

Electroweak scalar effects beyond dimension-six in SMEFT

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The Standard Model effective field theory (SMEFT) provides a robust framework for probing deviations in the couplings of Standard Model particles from their theoretical predictions. This framework relies on an expansion in higher-dimensional operators, often truncated at dimension-six. In this work, we compute the effective dimension-eight operators generated by integrating out heavy scalar fields at one-loop order in the Green's basis within two extended scalar sector models: the two Higgs doublet model and the complex triplet scalar model. In this work, we consider loops that consist of heavy field propagators only. We also investigate the impact of heavy scalar fields on the fermion sector, deriving the fermionic effective operators up to dimension-eight for these models, and detail how contributions can be mapped onto nonredundant bases. To assess the importance of higher-order contributions in the SMEFT expansion, we analyze dimension-eight effects for electroweak precision observables at the next frontier of precision lepton machines such as GigaZ.

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I. INTRODUCTION

In the past few decades, high-energy physics has made tremendous strides in understanding our Universe's fundamental forces and particles. Central to this understanding is the Standard Model (SM) of particle physics, a remarkably successful framework for describing the known particles and interactions, most notably through experimental verification at facilities such as the Large Hadron Collider (LHC). However, known limitations of the SM have led physicists to look for traces of physics beyond the Standard Model (BSM), which might manifest at higher energy scales that are not yet accessible through direct experiments.

Effective field theories (EFTs) have become a prevalent tool in this context. EFTs allow us to describe the effects of unknown high-energy physics at low energies [1,2], where direct measurements are possible, by expanding the known

theory in terms of higher-dimensional operators. Typically, these expansions include terms that capture the influence of new physics, even if the fundamental theory is not directly observed. The most commonly used EFT for collider experiments like those at the LHC is the SM effective field theory (SMEFT). This formalism expands the SM by adding higher-dimension operators that encapsulate the possible deviations from the SM predictions due to unknown high-energy phenomena. These operators consist of only SM fields and respect the gauge symmetries of the SM.

The dimension-six operators [3–5] are often the leading terms considered in these expansions, as they offer the first corrections to the SM predictions. In practice, SMEFT analyses often focus on dimension-six operators up to linear order (e.g., [6–10]), corresponding to $\mathcal{O}(1/\Lambda^2)$ in the new physics scale Λ (e.g., [11–15]). Some studies extend to quadratic contributions at $\mathcal{O}(1/\Lambda^4)$ (e.g., [16–22]), but a consistent treatment at this order generally requires accounting for terms linear in dimension-eight operators. In many cases, such as rare processes or regions of parameter space with limited data, these dimension-six terms might not capture all relevant effects, especially when the data lack precision [23–25]. As experimental searches, such as those conducted at the high-luminosity LHC (HL-LHC), push the boundaries of sensitivity, it becomes increasingly important to explore higher-order effects. This is where dimension-eight operators, e.g., [26–28], come into play [25,29–33]. These terms can provide a more

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refined description of possible BSM effects, offering corrections to processes where the dimension-six terms may be insufficient [34]. The impact of dimension-eight operators is crucial to validate the truncation of SMEFT at a fixed order in light of collider searches. This has been facilitated by the recent development of comprehensive and nonredundant bases for dimension-eight operators [35,36]. Another useful approach involves matching the SMEFT framework with a particular complete UV model containing new particles on the high-energy scale, Λ . This process often results in a limited set of effective operators, whose impact can be analyzed in low-energy and electroweak-scale phenomena [37–41].

This work investigates the role of dimension-eight operators within the EFT framework to enhance the predictive accuracy of SMEFT and its ability to constrain or identify BSM physics. Specifically, we focus on two well-motivated BSM scenarios that extend the SM with additional heavy scalar fields, one involving a complex scalar triplet and the other involving a complex scalar doublet. These models are of interest because such additional scalar fields are common in various extensions of the SM, including models related to neutrino mass generation and grand unification theories [42–48] and can leave characteristic signatures at different collider experiments [49–58]. The matching of dimension-six operators to UV models with a single heavy particle is well established at both tree level and one-loop [59–67], where the WCs are computed assuming only heavy particles in the loop. There are additional contributions from the loops that consist of light and heavy (LH) propagators. The LH-mixing effects up to dimension-six are discussed in [68–71]. It has been noted that the inclusion of dimension-eight operators in more complex models remains relatively unexplored. Previous studies have addressed only a limited number of cases [25,72–74]. In this study, we extend the framework by integrating out heavy scalars up to one-loop, focusing on complex triplet and doublet scalar scenarios.

We employ the model-independent dimension-eight effective action, derived in [75], to integrate out heavy scalar fields at the one-loop level for specific UV models, namely the electroweak complex triplet and doublet. Reference [75] takes care of the contributions from the loops that consist of heavy scalar propagators only. We utilize this model-independent effective action to calculate the local effective operators up to dimension-eight. Thus, our result is restricted to heavy particle loops. To capture the effects of light-heavy mixed propagator loops, we need to develop the universal effective action up to dimension-eight, which has yet to be done. We obtain the effective operators up to dimension-eight and the associated coefficients in Sec. II. We further capture the imprint of heavy field interactions with the SM fermions in Sec. III. In general, these results serve a dual purpose. First, they enable us to understand the impact of extending the analysis to dimension-eight, without increasing the number of free parameters, as they are functions of the same UV model parameters. We expect these additional effects to enable us to adjust the model parameter space more precisely and enlarge the parameter space where the EFT truncation is still validated. We impose constraints on the EFT parameter space for both models by utilizing electroweak precision data in Sec. IV. A specific focus is placed on the analysis of future precision lepton machines. Finally, we conclude in Sec. V. We put all the details of the computations in Appendix A for the complex triplet and doublet scalar models. In Appendix C, we tabulate the contributions for dimension-eight structures after removing redundancies at dimension-eight.

II. A MODEL-INDEPENDENT APPROACH TO INTEGRATING OUT A HEAVY SCALAR

Consider a UV-complete action S , which depends on heavy (Φ) and light (ϕ) fields. The effective action at one-loop after integrating out the fluctuations¹ around its classical solution (Φ_c) is given by

$$e^{iS_{\text{eff}}[\phi]} = \int [\mathcal{D}\Phi] e^{iS[\Phi, \phi]} = \int [\mathcal{D}\eta] \exp \left[i \left(S[\Phi_c, \phi] + \frac{1}{2} \frac{\delta^2 S}{\delta \Phi^2} \Big|_{\Phi=\Phi_c} \eta^2 + \mathcal{O}(\eta^3) \right) \right],$$

$$S_{\text{eff}} \approx S[\Phi_c] + \frac{i}{2} \ln \left(\text{Det} \frac{\delta^2 S}{\delta \Phi^2} \Big|_{\Phi=\Phi_c} \right). \quad (2.1)$$

In our analysis, the heavy field does not acquire any vacuum expectation value (VEV). Thus, all the component fields in the heavy scalar multiplet are degenerate. This allows us to integrate out the heavy scalar multiplet in a single step. After integrating out heavy scalars at one-loop, which consists of heavy-field propagators, the effective action consisting of operators up to mass dimension-eight is computed for the first time in [75] employing the heat-kernel method. The

model-independent one-loop effective action² up to dimension-eight is given as [75]

¹We assume $\Phi = \Phi_c + \eta$, where η are the quantum fluctuations.

²In Ref. [76], the same has been derived for fermions and noted its equivalence with the scalar case.

$$\begin{aligned}
\mathcal{L}_{\text{eff}}^{d \leq 8} &= \frac{c_s}{(4\pi)^2} M^4 \left[-\frac{1}{2} \left(\ln \left[\frac{M^2}{\mu^2} \right] - \frac{3}{2} \right) \right] + \frac{c_s}{(4\pi)^2} \text{Tr} \left\{ M^2 \left[- \left(\ln \left[\frac{M^2}{\mu^2} \right] - 1 \right) U \right] + M^0 \frac{1}{2} \left[-\ln \left[\frac{M^2}{\mu^2} \right] U^2 - \frac{1}{6} \ln \left[\frac{M^2}{\mu^2} \right] (G_{\mu\nu})^2 \right] \right. \\
&\quad + \frac{1}{M^2} \frac{1}{6} \left[-U^3 - \frac{1}{2} (P_\mu U)^2 - \frac{1}{2} U (G_{\mu\nu})^2 - \frac{1}{10} (J_\nu)^2 + \frac{1}{15} G^{\mu\nu} G_{\nu\rho} G^\rho{}_\mu \right] \\
&\quad + \frac{1}{M^4} \frac{1}{24} \left[U^4 - U^2 (P^2 U) + \frac{4}{5} U^2 (G_{\mu\nu})^2 + \frac{1}{5} (U G_{\mu\nu})^2 + \frac{1}{5} (P^2 U)^2 \right. \\
&\quad - \frac{2}{5} U (P_\mu U) J^\mu + \frac{2}{5} U (J_\mu)^2 - \frac{2}{15} (P^2 U) (G_{\rho\sigma})^2 + \frac{1}{35} (P_\nu J_\mu)^2 \\
&\quad - \frac{4}{15} U G^{\mu\nu} G_{\nu\rho} G^\rho{}_\mu - \frac{8}{15} (P_\mu P^\nu U) G^{\rho\mu} G_{\rho\nu} + \frac{16}{105} G^{\mu\nu} J_\mu J_\nu + \frac{16}{105} (P^\mu J_\nu) G^{\nu\sigma} G_{\sigma\mu} + \frac{1}{420} (G_{\mu\nu} G_{\rho\sigma})^2 \\
&\quad \left. + \frac{17}{210} (G_{\mu\nu})^2 (G_{\rho\sigma})^2 + \frac{2}{35} (G_{\mu\nu} G_{\nu\rho})^2 + \frac{1}{105} G^{\mu\nu} G_{\nu\rho} G^{\rho\sigma} G_{\sigma\mu} \right] \left. \right\} \\
&= \mathcal{L}_4 + \mathcal{L}_2 + \mathcal{L}_0 + \mathcal{L}_{-2} + \mathcal{L}_{-4},
\end{aligned} \tag{2.2}$$

where $c_s = 1/2$ for real scalar fields while $c_s = 1$ for complex scalar fields.

Here, $P_\mu = iD_\mu$, where D_μ denotes the covariant derivative in the Euclidean space. The stress tensor is given, as usual, $G_{\mu\nu} = [P_\mu, P_\nu]$; $J_\mu = D^\rho G_{\rho\mu} = D^\rho [D_\rho, D_\mu]$ denotes the gauge current. The elliptic operator $\Delta \equiv -\frac{\delta^2 \mathcal{L}}{\delta \Phi^\dagger \delta \Phi} \equiv -P^2 + M^2 + U \equiv D^2 + M^2 + U$, which encapsulates the interactions of the heavy scalar in theory, will be discussed later. The trace in the above expression accounts for all symmetry indices, including gauge, Lorentz (spinor and vector), flavor indices, etc. Additionally, Hermitian conjugates have been incorporated to ensure the self-Hermitian nature of the effective Lagrangian.

Let us consider a toy Lagrangian with a heavy scalar field Φ of mass M_s

$$\mathcal{L}^\Phi = \Phi^\dagger (P^2 - M_s^2) \Phi - \mathcal{L}_{\text{int}}. \tag{2.3}$$

Here, $\mathcal{L}_{\text{int}}$ represents the interaction part of the Lagrangian that involves both heavy (Φ) and light fields (ϕ). To demonstrate our methodology, we consider the interacting Lagrangian in the following form:

$$\begin{aligned}
\mathcal{L}_{\text{int}} &\supset \lambda (\phi^\dagger \phi)^2 + \lambda_1 (\Phi^\dagger \Phi)^2 + \eta (\phi^\dagger \phi) (\Phi^\dagger \Phi) \\
&\quad + (\kappa_1 (\phi^\dagger \phi) \Phi + \kappa_2 \phi^\dagger (\Phi^\dagger \Phi) + \text{H.c.}).
\end{aligned} \tag{2.4}$$

The parameters κ_1 and κ_2 have dimensions of mass and we assume $\kappa_1, \kappa_2 \ll M_s$. The effective action at the tree level arises from the term linear in the heavy field in the UV model. The Lagrangian in Eq. (2.3) takes the form

$$\begin{aligned}
\mathcal{L}^\Phi &\ni \{\Phi^\dagger B_s + \text{H.c.}\} + \Phi^\dagger (P^2 - M_s^2) \Phi \\
&\quad + \mathcal{F}(\Phi^\dagger \Phi, \phi^\dagger \phi) + \mathcal{O}(\Phi^3),
\end{aligned} \tag{2.5}$$

such that B_s and $\mathcal{F}$ contain interactions that are linear and quadratic in Φ , respectively; they are functions of light

fields only. For this sample Lagrangian, we define $B_s = -\kappa_1^* (\phi^\dagger \phi)$ and $U_s \equiv -\frac{\delta^2 \mathcal{F}}{\delta \Phi^\dagger \delta \Phi} = \eta (\phi^\dagger \phi) + 4\lambda_1 (\Phi^\dagger \Phi) + \kappa_2 \phi^\dagger + \kappa_2^* \phi$. We solve the equation of motion for the heavy field Φ and substitute it into the action. For the leading approximation, the classical solution of Φ reads as

$$\Phi_c = -\frac{1}{P^2 - M_s^2 - U_s} B_s. \tag{2.6}$$

Employing the covariant derivative expansion, we find³

$$\begin{aligned}
\Phi_c &= \left[1 - \frac{1}{M_s^2} (P^2 - U_s) \right]^{-1} \frac{1}{M_s^2} B_s \\
&= \frac{1}{M_s^2} B_s + \frac{1}{M_s^2} (P^2 - U_s) \frac{1}{M_s^2} B_s \\
&\quad + \frac{1}{M_s^2} (P^2 - U_s) \frac{1}{M_s^2} (P^2 - U_s) \frac{1}{M_s^2} B_s + \dots \\
&\equiv \Phi_c^{(0)} + \frac{\Phi_c^{(1)}}{M_s^2} + \frac{\Phi_c^{(2)}}{M_s^4} + \frac{\Phi_c^{(3)}}{M_s^6} + \dots
\end{aligned} \tag{2.7}$$

In the above equation, we highlight the series expansion of Φ_c . Here, U_s is a functional of ϕ , and Φ_c .⁴ Thus, Φ_c can be determined by inserting B_s and U_s in Eq. (2.7) and solving order by order in M_s^{-2} .

After integrating out Φ , a series of operators is obtained that can be systematically organized based on their canonical mass dimension. The effective Lagrangian can then be obtained up to the desired order by substituting Φ_c into

³Here, we assume the background field varies very slowly compared to the UV mass scale M_s and the local expansion of the UV-propagator is validated in the domain of the energy of our interest.

⁴A more detailed discussion on the dimension of U and Φ_c is presented when considering specific UV models in the subsequent sections.

Eq. (2.5) via Φ_c . Now, if the effective Lagrangian is to be truncated at order M_s^{-4} , we do not need to consider $\Phi_c^{(2)}$, which is the coefficient $\sim M_s^{-4}$ in the solution of Φ_c . To see this, we note that at order M_s^0 and M_s^{-2} , we have $\Phi_c^{(0)} = 0$ and $\Phi_c^{(1)} = B_s$, respectively. It is worth noting that the dimensions of the operators within $\Phi_c^{(i)}$ in Eq. (2.7) for different i 's are different. We have highlighted the effective Lagrangian with different colors, depending on its heavy mass $M \equiv M_s$ suppression factor. This helps us to identify up to which order in Φ_c we should feed in $U[\Phi_c]$ in Eq. (2.2). For example, setting $\Phi_c \equiv \Phi_c^{(1)}$ is sufficient to obtain dimension-eight operators from the blue-tinted part ($\mathcal{L}_{-4}$) of the effective Lagrangian in Eq. (2.2). Other parts, e.g., black ($\mathcal{L}_0$), green ($\mathcal{L}_2$), and magenta ($\mathcal{L}_{-2}$) colored texts, can offer dimension-eight operators if we employ higher order expansions in Φ_c . This works as $U \equiv U_s$ in Eq. (2.2) is not only a function of the light field ϕ , but also a heavy field-classical solution (Φ_c). Thus, dimension-eight operators may receive contributions from terms like $U_s[\Phi_c]$, $U_s^2[\Phi_c]$, see terms in $\mathcal{L}_2$ and $\mathcal{L}_0$ in Eq. (2.2). Here, the term $\Phi_c^{(3)}$ may contribute to the dimension-eight operator via structures as follows: $U_s[\Phi_c] \sim |\Phi_c|^2 \sim \Phi_c^{(3)} \Phi_c^{(1)}$ which leads to terms such as $\{U_s[\phi]^2 B_s[\phi]^2; P^2 U_s[\phi] B_s[\phi]^2; P^4 B_s[\phi]^2\} \sim \{\phi^8; P^2 \phi^6; P^4 \phi^4\}$.

We are ready to suitably adopt and apply this procedure to the particular cases of two BSM-relevant UV models, namely the complex scalar triplet model and the 2HDM, to enumerate the possible operators up to dimension-eight. We compute the corresponding operator Wilson coefficients in terms of the parameters of the UV theory (note that these results do not include mixed heavy-light effects). An implicit assumption throughout is that the heavy-mass scales are much greater than the weak scale. This condition is essential for the SMEFT to be valid for UV models, ensuring the decoupling of heavy degrees of freedom. In the following subsections, we present the leading contributions from loop-generated operators with integrated-out contributions at the dimension-eight level.

A. Case I: Electroweak complex triplet

We consider an electroweak $SU(2)_L$ scalar triplet $\Delta = \Delta^i T^i$ with hypercharge $Y = 1$. The generators of $SU(2)_L$ in the fundamental representation are given by $T^i = \sigma^i/2$ with the Pauli matrices σ^i with $i = 1, 2, 3$. The relevant part of the Lagrangian that involves a heavy complex scalar triplet [77] is

$$\mathcal{L}_{\text{BSM}} = (\text{Tr}[(\mathcal{D}_\mu \Delta)^\dagger (\mathcal{D}^\mu \Delta)] - M_\Delta^2 \text{Tr}[\Delta^\dagger \Delta] - \mathcal{L}_Y - V(H, \Delta)), \quad (2.8)$$

where the scalar potential is

$$V(H, \Delta) = \lambda_1 (H^\dagger H) \text{Tr}[\Delta^\dagger \Delta] + \lambda_2 (\text{Tr}[\Delta^\dagger \Delta])^2 + \lambda_3 \text{Tr}[(\Delta^\dagger \Delta)^2] + \lambda_4 (H^\dagger \Delta \Delta^\dagger H) + [\mu_\Delta (H^T i \sigma^2 \Delta^\dagger H) + \text{H.c.}]. \quad (2.9)$$

The covariant derivative has the usual form as $\mathcal{D}_\mu \equiv \partial_\mu - ig' Y B_\mu - ig T^i W_\mu^i$. Here, Y and T^i represent the generators of $U(1)_Y$ and $SU(2)_L$, respectively, with $i = 1, 2, 3$. In the adjoint representation $(T^i)_{jk} = i\epsilon^{ijk}$ with $i, j, k = 1, 2, 3$. The Yukawa term in the Lagrangian is

$$\mathcal{L}_Y = Y_{\Delta ab} L_a^T C i \sigma^2 \Delta L_b + \text{H.c.}, \quad (2.10)$$

where C represents the charge conjugation matrix, while $a, b = e, \mu, \tau$ denotes the flavor indices. This triplet model allows for the generation of the lepton-number-violating Majorana masses for the SM neutrinos, and that feature is also reflected through the effective operators.

In our setup, the BSM scalar triplet Δ is heavy and does not acquire a VEV. Consequently, all components of Δ are degenerate in mass, and we integrate out the entire multiplet in a single step. This ensures that the effective field theory matches the SMEFT rather than the HEFT framework. Electroweak symmetry breaking is not invoked until after the integration is performed. The functional $B \equiv \mathcal{L}/\text{mod}(\Delta)$ is derived from the interaction in which the heavy field Δ couples with the lighter fields linearly. For the scalar Lagrangian Eq. (2.9), it is given as

$$B^i = -\mu_\Delta H^T i \sigma_2 \tau^i H = \mu_\Delta \tilde{H}^\dagger \tau^i H, \quad (2.11)$$

where $\tilde{H} = i\sigma_2 H^*$ is the charge-conjugated Higgs field. Note that $\mu_\Delta \ll M_\Delta$ is a mass-dimension one coupling.

The scalar potential for the heavy triplet field [using Eq. (A1)] can be written in the following basis of heavy fields:

$$\mathcal{L}_{\text{int}} \supset \frac{1}{2} \begin{pmatrix} \Delta_i^* & \Delta_i \end{pmatrix} \begin{pmatrix} (U_{11})_{ij} & (U_{12})_{ij} \\ (U_{21})_{ij} & (U_{22})_{ij} \end{pmatrix} \begin{pmatrix} \Delta_j \\ \Delta_j^* \end{pmatrix}, \quad (2.12)$$

where the indices i, j range from 1 to 3, corresponding to the three component fields. Thus, U is a 6×6 Hermitian matrix, where each 3×3 block [Eq. (2.12)] in the (Δ_i, Δ_j^*) basis is given by

$$(U_{11})_{ij} = \frac{\delta^2 V}{\delta \Delta_i^* \delta \Delta_j}, \quad (U_{12})_{ij} = \frac{\delta^2 V}{\delta \Delta_i \delta \Delta_j}, \\ (U_{21})_{ij} = \frac{\delta^2 V}{\delta \Delta_i^* \delta \Delta_j^*}, \quad (U_{22})_{ij} = \frac{\delta^2 V}{\delta \Delta_i \delta \Delta_j^*}.$$

Hence, we obtain these matrix elements as

TABLE I. The matching results for relevant dimension-eight operators in the SMEFT for the complex triplet model with the heavy triplet integrated up to one-loop consist of heavy propagators only. The terms in blue ($\mathcal{L}_{-4}$), magenta ($\mathcal{L}_{-2}$), black ($\mathcal{L}_0$), and green ($\mathcal{L}_2$) denote the origin of these contributions in Eq. (2.2), respectively.

EFT operators	Matching results at scale $M_\Delta (\gg v)$
$\mathcal{O}_{\phi^8} = (\phi^\dagger \phi)^4$	$\mathbb{C}_{\phi^8} = \frac{1}{16\pi^2} (1 - \ln(\frac{M_\Delta^2}{\mu^2})) c_{\phi^8}^{\llbracket U \rrbracket} - \frac{1}{32\pi^2} \ln(\frac{M_\Delta^2}{\mu^2}) c_{\phi^8}^{\llbracket U^2 \rrbracket} - \frac{1}{96\pi^2} c_{\phi^8}^{\llbracket U^3 \rrbracket} + \frac{1}{384\pi^2} c_{\phi^8}^{\llbracket U^4 \rrbracket}$
$\mathcal{O}_{\phi^6}^{(1)} = (\phi^\dagger \phi)^2 (D_\mu \phi^\dagger D^\mu \phi)$	$\mathbb{C}_{\phi^6}^{(1)} = \frac{1}{16\pi^2} (1 - \ln(\frac{M_\Delta^2}{\mu^2})) c_{\phi^6}^{(1), \llbracket U \rrbracket} - \frac{1}{384\pi^2} c_{\phi^6}^{(1), \llbracket U^2 (P^2 U) \rrbracket}$
$\mathcal{O}_{\phi^6}^{(2)} = (\phi^\dagger \phi) (\phi^\dagger \sigma^I \phi) (D_\mu \phi^\dagger \sigma^I D^\mu \phi)$	$\mathbb{C}_{\phi^6}^{(2)} = -\frac{1}{384\pi^2} c_{\phi^6}^{(2), \llbracket U^2 (P^2 U) \rrbracket}$
$\mathcal{O}_{\phi^6}^{(3)} = (\phi^\dagger \phi)^2 (\phi^\dagger D^2 \phi + \text{H.c.})$	$\mathbb{C}_{\phi^6}^{(3)} = \frac{1}{16\pi^2} (1 - \ln(\frac{M_\Delta^2}{\mu^2})) c_{\phi^6}^{(3), \llbracket U \rrbracket} - \frac{1}{192\pi^2} c_{\phi^6}^{(3), \llbracket (P_\mu U)^2 \rrbracket} - \frac{1}{384\pi^2} c_{\phi^6}^{(3), \llbracket U^2 (P^2 U) \rrbracket}$
$\mathcal{O}_{W^2 \phi^4}^{(1)} = (\phi^\dagger \phi)^2 W_{\mu\nu}^I W^{I\mu\nu}$	$\mathbb{C}_{W^2 \phi^4}^{(1)} = -\frac{1}{192\pi^2} c_{W^2 \phi^4}^{(1), \llbracket U (G_{\mu\nu})^2 \rrbracket} + \frac{1}{480\pi^2} c_{W^2 \phi^4}^{(1), \llbracket U^2 (G_{\mu\nu})^2 \rrbracket} + \frac{1}{1920\pi^2} c_{W^2 \phi^4}^{(1), \llbracket (U G_{\mu\nu})^2 \rrbracket}$
$\mathcal{O}_{W^2 \phi^4}^{(3)} = (\phi^\dagger \sigma^I \phi) (\phi^\dagger \sigma^J \phi) W_{\mu\nu}^I W^{J\mu\nu}$	$\mathbb{C}_{W^2 \phi^4}^{(3)} = -\frac{1}{192\pi^2} c_{W^2 \phi^4}^{(3), \llbracket U (G_{\mu\nu})^2 \rrbracket} + \frac{1}{480\pi^2} c_{W^2 \phi^4}^{(3), \llbracket U^2 (G_{\mu\nu})^2 \rrbracket} + \frac{1}{1920\pi^2} c_{W^2 \phi^4}^{(3), \llbracket (U G_{\mu\nu})^2 \rrbracket}$
$\mathcal{O}_{B^2 \phi^4}^{(1)} = (\phi^\dagger \phi)^2 B_{\mu\nu} B^{\mu\nu}$	$\mathbb{C}_{B^2 \phi^4}^{(1)} = -\frac{1}{192\pi^2} c_{B^2 \phi^4}^{(1), \llbracket U (G_{\mu\nu})^2 \rrbracket} + \frac{1}{480\pi^2} c_{B^2 \phi^4}^{(1), \llbracket U^2 (G_{\mu\nu})^2 \rrbracket} + \frac{1}{1920\pi^2} c_{B^2 \phi^4}^{(1), \llbracket (U G_{\mu\nu})^2 \rrbracket}$
$\mathcal{O}_{WB \phi^4}^{(1)} = (\phi^\dagger \phi) (\phi^\dagger \sigma^I \phi) W_{\mu\nu}^I B^{\mu\nu}$	$\mathbb{C}_{WB \phi^4}^{(1)} = \frac{1}{480\pi^2} c_{WB \phi^4}^{(1), \llbracket U^2 (G_{\mu\nu})^2 \rrbracket} + \frac{1}{1920\pi^2} c_{WB \phi^4}^{(1), \llbracket (U G_{\mu\nu})^2 \rrbracket}$
$\mathcal{O}_{\phi^4}^{(3)} = (D^\mu \phi^\dagger D_\mu \phi) (D^\nu \phi^\dagger D_\nu \phi)$	$\mathbb{C}_{\phi^4}^{(3)} = \frac{1}{16\pi^2} (1 - \ln(\frac{M_\Delta^2}{\mu^2})) c_{\phi^4}^{(3), \llbracket U \rrbracket} + \frac{1}{1920\pi^2} c_{\phi^4}^{(3), \llbracket (P^2 U)^2 \rrbracket}$
$\mathcal{O}_{\phi^4}^{(4)} = (D_\mu \phi^\dagger D^\mu \phi) (\phi^\dagger D^2 \phi + \text{H.c.})$	$\mathbb{C}_{\phi^4}^{(4)} = \frac{1}{16\pi^2} (1 - \ln(\frac{M_\Delta^2}{\mu^2})) c_{\phi^4}^{(4), \llbracket U \rrbracket} + \frac{1}{1920\pi^2} c_{\phi^4}^{(4), \llbracket (P^2 U)^2 \rrbracket}$
$\mathcal{O}_{\phi^4}^{(8)} = ((D^2 \phi^\dagger \phi) (D^2 \phi^\dagger \phi) + \text{H.c.})$	$\mathbb{C}_{\phi^4}^{(8)} = \frac{1}{16\pi^2} (1 - \ln(\frac{M_\Delta^2}{\mu^2})) c_{\phi^4}^{(8), \llbracket U \rrbracket} + \frac{1}{1920\pi^2} c_{\phi^4}^{(8), \llbracket (P^2 U)^2 \rrbracket}$

$$(U_{11})_{ij} = \frac{1}{4} (2\lambda_1 + \lambda_4) (H^\dagger H) \delta_{ij} + \frac{1}{2} \lambda_2 (\Delta_a^* \Delta_a \delta_{ij} + \Delta_i \Delta_j^*) + \frac{1}{2} \lambda_3 (\Delta_a^* \Delta_a \delta_{ij} + \Delta_i \Delta_j^* - \Delta_i^* \Delta_j) - \frac{i}{2} \lambda_4 \epsilon_{ijk} (H^\dagger \tau^k H), \quad (2.13)$$

$$(U_{12})_{ij} = \frac{1}{2} \lambda_2 (\Delta_i^* \Delta_j^*) + \frac{1}{4} \lambda_3 (2\Delta_i^* \Delta_j^* - \Delta_a^* \Delta_a \delta_{ij}), \quad (2.14)$$

$$(U_{21})_{ij} = \frac{1}{2} \lambda_2 (\Delta_i \Delta_j) + \frac{1}{4} \lambda_3 (2\Delta_i \Delta_j - \Delta_a \Delta_a \delta_{ij}), \quad (2.15)$$

$$(U_{22})_{ij} = \frac{1}{4} (2\lambda_1 + \lambda_4) (H^\dagger H) \delta_{ij} + \frac{1}{2} \lambda_2 (\Delta_a^* \Delta_a \delta_{ij} + \Delta_i \Delta_j^*) + \frac{1}{2} \lambda_3 (\Delta_a^* \Delta_a \delta_{ij} + \Delta_i^* \Delta_j - \Delta_i \Delta_j^*) + \frac{i}{2} \lambda_4 \epsilon_{ijk} (H^\dagger \tau^k H). \quad (2.16)$$

These matrix elements need to be evaluated for the classical heavy field solution, which is given by

$$(\Delta_c)_i = \frac{B_i}{M_\Delta^2} + \frac{(P^2 - U)_{ij}}{M_\Delta^4} B_j + \frac{(P^2 - U)_{ij}^2}{M_\Delta^6} B_j + \dots \quad (2.17)$$

As discussed in the model-independent case, here, the classical solution of the heavy triplet scalar is computed up to order M_Δ^{-6} to ensure that no dimension-eight operator contribution from Eq. (2.2) is missed. We follow

similar footsteps and compute the dimension-eight effective operators from $\mathcal{L}_2, \mathcal{L}_0, \mathcal{L}_{-2}$ parts of the effective Lagrangian, along with the usual $\mathcal{L}_{-4}$ in Eq. (2.2). Using the matrix U mentioned above and Eq. (2.17), we derive the effective Lagrangian, as shown in Eq. (2.2), up to dimension-eight. WCs are functions of the following BSM parameters: $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \mu_\Delta$ along with the SM. The dimension-eight operators in the Green's basis [78], generated by integrating out the heavy-scalar triplet at the one-loop level within the above model framework, are given in Tables I and II. In these Tables, we highlight the origins of the operators using the following color codes: the terms in blue ($\mathcal{L}_{-4}$), magenta ($\mathcal{L}_{-2}$), black ($\mathcal{L}_0$), and green ($\mathcal{L}_2$) denote the origin of these contributions in Eq. (2.2), respectively. Notably, a single operator may emerge from different terms in Eq. (2.2). For example, $\mathcal{O}_{\phi^8}$ originates from functionals U, U^2, U^3, U^4 , and the respective WCs ($c_{\phi^8}^{\llbracket U \rrbracket}, c_{\phi^8}^{\llbracket U^2 \rrbracket}, c_{\phi^8}^{\llbracket U^3 \rrbracket}, c_{\phi^8}^{\llbracket U^4 \rrbracket}$) are depicted by respective colors in Table I. We successfully encapsulated this feature for all the operators. The computation details, along with the WCs derived from the various operator structures are presented in Appendix A 1.

We also present the results for dimension-six operators arising from this procedure in the SILH [6] basis in Appendix A 1⁵ that are in agreement with [79]. In addition,

⁵Dimension-six operators and their matching relations are given in Eqs. (A45), (A46), (A49), (A50), (A54), (A55), (A62), and (A63).

TABLE II. Summary of matching results for relevant dimension-eight operators in the SMEFT for the complex triplet model with the heavy triplet integrated up to one-loop consists of heavy propagators only. The terms in blue ($\mathcal{L}_{-4}$), and green ($\mathcal{L}_2$) denote the origin of these contributions in Eq. (2.2), respectively.

