

Helicity amplitude and branching fraction measurement of $\chi_{cJ} \rightarrow \Lambda \bar{\Lambda}$

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(Received 2 September 2025; accepted 12 December 2025; published 12 January 2026)

Utilizing 2712.4 ± 14.3 million $\psi(3686)$ events accumulated by the BESIII experiment, we perform a partial wave analysis of $\psi(3686) \rightarrow \gamma \chi_{cJ} \rightarrow \gamma \Lambda \bar{\Lambda}$ decay ($J = 0, 1, 2$). The ratio of the helicity amplitudes with same (++) and opposite (+-) helicity for $\chi_{c2} \rightarrow \Lambda \bar{\Lambda}$ decay is determined for the first time to be $R_{\chi_{c2}} = 0.575 \pm 0.048 \pm 0.018$, with a relative phase angle $\Delta\Phi_{\chi_{c2}} = 0.37 \pm 0.15 \pm 0.05$ rad. The parameters of the angular distribution of χ_{c2} are determined to be $\alpha_{\chi_{c2}} = -0.211 \pm 0.100 \pm 0.050$ and $\beta_{\chi_{c2}} = -0.039 \pm 0.089 \pm 0.033$, based on the distribution $dN/d\cos\theta = 1 + \alpha_{\chi_{c2}} \cos^2\theta + \beta_{\chi_{c2}} \cos^4\theta$. The width of χ_{c0} is determined to be $12.31 \pm 0.26 \pm 0.12$ MeV. Additionally, the branching fractions for $\chi_{cJ} \rightarrow \Lambda \bar{\Lambda}$ are measured to be $(3.662 \pm 0.048 \pm 0.111) \times 10^{-4}$, $(1.182 \pm 0.026 \pm 0.042) \times 10^{-4}$, and $(1.704 \pm 0.035 \pm 0.057) \times 10^{-4}$ for χ_{c0} , χ_{c1} and χ_{c2} , respectively, where the first uncertainty is statistical and the second systematic.

DOI: 10.1103/64lr-8c9k

I. INTRODUCTION

The P -wave triplet charmonium states χ_{cJ} ($J = 0, 1, 2$), composed of a charm-anticharm quark pair, lying in a region between perturbative and nonperturbative quantum chromodynamics (QCD), have been known for more than forty years, yet their decay mechanisms remain unclear. To understand them, especially their structure and nonperturbative QCD properties, it is crucial to experimentally investigate these charmonium states to a greater extent. Their baryonic decay branching fractions, in particular, have been extensively studied in various experiments, achieving roughly a few percent precision [1–5].

In the asymptotic energy region of massless perturbative QCD, the vector coupling of gluons and quarks conserves the helicity of the quarks, which leads to the helicity selection rule (HSR) [6]. The helicity-violating processes, such as $\eta_c, \chi_{c0} \rightarrow B \bar{B}$, where B denotes baryon, are forbidden by HSR. However, experimental measurements have shown that the branching fractions of χ_{c0} decaying to baryonic final states are comparable to those of χ_{c1} and χ_{c2} decays [7]. Those observations suggest that nonperturbative effects may play a significant role in χ_{cJ} decays.

Some theories have explored this phenomenon from the perspective of charmonium structure, suggesting that charmonium can exhibit HSR-violating decays through mixing with glueballs [8] or the color octet mechanism [9]. HSR-violation due to nonvanishing quark mass has also been investigated. It was found that by assigning quarks a constituent mass rather than a current mass, a comparable branching fraction of $\chi_{c0} \rightarrow p \bar{p}$ can be obtained as for the nonforbidden processes $\chi_{c1,2} \rightarrow p \bar{p}$ [10]. Other mechanisms, including quark pair creation [11], quark-diquark model [12], and the long-distance contribution via charmed hadron loops [13], have also been investigated. More recently, with the availability of precise experimental data, a systematic investigation based on SU(3) flavor symmetry has provided a unified framework to understand these HSR-violating processes [14]. This approach successfully describes both $\chi_{cJ} \rightarrow B_8 \bar{B}_8$ and $B_{10} \bar{B}_{10}$ decays within 2 standard deviations after including SU(3) breaking effects. Here B_8 and B_{10} represent light octet and decuplet baryons, respectively.

Precise measurement results of χ_{cJ} decays can be used to test theoretically proposed decay mechanisms and provide a deeper insight into the structure and properties of χ_{cJ} states. The BESIII detector has accumulated (2712.4 ± 14.3) million $\psi(3686)$ events [15], which offers a good opportunity to study χ_{cJ} decays with high precision. In this paper, we present a partial wave analysis (PWA) for the process $\psi(3686) \rightarrow \gamma \Lambda \bar{\Lambda} \rightarrow \gamma p \pi^- \bar{p} \pi^+$, providing a precise measurement of the helicity amplitude ratio of $\chi_{c2} \rightarrow \Lambda \bar{\Lambda}$, the branching fractions of $\chi_{cJ} \rightarrow \Lambda \bar{\Lambda}$ ($J = 0, 1, 2$), and the width of χ_{c0} .

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II. BESIII DETECTOR AND MONTE CARLO SIMULATION

The BESIII detector [16] records symmetric e^+e^- collisions provided by the BEPCII storage ring [17] in the center-of-mass energy range from 1.84 to 4.95 GeV, with a peak luminosity of $1.1 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ achieved at $\sqrt{s} = 3.773 \text{ GeV}$. BESIII has collected large data samples in this energy region [18–20]. The cylindrical core of the BESIII detector covers 93% of the full solid angle and consists of a helium-based multilayer drift chamber (MDC), a plastic scintillator time-of-flight system (TOF), and a CsI(Tl) electromagnetic calorimeter (EMC), which are all enclosed in a superconducting solenoidal magnet providing a 1.0 T magnetic field. The solenoid is supported by an octagonal flux-return yoke with resistive plate counter muon identification modules interleaved with steel. The charged-particle momentum resolution at 1 GeV/c is 0.5%, and the dE/dx resolution is 6% for electrons from Bhabha scattering. The EMC measures photon energies with a resolution of 2.5% (5%) at 1 GeV in the barrel (end cap) region. The time resolution in the TOF barrel region is 68 ps, while that in the end cap region was 110 ps. The end cap TOF system was upgraded in 2015 using multigap resistive plate chamber technology, providing a time resolution of 60 ps, which benefits 83% of the data used in this analysis [21–23].

Monte Carlo (MC) samples are generated using the geant4-based [24] BESIII Offline Software System [16], which includes the geometric description of the BESIII detector and the detector response, to determine detection efficiencies and to estimate backgrounds. The simulation models the beam energy spread and initial state radiation (ISR) in the e^+e^- annihilations with the generator KKMC [25]. The inclusive MC sample includes the production of the $\psi(3686)$ resonance, the ISR production of the J/ψ , and the continuum processes incorporated in KKMC [25]. All particle decays are modeled with evtgen [26] using branching fractions either taken from the Particle Data Group (PDG) [7], when available, or otherwise estimated with lundcharm [27]. Three $\psi(3686)$ data samples collected in 2009, 2012, and 2021 with (107.7 ± 0.6) , (345.4 ± 2.6) , and (2259.3 ± 11.1) million events [15], respectively, and their corresponding inclusive MC events, are utilized in the analysis. Also 10 million signal MC events are generated using a PWA model for the decay $\psi(3686) \rightarrow \gamma\chi_{cJ} \rightarrow \gamma\Lambda\bar{\Lambda}$. Final state radiation from charged final state particles is incorporated using PHOTOS [28].

