

Pathologies of dimension-zero scalar fields

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(Received 23 March 2026; accepted 14 July 2026; published 24 August 2026)

It has been claimed in a series of papers that scalar fields with a fourth-order Lagrangian $\sim (\Box\varphi)^2$ can solve the cosmological constant problem by canceling the loop contributions from standard model fields, and that their fluctuations can be the source of the primordial density perturbations of the Universe, without the need for inflation. We dispute these claims. The spectrum of the theory includes a ghost, which leads to classical instabilities and quantum violation of unitarity. We show that the new scalar particles cannot cancel the standard model contributions to the cosmological constant, unless they include a unitarity-violating ghost at the quantum level. Further, the coupling of such scalars to the particles of the standard model induces a confining fifth force which rules it out as a source of density perturbations in the early Universe.

DOI: [10.1103/63ck-c9rh](https://doi.org/10.1103/63ck-c9rh)

I. INTRODUCTION

In our quantum field theory courses, we have been taught that theories whose Lagrangians have more than two powers of derivatives are to be avoided, except in the case of effective field theories where expansions in powers of derivatives are restricted to be small corrections. Allowing such terms to correspond to arbitrarily high momenta results in additional poles in the propagator whose residues have the wrong sign: ghosts. Such degrees of freedom lead to negative probabilities, hence loss of unitarity, or negative energies, depending upon boundary conditions imposed on the fields [1].

Nevertheless, a particular Weyl-invariant fourth-order action for scalar fields coupled to gravity has received some attention in the literature,

$$S = - \int d^4x \sqrt{-g} \varphi \Delta_4 \varphi, \quad (1)$$

where

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$$\Delta_4 = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}R\Box + \frac{1}{3}(\nabla^\mu R)\nabla_\mu, \quad (2)$$

being originally motivated by studies of conformal supergravity [2].¹ The action of conformal supergravity is a fourth-order theory containing squared curvatures, $\mathcal{L} \ni R_{\mu\nu}^2 - R^2$. As noted in Ref. [5], it gives rise to a propagating scalar graviton mode $\varphi \sim h_\mu^\mu$ whose action corresponds to Eq. (1). The authors of Refs. [4,5] were aware of the problem of ghosts in conformal supergravity, hoping that some solution would eventually be found, while investigating its interesting properties.

As noted in Ref. [3], the minus sign in Eq. (1) applies in Minkowskian signature $(-+++)$, and becomes $+$ when Wick-rotating to Euclidean signature. It will be clear from the results we derive that the sicknesses of this theory cannot be cured by choosing the opposite sign for the overall action, since the fourth-order Lagrangian typically describes both a ghost and a nonghost degree of freedom.

A scalar with the action (1) was used in Ref. [6] as an auxiliary field which when integrated out yields a nonlocal action whose variation gives the trace anomaly. This reference also included linear couplings between φ and curvature invariants that led to effects already at tree-level.

¹This is presented by Ref. [3] as being the unique such operator; however Ref. [4] notes that it can be supplemented with a contribution from the square of the Weyl tensor, with an arbitrary coefficient.

There was no suggestion there that φ should be treated as a physical field. Unlike a conventional scalar field, it has mass dimension zero.

The present work is prompted by a more recent series of papers in which it was proposed that adding 36 of such dimension-zero scalars to the standard model of particle physics (minus the Higgs field) will solve the cosmological constant problem [3,7,8], as well as provide the primordial density perturbations of the Universe. In these works, it is claimed that a gauge symmetry of the Lagrangian circumvents its ghost problem. Here we will give a correct analysis of the theory, in both a flat background and in the Friedmann-Robertson-Walker metric, and we will elucidate some of the errors made in Refs. [3,7,8], that invalidate such claims.

II. THE GHOST PROBLEM

The easiest way to see the presence of a ghost in the higher-derivative theory is to rewrite it as a lower-derivative Lagrangian using an auxiliary field. For later convenience, we first formulate it in a general gravitational background. Using $\nabla^\mu R_{\mu\nu} = \frac{1}{2} \nabla_\nu R$ and integration by parts, the higher-derivative scalar action is equal to

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} (\Box \varphi)^2 - R^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + \frac{1}{3} R (\partial \varphi)^2 \right]. \quad (3)$$

One can show that it is invariant under Weyl transformations $g_{\mu\nu}(x) \mapsto \Omega^2(x) g_{\mu\nu}(x)$, $\varphi(x) \mapsto \varphi(x)$. Rewriting the first term using a Hubbard-Stratonovich field σ , the action becomes

$$\begin{aligned} S &= \int d^4x \sqrt{-g} \left[-\partial_\mu \sigma \partial^\mu \varphi - R^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right. \\ &\quad \left. + \frac{1}{3} R (\partial \varphi)^2 - \frac{1}{2} 12 \sigma^2 \right] \\ &= \int d^4x \sqrt{-g} \left[-\frac{1}{2} \left(\frac{\partial_\mu \varphi}{\partial_\mu \sigma} \right)^T \begin{pmatrix} R^{\mu\nu} - \frac{1}{3} R g^{\mu\nu} & g^{\mu\nu} \\ g^{\mu\nu} & 0 \end{pmatrix} \right. \\ &\quad \left. \times \begin{pmatrix} \partial_\mu \varphi \\ \partial_\mu \sigma \end{pmatrix} - \frac{1}{2} \sigma^2 \right]. \end{aligned} \quad (4)$$

If the gravitational background is weak, one can ignore $R^{\mu\nu} - \frac{1}{3} R g^{\mu\nu}$ compared to $g^{\mu\nu}$ in Eq. (4). Then since the eigenvalues of the kinetic mixing matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ are ± 1 , there is clearly a ghost in the spectrum. Even if the gravitational field is strong, the determinant of the mixing matrix remains unchanged, so the ghost cannot be circumvented in this case either.

One can observe already at the classical level that the system is subject to an instability, owing to the negative energy carried by the ghost, since the Hamiltonian density (in Minkowski space),

$$\mathcal{H} = \dot{\varphi} \dot{\sigma} + \vec{\nabla} \varphi \cdot \vec{\nabla} \sigma + \frac{1}{2} \sigma^2, \quad (5)$$

is unbounded from below. This is again a consequence of the negative determinant of the kinetic matrix. The equations of motion are

$$\ddot{\varphi} - \nabla^2 \varphi = -\sigma, \quad \ddot{\sigma} - \nabla^2 \sigma = 0, \quad (6)$$

which have the homogeneous solution

$$\sigma = \sigma_0 + \omega t, \quad \dot{\varphi} = \dot{\varphi}_0 - \sigma_0 t - \frac{1}{2} \omega t^2. \quad (7)$$

The corresponding Hamiltonian is

$$\mathcal{H} = \omega \dot{\varphi}_0 + \frac{1}{2} \sigma_0^2, \quad (8)$$

which can take arbitrarily negative values.

Moreover, inhomogeneous configurations may fluctuate with arbitrarily negative local values of $\mathcal{H}$, as illustrated by the wavelike solution

$$\varphi = A p \cdot x \sin k \cdot x, \quad \sigma = -2A p \cdot k \cos k \cdot x, \quad (9)$$

where k^μ is a null vector ($k^2 = 0$) and p is any 4-vector such that $p \cdot k \neq 0$. Then there is an oscillating contribution to the energy density that is linear in $p \cdot x$,

$$\mathcal{H} \sim 4A^2 \omega^2 p \cdot k p \cdot x \cos k \cdot x \sin k \cdot x, \quad (10)$$

whose amplitude is unbounded as $|x| \rightarrow \infty$.