EFT operators	Matching results at scale $M_\Delta (\gg v)$
$\mathcal{O}_{\phi^4}^{(10)} = (D^2 \phi^\dagger D^2 \phi)(\phi^\dagger \phi)$	$\mathbb{C}_{\phi^4}^{(10)} = \frac{1}{16\pi^2} (1 - \ln(\frac{M_\Delta^2}{\mu^2})) c_{\phi^4}^{(10), \llbracket U \rrbracket} + \frac{1}{1920\pi^2} c_{\phi^4}^{(10), \llbracket (P^2 U)^2 \rrbracket}$
$\mathcal{O}_{\phi^4}^{(11)} = (\phi^\dagger D^2 \phi)(D^2 \phi^\dagger \phi)$	$\mathbb{C}_{\phi^4}^{(11)} = \frac{1}{16\pi^2} (1 - \ln(\frac{M_\Delta^2}{\mu^2})) c_{\phi^4}^{(11), \llbracket U \rrbracket} + \frac{1}{1920\pi^2} c_{\phi^4}^{(11), \llbracket (P^2 U)^2 \rrbracket}$
$\mathcal{O}_{W\phi^4 D^2}^{(6)} = (\phi^\dagger \phi) D_\nu W^{I\mu\nu} (D_\mu \phi^\dagger i\sigma^I \phi + \text{H.c.})$	$\mathbb{C}_{W\phi^4 D^2}^{(6)} = -\frac{1}{960\pi^2} c_{W\phi^4 D^2}^{(6), \llbracket U(P_\mu U)J_\mu \rrbracket}$
$\mathcal{O}_{W\phi^4 D^2}^{(7)} = \epsilon^{IJK} (D_\mu \phi^\dagger \sigma^I \phi)(\phi^\dagger \sigma^J D_\nu \phi) W^{K\mu\nu}$	$\mathbb{C}_{W\phi^4 D^2}^{(7)} = -\frac{1}{960\pi^2} c_{W\phi^4 D^2}^{(7), \llbracket U(P_\mu U)J_\mu \rrbracket}$
$\mathcal{O}_{B\phi^4 D^2}^{(1)} = i(\phi^\dagger \phi)(D^\mu \phi^\dagger D^\nu \phi) B_{\mu\nu}$	$\mathbb{C}_{B\phi^4 D^2}^{(1)} = -\frac{1}{960\pi^2} c_{B\phi^4 D^2}^{(1), \llbracket U(P_\mu U)J_\mu \rrbracket}$
$\mathcal{O}_{W^2 \phi^2 D^2}^{(4)} = i\epsilon^{IJK} (D^\mu \phi^\dagger \sigma^I D^\nu \phi) W_{\mu\rho}^J W_{\nu}^{K\rho}$	$\mathbb{C}_{W^2 \phi^2 D^2}^{(4)} = -\frac{1}{720\pi^2} c_{W^2 \phi^2 D^2}^{(4), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket}$
$\mathcal{O}_{W^2 \phi^2 D^2}^{(11)} = (\phi^\dagger D_\nu \phi + D_\nu \phi^\dagger \phi) D_\mu W^{I\mu\rho} W^{I\nu}_\rho$	$\mathbb{C}_{W^2 \phi^2 D^2}^{(11)} = -\frac{1}{720\pi^2} c_{W^2 \phi^2 D^2}^{(11), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket}$
$\mathcal{O}_{W^2 \phi^2 D^2}^{(13)} = \phi^\dagger \phi D_\mu W^{I\mu\rho} D_\nu W^{I\nu}_\rho$	$\mathbb{C}_{W^2 \phi^2 D^2}^{(13)} = \frac{1}{960\pi^2} c_{W^2 \phi^2 D^2}^{(13), \llbracket U(J_\mu)^2 \rrbracket}$
$\mathcal{O}_{W^2 \phi^2 D^2}^{(14)} = (D_\mu \phi^\dagger \phi + \phi^\dagger D_\mu \phi) W^{I\nu\rho} D^\mu W^{I\nu}_\rho$	$\mathbb{C}_{W^2 \phi^2 D^2}^{(14)} = -\frac{1}{2880\pi^2} c_{W^2 \phi^2 D^2}^{(14), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket}$
$\mathcal{O}_{B^2 \phi^2 D^2}^{(2)} = (D^\mu \phi^\dagger D_\mu \phi) B_{\nu\rho} B^{\nu\rho}$	$\mathbb{C}_{B^2 \phi^2 D^2}^{(2)} = -\frac{1}{2880\pi^2} c_{B^2 \phi^2 D^2}^{(14), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket}$
$\mathcal{O}_{B^2 \phi^2 D^2}^{(4)} = (D_\mu \phi^\dagger \phi + \phi^\dagger D_\mu \phi) D_\nu B^{\mu\rho} B_\rho^\nu$	$\mathbb{C}_{B^2 \phi^2 D^2}^{(4)} = -\frac{1}{720\pi^2} c_{B^2 \phi^2 D^2}^{(4), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket}$
$\mathcal{O}_{B^2 \phi^2 D^2}^{(6)} = \phi^\dagger \phi D_\mu D_\nu B^{\mu\rho} B_\rho^\nu$	$\mathbb{C}_{B^2 \phi^2 D^2}^{(6)} = \frac{1}{960\pi^2} c_{B^2 \phi^2 D^2}^{(6), \llbracket U(J_\mu)^2 \rrbracket}$
$\mathcal{O}_{B^2 \phi^2 D^2}^{(8)} = (\phi^\dagger D_\nu \phi + D_\nu \phi^\dagger \phi) D_\mu B^{\mu\rho} B_\rho^\nu$	$\mathbb{C}_{B^2 \phi^2 D^2}^{(8)} = \frac{1}{960\pi^2} c_{B^2 \phi^2 D^2}^{(8), \llbracket U(J_\mu)^2 \rrbracket}$
$\mathcal{O}_{WB\phi^2 D^2}^{(1)} = (D^\mu \phi^\dagger \sigma^I D_\mu \phi) B_{\nu\rho} W^{I\nu\rho}$	$\mathbb{C}_{WB\phi^2 D^2}^{(1)} = -\frac{1}{2880\pi^2} c_{WB\phi^2 D^2}^{(1), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket}$
$\mathcal{O}_{WB\phi^2 D^2}^{(8)} = (\phi^\dagger \sigma^I D^\nu \phi + D^\nu \phi^\dagger \sigma^I \phi) D_\mu B^{\mu\rho} W^{I\nu}_\rho$	$\mathbb{C}_{WB\phi^2 D^2}^{(8)} = -\frac{1}{720\pi^2} c_{WB\phi^2 D^2}^{(8), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket}$
$\mathcal{O}_{WB\phi^2 D^2}^{(10)} = (\phi^\dagger \sigma^I \phi) D^\mu B_{\mu\rho} D_\nu W^{I\nu\rho}$	$\mathbb{C}_{WB\phi^2 D^2}^{(10)} = \frac{1}{960\pi^2} c_{WB\phi^2 D^2}^{(10), \llbracket U(J_\mu)^2 \rrbracket}$
$\mathcal{O}_{WB\phi^2 D^2}^{(11)} = (D_\nu \phi^\dagger \sigma^I \phi + \phi^\dagger \sigma^I D_\nu \phi) B_{\mu\rho} D^\mu W^{I\nu\rho}$	$\mathbb{C}_{WB\phi^2 D^2}^{(11)} = -\frac{1}{720\pi^2} c_{WB\phi^2 D^2}^{(11), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket}$
$\mathcal{O}_{W^3 \phi^2}^{(1)} = \epsilon^{IJK} (\phi^\dagger \phi) W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$\mathbb{C}_{W^3 \phi^2}^{(1)} = -\frac{1}{1440\pi^2} c_{W^3 \phi^2}^{(1), \llbracket UG_{\mu\nu}G_{\nu\rho}G_{\rho\mu} \rrbracket}$
$\mathcal{O}_{W^2 D^4} = (D_\sigma D_\mu W^{I\mu\nu})(D^\sigma D^\rho W^{I\rho}_\nu)$	$\mathbb{C}_{W^2 D^4} = \frac{1}{13440\pi^2} c_{W^2 D^4}^{(1), \llbracket (P_\nu J_\nu)^2 \rrbracket}$
$\mathcal{O}_{B^2 D^4} = (D_\sigma D_\mu B^{\mu\nu})(D^\sigma D^\rho B_{\rho\nu})$	$\mathbb{C}_{B^2 D^4} = \frac{1}{13440\pi^2} c_{B^2 D^4}^{(1), \llbracket (P_\nu J_\nu)^2 \rrbracket}$
$\mathcal{Q}_{W^4}^{(3)} = (W_{\mu\nu}^I W^{J\mu\nu})(W_{\rho\sigma}^I W^{J\rho\sigma})$	$\mathbb{C}_{W^4}^{(3)} = \frac{1}{161280\pi^2} c_{W^4}^{(1), \llbracket (G_{\mu\nu}G_{\rho\sigma})^2 \rrbracket} + \frac{17}{161280\pi^2} c_{W^4}^{(3), \llbracket (G_{\mu\nu})^2(G_{\rho\sigma})^2 \rrbracket}$
$\mathcal{Q}_{B^4}^{(1)} = (B_{\mu\nu} B^{\mu\nu})(B_{\rho\sigma} B^{\rho\sigma})$	$\mathbb{C}_{B^4}^{(1)} = \frac{1}{161280\pi^2} c_{B^4}^{(1), \llbracket (G_{\mu\nu}G_{\rho\sigma})^2 \rrbracket} + \frac{17}{161280\pi^2} c_{B^4}^{(1), \llbracket (G_{\mu\nu})^2(G_{\rho\sigma})^2 \rrbracket}$
$\mathcal{Q}_{W^2 B^2}^{(1)} = (B_{\mu\nu} B^{\mu\nu})(W_{\rho\sigma}^I W^{I\rho\sigma})$	$\mathbb{C}_{W^2 B^2}^{(1)} = \frac{1}{161280\pi^2} c_{W^2 B^2}^{(1), \llbracket (G_{\mu\nu}G_{\rho\sigma})^2 \rrbracket} + \frac{17}{161280\pi^2} c_{W^2 B^2}^{(1), \llbracket (G_{\mu\nu})^2(G_{\rho\sigma})^2 \rrbracket}$
$\mathcal{Q}_{W^2 B^2}^{(3)} = (B_{\mu\nu} W^{I\mu\nu})(B_{\rho\sigma} W^{I\rho\sigma})$	$\mathbb{C}_{W^2 B^2}^{(3)} = \frac{1}{161280\pi^2} c_{W^2 B^2}^{(3), \llbracket (G_{\mu\nu}G_{\rho\sigma})^2 \rrbracket} + \frac{17}{161280\pi^2} c_{W^2 B^2}^{(3), \llbracket (G_{\mu\nu})^2(G_{\rho\sigma})^2 \rrbracket}$

in Sec. III, we capture the impact of fermionic interactions of the heavy triplet scalar within the effective operators, which are of great phenomenological interest [80–82].

B. Case II: Electroweak complex doublet

In this section, we consider the electroweak complex $SU(2)_L$ doublet with hypercharge $Y = -\frac{1}{2}$, also known as

2HDM. The part of the Lagrangian that involves the heavy scalar doublet $\mathcal{H}_2$ is [59,62]

$$\mathcal{L}_{\mathcal{H}_2} = |D_\mu \mathcal{H}_2|^2 - m_{\mathcal{H}_2}^2 |\mathcal{H}_2|^2 - V_{\text{scalar}} - \mathcal{L}_{\text{Yuk}}, \quad (2.18)$$

where V_{scalar} refers to the scalar potential term

$$V_{\text{scalar}} = \frac{\lambda_{\mathcal{H}_2}}{4} |\mathcal{H}_2|^4 - (\eta_H |\tilde{H}|^2 + \eta_{\mathcal{H}_2} |\mathcal{H}_2|^2) (\tilde{H}^\dagger \mathcal{H}_2 + \mathcal{H}_2^\dagger \tilde{H}) \\ + \lambda_{\mathcal{H}_2,1} |\tilde{H}|^2 |\mathcal{H}_2|^2 + \lambda_{\mathcal{H}_2,2} |\tilde{H}^\dagger \mathcal{H}_2|^2 + \lambda_{\mathcal{H}_2,3} [(\tilde{H}^\dagger \mathcal{H}_2)^2 + (\mathcal{H}_2^\dagger \tilde{H})^2], \quad (2.19)$$

and $\mathcal{L}_{\text{Yuk}}$ is the Yukawa interactions of the SM fermions with the heavy scalar

$$\mathcal{L}_{\text{Yuk}} = Y_{\mathcal{H}_2}^{(e)} \bar{\ell}_L \mathcal{H}_2 e_R + Y_{\mathcal{H}_2}^{(u)} \bar{q}_L \tilde{\mathcal{H}}_2 u_R + Y_{\mathcal{H}_2}^{(d)} \bar{q}_L \mathcal{H}_2 d_R + \text{H.c.}, \quad (2.20)$$

where $\tilde{\mathcal{H}}_2 = i\sigma_2 \mathcal{H}_2^*$. We suppress the family indices of the fermions.⁶ Here, the BSM Higgs doublet $\mathcal{H}_2$ is assumed to be heavy enough and does not acquire a VEV before integrating out. As a result, all of its components remain degenerate in mass, allowing it to be integrated out in a single step. The resulting low-energy theory is expressed in terms of SMEFT operators, which we discuss in detail.

The potential V_{scalar} has distinct interactions such as $(\tilde{H}^\dagger \mathcal{H}_2)^2$ and $(\mathcal{H}_2^\dagger \tilde{H})^2$, so it is convenient to treat $\mathcal{H}_2$ and $\mathcal{H}_2^*$ as different variables. U is a 4×4 Hermitian matrix, where each 2×2 block on the $(\mathcal{H}_2, \mathcal{H}_2^*)$ basis is given by

$$(U_{11}^{\mathcal{H}_2})_{ij} = \frac{\delta^2 V_{\text{scalar}}}{\delta \mathcal{H}_{2i}^* \delta \mathcal{H}_{2j}}, \quad (U_{12}^{\mathcal{H}_2})_{ij} = \frac{\delta^2 V_{\text{scalar}}}{\delta \mathcal{H}_{2i} \delta \mathcal{H}_{2j}}, \\ (U_{21}^{\mathcal{H}_2})_{ij} = \frac{\delta^2 V_{\text{scalar}}}{\delta \mathcal{H}_{2i}^* \delta \mathcal{H}_{2j}^*}, \quad (U_{22}^{\mathcal{H}_2})_{ij} = \frac{\delta^2 V_{\text{scalar}}}{\delta \mathcal{H}_{2i} \delta \mathcal{H}_{2j}^*}.$$

Here, indices i, j range from 1 to 2 and refer to the two-component fields of each complex scalar doublet. We, thus, obtain these matrix elements as follows:

$$(U_{11}^{\mathcal{H}_2})_{ij} = \frac{\lambda_{\mathcal{H}_2}}{2} [\delta_{ij} (\mathcal{H}_{2k}^* \mathcal{H}_{2k}) + \mathcal{H}_{2j}^* \mathcal{H}_{2i}] + \lambda_{\mathcal{H}_2,1} \delta_{ij} (H_k^* H_k) + \lambda_{\mathcal{H}_2,2} (\tilde{H}_j \tilde{H}_i^*) \\ - \eta_{\mathcal{H}_2} [\delta_{ij} (\tilde{H}_k^* \mathcal{H}_{2k} + \mathcal{H}_{2k}^* \tilde{H}_k) + \mathcal{H}_{2i} \tilde{H}_j^* + \mathcal{H}_{2j}^* \tilde{H}_i], \quad (2.21)$$

$$(U_{12}^{\mathcal{H}_2})_{ij} = \frac{\lambda_{\mathcal{H}_2}}{4} [\mathcal{H}_{2i}^* \mathcal{H}_{2j}^* + \mathcal{H}_{2j}^* \mathcal{H}_{2i}^*] - \eta_{\mathcal{H}_2} [\mathcal{H}_{2i}^* \tilde{H}_j^* + \mathcal{H}_{2j}^* \tilde{H}_i^*] + \lambda_{\mathcal{H}_2,3} [\tilde{H}_i^* \tilde{H}_j^* + \tilde{H}_j^* \tilde{H}_i^*], \quad (2.22)$$

$$(U_{21}^{\mathcal{H}_2})_{ij} = \frac{\lambda_{\mathcal{H}_2}}{4} [\mathcal{H}_{2i} \mathcal{H}_{2j} + \mathcal{H}_{2j} \mathcal{H}_{2i}] - \eta_{\mathcal{H}_2} [\mathcal{H}_{2i} \tilde{H}_j + \mathcal{H}_{2j} \tilde{H}_i] + \lambda_{\mathcal{H}_2,3} [\tilde{H}_i \tilde{H}_j + \tilde{H}_j \tilde{H}_i], \quad (2.23)$$

$$(U_{22}^{\mathcal{H}_2})_{ij} = \frac{\lambda_{\mathcal{H}_2}}{2} [\delta_{ij} (\mathcal{H}_{2k}^* \mathcal{H}_{2k}) + \mathcal{H}_{2j} \mathcal{H}_{2i}^*] + \lambda_{\mathcal{H}_2,1} \delta_{ij} (H_k^* H_k) + \lambda_{\mathcal{H}_2,2} (\tilde{H}_j^* \tilde{H}_i) \\ - \eta_{\mathcal{H}_2} [\delta_{ij} (\tilde{H}_k^* \mathcal{H}_{2k} + \mathcal{H}_{2k}^* \tilde{H}_k) + \mathcal{H}_{2i}^* \tilde{H}_j + \mathcal{H}_{2j} \tilde{H}_i^*]. \quad (2.24)$$

These matrix elements must be evaluated with the classical heavy field solution for $\mathcal{H}_2$.

We obtain the functional B from the linear term in the heavy field $\mathcal{H}_2$ of the scalar Lagrangian [Eq. (2.19)] as $B_i = \eta_H (H^\dagger H) \tilde{H}_i$ (and $B_i^\dagger = \eta_H (H^\dagger H) \tilde{H}_i^\dagger$). The mass dimension of B is three, and η_H is a dimensionless coupling.

The equation of motion for the heavy doublet field $\mathcal{H}_2$ is given by

$$(\mathcal{H}_{2,c})_i = \frac{B_i}{M_{\mathcal{H}_2}^2} + \frac{(P^2 - U)_{ij}}{M_{\mathcal{H}_2}^4} B_j + \frac{(P^2 - U)_{ij}^2}{M_{\mathcal{H}_2}^6} B_j + \dots \quad (2.25)$$

As discussed in the model-independent scenario, the classical solution of the heavy doublet scalar is computed up to order $M_{\mathcal{H}_2}^{-6}$ to capture all possible contributions to the dimension-eight operators arising from Eq. (2.2). Accordingly, we evaluate

⁶Note that certain interaction terms allowed in general scalar extensions, such as $(\mathcal{H}_2^\dagger \mathcal{H}_2) \mathcal{H}_2$ or derivative couplings in the potential are not included in our considered BSM scenario.

TABLE III. The matching results for relevant dimension-eight operators in the SMEFT for the complex doublet model with the heavy doublet integrated up to one-loop consist of heavy propagators only. The terms in blue ($\mathcal{L}_{-4}$), magenta ($\mathcal{L}_{-2}$), black ($\mathcal{L}_0$), and green ($\mathcal{L}_2$) denote the origin of these contributions in Eq. (2.2), respectively.

EFT operators	Matching results at scale $M_{\mathcal{H}_2} (\gg v)$
$\mathcal{O}_{\phi^8} = (\phi^\dagger \phi)^4$	$\mathbb{C}_{\phi^8} = \frac{1}{16\pi^2} (1 - \ln(\frac{M_{\mathcal{H}_2}^2}{\mu^2})) c_{\phi^8}^{[U]} - \frac{1}{32\pi^2} \ln(\frac{M_{\mathcal{H}_2}^2}{\mu^2}) c_{\phi^8}^{[U^2]} - \frac{1}{96\pi^2} c_{\phi^8}^{[U^3]} + \frac{1}{384\pi^2} c_{\phi^8}^{[U^4]}$
$\mathcal{O}_{\phi^6}^{(1)} = (\phi^\dagger \phi)^2 (D_\mu \phi^\dagger D^\mu \phi)$	$\mathbb{C}_{\phi^6}^{(1)} = -\frac{1}{192\pi^2} c_{\phi^6}^{(1), [P_\mu U]^2}$
$\mathcal{O}_{\phi^6}^{(3)} = (\phi^\dagger \phi)^2 (\phi^\dagger D^2 \phi + \text{H.c.})$	$\mathbb{C}_{\phi^6}^{(3)} = \frac{1}{16\pi^2} (1 - \ln(\frac{M_{\mathcal{H}_2}^2}{\mu^2})) c_{\phi^6}^{(3), [U]} - \frac{1}{32\pi^2} \ln(\frac{M_{\mathcal{H}_2}^2}{\mu^2}) c_{\phi^6}^{(3), [U^2]} - \frac{1}{192\pi^2} c_{\phi^6}^{(3), [P_\mu U]^2} - \frac{1}{384\pi^2} c_{\phi^6}^{(3), [U^2(P^2 U)]}$
$\mathcal{O}_{W^2\phi^4}^{(1)} = (\phi^\dagger \phi)^2 W_{\mu\nu}^I W^{I\mu\nu}$	$\mathbb{C}_{W^2\phi^4}^{(1)} = -\frac{1}{192\pi^2} c_{W^2\phi^4}^{(1), [U(G_{\mu\nu})^2]} + \frac{1}{480\pi^2} c_{W^2\phi^4}^{(1), [U^2(G_{\mu\nu})^2]} + \frac{1}{1920\pi^2} c_{W^2\phi^4}^{(1), [U(G_{\mu\nu})^2]}$
$\mathcal{O}_{B^2\phi^4}^{(1)} = (\phi^\dagger \phi)^2 B_{\mu\nu} B^{\mu\nu}$	$\mathbb{C}_{B^2\phi^4}^{(1)} = -\frac{1}{192\pi^2} c_{B^2\phi^4}^{(1), [U(G_{\mu\nu})^2]} + \frac{1}{480\pi^2} c_{B^2\phi^4}^{(1), [U^2(G_{\mu\nu})^2]} + \frac{1}{1920\pi^2} c_{B^2\phi^4}^{(1), [U(G_{\mu\nu})^2]}$
$\mathcal{O}_{WB\phi^4}^{(1)} = (\phi^\dagger \phi) (\phi^\dagger \sigma^I \phi) W_{\mu\nu}^I B^{\mu\nu}$	$\mathbb{C}_{WB\phi^4}^{(1)} = -\frac{1}{192\pi^2} c_{WB\phi^4}^{(1), [U(G_{\mu\nu})^2]} + \frac{1}{480\pi^2} c_{WB\phi^4}^{(1), [U^2(G_{\mu\nu})^2]} + \frac{1}{1920\pi^2} c_{WB\phi^4}^{(1), [U(G_{\mu\nu})^2]}$
$\mathcal{O}_{\phi^4}^{(3)} = (D^\mu \phi^\dagger D_\mu \phi) (D^\nu \phi^\dagger D_\nu \phi)$	$\mathbb{C}_{\phi^4}^{(3)} = \frac{1}{16\pi^2} (1 - \ln(\frac{M_{\mathcal{H}_2}^2}{\mu^2})) c_{\phi^4}^{(3), [U]}$
$\mathcal{O}_{\phi^4}^{(4)} = (D_\mu \phi^\dagger D^\mu \phi) (\phi^\dagger D^2 \phi + \text{H.c.})$	$\mathbb{C}_{\phi^4}^{(4)} = \frac{1}{16\pi^2} (1 - \ln(\frac{M_{\mathcal{H}_2}^2}{\mu^2})) c_{\phi^4}^{(4), [U]} + \frac{1}{1920\pi^2} c_{\phi^4}^{(4), [(P^2 U)^2]}$
$\mathcal{O}_{\phi^4}^{(8)} = ((D^2 \phi^\dagger \phi) (D^2 \phi^\dagger \phi) + \text{H.c.})$	$\mathbb{C}_{\phi^4}^{(8)} = \frac{1}{16\pi^2} (1 - \ln(\frac{M_{\mathcal{H}_2}^2}{\mu^2})) c_{\phi^4}^{(8), [U]} + \frac{1}{1920\pi^2} c_{\phi^4}^{(8), [(P^2 U)^2]}$

TABLE IV. Summary of matching results for relevant dimension-eight operators in the SMEFT for the complex doublet model with the heavy doublet integrated up to one-loop consists of heavy propagators only. The terms in blue ($\mathcal{L}_{-4}$), and green ($\mathcal{L}_2$) denote the origin of these contributions in Eq. (2.2), respectively.

EFT operators	Matching results at scale $M_{\mathcal{H}_2} (\gg v)$
$\mathcal{O}_{\phi^4}^{(10)} = (D^2 \phi^\dagger D^2 \phi) (\phi^\dagger \phi)$	$\mathbb{C}_{\phi^4}^{(10)} = \frac{1}{16\pi^2} (1 - \ln(\frac{M_{\mathcal{H}_2}^2}{\mu^2})) c_{\phi^4}^{(10), [U]}$
$\mathcal{O}_{\phi^4}^{(11)} = (\phi^\dagger D^2 \phi) (D^2 \phi^\dagger \phi)$	$\mathbb{C}_{\phi^4}^{(11)} = \frac{1}{16\pi^2} (1 - \ln(\frac{M_{\mathcal{H}_2}^2}{\mu^2})) c_{\phi^4}^{(11), [U]} + \frac{1}{1920\pi^2} c_{\phi^4}^{(11), [(P^2 U)^2]}$
$\mathcal{O}_{\phi^4}^{(12)} = (D_\mu \phi^\dagger \phi) (D^\mu \phi^\dagger D^2 \phi) + \text{H.c.}$	$\mathbb{C}_{\phi^4}^{(12)} = \frac{1}{1920\pi^2} c_{\phi^4}^{(12), [(P^2 U)^2]}$
$\mathcal{O}_{B\phi^4 D^2}^{(3)} = (\phi^\dagger \phi) D_\nu B^{\mu\nu} (D_\mu \phi^\dagger i\phi + \text{H.c.})$	$\mathbb{C}_{B\phi^4 D^2}^{(3)} = -\frac{1}{960\pi^2} c_{B\phi^4 D^2}^{(1), [U(P_\mu U) J_\mu]}$
$\mathcal{O}_{W^2\phi^2 D^2}^{(2)} = (D^\mu \phi^\dagger D_\mu \phi) W_{\nu\rho}^I W^{I\nu\rho}$	$\mathbb{C}_{W^2\phi^2 D^2}^{(2)} = -\frac{1}{2880\pi^2} c_{W^2\phi^2 D^2}^{(2), [(P^2 U)(G_{\rho\sigma})^2]}$
$\mathcal{O}_{W^2\phi^2 D^2}^{(4)} = i\epsilon^{IJK} (D^\mu \phi^\dagger \sigma^I D^\nu \phi) W_{\mu\rho}^J W_{\nu}^{K\rho}$	$\mathbb{C}_{W^2\phi^2 D^2}^{(4)} = -\frac{1}{720\pi^2} c_{W^2\phi^2 D^2}^{(4), [(P_\mu P_\nu U) G_{\rho\mu} G_{\rho\nu}]} - \frac{1}{2880\pi^2} c_{W^2\phi^2 D^2}^{(4), [(P^2 U)(G_{\rho\sigma})^2]}$
$\mathcal{O}_{W^2\phi^2 D^2}^{(13)} = \phi^\dagger \phi D_\mu W^{I\mu\rho} D_\nu W^{I\nu}_\rho$	$\mathbb{C}_{W^2\phi^2 D^2}^{(13)} = \frac{1}{960\pi^2} c_{W^2\phi^2 D^2}^{(13), [U(J_\mu)^2]}$
$\mathcal{O}_{W^2\phi^2 D^2}^{(14)} = (D_\mu \phi^\dagger \phi + \phi^\dagger D_\mu \phi) W^{I\nu\rho} D^\mu W_{\nu\rho}^I$	$\mathbb{C}_{W^2\phi^2 D^2}^{(14)} = -\frac{1}{720\pi^2} c_{W^2\phi^2 D^2}^{(14), [(P_\mu P_\nu U) G_{\rho\mu} G_{\rho\nu}]} - \frac{1}{2880\pi^2} c_{W^2\phi^2 D^2}^{(14), [(P^2 U)(G_{\rho\sigma})^2]}$
$\mathcal{O}_{B^2\phi^2 D^2}^{(2)} = (D^\mu \phi^\dagger D_\mu \phi) B_{\nu\rho} B^{\nu\rho}$	$\mathbb{C}_{B^2\phi^2 D^2}^{(2)} = -\frac{1}{2880\pi^2} c_{B^2\phi^2 D^2}^{(14), [(P^2 U)(G_{\rho\sigma})^2]}$
$\mathcal{O}_{W^2\phi^2 D^2}^{(11)} = (\phi^\dagger D_\nu \phi + D_\nu \phi^\dagger \phi) D_\mu W^{I\mu\rho} W^{I\nu}_\rho$	$\mathbb{C}_{W^2\phi^2 D^2}^{(11)} = -\frac{1}{720\pi^2} c_{W^2\phi^2 D^2}^{(11), [(P_\mu P_\nu U) G_{\rho\mu} G_{\rho\nu}]} - \frac{1}{2880\pi^2} c_{W^2\phi^2 D^2}^{(11), [(P^2 U)(G_{\rho\sigma})^2]}$
$\mathcal{O}_{B^2\phi^2 D^2}^{(4)} = (D_\mu \phi^\dagger \phi + \phi^\dagger D_\mu \phi) D_\nu B^{\mu\rho} B^\nu_\rho$	$\mathbb{C}_{B^2\phi^2 D^2}^{(4)} = -\frac{1}{720\pi^2} c_{B^2\phi^2 D^2}^{(4), [(P_\mu P_\nu U) G_{\rho\mu} G_{\rho\nu}]} - \frac{1}{2880\pi^2} c_{B^2\phi^2 D^2}^{(4), [(P^2 U)(G_{\rho\sigma})^2]}$
$\mathcal{O}_{B^2\phi^2 D^2}^{(6)} = \phi^\dagger \phi D_\mu D_\nu B^{\mu\rho} B^\nu_\rho$	$\mathbb{C}_{B^2\phi^2 D^2}^{(6)} = \frac{1}{960\pi^2} c_{B^2\phi^2 D^2}^{(6), [U(J_\mu)^2]}$
$\mathcal{O}_{B^2\phi^2 D^2}^{(8)} = (\phi^\dagger D_\nu \phi + D_\nu \phi^\dagger \phi) D_\mu B^{\mu\rho} B^\nu_\rho$	$\mathbb{C}_{B^2\phi^2 D^2}^{(8)} = \frac{1}{960\pi^2} c_{B^2\phi^2 D^2}^{(8), [U(J_\mu)^2]} - \frac{1}{720\pi^2} c_{B^2\phi^2 D^2}^{(8), [(P_\mu P_\nu U) G_{\rho\mu} G_{\rho\nu}]} - \frac{1}{2880\pi^2} c_{B^2\phi^2 D^2}^{(8), [(P^2 U)(G_{\rho\sigma})^2]}$
$\mathcal{O}_{WB\phi^2 D^2}^{(1)} = (D^\mu \phi^\dagger \sigma^I D_\mu \phi) B_{\nu\rho} W^{I\nu\rho}$	$\mathbb{C}_{WB\phi^2 D^2}^{(1)} = -\frac{1}{2880\pi^2} c_{WB\phi^2 D^2}^{(1), [(P^2 U)(G_{\rho\sigma})^2]}$
$\mathcal{O}_{WB\phi^2 D^2}^{(8)} = (\phi^\dagger \sigma^I D^\nu \phi + D^\nu \phi^\dagger \sigma^I \phi) D_\mu B^{\mu\rho} W_{\nu\rho}^I$	$\mathbb{C}_{WB\phi^2 D^2}^{(8)} = -\frac{1}{720\pi^2} c_{WB\phi^2 D^2}^{(8), [(P_\mu P_\nu U) G_{\rho\mu} G_{\rho\nu}]} - \frac{1}{2880\pi^2} c_{WB\phi^2 D^2}^{(8), [(P^2 U)(G_{\rho\sigma})^2]}$

(Table continued)

TABLE IV. (Continued)

EFT operators	Matching results at scale $M_{\mathcal{H}_2} (\gg v)$
$\mathcal{O}_{WB\phi^2 D^2}^{(10)} = (\phi^\dagger \sigma^I \phi) D^\mu B_{\mu\rho} D_\nu W^{I\nu\rho}$	$\mathbb{C}_{WB\phi^2 D^2}^{(10)} = \frac{1}{960\pi^2} c_{WB\phi^2 D^2}^{(10), \llbracket U(J_\mu)^2 \rrbracket} - \frac{1}{720\pi^2} c_{WB\phi^2 D^2}^{(10), \llbracket (P_\mu P_\nu U) G_{\rho\mu} G_{\rho\nu} \rrbracket}$
$\mathcal{O}_{WB\phi^2 D^2}^{(11)} = (D_\nu \phi^\dagger \sigma^I \phi + \phi^\dagger \sigma^I D_\nu \phi) B_{\mu\rho} D^\mu W^{I\nu\rho}$	$\mathbb{C}_{WB\phi^2 D^2}^{(11)} = -\frac{1}{720\pi^2} c_{WB\phi^2 D^2}^{(11), \llbracket (P_\mu P_\nu U) G_{\rho\mu} G_{\rho\nu} \rrbracket}$
$\mathcal{O}_{WB\phi^2 D^2}^{(13)} = (\phi^\dagger \sigma^I \phi) B_{\mu\rho} D_\nu D^\mu W^{I\nu\rho}$	$\mathbb{C}_{WB\phi^2 D^2}^{(13)} = \frac{1}{960\pi^2} c_{WB\phi^2 D^2}^{(13), \llbracket U(J_\mu)^2 \rrbracket} - \frac{1}{720\pi^2} c_{WB\phi^2 D^2}^{(13), \llbracket (P_\mu P_\nu U) G_{\rho\mu} G_{\rho\nu} \rrbracket}$
$\mathcal{O}_{W^3\phi^2}^{(1)} = \epsilon^{IJK} (\phi^\dagger \phi) W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$\mathbb{C}_{W^3\phi^2}^{(1)} = -\frac{1}{1440\pi^2} c_{W^3\phi^2}^{(1), \llbracket U G_{\mu\nu} G_{\nu\rho} G_{\rho\mu} \rrbracket}$
$\mathcal{O}_{W^2 D^4} = (D_\sigma D_\mu W^{I\mu\nu}) (D^\sigma D^\rho W_{\rho\nu}^I)$	$\mathbb{C}_{W^2 D^4} = \frac{1}{13440\pi^2} c_{W^2 D^4}^{(1), \llbracket (P_\nu J_\nu)^2 \rrbracket}$
$\mathcal{O}_{B^2 D^4} = (D_\sigma D_\mu B^{\mu\nu}) (D^\sigma D^\rho B_{\rho\nu})$	$\mathbb{C}_{B^2 D^4} = \frac{1}{13440\pi^2} c_{B^2 D^4}^{(1), \llbracket (P_\nu J_\nu)^2 \rrbracket}$
$\mathcal{Q}_{W^4}^{(3)} = (W_{\mu\nu}^I W^{J\mu\nu}) (W_{\rho\sigma}^I W^{J\rho\sigma})$	$\mathbb{C}_{W^4}^{(3)} = \frac{1}{161280\pi^2} c_{W^4}^{(3), \llbracket (G_{\mu\nu} G_{\rho\sigma})^2 \rrbracket} + \frac{17}{161280\pi^2} c_{W^4}^{(3), \llbracket (G_{\mu\nu})^2 (G_{\rho\sigma})^2 \rrbracket}$
$\mathcal{Q}_{B^4}^{(1)} = (B_{\mu\nu} B^{\mu\nu}) (B_{\rho\sigma} B^{\rho\sigma})$	$\mathbb{C}_{B^4}^{(1)} = \frac{1}{161280\pi^2} c_{B^4}^{(1), \llbracket (G_{\mu\nu} G_{\rho\sigma})^2 \rrbracket} + \frac{17}{161280\pi^2} c_{B^4}^{(1), \llbracket (G_{\mu\nu})^2 (G_{\rho\sigma})^2 \rrbracket}$
$\mathcal{Q}_{W^2 B^2}^{(1)} = (B_{\mu\nu} B^{\mu\nu}) (W_{\rho\sigma}^I W^{I\rho\sigma})$	$\mathbb{C}_{W^2 B^2}^{(1)} = \frac{1}{161280\pi^2} c_{W^2 B^2}^{(1), \llbracket (G_{\mu\nu} G_{\rho\sigma})^2 \rrbracket} + \frac{17}{161280\pi^2} c_{W^2 B^2}^{(1), \llbracket (G_{\mu\nu})^2 (G_{\rho\sigma})^2 \rrbracket}$
$\mathcal{Q}_{W^2 B^2}^{(3)} = (B_{\mu\nu} W^{I\mu\nu}) (B_{\rho\sigma} W^{I\rho\sigma})$	$\mathbb{C}_{W^2 B^2}^{(3)} = \frac{1}{161280\pi^2} c_{W^2 B^2}^{(3), \llbracket (G_{\mu\nu} G_{\rho\sigma})^2 \rrbracket} + \frac{17}{161280\pi^2} c_{W^2 B^2}^{(3), \llbracket (G_{\mu\nu})^2 (G_{\rho\sigma})^2 \rrbracket}$

the effective operators generated from the magenta ($\mathcal{L}_{-2}$), black ($\mathcal{L}_0$), and green ($\mathcal{L}_2$) colored parts in the effective Lagrangian, in addition to those from the blue ($\mathcal{L}_{-4}$), contributions in Eq. (2.2). We obtain the dimension-eight operators arising by integrating out the heavy doublet scalar at the one-loop level and list them in the Green's basis in Tables III–IV. In these Tables we highlight the origins of the operators using the following color codes: the terms in blue ($\mathcal{L}_{-4}$), magenta ($\mathcal{L}_{-2}$), black ($\mathcal{L}_0$), and green ($\mathcal{L}_2$) denote the origin of these contributions in Eq. (2.2), respectively. Notably, a single operator may emerge from different terms in Eq. (2.2). For example, $\mathcal{O}_{\phi^8}$ originates from functionals U, U^2, U^3, U^4 , and the respective WCs $(c_{\phi^8}^{\llbracket U \rrbracket}, c_{\phi^8}^{\llbracket U^2 \rrbracket}, c_{\phi^8}^{\llbracket U^3 \rrbracket}, c_{\phi^8}^{\llbracket U^4 \rrbracket})$ are depicted by respective colors in Table III. We successfully encapsulated this feature for all the operators.

The details of the derivation for the associated Wilson coefficients up to $\mathcal{O}(M_{\mathcal{H}_2}^{-4})$ are discussed in Appendix A 2. Furthermore, we present the results for dimension-six operators derived from this procedure in the SILH [6] basis in Appendix A 2⁷

III. IMPACT OF FERMIONIC INTERACTIONS WITH THE HEAVY SCALAR FIELD

In this section, we analyze the impact of introducing fermionic interactions with the heavy scalar field for the

two models under consideration: the complex scalar triplet model and the complex scalar doublet model. In the BSM Lagrangian of both models, there is a linear term in the heavy field that describes its interaction with fermions [see Eqs. (2.10) and (2.20)]. Including this interaction in the functional B will play a crucial role in not only giving intriguing collider signatures for each of the two models, but it will also be useful in distinguishing between the two UV models from an effective theory framework.