III. EVENT SELECTION

vents from $\psi(3686) \rightarrow \gamma\chi_{cJ}$, $\chi_{cJ} \rightarrow \Lambda\bar{\Lambda}$ decay are identified by selecting events containing one photon and a pair of $\Lambda - \bar{\Lambda}$ particles. The $\Lambda(\bar{\Lambda})$ particle is reconstructed by its dominant decay $\Lambda \rightarrow p\pi^-$ ($\bar{\Lambda} \rightarrow \bar{p}\pi^+$). Charged tracks detected in MDC are required to have $|\cos(\theta)| < 0.93$

where θ is the polar angle of the track with respect to the beam axis. The distance of closest approach between track and the interaction point is required to be less than 20 cm along the beam axis. This requirement ensures that the tracks originate from the e^+e^- annihilation point and suppresses contamination from cosmic ray tracks. Particle identification (PID) for charged tracks combines measurements of the energy deposited in the MDC (dE/dx) and the flight time in the TOF to form likelihoods $\mathcal{L}(h)(h = p, \pi)$ [29] for each hadron h hypothesis. Tracks are identified as protons when the proton hypothesis likelihood is larger than that of the pion and larger than zero ($\mathcal{L}(p) > \mathcal{L}(\pi)$ and $\mathcal{L}(p) > 0$), otherwise they will be regarded as pions. The PID efficiency is nearly 100% for high momentum protons [30]. At least one candidate for each of p , $\bar{p}$, π^- , and π^+ is required.

The Λ and $\bar{\Lambda}$ candidates are identified by examining all possible $(p\pi^-)(\bar{p}\pi^+)$ combinations and selecting the combination with the smallest $\chi^2 = (M_{p\pi^-} - m_\Lambda)^2 + (M_{\bar{p}\pi^+} - m_{\bar{\Lambda}})^2$, where $M_{p\pi^-}$ and $M_{\bar{p}\pi^+}$ are the invariant masses of $(p\pi^-)$ and $(\bar{p}\pi^+)$ systems, respectively, and m_Λ and $m_{\bar{\Lambda}}$ are the nominal masses of Λ and $\bar{\Lambda}$, respectively [31]. To improve the momentum resolution, vertex constrained fits are applied to the Λ and $\bar{\Lambda}$ candidates, followed by secondary vertex constrained fits [32]. Given the long lifetime of $\Lambda(\bar{\Lambda})$ particles, the decay length divided by its uncertainty is required to be greater than 2.0 to suppress non- $\Lambda(\bar{\Lambda})$ backgrounds. Furthermore, a mass window of $[1.108, 1.123] \text{ GeV}/c^2$ is applied to both Λ and $\bar{\Lambda}$ candidates, as shown in Fig. 1.

The radiative photon is identified using showers in EMC. The deposited energy of each shower must be greater than 25 MeV in the barrel region ($|\cos\theta| < 0.8$) and greater than 50 MeV in the end cap region ($0.86 < |\cos\theta| < 0.92$). The difference between the EMC time and the event start time of each shower is required to be within $[0, 700] \text{ ns}$ to suppress the electronic noise and energy deposition unrelated to the signal event. A four-constraint (4C) kinematic fit, where the total four-momentum is constrained to match the initial four-momentum of the $\psi(3686)$, is performed on the $\Lambda\bar{\Lambda}$ with each of the showers. The one with the

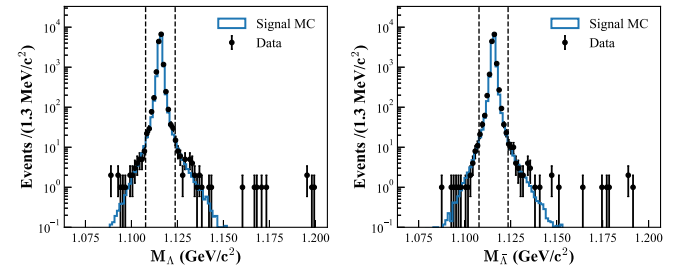


FIG. 1. Invariant mass distribution of Λ and $\bar{\Lambda}$ reconstructed by $p\pi^-$ and $\bar{p}\pi^+$ respectively. The two vertical dashed lines represent the mass window $[1.108, 1.123] \text{ GeV}/c^2$.

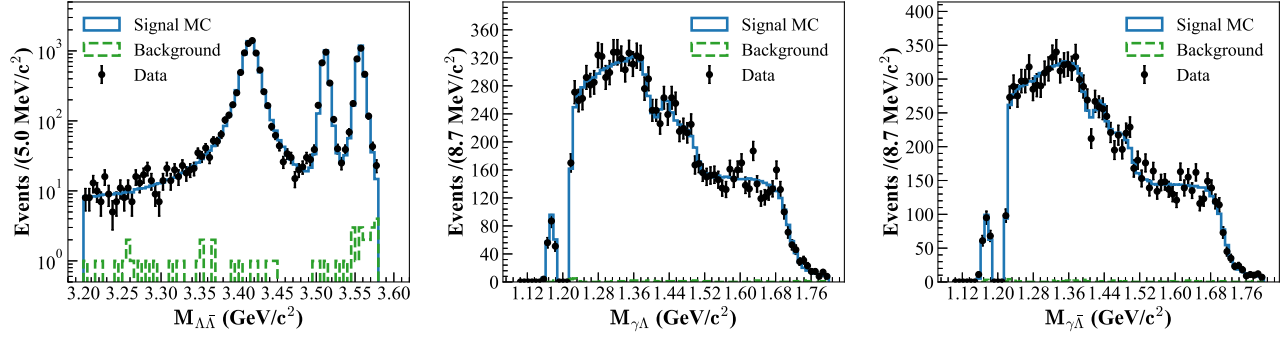


FIG. 2. Invariant mass distributions of $\Lambda\bar{\Lambda}$, $\gamma\Lambda$ and $\gamma\bar{\Lambda}$ for the selected events. The signal MC is normalized to the data by area normalization, excluding the background component. The three peaks in the left panel correspond to the χ_{c0} , χ_{c1} , and χ_{c2} signals, respectively. The dip around 1.2 GeV/ c^2 in the middle and right panels is caused by the Σ^0 and $\bar{\Sigma}^0$ veto requirements.

minimum χ_{4C}^2 is selected as the radiative photon candidate, and the χ_{4C}^2 is required to be less than 40 to suppress the background.