In this noninteracting theory, the sicknesses inherent to ghosts are ameliorated, since the energy is conserved. As soon as the negative energies can be offset by positive energies of normal particles interacting with φ , further instabilities become apparent, as we demonstrate in the next section, by studying the dynamics in cosmological backgrounds.

In Ref. [3], it was argued that there are actually no propagating physical states associated with the field φ once the “gauge symmetry” is taken into account, where $\varphi \rightarrow \varphi + \psi$ for harmonic functions $\Box \psi = 0$.² This argument comes from Sec. 10.2 of the textbook [9], but we believe that exposition was misinterpreted by Ref. [3]. A full discussion of this issue will be given in Sec. V, where we demonstrate that no dynamical degrees of freedom are eliminated by this symmetry, unless additional assumptions are invoked.

²One should note that in Ref. [3] this argument was inconsistently ignored, since in that case the field could not have fluctuations, that were supposed to replace those usually attributed to inflation.

III. FRIEDMANN-LEMAÎTRE-ROBERTSON-WALKER BACKGROUND

In curved spacetime, the characters of the degrees of freedom can be different than in Minkowski space. Nonvanishing background values for φ and the curvature R causes mixing between φ and the scalar components of the metric perturbations. This can produce qualitatively different properties for the fluctuations which we will demonstrate following the procedure of Ref. [10].

We consider a Friedmann-Lemaître-Robertson-Walker form for the metric, including a lapse function N , and a spatially homogeneous background value for φ ,

$$ds^2 = -N^2 dt^2 + a(t)^2 \delta_{ij} dx^i dx^j, \quad \varphi = \varphi(t). \quad (11)$$

By substituting this ansatz into the action, varying it with respect to the lapse, and then setting $N = 1$, we find the constraint equation,

$$2\dot{\varphi}\ddot{\varphi} - \dot{\varphi}^2 + \frac{4\dot{\varphi}\dot{a}\ddot{\varphi}}{a} + \frac{2\dot{\varphi}^2\ddot{a}}{a} + \frac{\dot{\varphi}^2\dot{a}^2}{a^2} = 0. \quad (12)$$

As a consequence of Weyl invariance, varying with respect to the scale factor gives the same equation. Variation with respect φ gives its equation of motion,

$$2a^3\ddot{\varphi} + (12\dot{a}\ddot{\varphi} + 2\dot{\varphi}\ddot{a} + 8\dot{a}\ddot{\varphi})a^2 + (14\dot{a}^2\ddot{\varphi} + 8\dot{\varphi}\ddot{a}\dot{a})a + 2\dot{\varphi}\dot{a}^3 = 0. \quad (13)$$

We can regard Eqs. (12) and (13) as determining the higher derivatives $\{\ddot{\varphi}(t), \ddot{\varphi}(t)\}$ of φ in terms of the remaining quantities.

We found three simple analytic solutions to the background equations, which will be useful for examples given below. For a matter dominated universe with $a(t) \sim t^{2/3}$, $\varphi(t) \sim t$ is an exact solution. For a de Sitter background with $a(t) \sim e^{Ht}$, there are two exact solutions, with $\varphi \sim 1/a$ and $\varphi \sim 1/a^3$, respectively. These cannot be superposed, even though the equation of motion for φ is linear, since the constraint equation is quadratic.

We next consider the dynamical fluctuations of φ and components of the metric around the above background, taking

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + \delta g_{\mu\nu}, \quad \varphi = \varphi^{(0)} + \chi, \quad (14)$$

where $g_{\mu\nu}^{(0)}$ and $\varphi^{(0)}$ are the background fields. The metric perturbations can be decomposed according to their transformations under spatial rotations,

$$\begin{aligned} \delta g_{00} &= -2\phi, \\ \delta g_{0i} &= a(t)(S_i + B_{,i}), \\ \delta g_{ij} &= a^2(t)(2\psi\delta_{ij} + 2E_{,ij} + F_{i,j}^T + F_{j,i}^T + h_{ij}^T), \end{aligned} \quad (15)$$

where the vector perturbations are divergenceless, $S_{i,i} = 0$ and $F_{i,i}^T = 0$, while tensor perturbations divergenceless and traceless, $h_{ij,j}^T = 0$ and $h_{ii}^T = 0$. At the leading order, the metric, vector and tensor perturbations decouple from each other, allowing us to study them separately.

A. Scalar fluctuations around the background

In this subsection, we analyze the scalar fluctuations around a nontrivial background. To avoid cumbersome expressions, we will outline the key steps that will allow the reader to reproduce the results if desired, and give only the main results. Adopting the Newtonian gauge with $E = 0$ and $B = 0$, we expand the action to the second order in the scalar perturbations. After integrating by parts, the Lagrangian density involves terms with higher-order time-derivatives. These are given by

$$\mathcal{L}_S = -a^3\ddot{\chi}^2 - 2a^3\dot{\varphi}\ddot{\chi}(\psi - \dot{\varphi}) + \dots \quad (16)$$

where $\dots$ denotes all of the remaining terms in the Lagrangian density. For brevity, we drop the $^{(0)}$ superscript of the background field. The second-order derivatives of χ are coupled to the gravitational potentials, and this interaction is multiplied $\dot{\varphi}$. Therefore, when $\dot{\varphi} \neq 0$, the theory may have qualitatively different behavior than the case $\dot{\varphi} = 0$, as we shall demonstrate.

By substituting

$$\psi = \psi' + \phi, \quad (17)$$

the scalar perturbation ϕ entirely drops out from the action. To make the content of the theory more manifest, we introduce an auxiliary field σ that reduces the overall order of derivatives in the Lagrangian density. In particular, we can rewrite the following part of the Lagrangian density,

$$\mathcal{L}_S = -a^3\ddot{\chi}^2 - 2a^3\dot{\varphi}\ddot{\chi}\psi' - a^3\dot{\varphi}^2\dot{\psi}'^2 + \dots \quad (18)$$

as

$$\tilde{\mathcal{L}}_S = a^3\sigma^2 - 2a^3\sigma(\ddot{\chi} + \dot{\varphi}\psi') + \dots \quad (19)$$

Integrating out σ shows that the two expressions are equivalent. σ obeys the equation of constraint $\sigma = \ddot{\chi} + \dot{\varphi}\psi'$. By substituting it into (19), one recovers Eq. (18). By partially integrating and further substituting

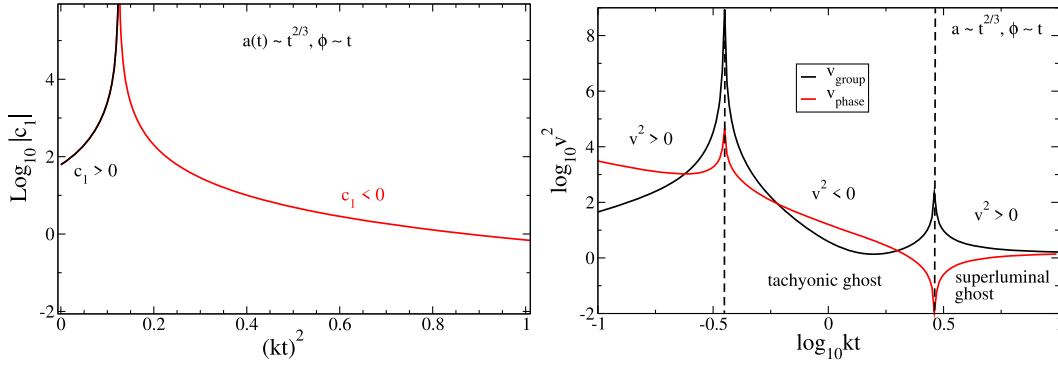


FIG. 1. Left: c_1 coefficient for scalar perturbations, Eq. (23), versus dimensionless wave number squared, $\hat{k}^2 = k^2 t^2$, in the cosmological background $a(t) \sim t^{2/3}$, $\phi(t) \sim t$. For small $\hat{k}$, it is negative, indicating that the perturbations are ghosts. Right: Squared group and phase velocities of the perturbations in this case.