A. Fermionic interactions mediated by the heavy triplet scalar field

Until now, we derived the functional B from the linear term in the heavy field Δ of the scalar Lagrangian of the complex triplet model [Eq. (2.9)]. We now focus on the interactions between fermions and the heavy field and derive the EFT operators up to dimension-eight for the fermionic interactions after the heavy triplet scalar field has been integrated out at one-loop. The Yukawa term in the Lagrangian for the interaction of the SM leptons with the heavy field Δ is expressed as

$$\mathcal{L}_Y = Y_{\Delta\alpha\beta} \ell_\alpha^T C i \sigma^2 \Delta \ell_\beta + \text{H.c.}, \quad (3.1)$$

Considering the total Lagrangian of Eq. (2.8), one obtains the functional $\hat{B}$ as

$$\begin{aligned} \hat{B}_i &= \mu_\Delta \tilde{H}^\dagger \tau^i H - Y_{\Delta\alpha\beta}^* \ell_{L_\alpha}^{r\dagger} \tau_i \sigma_2 C \ell_{L_\beta}^{s*} \\ &= \mu_\Delta \tilde{H}^\dagger \tau^i H - Y_{\Delta\alpha\beta}^* \bar{\ell}_{L_\alpha}^r \tau_i \tilde{\ell}_{L_\beta}^s, \end{aligned} \quad (3.2)$$

⁷Dimension-six operators and their matching relations are given in Eqs. (A110), (A111), (A114), (A115), (A118), (A119), (A122), (A123), (A126), (A127), (A136), (A137), (A140), and (A141).

TABLE V. The matching results for relevant fermionic dimension-eight, -seven, -six, -five operators in the SMEFT for the complex triplet model with the heavy triplet integrated up to one-loop (heavy propagators only). The terms in black ($\mathcal{L}_0$), and green ($\mathcal{L}_2$) denote the origin of these contributions in Eq. (2.2), respectively.

Dim-8 EFT operators	Matching results at scale $M_\Delta (\gg v)$
$\mathcal{Q}_{l^4\phi^2}^{(1)} = \epsilon_{ij}\epsilon_{mn}(\ell^i C \ell^m)\phi^j\phi^n(\phi^\dagger\phi)$	$\mathbb{C}_{l^4\phi^2}^{(1)} = -\frac{1}{32\pi^2}\ln\left(\frac{M_\Delta^2}{\mu^2}\right)c_{l^4\phi^2}^{(1),\llbracket U^2 \rrbracket} + \frac{1}{16\pi^2}(1 - \ln\left(\frac{M_\Delta^2}{\mu^2}\right))c_{l^4\phi^2}^{(1),\llbracket U \rrbracket}$
$\mathcal{Q}_{l^4\phi^2}^{(2)} = \epsilon_{ij}\epsilon_{mn}(\ell^i C \ell^m)\phi^j\phi^n(\phi^\dagger\phi)$	$\mathbb{C}_{l^4\phi^2}^{(2)} = -\frac{1}{32\pi^2}\ln\left(\frac{M_\Delta^2}{\mu^2}\right)c_{l^4\phi^2}^{(2),\llbracket U^2 \rrbracket} + \frac{1}{16\pi^2}(1 - \ln\left(\frac{M_\Delta^2}{\mu^2}\right))c_{l^4\phi^2}^{(2),\llbracket U \rrbracket}$
$\mathcal{Q}_{l^4D^2}^{(1)} = D^\nu(\bar{\ell}_p\gamma^\mu\ell_r)D_\nu(\bar{\ell}_s\gamma_\mu\ell_t)$	$\mathbb{C}_{l^4D^2}^{(1)} = \frac{1}{16\pi^2}(1 - \ln\left(\frac{M_\Delta^2}{\mu^2}\right))c_{l^4D^2}^{(1),\llbracket U \rrbracket}$
Dim-7 EFT operators	Matching results at scale $M_\Delta (\gg v)$
$\mathcal{Q}_{L\phi D}^{(1)} = \epsilon_{ij}\epsilon_{mn}\ell^i C(D^\mu\ell^j)\phi^m(D_\mu\phi^n)$	$\mathbb{C}_{L\phi D}^{(1)} = \frac{1}{16\pi^2}(1 - \ln\left(\frac{M_\Delta^2}{\mu^2}\right))c_{L\phi D}^{(1),\llbracket U \rrbracket}$
$\mathcal{Q}_{L\phi D}^{(2)} = \epsilon_{im}\epsilon_{jn}\ell^i C(D^\mu\ell^j)\phi^m(D_\mu\phi^n)$	$\mathbb{C}_{L\phi D}^{(2)} = \frac{1}{16\pi^2}(1 - \ln\left(\frac{M_\Delta^2}{\mu^2}\right))c_{L\phi D}^{(2),\llbracket U \rrbracket}$
$\mathcal{Q}_{L\phi} = \epsilon_{ij}\epsilon_{mn}(\ell^i C \ell^m)\phi^j\phi^n(\phi^\dagger\phi)$	$\mathbb{C}_{L\phi} = -\frac{1}{32\pi^2}\ln\left(\frac{M_\Delta^2}{\mu^2}\right)c_{L\phi}^{\llbracket U^2 \rrbracket}$
Dim-6 EFT operators	Matching results at scale $M_\Delta (\gg v)$
$\mathcal{Q}_{ll} = (\bar{\ell}_{L_a}^i\gamma_\mu\ell_{L_r}^j)(\bar{\ell}_{L_b}^s\gamma^\mu\ell_{L_\delta}^q)$	$\mathbb{C}_{ll} = \frac{1}{16\pi^2}(1 - \ln\left(\frac{M_\Delta^2}{\mu^2}\right))c_{ll}^{\llbracket U \rrbracket}$
Dim-5 EFT operators	Matching results at scale $M_\Delta (\gg v)$
$\mathcal{Q}_{\phi^2 L^2} = \epsilon_{ij}\epsilon_{mn}\phi^i\phi^m(\ell_p^j)^T C \ell_r^n$	$\mathbb{C}_{\phi^2 L^2} = \frac{1}{16\pi^2}(1 - \ln\left(\frac{M_\Delta^2}{\mu^2}\right))c_{\phi^2 L^2}^{\llbracket U \rrbracket}$

where the indices p, q, r, s denote the components of the lepton doublet ℓ_L , Y_Δ represents a symmetric matrix in the generation space, and $\tilde{\ell}_L = i\sigma_2(\ell_L)^c$ (i.e., $\tilde{\ell}_L = -\ell_L^T C i\sigma_2$).

Like in the previous section, we derive the resulting dimension-eight, seven, and six operators, emerging from the total effective Lagrangian (2.2). The details are given in Appendix A 1 a. We find that the Weinberg operator of dimension-five, the four-Fermi operator of the dimension-six Warsaw basis [4] from $\hat{B}_i^\dagger \hat{B}_i$, the dimension-seven, and -eight operators are generated, and the results are depicted in Table V. The complex triplet model possesses lepton number-violating (LNV) interaction, which gets reflected through the emergence of dimension-five and -seven LNV operators.

B. Fermionic interactions mediated by the heavy doublet scalar field

We now summarize the description of fermionic interactions with the heavy complex Higgs doublet in the EFT framework by truncating the EFT expansion up to dimension-eight operators. The Yukawa Lagrangian in the 2HDM model is given by

$$\begin{aligned}\mathcal{L}_{\text{Yuk}} &= Y_{\mathcal{H}_2}^{(e)}\bar{\ell}_L\tilde{\mathcal{H}}_2 e_R + Y_{\mathcal{H}_2}^{(u)}\bar{q}_L\mathcal{H}_2 u_R + Y_{\mathcal{H}_2}^{(d)}\bar{q}_L\tilde{\mathcal{H}}_2 d_R + \text{H.c.} \\ &= Y_{\mathcal{H}_2}^{(e)}\mathcal{H}_2^\dagger\tilde{\ell}_L e_R + Y_{\mathcal{H}_2}^{(u)}\bar{q}_L\mathcal{H}_2 u_R + Y_{\mathcal{H}_2}^{(d)}\mathcal{H}_2^\dagger\hat{q}_L d_R + \text{H.c.},\end{aligned}\quad (3.3)$$

where we define $\hat{\ell}_L \equiv -i\sigma_2(\bar{\ell}_L)^T$ and $\hat{q}_L \equiv -i\sigma_2(\bar{q}_L)^T$. We derive the functional $\hat{B}$ from the linear term in the heavy field $\mathcal{H}_2$ of the total Lagrangian [Eq. (2.18)], which is given by

$$\begin{aligned}\hat{B}_i &= \eta_H(H^\dagger H)\tilde{H}_i - ((Y_{\mathcal{H}_2}^{(e)})\hat{\ell}_L e_R \\ &\quad + (Y_{\mathcal{H}_2}^{(u)})^\dagger\bar{u}_R q_L + (Y_{\mathcal{H}_2}^{(d)})\hat{q}_L d_R)_i.\end{aligned}\quad (3.4)$$

Note that $\hat{B}$ has mass dimension three and $\{Y_{\mathcal{H}_2}^{(e)}, Y_{\mathcal{H}_2}^{(u)}, Y_{\mathcal{H}_2}^{(d)}\}$ are dimensionless couplings. The corresponding classical solution for the heavy doublet field $\mathcal{H}_2$ is given by

$$(\mathcal{H}_{2,c})_i = \frac{\hat{B}_i}{M_{\mathcal{H}_2}^2} + \frac{(P^2 - U)_{ij}}{M_{\mathcal{H}_2}^4}\hat{B}_j + \frac{(P^2 - U)_{ij}^2}{M_{\mathcal{H}_2}^6}\hat{B}_j + \dots \quad (3.5)$$

Substituting this solution back into action, we derive the possible higher-dimensional fermionic operators at dimension-eight (in Murphy basis [36]) and at dimension-six (in Warsaw basis [4]). The complex doublet model possesses lepton number conserving interactions, which explains the absence of odd-dimensional LNV operators, unlike the complex triplet case. The results are summarized in

⁸Explicitly writing the components of the $SU(2)$ doublet for the 1st generation lepton doublet gives $\hat{\ell}_L = (-\bar{e}_L, \bar{\nu}_{eL})^T$. This implies the relation $\bar{e}_R\hat{\ell}_L^\dagger H_j = \bar{e}_R\tilde{H}_j^\dagger\ell_L$, along with its Hermitian conjugate $H_j^\dagger\hat{\ell}_L e_R = \bar{\ell}_L\tilde{H}_j e_R$.

TABLE VI. The matching results for relevant fermionic dimension-eight, and -six operators in the SMEFT for the complex doublet model with the heavy doublet integrated up to one-loop (heavy propagators only). The terms in magenta ($\mathcal{L}_{-2}$), black ($\mathcal{L}_0$), and green ($\mathcal{L}_2$) denote the origin of these contributions in Eq. (2.2), respectively.

Dim-8 EFT operators	Matching results at scale $M_{\mathcal{H}_2} (\gg v)$
$\mathcal{Q}_{le\phi^5} = (\phi^\dagger \phi)^2 (\bar{l}_p e_r \phi)$	$\mathbb{C}_{le\phi^5} = \frac{-1}{96\pi^2} c_{le\phi^5}^{\llbracket U^3 \rrbracket} - \frac{1}{32\pi^2} \ln\left(\frac{M_{\mathcal{H}_2}^2}{\mu^2}\right) c_{le\phi^5}^{\llbracket U^2 \rrbracket} + \frac{1}{16\pi^2} (1 - \ln\left(\frac{M_{\mathcal{H}_2}^2}{\mu^2}\right)) c_{le\phi^5}^{\llbracket U \rrbracket}$
$\mathcal{Q}_{le\phi^3 D^2}^{(1)} = (D_\mu \phi^\dagger D^\mu \phi) (\bar{l}_p e_r \phi)$	$\mathbb{C}_{le\phi^3 D^2}^{(1)} = \frac{-1}{96\pi^2} c_{le\phi^3 D^2}^{(1), \llbracket U^3 \rrbracket} - \frac{1}{32\pi^2} \ln\left(\frac{M_{\mathcal{H}_2}^2}{\mu^2}\right) c_{le\phi^3 D^2}^{(1), \llbracket U^2 \rrbracket} + \frac{1}{16\pi^2} (1 - \ln\left(\frac{M_{\mathcal{H}_2}^2}{\mu^2}\right)) c_{le\phi^3 D^2}^{(1), \llbracket U \rrbracket}$
$\mathcal{Q}_{le\phi^3 D^2}^{(5)} = (\phi^\dagger D_\mu \phi) (\bar{l}_p e_r D^\mu \phi)$	$\mathbb{C}_{le\phi^3 D^2}^{(5)} = \frac{-1}{96\pi^2} c_{le\phi^3 D^2}^{(5), \llbracket U^3 \rrbracket} - \frac{1}{32\pi^2} \ln\left(\frac{M_{\mathcal{H}_2}^2}{\mu^2}\right) c_{le\phi^3 D^2}^{(5), \llbracket U^2 \rrbracket} + \frac{1}{16\pi^2} (1 - \ln\left(\frac{M_{\mathcal{H}_2}^2}{\mu^2}\right)) c_{le\phi^3 D^2}^{(5), \llbracket U \rrbracket}$
$\mathcal{Q}_{leW^2\phi}^{(1)} = (\bar{l}_p e_r) \phi W_{\mu\nu}^I W^{I\mu\nu}$	$\mathbb{C}_{leW^2\phi}^{(1)} = \frac{-1}{96\pi^2} c_{leW^2\phi}^{(1), \llbracket U^3 \rrbracket}$
$\mathcal{Q}_{leB^2\phi}^{(1)} = (\bar{l}_p e_r) \phi B_{\mu\nu} B^{\mu\nu}$	$\mathbb{C}_{leB^2\phi}^{(1)} = \frac{-1}{96\pi^2} c_{leB^2\phi}^{(1), \llbracket U^3 \rrbracket}$
$\mathcal{Q}_{leWB\phi}^{(1)} = (\bar{l}_p e_r) \tau^I \phi W_{\mu\nu}^I B^{\mu\nu}$	$\mathbb{C}_{leWB\phi}^{(1)} = \frac{-1}{96\pi^2} c_{leWB\phi}^{(1), \llbracket U^3 \rrbracket}$
$\mathcal{Q}_{l^2 e^2 \phi^2}^{(1)} = (\bar{l}_p e_r \phi) (\bar{l}_s e_t \phi)$	$\mathbb{C}_{l^2 e^2 \phi^2}^{(1)} = -\frac{1}{32\pi^2} \ln\left(\frac{M_{\mathcal{H}_2}^2}{\mu^2}\right) c_{l^2 e^2 \phi^2}^{(1), \llbracket U^2 \rrbracket} + \frac{1}{16\pi^2} (1 - \ln\left(\frac{M_{\mathcal{H}_2}^2}{\mu^2}\right)) c_{l^2 e^2 \phi^2}^{(1), \llbracket U \rrbracket}$
$\mathcal{Q}_{l^2 e^2 \phi^2}^{(3)} = (\bar{l}_p \gamma_\mu l_r) (\bar{e}_s \gamma^\mu e_t) \phi^\dagger \phi$	$\mathbb{C}_{l^2 e^2 \phi^2}^{(3)} = \frac{1}{16\pi^2} (1 - \ln\left(\frac{M_{\mathcal{H}_2}^2}{\mu^2}\right)) c_{l^2 e^2 \phi^2}^{(3), \llbracket U \rrbracket}$
$\mathcal{Q}_{lequ\phi^2}^{(1)} = (\bar{l}_p^j e_r) \epsilon_{jk} (\bar{q}_s^i u_t) \phi^\dagger \phi$	$\mathbb{C}_{lequ\phi^2}^{(1)} = -\frac{1}{32\pi^2} \ln\left(\frac{M_{\mathcal{H}_2}^2}{\mu^2}\right) c_{lequ\phi^2}^{\llbracket U^2 \rrbracket} + \frac{1}{16\pi^2} (1 - \ln\left(\frac{M_{\mathcal{H}_2}^2}{\mu^2}\right)) c_{lequ\phi^2}^{\llbracket U \rrbracket}$
$\mathcal{Q}_{leqd\phi^2}^{(3)} = (\bar{l}_p e_r \phi) (\bar{q}_s d_t \phi)$	$\mathbb{C}_{leqd\phi^2}^{(3)} = -\frac{1}{32\pi^2} \ln\left(\frac{M_{\mathcal{H}_2}^2}{\mu^2}\right) c_{leqd\phi^2}^{(3), \llbracket U^2 \rrbracket} + \frac{1}{16\pi^2} (1 - \ln\left(\frac{M_{\mathcal{H}_2}^2}{\mu^2}\right)) c_{leqd\phi^2}^{(3), \llbracket U \rrbracket}$
$\mathcal{Q}_{l^2 e^2 D^2} = (\bar{l}_p e_r \phi) (\bar{q}_s d_t \phi)$	$\mathbb{C}_{l^2 e^2 D^2} = \frac{1}{16\pi^2} (1 - \ln\left(\frac{M_{\mathcal{H}_2}^2}{\mu^2}\right)) c_{l^2 e^2 D^2}^{\llbracket U \rrbracket}$
Dim-6 EFT operators	Matching results at scale $M_{\mathcal{H}_2} (\gg v)$
$\mathcal{Q}_{e\phi} = (\phi^\dagger \phi) (\bar{l}_p e_r \phi)$	$\mathbb{C}_{e\phi} = -\frac{1}{32\pi^2} \ln\left(\frac{M_{\mathcal{H}_2}^2}{\mu^2}\right) c_{e\phi}^{\llbracket U^2 \rrbracket} (1 + m_H^2) + \frac{1}{16\pi^2} (1 - \ln\left(\frac{M_{\mathcal{H}_2}^2}{\mu^2}\right)) c_{e\phi}^{\llbracket U \rrbracket} (1 + m_H^2)$
$\mathcal{Q}_{le} = (\bar{l}_p \gamma_\mu l_r) (\bar{e}_s \gamma^\mu e_t)$	$\mathbb{C}_{le} = \frac{1}{16\pi^2} (1 - \ln\left(\frac{M_{\mathcal{H}_2}^2}{\mu^2}\right)) c_{le}^{\llbracket U \rrbracket}$
$\mathcal{Q}_{ledq} = (\bar{l}_p^j e_r) (\bar{d}_s q_t^j)$	$\mathbb{C}_{ledq} = \frac{1}{16\pi^2} (1 - \ln\left(\frac{M_{\mathcal{H}_2}^2}{\mu^2}\right)) c_{ledq}^{(1), \llbracket U \rrbracket}$
$\mathcal{Q}_{lequ}^{(1)} = (\bar{l}_p^j e_r) \epsilon_{jk} (\bar{q}_s^k u_t)$	$\mathbb{C}_{lequ}^{(1)} = \frac{1}{16\pi^2} (1 - \ln\left(\frac{M_{\mathcal{H}_2}^2}{\mu^2}\right)) c_{lequ}^{(1), \llbracket U \rrbracket}$

Table VI, and details of the computation are given in Appendix A 2 a. Although we present the operators considering the leptonic contribution only in $\hat{B}$, operators for the quark involving interactions with the heavy doublet can be obtained similarly. The nonredundant operator bases can be derived using the relations provided in Appendix C.

IV. RELEVANCE OF DIMENSION-8 EFFECTIVE OPERATORS AN EWPO(OBLIQUE-PARAMETER) CASE STUDY

Different observables and precision measurements serve as powerful tools to differentiate between UV scenarios when analyzed through matched EFT frameworks. In this context, dimension-eight effects offer a diverse and rich phenomenology with significant information. We consider electroweak precision observables to explore EFT-based discrimination of UV scenarios. Throughout this section, we work within the framework of universal theories, as

discussed in Refs. [83,84]. After applying the SM equations of motion and field redefinitions, the BSM effects that modify gauge-boson self-energies can be expressed entirely in terms of bosonic operators, and the oblique parameters provide a consistent way to describe these effects within this universal EFT subspace [83]. The oblique parameters [85–89] are a relevant phenomenological sector having supreme sensitivity potential in future lepton colliders such as FCC-ee with a potential GigaZ option. The high precision expected in these environments may make dimension-eight contributions relevant, although their importance depends on the outcome of the measurements. If data remain compatible with the SM, strong bounds on dimension-six operators will render dimension-eight effects negligible (except where they arise at leading order). If deviations from the SM are observed, then dimension-eight effects must be included for a consistent EFT description, which in turn may offer sensitivity to the different UV scenarios. Nevertheless, even with higher

precision and the inclusion of higher-dimensional operators, EFT analyses face intrinsic limitations due to blind directions. These arise when the number of independent observables is smaller than the number of independent operator coefficients at a given order. This motivates the use of differential measurements at collider processes [90], which probe additional kinematic dependencies and help reduce such degeneracies. A well-known example is the degeneracy of Higgs-gluon contact interactions that compete with top-quark Yukawa coupling modifications in inclusive measurements (see also [91]). Disentangling these interactions by resolving the Compton wavelengths of the relevant states requires differential information related to the process of energy-momentum transfer. Given that higher-dimensional operators can be increasingly relevant at large momentum transfers, their inclusion is phenomenologically motivated (see also recent studies

[92–95]). We leave a more detailed investigation of these latter points for future work.

At tree-level, dimension-six contributions are expressed in the Warsaw basis as

$$\frac{gg'}{16\pi}\Delta S = \frac{v^2}{\Lambda^2}C_{\phi WB}, \quad -\frac{g^2g'}{2\pi(g^2+g'^2)}\Delta T = \frac{v^2}{\Lambda^2}C_{\phi D}, \quad (4.1)$$

while in the SILH basis these relations are given by $\alpha\Delta T = v^2/\Lambda^2 c_T$ and $\alpha\Delta S = 4s_W^2 v^2/\Lambda^2(c_W + c_B)$, in the leading order. At one-loop order in the EFT, more operators can contribute to the oblique parameters, and the full one-loop SMEFT computation can be found in Ref. [96]. Numerically, the contributions from dimension-six up to one-loop, in the SILH basis, read as [96]⁹

$$\begin{aligned} \Delta S &= \left\{ 0.76(c_B + c_W) + 0.001 \left[0.2(c_B + 4c_{BB}) + 1.48(2c_B + 2c_{HB}) + 1.96(c_H + c_T) \right. \right. \\ &\quad \left. \left. + 0.71c_W - 1.4(2c_{HW} + 2c_W) + 0.66 \left(-\frac{2c_{2W}}{g^2} + \frac{2c_{3W}}{3g^2} \right) \right] \right\} \left(\frac{1 \text{ TeV}^2}{\Lambda^2} \right), \\ \Delta T &= \{ 8c_T - 0.001[0.13(2c_B + 2c_{HB}) - 3.97(c_H + c_T) + 0.12(2c_{HW} + 2c_W)] \} \left(\frac{1 \text{ TeV}^2}{\Lambda^2} \right). \end{aligned} \quad (4.2)$$

In performing the NLO fit to the oblique parameters, we allow only the Wilson coefficients $C_{\phi WB}$ and $C_{\phi D}$ to vary, with all others set to zero. In this scenario, the impact of NLO corrections within the SMEFT framework remains small. Considering the parameters having effects up to dimension-eight operators that incorporate both the tree-level dimension-six and-eight terms and the known one-loop SMEFT contributions at $\mathcal{O}(1/\Lambda^2)$ [96]

$$\begin{aligned} \frac{gg'}{16\pi}\Delta S &= \frac{v^2}{\Lambda^2}\tilde{C}_{\phi WB} + [\Delta S]_{\text{IL}}^{(6)}, \\ -\frac{g^2g'}{2\pi(g^2+g'^2)}\Delta T &= \frac{v^2}{\Lambda^2}\tilde{C}_{\phi D} + [\Delta T]_{\text{IL}}^{(6)}, \\ \frac{g^2}{16\pi}\Delta U &= \frac{v^4}{\Lambda^4}C_{W^2\phi^4}^{(3)}, \end{aligned} \quad (4.3)$$

with

$$\tilde{C}_{\phi WB} \equiv C_{\phi WB} + \frac{v^2}{2\Lambda^2}C_{WB\phi^4}^{(1)}, \quad \tilde{C}_{\phi D} \equiv C_{\phi D} + \frac{v^2}{\Lambda^2}C_{\phi^6}^{(1)}.$$

Here, $[\Delta S]_{\text{IL}}^{(6)}$ and $[\Delta T]_{\text{IL}}^{(6)}$ denote the one-loop SMEFT corrections at $\mathcal{O}(1/\Lambda^2)$. The oblique parameter U receives the first non-negligible contribution at dimension-eight.

Note that in this analysis, we evolve the universal bosonic operators at dimension-six contributing to the oblique parameters, but we neglect renormalization group (RG)-induced departures from universality, which arise at order $g^2/(16\pi^2)\ln(\Lambda/\mu_{\text{EW}})$ [84] and would require extending the fit beyond the $\{S, T, U\}$ basis. We do not evolve the dimension-eight Wilson coefficients. RG effects at this order would modify their contributions only by terms of order $(16\pi^2)^{-1}\ln(\Lambda/m_Z)v^4/\Lambda^4$, which are numerically subleading compared to the matching contributions included in this analysis.

To quantify the relevance of our matching computations, we adopt the GFitter confidence levels (CLs) of [97] and take on board the future collider option through GigaZ. Especially when constraints are high quality, the results of going beyond the leading order approximation in the matching computation become relevant. In parallel, the level of agreement and disagreement with the full theory computation then informs the associated systematic effects quantitatively.

⁹Note that for the complex scalar triplet model, operators corresponding to c_T and c_H can be generated at tree level while the other operators in Eq. (4.2) are generated at one-loop level. For the 2HDM, all operators in Eq. (4.2) are generated at one loop-level.

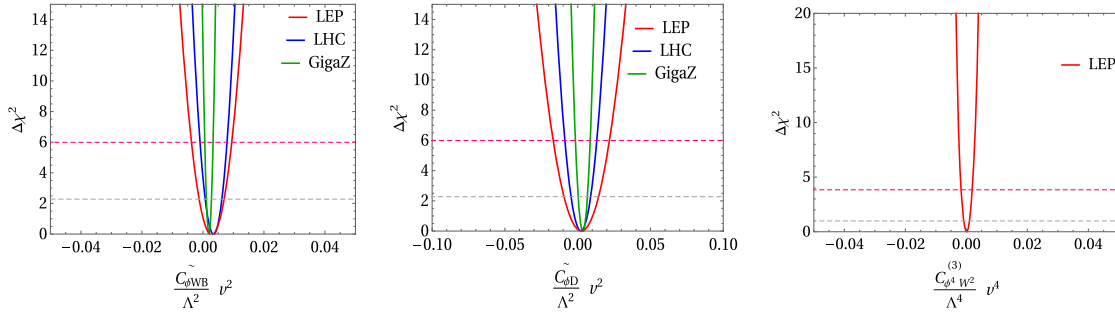


FIG. 1. One-dimensional fit and projections for the oblique parameters expressed in terms of coefficients $\tilde{C}_{\phi WB}^{(1)}/\Lambda^2$, $\tilde{C}_{\phi D}/\Lambda^2$, $\tilde{C}_{\phi^4 W^2}^{(3)}/\Lambda^4$. The horizontal dashed lines represent the allowed ranges at 68% CL (gray) and 95% CL (pink), respectively.

The results of a generic fit to the Wilson coefficients (WCs) at the LEP, expected LHC, and ILC/GigaZ are shown in Figs. 1 and 2 for completeness.¹⁰ A modest enhancement in precision is observed for the LHC, represented by the blue ellipse in Fig. 2 when comparing direct measurements with the SM expectation. In contrast, the GigaZ option (green ellipse in Fig. 2) demonstrates a significantly greater improvement in precision and sensitivity by an order of magnitude. We present the 95% marginalized limits on the dimension-six and eight operators for LHC, and GigaZ runs in Table VII.¹¹ Note that we first perform a dimension-six fit with $C_{\phi WB}$ and $C_{\phi D}$ coefficients against the oblique parameter constraints, while at dimension-eight, we work in the reduced three-parameter space $(\tilde{C}_{\phi WB}, \tilde{C}_{\phi D}, C_{W^2 \phi^4}^{(3)})$. In completely model-independent fits, it is difficult to assess the individual presence of dimension-eight operators, and marginalization leads to a severe degradation of the sensitivity. This observation is not new: adding more free parameters to the analysis of a fixed dataset introduces blind directions, and sensitivity, to some degree artificially and without motivation, is watered down. The relevant observation here is that moving forward to the precision frontier of the HL-LHC or a future Lepton collider, the contextualization of EFT constraints with matching calculations at high precision will be necessary to exploit

the quality of available data fully. Generic fits that leave all parameters free can inform this program through the available likelihoods. Still, taken in isolation, they provide neither a realistic nor an adequate measure of precision. Only matching to specific models provides clarity on their scale and effects.

We consider that the heavy scalars in the UV models do not acquire a vacuum expectation value and their renormalisable Yukawa couplings to SM fermions are switched off. The heavy fields then interact only with the bosonic sector. In this limit, the contributions relevant for the oblique parameter analysis arise from bosonic operators. Integrating them out at tree level and at one loop yields operators that can be mapped, using the SM equations of motion and field redefinitions, onto the universal SMEFT subspace discussed in Refs. [83,84]. The fermionic operators listed in Tables V and VI do not

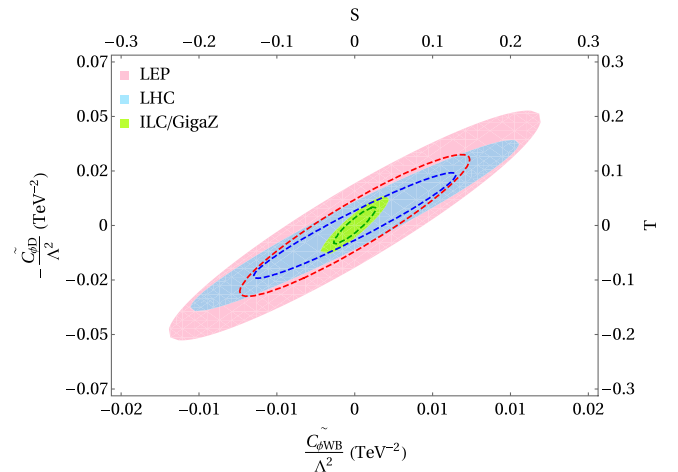


FIG. 2. Two-dimensional fit for the oblique parameters expressed in the S - T plane for terms of the present uncertainties (red), the LHC (blue), and the ILC/GigaZ prospects (green). These are equivalent to the coefficients $\tilde{C}_{\phi WB}^{(1)}/\Lambda^2$ and $\tilde{C}_{\phi D}/\Lambda^2$ via the map in Eq. (4.3). The shaded regions show the 2 σ preferred regions, while the dashed curves mark the 1 σ contours.

¹⁰For clarity, the Figs. 1 and 2 are shown with the tree-level map (including dimension-eight terms); the NLO corrections shift the allowed ranges only slightly, as quantified in Table VII, without changing the qualitative features.

¹¹We note that a TeraZ factory at the FCC-ee will produce 10^3 more Z bosons than GigaZ and can, therefore, in principle, constrain the oblique parameters at the level of 10^{-5} . A plethora of electroweak contributions need to be under control within the SM to fully exploit the BSM sensitivity that such an environment provides. Hence, we will focus on the more conservative GigaZ option in the following. Taken at face value, without reference to the accuracy of the SM hypothesis, TeraZ improves over GigaZ by an order of magnitude for the parameter choices discussed below.

TABLE VII. 95% CL constraints on the model-independent effective couplings contributing to the oblique parameters: (left column) only dimension-six, and (right column) both the dimension-six and -eight contributions for both LHC and FCC ee (GigaZ) scenarios. Constraints on each coefficient of the first column are obtained after marginalizing the other coefficients. The first row for each dimension-six coefficient presents the LO constraints, while the bracketed row presents the corresponding NLO constraints. The U parameter constrains the dimension eight operator coefficient $\frac{v^4}{\Lambda^4} C_{W^2\phi^4}^{(3)}$ to $[-0.0021, 0.0019]$ at 95% CL from current GFitter results [97].

i	Oblique parameter fit: 95% CL allowed range			
	LHC		FCC-ee (GigaZ)	
	$\frac{v^2}{\Lambda^2} C_i$	$\frac{v^2}{\Lambda^2} \tilde{C}_i$	$\frac{v^2}{\Lambda^2} C_i$	$\frac{v^2}{\Lambda^2} \tilde{C}_i$
ϕWB	$[-0.00043, 0.00068]$ ($[-0.00049, 0.00062]$)	$[-0.00141, 0.00758]$	$[-0.00010, 0.00292]$ ($[-0.00010, 0.00278]$)	$[-0.00020, 0.00394]$
ϕD	$[-0.00762, 0.01020]$ ($[-0.00786, 0.01011]$)	$[-0.00980, 0.00980]$	$[-0.00120, 0.00626]$ ($[-0.00129, 0.00623]$)	$[-0.00190, 0.00840]$

constitute independent directions for the oblique parameter analysis.¹² We therefore set their Wilson coefficients to zero by assumption when performing the fit.

We investigate the deviation of the EFT with the full theory after performing the one-loop level matching to the SMEFT up to dimension-eight.¹³ When the relatively conservative projections of the GigaZ EW precision constraints on the parameters S and T contrast with the model-specific circumstances of the models discussed in this paper, we obtain Fig. 3. There, the dimension-six contributions contain terms in the linear dimension-six WCs, where the corrections up to dimension-eight operators include dimensions-six and -eight in quadratic and linear, respectively.

We first consider the complex triplet model and study the impact on the quartic couplings λ_1 and λ_4 , which is illustrated in Fig. 3. Here, we assume the dimensionful trilinear coupling μ_Δ is much less than the mass of the triplet scalar: $\mu_\Delta/M_\Delta = 0.1$, and also set $\lambda_2 = \lambda_3 = 1$. While from Fig. 3, differences between the full model and the EFT curves may appear modest, quantitative deviations, especially in the T parameter, remain sizeable at lower masses and couplings. As shown in Table VIII for the triplet benchmark with $M_\Delta = 700$ GeV, the dimension-six truncation alone can lead to discrepancies of 11.58% deviation in the S parameter and 18.86% deviation in the T parameter. However, incorporating higher-dimensional operators (dimension-eight) reduces these deviations to 5.90% for S and 5.24% for T , improving the agreement

between the EFT and the full theory. The inclusion of dimension-eight operators systematically improves the agreement and is crucial for percent-level precision, as targeted by experiments such as the HL-LHC and GigaZ. Furthermore, the agreement continues to improve as the mass of the heavy triplet increases. In our framework, the triplet scalar does not acquire any vacuum expectation value and is integrated out. Thus, their passive presence is captured through the higher-dimensional effective operators that may result in final states of multilepton or multi- W bosons. Here, the mass of the heavy scalar controls the

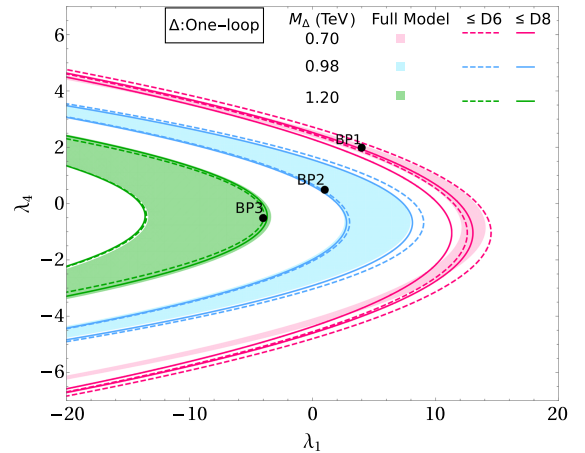


FIG. 3. The 95% CL GigaZ exclusion regions in $\lambda_1 - \lambda_4$ plane for different values of heavy complex triplet masses, M_Δ (pink for $M_\Delta = 0.7$ TeV, blue for $M_\Delta = 0.98$ TeV, green for $M_\Delta = 1.2$ TeV), where we have fixed $\mu_\Delta/M = 0.1$, $\lambda_2 = \lambda_3 = 1$. {BP1, BP2, BP3}, marked by the black dots are summarized in Table VIII. The shaded region depicts the allowed region from the oblique parameter constraints corresponding to a given triplet mass in the full model, and the solid (dashed) contours depict the SMEFT contribution up to dimension-eight (dimension-six) after matching to the complex triplet model up to one-loop.

