

First determination of fragmentation functions in an exotic-hadron candidate

S. Kumano 

Quark Matter Research Center, Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, People's Republic of China,
Southern Center for Nuclear Science Theory, Institute of Modern Physics, Chinese Academy of Sciences, Huizhou 516000, People's Republic of China,
and KEK Theory Center, Institute of Particle and Nuclear Studies, KEK, Oho 1-1, Tsukuba 305-0801, Japan



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In recent years, there are experimental reports on exotic-hadron candidates, which have different quark configurations from ordinary $q\bar{q}$ and qqq constituents. However, it is not easy to confirm their exotic nature from global observables such as masses, spins, parities, and decay widths. At high energies, internal quark and gluon configurations could become more apparent because hadrons should be described by fundamental degrees of freedom of quarks and gluons in quantum chromodynamics. One of such possibilities is to use the fragmentation functions (FFs). In this work, accurate FFs of an exotic-hadron candidate $f_0(980)$ are determined for the first time by a global analysis of experimental data on $e^+ + e^- \rightarrow f_0(980) + X$ with recent precise measurements of the Belle Collaboration. From the global analysis, we found that their second moments have a relation $M_u = M_d \ll M_s \sim M_g$ for up-quark, down-quark, strange-quark, and gluon FFs. Furthermore, the function $D_s^{f_0}(z)$ is distributed in the larger- z region in comparison with the functions $D_u^{f_0}(z)$, $D_d^{f_0}(z)$, and $D_g^{f_0}(z)$. These facts support that the $f_0(980)$ has the $s\bar{s}$ configuration at high energies. This is a new finding that $f_0(980)$ should be considered mainly as the $s\bar{s}$ state, which is different from our usual understanding as a tetraquark (or $K\bar{K}$) hadron from low-energy studies. Our results could indicate the transition of the internal configuration picture that $f_0(980)$ looks like a $q\bar{q}$ state at high energies, although it is described by tetraquark or $K\bar{K}$ molecule state at low energies. It sheds light on a new direction in exotic-hadron physics.

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Introduction. Many hadrons are described by quark models by assuming ordinary qqq and $q\bar{q}$ configurations as proposed in the original quark model of 1964. In recent years, there have been reports on exotic-hadron candidates (e.g., $qq\bar{q}\bar{q}$ and $qqqq\bar{q}$), which are different from these basic configurations. They are called exotic hadrons [1]. Recent investigations focus on heavy-quark systems. On the other hand, there are longstanding exotic-hadron candidates in the 1 GeV mass region, and $f_0(980)$ is such an example [1–3]. It is considered as a tetraquark or $K\bar{K}$ -moleculelike hadron from low-energy studies [1–3].

In general, it is difficult to identify internal configurations of exotic-hadron candidates by global observables such as masses, spins, parities, and decay widths. On the

contrary, high-energy reaction observables could be appropriate for such identifications because quarks and gluons are explicit degrees of freedom at high energies. Along this line, we proposed fragmentation functions (FFs) [4], constituent counting rule [5,6], and generalized parton distributions [7] as such high-energy observables.

A fragmentation function D_i^h indicates the probability for producing the hadron h from the parton i , which is up-, down-, strange-, charm-, bottom-quark, or gluon. Now, the FFs are relatively well determined by global analyses for major hadrons such as proton, pion, kaon, and other light hadrons [8,9]. In general, the FFs are classified into favored and disfavored functions as the parton distributions are classified into valence- and sea-quark distributions. The favored fragmentation indicates a fragmentation from a quark or an antiquark which exists within a hadron as a constituent in a quark model, and the disfavored means a fragmentation from a sea quark. Therefore, the favored and disfavored functions reflect internal quark and gluon configuration, which can be used for finding whether or not the hadron is really exotic. If the FFs are determined for

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exotic-hadron candidates by a global analysis of experimental data, we should be able to determine their internal structure. In particular, if the $f_0(980)$ has a tetraquark or $K\bar{K}$ -molecule structure, the favored FFs should be very different from the FFs of an ordinary $q\bar{q}$ meson.

