H_0 , which accounts for the first spin-independent relativistic correction to the kinetic energy,

$$\begin{aligned} H_0 &= \frac{\pi_{\bar{s}}^2}{2m_s} + \frac{\pi_c^2}{2m_c} + U_g(r) + U_{\text{conf}}(r) - \frac{(\pi_{\bar{s}}^2)^2}{8m_s^3} - \frac{(\pi_c^2)^2}{8m_c^3}, \\ \pi_{\bar{s}} &= \mathbf{p} - e_{\bar{s}}\mathbf{A}(\mathbf{r}_{\bar{s}}, t), & \pi_c &= -\mathbf{p} - e_c\mathbf{A}(\mathbf{r}_c, t), \\ U_g(r) &= -\frac{g}{r}, & U_{\text{conf}}(r) &= br, & g &= \frac{4}{3}\alpha_s, \end{aligned} \quad (3)$$

where $\mathbf{p} = -i\nabla$, and g and b are the parameters of our model. The values of these parameters are fixed by comparing the predictions for the masses of states with known experimental values [9].

The potential energy $U_g(r)$ of the c and $\bar{s}$ interaction is the zero component of a Lorentz vector, and $U_{\text{conf}}(r)$ is a Lorentz scalar [3]. This is why $U_g(r)$ and $U_{\text{conf}}(r)$ enter differently into the spin-dependent relativistic correction H_S to the Hamiltonian,

$$\begin{aligned} H_S &= \frac{1}{2} \left(\frac{g}{r^3} - \frac{b}{r} \right) \left(\frac{[\mathbf{r} \times \pi_{\bar{s}}] \cdot \mathbf{S}_{\bar{s}}}{m_s^2} - \frac{[\mathbf{r} \times \pi_c] \cdot \mathbf{S}_c}{m_c^2} \right) \\ &\quad - \frac{g}{m_s m_c r^3} ([\mathbf{r} \times \pi_c] \cdot \mathbf{S}_{\bar{s}} - [\mathbf{r} \times \pi_{\bar{s}}] \cdot \mathbf{S}_c) \\ &\quad + \frac{g}{m_s m_c} \left[\frac{3(\mathbf{n} \cdot \mathbf{S}_{\text{tot}})^2 - \mathbf{S}_{\text{tot}}^2}{2r^3} + \frac{4\pi}{3} \delta(r) \left(\mathbf{S}_{\text{tot}}^2 - \frac{3}{2} \right) \right], \end{aligned} \quad (4)$$

where $\mathbf{n} = \mathbf{r}/r$. Recall that $\mathbf{S}_{\text{tot}}^2 - 3/2 = 2\mathbf{S}_{\bar{s}} \cdot \mathbf{S}_c$. Finally, there are two more corrections to the Hamiltonian that do not explicitly include the potentials $U_g(r)$ and $U_{\text{conf}}(r)$. The first correction H_M is related to the interaction of the magnetic field with the magnetic moments of quarks,

$$H_M = -\frac{e_{\bar{s}}}{m_s} \mathbf{S}_{\bar{s}} \cdot \mathcal{H}(\mathbf{r}_{\bar{s}}, t) - \frac{e_c}{m_c} \mathbf{S}_c \cdot \mathcal{H}(\mathbf{r}_c, t). \quad (5)$$

The second correction H_T is related to the Thomas precession, which occurs due to the acceleration of a particle in the electric field of radiation,

$$\begin{aligned} H_T &= -\frac{e_{\bar{s}}}{4m_s^2} ([\pi_{\bar{s}} \times \mathcal{E}(\mathbf{r}_{\bar{s}}, t)] - [\mathcal{E}(\mathbf{r}_{\bar{s}}, t) \times \pi_{\bar{s}}]) \cdot \mathbf{S}_{\bar{s}} \\ &\quad - \frac{e_c}{4m_c^2} ([\pi_c \times \mathcal{E}(\mathbf{r}_c, t)] - [\mathcal{E}(\mathbf{r}_c, t) \times \pi_c]) \cdot \mathbf{S}_c. \end{aligned} \quad (6)$$

For $U_{\text{conf}}(r) = 0$, the Hamiltonian of a system with different masses and charges in a radiation field can be found in Refs. [10,11]. The contribution of $U_{\text{conf}}(r)$ to the spin-dependent part of the Hamiltonian is proportional to the constant b and is contained only in the correction H_S .

Note that the Hamiltonians H_0 , H_S , H_M , and H_T are written in the gauge $A^0(\mathbf{r}, t) = 0$, $\text{div}\mathbf{A}(\mathbf{r}, t) = 0$. Since $\pi_{\bar{s}} = m_s \mathbf{v}_{\bar{s}}$ and $\pi_c = m_c \mathbf{v}_c$, where $\mathbf{v}_{\bar{s}}$ and $\mathbf{v}_c$ are velocities, the operators $\pi_{\bar{s}}$ and π_c are gauge invariant. Therefore, the Hamiltonians H_0 , H_S , H_M , and H_T are expressed via the gauge invariant operators $\mathcal{E}(\mathbf{r}, t)$, $\mathcal{H}(\mathbf{r}, t)$, $\pi_{\bar{s}}$, and π_c . Thus, transition amplitudes following from these Hamiltonians are gauge invariant.

We now have everything we need to study both the structure of p -wave mesons and their radiative transitions.

III. STRUCTURE OF D_{s1} MESONS

We set $\mathbf{A}(\mathbf{r}, t) = 0$ and obtain the Hamiltonian describing the fine structure of the $c\bar{s}$ system with $L = 1$,

$$\begin{aligned}
H &= H_{nr} + \Delta H_{nr} + \Delta H_S, \\
H_{nr} &= \frac{\mathbf{p}^2}{2M_R} - \frac{g}{r} + br, \quad \Delta H_{nr} = -\frac{(\mathbf{p}^2)^2}{8m_s^3} - \frac{(\mathbf{p}^2)^2}{8m_c^3}, \\
\Delta H_S &= \frac{1}{4} \left(\frac{g}{r^3} - \frac{b}{r} \right) \mathbf{L} \cdot \left[\left(\frac{1}{m_s^2} + \frac{1}{m_c^2} \right) \mathbf{S}_{\text{tot}} + \left(\frac{1}{m_s^2} - \frac{1}{m_c^2} \right) \mathbf{\Sigma} \right] \\
&\quad + \frac{g}{m_s m_c r^3} \mathbf{L} \cdot \mathbf{S}_{\text{tot}} + \frac{g}{2m_s m_c r^3} [3(\mathbf{n} \cdot \mathbf{S}_{\text{tot}})^2 - \mathbf{S}_{\text{tot}}^2]. \quad (7)
\end{aligned}$$

Recall that $\mathbf{p}^2 = p_r^2 + \mathbf{L}^2/r^2$, where $-p_r^2$ is the radial part of the Laplacian. The Hamiltonian (7) coincides with the Hamiltonian in the model [3], with one difference—in our model, only the first correction ΔH_{nr} to the kinetic energy is taken into account. We have checked that such a replacement does not significantly affect the numerical results, but essentially simplifies the calculations.