¹²At dimension-eight, the bosonic matching results belong to the universal SMEFT basis of Ref. [98]. After employing the equations of motion, this corresponds to the specific bosonic and fermionic operator combinations that characterize universal theories, rather than a general fermionic dimension-eight SMEFT.

¹³The matching relations used are given in Appendix B, derived using the results given in Appendixes A 1 and A 2.

TABLE VIII. Comparison of S and T parameters for some benchmark points (BPs) with different masses and models. The deviation is given by $|\frac{\mathcal{O}_{\text{full}} - \mathcal{O}_{\text{EFT}}}{\mathcal{O}_{\text{full}}}| \times 100$, i.e., differences between full theory expectation and the EFT, normalized to the full theory expectation.

Heavy scalar model	Framework	S parameter		T parameter	
		Value	Deviation (%)	Value	Deviation (%)
BP1: $M_\Delta = 0.7$ TeV, $\lambda_2 = \lambda_3 = 1, \lambda_1 = 1$, $\lambda_4 = 2, \mu_\Delta/M_\Delta = 0.1$	Full model	-2.911×10^{-3}	...	-3.227×10^{-2}	...
	Dim-6	-2.574×10^{-3}	11.58	-3.835×10^{-2}	18.86
	Dim-6 + Dim-8	-2.740×10^{-3}	5.90	-3.396×10^{-2}	5.24
BP2: $M_\Delta = 0.98$ TeV, $\lambda_2 = \lambda_3 = 1, \lambda_1 = 0.5$, $\lambda_4 = 0.5, \mu_\Delta/M_\Delta = 0.1$	Full model	-1.451×10^{-3}	...	-1.509×10^{-2}	...
	Dim-6	-1.377×10^{-3}	5.10	-1.374×10^{-2}	8.94
	Dim-6 + Dim-8	-1.481×10^{-3}	2.09	-1.147×10^{-2}	2.20
BP3: $M_\Delta = 1.2$ TeV, $\lambda_2 = \lambda_3 = 1, \lambda_1 = -4$, $\lambda_4 = -0.5, \mu_\Delta/M_\Delta = 0.1$	Full model	-6.623×10^{-4}	...	-1.324×10^{-2}	...
	Dim-6	-6.318×10^{-4}	4.61	-1.434×10^{-2}	6.19
	Dim-6 + Dim-8	-6.444×10^{-4}	2.70	-1.283×10^{-2}	1.23
BP4: $M_{\mathcal{H}_2} = 0.7$ TeV, $\eta_H = 0.3, \eta_{\mathcal{H}_2} = 0.2, \lambda_{\mathcal{H}_2,1} = 1.0$, $\lambda_{\mathcal{H}_2,2} = 0.5, \lambda_{\mathcal{H}_2,3} = 0.4$	Full model	5.788×10^{-5}	...	-5.389×10^{-4}	...
	Dim-6	6.939×10^{-5}	19.89	-6.388×10^{-4}	18.54
	Dim-6 + Dim-8	5.574×10^{-5}	3.69	-5.163×10^{-4}	4.19
BP5: $M_{\mathcal{H}_2} = 1$ TeV, $\eta_H = 0.3, \eta_{\mathcal{H}_2} = 0.2, \lambda_{\mathcal{H}_2,1} = 1.0$, $\lambda_{\mathcal{H}_2,2} = 0.5, \lambda_{\mathcal{H}_2,3} = 0.4$	Full model	3.020×10^{-5}	...	-2.142×10^{-4}	...
	Dim-6	3.361×10^{-5}	6.67	-2.291×10^{-4}	6.93
	Dim-6 + Dim-8	3.001×10^{-5}	2.58	-2.203×10^{-4}	2.87
BP6: $M_{\mathcal{H}_2} = 1.2$ TeV, $\eta_H = 0.3, \eta_{\mathcal{H}_2} = 0.2, \lambda_{\mathcal{H}_2,1} = 1.0$, $\lambda_{\mathcal{H}_2,2} = 0.5, \lambda_{\mathcal{H}_2,3} = 0.4$	Full model	2.188×10^{-5}	...	-1.533×10^{-4}	...
	Dim-6	2.084×10^{-5}	4.76	-1.607×10^{-4}	4.74
	Dim-6 + Dim-8	2.228×10^{-5}	1.81	-1.506×10^{-4}	1.79

strength of the effective interactions and thus, in turn, affects the signal cross sections. In our analysis, unlike the full theory, the reconstruction of final states will not lead to any resonance peak corresponding to the heavy scalar mass. Thus, in the context of EFT, the direct production of heavy scalars, e.g., $\Delta^{\pm\pm}$, in collider experiments is not the relevant search channel.

Table VIII provides a more detailed comparison between the full theory, dimension-six, and dimension-eight effective approaches, illustrating the improvement in the EFT description of the full model when higher-dimensional operators are included. While naive scaling would suggest that the relative deviations from the full model decrease strictly as v^2/Λ^2 (dimension-six) and $(v^2/\Lambda^2)^2$ (dimension-eight), this relies on the assumption of fixed Wilson coefficients and a linear EFT expansion. In our UV completions, for instance, in the case of the complex triplet model, however, the matching coefficients depend non-trivially on quartic couplings λ_i and the dimensionful trilinear parameter μ_Δ , leading to deviations from the simplest scaling estimates. The S parameter deviations follow the expected v^2/M_Δ^2 behavior rather closely. In the case of the T parameter, the trend is less sharp because both tree-level terms $\propto \mu_\Delta^2/M_\Delta^4$ and loop-induced terms $\propto \lambda_i/M_\Delta^2$ contribute, and their interplay can either enhance or partially cancel the scaling behavior. At dimension-eight, the deviations systematically decrease further (to the

few-percent level). Similar observations hold for the 2HDM, see Fig. 4. The inclusion of dimension-eight terms leads to moderate but systematic improvements across benchmark points. Although not prominent in Fig. 4, the deviations shown in Table VIII highlight that the S and T parameters can differ from the full model at the level of 5–20% in the dimension-six approximation, even at 700 GeV. Dimension-eight terms reduce these to the 5% level, consistent with the expected goals of future precision programs. At $M_{\mathcal{H}_2} = 1$ TeV, the dimension-six approximation improves, but dimension-eight terms still refine the agreement, especially in the T parameter. Note that the benchmark points in Figs. 3 and 4, and Table VIII are chosen only for illustration, and the results remain unchanged under moderate variations of these parameters. The inclusion of properly matched dimension-eight contributions enables an adequate comparison with the full model over a broad range of masses and couplings. This extends to parameter ranges that also would start to push the perturbativity of the UV-complete model. The UV-complete model is no longer under control in the context of how the (experimental) limit setting is obtained in this region, yet the matched effective approach accurately reproduces this nonperturbative range.

For the discussed scenarios, and given the expected sensitivity for the parameters considered in our analysis, the precise determination of the microscopic parameters of the

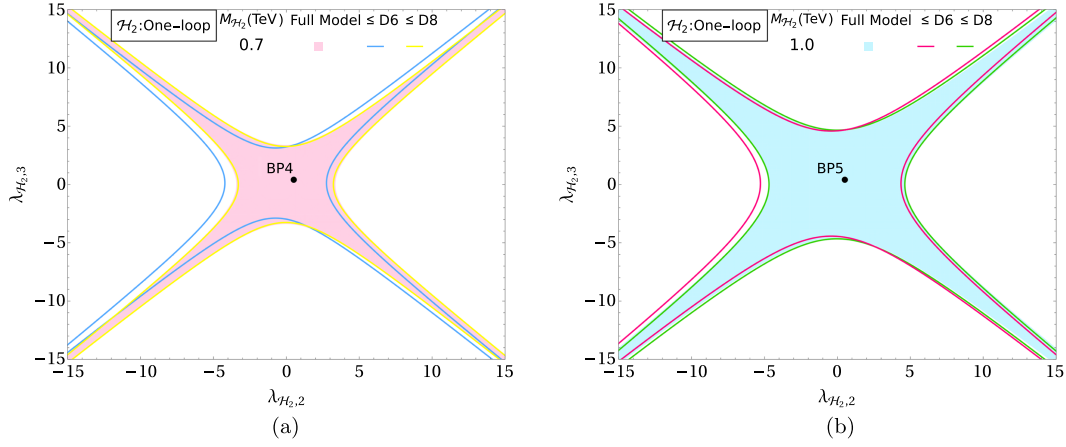


FIG. 4. The 95% CL GigaZ exclusion regions in $\lambda_{\mathcal{H}_{2,2}} - \lambda_{\mathcal{H}_{2,3}}$ plane for different values of heavy complex doublet masses, $M_{\mathcal{H}_{2,2}}$, where we have fixed $\eta_{\mathcal{H}} = 0.3$, $\eta_{\mathcal{H}_2} = 0.2$, $\lambda_{\mathcal{H}_{2,1}} = 1$. {BP4, BP5}, marked by the black dots are summarized in Table VIII. (a) The blue and yellow contours for $M_{\mathcal{H}_{2,2}} = 0.7$ TeV show the preferred regions for contributions from dimension-six operators and up to dimension-eight operators, respectively. (b) Pink and green contours with $M_{\mathcal{H}_{2,2}} = 1.0$ TeV represent contributions from dimension-six operators and up to dimension-eight operators, respectively.

BSM model using model-independent techniques crucially rests on precise matching. This creates significant theoretical overhead, but proves necessary to analyze scenarios over a broad parameter range when the data quality is high. Of course, in particular when experimental observations remain consistent with the SM expectation, matching improvements and the improved determination of the models' UV parameter space are not directly phenomenologically relevant. However, if an anomaly is observed, it is the very quality of UV parameter determination that feeds into the models' viability in other phenomenological arenas, e.g., the determination of first-order phase transitions in the early universe as part of Sakharov's criteria that are sensitively dependent on the model parameters, see, e.g., [99].

V. CONCLUSIONS

We explored the implications of dimension-eight operators within the SMEFT framework by systematically deriving their contributions from two extensions of the Standard Model: the complex scalar triplet model and the two Higgs doublet model. Using the heat kernel method, we performed a one-loop matching assuming only heavy scalars in the loop. This yielded the effective operators and their corresponding Wilson coefficients, shedding light on the interplay between heavy scalar fields and light-sector interactions. The resulting effective interactions, computed up to dimension-eight and one-loop, are systematically displayed in Tables I to VI. Furthermore, Tables IX and X provide a mapping of these interactions to a nonredundant basis, facilitating their application in further theoretical and phenomenological studies.

Our results highlight that dimension-eight operators play a systematic role in improving the precision of SMEFT

description. While the overall corrections remain moderate, they become relevant in high-precision contexts such as electroweak precision tests and percent-level measurements expected at future colliders. Moreover, our one-loop matching shows that dimension-eight operators can induce sizeable effects in UV scenarios, particularly in energy-enhanced processes such as Higgs production, vector boson scattering, and Drell-Yan. Parameter regions of models can be relevant to models' viability in other phenomenological arenas beyond those discussed here. Contextualizing, in model-independent approaches, a specific scenario's viability in addressing Sakharov's criteria for the experimentally established BSM nature of baryogenesis, for instance, sensitively depends on the parameter choices of the UV model (see, e.g., [100,101] for recent case studies).

Furthermore, we demonstrate the relevance of these contributions to electroweak precision observables, providing quantitative insights to their impact on parameters such as ΔS , ΔT , and ΔU . The sensitivity to deviations from the Standard Model is primarily dictated by measurement precision. Thus, the incorporation of dimension-eight operators provides a more precise theoretical description, leading to a better interpretation of potential deviations.

We also investigate the fermionic interactions mediated by heavy scalar fields. We identified and categorized the effective operators, including those affecting four-fermion interactions and neutrino mass terms. These operators include corrections to known dimension-six terms and generate new contributions that first appear at dimension-seven and -eight. We find that fermionic interactions mediated by heavy scalar fields enrich the structure of the effective field theory and introduce distinctive signatures in precision observables and collider processes, enabling potential discrimination between different ultraviolet-

TABLE IX. Translation of redundant dimension-eight *Green's basis* coefficients, emerging in the complex triplet model and complex doublet model into the other nonredundant dimension-eight operators and dimension-eight fermionic operators of *Murphy-basis* form. Here, Y_Φ refers to the hypercharge of the heavy scalar and Y_{SM}^{pq} denotes the SM Yukawa coupling, $\{p, q\} \in (1, 2, 3)$ are the flavor indices.

Green's basis operator	Green's basis coefficient	Relation of Green's basis operator in terms of Murphy basis operators
$\mathcal{O}_{\phi^6}^{(3)}$	$c_{\phi^6}^{(3)}$	$-4\lambda_{\text{SM}}\mathcal{O}_{\phi^8} + 2\lambda_{\text{SM}}v^2\mathcal{O}_6 - Y_{\text{SM}}^{pq}\mathcal{Q}_{\psi^2\phi^5} + \text{H.c.}$
$\mathcal{O}_{\phi^4}^{(4)}$	$c_{\phi^4}^{(4)}$	$-4\lambda_{\text{SM}}\mathcal{O}_{\phi^6}^{(1)} + 2\lambda_{\text{SM}}v^2(\phi^\dagger\phi)(D_\mu\phi^\dagger)(D^\mu\phi) - Y_{\text{SM}}^{pq}\mathcal{Q}_{\psi^2\phi^3D^2} + \text{H.c.}$
$\mathcal{O}_{\phi^4}^{(8)}$	$c_{\phi^4}^{(8)}$	$2\lambda_{\text{SM}}(v^4(\phi^\dagger\phi)^2 + 4\mathcal{O}_{\phi^8} - 4v^2\mathcal{O}_6) - 2\lambda_{\text{SM}}(\phi^\dagger\phi)J_\phi\phi + 4\lambda_{\text{SM}}Y_{\text{SM}}^{pq}\mathcal{Q}_{\psi^2\phi^5} + Y_{\text{SM}}^{pq^2}\mathcal{Q}_{\psi^4\phi^2}^{(2)} + \text{H.c.}$
$\mathcal{O}_{\phi^4}^{(10)}$	$c_{\phi^4}^{(10)}$	$\lambda_{\text{SM}}(v^4(\phi^\dagger\phi)^2 + 4\mathcal{O}_{\phi^8} - 4v^2\mathcal{O}_6) - \lambda_{\text{SM}}(\phi^\dagger\phi)J_\phi\phi + 2\lambda_{\text{SM}}Y_{\text{SM}}^{pq}\mathcal{Q}_{\psi^2\phi^5} + Y_{\text{SM}}^{pq^2}\mathcal{Q}_{\psi^4\phi^2}^{(1)} + \text{H.c.}$
$\mathcal{O}_{\phi^4}^{(11)}$	$c_{\phi^4}^{(11)}$	$\lambda_{\text{SM}}(v^4(\phi^\dagger\phi)^2 + 4\mathcal{O}_{\phi^8} - 4v^2\mathcal{O}_6) - \lambda_{\text{SM}}(\phi^\dagger\phi)J_\phi\phi + 2\lambda_{\text{SM}}Y_{\text{SM}}^{pq}\mathcal{Q}_{\psi^2\phi^5} + Y_{\text{SM}}^{pq^2}\mathcal{Q}_{\psi^4\phi^2}^{(3)} + \frac{1}{2}Y_{\text{SM}}^{pq^2}\mathcal{Q}_{\psi^4\phi^2}^{(1)} + \text{H.c.}$
$\mathcal{O}_{W\phi^4D^2}^{(6)}$	$c_{W\phi^4D^2}^{(6)}$	$\frac{g}{2}(\lambda_{\text{SM}}[v^2\mathcal{O}_6 - 2\mathcal{O}_{\phi^8}] + 5\mathcal{O}_{\phi^6}^{(1)}) + \frac{1}{2}Y_{\text{SM}}^{pq}(\mathcal{Q}_{\psi^2\phi^4D}^{(2)} - i\mathcal{Q}_{\psi^2\phi^4D}^{(4)} + \text{H.c.}) - \frac{g}{4}Y_{\text{SM}}^{pq}\mathcal{Q}_{\psi^2\phi^5}^{(1)}$
$\mathcal{O}_{W\phi^4D^2}^{(7)}$	$c_{W\phi^4D^2}^{(7)}$	$-4g(\mathcal{Q}_{\phi^6}^{(1)} - \mathcal{Q}_{\phi^6}^{(2)}) - 4Y_{\text{SM}}^{pq}(\mathcal{Q}_{\psi^2\phi^4D}^{(4)} - i\mathcal{Q}_{\psi^2\phi^4D}^{(3)} + \text{H.c.}) + i4\mathcal{O}_{W\phi^4D^2}^{(3)}$
$\mathcal{O}_{B\phi^4D^2}^{(3)}$	$c_{B\phi^4D^2}^{(3)}$	$i\frac{g'}{2}(\lambda_{\text{SM}}[v^2\mathcal{O}_6 - 2\mathcal{O}_{\phi^8}] + 3\mathcal{O}_{\phi^6}^{(1)} + 2\mathcal{O}_{\phi^6}^{(2)}) + iY_{\text{SM}}^{pq}\mathcal{Q}_{\psi^2\phi^4D}^{(1)} - i\frac{g'}{4}Y_{\text{SM}}^{pq}\mathcal{Q}_{\psi^2\phi^5}^{(1)} + \text{H.c.}$
$\mathcal{O}_{W^2\phi^2D^2}^{(13)}$	$c_{W^2\phi^2D^2}^{(13)}$	$\frac{g^2}{4}(\lambda_{\text{SM}}[v^2\mathcal{O}_6 - 2\mathcal{O}_{\phi^8}] + 5\mathcal{O}_{\phi^6}^{(1)}) + \frac{g}{2}Y_{\text{SM}}^{pq}(\mathcal{Q}_{\psi^2\phi^4D}^{(2)} + \mathcal{Q}_{\psi^2\phi^4D}^{(4)}) - \frac{g^2}{8}Y_{\text{SM}}^{pq}\mathcal{Q}_{\psi^2\phi^5}^{(1)} + Y_{\text{SM}}^{pq^2}\mathcal{Q}_{\psi^4\phi^2}^{(5)} + \text{H.c.}$
$\mathcal{O}_{W^2\phi^2D^2}^{(14)}$	$c_{W^2\phi^2D^2}^{(14)}$	$-\lambda_{\text{SM}}v^2\mathcal{O}_{\phi W} + 2\lambda_{\text{SM}}\mathcal{O}_{W^2\phi^4}^{(1)} + Y_{\text{SM}}^{pq}\mathcal{Q}_{\psi^2W^2\phi}^{(1)} + \text{H.c.}$
$\mathcal{O}_{B^2\phi^2D^2}^{(8)}$	$c_{B^2\phi^2D^2}^{(8)}$	$-\frac{g'^2}{4}(\lambda_{\text{SM}}[v^2\mathcal{O}_6 - 2\mathcal{O}_{\phi^8}] + 3\mathcal{O}_{\phi^6}^{(1)} + 2\mathcal{O}_{\phi^6}^{(2)}) + ig'Y_\Phi\mathcal{O}_{B\phi^4D^2}^{(1)} + \frac{g'}{4}(g'Y_\Phi^2\mathcal{O}_{B^2\phi^4}^{(1)} + gY_\Phi\mathcal{O}_{WB\phi^4}^{(1)}) - \frac{1}{2}Y_{\text{SM}}^{pq}g'Y_\Phi(\mathcal{Q}_{\psi^2B\phi^2D}^{(1)} + i\mathcal{Q}_{\psi^2B\phi^2D}^{(3)}) - \frac{g'}{2}Y_{\text{SM}}^{pq}\mathcal{Q}_{\psi^2\phi^4D}^{(1)} + \frac{g^2}{8}Y_{\text{SM}}^{pq}\mathcal{Q}_{\psi^2\phi^5}^{(1)} + \text{H.c.}$
$\mathcal{O}_{W^2\phi^2D^2}^{(11)}$	$c_{W^2\phi^2D^2}^{(11)}$	$-\frac{g^2}{4}(\lambda_{\text{SM}}[v^2\mathcal{O}_6 - 2\mathcal{O}_{\phi^8}] + 5\mathcal{O}_{\phi^6}^{(1)}) + g(i\mathcal{O}_{W\phi^4D^2}^{(1)} - \mathcal{Q}_{W\phi^4D^2}^{(3)}) + \frac{g}{4}(g'Y_\Phi\mathcal{O}_{WB\phi^4}^{(1)} + 2g\mathcal{O}_{W^2\phi^4}^{(1)}) - \frac{1}{2}Y_{\text{SM}}^{pq}(\mathcal{Q}_{\psi^2W\phi^2D}^{(5)} + i\mathcal{Q}_{\psi^2W\phi^2D}^{(7)}) - \frac{g}{8}Y_{\text{SM}}^{pq}(\mathcal{Q}_{\psi^2\phi^4D}^{(2)} + \mathcal{Q}_{\psi^2\phi^4D}^{(4)} - i\mathcal{Q}_{\psi^2\phi^4D}^{(3)}) + \frac{g^2}{16}Y_{\text{SM}}^{pq}(\mathcal{Q}_{\psi^2\phi^5}^{(1)} + \mathcal{Q}_{\psi^2\phi^5}^{(1)}) + \text{H.c.}$
$\mathcal{O}_{WB\phi^2D^2}^{(10)}$	$c_{WB\phi^2D^2}^{(10)}$	$\frac{gg'}{4}(\lambda_{\text{SM}}[v^2\mathcal{O}_6 - 2\mathcal{O}_{\phi^8}] + 3\mathcal{O}_{\phi^6}^{(1)} + 2\mathcal{O}_{\phi^6}^{(2)}) - \frac{gg'}{8}Y_{\text{SM}}^{pq}(\mathcal{Q}_{\psi^2\phi^5}^{(1)} + \mathcal{Q}_{\psi^2\phi^5}^{(1)}) + \frac{g}{2}Y_{\text{SM}}^{pq}\mathcal{Q}_{\psi^2\phi^4D}^{(1)} + \frac{g'}{4}Y_{\text{SM}}^{pq}(\mathcal{Q}_{\psi^2\phi^4D}^{(2)} - \mathcal{Q}_{\psi^2\phi^4D}^{(4)}) + Y_{\text{SM}}^{pq^2}\mathcal{Q}_{\psi^4\phi^2}^{(7)} + \text{H.c.}$

TABLE X. Translation of redundant dimension-eight *Green's basis* coefficients, emerging in the complex triplet model and complex doublet model into the other nonredundant dimension-eight operators and dimension-eight fermionic operators of *Murphy-basis* form. Here, Y_Φ refers to the hypercharge of the heavy scalar and Y_{SM}^{pq} denotes the SM Yukawa coupling, $\{p, q\} \in (1, 2, 3)$ are the flavor indices.

Green's basis operator	Green's basis coefficient	Relation of Green's basis operator in terms of Murphy basis operators
$\mathcal{O}_{WB\phi^2D^2}^{(11)}$	$c_{WB\phi^2D^2}^{(11)}$	$-2g'(i\mathcal{Q}_{W\phi^4D^2}^{(2)} + \mathcal{Q}_{W\phi^4D^2}^{(4)}) - \frac{gg'}{4}(\mathcal{Q}_{W^2\phi^4}^{(2)} - \mathcal{Q}_{W^2\phi^4}^{(4)}) - 2Y_{\text{SM}}^{pq}(\mathcal{Q}_{\psi^2W\phi^2D}^{(2)} - i\mathcal{Q}_{\psi^2W\phi^2D}^{(4)}) + \frac{gg'}{2}(\lambda_{\text{SM}}[v^2\mathcal{O}_6 - 2\mathcal{O}_{\phi^8}] + 3\mathcal{O}_{\phi^6}^{(1)} + 2\mathcal{O}_{\phi^6}^{(2)}) - 2gg'Y_\Phi\mathcal{O}_{WB\phi^2D^2}^{(4)} - i6gY_\Phi\mathcal{O}_{B\phi^4D^2}^{(1)} - \frac{g}{2}(gY_\Phi\mathcal{O}_{WB\phi^4}^{(1)} + g'Y_\Phi^2\mathcal{O}_{B^2\phi^4}^{(1)}) + g\mathcal{Q}_{\psi^2\phi^4D}^{(1)} - \frac{gg'}{4}(\mathcal{Q}_{\psi^2\phi^5}^{(1)} + \text{H.c.}) - gg'(\lambda_{\text{SM}}[v^2\mathcal{O}_6 - 2\mathcal{O}_{\phi^8}] + 3\mathcal{O}_{\phi^6}^{(1)} + 2\mathcal{O}_{\phi^6}^{(2)}) - 4\mathcal{Q}_{\psi^4\phi^2}^{(7)} + \frac{gg'}{2}Y_{\text{SM}}^{pq}(\mathcal{Q}_{\psi^2\phi^5}^{(1)} + \mathcal{Q}_{\psi^2\phi^5}^{(1)}) - 2gY_{\text{SM}}^{pq}\mathcal{Q}_{\psi^2\phi^4D}^{(1)} - g'Y_{\text{SM}}^{pq}(\mathcal{Q}_{\psi^2\phi^4D}^{(2)} - \mathcal{Q}_{\psi^2\phi^4D}^{(4)}) + \text{H.c.}$

(Table continued)

TABLE X. (Continued)

Green's basis operator	Green's basis coefficient	Relation of Green's basis operator in terms of Murphy basis operators
$\mathcal{O}_{WB\phi^2 D^2}^{(13)}$	$c_{WB\phi^2 D^2}^{(13)}$	$ \begin{aligned} & + \frac{gg'}{4} (\lambda_{\text{SM}} [v^2 \mathcal{O}_6 - 2\mathcal{O}_{\phi^8}] + 3\mathcal{O}_{\phi^6}^{(1)} + 2\mathcal{O}_{\phi^6}^{(2)}) - i3gY_{\Phi} \mathcal{O}_{B\phi^4 D^2}^{(1)} - \frac{g}{4} (gY_{\Phi} \mathcal{O}_{WB\phi^4}^{(1)} + g'Y_{\Phi}^2 \mathcal{O}_{B^2\phi^4}^{(1)}) \\ & + \frac{g}{2} Y_{\text{SM}}^{pq} \mathcal{Q}_{\psi^2 \phi^4 D}^{(1)} - Y_{\text{SM}}^{pq} Y_{\Phi} (\mathcal{Q}_{\psi^2 B\phi^2 D}^{(5)} + i\mathcal{Q}_{\psi^2 B\phi^2 D}^{(7)}) - \frac{gg'}{8} Y_{\text{SM}}^{pq} (\mathcal{Q}_{\psi^2 \phi^5}^{(1)} + \text{H.c.}) \\ & - \frac{gg'}{4} (\lambda_{\text{SM}} [v^2 \mathcal{O}_6 - 2\mathcal{O}_{\phi^8}] + 3\mathcal{O}_{\phi^6}^{(1)} + 2\mathcal{O}_{\phi^6}^{(2)}) + \frac{gg'}{8} Y_{\text{SM}}^{pq} (\mathcal{Q}_{\psi^2 \phi^5}^{(1)} + \mathcal{Q}_{\psi^2 \phi^5}^{\dagger(1)}) \\ & - \frac{g}{2} Y_{\text{SM}}^{pq} \mathcal{Q}_{\psi^2 \phi^4 D}^{(1)} - \frac{g'}{4} Y_{\text{SM}}^{pq} (\mathcal{Q}_{\psi^2 \phi^4 D}^{(2)} - \mathcal{Q}_{\psi^2 \phi^4 D}^{(4)}) - Y_{\text{SM}}^{pq^2} \mathcal{Q}_{\psi^4 \phi^2}^{(7)} \end{aligned} $
$\mathcal{O}_{B^2 D^4}$	$c_{B^2 D^4}$	$ \begin{aligned} & \frac{g^2 g^2}{4} \lambda_{\text{SM}} ((v^2 \mathcal{O}_6 - 2\mathcal{O}_{\phi^8}) + 2\mathcal{O}_{\phi^6}^{(1)} + 2\mathcal{O}_{\phi^6}^{(2)}) + g^2 (\mathcal{Q}_{\phi^4}^{(1)} - \mathcal{Q}_{\phi^4}^{(2)}) \\ & + \frac{g^2}{8} (g^2 Y_{\Phi}^2 \mathcal{O}_{B^2 \phi^4}^{(1)} - g^2 \mathcal{O}_{W^2 \phi^4}^{(1)}) + i g^2 (g' Y_{\Phi} \mathcal{O}_{B\phi^4 D^2}^{(1)} - g \mathcal{O}_{W\phi^4 D^2}^{(1)}) - \frac{g'}{2} Y_{\text{SM}}^{pq} \mathcal{Q}_{\psi^2 \phi^2 D^3}^{(1)} + \mathcal{Q}_{\psi^4 D^2}^{(2)} \\ & - \frac{g^2 g}{4} Y_{\text{SM}}^{pq} \mathcal{Q}_{\psi^2 \phi^4 D}^{(2)} - \frac{g^2 g^2}{8} Y_{\text{SM}}^{pq} \mathcal{Q}_{\psi^2 \phi^5}^{(1)} - \frac{3g^2}{4} Y_{\text{SM}}^{pq^2} (-\mathcal{Q}_{\psi^4 \phi^2}^{(1)} + \mathcal{Q}_{\psi^4 \phi^2}^{(2)} - 2\mathcal{Q}_{\psi^4 \phi^2}^{(3)}) \end{aligned} $
$\mathcal{O}_{W^2 D^4}$	$c_{W^2 D^4}$	$ \begin{aligned} & \frac{g^4}{4} \lambda_{\text{SM}} ((v^2 \mathcal{O}_6 - 2\mathcal{O}_{\phi^8}) + 5\mathcal{O}_{\phi^6}^{(1)}) + \frac{g^2 g^2}{2} \lambda_{\text{SM}} ((v^2 \mathcal{O}_6 - 2\mathcal{O}_{\phi^8}) + 3\mathcal{O}_{\phi^6}^{(1)} + 2\mathcal{O}_{\phi^6}^{(2)}) \\ & - g^2 (\mathcal{O}_{\phi^4}^{(1)} + \mathcal{O}_{\phi^4}^{(2)} - 2\mathcal{O}_{\phi^4}^{(3)}) - \frac{g^2}{8} (3g^2 Y_{\Phi}^2 \mathcal{O}_{B^2 \phi^4}^{(1)} + g^2 \mathcal{O}_{W^2 \phi^4}^{(1)} + 4gg' Y_{\Phi} \mathcal{O}_{WB\phi^4}^{(1)}) \\ & - i g^2 (3g' Y_{\Phi} \mathcal{O}_{B\phi^4 D^2}^{(1)} + g \mathcal{O}_{W\phi^4 D^2}^{(1)}) + \frac{g^2}{4} Y_{\text{SM}}^{pq} (g \mathcal{Q}_{\psi^2 \phi^4 D}^{(2)} + g \mathcal{Q}_{\psi^2 \phi^4 D}^{(4)} + 4g' \mathcal{Q}_{\psi^2 \phi^4 D}^{(1)}) + \text{H.c.} \\ & - \frac{g}{2} Y_{\text{SM}}^{pq} \mathcal{Q}_{\psi^2 \phi^2 D^3}^{(2)} - g^2 Y_{\text{SM}}^{pq} \mathcal{Q}_{\psi^2 W\phi^2 D}^{(11)} - \frac{g^2 (g^2 + 2g^2)}{8} Y_{\text{SM}}^{pq} \mathcal{Q}_{\psi^2 \phi^5}^{(1)} \\ & - \frac{3g^2}{4} Y_{\text{SM}}^{pq^2} (-3\mathcal{Q}_{\psi^4 \phi^2}^{(1)} + \mathcal{Q}_{\psi^4 \phi^2}^{(2)} + \mathcal{Q}_{\psi^4 \phi^2}^{\dagger(2)} + 2\mathcal{Q}_{\psi^4 \phi^2}^{(3)}) + Y_{\text{SM}}^{pq^2} \mathcal{Q}_{\psi^4 D^2}^{(3)} + \text{H.c.} \end{aligned} $

complete models. Thus, this shows a nontrivial interplay between scalar and fermionic sectors in effective descriptions of BSM physics.

As future experiments such as the high-luminosity LHC and prospective lepton colliders, e.g., the FCC-ee or ILC, promise unprecedented sensitivity, our findings lay a foundation for probing new physics through higher-order effects. These generalize to differential distributions where they are experimentally available, e.g., during the HL-LHC phase. Our work also serves as a stepping stone for extending EFT methodologies to include even higher-dimensional operators, further refining our ability to interpret physics signals beyond the Standard Model.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: MATCHING DETAILS

1. Case I: Integrating out complex triplet scalars

In this appendix, we present the computation details after applying the generic procedure described in Sec. II to the particular case of the complex triplet model. We can express terms in the Lagrangian Eq. (2.9) as

$$\begin{aligned}
\text{Tr}[\Delta^\dagger \Delta] &= \text{Tr}[\Delta_i^* \Delta_j \tau^i \tau^j] = \frac{1}{2} \Delta_i^* \Delta_i = \frac{1}{2} |\Delta|^2, \\
(\text{Tr}[\Delta^\dagger \Delta])^2 &= (\text{Tr}[\Delta_i^* \Delta_j \tau^i \tau^j])^2 = \frac{1}{4} (\Delta_i^* \Delta_i)^2 = \frac{1}{4} |\Delta|^4, \\
\text{Tr}[(\Delta^\dagger \Delta)^2] &= \Delta_i^* \Delta_j \Delta_k^* \Delta_l \text{Tr}[\tau^i \tau^j \tau^k \tau^l] \\
&= \frac{1}{8} \Delta_i^* \Delta_j \Delta_k^* \Delta_l (\delta_{ij} \delta_{kl} + \delta_{il} \delta_{jk} - \delta_{ik} \delta_{jl}) \\
&= \frac{1}{4} |\Delta|^4 - \frac{1}{8} (\Delta^*)^2 (\Delta)^2. \tag{A1}
\end{aligned}$$

We obtain the classical solution for the heavy field using the covariant derivative expansion method. In analogy to the basis in (Δ, Δ^*) defined in Eq. (2.17), we define a basis for B such that

$$\begin{pmatrix} \Delta_i \\ \Delta_i^* \end{pmatrix} = \frac{1}{M_\Delta^2} \begin{pmatrix} B_i \\ B_i^* \end{pmatrix} + \frac{1}{M_\Delta^4} \left((P^2 \delta_{ij}) \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} - \begin{pmatrix} (U_{11})_{ij} & (U_{12})_{ij} \\ (U_{21})_{ij} & (U_{22})_{ij} \end{pmatrix} \right) \begin{pmatrix} B_j \\ B_j^* \end{pmatrix} \\ + \frac{1}{M_\Delta^6} \left((P^2 \delta_{ij}) \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} - \begin{pmatrix} (U_{11})_{ij} & (U_{12})_{ij} \\ (U_{21})_{ij} & (U_{22})_{ij} \end{pmatrix} \right)^2 \begin{pmatrix} B_j \\ B_j^* \end{pmatrix} + \dots \quad (\text{A2})$$

A few comments on the dimensions of U and Δ , required to generate the operators at a particular mass dimension, are in order:

- (i) The scalar functional U , a double derivative of the Lagrangian with respect to the heavy fields Δ_i , always has a dimension two for our four-dimensional spacetime. However, it also contains the heavy field dependence in quadrature, as given in the underlined parts of $(U_{11})_{ij}$ and $(U_{12})_{ij}$ in Eq. (A3).

$$\begin{aligned} (U_{11})_{ij} &= \frac{1}{4}(2\lambda_1 + \lambda_4)(H^\dagger H)\delta_{ij} + \frac{1}{2}\lambda_2(\Delta_a^* \Delta_a \delta_{ij} + \Delta_i \Delta_j^*) + \frac{1}{2}\lambda_3(\Delta_a^* \Delta_a \delta_{ij} + \Delta_i \Delta_j^* - \Delta_i^* \Delta_j) - \frac{i}{2}\lambda_4 \epsilon_{ijk}(H^\dagger \tau^k H), \\ (U_{12})_{ij} &= \frac{1}{2}\lambda_2(\Delta_i^* \Delta_j^*) + \frac{1}{4}\lambda_3(2\Delta_i^* \Delta_j^* - \Delta_a^* \Delta_a^* \delta_{ij}), \end{aligned} \quad (\text{A3})$$

Thus, substituting the solution of the heavy field in the U matrix, we find the local operators in light fields having various mass dimensions.