In this work, we report a global analysis of $f_0(980)$ fragmentation function data for determining its internal configuration. As of 2008 [4], this kind of analysis was too early because there were not so many experimental data and their errors are very large on the measured fragmentation functions. The situation changed recently because accurate measurements have been done for light hadrons, including $f_0(980)$ by the Belle Collaboration in 2025 [10]. The high-intensity KEK B-Factor factory is suitable for such precision studies. With these recent KEKB data, it became realistic to propose accurate FFs of an exotic-hadron candidate $f_0(980)$ for the first time, and it enables us to find its internal exotic structure from the FFs.

The purpose of this work is to report analysis results including the KEKB data for determining the internal configuration of $f_0(980)$. By the determination of the $f_0(980)$ FFs, we expect that it is possible to distinguish among possible internal configurations, a nonstrange $q\bar{q}$ ($(u\bar{u} + d\bar{d})/\sqrt{2}$), a strange $q\bar{q}$ ($s\bar{s}$), a tetraquark or $K\bar{K}$ ($(u\bar{u}s\bar{s} + d\bar{d}s\bar{s})/\sqrt{2}$), and a glueball (gg). In the following, we explain the analysis method of the $f_0(980)$ FFs and its results to discuss the internal configuration picture. As shown in the following, we found that $f_0(980)$ looks like an $s\bar{s}$ state, which is contradictory to the usual understanding that it is a tetraquark or $K\bar{K}$ meson. In the end of this paper, we suggest a possible explanation how these seemingly inconsistent pictures could be reconciled without a contradiction.

Analysis method. The fragmentation functions are measured by high-energy hadron production processes. A typical one is the inclusive hadron production in the electron-positron collision ($e^+e^- \rightarrow hX$). The fragmentation function $F^h(z, Q^2)$ is defined by the e^+e^- total cross section σ_{tot} and the $e^+e^- \rightarrow hX$ cross section $\sigma(e^+e^- \rightarrow hX)$ as $F^h(z, Q^2) = d\sigma(e^+e^- \rightarrow hX)/dz/\sigma_{\text{tot}}$ [11] where the variable z is defined as $z = E_h/(\sqrt{s}/2)$ with the hadron energy E_h and the c.m. energy $\sqrt{s} = \sqrt{Q^2}$ of e^+e^- . The fragmentation occurs from primary partons, so that it is expressed theoretically by the sum of their contributions $F^h(z, Q^2) = \sum_i C_i(z, \alpha_s) \otimes D_i^h(z, Q^2)$, where $\otimes$ indicates the convolution integral $f(z) \otimes g(z) = \int_z^1 dy f(y)g(z/y)/y$. Here, $D_i^h(z, Q^2)$ is the fragmentation function of the hadron h from a parton i ($= u, d, s, \dots, g$), $C_i(z, \alpha_s)$ is a coefficient function to include perturbative QCD corrections, and α_s is the running coupling constant.

From theoretical and experimental studies at low energies, it is likely that $f_0(980)$ is a tetraquark or $K\bar{K}$ molecule [1]. However, let us consider all the possible configurations without prejudice. Then, a scalar meson in the 1 GeV

region could be formed as a nonstrange $q\bar{q}$ meson ($(u\bar{u} + d\bar{d})/\sqrt{2}$), a strange $q\bar{q}$ ($s\bar{s}$), tetraquark or $K\bar{K}$ ($(u\bar{u}s\bar{s} + d\bar{d}s\bar{s})/\sqrt{2}$), or a glueball (gg) [2]. In all these configurations, the quarks $u, d, \bar{u}$, and $\bar{d}$ are contained in the same fraction, which indicates that the parameters of these FFs could be assumed to be the same. The quarks s and $\bar{s}$ are also same, so that the common FFs could be used for both s and $\bar{s}$. Therefore, we express the fragmentation functions $D_u^{f_0}(z, Q_0^2) = D_{\bar{u}}^{f_0}(z, Q_0^2) = D_d^{f_0}(z, Q_0^2) = D_{\bar{d}}^{f_0}(z, Q_0^2)$, $D_s^{f_0}(z, Q_0^2)$, $D_g^{f_0}(z, Q_0^2)$, $D_c^{f_0}(z, m_c^2)$, and $D_b^{f_0}(z, m_b^2)$ in terms of parameters, which are determined by a χ^2 analysis of the data for $e^+ + e^- \rightarrow f_0 + X$. Each function is expressed by the three parameters N_i , α_i , and β_i as $D_i(z, Q_0^2) = N_i z^{\alpha_i} (1-z)^{\beta_i}$ where $i = u$ (or d), s , c , b , and g . This parametrization can account both the favored and disfavored FFs and the disfavored ones are suppressed at large z according to Figs. 1 and 2 of Ref. [4]. The initial scale is $Q_0^2 = 1 \text{ GeV}^2$, and m_c and m_b are taken $m_c = 1.43 \text{ GeV}$ and $m_b = 4.3 \text{ GeV}$. Q^2 variations to the experimental Q^2 points are calculated by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi equation [12]. The second moment of a fragmentation function $D_i(z, Q^2)$ is given by the integral $M_i = \int_0^1 dz z D_i(z, Q^2)$. The parameters N_i and M_i are related by $N_i^h = M_i/B(\alpha_i^h + 2, \beta_i + 1)$ with the β function $B(\alpha_i + 2, \beta_i + 1)$. The χ^2 analysis is done in the next-to-leading order (NLO) of α_s and uncertainties of the FFs are calculated by the Hessian method.