The eigenfunctions of the Hamiltonian with $J = 1$, $L = 1$, and $J_z = \mu$, Ψ_1 , and Ψ_2 are a linear combination of the functions ψ_1 and ψ_2 ,

$$\begin{aligned}
\Psi_1 &= c_1 \psi_1 + c_2 \psi_2, \quad \Psi_2 = c_2 \psi_1 - c_1 \psi_2, \\
\psi_1 &= Y_{1\mu}(\theta, \varphi) |S=0, S_z=0\rangle R_1(r), \\
\psi_2 &= \sum_{\mu'} C_{1\mu', 1(\mu-\mu')}^{1\mu} Y_{1\mu'}(\theta, \varphi) |S=1, S_z=\mu-\mu'\rangle R_1(r), \quad (8)
\end{aligned}$$

where $Y_{1\mu}(\theta, \varphi)$ is the spherical function, $C_{1\mu', 1(\mu-\mu')}^{1\mu}$ are the Clebsch-Gordan coefficients, and $R_1(r)$ is the radial wave function with the angular momentum $L = 1$. The corresponding energy and the coefficients $c_{1,2}$ are found from the solution of the secular equation [12].

The matrix elements of the operator ΔH_S read

$$\begin{aligned}
(\Delta H_S)_{11} &= 0, \\
(\Delta H_S)_{22} &= -\frac{1}{4} \left(\frac{1}{m_s^2} + \frac{1}{m_c^2} \right) \left(\left\langle \frac{g}{r^3} \right\rangle - \left\langle \frac{b}{r} \right\rangle \right) - \frac{1}{2m_s m_c} \left\langle \frac{g}{r^3} \right\rangle, \\
(\Delta H_S)_{21} &= (\Delta H_S)_{12} = \frac{1}{2\sqrt{2}} \left(\frac{1}{m_s^2} - \frac{1}{m_c^2} \right) \left(\left\langle \frac{g}{r^3} \right\rangle - \left\langle \frac{b}{r} \right\rangle \right). \quad (9)
\end{aligned}$$

Let Ψ_1 correspond to the lower energy $E_1 = E_1^{(0)} + \Delta E_1$ with $\Delta E_1 < 0$, and Ψ_2 to the higher energy $E_2 = E_1^{(0)} + \Delta E_2$ with $E_2 > 0$, where $E_1^{(0)}$ is the eigenvalue of the Hamiltonian H_{nr} . The state Ψ_1 corresponds to the $D_{s1}(2460)$ meson, and Ψ_2 to the $D_{s1}(2536)$ meson. Solving the secular equation, we obtain

$$\begin{aligned}
\Delta E_1 &= \frac{(\Delta H_S)_{22}}{2} - \gamma, \quad \Delta E_2 = \frac{(\Delta H_S)_{22}}{2} + \gamma, \\
\gamma &= \sqrt{\frac{(\Delta H_S)_{22}^2}{4} + (\Delta H_S)_{12}^2}, \\
c_1 &= -\text{sgn}(\Delta H_S)_{12} \sqrt{\frac{\Delta E_2}{2\gamma}}, \quad c_2 = \sqrt{\frac{|\Delta E_1|}{2\gamma}}. \quad (10)
\end{aligned}$$

To calculate the matrix elements of radiative transitions, we need to know the wave functions not only of D_{s1} mesons, but also of D_s mesons with $L = 0$, $S_{\text{tot}} = 0$, and $J = 0$. We use the wave functions calculated by means of the variational principle [12]. For states with $L = 0$, we write the radial wave function $R_0(r)$ as

$$R_0(r) = \sqrt{N_0} \exp(-\omega_0^2 r^2/2), \quad N_0 = \frac{4}{\sqrt{\pi}} \omega_0^3, \quad (11)$$

where ω_0 is a variational parameter. Then,

$$\begin{aligned}
E_0^{(0)} &= \frac{3\omega_0^2}{4M_R} + \frac{2}{\sqrt{\pi}} \left(\frac{b}{\omega_0} - g\omega_0 \right), \\
\frac{\partial}{\partial \omega_0} E_0^{(0)} &= \frac{3\omega_0}{2M_R} - \frac{2}{\sqrt{\pi}} \left(\frac{b}{\omega_0^2} + g \right) = 0. \quad (12)
\end{aligned}$$

Here, $E_0^{(0)}$ is the average value of the Hamiltonian H_{nr} over the wave function (11) at $L = 0$. Similarly, for $L = 1$, the variational radial wave function $R_1(r)$ has the form

$$R_1(r) = \sqrt{N_1} r \exp(-\omega_1^2 r^2/2), \quad N_1 = \frac{8}{3\sqrt{\pi}} \omega_1^5, \quad (13)$$

where the variational parameter ω_1 is found from the relations

$$\begin{aligned}
E_1^{(0)} &= \frac{5\omega_1^2}{4M_R} + \frac{4}{3\sqrt{\pi}} \left(\frac{2b}{\omega_1} - g\omega_1 \right), \\
\frac{\partial}{\partial \omega_1} E_1^{(0)} &= \frac{5\omega_1}{2M_R} - \frac{4}{3\sqrt{\pi}} \left(\frac{2b}{\omega_1^2} + g \right) = 0. \quad (14)
\end{aligned}$$

In these formulas, ω_0 and ω_1 are variational parameters that are found by minimizing the corresponding energies. For the average values in the matrix elements that determine the mixing coefficients of the functions ψ_1 and ψ_2 , we obtain

$$\int_0^\infty \frac{R_1^2(r)}{r^3} dr = \frac{4}{3\sqrt{\pi}} \omega_1^3, \quad \int_0^\infty \frac{R_1^2(r)}{r} dr = \frac{4}{3\sqrt{\pi}} \omega_1. \quad (15)$$

As a result, for the matrix elements, we have

$$\begin{aligned}
(\Delta H_S)_{11} &= 0, \\
(\Delta H_S)_{22} &= \frac{1}{3\sqrt{\pi}} \left[(b\omega_1 - g\omega_1^3) \left(\frac{1}{m_s^2} + \frac{1}{m_c^2} \right) - \frac{2g}{m_s m_c} \omega_1^3 \right], \\
(\Delta H_S)_{12} &= (\Delta H_S)_{21} = -\frac{\sqrt{2}}{3\sqrt{\pi}} (b\omega_1 - g\omega_1^3) \left(\frac{1}{m_s^2} - \frac{1}{m_c^2} \right).
\end{aligned} \tag{16}$$

Note that the spin-independent relativistic corrections,

$$(\Delta H_{nr})_{11} = (\Delta H_{nr})_{22} = -\frac{35\omega_1^4}{32} \left(\frac{1}{m_s^3} + \frac{1}{m_c^3} \right), \tag{17}$$

do not contribute to the energy difference $E_2 - E_1$. The corresponding shift of energy for the state with $L = 0$ is

$$(\Delta H_{nr})_{00} = -\frac{15\omega_0^4}{32} \left(\frac{1}{m_s^3} + \frac{1}{m_c^3} \right). \tag{18}$$

In addition, for $L = 0$, it is necessary to take into account the correction

$$(\Delta H_S)_{00} = -\frac{2g\omega_0^3}{\sqrt{\pi} m_s m_c}. \tag{19}$$

These shifts must be taken into account when comparing the prediction for energies of transitions with the experimental values.