- (ii) For dimension-six, we only need to consider the first two terms of the heavy triplet field solution [Eq. (A2)]

$$(\Delta_c)_i = \frac{B_i}{M_\Delta^2} + \underbrace{\frac{(P^2 - U)_{ij}}{M_\Delta^4} B_j}_{\text{wave-underlined}} + \underbrace{\frac{(P^2 - U)_{ij}^2}{M_\Delta^6} B_j}_{\text{double underlined}} + \dots \quad (\text{A4})$$

As these elements of the series contribute at $\mathcal{O}(\Delta_c^2)$ via U , we will have contributions up to order $1/M^6$ (U depends on the light fields only).

- (iii) In this work, first three terms in the heavy field expansion are sufficient and thus we truncate the series in $\Delta_c^* \Delta_c$ at order $1/M^8$. In the second term of Eq. (A4) (wave-underlined), linearly proportional to $(P^2 - U)$, U depends on the heavy field, see Eq. (2.16). However, as part of the solution for this heavy field, we must substitute the classical solution up to dimension-two in the third term in $\Delta_c^* \Delta_c$ [double underlined in Eq. (A4)] to get dimension-eight operators.

Keeping this strategy in mind, the dimension-eight operators are categorized into the following classes, depending on how they contribute in Eq. (2.2) that reads [75]

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{d \leq 8} &= \frac{c_s}{(4\pi)^2} M^4 \left[-\frac{1}{2} \left(\ln \left[\frac{M^2}{\mu^2} \right] - \frac{3}{2} \right) \right] + \frac{c_s}{(4\pi)^2} \text{Tr} \left\{ M^2 \left[-\left(\ln \left[\frac{M^2}{\mu^2} \right] - 1 \right) U \right] + M^0 \frac{1}{2} \left[-\ln \left[\frac{M^2}{\mu^2} \right] U^2 - \frac{1}{6} \ln \left[\frac{M^2}{\mu^2} \right] (G_{\mu\nu})^2 \right] \right. \\ &\quad + \frac{1}{M^2} \frac{1}{6} \left[-U^3 - \frac{1}{2} (P_\mu U)^2 - \frac{1}{2} U (G_{\mu\nu})^2 - \frac{1}{10} (J_\nu)^2 + \frac{1}{15} G^{\mu\nu} G_{\nu\rho} G^\rho{}_\mu \right] \\ &\quad + \frac{1}{M^4} \frac{1}{24} \left[U^4 - U^2 (P^2 U) + \frac{4}{5} U^2 (G_{\mu\nu})^2 + \frac{1}{5} (U G_{\mu\nu})^2 + \frac{1}{5} (P^2 U)^2 \right. \\ &\quad - \frac{2}{5} U (P_\mu U) J^\mu + \frac{2}{5} U (J_\mu)^2 - \frac{2}{15} (P^2 U) (G_{\rho\sigma})^2 + \frac{1}{35} (P_\nu J_\mu)^2 \\ &\quad - \frac{4}{15} U G^{\mu\nu} G_{\nu\rho} G^\rho{}_\mu - \frac{8}{15} (P_\mu P^\nu U) G^{\rho\mu} G_{\rho\nu} + \frac{16}{105} G^{\mu\nu} J_\mu J_\nu + \frac{16}{105} (P^\mu J_\nu) G^{\nu\sigma} G_{\sigma\mu} + \frac{1}{420} (G_{\mu\nu} G_{\rho\sigma})^2 \\ &\quad \left. + \frac{17}{210} (G_{\mu\nu})^2 (G_{\rho\sigma})^2 + \frac{2}{35} (G_{\mu\nu} G_{\nu\rho})^2 + \frac{1}{105} G^{\mu\nu} G_{\nu\rho} G^{\rho\sigma} G_{\sigma\mu} \right] \Big\} \\ &= \mathcal{L}_4 + \mathcal{L}_2 + \mathcal{L}_0 + \mathcal{L}_{-2} + \mathcal{L}_{-4}, \end{aligned} \quad (\text{A5})$$

where $c_s = 1/2$ for real scalar fields while $c_s = 1$ for complex scalar fields.

- (i) Contributions from operators $\in \mathcal{L}_{-4}$: Each operator term in Eq. (2.2) in blue-tinted text ($\mathcal{L}_{-4}$) has dimension-eight. The possibilities for dimension-eight operators in the Green's basis [78] are

$$\phi^8, \phi^6 D^2, \phi^4 D^4, X^3 \phi^2, X^2 \phi^4, X \phi^2 D^4, X \phi^4 D^2, X^2 \phi^2 D^2, X^4, X^2 X'^2, X^3 D^2, X^2 D^4. \quad (\text{A6})$$

where D denotes covariant derivative P_μ , and X represents the field strength tensor $X_{\mu\nu}$.

- (ii) Contributions from operators $\in \mathcal{L}_{-2}$: The total effective Lagrangian up to dimension-eight at one-loop has $\mathcal{O}(1/M^2)$ terms as shown in magenta color ($\mathcal{L}_{-2}$) in Eq. (2.2). Here, the U -dependent terms are

$$\frac{1}{6} \left[-U^3 - \frac{1}{2} (P_\mu U)^2 - \frac{1}{2} U (G_{\mu\nu})^2 \right]. \quad (\text{A7})$$

The heavy field solution truncated at B_i/M^2 generates dimension-eight operators when plugged in U -matrix, see Eq. (2.16). The possible dimension-eight operators in the Green's basis [78] are

$$\phi^8, \phi^6 D^2, \phi^4 X^2. \quad (\text{A8})$$

If we restrict U to be a functional of light fields only, we generate dimension-six ones. The operators arising in this set [see Eq. (2.2); in magenta ($\mathcal{L}_{-2}$)] belong to the following SILH-basis operators [6]

$$\phi^6, \phi^4 D^2, \phi^2 X^2, X^3. \quad (\text{A9})$$

- (iii) Contributions from operators $\in \mathcal{L}_0$: As U is a functional of heavy fields, the higher order local expansion in heavy fields may generate higher mass dimensional operators. For example, from

- (2) Covariant operator: $U^2(P^2 U)$

$$\text{Tr}[U^2(P^2 U)] \supset c_{\phi^6}^{(1), \|U^2(P^2 U)\|} \mathcal{O}_{\phi^6}^{(1)} + c_{\phi^6}^{(2), \|U^2(P^2 U)\|} \mathcal{O}_{\phi^6}^{(2)} + c_{\phi^6}^{(3), \|U^2(P^2 U)\|} \mathcal{O}_{\phi^6}^{(3)}. \quad (\text{A16})$$

The matching relations are

$$\begin{aligned} c_{\phi^6}^{(1), \|U^2(P^2 U)\|} &= \frac{3\lambda_1^3}{2} + \frac{9\lambda_1^2 \lambda_4}{4} + \frac{11\lambda_1 \lambda_4^2}{8} + \frac{5\lambda_4^3}{16}, & c_{\phi^6}^{(2), \|U^2(P^2 U)\|} &= \frac{\lambda_1 \lambda_4^2}{2} + \frac{\lambda_4^3}{4}, \\ c_{\phi^6}^{(3), \|U^2(P^2 U)\|} &= \frac{3\lambda_1^3}{4} + \frac{9\lambda_1^2 \lambda_4}{8} + \frac{15\lambda_1 \lambda_4^2}{16} + \frac{9\lambda_4^3}{32}. \end{aligned} \quad (\text{A17})$$

- (3) Covariant operator: $U^2(G_{\mu\nu})^2$

$$\text{Tr}[U^2(G_{\mu\nu})^2] \supset c_{W^2 \phi^4}^{(1), \|U^2(G_{\mu\nu})^2\|} \mathcal{O}_{W^2 \phi^4}^{(1)} + c_{W^2 \phi^4}^{(3), \|U^2(G_{\mu\nu})^2\|} \mathcal{O}_{W^2 \phi^4}^{(3)} + c_{B^2 \phi^4}^{(1), \|U^2(G_{\mu\nu})^2\|} \mathcal{O}_{B^2 \phi^4}^{(1)} + c_{WB \phi^4}^{(1), \|U^2(G_{\mu\nu})^2\|} \mathcal{O}_{WB \phi^4}^{(1)}. \quad (\text{A18})$$

$\text{Tr}[1/2(-\ln[M^2/\mu^2]U^2)]$, the dimension-eight operators

$$\phi^8, \phi^6 D^2, \quad (\text{A10})$$

and the dimension-six operators

$$\phi^6, \phi^4 D^2. \quad (\text{A11})$$

can emerge.

- (iv) Contributions from operators $\in \mathcal{L}_2$: Similarly, the term $\text{Tr}(U)$ in effective action offers the dimension-eight

$$\phi^8, \phi^6 D^2, \phi^4 D^4, \quad (\text{A12})$$

and the dimension-six

$$\phi^6, \phi^4 D^2 \quad (\text{A13})$$

operator. In this case, $\mathcal{O}(B^\dagger(P^2 - U)^2 B) \subset \Delta_c$ is considered.

Here, we enlist the higher mass-dimension effective operators, generated at the one-loop when U is a functional of light fields only. The dimension-eight operators are displayed in Green's basis.

Dimension-eight operators from terms $\in \mathcal{L}_{-4}$

- (1) Covariant operator: U^4

$$\text{Tr}[U^4] \supset c_{\phi^8}^{\|U^4\|} \mathcal{O}_{\phi^8}. \quad (\text{A14})$$

The matching relation is

$$c_{\phi^8}^{\|U^4\|} = \frac{3}{8} \lambda_4^4 + \frac{3}{4} \lambda_1^3 \lambda_4 + \frac{15}{16} \lambda_1^2 \lambda_4^2 + \frac{9}{16} \lambda_1 \lambda_4^3 + \frac{17}{128} \lambda_4^4. \quad (\text{A15})$$

The matching relations are

$$\begin{aligned} c_{W^2\phi^4}^{(1),\llbracket U^2(G_{\mu\nu})^2 \rrbracket} &= -g^2 \left(\frac{1}{4} \lambda_4^2 + \lambda_1^2 + \lambda_1 \lambda_4 \right), & c_{W^2\phi^4}^{(3),\llbracket U^2(G_{\mu\nu})^2 \rrbracket} &= -g^2 \lambda_4^2, \\ c_{B^2\phi^4}^{(1),\llbracket U^2(G_{\mu\nu})^2 \rrbracket} &= -g'^2 \left(\frac{3}{2} \lambda_1^2 + \frac{3}{2} \lambda_1 \lambda_4 + \frac{3}{4} \lambda_4^2 \right), & c_{WB\phi^4}^{(1),\llbracket U^2(G_{\mu\nu})^2 \rrbracket} &= gg' \left(\frac{3}{2} \lambda_1^2 - \frac{3}{4} \lambda_1 \lambda_4 - \frac{3}{4} \lambda_4^2 \right). \end{aligned} \quad (\text{A19})$$

(4) Covariant operator: $(UG_{\mu\nu})^2$

$$\text{Tr}[(UG_{\mu\nu})^2] \supset c_{W^2\phi^4}^{(1),\llbracket (UG_{\mu\nu})^2 \rrbracket} \mathcal{O}_{W^2\phi^4}^{(1)} + c_{W^2\phi^4}^{(3),\llbracket (UG_{\mu\nu})^2 \rrbracket} \mathcal{O}_{W^2\phi^4}^{(3)} + c_{B^2\phi^4}^{(1),\llbracket (UG_{\mu\nu})^2 \rrbracket} \mathcal{O}_{B^2\phi^4}^{(1)} + c_{WB\phi^4}^{(1),\llbracket (UG_{\mu\nu})^2 \rrbracket} \mathcal{O}_{WB\phi^4}^{(1)}. \quad (\text{A20})$$

The matching relations are

$$\begin{aligned} c_{W^2\phi^4}^{(1),\llbracket (UG_{\mu\nu})^2 \rrbracket} &= -\frac{1}{4} g^2 (2\lambda_1 + \lambda_4)^2, & c_{W^2\phi^4}^{(3),\llbracket (UG_{\mu\nu})^2 \rrbracket} &= -g^2 \lambda_4^2, \\ c_{B^2\phi^4}^{(1),\llbracket (UG_{\mu\nu})^2 \rrbracket} &= -\frac{1}{4} g'^2 \lambda_4^2 - \frac{3}{4} g'^2 (2\lambda_1 + \lambda_4)^2, & c_{WB\phi^4}^{(1),\llbracket (UG_{\mu\nu})^2 \rrbracket} &= 2gg' \lambda_4 (2\lambda_1 + \lambda_4). \end{aligned} \quad (\text{A21})$$

(5) Covariant operator: $(P^2 U)^2$

$$\text{Tr}[(P^2 U)^2] \supset c_{\phi^4}^{(3),\llbracket (P^2 U)^2 \rrbracket} \mathcal{O}_{\phi^4}^{(3)} + c_{\phi^4}^{(4),\llbracket (P^2 U)^2 \rrbracket} \mathcal{O}_{\phi^4}^{(4)} + c_{\phi^4}^{(8),\llbracket (P^2 U)^2 \rrbracket} \mathcal{O}_{\phi^4}^{(8)} + c_{\phi^4}^{(10),\llbracket (P^2 U)^2 \rrbracket} \mathcal{O}_{\phi^4}^{(10)} + c_{\phi^4}^{(11),\llbracket (P^2 U)^2 \rrbracket} \mathcal{O}_{\phi^4}^{(11)}. \quad (\text{A22})$$

The matching relations are

$$\begin{aligned} c_{\phi^4}^{(3),\llbracket (P^2 U)^2 \rrbracket} &= 6\lambda_1^2 + 6\lambda_1 \lambda_4 + \frac{5}{2} \lambda_4^2, & c_{\phi^4}^{(4),\llbracket (P^2 U)^2 \rrbracket} &= 6\lambda_1^2 + 6\lambda_1 \lambda_4 + \frac{5}{2} \lambda_4^2, \\ c_{\phi^4}^{(8),\llbracket (P^2 U)^2 \rrbracket} &= \frac{3}{2} \lambda_1^2 + \frac{3}{2} \lambda_1 \lambda_4 + \frac{5}{8} \lambda_4^2, & c_{\phi^4}^{(10),\llbracket (P^2 U)^2 \rrbracket} &= \lambda_4^2, \\ c_{\phi^4}^{(11),\llbracket (P^2 U)^2 \rrbracket} &= 3\lambda_1^2 + 3\lambda_1 \lambda_4 + \frac{1}{4} \lambda_4^2. \end{aligned} \quad (\text{A23})$$

(6) Covariant operator: $U(P_\mu U)J_\mu$

$$\text{Tr}[U(P_\mu U)J_\mu] \supset c_{W\phi^4 D^2}^{(6),\llbracket U(P_\mu U)J_\mu \rrbracket} \mathcal{O}_{W\phi^4 D^2}^{(6)} + c_{W\phi^4 D^2}^{(7),\llbracket U(P_\mu U)J_\mu \rrbracket} \mathcal{O}_{W\phi^4 D^2}^{(7)} + c_{B\phi^4 D^2}^{(3),\llbracket U(P_\mu U)J_\mu \rrbracket} \mathcal{O}_{B\phi^4 D^2}^{(3)}. \quad (\text{A24})$$

The matching relations are

$$\begin{aligned} c_{W\phi^4 D^2}^{(6),\llbracket U(P_\mu U)J_\mu \rrbracket} &= \frac{-3}{4} g \lambda_4 (2\lambda_1 + \lambda_4), & c_{W\phi^4 D^2}^{(7),\llbracket U(P_\mu U)J_\mu \rrbracket} &= g \lambda_4^2, \\ c_{B\phi^4 D^2}^{(3),\llbracket U(P_\mu U)J_\mu \rrbracket} &= \frac{3}{8} g' (2\lambda_1 + \lambda_4)^2 - g' \lambda_4^2. \end{aligned} \quad (\text{A25})$$

(7) Covariant operator: $U(J_\mu)^2$

$$\text{Tr}[U(J_\mu)^2] \supset c_{B^2\phi^2 D^2}^{(6),\llbracket U(J_\mu)^2 \rrbracket} \mathcal{O}_{B^2\phi^2 D^2}^{(6)} + c_{B^2\phi^2 D^2}^{(8),\llbracket U(J_\mu)^2 \rrbracket} \mathcal{O}_{B^2\phi^2 D^2}^{(8)} + c_{WB\phi^2 D^2}^{(10),\llbracket U(J_\mu)^2 \rrbracket} \mathcal{O}_{WB\phi^2 D^2}^{(10)} + c_{W^2\phi^2 D^2}^{(13),\llbracket U(J_\mu)^2 \rrbracket} \mathcal{O}_{W^2\phi^2 D^2}^{(13)}. \quad (\text{A26})$$

The matching relations are

$$\begin{aligned} c_{B^2\phi^2 D^2}^{(6),\llbracket U(J_\mu)^2 \rrbracket} &= \frac{3}{2} g^2 (2\lambda_1 + \lambda_4), & c_{B^2\phi^2 D^2}^{(8),\llbracket U(J_\mu)^2 \rrbracket} &= \frac{3}{2} g^2 (2\lambda_1 + \lambda_4), \\ c_{WB\phi^2 D^2}^{(10),\llbracket U(J_\mu)^2 \rrbracket} &= -4gg' \lambda_4, & c_{W^2\phi^2 D^2}^{(13),\llbracket U(J_\mu)^2 \rrbracket} &= -g^2 (2\lambda_1 + \lambda_4). \end{aligned} \quad (\text{A27})$$

(8) Covariant operator: $(P^2 U)(G_{\rho\sigma})^2$

$$\begin{aligned} \text{Tr}[(P^2 U)(G_{\rho\sigma})^2] \supset & c_{W^2\phi^2 D^2}^{(14), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} \mathcal{O}_{W^2\phi^2 D^2}^{(14)} + c_{B^2\phi^2 D^2}^{(2), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} \mathcal{O}_{B^2\phi^2 D^2}^{(2)} + c_{WB\phi^2 D^2}^{(1), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} \mathcal{O}_{WB\phi^2 D^2}^{(1)} \\ & + c_1^{\llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} (D^2 \phi^\dagger \phi + \phi^\dagger D^2 \phi) B_{\rho\sigma} B^{\rho\sigma} + c_2^{\llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} (D^2 \phi^\dagger \sigma^a \phi + \phi^\dagger \sigma^a D^2 \phi) W_{\rho\sigma}^a B^{\rho\sigma}. \end{aligned} \quad (\text{A28})$$

$$\begin{aligned} \text{Tr}[(P^2 U)(G_{\rho\sigma})^2] \supset & c_{W^2\phi^2 D^2}^{(14), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} \mathcal{O}_{W^2\phi^2 D^2}^{(14)} + c_{B^2\phi^2 D^2}^{(2), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} \mathcal{O}_{B^2\phi^2 D^2}^{(2)} + c_{WB\phi^2 D^2}^{(1), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} \mathcal{O}_{WB\phi^2 D^2}^{(1)} \\ & + c_{WB\phi^4}^{(1), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} \mathcal{O}_{WB\phi^4}^{(1)} + c_{B^2\phi^4}^{(1), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} \mathcal{O}_{B^2\phi^4}^{(1)} + c_{BB}^{\llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} \mathcal{O}_{BB} \\ = & c_{W^2\phi^2 D^2}^{(14), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} (D_\mu \phi^\dagger \phi + \phi^\dagger D_\mu \phi) W^{I\nu\rho} D^\mu W_{\nu\rho}^I + c_{B^2\phi^2 D^2}^{(2), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} (D^\mu \phi^\dagger D_\mu \phi) B_{\nu\rho} B^{\nu\rho} \\ & + c_{WB\phi^2 D^2}^{(1), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} (D^\mu \phi^\dagger \sigma^I D_\mu \phi) B_{\nu\rho} W^{I\nu\rho} + c_{WB\phi^4}^{(1), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} (\phi^\dagger \phi) (\phi^\dagger \sigma^I \phi) W_{\mu\nu}^I B^{\mu\nu} \\ & + c_{WB}^{\llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} (\phi^\dagger \sigma^I \phi) W_{\mu\nu}^I B^{\mu\nu} + c_{BB}^{\llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} (\phi^\dagger \phi) B_{\mu\nu} B^{\mu\nu}. \end{aligned} \quad (\text{A29})$$

The matching relations are

$$\begin{aligned} c_{W^2\phi^2 D^2}^{(14), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} &= -g^2(2\lambda_1 + \lambda_4), & c_{B^2\phi^2 D^2}^{(2), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} &= 3g^2(2\lambda_1 + \lambda_4), \\ c_{WB\phi^2 D^2}^{(1), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} &= 2gg'\lambda_4, & c_{B^2\phi^4}^{(1), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} &= -3g^2(2\lambda_1 + \lambda_4)\lambda_{\text{SM}}, \\ c_{WB\phi^4}^{(1), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} &= -2\lambda_4\lambda_{\text{SM}}gg', & c_{WB}^{\llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} &= \lambda_4 m_H^2, \\ c_{BB}^{\llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} &= 3g^2(2\lambda_1 + \lambda_4)m_H^2. \end{aligned} \quad (\text{A30})$$

(9) Covariant operator: $UG_{\mu\nu}G_{\nu\rho}G_{\rho\mu}$

$$\text{Tr}[UG_{\mu\nu}G_{\nu\rho}G_{\rho\mu}] \supset c_{W^3\phi^2}^{(1), \llbracket UG_{\mu\nu}G_{\nu\rho}G_{\rho\mu} \rrbracket} \mathcal{O}_{W^3\phi^2}^{(1)}. \quad (\text{A31})$$

The matching relation is

$$c_{W^3\phi^2}^{(1), \llbracket UG_{\mu\nu}G_{\nu\rho}G_{\rho\mu} \rrbracket} = \frac{3}{2}(2\lambda_1 + \lambda_4)g^3. \quad (\text{A32})$$

(10) Covariant operator: $(P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu}$

$$\begin{aligned} \text{Tr}[(P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu}] \supset & c_{W^2\phi^2 D^2}^{(4), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} \mathcal{O}_{W^2\phi^2 D^2}^{(4)} + c_{W^2\phi^2 D^2}^{(11), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} \mathcal{O}_{W^2\phi^2 D^2}^{(11)} \\ & + c_{W^2\phi^2 D^2}^{(14), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} \mathcal{O}_{W^2\phi^2 D^2}^{(14)} + c_{B^2\phi^2 D^2}^{(4), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} \mathcal{O}_{B^2\phi^2 D^2}^{(4)} \\ & + c_{B^2\phi^2 D^2}^{(8), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} \mathcal{O}_{B^2\phi^2 D^2}^{(8)} + c_{WB\phi^2 D^2}^{(8), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} \mathcal{O}_{WB\phi^2 D^2}^{(8)} \\ & + c_{WB\phi^2 D^2}^{(11), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} \mathcal{O}_{WB\phi^2 D^2}^{(11)}. \end{aligned} \quad (\text{A33})$$

The matching relations are

$$\begin{aligned} c_{W^2\phi^2 D^2}^{(4), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} &= -g^2\lambda_4, & c_{W^2\phi^2 D^2}^{(11), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} &= g^2(2\lambda_1 + \lambda_4), \\ c_{W^2\phi^2 D^2}^{(14), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} &= g^2(2\lambda_1 + \lambda_4), & c_{B^2\phi^2 D^2}^{(4), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} &= \frac{3}{2}g^2(2\lambda_1 + \lambda_4), \\ c_{B^2\phi^2 D^2}^{(8), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} &= \frac{3}{2}g^2(2\lambda_1 + \lambda_4), & c_{WB\phi^2 D^2}^{(8), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} &= -2gg'\lambda_4, \\ c_{WB\phi^2 D^2}^{(11), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} &= -2gg'\lambda_4. \end{aligned} \quad (\text{A34})$$

(11) Covariant operator: $(P_\nu J_\mu)^2$

$$\text{Tr}[(P_\nu J_\mu)^2] \supset c_{B^2 D^4}^{[(P_\nu J_\mu)^2]} \mathcal{O}_{B^2 D^4} + c_{W^2 D^4}^{[(P_\nu J_\mu)^2]} \mathcal{O}_{W^2 D^4}. \quad (\text{A35})$$

The matching relations are

$$c_{B^2 D^4}^{[(P_\nu J_\mu)^2]} = -3g'^2, \quad c_{W^2 D^4}^{[(P_\nu J_\mu)^2]} = 2g^2. \quad (\text{A36})$$

(12) Covariant operator: $(G_{\mu\nu} G_{\rho\sigma})^2$

$$\text{Tr}[(G_{\mu\nu} G_{\rho\sigma})^2] \supset c_{W^4}^{(3), [(G_{\mu\nu} G_{\rho\sigma})^2]} \mathcal{Q}_{W^4}^{(3)} + c_{B^4}^{(1), [(G_{\mu\nu} G_{\rho\sigma})^2]} \mathcal{Q}_{B^4}^{(1)} + c_{W^2 B^2}^{(1), [(G_{\mu\nu} G_{\rho\sigma})^2]} \mathcal{Q}_{W^2 B^2}^{(1)} + c_{W^2 B^2}^{(3), [(G_{\mu\nu} G_{\rho\sigma})^2]} \mathcal{Q}_{W^2 B^2}^{(3)}. \quad (\text{A37})$$

The matching relations are

$$\begin{aligned} c_{W^4}^{(3), [(G_{\mu\nu} G_{\rho\sigma})^2]} &= 2g^4, & c_{B^4}^{(1), [(G_{\mu\nu} G_{\rho\sigma})^2]} &= 3g'^4, \\ c_{W^2 B^2}^{(1), [(G_{\mu\nu} G_{\rho\sigma})^2]} &= 4g^2 g'^2, & c_{W^2 B^2}^{(3), [(G_{\mu\nu} G_{\rho\sigma})^2]} &= 8g^2 g'^2. \end{aligned} \quad (\text{A38})$$

(13) Covariant operator: $(G_{\mu\nu})^2 (G_{\rho\sigma})^2$

$$\text{Tr}[(G_{\mu\nu})^2 (G_{\rho\sigma})^2] \supset c_{W^4}^{(3), [(G_{\mu\nu})^2 (G_{\rho\sigma})^2]} \mathcal{Q}_{W^4}^{(3)} + c_{B^4}^{(1), [(G_{\mu\nu})^2 (G_{\rho\sigma})^2]} \mathcal{Q}_{B^4}^{(1)} + c_{W^2 B^2}^{(1), [(G_{\mu\nu})^2 (G_{\rho\sigma})^2]} \mathcal{Q}_{W^2 B^2}^{(1)} + c_{W^2 B^2}^{(3), [(G_{\mu\nu})^2 (G_{\rho\sigma})^2]} \mathcal{Q}_{W^2 B^2}^{(3)}. \quad (\text{A39})$$

The matching relations are

$$\begin{aligned} c_{W^4}^{(3), [(G_{\mu\nu})^2 (G_{\rho\sigma})^2]} &= 2g^4, & c_{B^4}^{(1), [(G_{\mu\nu})^2 (G_{\rho\sigma})^2]} &= 3g'^4, \\ c_{W^2 B^2}^{(1), [(G_{\mu\nu})^2 (G_{\rho\sigma})^2]} &= 4g^2 g'^2, & c_{W^2 B^2}^{(3), [(G_{\mu\nu})^2 (G_{\rho\sigma})^2]} &= 8g^2 g'^2. \end{aligned} \quad (\text{A40})$$

(14) Covariant operators: $(G_{\mu\nu} G_{\nu\rho})^2$, $G_{\mu\nu} G_{\nu\rho} G_{\rho\sigma} G_{\sigma\mu}$, $G_{\mu\nu} J_\mu J_\nu$, $P_\mu J_\nu G_{\nu\sigma} G_{\sigma\mu}$

$$\text{Tr}[(G_{\mu\nu} G_{\nu\rho})^2] \supset -(D_\nu G_{\nu\rho})(D_\mu G_{\mu\nu})G_{\nu\rho} - (D_\nu G_{\nu\rho})G_{\mu\nu}(D_\mu G_{\nu\rho}) + (D_\mu G_{\nu\rho})(D_\nu G_{\mu\nu})G_{\nu\rho} + (D_\mu G_{\nu\rho})G_{\mu\nu}(D_\nu G_{\nu\rho}). \quad (\text{A41})$$

$$\text{Tr}[G_{\mu\nu} J_\mu J_\nu] = g^3 \epsilon^{IJK} W_{\mu\nu}^I D_\rho W_{\rho\mu}^J D_\sigma W_{\sigma\nu}^K + 4ig^2 g' W_{\mu\nu}^I D_\rho W_{\rho\mu}^I D_\sigma B_{\sigma\nu} + 2ig^2 g' W_{\mu\nu}^I D_\rho B_{\rho\mu} D_\sigma W_{\sigma\nu}^I. \quad (\text{A42})$$

Employing the classical EOM for the field strength tensor, each of the terms in the above equations, we can construct the nonredundant dimension-eight operators involving light scalar, and fermionic fields.

Dimension-eight and dimension-six operators from terms $\in \mathcal{L}_{-2}$

(1) Covariant operator: U^3

$$\text{Tr}[U^3] \supset c_{\phi^8}^{[U^3]} \mathcal{O}_{\phi^8}. \quad (\text{A43})$$

The matching relation at dimension-eight is

$$c_{\phi^8}^{[U^3]} = \frac{3\mu_\Delta^2}{4M_\Delta^4} \left(\lambda_1^2 \lambda_2 + \frac{3}{4} \lambda_1^2 \lambda_3 + \lambda_1 \lambda_2 \lambda_4 + \frac{3}{4} \lambda_1 \lambda_2 \lambda_4 + \frac{5}{16} \lambda_2 \lambda_4^2 + \frac{1}{4} \lambda_3 \lambda_4^2 \right). \quad (\text{A44})$$

$$\text{Tr}[U^3] \supset c_6^{[U^3]} \mathcal{O}_6 = c_6^{[U^3]} (\phi^\dagger \phi)^3, \quad (\text{A45})$$

with the matching relation at dimension-six as

$$c_6^{\llbracket U^3 \rrbracket} = \frac{3}{32} (8\lambda_1^3 + 3\lambda_4^3 + 12\lambda_1^2\lambda_4 + 10\lambda_1\lambda_4^2). \quad (\text{A46})$$

(2) Covariant operator: $(P_\mu U)^2$

$$\text{Tr}[(P_\mu U)^2] \supset c_{\phi^6}^{(3), \llbracket (P_\mu U)^2 \rrbracket} \mathcal{O}_{\phi^6}^{(3)} \quad (\text{A47})$$

The matching relation at dimension-eight is

$$c_{\phi^6}^{(3), \llbracket (P_\mu U)^2 \rrbracket} = -\frac{\mu_\Delta^2}{M_\Delta^4} \left(\lambda_1\lambda_2 + \frac{15}{8}\lambda_1\lambda_3 + \frac{1}{2}\lambda_2\lambda_4 + \frac{15}{16}\lambda_3\lambda_4 \right). \quad (\text{A48})$$

$$\begin{aligned} \text{Tr}[(P_\mu U)^2] &\supset c_H^{\llbracket (P_\mu U)^2 \rrbracket} \mathcal{O}_H + c_T^{\llbracket (P_\mu U)^2 \rrbracket} \mathcal{O}_T + c_R^{\llbracket (P_\mu U)^2 \rrbracket} \mathcal{O}_R \\ &= c_H^{\llbracket (P_\mu U)^2 \rrbracket} \frac{1}{2} (\partial_\mu |\phi|^2)^2 + c_T^{\llbracket (P_\mu U)^2 \rrbracket} \frac{1}{2} (\phi^\dagger \overleftrightarrow{D}^\mu \phi)^2 + c_R^{\llbracket (P_\mu U)^2 \rrbracket} |\phi|^2 |D_\mu \phi|^2 \end{aligned} \quad (\text{A49})$$

with the matching relation at dimension-six as

$$c_H^{\llbracket (P_\mu U)^2 \rrbracket} = -\frac{3}{4} (4\lambda_1^2 + \lambda_4^2 + 4\lambda_1\lambda_4), \quad c_T^{\llbracket (P_\mu U)^2 \rrbracket} = -\frac{1}{2} (\lambda_4^2), \quad c_R^{\llbracket (P_\mu U)^2 \rrbracket} = -\lambda_4^4. \quad (\text{A50})$$

(3) Covariant operator: $U(G_{\mu\nu})^2$

$$\text{Tr}[U(G_{\mu\nu})^2] \supset c_{W^2\phi^4}^{(1), \llbracket U(G_{\mu\nu})^2 \rrbracket} \mathcal{O}_{W^2\phi^4}^{(1)} + c_{W^2\phi^4}^{(3), \llbracket U(G_{\mu\nu})^2 \rrbracket} \mathcal{O}_{W^2\phi^4}^{(3)} + c_{B^2\phi^4}^{(1), \llbracket U(G_{\mu\nu})^2 \rrbracket} \mathcal{O}_{B^2\phi^4}^{(1)}. \quad (\text{A51})$$

The matching relations at dimension-eight are

$$\begin{aligned} c_{W^2\phi^4}^{(1), \llbracket U(G_{\mu\nu})^2 \rrbracket} &= -\frac{1}{M_\Delta^4} \mu_\Delta^2 g^2 (\lambda_2 + \lambda_3) - \frac{1}{2M_\Delta^4} g^2 \mu_\Delta^2 (\lambda_2 + 2\lambda_3) \\ c_{W^2\phi^4}^{(3), \llbracket U(G_{\mu\nu})^2 \rrbracket} &= \frac{1}{M_\Delta^4} \mu_\Delta^2 g^2 (\lambda_2 + \lambda_3) + \frac{1}{2M_\Delta^4} \mu_\Delta^2 g^2 \lambda_3, \\ c_{B^2\phi^4}^{(1), \llbracket U(G_{\mu\nu})^2 \rrbracket} &= -\frac{2}{M_\Delta^4} \mu_\Delta^2 g'^2 \lambda_2 - \frac{3}{2M_\Delta^4} \mu_\Delta^2 g'^2 \lambda_3. \end{aligned} \quad (\text{A52})$$

$$\begin{aligned} \text{Tr}[U(G_{\mu\nu})^2] &\supset c_{BB}^{\llbracket U(G_{\mu\nu})^2 \rrbracket} \mathcal{O}_{BB} + c_{WW}^{\llbracket U(G_{\mu\nu})^2 \rrbracket} \mathcal{O}_{WW} + c_{WB}^{\llbracket U(G_{\mu\nu})^2 \rrbracket} \mathcal{O}_{WB} \\ &= c_{BB}^{\llbracket U(G_{\mu\nu})^2 \rrbracket} g'^2 (\phi^\dagger \phi) B_{\mu\nu} B^{\mu\nu} + c_{WW}^{\llbracket U(G_{\mu\nu})^2 \rrbracket} g^2 (\phi^\dagger \phi) W_{\mu\nu}^I W^{I\mu\nu} + c_{WB}^{\llbracket U(G_{\mu\nu})^2 \rrbracket} 2gg' (\phi^\dagger \sigma^I \phi) W_{\mu\nu}^I B^{\mu\nu}, \end{aligned} \quad (\text{A53})$$

with the matching relation at dimension-six as

$$c_{BB}^{\llbracket U(G_{\mu\nu})^2 \rrbracket} = -\frac{3}{2} (2\lambda_1 + \lambda_4), \quad c_{WW}^{\llbracket U(G_{\mu\nu})^2 \rrbracket} = -(2\lambda_1 + \lambda_4), \quad c_{WB}^{\llbracket U(G_{\mu\nu})^2 \rrbracket} = 2\lambda_4. \quad (\text{A54})$$

Dimension-eight and dimension-six operators from terms $\in \mathcal{L}_0$

(1) Covariant operator: U^2

$$\text{Tr}[U^2] \supset c_{\phi^8}^{\llbracket U^2 \rrbracket} \mathcal{O}_{\phi^8}. \quad (\text{A55})$$