The main backgrounds remaining arise from the processes $\psi(3686) \rightarrow \Sigma^0 \bar{\Sigma}^0$ and $\psi(3686) \rightarrow \Lambda \bar{\Sigma}^0 + \text{c.c.}$ By requiring the invariant mass of $\gamma\Lambda(\gamma\bar{\Lambda})$ to be outside of the Σ^0 ($\bar{\Sigma}^0$) mass window $[1.183, 1.216]([1.183, 1.219])$ GeV/ c^2 , 93% $\psi(3686) \rightarrow \Sigma^0 \bar{\Sigma}^0$ backgrounds and 99% $\psi(3686) \rightarrow \Lambda \bar{\Sigma}^0 + \text{c.c.}$ backgrounds are removed. The mass windows for Σ^0 and $\bar{\Sigma}^0$ differ slightly due to different mass resolutions.

Figure 2 shows the invariant mass distribution of $\Lambda\bar{\Lambda}$, $\gamma\Lambda$ and $\gamma\bar{\Lambda}$ after using the above selection criteria. The signal range for χ_{cJ} is $M_{\Lambda\bar{\Lambda}} \in [3.20, 3.58]$ GeV/ c^2 , and the number of selected data candidates is 12880. The number of background events is estimated to be 49 from the inclusive MC sample, corresponding to a background rate of 0.4%.

A PWA fitting is performed on the selected events. The amplitude formula and the fitting results are presented in Secs. IV and V, respectively.

IV. AMPLITUDE FORMULA

We construct the amplitude using the helicity formalism [33,34], which defines the amplitude for a decay process using helicity eigenstates. Helicity is the projection of a particle's spin onto the direction of its momentum, commonly denoted by λ . The decay amplitude for each sequential decay can be characterized by the helicity angles $\Omega_i = (\theta_i, \phi_i)$ and helicities of particles, defined as follows:

- (i) In the $e^+e^- \rightarrow \psi(3686) \rightarrow \gamma(\lambda_1)\chi_{cJ}(\lambda_2)$ ($\lambda_1 = -1, 1; -J \leq \lambda_2 \leq J$) process, the helicity system is established with the z -axis aligned with the e^+ particle's direction. The angles (θ_0, ϕ_0) describe the orientation of the χ_{cJ} particle in the center-of-mass frame of the e^+e^- collision. The helicity amplitude of this decay is denoted by $A_{\lambda_1, \lambda_2}^{(J)}$. This electromagnetic

radiation decay conserves parity and angular momentum, resulting in the following relations: $A_{\lambda_1, \lambda_2}^{(0)/(2)} = A_{-\lambda_1, -\lambda_2}^{(0)/(2)}$, $A_{\lambda_1, \lambda_2}^{(1)} = -A_{-\lambda_1, -\lambda_2}^{(1)}$, and $A_{\lambda_1, \lambda_2}^{(J)} = 0$ if $|\lambda_1 - \lambda_2| > J$.

- (ii) In the $\chi_{cJ} \rightarrow \Lambda(\lambda_3)\bar{\Lambda}(\lambda_4)$ ($\lambda_3, \lambda_4 = -1/2, 1/2$) decay, the azimuthal angle ϕ_1 denotes the separation between the decay and production planes of χ_{cJ} . θ_1 is then the angle between the momentum of Λ in the rest frame of χ_{cJ} and the momentum of χ_{cJ} in the center-of-mass frame. The helicity amplitude of this decay is denoted by $B_{\lambda_3, \lambda_4}^{(J)}$. Parity conservation leads to the following relations: $B_{\lambda_3, \lambda_4}^{(0)/(2)} = B_{-\lambda_3, -\lambda_4}^{(0)/(2)}$ and $B_{\lambda_3, \lambda_4}^{(1)} = -B_{-\lambda_3, -\lambda_4}^{(1)}$, while CP conservation implies $B_{\lambda_3, \lambda_4}^{(1)} = 0$ if $\lambda_3 = \lambda_4$ for χ_{c1} decays.
- (iii) For the $\Lambda \rightarrow p(\lambda_5)\pi^-(\lambda_6)$ ($\lambda_5 = -1/2, 1/2$) decay, ϕ_2 is the angle between the production and decay planes of Λ , while θ_2 is the angle between the proton's momentum in the rest frame of Λ and the momentum of Λ in the rest frame of χ_{cJ} . The helicity amplitude of this decay is denoted by F_{λ_5} , which is related to the Λ decay asymmetry parameter by $\alpha_\Lambda = \frac{|F_+|^2 - |F_-|^2}{|F_+|^2 + |F_-|^2}$ [35], with $F_\pm \equiv F_{\pm 1/2}$. In the fit, α_Λ is fixed to the value measured by BESIII [36].
- (iv) In the $\bar{\Lambda} \rightarrow \bar{p}(\lambda_6)\pi^+(\lambda_7)$ ($\lambda_6 = -1/2, 1/2$) decay, ϕ_3 represents the angle between the production and decay planes of $\bar{\Lambda}$, and θ_3 is the angle between the antiproton's momentum in the rest frame of $\bar{\Lambda}$ and the momentum of $\bar{\Lambda}$ in the rest frame of χ_{cJ} . The helicity amplitude of this decay is denoted by G_{λ_6} , which is related to the $\bar{\Lambda}$ decay asymmetry parameter by $\alpha_{\bar{\Lambda}} = \frac{|G_+|^2 - |G_-|^2}{|G_+|^2 + |G_-|^2}$ [35], with $G_\pm \equiv G_{\pm 1/2}$. The $\alpha_{\bar{\Lambda}}$ is constrained to equal α_Λ by CP symmetry.

The definitions of these amplitudes are summarized in Table I. The decay topology for each sequential decay is shown in Fig. 3. The procedure for constructing the total

TABLE I. Helicity angles and amplitude definition for the χ_{cJ} states with $J = 0, 1, 2$ corresponding to χ_{c0} , χ_{c1} , and χ_{c2} , respectively.

Decays	Angles	Amplitudes
$\psi(3686)(M) \rightarrow \gamma(\lambda_1)\chi_{cJ}(\lambda_2)$	$\Omega_0 = (\theta_0, \phi_0)$	$A_{\lambda_1, \lambda_2}^{(J)}$
$\chi_{cJ} \rightarrow \Lambda(\lambda_3)\bar{\Lambda}(\lambda_4)$	$\Omega_1 = (\theta_1, \phi_1)$	$B_{\lambda_3, \lambda_4}^{(J)}$
$\Lambda \rightarrow p(\lambda_5)\pi^-$	$\Omega_2 = (\theta_2, \phi_2)$	F_{λ_5}
$\bar{\Lambda} \rightarrow \bar{p}(\lambda_6)\pi^+$	$\Omega_3 = (\theta_3, \phi_3)$	G_{λ_6}

$$\mathcal{A}_k(\mathbf{x}, M, \lambda_1, \lambda_5, \lambda_6) = \sum_{\lambda_2, \lambda_3, \lambda_4} D_{M, \lambda_1 - \lambda_2}^{1*}(\Omega_0) D_{\lambda_2, \lambda_3 - \lambda_4}^{J*}(\Omega_1) D_{\lambda_3, \lambda_5}^{1/2*}(\Omega_2) D_{\lambda_4, \lambda_6}^{1/2*}(\Omega_3) BW_k(m_{\Lambda\bar{\Lambda}}) A_{\lambda_1, \lambda_2}^{(J)} B_{\lambda_3, \lambda_4}^{(J)} F_{\lambda_5} G_{\lambda_6}, \quad (1)$$

where $\mathbf{x} = (\Omega_0, \Omega_1, \Omega_2, \Omega_3, m_{\Lambda\bar{\Lambda}})$; $D_{m,n}^J(\Omega_i) = D_{m,n}^J(\phi_i, \theta_i, 0)$ is the Wigner- D matrix. BW_k is Breit-Wigner propagator for k th resonance, defined as

$$BW(m) = \frac{1}{m_0^2 - m^2 - im_0\Gamma_0}, \quad (2)$$

where m_0 is the mass of the resonance, and Γ_0 is the width of the resonance.