$$\sigma = \sigma' - (2\ddot{\phi}a + \dot{a}\dot{\phi}) \frac{\psi'}{a}, \quad (20)$$

the kinetic term for ψ' is eliminated, and it becomes another auxiliary field whose equation of motion is a constraint. Using it to eliminate ψ' from the action, the latter becomes a function of only σ' and χ . Then expressing the fields in the momentum space as

$$X = \int \frac{d^3k}{(2\pi)^{3/2}} X_{\vec{k}} e^{i\vec{k}\vec{x}}, \quad (21)$$

where X stands for either σ' or χ , and substituting

$$\sigma'_{\vec{k}} = \tilde{\sigma}_{\vec{k}} - \frac{1}{6a^2\dot{\phi}^2} (2\dot{\phi}^2 k^2 - 3a^2\ddot{\phi}^2 + 3\dot{\phi}^2\dot{a}^2) \chi_{\vec{k}}, \quad (22)$$

the kinetic term for χ is also removed, making it too an auxiliary field. Integrating out the χ field, the final form of the scalar perturbation Lagrangian has only a single dynamical scalar fluctuation $\tilde{\sigma}$,

$$\tilde{\mathcal{L}}_{Sk} = c_1(t, k^2) \dot{\tilde{\sigma}}_{\vec{k}} \dot{\tilde{\sigma}}_{-\vec{k}} - c_2(t, k^2) k^2 \tilde{\sigma}_{-\vec{k}} \tilde{\sigma}_{\vec{k}}, \quad (23)$$

where $c_{1,2}(t, k^2)$ are complicated functions of the background fields $\phi(t, \vec{x})$ and $a(t)$, and the momentum of the perturbations, whose full expressions are given in the Appendix. They simplify in the limit $k \rightarrow \infty$, and lead to a superluminal propagation speed,

$$c_s = \sqrt{\frac{a^2 c_2}{c_1}} = \sqrt{\frac{3}{2}}, \quad (24)$$

where the factor of a^2 takes into account the redshifting of the physical momentum.

For finite k , the phase and group velocities differ from each other, and their behavior depends upon the background $a(t)$ and $\phi(t)$ solutions. As an example, consider

the matter-dominated background $a(t) \sim t^{2/3}$, where $\phi(t) = \dot{\phi}_0 t$.³ The propagation speed, as shown in Fig. 1, can be expressed as a function of a dimensionless wave number $kt = 3k/2H$, where t is cosmic time and H is the Hubble parameter. For this background, c_1 is negative at large k , indicating the perturbations are (superluminal) ghosts. For intermediate k , there is a region where both c_1 and c_2 are negative, resulting in tachyonic ghosts. At small k , these pathologies are avoided, but the phase velocity diverges as $k \rightarrow 0$, while the group velocity goes to zero.

For an inflationary background with $a \sim e^{Ht}$ and $\phi \sim 1/a$, the scalar perturbations are ghosts at all wave numbers, and like the matter-dominated case, they become superluminal at wave numbers $k/H \gtrsim 10$, reaching the same asymptotic speed of $\sqrt{3/2}$ as in that case. The phase and group velocities diverge or approach zero, respectively, as $k \rightarrow 0$, as shown in Fig. 2. The other inflationary solution with $\phi \sim 1/a^3$ has similar behavior.

B. Vector fluctuations

It turns out that there are no physical vector fluctuations in the theory, even though S_i is *a priori* present in the metric decomposition (15). By expanding its action to second order and integrating several times by parts, we its Lagrangian density,

$$\mathcal{L}_V = \frac{2}{3} \dot{\phi}^2 a k^2 S_i \Delta S_i, \quad (25)$$

in the Poisson gauge with $F_i = 0$, which corresponds to the gauge invariant variable $V_i = S_i - \dot{S}_i$. Clearly, S_i does not propagate since there are no time derivatives. It satisfies the constraint $\Delta S_i = 0$, and can therefore be set to zero.

³Here we assume that ϕ is a spectator field, not dominating the energy density of the Universe, hence its equation of state need not be that of nonrelativistic matter.

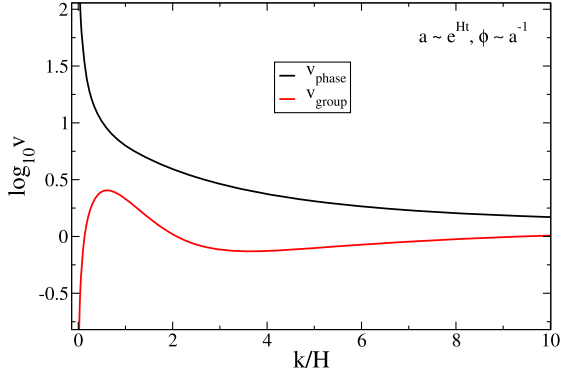


FIG. 2. Magnitude of phase and group velocities for scalar ghost perturbations around the inflationary background with $a \sim e^{Ht}$ and $\phi \sim 1/a$.

C. Tensor fluctuations

For simplicity, we have not included the $\sqrt{-g}R$ term of the gravitational action in this section; see Sec. IV. It contributes a large positive kinetic term to the tensor fluctuations, which would be negative without it (see the Appendix). Hence we do not further consider them here.

D. Limit of minkowski space

If one takes the limit $a(t) \rightarrow \text{constant}$, the theory should revert to that of the Minkowski background; however, it is possible to still have a nontrivial $\varphi(t)$ field. The equation of constraint (12) reduces to $2\dot{\varphi}\ddot{\varphi} = \dot{\varphi}^2$, while the equation of motion is $\ddot{\varphi} = 0$. Supposing for simplicity a power law solution $\varphi \sim t^p$, the only nontrivial solution is with $p = 1$. In this case, the scalar fluctuation is a ghost with propagation speed $\sqrt{3/2}$, independently of k , and the tensor fluctuation is a tachyonic ghost, as in the previous subsection. A strong-coupling obstruction is expected in the limit $\dot{\varphi} \rightarrow 0$ as discussed there.

IV. INCLUSION OF DYNAMICAL GRAVITY

In the foregoing section, we neglected the usual graviton kinetic term $\sqrt{-g}R$ from the Einstein-Hilbert action, which can mix with the kinetic operators considered above. This neglect is expected to be a good approximation when the background $\dot{\varphi} \ll M_P$ (the reduced Planck mass), since the new fluctuations become more strongly coupled than gravity in that regime.