The matching relation at dimension-eight is

$$c_{\phi^8}^{\llbracket U^2 \rrbracket} = \frac{1}{2} \frac{\mu_\Delta^2}{M_\Delta^6} \lambda_1^2 \lambda_2 + \frac{3}{8} \frac{\mu_\Delta^2}{M_\Delta^6} \lambda_1^2 \lambda_3 + \frac{1}{2} \frac{\mu_\Delta^2}{M_\Delta^6} \lambda_1 \lambda_2 \lambda_4 + \frac{3}{8} \frac{\mu_\Delta^2}{M_\Delta^6} \lambda_1 \lambda_3 \lambda_4 + \frac{7}{8} \frac{\mu_\Delta^4}{M_\Delta^8} \lambda_2^2 + \frac{11}{8} \frac{\mu_\Delta^4}{M_\Delta^8} \lambda_2 \lambda_3 + \frac{1}{8} \frac{\mu_\Delta^2}{M_\Delta^6} \lambda_2 \lambda_4^2 + \frac{33}{32} \frac{\mu_\Delta^4}{M_\Delta^8} \lambda_3^2 + \frac{3}{32} \frac{\mu_\Delta^2}{M_\Delta^6} \lambda_3 \lambda_4^2. \quad (\text{A56})$$

$$\text{Tr}[U^2] \supset c_6^{\llbracket U^2 \rrbracket} \mathcal{O}_6, \quad (\text{A57})$$

with the matching relation at dimension-six as

$$c_6^{\llbracket U^2 \rrbracket} = \frac{\mu_\Delta^2}{M_\Delta^4} \left(2\lambda_1 \lambda_2 + \frac{3}{2} \lambda_1 \lambda_3 + \lambda_2 \lambda_4 + \frac{3}{4} \lambda_3 \lambda_4 \right). \quad (\text{A58})$$

Dimension-eight and dimension-six operators arising from terms $\in \mathcal{L}_2$

(i) Covariant operator: U

$$\text{Tr}[U] \supset c_{\phi^8}^{\llbracket U \rrbracket} \mathcal{O}_{\phi^8} + c_{\phi^6}^{(1), \llbracket U \rrbracket} \mathcal{O}_{\phi^6}^{(1)} + c_{\phi^6}^{(3), \llbracket U \rrbracket} \mathcal{O}_{\phi^6}^{(3)} + c_{\phi^4}^{(10), \llbracket U \rrbracket} \mathcal{O}_{\phi^4}^{(10)} + c_{\phi^4}^{(11), \llbracket U \rrbracket} \mathcal{O}_{\phi^4}^{(11)}. \quad (\text{A59})$$

The matching relations at dimension-eight are

$$\begin{aligned} c_{\phi^8}^{\llbracket U \rrbracket} &= \frac{\mu_\Delta^2}{M_\Delta^8} \left(2\lambda_1^2 \lambda_4^2 + 2\lambda_1 \lambda_4^3 + \frac{1}{2} \lambda_4^4 \right) - \frac{\mu_\Delta^4}{M_\Delta^{10}} \left(\lambda_2^2 + \frac{11}{4} \lambda_2 \lambda_3 - \frac{3}{4} \lambda_3^2 \right), \\ c_{\phi^6}^{(1), \llbracket U \rrbracket} &= -\frac{\mu_\Delta^2}{M_\Delta^8} \left(6\lambda_1 \lambda_2 + 9\lambda_1 \lambda_3 + 3\lambda_2 \lambda_4 + \frac{9}{4} \lambda_3 \lambda_4 \right), \\ c_{\phi^6}^{(3), \llbracket U \rrbracket} &= -\frac{\mu_\Delta^2}{M_\Delta^8} \left(6\lambda_1 \lambda_2 + \frac{9}{2} \lambda_1 \lambda_3 + 3\lambda_2 \lambda_4 + \frac{9}{4} \lambda_3 \lambda_4 \right), \\ c_{\phi^4}^{(10), \llbracket U \rrbracket} &= -\frac{\mu_\Delta^2}{M_\Delta^8} (8\lambda_2 + 6\lambda_3), \\ c_{\phi^4}^{(11), \llbracket U \rrbracket} &= -\frac{\mu_\Delta^2}{M_\Delta^8} (8\lambda_2 + 6\lambda_3). \end{aligned} \quad (\text{A60})$$

$$\text{Tr}[U] \supset c_H^{\llbracket U \rrbracket} \mathcal{O}_H + c_6^{\llbracket U \rrbracket} \mathcal{O}_6, \quad (\text{A61})$$

with the matching relations at dimension-six as

$$c_H^{\llbracket U \rrbracket} = \frac{\mu_\Delta^2}{M^6} (8\lambda_2 + 6\lambda_3), \quad c_6^{\llbracket U \rrbracket} = -\frac{\mu_\Delta^2}{M^6} \left(2\lambda_1 \lambda_2 + \lambda_2 \lambda_4 + \frac{3}{2} \lambda_1 \lambda_3 + \frac{3}{4} \lambda_3 \lambda_4 \right). \quad (\text{A62})$$

The results discussed until now are summarized in Tables I and II.

a. Incorporating the effect of fermion interactions: Complex scalar triplet

The Yukawa interaction between the SM fermions and the triplet scalar is depicted in Eq. (2.10). The functionals $\hat{B}$ and $\hat{B}^\dagger$, derived from the relevant part of the Lagrangian, Eq. (2.8), are given as

$$\hat{B}_i = \mu_\Delta \tilde{H}^\dagger \tau^i H - Y_{\Delta_{\alpha\beta}}^* \ell_{L_\alpha}^{r\dagger} \tau_i i \sigma_2 C \ell_{L_\beta}^{s*} = \mu_\Delta \tilde{H}^\dagger \tau^i H - Y_{\Delta_{\alpha\beta}}^* \bar{\ell}_{L_\alpha}^r \tau_i \tilde{\ell}_{L_\beta}^s, \quad (\text{A63})$$

$$\hat{B}_i^\dagger = \mu_\Delta H^\dagger \tau^i \tilde{H} - Y_{\Delta_{\gamma\delta}} \ell_{L_\gamma}^{pT} C i \sigma_2 \tau_i \ell_{L_\delta}^q = \mu_\Delta H^\dagger \tau^i \tilde{H} - Y_{\Delta_{\gamma\delta}} \bar{\ell}_{L_\gamma}^p \tau_i \ell_{L_\delta}^q. \quad (\text{A64})$$

These functionals have mass dimension three. Encapsulating these additional contributions, we find

$$\Delta^* \Delta = \hat{B}^\dagger \frac{1}{M_\Delta^4} \hat{B} + \hat{B}^\dagger \frac{1}{M_\Delta^4} (P^2 - U) \frac{1}{M_\Delta^2} \hat{B} + \hat{B}^\dagger \frac{1}{M_\Delta^4} (P^2 - U) \frac{1}{M_\Delta^2} (P^2 - U) \frac{1}{M_\Delta^2} \hat{B} + \dots \quad (\text{A65})$$

where $\hat{B}_i^\dagger \hat{B}_i$ is given by¹⁴

$$\begin{aligned} \hat{B}_i^\dagger \hat{B}_i &= \frac{1}{4} \mu_\Delta^2 (H^\dagger H)^2 + \frac{1}{2} Y_{\Delta\gamma\delta} Y_{\Delta\alpha\beta}^* (\bar{\ell}_{L_\alpha}^r \ell_{L_\delta}^q)_i (\bar{\ell}_{L_\gamma}^p \tilde{\ell}_{L_\beta}^s)_i \\ &\quad - \frac{1}{4} Y_{\Delta\gamma\delta} Y_{\Delta\alpha\beta}^* (\bar{\ell}_{L_\alpha}^r \tilde{\ell}_{L_\beta}^s)_i (\bar{\ell}_{L_\gamma}^p \ell_{L_\delta}^q)_i - \mu_\Delta Y_{\Delta\gamma\delta} \tilde{H}^\dagger \tau^i H \ell_{L_\gamma}^{pT} C i \sigma_2 \tau_i \ell_{L_\delta}^q + \text{H.c.} \\ &= \frac{1}{4} \mu_\Delta^2 (H^\dagger H)^2 + Y_{\Delta\gamma\delta} Y_{\Delta\alpha\beta}^* (\bar{\ell}_{L_\alpha}^r \ell_{L_\delta}^q)_i (\bar{\ell}_{L_\beta}^s \ell_{L_\gamma}^p)_i - \frac{1}{2} \mu_\Delta Y_{\Delta\alpha\beta} (\tilde{H}^\dagger \ell_{L_\gamma}^{pT}) C (\tilde{H}^\dagger \ell_{L_\delta}^q). \end{aligned} \quad (\text{A66})$$

After Fierz transformations,¹⁵ the four-fermion part in Eq. (A66) can be further reduced to the form:

$$(\hat{B}_i^\dagger \hat{B}_i)^{\text{lep}} = -\frac{1}{2} Y_{\Delta\gamma\delta} Y_{\Delta\alpha\beta}^* (\bar{\ell}_{L_\alpha}^r \gamma_\mu \ell_{L_\gamma}^p)_i (\bar{\ell}_{L_\beta}^s \gamma^\mu \ell_{L_\delta}^q)_i. \quad (\text{A67})$$

Below, we systematically recollect all the additional fermionic effective operators following the earlier principle.

Dimension-five operator

We have only one dimension-five operator arising from the trace of U in the effective Lagrangian (2.2), which is the Weinberg operator

$$\text{Tr}[U] \supset c_{\phi^2 L^2}^{[U]} \mathcal{Q}_{\phi^2 L^2} = c_{\phi^2 L^2}^{[U]} (\tilde{\phi}^\dagger \ell_{L_\gamma}^{pT}) C (\tilde{\phi}^\dagger \ell_{L_\delta}^q) \equiv c_{\phi^2 L^2}^{[U]} \epsilon^{ij} \epsilon^{mn} \phi_i \phi_m (\ell_j^p)^T C \ell_n^q$$

with

$$c_{\phi^2 L^2}^{[U]} = \frac{-1}{4M_\Delta^4} \mu_\Delta Y_{\Delta pq} (4\lambda_2 + 3\lambda_3). \quad (\text{A68})$$

It is worthy to mention that the operator $\mathcal{Q}_{\phi^2 L^2}$ violates the lepton number L by two units.

Dimension-six operator

$$\text{Tr}[U] \supset c_{ll}^{[U]} \mathcal{Q}_{ll} = c_{ll}^{[U]} (\bar{\ell}_{L_\alpha}^r \gamma_\mu \ell_{L_\gamma}^p)_i (\bar{\ell}_{L_\beta}^s \gamma^\mu \ell_{L_\delta}^q) \quad \text{with} \quad c_{ll}^{[U]} = -\frac{1}{4M_\Delta^2} Y_{\Delta pq} Y_{\Delta rs}^* (4\lambda_2 + 3\lambda_3). \quad (\text{A69})$$

Dimension-seven operators

$$\begin{aligned} \text{Tr}[U] &\supset c_{L\phi D}^{(1), [U]} \mathcal{Q}_{L\phi D}^{(1)} + c_{L\phi D}^{(2), [U]} \mathcal{Q}_{L\phi D}^{(2)} + c_{L\phi}^{[U]} \mathcal{Q}_{L\phi} \\ &= c_{L\phi D}^{(1), [U]} \epsilon^{ij} \epsilon^{mn} \ell_i^p C (D^\mu \ell_j^q) \phi_m (D_\mu \phi_n) + c_{L\phi D}^{(2), [U]} \epsilon^{im} \epsilon^{jn} \ell_i^p C (D^\mu \ell_j^q) \phi_m (D_\mu \phi_n) \\ &\quad + c_{L\phi}^{[U]} \epsilon^{ij} \epsilon^{mn} (\ell_i^p C \ell_m^q) \phi_j \phi_n (\phi^\dagger \phi). \end{aligned} \quad (\text{A70})$$

The matching relations are

$$\begin{aligned} c_{L\phi D}^{(1), [U]} &= \frac{-1}{2M_\Delta^6} \mu_\Delta Y_{\Delta pq} (4\lambda_2 + 3\lambda_3), \quad c_{L\phi D}^{(2), [U]} = \frac{-1}{2M_\Delta^6} \mu_\Delta Y_{\Delta pq} (4\lambda_2 + 3\lambda_3), \\ c_{L\phi}^{[U]} &= \frac{-1}{2M_\Delta^6} \mu_\Delta Y_{\Delta pq} \left(\lambda_2 \lambda_4 - \frac{3}{2} (\lambda_2 + \lambda_3) \lambda_4 + \lambda_2 (2\lambda_1 + \lambda_4) + \frac{3}{2} (\lambda_2 + \lambda_3) (2\lambda_1 + \lambda_4) \right). \end{aligned} \quad (\text{A71})$$

$$\text{Tr}[U^2] \supset c_{L\phi}^{[U^2]} \mathcal{Q}_{L\phi}. \quad (\text{A72})$$

¹⁴We used the completeness relation for Pauli matrices $\tau_{\alpha\beta}^i \tau_{\gamma\delta}^i = \frac{1}{2} \delta_{\alpha\delta} \delta_{\beta\gamma} - \frac{1}{4} \delta_{\alpha\beta} \delta_{\gamma\delta}$.

¹⁵These read $(\bar{\psi}_{1L} \gamma_\mu \psi_{2L})(\bar{\psi}_{3L} \gamma^\mu \psi_{4L}) = (\bar{\psi}_{1L} \gamma_\mu \psi_{4L})(\bar{\psi}_{3L} \gamma^\mu \psi_{2L})$.

The matching relation is

$$c_{L\phi}^{\llbracket U^2 \rrbracket} = \frac{1}{2M_\Delta^4} \mu_\Delta Y_{\Delta pq} \left(2\lambda_1\lambda_2 + \frac{3}{2}\lambda_1\lambda_3 + \lambda_2\lambda_4 + \frac{3}{4}\lambda_3\lambda_4 \right).$$

Dimension-eight operators

$$\text{Tr}[U] \supset c_{l^4\phi^2}^{(1),\llbracket U \rrbracket} \mathcal{Q}_{l^4\phi^2}^{(1)} + c_{l^4\phi^2}^{(2),\llbracket U \rrbracket} \mathcal{Q}_{l^4\phi^2}^{(2)} + c_{l^4D^2}^{(1),\llbracket U \rrbracket} \mathcal{Q}_{l^4D^2}^{(1)}. \quad (\text{A73})$$

The matching relations are

$$\begin{aligned} c_{l^4\phi^2}^{(1),\llbracket U \rrbracket} &= \frac{1}{4M_\Delta^6} Y_{\Delta r} Y_{\Delta ps}^* (10\lambda_1\lambda_2 + 6\lambda_1\lambda_3 + 5\lambda_2\lambda_4 + 3\lambda_3\lambda_4), \\ c_{l^4\phi^2}^{(2),\llbracket U \rrbracket} &= \frac{1}{4M_\Delta^6} Y_{\Delta r} Y_{\Delta ps}^* (6\lambda_1\lambda_4 + 2\lambda_2\lambda_4 + 3\lambda_3\lambda_4), \\ c_{l^4D^2}^{\llbracket U \rrbracket} &= \frac{1}{4M_\Delta^6} Y_{\Delta r} Y_{\Delta ps}^* (20\lambda_2 + 12\lambda_3). \end{aligned} \quad (\text{A74})$$

$$\text{Tr}[U^2] \supset c_{l^4\phi^2}^{(1),\llbracket U^2 \rrbracket} \mathcal{Q}_{l^4\phi^2}^{(1)} + c_{l^4\phi^2}^{(2),\llbracket U^2 \rrbracket} \mathcal{Q}_{l^4\phi^2}^{(2)}. \quad (\text{A75})$$

The matching relations are

$$c_{l^4\phi^2}^{(1),\llbracket U^2 \rrbracket} = -\frac{1}{M_\Delta^4} Y_{\Delta r} Y_{\Delta ps}^* \left(\lambda_1\lambda_2 + \frac{3}{4}\lambda_1\lambda_3 + \frac{1}{8}\lambda_3\lambda_4 + \frac{1}{2}\lambda_2\lambda_4 \right), \quad c_{l^4\phi^2}^{(2),\llbracket U^2 \rrbracket} = \frac{1}{4M_\Delta^4} Y_{\Delta r} Y_{\Delta ps}^* \lambda_4\lambda_2. \quad (\text{A76})$$

The effective operators for fermionic interactions discussed so far are summarized in Table V.

2. Case II: Integrating out complex doublet scalar

In this section, we discuss the integration out of the heavy complex doublet. We follow the same strategy similar to the complex triplet case. We define a basis for the heavy doublet field $(\mathcal{H}_2, \mathcal{H}_2^*)$, and the classical solutions as

$$\begin{aligned} \begin{pmatrix} \mathcal{H}_{2i} \\ \mathcal{H}_{2i}^* \end{pmatrix} &= \frac{1}{M_{\mathcal{H}_2}^2} \begin{pmatrix} B_i \\ B_i^* \end{pmatrix} + \frac{1}{M_{\mathcal{H}_2}^4} \left((P^2\delta_{ij}) \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} - \begin{pmatrix} (U_{11})_{ij} & (U_{12})_{ij} \\ (U_{21})_{ij} & (U_{22})_{ij} \end{pmatrix} \right) \begin{pmatrix} B_j \\ B_j^* \end{pmatrix} \\ &+ \frac{1}{M_{\mathcal{H}_2}^6} \left((P^2\delta_{ij}) \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} - \begin{pmatrix} (U_{11})_{ij} & (U_{12})_{ij} \\ (U_{21})_{ij} & (U_{22})_{ij} \end{pmatrix} \right)^2 \begin{pmatrix} B_j \\ B_j^* \end{pmatrix} + \dots \end{aligned} \quad (\text{A77})$$

The heavy field is gone through the covariant derivative expansion, and we truncate that expansion tracking their contributions up to dimension-eight effective operators.

- (i) The quadratic contribution of the heavy field in U as underlined in the following equation for U_{11} receives contribution of the first term of the solution of $\mathcal{H}_2$ from Eq. (A77), i.e., $\mathcal{O}(B/M_{\mathcal{H}_2}^2)$

$$\begin{aligned} \left(U_{11}^{\mathcal{H}_2} \right)_{ij} &= \frac{\lambda_{\mathcal{H}_2}}{2} [\delta_{ij} (\mathcal{H}_{2k}^* \mathcal{H}_{2k}) + \mathcal{H}_{2j}^* \mathcal{H}_{2i}] \\ &\quad - \underbrace{\eta_{\mathcal{H}_2} \left[\delta_{ij} \left(\tilde{H}_k^* \mathcal{H}_{2k} + \mathcal{H}_{2k}^* \tilde{H}_k \right) + \mathcal{H}_{2i} \tilde{H}_j^* + \mathcal{H}_{2j}^* \tilde{H}_i \right]}_{\text{wave-underlined}} \\ &\quad + \lambda_{\mathcal{H}_2,1} \delta_{ij} (H_k^* H_k) + \lambda_{\mathcal{H}_2,2} \left(\tilde{H}_j \tilde{H}_i^* \right). \end{aligned} \quad (\text{A78})$$

- (ii) For the wave-underlined part of U_{11} , we can get dimension-eight operators in two ways. The solution for the heavy field of $\mathcal{H}_2$ needs to be truncated at $\mathcal{O}(B^\dagger(P^2 - U)^2 B)$ term. First, we consider the linear term in U of this solution ($\mathcal{O}(B^\dagger(P^2 - U)B)$) wherein U receives the contribution from this wave-underlined part of U and we restrict to

$\mathcal{O}(B/M_{\mathcal{H}_2}^2)$ in solution of $\mathcal{H}_2$. The second way in which the dimension-eight operators can be generated is from the quadratic term in U , i.e., $\mathcal{O}(B^\dagger(P^2 - U)^2 B)$, where U gets contribution from the light fields only.

- (iii) The trace of U^2 can give rise to dimension-eight operators in two ways: (i) One of the U 's will carry dependence from heavy field solution up to dimension-six, i.e., up to a $B^\dagger U B$ term, while the other U will have dimension-two terms from the light fields only. (ii) The other way is that both the U 's interact via the heavy fields only with the heavy field solution being truncated up to first term $\frac{B}{M^2}$, so that each U having quadratic depend-

ence on $|H_{2c}|^2$ will have dimension-four operators, from the $B^\dagger B$ term. Similarly, we can compute dimension-eight operators from terms U^3 , $(P_\mu U)^2$ and $U(G_{\mu\nu})^2$ of the total effective Lagrangian.

The WCs associated with the effective operators are functions of the following BSM parameters: $\lambda_{\mathcal{H}_2}, \lambda_{\mathcal{H}_2,1}, \lambda_{\mathcal{H}_2,2}, \lambda_{\mathcal{H}_2,3}, \eta_{\mathcal{H}_2}, \eta_H, Y_{\mathcal{H}_2}^{(e)}, Y_{\mathcal{H}_2}^{(u)}, Y_{\mathcal{H}_2}^{(d)}$ along with the SM ones.

Dimension-eight operators from terms $\in \mathcal{L}_{-4}$

- (1) Covariant operator: U^4

$$\text{Tr}[U^4] \supset c_{\phi^8}^{\llbracket U^4 \rrbracket} \mathcal{O}_{\phi^8}. \quad (\text{A79})$$

$$\begin{aligned} c_{\phi^8}^{\llbracket U^4 \rrbracket} = & 4\lambda_{\mathcal{H}_2,1}^4 + 8\lambda_{\mathcal{H}_2,1}^3 \lambda_{\mathcal{H}_2,2} + 12\lambda_{\mathcal{H}_2,1}^2 \lambda_{\mathcal{H}_2,2}^2 + 48\lambda_{\mathcal{H}_2,1}^2 \lambda_{\mathcal{H}_2,3}^2 + 8\lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,2}^3 \\ & + 96\lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,2} \lambda_{\mathcal{H}_2,3}^2 + 4\lambda_{\mathcal{H}_2,2}^4 + 48\lambda_{\mathcal{H}_2,2}^2 \lambda_{\mathcal{H}_2,3}^2 + 32\lambda_{\mathcal{H}_2,3}^4. \end{aligned} \quad (\text{A80})$$

- (2) Covariant operator: $U^2(P^2 U)$

$$\text{Tr}[U^2(P^2 U)] \supset c_{\phi^6}^{(3), \llbracket U^2(P^2 U) \rrbracket} \mathcal{O}_{\phi^6}^{(3)}. \quad (\text{A81})$$

$$c_{\phi^6}^{(3), \llbracket U^2(P^2 U) \rrbracket} = 4\lambda_{\mathcal{H}_2,1}^3 + 6\lambda_{\mathcal{H}_2,1}^2 \lambda_{\mathcal{H}_2,2} + 6\lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,2}^2 + 24\lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,3}^2 + 4\lambda_{\mathcal{H}_2,2}^3 + 24\lambda_{\mathcal{H}_2,2} \lambda_{\mathcal{H}_2,3}^2. \quad (\text{A82})$$

- (3) Covariant operator: $U^2(G_{\mu\nu})^2$

$$\text{Tr}[U^2(G_{\mu\nu})^2] \supset c_{W^2\phi^4}^{(1), \llbracket U^2(G_{\mu\nu})^2 \rrbracket} \mathcal{O}_{W^2\phi^4}^{(1)} + c_{B^2\phi^4}^{(1), \llbracket U^2(G_{\mu\nu})^2 \rrbracket} \mathcal{O}_{B^2\phi^4}^{(1)} + c_{WB\phi^4}^{(1), \llbracket U^2(G_{\mu\nu})^2 \rrbracket} \mathcal{O}_{WB\phi^4}^{(1)}. \quad (\text{A83})$$

$$\begin{aligned} c_{W^2\phi^4}^{(1), \llbracket U^2(G_{\mu\nu})^2 \rrbracket} &= -g^2 \left(\lambda_{\mathcal{H}_2,1}^2 + \lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,2} + \frac{1}{2} \lambda_{\mathcal{H}_2,2}^2 + \lambda_{\mathcal{H}_2,3}^2 \right), \\ c_{B^2\phi^4}^{(1), \llbracket U^2(G_{\mu\nu})^2 \rrbracket} &= -2g^2 (2\lambda_{\mathcal{H}_2,1}^2 + \lambda_{\mathcal{H}_2,2}^2 + 2\lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,2} + 2\lambda_{\mathcal{H}_2,3}^2), \\ c_{WB\phi^4}^{(1), \llbracket U^2(G_{\mu\nu})^2 \rrbracket} &= -8gg' \lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,2}. \end{aligned} \quad (\text{A84})$$

- (4) Covariant operator: $(UG_{\mu\nu})^2$

$$\text{Tr}[(UG_{\mu\nu})^2] \supset c_{W^2\phi^4}^{(1), \llbracket (UG_{\mu\nu})^2 \rrbracket} \mathcal{O}_{W^2\phi^4}^{(1)} + c_{B^2\phi^4}^{(1), \llbracket (UG_{\mu\nu})^2 \rrbracket} \mathcal{O}_{B^2\phi^4}^{(1)} + c_{WB\phi^4}^{(1), \llbracket (UG_{\mu\nu})^2 \rrbracket} \mathcal{O}_{WB\phi^4}^{(1)}. \quad (\text{A85})$$

$$\begin{aligned} c_{W^2\phi^4}^{(1), \llbracket (UG_{\mu\nu})^2 \rrbracket} &= -g^2 \left(\lambda_{\mathcal{H}_2,1}^2 + \lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,2} + \frac{1}{2} \lambda_{\mathcal{H}_2,2}^2 + \lambda_{\mathcal{H}_2,3}^2 \right), \\ c_{B^2\phi^4}^{(1), \llbracket (UG_{\mu\nu})^2 \rrbracket} &= -2g^2 (2\lambda_{\mathcal{H}_2,1}^2 + \lambda_{\mathcal{H}_2,2}^2 + 2\lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,2} + 2\lambda_{\mathcal{H}_2,3}^2), \\ c_{WB\phi^4}^{(1), \llbracket (UG_{\mu\nu})^2 \rrbracket} &= -8gg' \lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,2}. \end{aligned} \quad (\text{A86})$$

- (5) Covariant operator: $(P^2 U)^2$

$$\begin{aligned} \text{Tr}[(P^2 U)^2] \supset & c_{\phi^4}^{(4), \llbracket (P^2 U)^2 \rrbracket} \mathcal{O}_{\phi^4}^{(4)} + c_{\phi^4}^{(8), \llbracket (P^2 U)^2 \rrbracket} \mathcal{O}_{\phi^4}^{(8)} + c_{\phi^4}^{(10), \llbracket (P^2 U)^2 \rrbracket} \mathcal{O}_{\phi^4}^{(10)} \\ & + c_{\phi^4}^{(11), \llbracket (P^2 U)^2 \rrbracket} \mathcal{O}_{\phi^4}^{(11)} + c_{\phi^4}^{(12), \llbracket (P^2 U)^2 \rrbracket} \mathcal{O}_{\phi^4}^{(12)}. \end{aligned} \quad (\text{A87})$$

$$\begin{aligned}
c_{\phi^4}^{(4), \llbracket (P^2 U)^2 \rrbracket} &= 8(4\lambda_{\mathcal{H}_2,1}^2 + 4\lambda_{\mathcal{H}_2,1}\lambda_{\mathcal{H}_2,2} + \lambda_{\mathcal{H}_2,2}^2), \\
c_{\phi^4}^{(8), \llbracket (P^2 U)^2 \rrbracket} &= 2(4\lambda_{\mathcal{H}_2,1}^2 + 4\lambda_{\mathcal{H}_2,1}\lambda_{\mathcal{H}_2,2} + \lambda_{\mathcal{H}_2,2}^2), \\
c_{\phi^4}^{(10), \llbracket (P^2 U)^2 \rrbracket} &= 8\lambda_{\mathcal{H}_2,3}^2, \\
c_{\phi^4}^{(11), \llbracket (P^2 U)^2 \rrbracket} &= 4(4\lambda_{\mathcal{H}_2,1}^2 + 4\lambda_{\mathcal{H}_2,1}\lambda_{\mathcal{H}_2,2} + \lambda_{\mathcal{H}_2,2}^2 + \lambda_{\mathcal{H}_2,3}^2), \\
c_{\phi^4}^{(12), \llbracket (P^2 U)^2 \rrbracket} &= -8(4\lambda_{\mathcal{H}_2,1}^2 + 4\lambda_{\mathcal{H}_2,1}\lambda_{\mathcal{H}_2,2} + \lambda_{\mathcal{H}_2,2}^2). \tag{A88}
\end{aligned}$$

(6) Covariant operator: $U(J_\mu)^2$

$$\text{Tr}[U(J_\mu)^2] \supset c_{B^2\phi^2 D^2}^{(6), \llbracket U(J_\mu)^2 \rrbracket} \mathcal{O}_{B^2\phi^2 D^2}^{(6)} + c_{B^2\phi^2 D^2}^{(8), \llbracket U(J_\mu)^2 \rrbracket} \mathcal{O}_{B^2\phi^2 D^2}^{(8)} + c_{WB\phi^2 D^2}^{(10), \llbracket U(J_\mu)^2 \rrbracket} \mathcal{O}_{WB\phi^2 D^2}^{(10)} + c_{W^2\phi^2 D^2}^{(13), \llbracket U(J_\mu)^2 \rrbracket} \mathcal{O}_{W^2\phi^2 D^2}^{(13)}. \tag{A89}$$

$$\begin{aligned}
c_{B^2\phi^2 D^2}^{(6), \llbracket U(J_\mu)^2 \rrbracket} &= -4g^2(2\lambda_{\mathcal{H}_2,1} + \lambda_{\mathcal{H}_2,2}), & c_{B^2\phi^2 D^2}^{(8), \llbracket U(J_\mu)^2 \rrbracket} &= -4g'^2(2\lambda_{\mathcal{H}_2,1} + \lambda_{\mathcal{H}_2,2}), \\
c_{WB\phi^2 D^2}^{(10), \llbracket U(J_\mu)^2 \rrbracket} &= -4gg'\lambda_{\mathcal{H}_2,2}, & c_{W^2\phi^2 D^2}^{(13), \llbracket U(J_\mu)^2 \rrbracket} &= -g^2(2\lambda_{\mathcal{H}_2,1} + \lambda_{\mathcal{H}_2,2}). \tag{A90}
\end{aligned}$$

(7) Covariant operator: $U(P_\mu U)J_\mu$ $\text{Tr}[U(P_\mu U)J_\mu]$ gets contributions from $\mathcal{O}_{B^2\phi^2 D^2}^{(3)}$ of the Green's basis

$$\text{Tr}[U(P_\mu U)J_\mu] \supset c_{B^2\phi^2 D^2}^{(3), \llbracket U(P_\mu U)J_\mu \rrbracket} \mathcal{O}_{B^2\phi^2 D^2}^{(3)}. \tag{A91}$$

The matching relation is

$$c_{B\phi^4 D^2}^{(3), \llbracket U(P_\mu U)J_\mu \rrbracket} = -2g'(2\lambda_{\mathcal{H}_2,1}^2 + \lambda_{\mathcal{H}_2,2}^2 - \lambda_{\mathcal{H}_2,1}\lambda_{\mathcal{H}_2,2} - 2\lambda_{\mathcal{H}_2,3}^2). \tag{A92}$$

(8) Covariant operator: $(P^2 U)(G_{\rho\sigma})^2$

$$\begin{aligned}
\text{Tr}[(P^2 U)(G_{\rho\sigma})^2] \supset & c_{W^2\phi^2 D^2}^{(2), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} \mathcal{O}_{W^2\phi^2 D^2}^{(4)} + c_{W^2\phi^2 D^2}^{(4), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} \mathcal{O}_{W^2\phi^2 D^2}^{(2)} \\
& + c_{B^2\phi^2 D^2}^{(2), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} \mathcal{O}_{B^2\phi^2 D^2}^{(2)} + c_{WB\phi^2 D^2}^{(1), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} \mathcal{O}_{WB\phi^2 D^2}^{(1)} \\
& - \frac{g^2}{2}(2\lambda_{\mathcal{H}_2,1} + \lambda_{\mathcal{H}_2,2})(\underline{D^2\phi^\dagger\phi + \phi^\dagger D^2\phi}) \underline{W_{\rho\sigma}^a W^{a,\rho\sigma}} - ig^2\lambda_{\mathcal{H}_2,2}\epsilon_{abc}(\underline{D^2\phi^\dagger\tau^c\phi + \phi^\dagger\tau^c D^2\phi}) \underline{W_{\rho\sigma}^a W^{b,\rho\sigma}} \\
& + 4gg'\lambda_{\mathcal{H}_2,2}(\underline{D^2\phi^\dagger\tau^a\phi + \phi^\dagger\tau^a D^2\phi}) \underline{B_{\rho\sigma} W_{\rho\sigma}^a} - 2(2\lambda_{\mathcal{H}_2,1} + \lambda_{\mathcal{H}_2,2})g'^2(\underline{D^2\phi^\dagger\phi + \phi^\dagger D^2\phi}) \underline{B_{\rho\sigma} B^{\rho\sigma}} \tag{A93}
\end{aligned}$$

$$\begin{aligned}
& = c_{W^2\phi^2 D^2}^{(2), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} (D^\mu\phi^\dagger D_\mu\phi) W_{\nu\rho}^I W^{I\nu\rho} + c_{W^2\phi^2 D^2}^{(4), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} i\epsilon^{IJK} (D^\mu\phi^\dagger \sigma^I D^\nu\phi) W_{\mu\rho}^J W_\nu^{K\rho} \\
& + c_{B^2\phi^2 D^2}^{(2), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} (D_\mu\phi^\dagger\phi + \phi^\dagger D_\mu\phi) D_\nu B^{\mu\rho} B_\rho^\nu + c_{WB\phi^2 D^2}^{(1), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} (D^\mu\phi^\dagger \sigma^I D_\mu\phi) B_{\nu\rho} W^{I\nu\rho} + \text{EOM}. \tag{A94}
\end{aligned}$$

The underlined terms in Eq. (A93) get reduced by the EOM for the scalar field to other classes.

$$\begin{aligned}
c_{W^2\phi^2 D^2}^{(2), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} &= -g^2(2\lambda_{\mathcal{H}_2,1} + \lambda_{\mathcal{H}_2,2}), & c_{W^2\phi^2 D^2}^{(4), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} &= -2g^2\lambda_{\mathcal{H}_2,2}, \\
c_{B^2\phi^2 D^2}^{(2), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} &= -4g'^2(2\lambda_{\mathcal{H}_2,1} + \lambda_{\mathcal{H}_2,2}), & c_{WB\phi^2 D^2}^{(1), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} &= -8gg'\lambda_{\mathcal{H}_2,2}. \tag{A95}
\end{aligned}$$

(9) Covariant operator: $(P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu}$

$$\begin{aligned} \text{Tr}[(P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu}] \supset & c_{W^2\phi^2D^2}^{(4), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} \mathcal{O}_{W^2\phi^2D^2}^{(4)} + c_{W^2\phi^2D^2}^{(11), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} \mathcal{O}_{W^2\phi^2D^2}^{(11)} \\ & + c_{W^2\phi^2D^2}^{(14), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} \mathcal{O}_{W^2\phi^2D^2}^{(14)} + c_{B^2\phi^2D^2}^{(4), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} \mathcal{O}_{B^2\phi^2D^2}^{(4)} \\ & + c_{B^2\phi^2D^2}^{(8), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} \mathcal{O}_{B^2\phi^2D^2}^{(8)} + c_{WB\phi^2D^2}^{(8), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} \mathcal{O}_{WB\phi^2D^2}^{(8)} \\ & + c_{WB\phi^2D^2}^{(10), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} \mathcal{O}_{WB\phi^2D^2}^{(10)} + c_{WB\phi^2D^2}^{(11), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} \mathcal{O}_{WB\phi^2D^2}^{(11)} + c_{WB\phi^2D^2}^{(13), \llbracket (P^2 U)(G_{\rho\sigma})^2 \rrbracket} \mathcal{O}_{WB\phi^2D^2}^{(13)}. \end{aligned} \quad (\text{A96})$$

$$\begin{aligned} c_{W^2\phi^2D^2}^{(4), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} &= -g^2\lambda_{\mathcal{H}_{2,2}}, & c_{W^2\phi^2D^2}^{(11), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} &= \frac{g^2}{2}(2\lambda_{\mathcal{H}_{2,1}} + \lambda_{\mathcal{H}_{2,2}}), \\ c_{W^2\phi^2D^2}^{(14), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} &= \frac{g^2}{2}(2\lambda_{\mathcal{H}_{2,1}} + \lambda_{\mathcal{H}_{2,2}}), & c_{B^2\phi^2D^2}^{(4), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} &= g^2(2\lambda_{\mathcal{H}_{2,1}} + \lambda_{\mathcal{H}_{2,2}}), \\ c_{B^2\phi^2D^2}^{(8), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} &= g'^2(2\lambda_{\mathcal{H}_{2,1}} + \lambda_{\mathcal{H}_{2,2}}), & c_{WB\phi^2D^2}^{(8), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} &= -2gg'\lambda_{\mathcal{H}_{2,2}}, \\ c_{WB\phi^2D^2}^{(10), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} &= -2gg'\lambda_{\mathcal{H}_{2,2}}, & c_{WB\phi^2D^2}^{(11), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} &= 2gg'\lambda_{\mathcal{H}_{2,2}}, \\ c_{WB\phi^2D^2}^{(13), \llbracket (P_\mu P_\nu U)G_{\rho\mu}G_{\rho\nu} \rrbracket} &= -2gg'\lambda_{\mathcal{H}_{2,2}}. \end{aligned} \quad (\text{A97})$$