IV. RADIATIVE TRANSITIONS OF D_{s1} MESONS

To derive the Hamiltonian H_{rad} corresponding to the emission of one photon, it is necessary to keep in H_0 , H_S , H_M , and H_T the terms linear in $A(\mathbf{r}, t)$. Let us represent H_{rad} as follows:

$$H_{\text{rad}} = G_0 + \frac{1}{2} \mathbf{S}_{\text{tot}} \cdot \mathbf{G}_{\text{tot}} + \frac{1}{2} \mathbf{\Sigma} \cdot \mathbf{G}_{\Sigma}, \tag{20}$$

where

$$\begin{aligned}
G_0 &= -(\mathbf{A}_W \cdot \mathbf{p}) F_1 + \frac{1}{4} \{ \mathbf{p}^2, F_3 \} (\mathbf{A}_W \cdot \mathbf{p}), \\
\mathbf{G}_{\text{tot}} &= -[\mathbf{r} \times \mathbf{A}_W] \left[\frac{1}{2} \left(\frac{g}{r^3} - \frac{b}{r} \right) f_2 + \frac{g}{m_s m_c r^3} f_0 \right] \\
&\quad + i[\mathbf{k} \times \mathbf{A}_W] f_1 + \frac{i\omega}{4} \{ [\mathbf{p} \times \mathbf{A}_W], f_2 \}, \\
\mathbf{G}_{\Sigma} &= [\mathbf{r} \times \mathbf{A}_W] \left[-\frac{1}{2} \left(\frac{g}{r^3} - \frac{b}{r} \right) F_2 + \frac{g}{m_s m_c r^3} F_0 \right] \\
&\quad + i[\mathbf{k} \times \mathbf{A}_W] F_1 + \frac{i\omega}{4} \{ [\mathbf{p} \times \mathbf{A}_W], F_2 \}.
\end{aligned} \tag{21}$$

Here,

$$\begin{aligned}
f_0 &= e_{\bar{s}} e^{-iq_{\bar{s}} \cdot \mathbf{r}} - e_c e^{iq_c \cdot \mathbf{r}}, & f_1 &= \frac{e_{\bar{s}}}{m_s} e^{-iq_{\bar{s}} \cdot \mathbf{r}} + \frac{e_c}{m_c} e^{iq_c \cdot \mathbf{r}}, \\
f_2 &= \frac{e_{\bar{s}}}{m_s^2} e^{-iq_{\bar{s}} \cdot \mathbf{r}} - \frac{e_c}{m_c^2} e^{iq_c \cdot \mathbf{r}}, & F_0 &= e_{\bar{s}} e^{-iq_{\bar{s}} \cdot \mathbf{r}} + e_c e^{iq_c \cdot \mathbf{r}}, \\
F_1 &= \frac{e_{\bar{s}}}{m_s} e^{-iq_{\bar{s}} \cdot \mathbf{r}} - \frac{e_c}{m_c} e^{iq_c \cdot \mathbf{r}}, & F_2 &= \frac{e_{\bar{s}}}{m_s^2} e^{-iq_{\bar{s}} \cdot \mathbf{r}} + \frac{e_c}{m_c^2} e^{iq_c \cdot \mathbf{r}}, \\
F_3 &= \frac{e_{\bar{s}}}{m_s^3} e^{-iq_{\bar{s}} \cdot \mathbf{r}} - \frac{e_c}{m_c^3} e^{iq_c \cdot \mathbf{r}}, & \mathbf{q}_{\bar{s}} &= \frac{M_R}{m_s} \mathbf{k}, \quad \mathbf{q}_c = \frac{M_R}{m_c} \mathbf{k},
\end{aligned} \tag{22}$$

and $\{A, B\} = AB + BA$.

A. Radiative transitions of D_{s1} mesons to $D_s \gamma$

The state of the D_s meson is described by the wave function ψ_0 ,

$$\psi_0 = Y_{00} |S=0, S_z=0\rangle R_0(r). \tag{23}$$

To calculate the amplitudes of radiative transitions from the states $\Psi_{1,2}$ to the state ψ_0 , it is necessary to calculate two matrix elements,

$$T_1 = \langle \psi_0 | H_{\text{rad}} | \psi_1 \rangle, \quad T_2 = \langle \psi_0 | H_{\text{rad}} | \psi_2 \rangle. \tag{24}$$

In the matrix element T_1 , the only nonzero contribution comes from the term G_0 in Eq. (20),

$$T_1 = i \frac{\sqrt{3}}{4\pi} \int d^3 r R_0 G_0 R_1 (\mathbf{e}_{\mu} \cdot \mathbf{n}), \quad \mu = J_z, \tag{25}$$

where the following relations are used:

$$\begin{aligned}
Y_{1\mu} &= i \sqrt{\frac{3}{4\pi}} (\mathbf{e}_{\mu} \cdot \mathbf{n}), & Y_{00} &= \frac{1}{\sqrt{4\pi}}, & \mathbf{n} &= \mathbf{r}/r, \\
\mathbf{e}_0 &= \mathbf{e}_z, & \mathbf{e}_1 &= -\frac{(\mathbf{e}_x + i\mathbf{e}_y)}{\sqrt{2}}, & \mathbf{e}_{-1} &= \frac{(\mathbf{e}_x - i\mathbf{e}_y)}{\sqrt{2}}.
\end{aligned} \tag{26}$$

In T_2 , the only nonzero contribution comes from the term $\mathbf{\Sigma} \cdot \mathbf{G}_{\Sigma}/2$ in Eq. (20). After simple transformations, we arrive at the result

$$T_2 = \frac{\sqrt{3}}{8\pi\sqrt{2}} \int d^3 r R_0 (\mathbf{e}_{\mu} \cdot [\mathbf{G}_{\Sigma} \times \mathbf{n}]) R_1. \tag{27}$$

Using the obtained wave functions, we find

$$\begin{aligned}
T_1 &= Z (\mathbf{A}_W \cdot \mathbf{e}_{\mu}) t_1, & T_2 &= Z (\mathbf{A}_W \cdot \mathbf{e}_{\mu}) t_2, \\
Z &= \frac{\omega_0 \omega_1^2 \sqrt{\omega_0 \omega_1}}{\bar{\omega}^4}, & \bar{\omega} &= \sqrt{\frac{\omega_0^2 + \omega_1^2}{2}}.
\end{aligned} \tag{28}$$