Here we relax that restriction and include the $\sqrt{-g}R$ contribution, considering the action

$$S_{g\Delta_4} = \int d^4x \sqrt{-g} \left(\frac{1}{2} M_P^2 R - \square\varphi\square\varphi + 2R^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi - \frac{2}{3}R\nabla_\mu\varphi\nabla^\mu\varphi \right). \quad (26)$$

We will consider three cosmological regimes: (1) φ has a background solution that dominates the energy density of

the Universe; (2) φ as well as other sources of stress-energy contribute significantly to the expansion rate; and (3) is a spectator field.

The action (26) is not the full higher-dimensional gravitational action considered in [8], but contains only the leading term, the Ricci scalar. We have omitted the square of the Weyl tensor as well as the derivatives of the Ricci scalar of the form $\square R$. Including them would remove the constraint from the gravitational potential ϕ which otherwise holds [see below Eq. (35)], exacerbating the ghost problem in the scalar sector. Including the square of the Weyl tensor introduces a ghost mode for the tensor modes, as well as two vector modes [11].⁴ In the following therefore, we take the Lagrangian to be Eq. (26).

A. Scalar field with nonvanishing background

The inclusion of the Ricci scalar breaks the conformal invariance that was present in the original Δ_4 theory. Assuming the background (11), varying the action with respect to the lapse N , and then setting $N = 1$, we find the constraint equation,

$$3M_P^2 H^2 + \dot{\varphi}\ddot{\varphi} + \dot{\varphi}^2 - \frac{2\ddot{a}\dot{\varphi}^2}{a} - 4H\dot{\varphi}\ddot{\varphi} - H^2\dot{\varphi}^2 = 0, \quad (27)$$

where $H = \dot{a}/a$. Variation with respect to the scale factor gives

$$-\dot{\varphi}^2 + 2\dot{\varphi}\ddot{\varphi} + 3M_P^2 H^2 + H^2\dot{\varphi}^2 + (6M_P^2 + 2\dot{\varphi}^2)\frac{\ddot{a}}{a} + 4H\dot{\varphi}\ddot{\varphi} = 0, \quad (28)$$

while the equation of motion for the scalar field is given by

$$2\ddot{\varphi} + 12\ddot{\varphi}H + 2\dot{\varphi}\frac{\ddot{a}}{a} + 8\ddot{\varphi}\frac{\ddot{a}}{a} + 14\ddot{\varphi}H^2 + 8H\frac{\ddot{a}}{a}\dot{\varphi} + 2\dot{a}^3\dot{\varphi} = 0. \quad (29)$$

Equations (28) and (29) can be combined to show that

$$\frac{\ddot{a}}{a} = -H^2, \quad (30)$$

which describes a radiation-dominated Universe. Since φ is scale-invariant, this result could be anticipated, and we find the solutions,

$$a(t) = c_0\sqrt{t} \\ \varphi(t) = c_1 + c_3\left(M_P^2 + \frac{c_2^2}{3}\right)\sqrt{t} + c_2t + \frac{t^{3/2}}{c_3}, \quad (31)$$

where a_0 , c_1 , c_2 , and c_3 are integration constants.

⁴See however [12,13], for the potential resolution of the gravitational ghost via boundary conditions.

To find the action for the scalar modes, we decompose the metric and the scalar field according to (14) and (15), and expand the action up to second order in the perturbations, while fixing the longitudinal gauge. After performing several integrations by parts, and substituting

$$\psi = \psi' + \phi, \quad (32)$$

ϕ does not decouple from the remaining fields, as it did in the case of the pure Δ_4 theory, and higher derivatives of fields still remain. To reduce their order, we introduce the auxiliary field σ as before, which now satisfies the constraint

$$\sigma = \ddot{\chi} + \dot{\phi}\psi', \quad (33)$$

obtained by varying the modified action with respect to it. After several integrations by parts, and by substituting

$$\sigma = \sigma' - (\dot{\phi}\dot{a} + 2\dot{\phi}a)\frac{\psi'}{a}, \quad (34)$$

followed by

$$\psi' = \psi'' - \phi, \quad (35)$$

we find that ϕ becomes a nonpropagating field. Its equation of motion gives a new constraint, which upon substitution back into the action renders the equations of motion second order, involving only (ψ'', χ, σ') . Fourier transforming the fields using the convention (21), and introducing further substitutions

$$\psi''_{\vec{k}} = \tilde{\psi}_{\vec{k}} + \frac{\dot{a}}{\dot{\phi}}\chi_{\vec{k}}, \quad (36)$$

and

$$\sigma'_k = \sigma''_k + \frac{1}{6a^2\dot{\phi}^2}(9M_P^2\dot{a}^2 - 3\dot{a}^2\dot{\phi}^2 + 3\dot{\phi}^2a^2 + 2\dot{\phi}^2k^2)\chi_{\vec{k}}, \quad (37)$$

causes χ to also become nonpropagating. Using its constraint, the action becomes a function of only the two fields σ'' and $\tilde{\psi}$.

To diagonalize the kinetic terms, we substitute

$$\sigma''_k = \tilde{\sigma}_{\vec{k}} - \frac{3(3M_P^2\dot{a} - 9\dot{\phi}^2\dot{a} - 2\dot{\phi}\dot{\phi}a)M_P^2}{8a\dot{\phi}^3}\tilde{\psi}_{\vec{k}} \quad (38)$$

and find the Lagrangian density in momentum space,

$$\begin{aligned} \tilde{\mathcal{L}}_{Sk} = & d_1(t, k)\tilde{\psi}_{\vec{k}}\tilde{\psi}_{-\vec{k}} + d_2(t, k)\tilde{\sigma}_{\vec{k}}\tilde{\sigma}_{-\vec{k}} \\ & + \frac{1}{2}d_3(t, k)(\tilde{\psi}_{\vec{k}}\dot{\tilde{\sigma}}_{-\vec{k}} + \dot{\tilde{\sigma}}_{\vec{k}}\tilde{\psi}_{-\vec{k}}) \\ & + d_4(t, k)\tilde{\psi}_{\vec{k}}\tilde{\psi}_{-\vec{k}} + d_5(t, k)\tilde{\sigma}_{\vec{k}}\tilde{\sigma}_{-\vec{k}} \\ & + \frac{1}{2}d_6(t, k)(\tilde{\psi}_{\vec{k}}\dot{\tilde{\sigma}}_{-\vec{k}} + \dot{\tilde{\sigma}}_{\vec{k}}\tilde{\psi}_{-\vec{k}}), \end{aligned} \quad (39)$$

where $d_1, \dots, d_6$ are time and momentum dependent functions. Only d_1 and d_2 are needed to determine whether there are ghosts,

$$d_1(t, k) = \frac{3(\frac{3}{2}M_P^2 - 4\dot{\phi}^2)M_P^2a^3}{4\dot{\phi}^2} \quad (40)$$

and

$$d_2(t, k) = -\frac{24a^3\dot{\phi}^4}{54M_P^4H^2 - 18M_P^2\dot{\phi}^2H^2 - 36M_P^2\dot{\phi}\ddot{\phi}H - 21\dot{\phi}^4H^2 + 32\dot{\phi}^4k^2 + 12\dot{\phi}^3\ddot{\phi}H + 36\dot{\phi}^2\dot{\phi}^2}. \quad (41)$$

Using the solutions for the background fields (31), one can show that $d_1 < 0$ for any value of c_3 , and $d_2 < 0$ in all simplifying regimes that we considered. For example, at high momentum k ,

$$d_2(t, k) \sim -\frac{3a^5}{4k^2}. \quad (42)$$

Moreover, by turning on only c_3 in the solution (31), one can show that it dominates the denominator of d_2 at both early and late times, leading to $d_2 < 0$. Since like d_1 it is negative, two ghosts are present in this version of the model.