The helicity amplitude for the two-body decay, e.g., $A_{\lambda_1, \lambda_2}^{(J)}$ and $B_{\lambda_3, \lambda_4}^{(J)}$, involving spin and helicity states $J_0 \rightarrow J_1(\lambda_1) + J_2(\lambda_2)$, is described within the L - S coupling scheme as

$$H_{\lambda_1 \lambda_2} = \sum_{ls} g_{ls} \sqrt{\frac{2l+1}{2J_0+1}} \langle l0, s\delta | J_0\delta \rangle \langle J_1 \lambda_1, J_2 - \lambda_2 | s\delta \rangle \times \left(\frac{q}{q_0}\right)^l B_l'(q, q_0, d), \quad (3)$$

where l is the orbital angular momentum quantum number, s is the total spin of the two-body system, $\delta = \lambda_1 - \lambda_2$ is the helicity difference, g_{ls} is the coupling constant for each partial wave, q is the momentum of the daughter particle in the rest frame of the parent particle, and q_0 is calculated at the nominal mass of the parent particle, d is the radius of the barrier factor, and B_l' is the reduced Blatt-Weisskopf barrier factor [40], which is explicitly expressed as

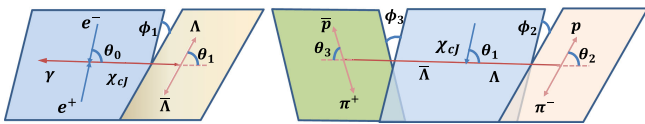


FIG. 3. Definition of helicity angles for each sequential decays in $e^+e^- \rightarrow \psi(3686) \rightarrow \gamma\chi_{cJ}, \chi_{cJ} \rightarrow \Lambda\bar{\Lambda}, \Lambda \rightarrow p\pi^-$ and $\bar{\Lambda} \rightarrow \bar{p}\pi^+$.

amplitude follows the analysis presented in Ref. [37] and is automatically implemented in the software TF-PWA [38], developed in the TensorFlow framework [39]. TF-PWA includes mass resolution effects for describing the χ_{cJ} line shape.

The amplitude for the sequential decay $\psi(3686) \rightarrow \gamma\Lambda\bar{\Lambda}$ with $\Lambda(\bar{\Lambda}) \rightarrow p\pi^-(\bar{p}\pi^+)$ is the coherent sum of all resonant and nonresonant (NR) components with spin-parity J^P , expressed as

$$B_0'(q, q_0, d) = 1, \\ B_1'(q, q_0, d) = \sqrt{\frac{1 + (q_0d)^2}{1 + (qd)^2}}, \\ B_2'(q, q_0, d) = \sqrt{\frac{9 + 3(q_0d)^2 + (q_0d)^4}{9 + 3(qd)^2 + (qd)^4}}, \quad (4)$$

where d takes 3.0 GeV^{-1} in the nominal fit. For NR components, we take $BW_k = 1$.

Equation (3) can be utilized to decompose the helicity amplitudes into L - S couplings, with an equivalent number of parameters for both hadronic and weak decays. For instance, the two independent helicity amplitudes can be expanded in terms of the $L = 1, 3$ waves for the $\chi_{c2} \rightarrow \Lambda\bar{\Lambda}$ decay at $q = q_0$, and the barrier factor is canceled

$$B_{+,+}^{(2)} = \frac{\sqrt{5}}{5} g_{11} - \frac{\sqrt{30}}{10} g_{31}, \\ B_{+,-}^{(2)} = \frac{\sqrt{30}}{10} g_{11} + \frac{\sqrt{5}}{5} g_{31}, \quad (5)$$

where $+$ ($-$) represents the $\Lambda(\bar{\Lambda})$ helicity $1/2(-1/2)$. However, in the radiative decay $\psi(3686) \rightarrow \gamma\chi_{cJ}$, the L - S expansion results in redundant terms, requiring additional gauge-invariant conditions to constrain the additional degrees of freedom. In the covariant amplitude method, the surplus photon polarizations can be constrained by employing the Coulomb gauge [41]. Here, we eliminate the surplus terms as demonstrated in Ref. [42].

The Λ angular distribution in χ_{cJ} decays can be formulated as [35]

$$\frac{dN}{d\cos\theta_1} = \begin{cases} 1 & \text{for } \chi_{c0}, \\ 1 + \alpha_{\chi_{c1}} \cos^2\theta_1 & \text{for } \chi_{c1}, \\ 1 + \alpha_{\chi_{c2}} \cos^2\theta_1 + \beta_{\chi_{c2}} \cos^4\theta_1 & \text{for } \chi_{c2}, \end{cases} \quad (6)$$

with

$$\begin{aligned} \alpha_{\chi_{c1}} &= \frac{-2r_1^2 R_1^2 + r_1^2 + 4R_1^2 - 2}{2r_1^2 R_1^2 + r_1^2 + 2}, \\ \alpha_{\chi_{c2}} &= \frac{12r_2^2 R_2^2 - 6r_3^2 R_2^2 - 6r_2^2 - 12R_2^2 + 12}{3r_3^2 R_2^2 + 2r_2^2 + 2r_3^2 + 2R_2^2}, \\ \beta_{\chi_{c2}} &= \frac{-12r_2^2 R_2^2 + 3r_3^2 R_2^2 + 8r_2^2 - 2r_3^2 + 18R_2^2 - 12}{3r_3^2 R_2^2 + 2r_2^2 + 2r_3^2 + 2R_2^2}, \end{aligned} \quad (7)$$

where $r_1 = |A_{1,1}^{(1)}|/|A_{1,0}^{(1)}|$, $r_2 = |A_{1,1}^{(2)}|/|A_{1,0}^{(2)}|$, $r_3 = |A_{1,2}^{(2)}|/|A_{1,0}^{(2)}|$, $R_1 = |B_{++}^{(1)}|/|B_{+-}^{(1)}|$, $R_2 = |B_{++}^{(2)}|/|B_{+-}^{(2)}|$, $R_1 = 0$ due to the CP conservation in $\chi_{c1} \rightarrow \Lambda\bar{\Lambda}$ decay.