For the coefficient t_1 , the straightforward calculation gives $t_1 = t_{11} + t_{12}$, where

$$\begin{aligned}
t_{11} &= -\frac{\omega_0^2}{\sqrt{2}\bar{\omega}} \left(\frac{e_{\bar{s}}}{m_s} e^{-Q_s^2/4} - \frac{e_c}{m_c} e^{-Q_c^2/4} \right), \\
t_{12} &= \frac{5\omega_0^4\omega_1^2}{4\sqrt{2}\bar{\omega}^3} \left\{ \frac{e_{\bar{s}}}{m_s^3} e^{-Q_s^2/4} \left[1 + \frac{Q_s^2(\omega_0^4 + \omega_1^4)}{20\omega_0^2\omega_1^2} \right] \right. \\
&\quad \left. - \frac{e_c}{m_c^3} e^{-Q_c^2/4} \left[1 + \frac{Q_c^2(\omega_0^4 + \omega_1^4)}{20\omega_0^2\omega_1^2} \right] \right\}, \\
Q_s &= \frac{M_R k}{m_s \bar{\omega}}, \quad Q_c = \frac{M_R k}{m_c \bar{\omega}}. \quad (29)
\end{aligned}$$

Note that accounting for retardation, i.e., $e^{-iq_s \cdot r}$ and $e^{iq_c \cdot r}$ in the photon wave function, and compensation in the difference $e_{\bar{s}}/m_s - e_c/m_c$ lead to a decrease in the value of nonrelativistic contribution t_{11} by half. Therefore, it is important to take into account the relativistic corrections t_{12} and t_2 . For t_2 , we find $t_2 = t_{21} + t_{22} + t_{23}$, where

$$\begin{aligned}
t_{21} &= \frac{1}{\sqrt{\pi}} \int_0^\infty r e^{-r^2} \left\{ e_{\bar{s}} \left[\frac{(br^2 - g\bar{\omega}^2)}{m_s^2} + \frac{2g\bar{\omega}^2}{m_s m_c} \right] \mathcal{F}(Q_s r) \right. \\
&\quad \left. + e_c \left[\frac{(br^2 - g\bar{\omega}^2)}{m_c^2} + \frac{2g\bar{\omega}^2}{m_s m_c} \right] \mathcal{F}(Q_c r) \right\} dr, \\
\mathcal{F}(x) &= \frac{\sin x}{x} + \frac{\cos x}{x^2} - \frac{\sin x}{x^3}, \\
t_{22} &= \frac{k}{4} \left(\frac{e_{\bar{s}}}{m_s} Q_s e^{-Q_s^2/4} + \frac{e_c}{m_c} Q_c e^{-Q_c^2/4} \right), \\
t_{23} &= \frac{k}{16\bar{\omega}} \left[4\omega_0^2 \left(\frac{e_{\bar{s}}}{m_s^2} e^{-Q_s^2/4} + \frac{e_c}{m_c^2} e^{-Q_c^2/4} \right) \right. \\
&\quad \left. + \bar{\omega}^2 \left(\frac{e_{\bar{s}}}{m_s^2} Q_s^2 e^{-Q_s^2/4} + \frac{e_c}{m_c^2} Q_c^2 e^{-Q_c^2/4} \right) \right]. \quad (30)
\end{aligned}$$

As a result, we obtain the transition probabilities,

$$\begin{aligned}
\Gamma(D_{s1}(2460) \rightarrow D_s \gamma) &= \frac{4}{3} k_1 Z^2 (c_1 t_1 + c_2 t_2)^2, \\
\Gamma(D_{s1}(2536) \rightarrow D_s \gamma) &= \frac{4}{3} k_2 Z^2 (c_2 t_1 - c_1 t_2)^2. \quad (31)
\end{aligned}$$

When calculating these quantities, it is necessary to substitute the corresponding photon energies,

$$\begin{aligned}
k_1 &= k_{10} - \frac{k_{10}^2}{2M_0}, \quad k_2 = k_{20} - \frac{k_{20}^2}{2M_0}, \\
k_{10} &= [E_1^{(0)} + \Delta E_1 + (\Delta H_{nr})_{11}] \\
&\quad - [E_0^{(0)} + (\Delta H_{nr})_{00} + (\Delta H_s)_{00}], \\
k_{20} &= k_1 + \Delta E_2 - \Delta E_1. \quad (32)
\end{aligned}$$

In these formulas, the recoil energies are taken into account, and M_0 is the mass of the D_s meson. Explicit expressions for all the quantities included here are given in Eqs. (10), (12), (14), and (16)–(19).

For numerical estimates, we use the values of parameters

$$\begin{aligned}
m_s &= 0.5 \text{ GeV}, \quad m_c = 1.7 \text{ GeV}, \\
b &= 0.18 \text{ GeV}^2, \quad g = 0.8.
\end{aligned}$$

With these parameters, we have

$$\begin{aligned}
\omega_0 &= 0.47 \text{ GeV}, \quad \omega_1 = 0.38 \text{ GeV}, \\
c_1 &= 0.71, \quad c_2 = 0.71, \quad \varphi = \arctan(c_2/c_1) = 45^\circ, \\
M(D_s^*) - M(D_s) &= 147 \text{ MeV}, \\
M(D_{s1}(2536)) - M(D_{s1}(2460)) &= 50 \text{ MeV}, \\
M(D_{s1}(2460)) - M(D_s) &= 600 \text{ MeV}. \quad (33)
\end{aligned}$$

For the transitions $D_{s1}(2460) \rightarrow D_s \gamma$ and $D_{s1}(2536) \rightarrow D_s \gamma$, we find from (31)

$$\begin{aligned}
\Gamma(D_{s1}(2460) \rightarrow D_s \gamma) &= 297 \text{ KeV}, \\
\Gamma(D_{s1}(2536) \rightarrow D_s \gamma) &= 12 \text{ KeV}. \quad (34)
\end{aligned}$$

This result answers the question of [7] on why the decay $D_{s1}(2536) \rightarrow D_s \gamma$ is not observed, in contrast to the decay $D_{s1}(2460) \rightarrow D_s \gamma$.

In the nonrelativistic approximation, only the contribution t_{11} , Eq. (29), should be taken into account. In this case,

$$\begin{aligned}
\Gamma(D_{s1}(2460) \rightarrow D_s \gamma) &= 4.9 \text{ KeV}, \\
\Gamma(D_{s1}(2536) \rightarrow D_s \gamma) &= 5.2 \text{ KeV}. \quad (35)
\end{aligned}$$

B. Radiative transitions of D_{s1} mesons to $D_s^* \gamma$

The state of the D_s^* meson is described by the wave function ψ'_0 ,

$$\psi'_0 = Y_{00} |S=1, S_z=\mu'\rangle R_0(r). \quad (36)$$

We need to calculate two matrix elements,

$$T'_1 = \langle \psi'_0 | H_{\text{rad}} | \psi_1 \rangle, \quad T'_2 = \langle \psi'_0 | H_{\text{rad}} | \psi_2 \rangle. \quad (37)$$