B. Inclusion of matter

In the previous subsection, only ϕ was assumed to contribute to the energy density of the Universe, leading to the restriction of $a \sim t^{1/2}$ cosmological expansion. Here we consider a toy model in which both ϕ and another component, taken to be an additional scalar field Φ , can contribute. This alleviates the restriction on $a(t)$ and leads to qualitatively different behavior, as we will show. The methodology is similar to the previous case; we here summarize the main steps. The action is given by

$$S = S_{g\Delta_4} - \frac{1}{2} \int d^4x \sqrt{-g}(\partial_\mu \Phi \partial^\mu \Phi - V(\Phi)), \quad (43)$$

where the external scalar Φ EOM is

$$-V_{,\Phi} - 3H\dot{\Phi} - \ddot{\Phi} = 0. \quad (44)$$

Here, $\dot{}$ denotes the derivative with respect to Φ , and $H = \dot{a}/a$. Variation of the action with respect to the lapse gives the first Friedmann equation,

$$\begin{aligned} -V + H^2(3M_P^2 - \dot{\Phi}^2) - \frac{\ddot{\Phi}^2}{2} - 2\frac{\ddot{a}}{a}\dot{\Phi}^2 \\ - 2\dot{\Phi}\ddot{\Phi} + \ddot{\Phi}^2 - 4H\dot{\Phi}\ddot{\Phi} = 0, \end{aligned} \quad (45)$$

while the variation with respect to the scale factor gives

$$\begin{aligned} -6V - 2\ddot{\Phi}^2 + 4\dot{\Phi}\ddot{\Phi} + 3\ddot{\Phi}^2 + 8\dot{\Phi}\ddot{\Phi}H \\ + 2(3M_P^2 + \dot{\Phi}^2)\left(2\frac{\ddot{a}}{a} + H^2\right) = 0. \end{aligned} \quad (46)$$

By combining the these two equations, one can find that the equation for the scale factor which was in the absence of matter given by (30) now generalizes to

$$6M_P^2\left(\frac{\ddot{a}}{a} + H^2\right) + \dot{\Phi}^2 - 4V = 0. \quad (47)$$

To analyze the degrees of freedom, we perturb around the background values, including the external scalar field

$\Phi(t) + \delta\Phi$, and assume that the background equations of motion, solved in terms of $\{\ddot{\Phi}, \ddot{a}, V_{,\Phi}(\Phi)\}$ are satisfied. The analysis is similar to the previous subsection; we follow the same steps as in Eqs. (32)–(37), followed by

$$\chi_{\vec{k}} = \chi'_{\vec{k}} + \frac{\dot{\Phi}}{\Phi}\delta\Phi_{\vec{k}} - \frac{4\dot{\Phi}^2}{\dot{\Phi}^2 + 2V}\sigma''_{\vec{k}}, \quad (48)$$

which makes χ' a nonpropagating field. We find its constraint, solve it, and substitute back to the action, which then becomes a function of three propagating fields, $\tilde{\psi}$, σ'' , and Φ . One can show that the determinant of the kinetic matrix no longer vanishes, meaning that these three fields are the degrees of freedom of the theory.

Since the final Lagrangian takes quite a complicated form, we will omit its expression, and state the conditions under which the above modes are not ghosts. These conditions are determined by the determinant of the total kinetic matrix, determinant of the submatrix, corresponding to σ''' and Φ , and the kinetic term that multiplies the $\dot{\Phi}^2$ term in the Lagrangian density.⁵ Specifically, the kinetic matrix is denoted by $\mathcal{L} \supset K_{ij}\dot{V}_{i,\vec{k}}\dot{V}_{j,-\vec{k}}$, where $V_i = (\tilde{\psi}, \sigma'', \Phi)$. We define the (sub)determinants $K = \det(K_{ij})$, $\det K_1^1 = \det(K_{22}K_{33} - K_{23}K_{32})$, and $K_{1,2}^{1,2} = K_{33}$. These quantities are given by

$$K = \frac{9M_P^4 a^9 \dot{\Phi}^2}{2(-3M_P^2 + 2\dot{\Phi}^2)V + (8M_P^2 k^2/a^2 - 6M_P^2 H^2 - \dot{\Phi}^2)\dot{\Phi}^2 + 6\ddot{\Phi}^2 M_P^2}, \quad (49)$$

$$\det K_1^1 = -\frac{3\dot{\Phi}^2 a^6 M_P^2}{2(-3M_P^2 + 2\dot{\Phi}^2)V + (8M_P^2 k^2/a^2 - 6M_P^2 H^2 - \dot{\Phi}^2)\dot{\Phi}^2 + 6\ddot{\Phi}^2 M_P^2}, \quad (50)$$

and

$$K_{1,2}^{1,2} = a^3 \frac{(4\dot{\Phi}^2 - 6M_P^2)V + (6\ddot{\Phi}^2 M_P^2 - 3\dot{\Phi}^2 M_P^2 - \dot{\Phi}^2 \dot{\Phi}^2) + (8k^2/a^2 - 6H^2)\dot{\Phi}^2 M_P^2}{(8\dot{\Phi}^2 - 12M_P^2)V + (12\ddot{\Phi}^2 M_P^2 - 2\dot{\Phi}^2 \dot{\Phi}^2) + (16k^2/a^2 - 12H^2)\dot{\Phi}^2 M_P^2}. \quad (51)$$

Using Sylvester's criterion, the kinetic matrix is positive definite only if all three quantities are positive,

$$K > 0, \quad \det K_1^1 > 0, \quad \text{and} \quad K_{1,2}^{1,2} > 0. \quad (52)$$

However, K and $\det K_1^1$ are proportional, with a negative ratio, since the scale factor is positive,

$$K = -3M_P^2 a^3 \det K_1^1. \quad (53)$$

Therefore, since the scale-factor is always positive, we can see that regardless of the background, at least one of the scalar modes will always be a ghost.

We have checked that setting $\Phi = 0$ gives back the results of the previous subsection, which found two ghost modes. By the analogous procedure, one can show that the same result holds if Φ is replaced by a k -essence field,

$$S = \int d^4x \sqrt{-g} p(X, \Phi), \quad (54)$$

where $X = -g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi$.

⁵One can also choose analogously kinetic terms for $\tilde{\psi}$ and σ''' and the corresponding sub-matrices to find an equivalent result.

C. Δ_4 scalar as a spectator

Lastly, we consider the case when the Universe is dominated by a perfect fluid, while treating the Δ_4 scalar as a spectator field. In this case, the constraint equation becomes

$$3M_P^2 H^2 = \varepsilon, \quad (55)$$

where ε is the energy density, while the acceleration equation simplifies to the standard expression,

$$6M_P^2 \frac{\ddot{a}}{a} = -(\varepsilon + 3p). \quad (56)$$

Therefore, depending on the external matter with equation of state w , with $p = w\varepsilon$, the scale factor expands as $a \sim t^p$ with $p = 2/(3(1+w))$. Assuming that φ has vanishing background, it decouples from the gravitational and matter perturbations at the level of quadratic action, which we will thus ignore. Then the kinetic term for φ is given by

$$\begin{aligned} \mathcal{L}_\chi = & -a^3 \dot{\chi}^2 + \frac{-a^3(\varepsilon + p) + 4M_P^2 \dot{a}^2}{2M_P^2} \chi^2 \\ & - \frac{a}{2} \dot{\chi} \Delta \chi - \frac{1}{a} \Delta \chi \Delta \chi. \end{aligned} \quad (57)$$

We can see that the first term is higher order in time derivatives, thus indicating that there are two scalar modes corresponding to χ —a healthy mode, and a ghost.