V. AMPLITUDE FIT PROCESS AND RESULT

Using the amplitude defined in Eq. (1), the total amplitude is the coherent sum of all components, as expressed by

$$|\mathcal{A}(\mathbf{x})|^2 = \frac{1}{2} \sum_{M=\pm 1, \lambda_1, \lambda_5, \lambda_6} \left| \sum_k \mathcal{A}_k(\mathbf{x}, M, \lambda_1, \lambda_5, \lambda_6) \right|^2, \quad (8)$$

where we choose $M = \pm 1$ to ensure the helicity conservation of the $\psi(3686)$ particle generated from the e^+e^- annihilation process. The probability density function is defined as

$$P'(\mathbf{x}) = \frac{|\mathcal{A}(\mathbf{x})|^2 \epsilon(\mathbf{x})}{\int |\mathcal{A}(\mathbf{x})|^2 \epsilon(\mathbf{x}) d\Phi} = \frac{N'(\mathbf{x})}{D}, \quad (9)$$

where $N'(\mathbf{x}) = |\mathcal{A}(\mathbf{x})|^2 \epsilon(\mathbf{x})$ is the numerator, $D = \int |\mathcal{A}(\mathbf{x})|^2 \epsilon(\mathbf{x}) d\Phi$ is the normalization factor (denominator), $\epsilon(\mathbf{x})$ is the detection efficiency, and Φ is the phase space for $\psi(3686) \rightarrow \gamma\Lambda\bar{\Lambda}$ with $\Lambda(\bar{\Lambda}) \rightarrow p\pi^-(\bar{p}\pi^+)$. The normalization factor D can be calculated by MC integration over a phase space MC sample.

Considering the mass resolution effects of the observed χ_{cJ} states in the detected $\Lambda\bar{\Lambda}$ mass distribution, the numerator in Eq. (9) changes to

$$N(\mathbf{x}) = \int |\mathcal{A}(\mathbf{x}_i)|^2 f(m_i|m) \epsilon(\mathbf{x}) dm_i, \quad (10)$$

where m_i is the true value of the invariant mass of $\Lambda\bar{\Lambda}$ and m is the measured value. $f(m_i|m)$ represents the probability of the true mass being m_i given the measured mass m . The efficiency term $\epsilon(\mathbf{x})$ can be discarded because it only contributes a constant term to the log-likelihood. By

integrating over all possible m_i , we obtain the probability of observing an event with a measured mass of m , and $f(m_i|m)$ can be obtained from the phase space MC simulation. The probability density function with mass resolution is defined as

$$P(\mathbf{x}) = \frac{N(\mathbf{x})}{D}. \quad (11)$$

The amplitude model parameters are determined by minimizing the negative log-likelihood (NLL). The total NLL is obtained by subtracting the NLL of background contributions from the NLL of the data

$$\mathcal{S} = -\ln L = -\sum_{i \in \text{data}} \ln P(x_i) + \sum_{i \in \text{bkg}} \ln P(x_i). \quad (12)$$

The fit fraction (FF) for each component can be calculated as

$$FF_i = \frac{\int |\mathcal{A}_i|^2 d\Phi}{\int |\mathcal{A}(\mathbf{x})|^2 d\Phi}, \quad (13)$$

where $\mathcal{A}_i$ is the amplitude of the i th component and the integration is calculated using the truth-level phase space MC events.

To model the data, we incorporate the three χ_{cJ} states with their masses and widths fixed to PDG values [31], except for the χ_{c0} width, which is treated as a free parameter. Additionally, we include a NR component of $\Lambda\bar{\Lambda}$ with $J^P = 2^+$, determined by the significance test.

The mass and angular projections of the data and fit results are displayed in Fig. 4, where the fit results exhibit good agreement with data. The helicity amplitude ratio, fit fractions, angular parameters of χ_{c1} and χ_{c2} , and the phase angle of the amplitude are provided in Table II.

The branching fractions of $\chi_{cJ} \rightarrow \Lambda\bar{\Lambda}$ are calculated by

$$\mathcal{B}(\chi_{cJ} \rightarrow \Lambda\bar{\Lambda}) = \frac{N_{\text{obs}}}{\epsilon N_{\psi(3686)} \mathcal{B}(\psi(3686)) \mathcal{B}(\Lambda) \mathcal{B}(\bar{\Lambda})} \cdot FF_J, \quad (14)$$

where $N_{\text{obs}} = 12831 \pm 114$ represents the number of observed signal events after subtracting the background candidates from the total number of data events, ϵ denotes the detection efficiency, which is determined to be $17.64 \pm 0.05\%$ from a signal MC sample generated using the amplitude model with fixed parameters based on the PWA fit. $\mathcal{B}(\psi(3686))$, $\mathcal{B}(\Lambda) = 63.90 \pm 0.50\%$, and $\mathcal{B}(\bar{\Lambda}) = 63.90 \pm 0.50\%$, which are the branching fractions for $\psi(3686) \rightarrow \gamma\chi_{cJ}$, $\Lambda \rightarrow p\pi^-$, and $\bar{\Lambda} \rightarrow \bar{p}\pi^+$, respectively, are taken from the PDG [7].

The number of observed signal events for each χ_{cJ} state, measured product branching fractions of $\psi(3686) \rightarrow \gamma\chi_{cJ}$