The nonzero contribution to the matrix element T'_1 comes from the term $\mathbf{\Sigma} \cdot \mathbf{G}_\Sigma/2$ in Eq. (20),

$$T'_1 = i \frac{\sqrt{3}}{8\pi} \int d^3 r R_0 (\mathbf{G}_\Sigma \cdot \mathbf{e}_{\mu'}^*) (\mathbf{n} \cdot \mathbf{e}_\mu) R_1. \quad (38)$$

The nonzero contributions to T'_2 come from the terms G_0 and $\mathbf{S}_{\text{tot}} \cdot \mathbf{G}_{\text{tot}}/2$,

$$\begin{aligned}
T'_2 &= -\frac{\sqrt{3}}{4\pi\sqrt{2}} \int d^3 r R_0 G_0 ([\mathbf{e}_{\mu'}^* \times \mathbf{e}_\mu] \cdot \mathbf{n}) R_1, \\
&\quad - i \frac{\sqrt{3}}{8\pi\sqrt{2}} \int d^3 r R_0 ([\mathbf{G}_{\text{tot}} \times \mathbf{e}_{\mu'}^*] \cdot [\mathbf{n} \times \mathbf{e}_\mu]) R_1. \quad (39)
\end{aligned}$$

Calculating the matrix elements yields the following expression for the amplitude T'_1 :

$$T'_1 = iZ\{(\mathbf{A}_W \cdot [\mathbf{e}_{\mu'}^* \times \mathbf{e}_\mu])\tau_{11} + (\mathbf{A}_W \cdot [\mathbf{e}_{\mu'}^* \times \boldsymbol{\lambda}]) (\mathbf{e}_\mu \cdot \boldsymbol{\lambda})\tau_{12}\}, \quad (40)$$

where $\boldsymbol{\lambda} = \mathbf{k}/k$, Z is defined in (28), and the coefficients τ_{11} and τ_{12} are

$$\begin{aligned} \tau_{11} = & \frac{\sqrt{2}}{\sqrt{\pi}} \int_0^\infty r e^{-r^2} \left\{ e_{\bar{s}} \left[\frac{(br^2 - g\bar{\omega}^2)}{m_s^2} + \frac{2g\bar{\omega}^2}{m_s m_c} \right] \mathcal{F}_1(Q_s r) \right. \\ & + e_c \left[\frac{(br^2 - g\bar{\omega}^2)}{m_c^2} + \frac{2g\bar{\omega}^2}{m_s m_c} \right] \mathcal{F}_1(Q_c r) \Big\} dr \\ & + \frac{k\omega_0^2}{4\sqrt{2}\bar{\omega}} \left(\frac{e_{\bar{s}}}{m_s^2} e^{-Q_s^2/4} + \frac{e_c}{m_c^2} e^{-Q_c^2/4} \right), \\ \mathcal{F}_1(x) = & \frac{\sin x}{x^3} - \frac{\cos x}{x^2}, \end{aligned} \quad (41)$$

$$\begin{aligned} \tau_{12} = & \frac{\sqrt{2}}{\sqrt{\pi}} \int_0^\infty r e^{-r^2} \left\{ e_{\bar{s}} \left[\frac{(br^2 - g\bar{\omega}^2)}{m_s^2} + \frac{2g\bar{\omega}^2}{m_s m_c} \right] \mathcal{F}_2(Q_s r) \right. \\ & + e_c \left[\frac{(br^2 - g\bar{\omega}^2)}{m_c^2} + \frac{2g\bar{\omega}^2}{m_s m_c} \right] \mathcal{F}_2(Q_c r) \Big\} dr \\ & + \frac{M_R k^2}{2\sqrt{2}\bar{\omega}} \left(\frac{e_{\bar{s}}}{m_s^2} e^{-Q_s^2/4} + \frac{e_c}{m_c^2} e^{-Q_c^2/4} \right) \\ & + \frac{M_R^2 k^3}{8\sqrt{2}\bar{\omega}} \left(1 - \frac{\omega_0^2}{\bar{\omega}^2} \right) \left(\frac{e_{\bar{s}}}{m_s^4} e^{-Q_s^2/4} + \frac{e_c}{m_c^4} e^{-Q_c^2/4} \right), \\ \mathcal{F}_2(x) = & \frac{\sin x}{x} - \frac{3 \sin x}{x^3} + \frac{3 \cos x}{x^2}. \end{aligned} \quad (42)$$

The amplitude T'_2 reads

$$T'_2 = iZ\{(\mathbf{A}_W \cdot [\mathbf{e}_{\mu'}^* \times \mathbf{e}_\mu])\tau_{21} + (\mathbf{A}_W \cdot [\mathbf{e}_\mu \times \boldsymbol{\lambda}]) (\mathbf{e}_{\mu'}^* \cdot \boldsymbol{\lambda})\tau_{22}\}, \quad (43)$$

where the coefficients τ_{21} and τ_{22} are

$$\begin{aligned} \tau_{21} = & \frac{1}{\sqrt{\pi}} \int_0^\infty r e^{-r^2} \left\{ e_{\bar{s}} \left[-\frac{(br^2 - g\bar{\omega}^2)}{m_s^2} + \frac{2g\bar{\omega}^2}{m_s m_c} \right] \mathcal{F}_1(Q_s r) \right. \\ & - e_c \left[-\frac{(br^2 - g\bar{\omega}^2)}{m_c^2} + \frac{2g\bar{\omega}^2}{m_s m_c} \right] \mathcal{F}_1(Q_c r) \Big\} dr - \frac{k\omega_0^2}{8\bar{\omega}} \left(\frac{e_{\bar{s}}}{m_s^2} e^{-Q_s^2/4} - \frac{e_c}{m_c^2} e^{-Q_c^2/4} \right) \\ & - \frac{\omega_0^2}{2\bar{\omega}} \left(\frac{e_{\bar{s}}}{m_s} e^{-Q_s^2/4} - \frac{e_c}{m_c} e^{-Q_c^2/4} \right) + \frac{5\omega_0^4 \omega_1^2}{8\bar{\omega}^3 m_s^3} e_{\bar{s}} e^{-Q_s^2/4} \left[1 + \frac{Q_s^2(\omega_0^4 + \omega_1^4)}{20\omega_0^2 \omega_1^2} \right] \\ & - \frac{5\omega_0^4 \omega_1^2}{8\bar{\omega}^3 m_c^3} e_c e^{-Q_c^2/4} \left[1 + \frac{Q_c^2(\omega_0^4 + \omega_1^4)}{20\omega_0^2 \omega_1^2} \right], \end{aligned} \quad (44)$$

$$\begin{aligned} \tau_{22} = & -\frac{1}{\sqrt{\pi}} \int_0^\infty r e^{-r^2} \left\{ e_{\bar{s}} \left[-\frac{(br^2 - g\bar{\omega}^2)}{m_s^2} + \frac{2g\bar{\omega}^2}{m_s m_c} \right] \mathcal{F}_2(Q_s r) \right. \\ & - e_c \left[-\frac{(br^2 - g\bar{\omega}^2)}{m_c^2} + \frac{2g\bar{\omega}^2}{m_s m_c} \right] \mathcal{F}_2(Q_c r) \Big\} dr + \frac{M_R k^2}{4\bar{\omega}} \left(\frac{e_{\bar{s}}}{m_s^2} e^{-Q_s^2/4} - \frac{e_c}{m_c^2} e^{-Q_c^2/4} \right) \\ & + \frac{M_R k^2}{16} \left(1 - \frac{\omega_0^2}{\bar{\omega}^2} \right) \left(\frac{e_{\bar{s}}}{m_s^3} e^{-Q_s^2/4} - \frac{e_c}{m_c^3} e^{-Q_c^2/4} \right). \end{aligned} \quad (45)$$