V. THERE IS NO GAUGE SYMMETRY

The action (1), restricted to a Minkowski space background, with vanishing background value of φ , is invariant under transformations $\varphi \rightarrow \varphi + \alpha(x)$, for harmonic functions such that $\square\alpha = 0$, and which contribute no surface term after integration by parts.⁶ In Ref. [3] it was asserted that this “gauge symmetry” removes the dynamical degrees of freedom, thereby circumventing the problem of ghosts based on an argument in the textbook [9].

Here we will revisit these claims, and show that there is no reduction of degrees of freedom resulting from the symmetry, unless additional assumptions are made.

Clearly the transformation $\varphi \rightarrow \varphi + \alpha(x)$ is not a gauge transformation in the usual sense, under which common physical observables remain invariant. Let us take the Minkowski space limit of Eq. (4), whose Lagrangian is $-\partial_\mu \varphi \partial^\mu \sigma - \frac{1}{2} \sigma^2$. Then, for example, the stress-energy tensor,

$$T_{\mu\nu} = \partial_\mu \varphi \partial_\nu \sigma + \partial_\mu \sigma \partial_\nu \varphi - \eta_{\mu\nu} \left(\frac{\sigma^2}{2} + \partial_\gamma \varphi \partial^\gamma \sigma \right), \quad (58)$$

⁶This symmetry is broken by the curvature-dependent terms in Eq. (2) that do not involve $\square$. Therefore, here we consider only the linearized theory, and neglect the higher-order contributions of the metric perturbations.

is not invariant. One would have to construct gauge-invariant operators from $\square\varphi$ and insist that they alone are physically meaningful. However, this is an artificial additional requirement that is not forced upon us *a priori*, and moreover it leads to a physically uninteresting theory.

In the textbook Ref. [9], the $(\square\varphi)^2$ model is first treated in Sec. (10.1 A) as a theory in its own right, followed by Sec. (10.1 B) in which it is used to model QED with a covariant gauge fixing term. In the first case, Ref. [9] shows that proper quantization of the theory to eliminate the unphysical ghost states results in it being trivial; only the vacuum state exists. It is then coupled to a Dirac fermion and it is shown that following the same procedure, the resulting theory has no physical content beyond that of the original fermion. This clearly goes against the claims of Ref. [3] that such a field can have physical consequences.

In the second case, the full action, including the gauge fixing term σ^2 , where $\sigma = \partial_\mu A^\mu$ (called Λ in Ref. [9]) is invariant under gauge transformations α that are harmonic. This is an old pre-Becchi-Rouet-Stora-Tutyin (BRST) [14,15] approach where the gauge-fixed Lagrangian is treated as fundamental, and only harmonic gauge transformations are considered to be symmetries. In this approach, the QED stress tensor contains a term proportional to σ , but it does not contribute to physical correlators since σ commutes with all gauge-invariant operators and its vacuum expectation value is zero [by fiat, (10.94)], e.g., $\sigma \rightarrow 0$ in all observables, corresponding to the choice of gauge $\partial_\mu A^\mu = 0$.

One can see that the theory only becomes trivial, in the sense of having no dynamical degrees of freedom, when the extra condition $\sigma = 0$ is imposed. Consider general plane-wave solutions,

$$\begin{aligned} \sigma &= a_k e^{-ik \cdot x} + a_k^* e^{ik \cdot x}, \\ \varphi &= (a_k \bar{k} \cdot x + b_k f(k \cdot x)) e^{-ik \cdot x} + \text{H.c.}, \end{aligned} \quad (59)$$

where k^μ and $\bar{k}^\mu$ are null vectors ($k^2 = \bar{k}^2 = 0$) with $\bar{k}^\mu = i(k^0, -\vec{k})/(2|\vec{k}|^2)$, so that $i\bar{k} \cdot k = 1$. These satisfy the equations of motion (6), since the combination $f e^{-ik \cdot x}$ multiplied by b_k is harmonic. Without additional assumptions, the independent coefficients a_k and b_k correspond to two scalar degrees of freedom. If we treat $\varphi \rightarrow \varphi + \alpha(x)$ as a gauge symmetry, the b_k term can be set to zero, but the remaining a_k terms can only be eliminated by declaring they are zero. In other words, the above choice of gauge does not fix all of the degrees of freedom [16]. Moreover, if one were to remove half of the degrees of freedom in this way, then cancellation of the vacuum energy density as envisioned by Refs. [3,7,8] would require 72 such fields, rather than the 36 claimed in those papers.

In Refs. [7,8], another work which was cited in connection to the supposed gauge symmetry was Ref. [17].

This paper shows that certain higher-derivative scalar theories can be made trivial by supplementing them with a ghost field c that enables a BRST symmetry. In particular, the Lagrangian

$$\mathcal{L} = \frac{1}{2} \square \varphi \square \varphi - \bar{c} \square c \quad (60)$$

has such a symmetry, which renders it trivial, in the sense that the only state it possesses is the ground state. However, Refs. [7,8] do not include the ghost field. Doing so would lead to a different theory having no fluctuations, and thus having no effect on the vacuum energy nor on cosmological perturbations.

In summary, we find that this theory has no gauge symmetry, unless one adds to it an anticommuting ghost field, or by fiat declares that σ is not observable and should be set to zero, which Ref. [9] imposed in order to turn it into a model of partially gauge-fixed QED. Intrinsically both φ and σ are propagating degrees of freedom.

VI. THE COSMOLOGICAL CONSTANT

In Ref. [3], the one-loop contribution of φ to the vacuum energy was computed using the Euclidean Green's function, finding that φ contributes the same as two normal massless scalars to the cosmological constant Λ . By counting degrees of freedom, one can then cancel the ultraviolet contributions of the Standard Model particles to Λ , excluding those of the Higgs boson, by adding 36φ fields. (It was hypothesized that the Higgs could be a bound state of standard model fermions, and would therefore not make a separate contribution.) It is unclear how the results of this derivation would carry over to Minkowski space.

We again use the two-field formulation of Eq. (4) restricted to Minkowski space, in which computations simplify. One can readily show that the propagator in the (φ, σ) field space has the matrix form,

$$G_{ij}(x, x') = i \int \frac{d^4 k}{(2\pi)^4} \begin{pmatrix} \frac{1}{k^4} & \frac{1}{-k^2 - is_1 \epsilon} \\ \frac{1}{-k^2 - is_2 \epsilon} & 0 \end{pmatrix}, \quad (61)$$

$$\times e^{ik \cdot (x - x')},$$

where the factors $s_i = \pm 1$ allow for general sign choices for the $i\epsilon$ prescriptions of the two fields. Notice that φ and σ have different dimensions, leading to different powers of k^2 amongst the matrix elements. The $1/k^4$ term (shorthand for the product of the two off-diagonal propagators) does not contribute to the vacuum energy. One can compute the expectation value of the Hamiltonian (5) from the propagator,

$$\begin{aligned} \langle \mathcal{H} \rangle &= \frac{1}{2} \lim_{x \rightarrow x'} [\partial_t \partial_{t'} + \vec{\nabla}_x \cdot \vec{\nabla}_{x'}] (G_{12} + G_{21}) \\ &= \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} |\vec{k}| (s_1 + s_2). \end{aligned} \quad (62)$$

If $s_1 = s_2 = +1$, this gives the result of Ref. [3], namely twice the contribution of a normal scalar field. However, one of the fields is still a ghost, and choosing the sign of ϵ to give it positive energy endows it with negative probability, causing violation of unitarity in scattering amplitudes, as we will show in Sec. VII.