Past $e^+ + e^- \rightarrow f_0 + X$ experiments [10,13–15] are listed in Table I. Different kinematical variables are used in showing the data depending on the experimental collaborations, and they are defined by $z = z_E = 2E_h/\sqrt{s}$, $z_p = 2p_h/\sqrt{s - 4m_h^2}$, $\bar{z}_p = 2p_h/\sqrt{s}$, where p_h and m_h are the f_0 momentum and its mass, respectively. The experimental cross sections are shown by slightly different quantities as shown in Table I, so that they are converted to the same FFs $F^{f_0}(z, Q^2)$ for calculating χ^2 in comparison with theoretical FFs. The results are shown in Fig. 1. Although Q^2 values are different between the experimental

TABLE I. Experimental collaborations, laboratories, references, c.m. energies in GeV, numbers of data points, kinematical variables, and cross sections are listed for used datasets of $e^+ + e^- \rightarrow f_0(980) + X$. SLAC indicates SLAC National Accelerator Laboratory.

Exp.	Lab.	Ref.	$\sqrt{s}$	Data No.	Var.	Cross
HRS	SLAC	[13]	29	4	z_p	$\frac{s}{\beta} \frac{d\sigma}{dz}$
DELPHI	CERN	[14]	91.2	11	z_p	$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma}{dz_p}$
OPAL	CERN	[15]	91.2	8	$\bar{z}_p$	$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma}{d\bar{z}_p}$
Belle	KEK	[10]	10.58	40	z_p	$\frac{d\sigma}{dz_p}$
Total				63		

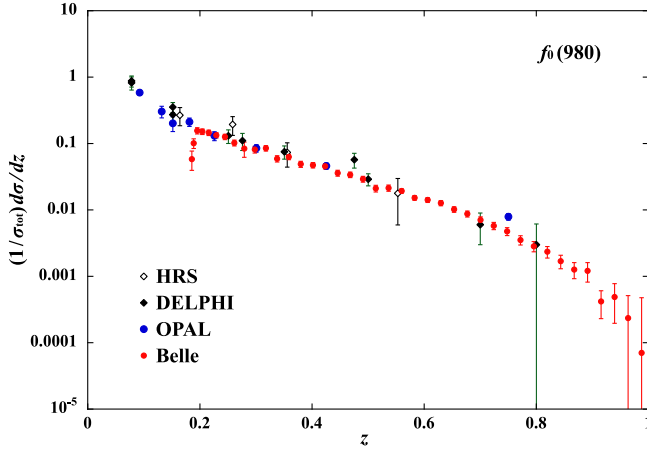


FIG. 1. Fragmentation-function data for $f_0(980)$ in the form of $F^{f_0} = d\sigma(e^+e^- \rightarrow f_0 X)/dz/\sigma_{\text{tot}}$.

groups, four experimental datasets are consistent with each other. It is obvious from this figure that the situation of the $f_0(980)$ FFs was improved significantly recently because of the Belle data in 2025. The previous high resolution spectrometer (HRS), detector with lepton, photon and hadron identification (DELPHI), and an omni-purpose apparatus at LEP (OPAL) data are taken in scattered kinematical points with large errors, so that one can imagine that the determination of the FFs was difficult from only these data. Including the precise Belle data in the wide kinematical region of z , we expect that the $f_0(980)$ FFs can be determined accurately.