(10) Covariant operator: $UG_{\mu\nu}G_{\nu\rho}G_{\rho\mu}$

$$\text{Tr}[UG_{\mu\nu}G_{\nu\rho}G_{\rho\mu}] \supset c_{W^3\phi^2}^{(1), \llbracket UG_{\mu\nu}G_{\nu\rho}G_{\rho\mu} \rrbracket} \mathcal{O}_{W^3\phi^2}^{(1)}. \quad (\text{A98})$$

$$c_{W^3\phi^2}^{(1), \llbracket UG_{\mu\nu}G_{\nu\rho}G_{\rho\mu} \rrbracket} = -g^3\lambda_{\mathcal{H}_{2,1}}. \quad (\text{A99})$$

(11) Covariant operator: $(P_\nu J_\mu)^2$

$$\text{Tr}[(P_\nu J_\mu)^2] \supset c_{B^2D^4}^{\llbracket (P_\nu J_\mu)^2 \rrbracket} \mathcal{O}_{B^2D^4} + c_{W^2D^4}^{\llbracket (P_\nu J_\mu)^2 \rrbracket} \mathcal{O}_{W^2D^4}. \quad (\text{A100})$$

The matching relations are

$$c_{B^2D^4}^{\llbracket (P_\nu J_\mu)^2 \rrbracket} = -\frac{3}{4}g^2, \quad c_{W^2D^4}^{\llbracket (P_\nu J_\mu)^2 \rrbracket} = 2g^2. \quad (\text{A101})$$

(12) Covariant operator: $(G_{\mu\nu}G_{\rho\sigma})^2$

$$\text{Tr}[(G_{\mu\nu}G_{\rho\sigma})^2] \supset c_{W^4}^{(3), \llbracket (G_{\mu\nu}G_{\rho\sigma})^2 \rrbracket} \mathcal{Q}_{W^4}^{(3)} + c_{B^4}^{(1), \llbracket (G_{\mu\nu}G_{\rho\sigma})^2 \rrbracket} \mathcal{Q}_{B^4}^{(1)} + c_{W^2B^2}^{(1), \llbracket (G_{\mu\nu}G_{\rho\sigma})^2 \rrbracket} \mathcal{Q}_{W^2B^2}^{(1)} + c_{W^2B^2}^{(3), \llbracket (G_{\mu\nu}G_{\rho\sigma})^2 \rrbracket} \mathcal{Q}_{W^2B^2}^{(3)}. \quad (\text{A102})$$

The matching relations are

$$\begin{aligned} c_{W^4}^{(3), \llbracket (G_{\mu\nu}G_{\rho\sigma})^2 \rrbracket} &= 2g^4, & c_{B^4}^{(1), \llbracket (G_{\mu\nu}G_{\rho\sigma})^2 \rrbracket} &= \frac{3}{16}g^4, \\ c_{W^2B^2}^{(1), \llbracket (G_{\mu\nu}G_{\rho\sigma})^2 \rrbracket} &= g^2g'^2, & c_{W^2B^2}^{(3), \llbracket (G_{\mu\nu}G_{\rho\sigma})^2 \rrbracket} &= 2g^2g'^2. \end{aligned} \quad (\text{A103})$$

(13) Covariant operator: $(G_{\mu\nu})^2(G_{\rho\sigma})^2$

$$\text{Tr}[(G_{\mu\nu})^2(G_{\rho\sigma})^2] \supset c_{W^4}^{(3), \llbracket (G_{\mu\nu})^2(G_{\rho\sigma})^2 \rrbracket} \mathcal{Q}_{W^4}^{(3)} + c_{B^4}^{(1), \llbracket (G_{\mu\nu})^2(G_{\rho\sigma})^2 \rrbracket} \mathcal{Q}_{B^4}^{(1)} + c_{W^2B^2}^{(1), \llbracket (G_{\mu\nu})^2(G_{\rho\sigma})^2 \rrbracket} \mathcal{Q}_{W^2B^2}^{(1)} + c_{W^2B^2}^{(3), \llbracket (G_{\mu\nu})^2(G_{\rho\sigma})^2 \rrbracket} \mathcal{Q}_{W^2B^2}^{(3)}. \quad (\text{A104})$$

The matching relations are

$$\begin{aligned} c_{W^4}^{(3), \llbracket (G_{\mu\nu})^2 (G_{\rho\sigma})^2 \rrbracket} &= 2g^4, & c_{B^4}^{(1), \llbracket (G_{\mu\nu})^2 (G_{\rho\sigma})^2 \rrbracket} &= \frac{3}{16}g^4, \\ c_{W^2 B^2}^{(1), \llbracket (G_{\mu\nu})^2 (G_{\rho\sigma})^2 \rrbracket} &= g^2 g'^2, & c_{W^2 B^2}^{(3), \llbracket (G_{\mu\nu})^2 (G_{\rho\sigma})^2 \rrbracket} &= 2g^2 g'^2. \end{aligned} \quad (\text{A105})$$

- (14) Covariant operator: $(G_{\mu\nu} G_{\nu\rho})^2$, $G_{\mu\nu} G_{\nu\rho} G_{\rho\sigma} G_{\sigma\mu}$, $G_{\mu\nu} J_\mu J_\nu$, $P_\mu J_\nu G_{\nu\sigma} G_{\sigma\mu}$

$$\begin{aligned} \text{Tr}[(G_{\mu\nu} G_{\nu\rho})^2] &\supset -(D_\nu G_{\nu\rho})(D_\mu G_{\mu\nu})G_{\nu\rho} \\ &\quad - (D_\nu G_{\nu\rho})G_{\mu\nu}(D_\mu G_{\nu\rho}) \\ &\quad + (D_\mu G_{\nu\rho})(D_\nu G_{\mu\nu})G_{\nu\rho} \\ &\quad + (D_\mu G_{\nu\rho})G_{\mu\nu}(D_\nu G_{\nu\rho}). \end{aligned} \quad (\text{A106})$$

$$\begin{aligned} \text{Tr}[G_{\mu\nu} J_\mu J_\nu] &= g^3 \epsilon^{IJK} W_{\mu\nu}^I D_\rho W_{\rho\mu}^J D_\sigma W_{\sigma\nu}^K \\ &\quad + 4ig^2 g' W_{\mu\nu}^I D_\rho W_{\rho\mu}^I D_\sigma B_{\sigma\nu} \\ &\quad + 2ig^2 g' W_{\mu\nu}^I D_\rho B_{\rho\mu} D_\sigma W_{\sigma\nu}^I. \end{aligned} \quad (\text{A107})$$

By employing the EOM of gauge field tensors and the notion of conserved currents, these dimension-eight operators can be written in terms of IR scalar and fermionic fields.

Dimension-eight and dimension-six operators from terms $\in \mathcal{L}_{-2}$

- (1) Covariant operator: U^3

$$\text{Tr}[U^3] \supset c_{\phi^8}^{\llbracket U^3 \rrbracket} \mathcal{O}_{\phi^8}. \quad (\text{A108})$$

The matching relation at dimension-eight is

$$c_{\phi^8}^{\llbracket U^3 \rrbracket} = \frac{-48\eta_H \eta_{\mathcal{H}_2}}{M_{\mathcal{H}_2}^2} \left(\lambda_{\mathcal{H}_2,1}^2 + \lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,2} + \lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,3} + \lambda_{\mathcal{H}_2,2} \lambda_{\mathcal{H}_2,3} + \frac{1}{2} \lambda_{\mathcal{H}_2,2}^2 + 4\lambda_{\mathcal{H}_2,3}^2 \right). \quad (\text{A109})$$

$$\text{Tr}[U^3] \supset c_6^{\llbracket U^3 \rrbracket} \mathcal{O}_6 = c_6^{\llbracket U^3 \rrbracket} (\phi^\dagger \phi)^3, \quad (\text{A110})$$

with the matching relation at dimension-six as

$$c_6^{\llbracket U^3 \rrbracket} = 4\lambda_{\mathcal{H}_2,1}^3 + 6\lambda_{\mathcal{H}_2,1}^2 \lambda_{\mathcal{H}_2,2} + 6\lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,2}^2 + 2\lambda_{\mathcal{H}_2,2}^3 + 24\lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,3}^2 + 24\lambda_{\mathcal{H}_2,3}^2 \lambda_{\mathcal{H}_2,2}. \quad (\text{A111})$$

- (2) Covariant operator: $(P_\mu U)^2$

$$\text{Tr}[(P_\mu U)^2] \supset c_{\phi^6}^{(1), \llbracket (P_\mu U)^2 \rrbracket} \mathcal{O}_{\phi^6}^{(1)} + c_{\phi^6}^{(3), \llbracket (P_\mu U)^2 \rrbracket} \mathcal{O}_{\phi^6}^{(3)}. \quad (\text{A112})$$

The matching relations at dimension-eight are

$$\begin{aligned} c_{\phi^6}^{(1), \llbracket (P_\mu U)^2 \rrbracket} &= \frac{2\eta_H \eta_{\mathcal{H}_2}}{M_{\mathcal{H}_2}^2} (6\lambda_{\mathcal{H}_2,1} + 4\lambda_{\mathcal{H}_2,2} + 2\lambda_{\mathcal{H}_2,3}), \\ c_{\phi^6}^{(3), \llbracket (P_\mu U)^2 \rrbracket} &= \frac{2\eta_H \eta_{\mathcal{H}_2}}{M_{\mathcal{H}_2}^2} (6\lambda_{\mathcal{H}_2,1} + 4\lambda_{\mathcal{H}_2,2} - \lambda_{\mathcal{H}_2,3}). \end{aligned} \quad (\text{A113})$$

$$\text{Tr}[(P_\mu U)^2] \supset c_H^{\llbracket (P_\mu U)^2 \rrbracket} \mathcal{O}_H + c_T^{\llbracket (P_\mu U)^2 \rrbracket} \mathcal{O}_T + c_R^{\llbracket (P_\mu U)^2 \rrbracket} \mathcal{O}_R \quad (\text{A114})$$

with the matching relation at dimension-six as

$$c_H^{\llbracket (P_\mu U)^2 \rrbracket} = -(4\lambda_{\mathcal{H}_2,1}^2 + 4\lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,2} + \lambda_{\mathcal{H}_2,2}^2), \quad c_T^{\llbracket (P_\mu U)^2 \rrbracket} = -\lambda_{\mathcal{H}_2,2}^2, \quad c_R^{\llbracket (P_\mu U)^2 \rrbracket} = -2\lambda_{\mathcal{H}_2,2}^2. \quad (\text{A115})$$

- (3) Covariant operator: $U(G_{\mu\nu})^2$

$$\text{Tr}[U(G_{\mu\nu})^2] \supset c_{W^2 \phi^4}^{(1), \llbracket U(G_{\mu\nu})^2 \rrbracket} \mathcal{O}_{W^2 \phi^4}^{(1)} + c_{B^2 \phi^4}^{(1), \llbracket U(G_{\mu\nu})^2 \rrbracket} \mathcal{O}_{B^2 \phi^4}^{(1)} + c_{WB \phi^4}^{(1), \llbracket U(G_{\mu\nu})^2 \rrbracket} \mathcal{O}_{WB \phi^4}^{(1)}. \quad (\text{A116})$$

The matching relations at dimension-eight are

$$c_{W^2\phi^4}^{(1),\llbracket U(G_{\mu\nu})^2 \rrbracket} = \frac{3\eta_H\eta_{\mathcal{H}_2}}{M_{\mathcal{H}_2}^2}g^2, \quad c_{B^2\phi^4}^{(1),\llbracket U(G_{\mu\nu})^2 \rrbracket} = \frac{12\eta_H\eta_{\mathcal{H}_2}}{M_{\mathcal{H}_2}^2}g^2, \quad c_{WB\phi^4}^{(1),\llbracket U(G_{\mu\nu})^2 \rrbracket} = \frac{8\eta_H\eta_{\mathcal{H}_2}}{M_{\mathcal{H}_2}^2}gg'. \quad (\text{A117})$$

$$\text{Tr}[U(G_{\mu\nu})^2] \supset c_{BB}^{\llbracket U(G_{\mu\nu})^2 \rrbracket} \mathcal{O}_{BB} + c_{WW}^{\llbracket U(G_{\mu\nu})^2 \rrbracket} \mathcal{O}_{WW} + c_{WB}^{\llbracket U(G_{\mu\nu})^2 \rrbracket} \mathcal{O}_{WB} \quad (\text{A118})$$

with the matching relation at dimension-six as

$$c_{BB}^{\llbracket U(G_{\mu\nu})^2 \rrbracket} = -\frac{g^2}{2}(2\lambda_{\mathcal{H}_2,1} + \lambda_{\mathcal{H}_2,2}), \quad c_{WW}^{\llbracket U(G_{\mu\nu})^2 \rrbracket} = -\frac{g^2}{2}(2\lambda_{\mathcal{H}_2,1} + \lambda_{\mathcal{H}_2,2}), \quad c_{WB}^{\llbracket U(G_{\mu\nu})^2 \rrbracket} = -gg'\lambda_{\mathcal{H}_2,2}. \quad (\text{A119})$$

Dimension-eight and dimension-six operators from terms $\in \mathcal{L}_0$

(1) Covariant operator: U^2

$$\text{Tr}[U^2] \supset c_{\phi^8}^{\llbracket U^2 \rrbracket} \mathcal{O}_{\phi^8} + c_{\phi^6}^{(3),\llbracket U^2 \rrbracket} \mathcal{O}_{\phi^6}^{(3)}. \quad (\text{A120})$$

The matching relation at dimension-eight is

$$c_{\phi^8}^{\llbracket U^2 \rrbracket} = \frac{4\eta_{\mathcal{H}_2}}{M_{\mathcal{H}_2}^4}(\eta_H(12\eta_{\mathcal{H}_2}\eta_H + 6\lambda_{\mathcal{H}_2,1}^2 + 10\lambda_{\mathcal{H}_2,1}\lambda_{\mathcal{H}_2,2} + 4\lambda_{\mathcal{H}_2,2}^2) + \frac{4\lambda_{\mathcal{H}_2}}{M_{\mathcal{H}_2}^4}(2\eta_H^2\lambda_{\mathcal{H}_2,3} + \eta_{\mathcal{H}_2}^2(3\lambda_{\mathcal{H}_2,1} + 2\lambda_{\mathcal{H}_2,2})),$$

$$c_{\phi^6}^{(3),\llbracket U^2 \rrbracket} = -\frac{12\eta_H\eta_{\mathcal{H}_2}}{M_{\mathcal{H}_2}^4}(3\lambda_{\mathcal{H}_2,1} + 2\lambda_{\mathcal{H}_2,2}) \quad (\text{A121})$$

$$\text{Tr}[U^2] \supset c_6^{\llbracket U^2 \rrbracket} \mathcal{O}_6 = c_6^{\llbracket U^2 \rrbracket} (\phi^\dagger \phi)^3 \quad (\text{A122})$$

with the matching relation at dimension-six as

$$c_6^{\llbracket U^2 \rrbracket} = -\frac{4\eta_{\mathcal{H}_2}\eta_H}{M_{\mathcal{H}_2}^2}(6\lambda_{\mathcal{H}_2,1} + 4\lambda_{\mathcal{H}_2,2} - 2\lambda_{\mathcal{H}_2,3}). \quad (\text{A123})$$

Dimension-eight and dimension-six operators from terms $\in \mathcal{L}_2$

(1) Covariant operator: U

$$\text{Tr}[U] \supset c_{\phi^8}^{\llbracket U \rrbracket} \mathcal{O}_{\phi^8} + c_{\phi^6}^{(3),\llbracket U \rrbracket} \mathcal{O}_{\phi^6}^{(3)} + c_{\phi^4}^{(3),\llbracket U \rrbracket} \mathcal{O}_{\phi^4}^{(3)} + c_{\phi^4}^{(4),\llbracket U \rrbracket} \mathcal{O}_{\phi^4}^{(4)} + c_{\phi^6}^{(8),\llbracket U \rrbracket} \mathcal{O}_{\phi^6}^{(8)} + c_{\phi^4}^{(10),\llbracket U \rrbracket} \mathcal{O}_{\phi^4}^{(10)} + c_{\phi^4}^{(11),\llbracket U \rrbracket} \mathcal{O}_{\phi^4}^{(11)}. \quad (\text{A124})$$

The matching relations at dimension-eight are

$$c_{\phi^8}^{\llbracket U \rrbracket} = -\frac{16\eta_H^2\eta_{\mathcal{H}_2}}{M_{\mathcal{H}_2}^6} - \frac{8\eta_H\eta_{\mathcal{H}_2}\lambda_{\mathcal{H}_2,1}\lambda_{\mathcal{H}_2,2}}{M_{\mathcal{H}_2}^6} - \frac{8\eta_H\eta_{\mathcal{H}_2}\lambda_{\mathcal{H}_2,2}^2}{M_{\mathcal{H}_2}^6} + \frac{12\eta_H\lambda_{\mathcal{H}_2,1}\lambda_H}{M_{\mathcal{H}_2}^6} + \frac{12\eta_H\lambda_{\mathcal{H}_2,2}\lambda_H}{M_{\mathcal{H}_2}^6},$$

$$c_{\phi^6}^{(3),\llbracket U \rrbracket} = -\frac{26\eta_H\eta_{\mathcal{H}_2}\lambda_{\mathcal{H}_2,1}}{M_{\mathcal{H}_2}^6} - \frac{12\eta_H\eta_{\mathcal{H}_2}\lambda_{\mathcal{H}_2,2}^2}{M_{\mathcal{H}_2}^6} - \frac{18\eta_H^2\lambda_H}{M_{\mathcal{H}_2}^6},$$

$$c_{\phi^4}^{(3),\llbracket U \rrbracket} = \frac{24\eta_H\eta_{\mathcal{H}_2}}{M_{\mathcal{H}_2}^6}, \quad c_{\phi^4}^{(4),\llbracket U \rrbracket} = \frac{24\eta_H\eta_{\mathcal{H}_2}}{M_{\mathcal{H}_2}^6},$$

$$c_{\phi^4}^{(8),\llbracket U \rrbracket} = \frac{9\eta_H\eta_{\mathcal{H}_2}}{M_{\mathcal{H}_2}^6}, \quad c_{\phi^4}^{(10),\llbracket U \rrbracket} = \frac{6\eta_H\eta_{\mathcal{H}_2}}{M_{\mathcal{H}_2}^6},$$

$$c_{\phi^4}^{(11),\llbracket U \rrbracket} = \frac{18\eta_H\eta_{\mathcal{H}_2}}{M_{\mathcal{H}_2}^6}. \quad (\text{A125})$$

$$\text{Tr}[U] \supset c_6^{\llbracket U \rrbracket} \mathcal{O}_6 + c_H^{\llbracket U \rrbracket} \mathcal{O}_H + c_R^{\llbracket U \rrbracket} \mathcal{O}_R, \quad (\text{A126})$$

with the matching relations at dimension-six as

$$c_6^{\llbracket U \rrbracket} = \frac{12\eta_H\eta_{\mathcal{H}_2}}{M_{\mathcal{H}_2}^4}(\lambda_{\mathcal{H}_2,1} + \lambda_{\mathcal{H}_2,2}) + \frac{3\lambda_{\mathcal{H}_2}\eta_H^2}{M_{\mathcal{H}_2}^4}, \quad c_H^{\llbracket U \rrbracket} = -\frac{12\eta_H\eta_{\mathcal{H}_2}}{M_{\mathcal{H}_2}^4}, \quad c_R^{\llbracket U \rrbracket} = -\frac{12\eta_H\eta_{\mathcal{H}_2}}{M_{\mathcal{H}_2}^4}. \quad (\text{A127})$$

The results discussed so far are summarized in Tables III and IV.

a. Incorporating the effect of fermion interactions: Complex Higgs doublet

We further consider the interactions between the heavy complex doublet and the SM fermions, see Eq. (3.3). This modifies the classical solution of the heavy field via the functional B , and leads to the emergence of new operators involving SM fermions that we discuss below.

Dimension-eight operators from terms $\in \mathcal{L}_{-2}$

(1) Covariant operator: U^3

$$\text{Tr}[U^3] \supset c_{le\phi^5}^{\llbracket U^3 \rrbracket} \mathcal{Q}_{le\phi^5}, \quad (\text{A128})$$

where, $\{p, r\}$ are the flavor indices. The matching relation is

$$c_{le\phi^5}^{\llbracket U^3 \rrbracket} = \frac{-24\eta_{\mathcal{H}_2}Y_{\mathcal{H}_2}^{(e)}}{M_{\mathcal{H}_2}^2} \left(\lambda_{\mathcal{H}_2,1}^2 + \lambda_{\mathcal{H}_2,1}\lambda_{\mathcal{H}_2,2} + \lambda_{\mathcal{H}_2,1}\lambda_{\mathcal{H}_2,3} + \lambda_{\mathcal{H}_2,2}\lambda_{\mathcal{H}_2,3} + \frac{1}{2}\lambda_{\mathcal{H}_2,2}^2 + 4\lambda_{\mathcal{H}_2,3}^2 \right). \quad (\text{A129})$$

(2) Covariant operator: $(P_\mu U)^2$

$$\begin{aligned} \text{Tr}[(P_\mu U)^2] &\supset c_{le\phi^3 D^2}^{(1), \llbracket (P_\mu U)^2 \rrbracket} \mathcal{Q}_{le\phi^3 D^2}^{(1)} + c_{le\phi^3 D^2}^{(5), \llbracket (P_\mu U)^2 \rrbracket} \mathcal{Q}_{le\phi^3 D^2}^{(5)} + \text{EOM} \\ &= c_{le\phi^3 D^2}^{(1), \llbracket (P_\mu U)^2 \rrbracket} (D_\mu \phi^\dagger D^\mu \phi) (\bar{l}_p e_r \phi) + c_{le\phi^3 D^2}^{(5), \llbracket (P_\mu U)^2 \rrbracket} (\phi^\dagger D_\mu \phi) (\bar{l}_p e_r D^\mu \phi) \\ &\quad - \frac{2(3\lambda_{\mathcal{H}_2,1} + 2\lambda_{\mathcal{H}_2,2})\eta_{\mathcal{H}_2}Y_{\mathcal{H}_2}^{(e)}}{M_{\mathcal{H}_2}^2} ((2m^2\phi^\dagger\phi - 2\lambda_{\text{SM}}(\phi^\dagger\phi)^2 - Y_e^{(SM)\dagger}(\bar{e}_r l \phi^\dagger) \\ &\quad + Y_u^{(SM)\dagger}\epsilon_{jk}\bar{q}^k u \phi^\dagger - Y_d^{(SM)\dagger}(\bar{d}_p q_r \phi^\dagger)) (\bar{l}_p e_r \phi) + \text{H.c.}). \end{aligned} \quad (\text{A130})$$

The matching relations are

$$c_{le\phi^3 D^2}^{(1), \llbracket (P_\mu U)^2 \rrbracket} = \frac{1}{M_{\mathcal{H}_2}^2} ((6\lambda_{\mathcal{H}_2,1} + 4\lambda_{\mathcal{H}_2,2})\eta_{\mathcal{H}_2}Y_{\mathcal{H}_2}^{(e)}), \quad c_{le\phi^3 D^2}^{(5), \llbracket (P_\mu U)^2 \rrbracket} = \frac{1}{M_{\mathcal{H}_2}^2} ((6\lambda_{\mathcal{H}_2,1} + 4\lambda_{\mathcal{H}_2,2})\eta_{\mathcal{H}_2}Y_{\mathcal{H}_2}^{(e)}). \quad (\text{A131})$$

(3) Covariant operator: $U(G_{\mu\nu})^2$

$$\text{Tr}[U(G_{\mu\nu})^2] \supset c_{leW^2\phi}^{(1), \llbracket U(G_{\mu\nu})^2 \rrbracket} \mathcal{Q}_{leW^2\phi}^{(1)} + c_{leB^2\phi}^{(1), \llbracket U(G_{\mu\nu})^2 \rrbracket} \mathcal{Q}_{leB^2\phi}^{(1)} + c_{leWB\phi}^{(1), \llbracket U(G_{\mu\nu})^2 \rrbracket} \mathcal{Q}_{leWB\phi}^{(1)}. \quad (\text{A132})$$

The matching relations are

$$\begin{aligned} c_{leW^2\phi}^{(1), \llbracket U(G_{\mu\nu})^2 \rrbracket} &= \frac{3}{2M_{\mathcal{H}_2}^2} \eta_{\mathcal{H}_2} g^2 Y_{\mathcal{H}_2}^{(e)}, & c_{leB^2\phi}^{(1), \llbracket U(G_{\mu\nu})^2 \rrbracket} &= \frac{6}{M_{\mathcal{H}_2}^2} \eta_{\mathcal{H}_2} g^2 Y_{\mathcal{H}_2}^{(e)}, \\ c_{leWB\phi}^{(1), \llbracket U(G_{\mu\nu})^2 \rrbracket} &= \frac{4}{M_{\mathcal{H}_2}^2} \eta_{\mathcal{H}_2} g g' Y_{\mathcal{H}_2}^{(e)}. \end{aligned} \quad (\text{A133})$$

No dimension-six fermionic operators emerge from terms $\in \mathcal{L}_{-2}$ of Eq. (2.2).

Dimension-eight and dimension-six operators from terms $\in \mathcal{L}_0$

(1) Covariant operator: U^2

$$\begin{aligned} \text{Tr}[U^2] \supset & c_{l^2 e^2 \phi^2}^{(1), \llbracket U^2 \rrbracket} \mathcal{Q}_{l^2 e^2 \phi^2}^{(1)} + c_{lequ\phi^2}^{(1), \llbracket U^2 \rrbracket} \mathcal{Q}_{lequ\phi^2}^{(1)} + c_{leqd\phi^2}^{(3), \llbracket U^2 \rrbracket} \mathcal{Q}_{leqd\phi^2}^{(3)} + c_{le\phi^3 D^2}^{(5), \llbracket U^2 \rrbracket} \mathcal{Q}_{le\phi^3 D^2}^{(5)} \\ & + c_{le\phi^5}^{\llbracket U^2 \rrbracket} \mathcal{Q}_{le\phi^5} + c_{e\phi}^{\llbracket U^2 \rrbracket} \mathcal{O}_{e\phi}. \end{aligned} \quad (\text{A134})$$

The matching relations at dimension-eight are

$$\begin{aligned} c_{l^2 e^2 \phi^2}^{(1), \llbracket U^2 \rrbracket} &= \frac{1}{M_{\mathcal{H}_2}^4} (40|Y_{\mathcal{H}_2}^{(e)}|^2 \eta_{\mathcal{H}_2}^2 + 12|Y_{\mathcal{H}_2}^{(e)}|^2 \eta_{\mathcal{H}_2} (\lambda_{\mathcal{H}_2,1} + \lambda_{\mathcal{H}_2,2}) + (4\lambda_{\mathcal{H}_2,1} + \lambda_{\mathcal{H}_2,2}) |Y_{\mathcal{H}_2}^{(e)}|^2 \lambda_{\mathcal{H}_2}), \\ c_{lequ\phi^2}^{(1), \llbracket U^2 \rrbracket} &= \frac{1}{M_{\mathcal{H}_2}^4} (-6Y_{\mathcal{H}_2}^{(e)} Y_{\text{SM}}^{(u)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,1} - 6Y_{\mathcal{H}_2}^{(e)} Y_{\text{SM}}^{(u)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,2}), \\ c_{leqd\phi^2}^{(3), \llbracket U^2 \rrbracket} &= \frac{1}{M_{\mathcal{H}_2}^4} (6Y_{\mathcal{H}_2}^{(e)} Y_{\text{SM}}^{(d)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,1} + 6Y_{\mathcal{H}_2}^{(e)} Y_{\text{SM}}^{(d)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,2}), \\ c_{le\phi^3 D^2}^{(5), \llbracket U^2 \rrbracket} &= \frac{1}{M_{\mathcal{H}_2}^4} (6Y_{\mathcal{H}_2}^{(e)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,1}^3 + 18Y_{\mathcal{H}_2}^{(e)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,1}^2 \lambda_{\mathcal{H}_2,2} + 18Y_{\mathcal{H}_2}^{(e)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,2}^2 \\ &\quad + 3Y_{\mathcal{H}_2}^{(e)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2} + Y_{\mathcal{H}_2}^{(e)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,2} \lambda_{\mathcal{H}_2} + 18Y_{\mathcal{H}_2}^{(e)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,1} \lambda_{\text{SM}} + 18Y_{\mathcal{H}_2}^{(e)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,2} \lambda_{\text{SM}} + 6Y_{\mathcal{H}_2}^{(e)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,2}^3), \\ c_{le\phi^5}^{\llbracket U^2 \rrbracket} &= \frac{1}{M_{\mathcal{H}_2}^4} (-12Y_{\mathcal{H}_2}^{(e)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,1} - 12Y_{\mathcal{H}_2}^{(e)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,2}), \\ c_{e\phi}^{\llbracket U^2 \rrbracket} &= \frac{1}{M_{\mathcal{H}_2}^4} (-18m_H^2 Y_{\mathcal{H}_2}^{(e)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,1} - 18m_H^2 Y_{\mathcal{H}_2}^{(e)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,2}). \end{aligned} \quad (\text{A135})$$

$$\text{Tr}[U^2] \supset c_{e\phi}^{\llbracket U^2 \rrbracket} \mathcal{O}_{e\phi} = c_{e\phi}^{\llbracket U^2 \rrbracket} (\phi^\dagger \phi) (\bar{l}_p e_r \phi), \quad (\text{A136})$$

with the matching relation at dimension-six as

$$c_{e\phi}^{\llbracket U^2 \rrbracket} = -\frac{2}{M_{\mathcal{H}_2}^2} (6\lambda_{\mathcal{H}_2,1} \eta_{\mathcal{H}_2} Y_{\mathcal{H}_2}^{(e)} + 4\lambda_{\mathcal{H}_2,2} \eta_{\mathcal{H}_2} Y_{\mathcal{H}_2}^{(e)} - 2\lambda_{\mathcal{H}_2,3} \eta_{\mathcal{H}_2} Y_{\mathcal{H}_2}^{(e)}). \quad (\text{A137})$$

Dimension-eight and dimension-six operators from terms $\in \mathcal{L}_2$

(1) Covariant operator: U

$$\begin{aligned} \text{Tr}[U] \supset & c_{l^2 e^2 \phi^2}^{(1), \llbracket U \rrbracket} \mathcal{Q}_{l^2 e^2 \phi^2}^{(1)} + c_{l^2 e^2 \phi^2}^{(3), \llbracket U \rrbracket} \mathcal{Q}_{l^2 e^2 \phi^2}^{(3)} + c_{lequ\phi^2}^{(1), \llbracket U \rrbracket} \mathcal{Q}_{lequ\phi^2}^{(1)} + c_{leqd\phi^2}^{(3), \llbracket U \rrbracket} \mathcal{Q}_{leqd\phi^2}^{(3)} \\ & + c_{l^2 e^2 D^2}^{\llbracket U \rrbracket} \mathcal{Q}_{l^2 e^2 D^2} + c_{le\phi^5}^{\llbracket U \rrbracket} \mathcal{Q}_{le\phi^5} + c_{e\phi}^{\llbracket U \rrbracket} \mathcal{O}_{e\phi} + c_{le\phi^3 D^2}^{(5), \llbracket U \rrbracket} \mathcal{Q}_{le\phi^3 D^2}^{(5)}. \end{aligned} \quad (\text{A138})$$

The matching relations at dimension-eight are

$$\begin{aligned} c_{l^2 e^2 \phi^2}^{(1), \llbracket U \rrbracket} &= -\frac{1}{M_{\mathcal{H}_2}^6} (6|Y_{\mathcal{H}_2}^{(e)}|^2 \eta_{\mathcal{H}_2}^2 + Y_{\mathcal{H}_2}^{(e)} Y_{\text{SM}}^{(e)} \eta_{\mathcal{H}_2} (\lambda_{\mathcal{H}_2,1} - \lambda_{\mathcal{H}_2,2}) - 3\lambda_{\mathcal{H}_2} |Y_{\mathcal{H}_2}^{(e)}|^2 (\lambda_{\mathcal{H}_2,1} + \lambda_{\mathcal{H}_2,2}) + 3Y_{\mathcal{H}_2}^{(e)} Y_{\text{SM}}^{(e)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2} \\ &\quad + 3Y_{\mathcal{H}_2}^{(e)} Y_{\text{SM}}^{(e)} \eta_{\mathcal{H}_2} (2\lambda_{\mathcal{H}_2,1} + \lambda_{\mathcal{H}_2,2})), \\ c_{l^2 e^2 \phi^2}^{(3), \llbracket U \rrbracket} &= -\frac{1}{M_{\mathcal{H}_2}^6} (6Y_{\mathcal{H}_2}^{(e)2} \eta_{\mathcal{H}_2} + Y_{\mathcal{H}_2}^{(e)} Y_{\text{SM}}^{(e)} \eta_{\mathcal{H}_2} (\lambda_{\mathcal{H}_2,1} - \lambda_{\mathcal{H}_2,2})), \\ c_{lequ\phi^2}^{(1), \llbracket U \rrbracket} &= \frac{1}{M_{\mathcal{H}_2}^6} (-7Y_{\mathcal{H}_2}^{(e)} Y_{\text{SM}}^{(u)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,1} - 2Y_{\mathcal{H}_2}^{(e)} Y_{\text{SM}}^{(u)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,2} - 3Y_{\mathcal{H}_2}^{(e)} Y_{\text{SM}}^{(u)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2}), \end{aligned}$$

$$\begin{aligned}
c_{led\phi^2}^{(3),\llbracket U \rrbracket} &= \frac{1}{M_{\mathcal{H}_2}^6} (-Y_{\mathcal{H}_2}^{(e)} Y_{\text{SM}}^{(d)} \eta_{\mathcal{H}_2} (\lambda_{\mathcal{H}_2,1} + \lambda_{\mathcal{H}_2,2}) - 3Y_{\mathcal{H}_2}^{(e)} Y_{\text{SM}}^{(d)*} \eta_{\mathcal{H}_2} (2\lambda_{\mathcal{H}_2,1} + \lambda_{\mathcal{H}_2,2}) + 3Y_{\mathcal{H}_2}^{(e)} Y_{\text{SM}}^{(d)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2}), \\
c_{le^2 D^2}^{\llbracket U \rrbracket} &= \frac{3}{M_{\mathcal{H}_2}^6} \lambda_{\mathcal{H}_2} |Y_{\mathcal{H}_2}^{(e)}|^2, \quad c_{le\phi^3 D^2}^{(5),\llbracket U \rrbracket} = \frac{1}{M_{\mathcal{H}_2}^6} (-6Y_{\mathcal{H}_2}^{(e)} \eta_{\mathcal{H}_2}^{(e)} \lambda_{\mathcal{H}_2}), \\
c_{le\phi^5}^{\llbracket U \rrbracket} &= \frac{1}{M_{\mathcal{H}_2}^6} (-3Y_{\mathcal{H}_2}^{(e)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2}^2 - 6Y_{\mathcal{H}_2}^{(e)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,2} - 3Y_{\mathcal{H}_2}^{(e)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,2}^2 \\
&\quad - 3Y_{\mathcal{H}_2}^{(e)} \eta_{\mathcal{H}_2} (2\lambda_{\mathcal{H}_2,1} + \lambda_{\mathcal{H}_2,2}) \lambda_{\text{SM}} - 2Y_{\mathcal{H}_2}^{(e)} \eta_{\mathcal{H}_2} (\lambda_{\mathcal{H}_2,1} + \lambda_{\mathcal{H}_2,2}) \lambda_{\text{SM}} \\
&\quad + 6Y_{\mathcal{H}_2}^{(e)} \lambda_{\mathcal{H}_2} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,1} + 6Y_{\mathcal{H}_2}^{(e)} \lambda_{\mathcal{H}_2} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2,2} + 9Y_{\mathcal{H}_2}^{(e)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2} \lambda_{\text{SM}}), \\
c_{e\phi}^{\llbracket U \rrbracket} &= \frac{1}{M_{\mathcal{H}_2}^6} (m_H^2 Y_{\mathcal{H}_2}^{(e)} \eta_{\mathcal{H}_2} (8\lambda_{\mathcal{H}_2,1} + \lambda_{\mathcal{H}_2,2}) - 9m_H^2 Y_{\mathcal{H}_2}^{(e)} \eta_{\mathcal{H}_2} \lambda_{\mathcal{H}_2}). \tag{A139}
\end{aligned}$$

$$\text{Tr}[U] \supset c_{e\phi}^{\llbracket U \rrbracket} \mathcal{Q}_{e\phi} + c_{le}^{\llbracket U \rrbracket} \mathcal{Q}_{le} + c_{ledq}^{\llbracket U \rrbracket} \mathcal{Q}_{ledq} + c_{lequ}^{(1),\llbracket U \rrbracket} \mathcal{Q}_{lequ}^{(1)} + \eta_{\mathcal{H}_2} m_H^2 Y_{\mathcal{H}_2}^{(e)} \bar{l}_p e_r \phi + \text{h.c.}, \tag{A140}$$

with the matching relations at dimension-six as

$$\begin{aligned}
c_{e\phi}^{\llbracket U \rrbracket} &= \frac{1}{M_{\mathcal{H}_2}^4} (-3\eta_{\mathcal{H}_2} \lambda_{\text{SM}} Y_{\mathcal{H}_2}^{(e)} + 3\eta_{\mathcal{H}_2} Y_{\mathcal{H}_2}^{(e)} (\lambda_{\mathcal{H}_2,1} + \lambda_{\mathcal{H}_2,2})), \\
c_{le}^{\llbracket U \rrbracket} &= \frac{1}{M_{\mathcal{H}_2}^4} \left(-3\eta_{\mathcal{H}_2} Y_{\text{SM}}^{(e)} Y_{\mathcal{H}_2}^{(e)} - \frac{3}{2} \lambda_{\mathcal{H}_2} Y_{\mathcal{H}_2}^{(e)} Y_{\mathcal{H}_2}^{(e)*} \right), \\
c_{ledq}^{\llbracket U \rrbracket} &= -\frac{1}{M_{\mathcal{H}_2}^4} (-3\eta_{\mathcal{H}_2} Y_{\text{SM}}^{(d)*} Y_{\mathcal{H}_2}^{(e)}), \quad c_{lequ}^{(1),\llbracket U \rrbracket} = \frac{1}{M_{\mathcal{H}_2}^4} (-3\eta_{\mathcal{H}_2} Y_{\text{SM}}^{(u)} Y_{\mathcal{H}_2}^{(e)}). \tag{A141}
\end{aligned}$$

We summarize all the operators and their associated WCs in Table VI.