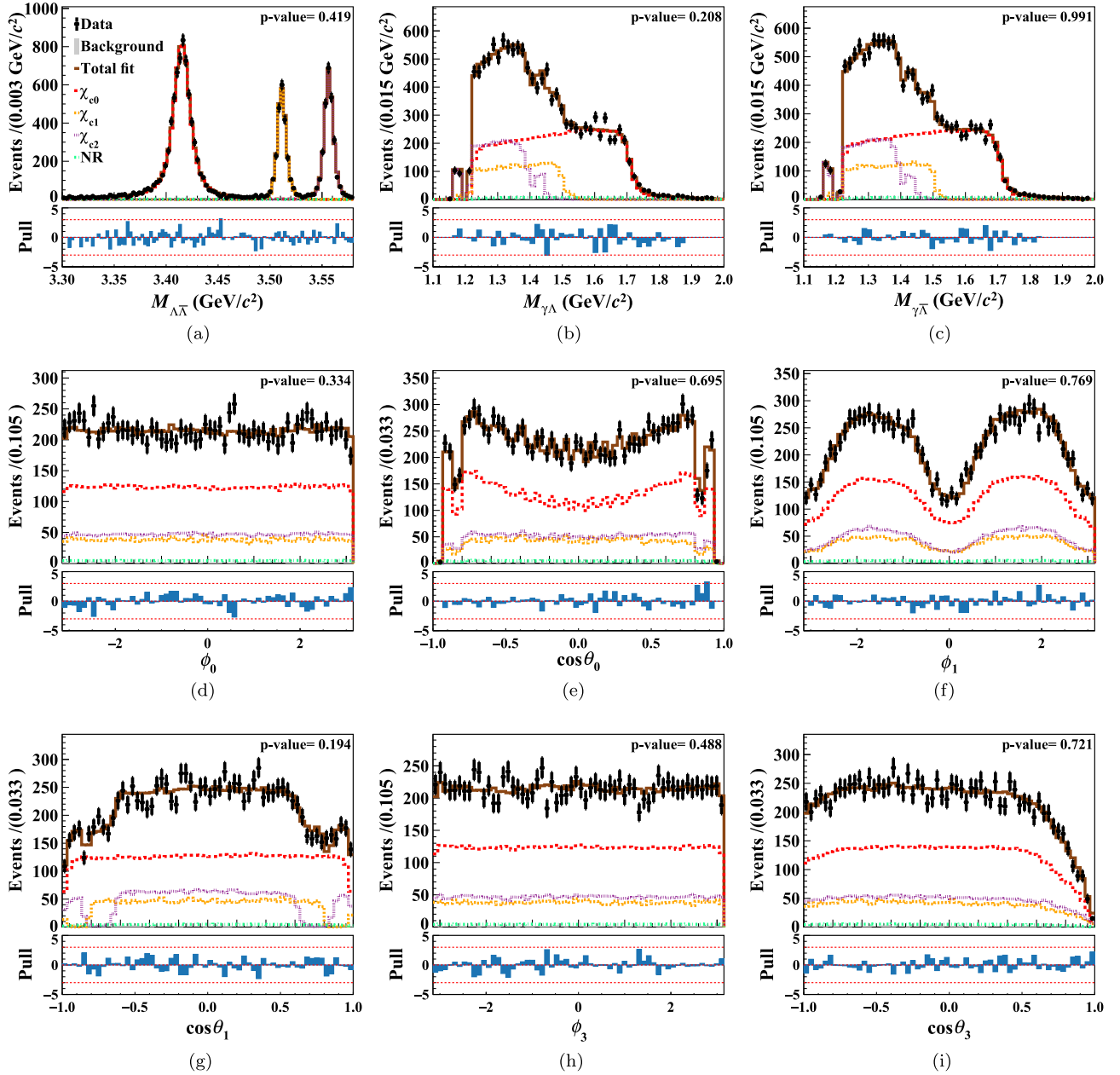


FIG. 4. Nominal fit results. (a), (b), (c) are the invariant mass distribution for $\Lambda\bar{\Lambda}$, $\gamma\Lambda$ and $\gamma\bar{\Lambda}$, respectively. (d), (e) are the helicity angle distribution of χ_{cJ} ($\phi_0, \cos\theta_0$), (f), (g) are the helicity angle distribution for Λ ($\phi_1, \cos\theta_1$), and (h), (i) are the helicity angle distribution for the proton ($\phi_3, \cos\theta_3$). The helicity angle distribution for antiproton are similar. In all figures, the black dots with error bars are data, the brown histograms are the total fit results, and the color histograms are the line shapes for each of the components, defined in the legend in figure (a). The gray histograms are the background contributions. Figure (a) uses 100 bins, while the other figures use 60 bins. The p -value shown in each figure is calculated assuming a χ^2 distribution with n_{bin} degrees of freedom, where n_{bin} is the number of bins in each projection. This value serves only to assess the consistency between data and model in each projection, and is not used in the parameter estimation.

and $\chi_{cJ} \rightarrow \Lambda\bar{\Lambda}$, the PDG values of $\psi(3686) \rightarrow \gamma\chi_{cJ}$, the measured values of $\chi_{cJ} \rightarrow \Lambda\bar{\Lambda}$, and the PDG values of $\chi_{cJ} \rightarrow \Lambda\bar{\Lambda}$ are listed in Table III. The number of signal events is provided as a statistical reference and is not used for the branching fraction calculation.

VI. SYSTEMATIC UNCERTAINTY OF THE AMPLITUDE ANALYSIS

In this section, uncertainties originating from multiple sources are estimated, encompassing photon detection, $\Lambda(\bar{\Lambda})$ reconstruction, Λ decay parameters,

TABLE II. Fit results with statistical and systematic uncertainties where r_1, r_2, r_3, R_2 are the modulus of amplitude ratios for $A_{1,1}^{(1)}/A_{1,0}^{(1)}, A_{1,1}^{(2)}/A_{1,0}^{(2)}, A_{1,2}^{(2)}/A_{1,0}^{(2)}$ and $B_{++}^{(2)}/B_{+-}^{(2)}, \Delta\Phi_{r_1}, \Delta\Phi_{r_2}, \Delta\Phi_{r_3}, \Delta\Phi_{R_2}$ are the corresponding relative phase angles. $FF_{\chi_{c0}}, FF_{\chi_{c1}}, FF_{\chi_{c2}}, FF_{NR(2^+)}$ are the fit fractions of $\chi_{c0}, \chi_{c1}, \chi_{c2}, NR(2^+)$, respectively. $\alpha_{\chi_{c1}}, \alpha_{\chi_{c2}}, \beta_{\chi_{c2}}$ are the angular parameters of χ_{c1} and χ_{c2} in Eq. (7) and $\Gamma_{\chi_{c0}}$ is the width of χ_{c0} .

Parameters	Value	Unit
r_1	$1.037 \pm 0.047 \pm 0.029$	...
r_2	$1.48 \pm 0.13 \pm 0.07$	...
r_3	$1.85 \pm 0.15 \pm 0.10$	...
$\Delta\Phi_{r_1}$	$0.042 \pm 0.107 \pm 0.032$	Rad
$\Delta\Phi_{r_2}$	$0.37 \pm 0.12 \pm 0.17$	Rad
$\Delta\Phi_{r_3}$	$0.13 \pm 0.12 \pm 0.05$	Rad
$R_{\chi_{c2}}$	$0.575 \pm 0.048 \pm 0.018$	...
$\Delta\Phi_{\chi_{c2}}$	$0.37 \pm 0.15 \pm 0.05$	Rad
$FF_{\chi_{c0}}$	$0.5459 \pm 0.0052 \pm 0.0076$	...
$FF_{\chi_{c1}}$	$0.1755 \pm 0.0036 \pm 0.0034$	...
$FF_{\chi_{c2}}$	$0.2470 \pm 0.0046 \pm 0.0047$	...
$FF_{NR(2^+)}$	$0.0265 \pm 0.0040 \pm 0.0098$	...
$\alpha_{\chi_{c1}}$	$-0.301 \pm 0.042 \pm 0.026$	...
$\alpha_{\chi_{c2}}$	$-0.211 \pm 0.100 \pm 0.050$	...
$\beta_{\chi_{c2}}$	$-0.039 \pm 0.089 \pm 0.033$	...
$\Gamma_{\chi_{c0}}$	$12.31 \pm 0.26 \pm 0.12$	MeV

Blatt-Weisskopf barrier factor, additional nonresonant contributions, resonance model, resolution model, E_1 transition, kinematic fit, and nonresonant model. The results are listed in Table IV.

(i) Photon detection and $\Lambda(\bar{\Lambda})$ reconstruction

To estimate the uncertainty due to the data-MC efficiency difference, we add an additional weight factor to correct the MC efficiency and perform a new PWA fit. For the photon detection efficiency, the correction factor is determined using the ISR process $J/\psi \rightarrow \mu\mu\gamma_{\text{ISR}}$ as a control sample. For the $\Lambda(\bar{\Lambda})$ reconstruction efficiency correction, we use angular-dependent efficiency correction factors, which are determined using the control sample $\psi(3686) \rightarrow \Lambda\bar{\Lambda}$. The differences of the PWA results between the new fitting and the nominal one are taken as the systematic uncertainties.