It is enlightening to repeat the calculation using field redefinitions that convert the Lagrangian to one involving conventionally normalized scalar fields with mass dimension one,

$$\varphi = \frac{1}{2M} (\eta + \psi), \quad \sigma = M(\eta - \psi), \quad (63)$$

where M is an arbitrary mass scale. Then the Lagrangian becomes

$$\mathcal{L} = \frac{1}{2} (-(\partial\eta)^2 + (\partial\psi)^2 - M^2(\eta - \psi)^2). \quad (64)$$

In the (η, ψ) basis, the matrix propagator in momentum space is

$$G_{ij} = \text{diag} \left(\frac{i}{-k^2 - is_1 \epsilon}, \frac{-i}{-k^2 - is_2 \epsilon} \right) + i \frac{M^2}{k^4} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad (65)$$

and the Hamiltonian density is

$$\mathcal{H} = \frac{1}{2} (\dot{\eta}^2 + |\nabla\eta|^2 - \dot{\psi}^2 - |\nabla\psi|^2 + M^2(\eta - \psi)^2). \quad (66)$$

Because of the wrong sign for the G_{22} element, the contribution of ψ to the expectation value $\langle \mathcal{H} \rangle$ is the same as that of η when $s_1 = s_2$, the sign in the propagator canceling the sign in $\mathcal{H}$, despite that ψ is a ghost while η is a normal excitation. The contributions going as M^2 must cancel since the choice of M is arbitrary and cannot affect any observable quantity. In particular, results should be continuous in the limit $M \rightarrow 0$. Hence this gives the same result as Eq. (62).

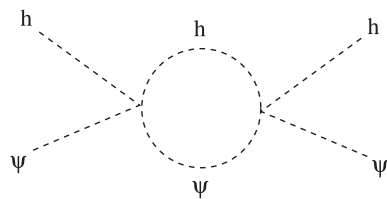
Even if we were to accept the presence of the ghost field, there are several other deficiencies in this mechanism. First, it approximates all the SM fields as being massless. Therefore only the leading quartic divergences to the cosmological constant are cancelled. The subleading ones that are quadratic in the cutoff and in the masses are uncanceled, leaving a residual contribution that is still many orders of magnitude above the observed value.

Second, the conclusions of Ref. [3] might be interpreted as a numerical coincidence, that the addition of 36 dimension-zero scalars simultaneously cancels the Weyl anomaly and the cosmological constant, if one assumes that there are three right-handed neutrinos, *and* that the Higgs field is a composite state made of fields already within the fermionic sector of the SM. The latter assumption is not supported by any theoretical studies that we are aware of; the known framework that comes closest is top quark condensation [18], but this requires new degrees of freedom beyond the SM. Moreover, the numerical coincidence only occurs if the possible linear couplings between φ and curvature invariants, mentioned in the introduction in connection with Ref. [6], are omitted. By including them, simultaneous cancellations of the Weyl anomaly and the quartically divergent contributions to the vacuum energy would be possible for any field content.

VII. LOSS OF UNITARITY

In this section, we will consider generic violations of unitarity that can occur in the higher-derivative theory as a consequence of the ghosts. The field reparametrization (63) makes it easier to appreciate these difficulties. Let us consider a hypothetical coupling $\lambda h^2 \psi^2$ of the Higgs field h to the ghost field ψ . Suppose that $s_2 > 0$ so that both ψ carries positive energy. Then the tree-level scattering $h\psi \rightarrow h\psi$ is allowed, but the ψ propagator in the one-loop contribution has the wrong sign, and the imaginary part of this contribution to $h\psi \rightarrow h\psi$ will inherit this wrong sign, in violation of the optical theorem. The Feynman diagram is shown in Fig. 3 (left).

In the case where $s_2 < 0$, so that ψ carries negative energy, the scattering amplitudes for $h\psi \leftrightarrow h\psi$ are still kinematically allowed. Unitarity is violated for the process $h\psi \rightarrow h\psi$ because of the wrong-sign propagator for ψ in the loop, even though ψ has a positive norm in the Hilbert space. The imaginary part of the loop diagram does not depend upon s_2 in this case, since the Higgs has a finite width Γ_h . When combining denominators using the Feynman parameter integral x , the imaginary part of the combined denominator becomes $i(x\Gamma_h + (1-x)s_2\epsilon)$, hence one can ignore ϵ in the loop.



We have illustrated the breakdown of unitarity in a model where φ couples to another scalar field for simplicity, but even in the absence of such couplings, analogous interactions with gravitons are inevitable, and we expect the same problems to occur.

VIII. COSMOLOGICAL FLUCTUATIONS AND FIFTH FORCES

Reference [3] noted that the vacuum fluctuations of φ take the form,

$$\begin{aligned} \langle \varphi^2 \rangle &= \lim_{x \rightarrow x'} G_{\varphi\varphi}(x, x') = \int \frac{d^4 k}{(2\pi)^4} \frac{i}{k^4} \\ &\sim \int \frac{d^3 k}{(2\pi)^3} \frac{1}{k^3}, \end{aligned} \quad (67)$$

using Eq. (61), and doing the integral over k^0 . These are scale-invariant fluctuations, and so it is tempting to think that φ could provide the cosmological seeds needed to get a nearly scale-invariant spectrum as observed in the cosmic microwave background (CMB), without the need to invoke inflation. This idea was developed in Ref. [8], where it was argued that φ (in fact, 36 copies φ_j of it) couples linearly to the energy density ρ_i of each standard model field as a consequence of the trace anomaly,

$$\mathcal{L} \sim \sum_{ij} c_{\chi} \rho_i \varphi_j, \quad (68)$$

where c_{χ} is dominated by the QCD contribution, $c_{\chi} \sim \alpha_s^2 / (36\pi^2)$. Then at high temperatures, the SM plasma sources fluctuations in φ_i which induce a curvature perturbation, whose magnitude is argued to match that needed to explain the observed temperature fluctuations of the CMB.