Results. By the χ^2 analysis of the data in Fig. 1, the optimum FFs are determined with $\chi^2/\text{d.o.f.} = 1.43$, where we fixed two parameters β_c and β_b , which control the function forms of $D_c^{f_0}(z)$ and $D_b^{f_0}(z)$ at large z , in the charm and bottom functions as $\beta_c = \beta_b = 10$. However, we do not fix the parameters of the up (down), strange, and gluon functions, because they are crucial for determining the internal configuration of $f_0(980)$. Therefore, the total number of the parameters is 13. As seen in Fig. 1, it is almost impossible to determine the FFs of $f_0(980)$ without the Belle data. In fact, we obtained huge uncertainty bands for the $f_0(980)$ FFs in 2008 [4]. However, the Belle data are accurate enough and they enable us to determine the FFs precisely.

By the NLO χ^2 analysis, the second moments of $D_u^{f_0}$ ($= D_d^{f_0}$), $D_s^{f_0}$, and $D_g^{f_0}$ are determined as $M_u = M_d = 0.00329 \pm 0.00012$, $M_s = 0.01936 \pm 0.00041$, $M_g = 0.01231 \pm 0.00074$ at $Q^2 = 1 \text{ GeV}^2$. These values indicate a relation among these second moments as

$$M_u = M_d \ll M_s \sim M_g. \quad (1)$$

This relation indicates that the u and d quarks are not the main components of $f_0(980)$, although they should be

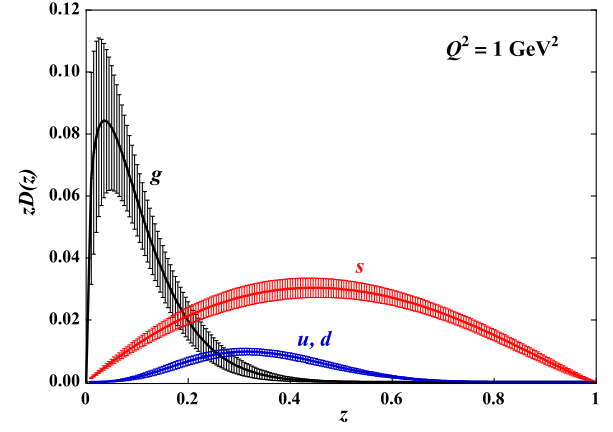


FIG. 2. Determined fragmentation functions for $f_0(980)$.

the main ingredients in the tetraquark or $K\bar{K}$ picture from the low-energy studies. The other parameters are obtained as $\alpha_g = -0.559 \pm 0.071$, $\beta_g = 11.86 \pm 0.94$, $\alpha_u = 2.33 \pm 0.11$, $\beta_u = 7.266 \pm 0.060$, $\alpha_s = 0.2623 \pm 0.0023$, $\beta_s = 1.276 \pm 0.043$, $M_c = 0.00073 \pm 0.00012$, $\alpha_c = 3.557 \pm 0.073$, $M_b = 0.000983 \pm 0.000074$, and $\alpha_b = 14.53 \pm 0.074$. In the pion case, according to a recent global analysis (J. Gao *et al.*, 2025 [9]), the relation is given as $M_{u,\bar{d}}^{\pi^+} = 0.481 \gg M_g^{\pi^+} = 0.102$, $M_{s,\bar{s}}^{\pi^+} = 0.234$, $M_{d,\bar{u}}^{\pi^+} = 0.119$ in the NLO analysis at $Q^2 = (1.4)^2 \text{ GeV}^2$, which indicates that the u and $\bar{d}$ are the main components in π^+ as suggested by the quark model.

The determined FFs $D_u^{f_0}(z) = D_d^{f_0}(z)$, $D_s^{f_0}(z)$, and $D_g^{f_0}(z)$ are shown at $Q^2 = 1 \text{ GeV}^2$ in Fig. 2 with uncertain bands with $\Delta\chi^2 = 14.8$ for the 13 parameter χ^2 fit [16]. It is obvious from this figure that the FFs of $f_0(980)$ are now determined accurately, whereas the FFs of the 2008 analysis had huge uncertainty bands. The s -quark function $D_s(z)$ is much larger than the light-quark ones $D_u(z)$ and $D_d(z)$, and it should be noted that $D_s(z)$ is distributed in the larger- z region than $D_u(z)$ and $D_d(z)$. We also notice that the gluon function $D_g(z)$ is distributed in the small- z region, although the function itself is larger than the s -quark one.