(ii) Decay parameter of $\Lambda(\bar{\Lambda})$

In the nominal fit, the decay parameter of $\Lambda(\bar{\Lambda})$ is fixed to the value measured by BESIII [36]. To account for the systematic uncertainty, we change the value of decay parameter to 1 standard deviation larger, and the differences between the new and nominal results are taken as the systematic uncertainties.

(iii) Blatt-Weisskopf barrier factor

In the nominal fit, we incorporate the Blatt-Weisskopf barrier factor into the amplitude model to account for the energy-dependent behavior of the helicity amplitude. To estimate the systematic uncertainty, we remove the Blatt-Weisskopf barrier factor, and the differences between the fitted and nominal values are taken as the systematic uncertainties.

(iv) Additional nonresonant components

To estimate the systematic uncertainty related to additional nonresonant components, we add $NR(0^-)$, $NR(0^+)$, and $NR(1^+)$ one at a time to the base model and perform alternative fits. The largest differences between the fitted and nominal results are taken as the systematic uncertainties.

(v) Resonance model

In the nominal fit, we use a constant width Breit-Wigner propagator for χ_{c0} . To estimate the systematic uncertainty due to the resonance model, we perform an alternative fit using a running width Breit-Wigner propagator for χ_{c0} , with the running width $\Gamma(m) = \Gamma_0(\frac{q}{q_0})^{2l+1} \frac{m_0}{m} B_l^2(q, q_0, d)$. The differences between the fitted and nominal values are taken as the systematic uncertainties.

(vi) Resolution model

The data-MC resolution difference is determined by fitting the χ_{c1} and χ_{c2} data using the signal MC shape convolved with a Gaussian function. The standard deviation of the Gaussian function is determined to be (1.47 ± 0.16) MeV. To evaluate the systematic uncertainty, we enlarge the data-MC resolution difference by 1 standard deviation and get a new resolution model. The differences between the PWA results using the new resolution model and the nominal results are taken as the systematic uncertainties.

TABLE III. The number of observed signal events for each χ_{cJ} state, the measured product branching fractions of $\psi(3686) \rightarrow \gamma\chi_{cJ}$ and $\chi_{cJ} \rightarrow \Lambda\bar{\Lambda}$ ($\mathcal{B}_1\mathcal{B}_2$), the PDG branching fractions of $\psi(3686) \rightarrow \gamma\chi_{cJ}$ ($\mathcal{B}_1$) from the PDG [7], the measured and the PDG [7] branching fractions of $\chi_{cJ} \rightarrow \Lambda\bar{\Lambda}$ ($\mathcal{B}_2$). The first uncertainty is statistical and the second is systematic.

Chan	Events	Meas. $\mathcal{B}_1\mathcal{B}_2$ ($\times 10^{-5}$)	PDG $\mathcal{B}_1$ ($\times 10^{-2}$)	Meas. $\mathcal{B}_2$ ($\times 10^{-4}$)	PDG $\mathcal{B}_2$ ($\times 10^{-4}$)
χ_{c0}	7437 ± 64	$3.585 \pm 0.047 \pm 0.080$	9.79 ± 0.20	$3.662 \pm 0.048 \pm 0.111$	3.61 ± 0.16
χ_{c1}	2278 ± 45	$1.152 \pm 0.025 \pm 0.030$	9.75 ± 0.24	$1.182 \pm 0.026 \pm 0.042$	1.27 ± 0.09
χ_{c2}	2787 ± 56	$1.622 \pm 0.033 \pm 0.042$	9.52 ± 0.20	$1.704 \pm 0.035 \pm 0.057$	1.86 ± 0.16

TABLE IV. The relative systematic uncertainty in the PWA results, expressed as a percentage. I is from the photon reconstruction and $\Lambda(\bar{\Lambda})$ reconstruction, II is from the $\Lambda(\bar{\Lambda})$ decay parameter, III is from using the Blatt-Weisskopf barrier factor in helicity amplitude, IV is from the nonresonant components, V is from the resonance model, VI is from the resolution difference between data and MC, VII is from the influence of the E_1 transition and damping factor, VIII is from the uncertainty of kinematic fit, and IX is from the uncertainty of NR model. The total systematic uncertainty is calculated as the quadrature sum of the individual uncertainties.

Parameter	I	II	III	IV	V	VI	VII	VIII	IX	Total
r_1	0.20	0.06	2.56	1.29	0.00	0.02	0.09	0.07	0.02	2.88
r_2	1.31	0.06	4.33	1.66	0.00	0.05	0.97	0.14	0.21	4.92
r_3	1.03	0.05	4.81	1.95	0.01	0.06	0.98	0.05	0.32	5.40
$\Delta\Phi_{r_1}$	6.26	0.10	41.26	79.69	0.39	4.67	24.97	7.52	4.09	93.86
$\Delta\Phi_{r_2}$	2.77	0.02	61.49	10.25	0.03	0.56	12.11	0.38	0.12	63.57
$\Delta\Phi_{r_3}$	6.87	0.38	15.21	31.05	0.41	0.13	61.62	0.55	0.85	71.00
$R_{\chi_{c2}}$	1.75	0.03	1.81	1.37	0.00	0.03	1.37	0.09	0.01	3.18
$\Delta\Phi_{\chi_{c2}}$	2.07	0.25	12.70	6.52	0.05	0.63	5.11	0.31	1.00	15.36
$FF_{\chi_{c0}}$	0.02	0.00	0.11	1.39	0.00	0.03	0.05	0.06	0.06	1.40
$FF_{\chi_{c1}}$	0.08	0.00	0.12	1.66	0.00	0.08	0.96	0.01	0.09	1.93
$FF_{\chi_{c2}}$	0.23	0.01	0.36	0.87	0.01	0.05	1.63	0.15	0.17	1.91
$FF_{NR(2^+)}$	0.74	0.09	9.86	34.46	0.11	0.39	1.61	0.73	1.83	35.95
$\alpha_{\chi_{c1}}$	0.59	0.17	7.39	3.74	0.00	0.06	0.26	0.22	0.05	8.31
$\alpha_{\chi_{c2}}$	4.68	0.27	23.34	9.15	0.03	0.29	4.16	0.41	1.58	25.89
$\beta_{\chi_{c2}}$	14.84	0.76	51.00	17.52	0.02	0.47	9.56	2.35	2.22	56.84
$\Gamma_{\chi_{c0}}$	0.10	0.01	0.05	0.18	0.01	0.54	0.70	0.00	0.26	0.94

(vii) E_1 transition

To estimate the systematic uncertainty due to the E_1 transition, we multiply the Breit-Wigner propagator with the E_1 transition factor $(\frac{E_\gamma}{E_\gamma^0})^3$ and damping factor $D(\sqrt{s}) = (\frac{(E_\gamma^0)^2}{E_\gamma^0 E_\gamma + (E_\gamma^0 - E_\gamma)^2})^{\frac{1}{2}}$ [43] and perform a new PWA fit. The differences between the new PWA and nominal PWA results are taken as the systematic uncertainties.

(viii) Kinematic fit

In order to take into account the impact of the kinematic fit on the PWA results, we perform PWA fits using two sets of MC samples: with and without helix parameter correction [44]. The differences between the two results are taken as the systematic uncertainties.