The probabilities of radiative transitions are of the forms

$$\begin{aligned} \Gamma(D_{s1}(2460) \rightarrow D_s^* \gamma) = & \frac{4}{3} k_1 Z^2 \{ [c_1 \tau_{11} + c_2 (\tau_{21} - \tau_{22})]^2 \\ & + [c_1 (\tau_{11} + \tau_{12}) + c_2 \tau_{21}]^2 \}, \\ \Gamma(D_{s1}(2536) \rightarrow D_s^* \gamma) = & \frac{4}{3} k_2 Z^2 \{ [-c_2 \tau_{11} + c_1 (\tau_{21} - \tau_{22})]^2 \\ & + [-c_2 (\tau_{11} + \tau_{12}) + c_1 \tau_{21}]^2 \}. \end{aligned} \quad (46)$$

The photon momenta in radiative transitions are given in Eq. (32) with $(\Delta H)_{00}$ replaced by

$$(\Delta H)_{01} = \frac{2g\omega_0^3}{3\sqrt{\pi} m_s m_c}. \quad (47)$$

For numerical estimates, we use the same parameters as before. As a result, we obtain

$$\begin{aligned} \Gamma(D_{s1}(2460) \rightarrow D_s^* \gamma) &= 104 \text{ KeV}, \\ \Gamma(D_{s1}(2536) \rightarrow D_s^* \gamma) &= 29 \text{ KeV}. \end{aligned} \quad (48)$$

In the nonrelativistic approximation,

$$\begin{aligned} \Gamma(D_{s1}(2460) \rightarrow D_s^* \gamma) &= 7.1 \text{ KeV}, \\ \Gamma(D_{s1}(2536) \rightarrow D_s^* \gamma) &= 7.4 \text{ KeV}. \end{aligned} \quad (49)$$

Thus, the importance of relativistic corrections is obvious.

C. Comparison with previous predictions

Numerous theoretical works [13–18] are devoted to calculation of radiative decay partial widths of D_{s1} mesons. The results of these studies, carried out within the framework of potential models, are presented in Table I together with the predictions of our work. It is evident that in all previous studies, the predictions for the width $\Gamma(D_{s1}(2536) \rightarrow D_s \gamma)$ are larger than that for $\Gamma(D_{s1}(2460) \rightarrow D_s \gamma)$, and the decay width $\Gamma(D_{s1}(2460) \rightarrow D_s^* \gamma)$ systematically exceeds $\Gamma(D_{s1}(2460) \rightarrow D_s \gamma)$, which significantly contradicts the experimental data [9]. Apparently, this is due to the fact that in all previous works, the spin-dependent contributions to the radiative transition amplitudes have been neglected. Therefore, the ratio of transition probabilities to D_s is determined only by the mixing angle, which is of the order of forty degrees. As can be seen from our calculations, the spin-independent contribution to the transition amplitudes is strongly suppressed. As a result, the contributions of relativistic corrections, dependent on quark spins and independent of them, become very important. As it turns out, the relativistic corrections to the amplitudes of radiative transitions are of the same order as the contribution of the nonrelativistic dipole moment operator. Since in the decay amplitude $D_{s1}(2536) \rightarrow D_s \gamma$ the corresponding contributions enter with opposite signs (46), a strong compensation occurs in the corresponding decay width. At the same time, an amplification takes place in the decay amplitude $D_{s1}(2460) \rightarrow D_s \gamma$. A similar phenomenon occurs in transitions to $D_s^* \gamma$.

Of course, our predictions are model-dependent, and the numerical values of widths are sensitive to such parameters as the quark masses and the constants in the interaction potential. However, at our values of the parameters, we reproduce well the experimentally known mass differences of D_{s1} , D_s , and D_s^* mesons. The sensitivity of radiative widths to small changes in model parameters is more significant. Since $\Gamma(D_{s1}(2536) \rightarrow D_s \gamma)$ turned out to be very small as a result of strong compensation, we believe that this value lies in the range from zero to 15 KeV. At the same time, we estimate the accuracy of the remaining predictions at 50%. Note that such a variation of widths does not significantly affect their ratios. Our statement, that the decay width of $D_{s1}(2460) \rightarrow D_s \gamma$ transition is much larger than that for $D_{s1}(2536) \rightarrow D_s \gamma$, is stable with respect to variations in the model parameters.

Since the decay probability of $D_{s1}(2460) \rightarrow D_s \gamma$ is known from the experiment, we can estimate the total width of the $D_{s1}(2460)$ meson, $\Gamma_{\text{tot}}(D_{s1}(2460)) \sim 1.5$ MeV. Therefore, measuring the total width of $D_{s1}(2460)$ is an important check of our estimates. In addition, measuring the ratio $\Gamma(D_{s1}(2460) \rightarrow D_s^* \gamma) / \Gamma(D_{s1}(2460) \rightarrow D_s \gamma)$ will also be a test of the model. This ratio is much less dependent on the choice of parameters than the partial widths and is easier to measure.

V. RADIATIVE TRANSITIONS OF B_{s1} AND B_{c1} MESONS

Using the obtained formulas, we can predict the partial widths for radiative transitions $B_{s1} \rightarrow B_s \gamma$, $B_{s1} \rightarrow B_s^* \gamma$, $B_{c1} \rightarrow B_c \gamma$, and $B_{c1} \rightarrow B_c^* \gamma$.