Gluon and quark-flavor separations in the analysis of the inclusive e^+e^- data are explained in Ref. [8]. Because the DELPHI/OPAL energy 91.2 GeV and the Belle energy 10.58 GeV are well separated, the gluon FF is constrained mainly by the scaling violation of the FF data. Because of the differences between the electric charges and weak couplings, it is possible to separate the two functions D_{u^+} and $D_{d^+} + D_{s^+}$, aside from the heavy-quark FFs. This separation means that the separation between $D_u = D_d$ and D_s in this work, because of $D_{u^+} = D_{d^+}$ in $f_0(980)$. To test effects of heavy-quark FFs, the analysis was done by changing the fixed parameters as $\beta_c = \beta_b = 5$; however, the moment relation and the distributions are not significantly changed for judging the internal structure. It should be noted that accurate data are needed in future at high

TABLE II. Possible $f_0(980)$ configurations and typical features of their fragmentation functions.

Configuration	Second moment	Function
$(u\bar{u} + d\bar{d})/\sqrt{2}$	$M_s < M_u < M_g$	$z_u^{\max} > z_s^{\max}$
$s\bar{s}$	$M_u < M_s \lesssim M_g$	$z_u^{\max} < z_s^{\max}$
$(u\bar{u}s\bar{s} + d\bar{d}s\bar{s})/\sqrt{2}$	$M_u \sim M_s \lesssim M_g$	$z_u^{\max} \sim z_s^{\max}$
gg	$M_u \sim M_s < M_g$	$z_u^{\max} \sim z_s^{\max}$

energies, especially with charm- and bottom-quark tagging, to obtain a solid fit result on the FFs.

General criteria are shown in Table II for judging internal configuration of $f_0(980)$ from the FFs by using the second moments and their functional forms [4]. For example, if $f_0(980)$ is an $s\bar{s}$ type, a gluon is emitted from a strange quark (or antiquark) and the gluon splits into an $s\bar{s}$ pair to form the $s\bar{s}$ bound state together with the initial strange quark (or strange antiquark). This is a favored fragmentation process. On the other hand, if an unfavored fragmentation occurs from a u quark, the $s\bar{s}$ from the gluon splitting could form the $s\bar{s}$ bound state by radiating another gluon. If it is the unfavored fragmentation from an initial gluon, the $s\bar{s}$ bound state is formed by the gluon splitting and another gluon radiation to maintain the color singlet nature. Because of the differences between these formation processes, there exist specific features in the second moments and the z -dependent functional forms. In the favored fragmentation without the gluon emission, the second moment of $D_s(z)$ is larger. In addition, more energy is transferred to $f_0(980)$ from the initial s (or $\bar{s}$) quark, the function $D_s(z)$ is distributed at large z . These relations are listed in the $s\bar{s}$ configuration part of Table II. The other relations of this table are derived by similar studies on the fragmentation processes to $f_0(980)$.

One should note, however, that these criteria are very rough ideas, especially on the functional form because quark-mass effects are not considered. In fact, the determined u and $\bar{s}$ FFs are very different in K^+ [8,9] because the strange-quark mass is heavier, although both are favored functions. The first moments are different in favored and disfavored functions in global analyses, for example, for pions and kaons; however, the difference between the functional shapes is not very clear at this stage. Therefore, these conditions should be considered as a starting point to initiate much detailed theoretical studies including the mass effect.

Considering the criteria of Table II, we decide the appropriate internal configuration of $f_0(980)$. The analysis results on the two factors,

- (i) the relation $M_u = M_d \ll M_s \sim M_g$ in Eq. (1), and
- (ii) the z -dependent functional forms of $D_u(z)$, $D_d(z)$, $D_s(z)$, and $D_g(z)$ in Fig. 2.