APPENDIX B: MATCHING RELATIONS FOR OBLIQUE PARAMETERS

In this appendix, we present the matching expressions for the SMEFT Wilson coefficients that contribute to the oblique parameters ΔS and ΔT , as defined in Eq. (4.3) in terms of the model parameters for the two cases we consider: the complex scalar triplet model and the 2HDM. These arise from integrating out the respective heavy scalar fields in the two models. The matching relations between the parameters of these models and the SMEFT Wilson coefficients for various operator structures appearing in Eq. (2.2) are provided in Appendixes A 1 and A 2, respectively.

For the complex scalar triplet model, the SMEFT contributions to the S parameter, including both dimension-six and dimension-eight operators, are given as

$$\Delta S^\Delta = C_S^{\text{D6},\Delta} + C_S^{\text{D8},\Delta}, \tag{B1}$$

where the dimension-six contribution $C_S^{\text{D6},\Delta}$, that is generated at one-loop only, reads as

$$\begin{aligned}
\left. \frac{C_S^{\text{D6},\Delta}}{\Lambda^2} \right|_{\Lambda=M_\Delta} &= 0.001 \left[-0.2 \times \left(\frac{12}{24\pi^2 M_\Delta^2} (2\lambda_1 + \lambda_4) \right) + 1.48 \times 2 \times \left(\frac{-3}{24\pi^2 M_\Delta^2} (2\lambda_1 + \lambda_4) \right) \right. \\
&\quad \left. + 1.96 \left(\frac{2\mu_\Delta^2}{M_\Delta^4} + 2 \frac{6}{16\pi^2 M_\Delta^2} (2\lambda_2 + 6\lambda_3) + \frac{2\mu_\Delta^2}{M_\Delta^4} + \frac{1}{48\pi^2 M_\Delta^2} \lambda_4^2 \right) + 1.4 \times \frac{4}{12\pi^2 M_\Delta^2} (2\lambda_1 + \lambda_4) \right] \tag{B2}
\end{aligned}$$

and $C_S^{\text{D8},\Delta} = (C_S^{\text{D6},\Delta})^2 + C_{WB\phi^4}^{(1),\Delta}$. Here, $C_{WB\phi^4}^{(1),\Delta}$, the dimension-eight operator coefficient, expressed as a function of the complex triplet model parameters, is given by

$$\begin{aligned} \left. \frac{C_{WB\phi^4}^{(1),\Delta}}{\Lambda^4} \right|_{\Lambda=M_\Delta} &= \frac{1}{480\pi^2 M_\Delta^4} g g' \left(\frac{3}{2} \lambda_1^2 - \frac{3}{4} \lambda_1 \lambda_4 - \frac{3}{4} \lambda_4^2 \right) + \frac{1}{1920\pi^2 M_\Delta^4} 2g g' \lambda_4 (2\lambda_1 + \lambda_4) \\ &+ \frac{1}{16\pi^2 M_\Delta^4} \left(1 - \ln \left(\frac{M_\Delta^2}{\mu^2} \right) \right) (-2\lambda_4 \lambda_{SM} g g') \end{aligned} \quad (B3)$$

Similarly, the contribution to the T parameter reads

$$\Delta T^\Delta = C_T^{D6,\Delta} + C_T^{D8,\Delta}, \quad (B4)$$

with the dimension-six contribution up to one-loop level

$$\begin{aligned} \left. \frac{C_T^{D6,\Delta}}{\Lambda^2} \right|_{\Lambda=M_\Delta} &= 8 \left(-\frac{2\mu_\Delta^2}{M_\Delta^4} + \frac{1}{48\pi^2} \frac{\lambda_4^2}{M_\Delta^2} \right) - 0.078 \times \left(\frac{-6}{16\pi^2 M_\Delta^2} (4\lambda_1 + 2\lambda_4) \right) \\ &- 0.001 \left[-3.97 \times \left(\frac{2\mu_\Delta^2}{M_\Delta^4} + 2 \frac{6\mu_\Delta^2}{16\pi^2 M_\Delta^4} (2\lambda_2 + 6\lambda_3) + \frac{2\mu_\Delta^2}{M_\Delta^4} + \frac{1}{48\pi^2} \frac{\lambda_4^2}{M_\Delta^2} \right) + 0.12 \times \left(\frac{-6}{12\pi^2 M_\Delta^2} (2\lambda_1 + \lambda_4) \right) \right] \end{aligned} \quad (B5)$$

and $C_T^{D8,\Delta} = (C_T^{D6,\Delta})^2 + C_{\phi^6}^{(1),\Delta}$. Here, $C_{\phi^6}^{(1),\Delta}$, the dimension-eight operator term coefficient, expressed as a function of the complex triplet model parameters is given by

$$\begin{aligned} \left. \frac{C_{\phi^6}^{(1),\Delta}}{\Lambda^4} \right|_{\Lambda=M_\Delta} &= -\frac{1}{384\pi^2 M_\Delta^4} \left(\frac{3\lambda_1^3}{2} + \frac{9\lambda_1^2 \lambda_4}{4} + \frac{11\lambda_1 \lambda_4^2}{8} + \frac{5\lambda_4^3}{16} \right) + \frac{1}{1920\pi^2 M_\Delta^4} (-4\lambda_{SM}) \left(6\lambda_1^2 + 6\lambda_1 \lambda_4 + \frac{5}{2} \lambda_4^2 \right) \\ &+ \frac{1}{16\pi^2} \left(1 - \ln \left(\frac{M_\Delta^2}{\mu^2} \right) \right) \left(-\frac{\mu_\Delta^2}{M_\Delta^6} \right) \left(6\lambda_1 \lambda_2 + 9\lambda_1 \lambda_3 + 3\lambda_2 \lambda_4 + \frac{9}{4} \lambda_3 \lambda_4 \right) \end{aligned} \quad (B6)$$

As evident from the above expressions, both ΔS^Δ and ΔT^Δ depend on quadratic structures such as $(2\lambda_1 + \lambda_4)/M_\Delta^2$ together with λ_4^2/M_Δ^2 and μ_Δ^2/M_Δ^4 . Consequently, the resulting constraints in the (λ_1, λ_4) plane take the form $a(2\lambda_1 + \lambda_4) + b\lambda_4^2 + c = 0$, which corresponds to parabolic exclusion contours, as seen in Fig. 3.

For the 2HDM, we express the relevant Wilson coefficients for oblique parameters in terms of the model parameters in the following. The SMEFT contributions to the S parameter is given by

$$\Delta S^{\mathcal{H}_2} = C_S^{D6,\mathcal{H}_2} + C_S^{D8,\mathcal{H}_2}, \quad (B7)$$

where the dimension-six contribution is

$$\begin{aligned} \left. \frac{C_S^{D6,\mathcal{H}_2}}{\Lambda^2} \right|_{\Lambda=M_{\mathcal{H}_2}} &= 0.001 \left[0.2 \times 0.133 \left(\frac{8}{24\pi^2 M_{\mathcal{H}_2}^2} (2\lambda_{\mathcal{H}_{2,1}} + \lambda_{\mathcal{H}_{2,2}}) \right) - 1.4 \times 0.133 \times \left(\frac{2}{12\pi^2 M_{\mathcal{H}_2}^2} (2\lambda_{\mathcal{H}_{2,1}} + \lambda_{\mathcal{H}_{2,2}}) \right) \right. \\ &\left. + 1.96 \frac{1}{12\pi^2 M_{\mathcal{H}_2}^2} (4\lambda_{\mathcal{H}_{2,1}}^2 + 4\lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,2} + \lambda_{\mathcal{H}_2,2}^2) \ln \left(\frac{M_{\mathcal{H}_2}^2}{\mu^2} \right) + 1.4 \times 0.442 \frac{1}{12\pi^2 M_{\mathcal{H}_2}^2} (2\lambda_{\mathcal{H}_{2,1}} + \lambda_{\mathcal{H}_{2,2}}) \right] \end{aligned} \quad (B8)$$

and $C_S^{D8,\mathcal{H}_2} = (C_S^{D6,\mathcal{H}_2})^2 + C_{WB\phi^4}^{(1),\mathcal{H}_2}$. Here, $C_{WB\phi^4}^{(1),\mathcal{H}_2}$, the dimension-eight operator coefficient expressed as a function of the heavy doublet model parameters, is given by

$$\left. \frac{C_{WB\phi^4}^{(1),\mathcal{H}_2}}{\Lambda^4} \right|_{\Lambda=M_{\mathcal{H}_2}} = -\frac{1}{192\pi^2 M_{\mathcal{H}_2}^4} (8\eta_H \eta_{\mathcal{H}_2} g g') + \frac{1}{480\pi^2 M_{\mathcal{H}_2}^4} (-8g g' \lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,2}) + \frac{1}{1920\pi^2 M_{\mathcal{H}_2}^4} (-8g g' \lambda_{\mathcal{H}_2,1} \lambda_{\mathcal{H}_2,2})$$

For the T parameter, we similarly write

$$\Delta T^{\mathcal{H}_2} = C_T^{\text{D6},\mathcal{H}_2} + C_T^{\text{D8},\mathcal{H}_2}, \quad (\text{B9})$$

where the dimension-six part is

$$\begin{aligned} \left. \frac{C_T^{\text{D6},\mathcal{H}_2}}{\Lambda^2} \right|_{\Lambda=M_{\mathcal{H}_2}} &= 8 \left(\frac{\lambda_{\mathcal{H}_2,2}^2}{192\pi^2 M_{\mathcal{H}_2}^2} - \frac{\lambda_{\mathcal{H}_2,3}^2}{48\pi^2 M_{\mathcal{H}_2}^2} \right) - 0.001 \left[0.13 \times \left(\frac{2 \times (-0.133)}{24\pi^2 M_{\mathcal{H}_2}^2} (2\lambda_{\mathcal{H}_2,1} + \lambda_{\mathcal{H}_2,2}) \right. \right. \\ &\quad + 3.97 \frac{1}{32\pi^2 M_{\mathcal{H}_2}^2} (4\lambda_{\mathcal{H}_2,1}^2 + 4\lambda_{\mathcal{H}_2,1}\lambda_{\mathcal{H}_2,2} + \lambda_{\mathcal{H}_2,2}^2) \ln \left(\frac{M_{\mathcal{H}_2}^2}{\mu^2} \right) \\ &\quad - \frac{12}{16\pi^2 M_{\mathcal{H}_2}^2} \eta_H \eta_{\mathcal{H}_2} + \frac{1}{192\pi^2 M_{\mathcal{H}_2}^2} \lambda_{\mathcal{H}_2,2}^2 - \frac{1}{48\pi^2 M_{\mathcal{H}_2}^2} \lambda_{\mathcal{H}_2,3}^2 \Big) \\ &\quad \left. \left. + 0.12 \times 2 \times 0.442 \left(\frac{1}{24\pi^2 M_{\mathcal{H}_2}^2} (2\lambda_{\mathcal{H}_2,1} + \lambda_{\mathcal{H}_2,2}) \right) \right] \right] \quad (\text{B10}) \end{aligned}$$

and $C_T^{\text{D8},\mathcal{H}_2} = (C_T^{\text{D6},\mathcal{H}_2})^2 + C_{\phi^6}^{(1),\mathcal{H}_2}$. Here, $C_{\phi^6}^{(1),\mathcal{H}_2}$, the dimension-eight operator coefficient expressed as a function of the heavy doublet model parameters, is given by

$$\left. \frac{C_{\phi^6}^{(1),\mathcal{H}_2}}{\Lambda^4} \right|_{\Lambda=M_{\mathcal{H}_2}} = -\frac{1}{192\pi^2} \frac{2\eta_H \eta_{\mathcal{H}_2}}{M_{\mathcal{H}_2}^4} (6\lambda_{\mathcal{H}_2,1} + 4\lambda_{\mathcal{H}_2,2} + 2\lambda_{\mathcal{H}_2,3}) \quad (\text{B11})$$

For the 2HDM, the S parameter predominantly constrains $\lambda_{\mathcal{H}_{2,2}}$ (apart from $\lambda_{\mathcal{H}_{2,1}}$) through linear and quadratic terms. In contrast, the T parameter involves terms $\propto (\lambda_{\mathcal{H}_{2,2}}^2 - c\lambda_{\mathcal{H}_{2,3}}^2)$ dependence, which generates the characteristic hyperbolic contours shown in Fig. 4.

APPENDIX C: PARAMETRIZING DIMENSION-8 TERMS AFTER REMOVING REDUNDANCIES

We provide effective operators and related WCs on a Green's basis, which contains redundant structures. These operators can be recast on a nonredundant operator basis following the conventions depicted in [36] after employing the equation of motions, Fierz and Bianchi identities, and related information discussed in [98,102]. The operator relations among the Green and Murphy bases are given in Tables IX, X that can be used to find the nonredundant dimension-eight effective operators.

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- [1] S. Weinberg, Phenomenological Lagrangians, *Physica A (Amsterdam)* **96**, 327 (1979).
 - [2] H. Georgi, On-shell effective field theory, *Nucl. Phys. B* **361**, 339 (1991).
 - [3] W. Buchmüller and D. Wyler, Effective Lagrangian analysis of new interactions and flavour conservation, *Nucl. Phys. B* **268**, 621 (1986).
 - [4] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, Dimension-six terms in the standard model Lagrangian, *J. High Energy Phys.* **10** (2010) 085.
 - [5] A. Dedes, M. Paraskevas, J. Rosiek, K. Suxho, and L. Trifyllis, SmeftFR—Feynman rules generator for the standard model effective field theory, *Comput. Phys. Commun.* **247**, 106931 (2020).
 - [6] G. F. Giudice, C. Grojean, A. Pomarol, and R. Rattazzi, The strongly-interacting light Higgs, *J. High Energy Phys.* **06** (2007) 045.
 - [7] C. Englert, R. Kogler, H. Schulz, and M. Spannowsky, Higgs coupling measurements at the LHC, *Eur. Phys. J. C* **76**, 393 (2016).
 - [8] J. Ellis, C. W. Murphy, V. Sanz, and T. You, Updated global SMEFT fit to Higgs, diboson and electroweak data, *J. High Energy Phys.* **06** (2018) 146.
 - [9] S. Banerjee, R. S. Gupta, J. Y. Reiness, S. Seth, and M. Spannowsky, Towards the ultimate differential SMEFT analysis, *J. High Energy Phys.* **09** (2020) 170.
 - [10] S. Banerjee, C. Englert, R. S. Gupta, and M. Spannowsky, Probing electroweak precision physics via boosted Higgs-strahlung at the LHC, *Phys. Rev. D* **98**, 095012 (2018).
 - [11] A. Pomarol and F. Riva, Towards the ultimate SM fit to close in on Higgs physics, *J. High Energy Phys.* **01** (2014) 151.
 - [12] A. Butter, O. J. P. Éboli, J. Gonzalez-Fraile, M. C. Gonzalez-Garcia, T. Plehn, and M. Rauch, The

- gauge-Higgs legacy of the LHC run I, *J. High Energy Phys.* **07** (2016) 152.
- [13] L. Berthier and M. Trott, Consistent constraints on the standard model effective field theory, *J. High Energy Phys.* **02** (2016) 069.
- [14] L. Berthier and M. Trott, Towards consistent electroweak precision data constraints in the SMEFT, *J. High Energy Phys.* **05** (2015) 024.
- [15] J. de Blas *et al.*, Higgs boson studies at future particle colliders, *J. High Energy Phys.* **01** (2020) 139.
- [16] A. Buckley, C. Englert, J. Ferrando, D. J. Miller, L. Moore, M. Russell, and C. D. White, Constraining top quark effective theory in the LHC Run II era, *J. High Energy Phys.* **04** (2016) 015.
- [17] I. Brivio, S. Bruggisser, F. Maltoni, R. Moutafis, T. Plehn, E. Vryonidou, S. Westhoff, and C. Zhang, O new physics, where art thou? A global search in the top sector, *J. High Energy Phys.* **02** (2020) 131.
- [18] J. Ellis, M. Madigan, K. Mimasu, V. Sanz, and T. You, Top, Higgs, diboson and electroweak fit to the standard model effective field theory, *J. High Energy Phys.* **04** (2021) 279.
- [19] G. Durieux, A. Irlles, V. Miralles, A. Peñuelas, R. Pöschl, M. Perelló, and M. Vos, The electro-weak couplings of the top and bottom quarks—Global fit and future prospects, *J. High Energy Phys.* **12** (2019) 98; **01** (2021) 195(E).
- [20] T. Biswas, A. Datta, and B. Mukhopadhyaya, Following the trail of new physics via the vector boson fusion Higgs boson signal at the large hadron collider, *Phys. Rev. D* **105**, 055028 (2022).
- [21] T. Biswas and A. Datta, Exploring Higgs-photon production at the LHC, *J. High Energy Phys.* **05** (2023) 104.
- [22] J. J. Ethier, G. Magni, F. Maltoni, L. Mantani, E. R. Nocera, J. Rojo, E. Slade, E. Vryonidou, and C. Zhang (SMEFT Collaboration), Combined SMEFT interpretation of Higgs, diboson, and top quark data from the LHC, *J. High Energy Phys.* **11** (2021) 089.
- [23] C. Hays, A. Martin, V. Sanz, and J. Setford, On the impact of dimension-eight SMEFT operators on Higgs measurements, *J. High Energy Phys.* **02** (2019) 123.
- [24] C. Degrande, A basis of dimension-eight operators for anomalous neutral triple gauge boson interactions, *J. High Energy Phys.* **02** (2014) 101.
- [25] S. Dawson, S. Homiller, and M. Sullivan, Impact of dimension-eight SMEFT contributions: A case study, *Phys. Rev. D* **104**, 115013 (2021).
- [26] A. Dedes, J. Rosiek, M. Ryczkowski, K. Suxho, and L. Trifyllis, SmeftFR v3—Feynman rules generator for the standard model effective field theory, *Comput. Phys. Commun.* **294**, 108943 (2024).
- [27] S. D. Bakshi, M. Chala, A. Díaz-Carmona, Z. Ren, and F. Vilches, Renormalization of the SMEFT to dimension eight: Fermionic interactions I, *J. High Energy Phys.* **12** (2025) 214.
- [28] S. Das Bakshi, M. Chala, A. Díaz-Carmona, and G. Guedes, Towards the renormalisation of the standard model effective field theory to dimension eight: Bosonic interactions II, *Eur. Phys. J. Plus* **137**, 973 (2022).
- [29] H.-L. Li, Y.-H. Ni, M.-L. Xiao, and J.-H. Yu, Complete UV resonances of the dimension-8 SMEFT operators, *J. High Energy Phys.* **05** (2024) 238.
- [30] C. Degrande and H.-L. Li, Impact of dimension-8 SMEFT operators on diboson productions, *J. High Energy Phys.* **06** (2023) 149.
- [31] Q. Chen, K. Mimasu, T. A. Wu, G.-D. Zhang, and S.-Y. Zhou, Capping the positivity cone: Dimension-8 Higgs operators in the SMEFT, *J. High Energy Phys.* **03** (2024) 180.
- [32] J. Ellis, K. Mimasu, and F. Zampedri, Dimension-8 SMEFT analysis of minimal scalar field extensions of the Standard Model, *J. High Energy Phys.* **10** (2023) 051.
- [33] S. Dawson, M. Forslund, and M. Schnubel, SMEFT matching to Z' models at dimension eight, *Phys. Rev. D* **110**, 015002 (2024).
- [34] N. Adhikary, J. Das, and D. Dey, Two-loop dimension six effective action: Integrating out heavy scalar, *Nucl. Phys. B* **1022**, 117273 (2026).
- [35] H.-L. Li, Z. Ren, J. Shu, M.-L. Xiao, J.-H. Yu, and Y.-H. Zheng, Complete set of dimension-8 operators in the standard model effective field theory, *Phys. Rev. D* **104**, 015026 (2021).
- [36] C. W. Murphy, Dimension-8 operators in the standard model effective field theory, *J. High Energy Phys.* **10** (2020) 174.
- [37] I. Brivio and M. Trott, The standard model as an effective field theory, *Phys. Rep.* **793**, 1 (2019).
- [38] G. Isidori, F. Wilsch, and D. Wyler, The standard model effective field theory at work, *Rev. Mod. Phys.* **96**, 015006 (2024).
- [39] Anisha, S. Das Bakshi, S. Banerjee, A. Biekötter, J. Chakraborty, S. Kumar Patra, and M. Spannowsky, Effective limits on single scalar extensions in the light of recent LHC data, *Phys. Rev. D* **107**, 055028 (2023).
- [40] M. Gorbahn, J. M. No, and V. Sanz, Benchmarks for Higgs effective theory: Extended Higgs sectors, *J. High Energy Phys.* **10** (2015) 036.
- [41] F. del Aguila, Z. Kunszt, and J. Santiago, One-loop effective Lagrangians after matching, *Eur. Phys. J. C* **76**, 244 (2016).
- [42] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher, and J. P. Silva, Theory and phenomenology of two-Higgs-doublet models, *Phys. Rep.* **516**, 1 (2012).
- [43] J. F. Gunion, H. E. Haber, G. L. Kane, and S. Dawson, *The Higgs Hunter's Guide* (CRC Press, Boca Raton, 2000), Vol. 80.
- [44] F. del Aguila, J. A. Aguilar-Saavedra, J. de Blas, and M. Perez-Victoria, Electroweak constraints on see-saw messengers and their implications for LHC, in *Proceedings of the 43rd Rencontres de Moriond on Electroweak Interactions and Unified Theories* (2008), pp. 45–52, [arXiv:0806.1023](https://arxiv.org/abs/0806.1023).
- [45] A. G. Akeroyd and M. Aoki, Single and pair production of doubly charged Higgs bosons at hadron colliders, *Phys. Rev. D* **72**, 035011 (2005).
- [46] A. G. Akeroyd and C.-W. Chiang, Doubly charged Higgs bosons and three-lepton signatures in the Higgs triplet model, *Phys. Rev. D* **80**, 113010 (2009).

- [47] Z.-L. Han, R. Ding, and Y. Liao, LHC phenomenology of the type II seesaw mechanism: Observability of neutral scalars in the nondegenerate case, *Phys. Rev. D* **92**, 033014 (2015).
- [48] A. G. Akeroyd, M. Aoki, and H. Sugiyama, Lepton flavour violating decays $\tau \rightarrow \bar{l}ll$ and $\mu \rightarrow e\gamma$ in the Higgs triplet model, *Phys. Rev. D* **79**, 113010 (2009).
- [49] A. Dey, J. Lahiri, and B. Mukhopadhyaya, Extended scalar sectors, effective operators and observed data, *J. High Energy Phys.* **11** (2018) 127.
- [50] J. Cao, P. Wan, L. Wu, and J. M. Yang, Lepton-specific two-Higgs doublet model: Experimental constraints and implication on Higgs phenomenology, *Phys. Rev. D* **80**, 071701 (2009).
- [51] G. Bhattacharyya and D. Das, Scalar sector of two-Higgs-doublet models: A minireview, *Pramana* **87**, 40 (2016).
- [52] G. Bambhaniya, J. Chakraborty, S. Goswami, and P. Konar, Generation of neutrino mass from new physics at TeV scale and multilepton signatures at the LHC, *Phys. Rev. D* **88**, 075006 (2013).
- [53] Y. Cai, T. Han, T. Li, and R. Ruiz, Lepton number violation: Seesaw models and their collider tests, *Front. Phys.* **6**, 40 (2018).
- [54] P. S. Bhupal Dev and Y. Zhang, Displaced vertex signatures of doubly charged scalars in the type-II seesaw and its left-right extensions, *J. High Energy Phys.* **10** (2018) 199.
- [55] B. Fuks, M. Nemeš, and R. Ruiz, Doubly charged Higgs boson production at hadron colliders, *Phys. Rev. D* **101**, 075022 (2020).
- [56] S. Antusch, O. Fischer, A. Hammad, and C. Scherb, Low scale type II seesaw: Present constraints and prospects for displaced vertex searches, *J. High Energy Phys.* **02** (2019) 157.
- [57] F. del Águila and M. Chala, LHC bounds on lepton number violation mediated by doubly and singly-charged scalars, *J. High Energy Phys.* **03** (2014) 027.
- [58] A. Tumasyan *et al.* (CMS Collaboration), Prospects for a Search for Doubly Charged Higgs Bosons at the HL-LHC, <https://cds.cern.ch/record/2808604>.
- [59] B. Henning, X. Lu, and H. Murayama, How to use the standard model effective field theory, *J. High Energy Phys.* **01** (2016) 023.
- [60] G. von Gersdorff and K. Santos, New covariant Feynman rules for effective field theories, *J. High Energy Phys.* **04** (2023) 025.
- [61] J. Fuentes-Martín, M. König, J. Pagès, A. E. Thomsen, and F. Wilsch, A proof of concept for matchete: An automated tool for matching effective theories, *Eur. Phys. J. C* **83**, 662 (2023).
- [62] S. Das Bakshi, J. Chakraborty, and S. K. Patra, CoDEX: Wilson coefficient calculator connecting SMEFT to UV theory, *Eur. Phys. J. C* **79**, 21 (2019).
- [63] T. Cohen, X. Lu, and Z. Zhang, STREAMlining EFT matching, *SciPost Phys.* **10**, 098 (2021).
- [64] A. Carmona, A. Lazopoulos, P. Olgoso, and J. Santiago, Matchmakereft: Automated tree-level and one-loop matching, *SciPost Phys.* **12**, 198 (2022).
- [65] J. C. Criado, MatchingTools: A Python library for symbolic effective field theory calculations, *Comput. Phys. Commun.* **227**, 42 (2018).
- [66] A. Celis, J. Fuentes-Martín, A. Vicente, and J. Virto, DsixTools: The standard model effective field theory toolkit, *Eur. Phys. J. C* **77**, 405 (2017).
- [67] U. Haisch, M. Ruhdorfer, E. Salvioni, E. Venturini, and A. Weiler, Singlet night in Feynman-ville: One-loop matching of a real scalar, *J. High Energy Phys.* **04** (2020) 164; **07** (2020) 066(E).
- [68] A. Drozd, J. Ellis, J. Quevillon, and T. You, The universal one-loop effective action, *J. High Energy Phys.* **03** (2016) 180.
- [69] J. Fuentes-Martín, J. Portoles, and P. Ruiz-Femenia, Integrating out heavy particles with functional methods: A simplified framework, *J. High Energy Phys.* **09** (2016) 156.
- [70] B. Henning, X. Lu, and H. Murayama, One-loop matching and running with covariant derivative expansion, *J. High Energy Phys.* **01** (2018) 123.
- [71] U. Banerjee, J. Chakraborty, S. U. Rahaman, and K. Ramkumar, One-loop effective action up to any mass-dimension for non-degenerate scalars and fermions including light-heavy mixing, *Eur. Phys. J. Plus* **139**, 169 (2024).
- [72] M. Jiang, N. Craig, Y.-Y. Li, and D. Sutherland, Complete one-loop matching for a singlet scalar in the Standard Model EFT, *J. High Energy Phys.* **02** (2019) 031.
- [73] V. Gherardi, D. Marzocca, and E. Venturini, Matching scalar leptoquarks to the SMEFT at one loop, *J. High Energy Phys.* **07** (2020) 225; **01** (2021) 006(E).
- [74] Y. Du, X.-X. Li, and J.-H. Yu, Neutrino seesaw models at one-loop matching: Discrimination by effective operators, *J. High Energy Phys.* **09** (2022) 207.
- [75] U. Banerjee, J. Chakraborty, S. U. Rahaman, and K. Ramkumar, One-loop effective action up to dimension eight: Integrating out heavy scalar(s), *Eur. Phys. J. Plus* **139**, 159 (2024).
- [76] J. Chakraborty, S. U. Rahaman, and K. Ramkumar, One-loop effective action up to dimension eight: Integrating out heavy fermion(s), *Nucl. Phys. B* **1000**, 116488 (2024).
- [77] N. D. Barrie, C. Han, and H. Murayama, Type II Seesaw leptogenesis, *J. High Energy Phys.* **05** (2022) 160.
- [78] M. Chala, A. Díaz-Carmona, and G. Guedes, A Green's basis for the bosonic SMEFT to dimension 8, *J. High Energy Phys.* **05** (2022) 138.
- [79] S. Das Bakshi, J. Chakraborty, and S. K. Patra, CoDEX: Wilson coefficient calculator connecting SMEFT to UV theory, *Eur. Phys. J. C* **79**, 21 (2019).
- [80] T. Li, Type II Seesaw and tau lepton at the HL-LHC, HE-LHC and FCC-hh, *J. High Energy Phys.* **09** (2018) 079.
- [81] P. D. Bolton and S. T. Petcov, Measurements of $\mu \rightarrow 3e$ decay with polarised muons as a probe of new physics, *Phys. Lett. B* **833**, 137296 (2022).
- [82] R. Primulando, J. Julio, and P. Uttayarat, Scalar phenomenology in type-II seesaw model, *J. High Energy Phys.* **08** (2019) 024.
- [83] J. D. Wells and Z. Zhang, Effective theories of universal theories, *J. High Energy Phys.* **01** (2016) 123.

- [84] J. D. Wells and Z. Zhang, Renormalization group evolution of the universal theories EFT, *J. High Energy Phys.* **06** (2016) 122.
- [85] M. E. Peskin and T. Takeuchi, Estimation of oblique electroweak corrections, *Phys. Rev. D* **46**, 381 (1992).
- [86] M. E. Peskin and T. Takeuchi, A new constraint on a strongly interacting Higgs sector, *Phys. Rev. Lett.* **65**, 964 (1990).
- [87] E. Bagnaschi, J. Ellis, M. Madigan, K. Mimasu, V. Sanz, and T. You, SMEFT analysis of m_W , *J. High Energy Phys.* **08** (2022) 308.
- [88] P. Asadi, C. Cesarotti, K. Fraser, S. Homiller, and A. Parikh, Oblique lessons from the W mass measurement at CDF II, *Phys. Rev. D* **108**, 055026 (2023).
- [89] T. Corbett, A. Helset, A. Martin, and M. Trott, EWPD in the SMEFT to dimension eight, *J. High Energy Phys.* **06** (2021) 076.
- [90] A. Belvedere, C. Englert, R. Kogler, and M. Spannowsky, Dispelling the $\sqrt{\mathcal{L}}$ myth for the High-Luminosity LHC, *Eur. Phys. J. C* **84**, 715 (2024).
- [91] A. Banfi, A. Martin, and V. Sanz, Probing top-partners in Higgs + jets, *J. High Energy Phys.* **08** (2014) 053.
- [92] T. Corbett, J. Desai, O. J. P. Éboli, M. C. Gonzalez-Garcia, M. Martinez, and P. Reimitz, Impact of dimension-eight SMEFT operators in the electroweak precision observables and triple gauge couplings analysis in universal SMEFT, *Phys. Rev. D* **107**, 115013 (2023).
- [93] S. Hamoudou, J. Kumar, and D. London, Dimension-8 SMEFT matching conditions for the low-energy effective field theory, *J. High Energy Phys.* **03** (2023) 157.
- [94] S. Dawson, D. Fontes, S. Homiller, and M. Sullivan, Role of dimension-eight operators in an EFT for the 2HDM, *Phys. Rev. D* **106**, 055012 (2022).
- [95] S. Das Bakshi, S. Dawson, D. Fontes, and S. Homiller, Relevance of one-loop SMEFT matching in the 2HDM, *Phys. Rev. D* **109**, 075022 (2024).
- [96] C.-Y. Chen, S. Dawson, and C. Zhang, Electroweak effective operators and Higgs physics, *Phys. Rev. D* **89**, 015016 (2014).
- [97] M. Baak, J. Cúth, J. Haller, A. Hoecker, R. Kogler, K. Mönig, M. Schott, and J. Stelzer (Gfitter Group Collaboration), The global electroweak fit at NNLO and prospects for the LHC and ILC, *Eur. Phys. J. C* **74**, 3046 (2014).
- [98] T. Corbett, J. Desai, O. J. P. Éboli, and M. C. Gonzalez-Garcia, Dimension-eight operator basis for universal standard model effective field theory, *Phys. Rev. D* **110**, 033003 (2024).
- [99] P. Basler, M. Mühlleitner, and J. Müller, Electroweak phase transition in non-minimal Higgs sectors, *J. High Energy Phys.* **05** (2020) 016.
- [100] Anisha, L. Biermann, C. Englert, and M. Mühlleitner, Two Higgs doublets, effective interactions and a strong first-order electroweak phase transition, *J. High Energy Phys.* **08** (2022) 091.
- [101] Anisha, D. Azevedo, L. Biermann, C. Englert, and M. Mühlleitner, Effective 2HDM Yukawa interactions and a strong first-order electroweak phase transition, *J. High Energy Phys.* **02** (2024) 045.
- [102] U. Banerjee, J. Chakraborty, C. Englert, S. U. Rahaman, and M. Spannowsky, Integrating out heavy scalars with modified equations of motion: Matching computation of dimension-eight SMEFT coefficients, *Phys. Rev. D* **107**, 055007 (2023).