A. Radiative transitions $B_{s1} \rightarrow B_s \gamma$ and $B_{s1} \rightarrow B_s^* \gamma$

Let us make the substitutions $m_c \rightarrow m_b = 4.8$ GeV and $e_c \rightarrow e_b = -e/3$ in the formulas obtained for D_{s1} mesons. In addition, for B_{s1} mesons we use the value $g = 0.7$. The small decrease in the constant g is due to the fact that with the growth of heavy quark mass, the reduced mass M_R increases, which leads to a decrease in the characteristic size of the system. As a result, the strong interaction coupling constant α_s and, consequently, the constant g , decrease. Then, we have

$$\begin{aligned} \omega_0 &= 0.49 \text{ GeV}, & \omega_1 &= 0.40 \text{ GeV}, \\ c_1 &= 0.78, & c_2 &= 0.62, & \varphi &= \arctan(c_2/c_1) = 38.3^\circ, \\ M(B_s^*) - M(B_s) &= 52 \text{ MeV}, \\ M(B_{s1}(5829)) - M(B_{s1}(5748)) &= 59 \text{ MeV}, \\ M(B_{s1}(5748)) - M(B_s) &= 490 \text{ MeV}. \end{aligned} \quad (50)$$

For the transitions $B_{s1}(5748) \rightarrow B_s \gamma$ and $B_{s1}(5829) \rightarrow B_s \gamma$, we obtain

$$\begin{aligned} \Gamma(B_{s1}(5748) \rightarrow B_s \gamma) &= 47 \text{ KeV}, \\ \Gamma(B_{s1}(5829) \rightarrow B_s \gamma) &= 28 \text{ KeV}. \end{aligned} \quad (51)$$

For the transitions of B_{s1} mesons to $B_s^* \gamma$, we find

$$\begin{aligned} \Gamma(B_{s1}(5748) \rightarrow B_s^* \gamma) &= 20 \text{ KeV}, \\ \Gamma(B_{s1}(5829) \rightarrow B_s^* \gamma) &= 41 \text{ KeV}. \end{aligned} \quad (52)$$

In the nonrelativistic approximation, we have

$$\begin{aligned} \Gamma(B_{s1}(5748) \rightarrow B_s \gamma) &= 111 \text{ KeV}, \\ \Gamma(B_{s1}(5829) \rightarrow B_s \gamma) &= 70 \text{ KeV}, \\ \Gamma(B_{s1}(5748) \rightarrow B_s^* \gamma) &= 67 \text{ KeV}, \\ \Gamma(B_{s1}(5829) \rightarrow B_s^* \gamma) &= 5 \text{ KeV}. \end{aligned} \quad (53)$$

TABLE II. Theoretical predictions for radiative decay widths $B_{s1} \rightarrow B_s^{(*)} \gamma$ in KeV.

Publication	[19]	[20]	[21]	This work
$\Gamma(B_{s1}(5748) \rightarrow B_s \gamma)$	37	70.6	97.7	47
$\Gamma(B_{s1}(5748) \rightarrow B_s^* \gamma)$	56	36.9	39.5	20
$\Gamma(B_{s1}(5829) \rightarrow B_s \gamma)$	27	47.8	56.6	28
$\Gamma(B_{s1}(5829) \rightarrow B_s^* \gamma)$	53	57.3	98.8	41

TABLE III. Theoretical predictions for radiative decay widths $B_{c1} \rightarrow B_c^{(*)}\gamma$ in KeV.

Publication	[22]	[23]	[24]	[25]	[26]	[27]	[28]	[29]	This work
$\Gamma(B_{c1}(6743) \rightarrow B_c\gamma)$	13	18.4	32.4	11.6	16.6	35	30.1	9.9	22
$\Gamma(B_{c1}(6743) \rightarrow B_c^*\gamma)$	60	78.9	75.6	77.8	49	70	47.8	62.5	75
$\Gamma(B_{c1}(6750) \rightarrow B_c\gamma)$	80	132	127.8	131.1	66.6	74	64	92.3	47
$\Gamma(B_{c1}(6750) \rightarrow B_c^*\gamma)$	11	13.6	26.2	8.1	10.5	40	25.6	7.5	2

A comparison of our results with the predictions of other authors is given in Table II.

B. Radiative transitions $B_{c1} \rightarrow B_c\gamma$ and $B_{c1} \rightarrow B_c^*\gamma$

The expressions for probabilities of radiative transitions $B_{c1} \rightarrow B_c\gamma$ and $B_{c1} \rightarrow B_c^*\gamma$ are obtained from that for transitions $D_{s1} \rightarrow D_s\gamma$ and $D_{s1} \rightarrow D_s^*\gamma$ by substituting $m_c \rightarrow m_b = 4.8$ GeV, $m_s \rightarrow m_c = 1.7$ GeV, $e_c \rightarrow e_b = -e/3$, and $e_{\bar{s}} \rightarrow e_{\bar{c}} = -2e/3$. In addition, we use the value $g = 0.6$ for these transitions. As a result, we have

$$\begin{aligned}
\omega_0 &= 0.82 \text{ GeV}, & \omega_1 &= 0.60 \text{ GeV}, \\
c_1 &= -0.22, & c_2 &= 0.98, & \varphi &= \arctan(c_2/c_1) = -77^\circ, \\
M(B_c^*) - M(B_c) &= 61 \text{ MeV}, \\
M(B_{c1}(6750)) - M(B_{c1}(6743)) &= 8 \text{ MeV}, \\
M(B_{c1}(6743)) - M(B_c) &= 497 \text{ MeV}.
\end{aligned} \tag{54}$$

For transitions $B_{c1}(6743) \rightarrow B_c\gamma$ and $B_{c1}(6750) \rightarrow B_c\gamma$, we obtain

$$\begin{aligned}
\Gamma(B_{c1}(6743) \rightarrow B_c\gamma) &= 22 \text{ KeV}, \\
\Gamma(B_{c1}(6750) \rightarrow B_c\gamma) &= 47 \text{ KeV}.
\end{aligned} \tag{55}$$

Interestingly, for the $B_{c1} \rightarrow B_c\gamma$ transition, the probability of emission from the lower level is very small, compared to the probability of emission from the upper level.

For transition $B_{c1} \rightarrow B_c^*\gamma$, we find

$$\begin{aligned}
\Gamma(B_{c1}(6743) \rightarrow B_c^*\gamma) &= 75 \text{ KeV}, \\
\Gamma(B_{c1}(6750) \rightarrow B_c^*\gamma) &= 2 \text{ KeV}.
\end{aligned} \tag{56}$$

In the nonrelativistic approximation, we obtain

$$\begin{aligned}
\Gamma(B_{c1}(6743) \rightarrow B_c\gamma) &= 5 \text{ KeV}, \\
\Gamma(B_{c1}(6750) \rightarrow B_c\gamma) &= 107 \text{ KeV}, \\
\Gamma(B_{c1}(6743) \rightarrow B_c^*\gamma) &= 96 \text{ KeV}, \\
\Gamma(B_{c1}(6750) \rightarrow B_c^*\gamma) &= 5 \text{ KeV}.
\end{aligned} \tag{57}$$

A comparison of our results with the predictions of other authors is given in Table III.

Unfortunately, there are currently no experimental data for radiative decays of B_{s1} and B_{c1} mesons.

Let us make a remark on the mixing angle $\varphi = \arctan(c_2/c_1)$. If we let the mass of a heavy quark go to infinity, then φ will be either $-\arctan\sqrt{2} = -54.7^\circ$ or $\arctan(1/\sqrt{2}) = 35.3^\circ$ [5], depending on the ratio of the parameters g and b . For $g = 0.5$ and $b = 0$, it will be -54.7° , and for $g = 0$ and $b = 0.18$, it will be 35.3° . Therefore, it is seen that for all the cases considered, the heavy quark approximation works rather well only for the system $b\bar{s}$, where $m_b \gg m_s$.