(ix) Nonresonant model

To evaluate the uncertainty due to the NR model, we performed a new fit with varying the NR propagator from $NR = 1.0$ to a first-order

polynomial, $NR = 1.0 + re^{i\phi}m$, where r and ϕ are arbitrary parameters and m is the invariant mass of $\Lambda\bar{\Lambda}$. The differences between the new PWA and nominal PWA results are taken as the systematic uncertainties.

VII. SYSTEMATIC UNCERTAINTY OF THE BRANCHING FRACTION

In this section, we estimate systematic uncertainties in the determination of the branching fraction. The sources of uncertainties taken into account encompass photon detection, the number of $\psi(3686)$ events, branching fractions for Λ decay, fit fractions, $\Lambda\bar{\Lambda}$ reconstruction, kinematic fit, and signal model. The results are listed in Table V.

(i) Photon reconstruction

The uncertainty in photon detection efficiency is estimated to be 0.2% per photon, determined using a control sample of the ISR process, $J/\psi \rightarrow \mu\mu\gamma_{\text{ISR}}$.

TABLE V. The relative systematic uncertainty of the branching fraction, expressed as a percentage. I is from photon reconstruction, II is from the number of $\psi(3686)$ events, III is from the branching fraction of $\Lambda \rightarrow p\pi^-$, IV is from the branching fraction of $\psi(3686) \rightarrow \gamma\chi_{cJ}$, V is from the FFs of the PWA, VI is from the $\Lambda(\bar{\Lambda})$ reconstruction, VII is from the kinematic fit, and VIII is from the signal model. The total systematic uncertainty is calculated as the quadrature sum of the individual uncertainties.

Decay	I	II	III	IV	V	VI	VII	VIII	Total
$\mathcal{B}(\chi_{c0} \rightarrow \Lambda\bar{\Lambda})$	0.20	0.53	1.56	2.26	1.40	0.29	0.41	0.20	3.18
$\mathcal{B}(\chi_{c1} \rightarrow \Lambda\bar{\Lambda})$	0.20	0.53	1.56	2.77	1.93	0.29	0.41	0.20	3.80
$\mathcal{B}(\chi_{c2} \rightarrow \Lambda\bar{\Lambda})$	0.20	0.53	1.56	2.45	1.92	0.29	0.41	0.20	3.57

- (ii) Number of $\psi(3686)$ events and branching fraction of Λ decay

The uncertainties of the number of $\psi(3686)$ events [15], the branching fraction of $\psi(3686) \rightarrow \gamma\chi_{cJ}$ [7], and the branching fraction of $\Lambda \rightarrow p\pi^- (\bar{\Lambda} \rightarrow \bar{p}\pi^+)$ [7] are propagated as systematic uncertainties.

- (iii) Fit fraction

The systematic uncertainties in the FF s are propagated to the branching fractions as systematic uncertainties.

- (iv) $\Lambda\bar{\Lambda}$ reconstruction

For the $\Lambda(\bar{\Lambda})$ reconstruction efficiency correction, we use angular-dependent efficiency correction factors, which are determined using the control sample $\psi(3686) \rightarrow \Lambda\bar{\Lambda}$. The uncertainty of efficiency propagated from the efficiency correction factor is taken as the systematic uncertainty.

- (v) Kinematic fit

Due to the data-MC differences in helix parameters [44], kinematic fits may lead to discrepancies in the efficiency. To account for this effect, we calculate the efficiency using two MC samples: one directly using the MC helix parameters, and another using control samples to correct the MC helix parameters before performing the kinematic fit. The difference between these two efficiencies on the branching fractions are assigned as the systematic uncertainties.

- (vi) Signal model

The systematic uncertainty due to the signal model is estimated to be 0.2% by varying the PWA model within the uncertainty of the model parameters.

VIII. SUMMARY

Using 2712.4 ± 14.3 million $\psi(3686)$ events collected by the BESIII detector, we conduct a partial wave analysis of the decay process $\psi(3686) \rightarrow \gamma\chi_{cJ} \rightarrow \gamma\Lambda\bar{\Lambda}$. The helicity amplitude ratio for χ_{c2} is measured for the first time to be $R_{\chi_{c2}} = 0.575 \pm 0.048 \pm 0.018$, with a relative phase angle of $\Delta\Phi_{\chi_{c2}} = 0.37 \pm 0.15 \pm 0.05$ rad. We also report angular distribution parameters for χ_{c2} : $\alpha_{\chi_{c2}} = -0.211 \pm 0.100 \pm 0.050$ and $\beta_{\chi_{c2}} = -0.039 \pm 0.089 \pm 0.033$. The width of χ_{c0} is determined to be $12.31 \pm 0.26 \pm 0.12$ MeV, providing a precise measurement of this parameter. Branching fractions for $\chi_{cJ} \rightarrow \Lambda\bar{\Lambda}$ ($J = 0, 1, 2$) are measured to be $(3.662 \pm 0.048 \pm 0.111) \times 10^{-4}$, $(1.182 \pm 0.026 \pm 0.042) \times 10^{-4}$, and $(1.704 \pm 0.035 \pm 0.057) \times 10^{-4}$, respectively. These results demonstrate a notable

enhancement in precision compared to current world average values [7].

ACKNOWLEDGMENTS

The BESIII Collaboration thanks the staff of BEPCII [45] and the IHEP computing center for their strong support. This work is supported in part by National Key R&D Program of China under Contracts No. 2025YFA1613900, No. 2023YFA1606000, No. 2023YFA1606704; National Natural Science Foundation of China (NSFC) under Contracts No. 12175244, No. 11635010, No. 11935015, No. 11935016, No. 11935018, No. 12025502, No. 12035009, No. 12035013, No. 12061131003, No. 12192260, No. 12192261, No. 12192262, No. 12192263, No. 12192264, No. 12192265, No. 12221005, No. 12225509, No. 12235017, No. 12361141819; Beijing Natural Science Foundation of China (BNSF) under Contract No. IS23014; the Chinese Academy of Sciences (CAS) Large-Scale Scientific Facility Program; CAS under Contract No. YSBR-101; 100 Talents Program of CAS; CAS Project for Young Scientists in Basic Research No. YSBR-117; The Institute of Nuclear and Particle Physics (INPAC) and Shanghai Key Laboratory for Particle Physics and Cosmology; German Research Foundation DFG under Contract No. FOR5327; Istituto Nazionale di Fisica Nucleare, Italy; Knut and Alice Wallenberg Foundation under Contracts Nos. 2021.0174, 2021.0299; Ministry of Development of Turkey under Contract No. DPT2006K-120470; National Research Foundation of Korea under Contract No. NRF-2022R1A2C1092335; National Science and Technology fund of Mongolia; National Science Research and Innovation Fund (NSRF) via the Program Management Unit for Human Resources & Institutional Development, Research and Innovation of Thailand under Contract No. B50G670107; Polish National Science Centre under Contract No. 2024/53/B/ST2/00975; Swedish Research Council under Contract No. 2019.04595; U.S. Department of Energy under Contract No. DE-FG02-05ER41374.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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