Here we will not argue with the logic of these assertions; rather we point out that the interaction (68) is ruled out by low-energy observables. Extrapolating it to low energies, where $\alpha_s \sim 1$ and quarks become hadronized (coupling with similar strength to φ_i), the interaction with electrons and protons is of order

$$\mathcal{L} \sim c_{\chi} \sum_i \varphi_i (m_e \bar{e}e + m_p \bar{p}p), \quad (69)$$

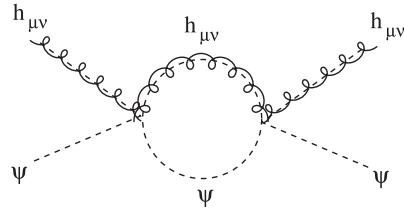


FIG. 3. Left: hypothetical loop contribution to ψ -Higgs scattering, that violates unitarity because of the wrong-sign ψ propagator. Right: analogous unavoidable contribution to graviton- ψ scattering, that similarly violates unitarity.

with $c_\chi \sim 1/(36\pi^2)$. The exchange of φ_i creates a fifth force between the electron and the proton, whose matrix element at low momentum transfer $\vec{k}^2$ goes as

$$\mathcal{M} \sim 36c_\chi^2 m_e m_p (\bar{u}_e u_e)(\bar{u}_p u_p) \frac{1}{\vec{k}^4} \quad (70)$$

in terms of the Dirac spinors, using the $G_{\varphi\varphi}$ element of the propagator (61). The corresponding potential energy is found by Fourier transforming $\mathcal{M}$, which by dimensional analysis corresponds to a linearly confining potential between the electron and the proton,

$$V(\vec{x}) \sim 36c_\chi^2 m_e m_p |\vec{x}|. \quad (71)$$

One can readily show that this is many orders of magnitude stronger than the Coulomb potential at distances of order the Bohr radius; hence the properties of atoms would be drastically changed by such a force.

IX. CONCLUSIONS

The appearance of zero-dimension scalar fields in conformal supergravity, a theory with interesting properties that might have recommended it as a consistent unification with gravity, motivates their further study. In this paper we have made a thorough analysis of the conformal $\varphi\Delta_4\varphi$ theory, both in Minkowski space and in gravitational backgrounds, either considering the background to be fixed or fluctuating.

As one would expect, the theory, being fourth order in derivatives, has a ghost in any of these settings. In the general case, the theory propagates two scalar degrees of freedom, out of which at least one is a ghost mode. When the system is coupled to gravity, it can happen that both scalars become ghosts. This is the case when φ has a classical background that dominates the expansion, which we showed simulates a radiation-dominated Universe. If φ partially dominates the expansion, the situation is complicated, but always contains at least one ghost. In the limit where φ becomes a spectator field, the system has similar behavior to Minkowski space, with one ghost and one nonghost field.

At the quantum level, we have shown that the ghosts indeed lead to violation of unitarity of scattering

amplitudes, and that the cancellation of the cosmological constant is incomplete at best. Moreover, the couplings of φ to the SM, needed to get the observed spectrum of CMB fluctuations, is ruled out by the drastic effect that φ exchange would have on atomic structure. The dimension zero scalar theory thus creates serious problems of its own, without offering any viable solutions to existing ones.

ACKNOWLEDGMENTS

We thank Simon Caron-Huot for valuable contributions at the start of this work and for comments on the manuscript; also A. Tseytlin for helpful comments and pertinent information. The work of J. C. is supported by the Natural Sciences and Engineering Research Council (NSERC) of Canada. J. C. thanks the Niels Bohr International Academy for its hospitality during the inception of this study. A. H. would like to thank Robert Brandenberger for useful discussions, and the McGill University for hospitality, where part of this work was carried out. The work of A. H. was supported in part by the JSPS KAKENHI Grant No. JP26K17133, KAKENHI Grant No. JP25H00403, the CD3 Google Seed grant, and by the World Premier International Research Center Initiative (WPI), MEXT, Japan.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX: COEFFICIENTS FOR SEC. III

Here we provide the full expressions for the coefficients for the scalar modes corresponding to Eq. (23). They are given by

$$c_1(t, k^2) = -\frac{24\dot{\varphi}^2 a^5}{(32k^2 + 24a\ddot{a} + 3\dot{a}^2)\dot{\varphi}^2 + 12\dot{\varphi}\dot{a}\ddot{\varphi}a + 36\ddot{\varphi}^2 a^2}, \quad (A1)$$

and

$$\begin{aligned} \gamma(t, k^2)c_2(t, k^2) = & -\left(\left(-\frac{15\ddot{a}\dot{a}}{16} + \frac{17\ddot{a}^2}{8}\right)a^2 + \frac{43(k^2 + \frac{93\dot{a}^2}{172})\ddot{a}a}{12} + k^4 + \frac{313k^2\dot{a}^2}{96} + \frac{17\dot{a}^4}{32}\right)\dot{\varphi}^4 \\ & + \frac{11a\ddot{\varphi}(\dot{a}k^2 + \frac{5}{11}\ddot{a}a^2 + \frac{37}{11}\dot{a}\ddot{a}a + \frac{24}{11}\dot{a}^3)\dot{\varphi}^3}{8} + \frac{13a^2(\frac{16a\ddot{a}}{13} + k^2 + \frac{185\dot{a}^2}{52})\ddot{\varphi}^2\dot{\varphi}^2}{8} + \frac{11\ddot{\varphi}^3 a^3 \dot{a} \dot{\varphi}}{16} - \frac{3a^4 \ddot{\varphi}^4}{8}, \end{aligned} \quad (A2)$$

with

$$\gamma(t, k^2) = \frac{k^2(36\dot{\phi}^2 a^2 + 12\dot{a} \dot{\phi} \ddot{\phi} a + 24\ddot{a}\dot{\phi}^2 a + 32\dot{\phi}^2 k^2 + 3\dot{a}^2 \dot{\phi}^2)^2}{1152a^3}. \quad (\text{A3})$$

In the high- k limit, the above two expressions simplify, giving

$$c_1(t, k^2) \sim -\frac{3a^5}{4k^2} \quad (\text{A4})$$

and

$$c_2(t, k^2) \sim -\frac{9a^3}{8k^2}. \quad (\text{A5})$$

To find the speed of propagation, we find the corresponding equation of motion, and assume the plane-wave solution $\sigma \sim e^{-i\omega t}$. By further setting $\omega = \frac{k}{a} \sqrt{c_s}$, where c_s is the speed of propagation, we arrive to the following value in the high- k limit,

$$c_s^2 = \frac{3}{2}. \quad (\text{A6})$$

For the special case of Minkowski space $a = 1$ with the exact solution $\dot{\phi} = 1$, we find

$$c_1 = -\frac{3}{4}, \quad c_2 = -\frac{9}{8}, \quad (\text{A7})$$

giving a propagation speed of $\sqrt{3/2}$ and a ghostlike kinetic term.

1. Tensor fluctuations in the Δ_4 theory without the Einstein term

The Lagrangian density for the tensor perturbations corresponding to the action (1) is given by

$$\mathcal{L}_T = -a\dot{\phi}^2 \left(\frac{1}{3} a^2 h_{ij}^T h_{ij}^T + \frac{1}{6} h_{ij,k}^T h_{ij,k}^T \right). \quad (\text{A8})$$

The tensor fluctuation is thus a ghost at all wavenumbers, and it is also tachyonic because of the relative $+$ sign between the two terms. The squared propagation speed $v_s^2 = -1/2$ is subluminal in magnitude. The kinetic term vanishes as $\dot{\phi} \rightarrow 0$, signaling a possible strong-coupling obstruction when canonically normalizing $h_{ij}^T \rightarrow h_{ij}^T/|\dot{\phi}|$ and accounting for the gravitational couplings of h_{ij}^T at cubic and higher order. This is analogous to the van Dam-Veltman-Zakharov discontinuity [19,20] that separates massive and massless gravity theories at linear order in the perturbations. Although this discontinuity might be resolved through the Vainshtein mechanism similarly to the massive gravity, or a similar mechanism in the massive Yang-Mills theory [21,22], here we will not further pursue this aspect of the theory in the light of the sicknesses which afflict it already on the level of the scalar perturbations.

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