VI. CONCLUSION

Within the phenomenological approach, predictions were obtained for the probabilities of radiative transitions $D_{s1} \rightarrow D_s\gamma$, $D_{s1} \rightarrow D_s^*\gamma$, $B_{s1} \rightarrow B_s\gamma$, $B_{s1} \rightarrow B_s^*\gamma$, $B_{c1} \rightarrow B_c\gamma$, and $B_{c1} \rightarrow B_c^*\gamma$. These predictions explained the unusual ratio between the widths of different radiative transitions of D_{s1} mesons noted in [7]. The effect appeared due to a large suppression of nonrelativistic contribution from the electric dipole moment of the $c\bar{s}$ system to the decay amplitude. As a result, the relativistic corrections to the radiation amplitude turned out to be dominant, and the amplitudes (24) became identical in sign and close in magnitude. Since the states $\Psi_{1,2}$ corresponding to D_{s1} mesons are superposition of $\psi_{1,2}$ states with the coefficients $c_{1,2}$ close in value, a large compensation of contributions occurred in the transition amplitude $\Gamma(D_{s1}(2536) \rightarrow D_s\gamma)$. As a result, the width $\Gamma(D_{s1}(2536) \rightarrow D_s\gamma)$ became strongly suppressed.

All obtained predictions were in qualitative agreement with the available experimental results and constraints. All predictions were obtained with one standard set of parameters under natural variation of the coupling constant g . Our predictions can be tested in future measurements at BESIII, BELLE-II, and LHCb Collaborations.

DATA AVAILABILITY

No data were created or analyzed in this study.

- [1] G. W. F. Drake, Theory of relativistic magnetic dipole transitions: Lifetime of the metastable 2^3S state of the heliumlike ions, *Phys. Rev. A* **3**, 908 (1971).
- [2] I. V. Obraztsov and A. I. Milstein, Quadrupole radiation and e^+e^- pair production in the collision of nonrelativistic nuclei, *Phys. Lett. B* **820**, 136514 (2021).
- [3] S. Godfrey and N. Isgur, Mesons in a relativized quark model with chromodynamics, *Phys. Rev. D* **32**, 189 (1985).
- [4] T. Barnes, S. Godfrey, and E. S. Swanson, Higher charmonia, *Phys. Rev. D* **72**, 054026 (2005).
- [5] T. Matsuki, T. Morii, and K. Seo, Mixing angle between $^3P_1$ and $^1P_1$ in HQET, *Prog. Theor. Phys.* **124**, 285 (2010).
- [6] Q. T. Song, D. Y. Chen, X. Liu, and T. Matsuki, Charmed-strange mesons revisited: Mass spectra and strong decays, *Phys. Rev. D* **91**, 054031 (2015).
- [7] A. Bondar, Why we do not see the radiative decays of the $D_{s1}^+(2536)$ meson?, *JETP Lett.* **121**, 231 (2025).
- [8] B. Aubert, R. Barate, M. Bona *et al.* (BABAR Collaboration), A study of the $D_{sJ}^*(2317)^+$ and $D_{sJ}(2460)^+$ mesons in inclusive $c\bar{c}$ production near $\sqrt{s} = 10.6$ GeV, *Phys. Rev. D* **74**, 032007 (2006).
- [9] S. Navas *et al.* (Particle Data Group), Review of particle physics, *Phys. Rev. D* **110**, 030001 (2024).
- [10] H. M. Pilkuhn, *Relativistic Particle Physics* (Springer-Verlag, New York, 1979).
- [11] R. N. Lee, A. I. Milstein, and M. Schumacher, Relativistic corrections to the electromagnetic polarizabilities of compound systems, *Phys. Rev. Lett.* **87**, 051601 (2001).
- [12] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics, Non-Relativistic Theory* (Pergamon Press, Oxford, 1977).
- [13] S. Godfrey, Properties of the charmed P -wave mesons, *Phys. Rev. D* **72**, 054029 (2005).
- [14] J. L. Goity and W. Roberts, Radiative transitions in heavy mesons in a relativistic quark model, *Phys. Rev. D* **64**, 094007 (2001).
- [15] N. Green, W. W. Repko, and S. F. Radford, Note on predictions for $c\bar{s}$ quarkonia using a three-loop static potential, *Nucl. Phys.* **A958**, 71 (2017).
- [16] S. F. Radford, W. W. Repko, and M. J. Saelim, Potential model calculations and predictions for $c\bar{s}$ quarkonia, *Phys. Rev. D* **80**, 034012 (2009).
- [17] S. F. Chen, J. Liu, H. Q. Zhou, and D. Y. Chen, Electric transitions of the charmed-strange mesons in a relativistic quark model, *Eur. Phys. J. C* **80**, 290 (2020).
- [18] J. G. Korner, D. Pirjol, and K. Schilcher, Radiative decays of the p wave charmed mesons, *Phys. Rev. D* **47**, 3955 (1993).
- [19] Q. Li, R. H. Ni, and X. H. Zhong, Towards establishing an abundant B and B_s spectrum up to the second orbital excitations, *Phys. Rev. D* **103**, 116010 (2021).
- [20] S. Godfrey, K. Moats, and E. S. Swanson, B and B_s meson spectroscopy, *Phys. Rev. D* **94**, 054025 (2016).
- [21] Q. F. Lü, T. T. Pan, Y. Y. Wang, E. Wang, and D. M. Li, Excited bottom and bottom-strange mesons in the quark model, *Phys. Rev. D* **94**, 074012 (2016).
- [22] S. Godfrey, Spectroscopy of B_c mesons in the relativized quark model, *Phys. Rev. D* **70**, 054017 (2004).
- [23] D. Ebert, R. N. Faustov, and V. O. Galkin, Properties of heavy quarkonia and B_c mesons in the relativistic quark model, *Phys. Rev. D* **67**, 014027 (2003).
- [24] L. P. Fulcher, Phenomenological predictions of the properties of the B_c system, *Phys. Rev. D* **60**, 074006 (1999).
- [25] S. S. Gershtein, V. V. Kiselev, A. K. Likhoded, and A. V. Tkabladze, B_c spectroscopy, *Phys. Rev. D* **51**, 3613 (1995).
- [26] T. Y. Li, L. Tang, Z. Y. Fang, C. H. Wang, C. Q. Pang, and X. Liu, Higher states of the B_c meson family, *Phys. Rev. D* **108**, 034019 (2023).
- [27] Q. Li, M. S. Liu, L. S. Lu, Q. F. Lü, L. C. Gui, and X. H. Zhong, Excited bottom-charmed mesons in a nonrelativistic quark model, *Phys. Rev. D* **99**, 096020 (2019).
- [28] X. J. Li, Y. S. Li, F. L. Wang, and X. Liu, Spectroscopic survey of higher-lying states of B_c meson family, *Eur. Phys. J. C* **83**, 1080 (2023).
- [29] E. J. Eichten and C. Quigg, Mesons with beauty and charm: New horizons in spectroscopy, *Phys. Rev. D* **99**, 054025 (2018).