indicate that the $f_0(980)$ looks like an $s\bar{s}$ meson from its fragmentation functions from Table II because the s -quark

moment M_s is much larger than the u - and d -quark ones $M_{u,d}$ and because the $D_s(z)$ is distributed in the larger- z region than the other FFs. In Eq. (1), we find that the gluon moment M_g is as large as M_s ; however, it is not the major constituent by considering the functional form distributed in the small- z region. These facts indicate that the main constituent of $f_0(980)$ is $s\bar{s}$, and the light quarks (u , d) and the gluon are not the main components. In order to check the effect of the $Q_0^2 = 1 \text{ GeV}^2$ choice, the analysis was repeated by taking a larger value $Q_0^2 = 4 \text{ GeV}^2$. The obtained results are similar ($\chi^2/\text{d.o.f.} = 1.33$, $M_u = M_d = 0.00286 \pm 0.00034 \ll M_s = 0.01500 \pm 0.00041 \sim M_g = 0.0183 \pm 0.0015$) and the FFs are softer than the ones at $Q^2 = 1 \text{ GeV}^2$ due to the Q^2 evolution [12], which supports the suggested picture for $f_0(980)$.

This is the first clear indication on the internal structure of the $f_0(980)$ from an analysis of high-energy experimental data on the fragmentation functions. However, it makes us wonder about the difference from our “common sense” from the low-energy studies that the $f_0(980)$ is a tetraquark or $K\bar{K}$ molecule. Properties of $f_0(980)$ have been studied experimentally at low energies for a long time, and it is our understanding that the $f_0(980)$ is now considered as a tetraquark or $K\bar{K}$ molecule. This picture is now confirmed, for example, by two-photon decay and radiative-decay widths of $f_0(980)$ [1]. This picture and our analysis result of the FFs look like a paradox.

These seemingly contradictory results could be understood in the following way: the $\Lambda(1405)$ is considered as a pentaquark or $\bar{K}N$ molecule from theoretical and experimental studies at low energies. The perturbative QCD indicates a specific feature, called the constituent counting rule, on cross sections of high-energy exclusive hadron processes. This constituent counting analysis of $\Lambda(1405)$ photo-production data suggested the transition that $\Lambda(1405)$ could be a pentaquark state at low energies but it becomes an ordinary qqq state at high energies [5]. In the same way, $f_0(980)$ could be a tetraquark state at low energies and it becomes a $q\bar{q}$ state at high energies. Our FF analysis supports this scenario:

$$f_0(980) = \text{tetraquark state} \rightarrow q\bar{q} \text{ state} \\ (\text{at low energy} \rightarrow \text{high energy}).$$

If this interpretation is right, this work could open a new view of hadrons and a new field of hadron physics. Exotic-hadron candidates could be interpreted as exotic hadrons at low energies, whereas they could be viewed as ordinary hadrons with the $q\bar{q}$ and qqq configurations at high energies. Obviously, additional experimental confirmations are needed to confirm this idea on the transition scenario. The original naïve quark model could be valid at high energies, where quarks and gluons are explicit fundamental

degrees of freedom of QCD, whereas some hadrons are considered as exotic ones at low energies.

Summary. The accurate FFs of the $f_0(980)$ were determined by a global analysis of the experimental data for the first time. In particular, the measurements of the Belle Collaboration of 2025 on the process $e^+ + e^- \rightarrow f_0(980) + X$ made the accurate determination possible. From the second moments of the determined FFs, we found the relation $M_u = M_d \ll M_s \sim M_g$. Furthermore, we found that the strange-quark function $D_s(z)$ is distributed in the large- z region in comparison with the light-quark functions, $D_u(z)$ and $D_d(z)$, and also the gluon function $D_g(z)$. These relations on the second moments and the functional forms indicate that $f_0(980)$ looks like an $s\bar{s}$ state from the FFs. There is also an $s\bar{s}$ -state indication in high-energy heavy-ion reactions [17].

This fact seems to be contradictory to the common sense that $f_0(980)$ is a tetraquark state or $K\bar{K}$ molecule from low-energy studies. The only way to reconcile these different

pictures of the same hadron is that the internal configuration looks different depending on the energy with which we look at the hadron. Namely, $f_0(980)$ is a tetraquark state or $K\bar{K}$ molecule at low energies, but it is an $s\bar{s}$ state at high energies. This transition picture agrees with the phenomena seen in the photon production of $\Lambda(1405)$ by using the constituent counting rule of perturbative QCD.

In general, some exotic hadrons may be described by the $q\bar{q}$ and qqq pictures at high energies, although they could be considered as exotic hadrons at low energies. From this work, we expect that a new field of hadron physics could be created for the transition from exotic hadrons to the standard $q\bar{q}$ and qqq configurations as the energy increases.

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Data availability. The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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