

Lepton-flavor violation in the minimal R -symmetric SSM with a seesaw mechanism

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The minimal R -symmetric supersymmetric Standard Model with a seesaw mechanism extends the minimal R -symmetric supersymmetric Standard Model by incorporating right-handed neutrinos to generate neutrino masses via the type-I seesaw mechanism. This work presents a detailed analysis of lepton-flavor-violating (LFV) processes, including $\ell_i \rightarrow \ell_j \gamma$, $\ell_i \rightarrow 3\ell_j$, and Higgs decays $h \rightarrow \ell_i \ell_j$. Based on the current experimental limitations, we carry out detailed parameter scanning and numerical calculations to analyze the effects of different sensitive parameters on LFV. The numerical results show that the nondiagonal elements involving the initial and final leptons are the main sensitive parameters and LFV sources.

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I. INTRODUCTION

Supersymmetry (SUSY) stands as one of the most theoretically compelling frameworks for physics beyond the Standard Model (BSM) [1]. However, SUSY might not be realized in its minimal form, the minimal supersymmetry Standard Model (MSSM). Recent developments have highlighted the minimal R -symmetric SUSY Standard Model (MRSSM) as a particularly promising alternative of beyond MSSM. It is based on a continuous unbroken $U(1)_R$ symmetry under which the superparticles are R -charged [2]. It involves Dirac gauginos, $N = 2$ SUSY multiplets in the gauge and Higgs sectors, and supersoft SUSY breaking [3–7]. Furthermore, the model can provide many more candidates for dark matter comparing that with the MSSM [8–10]. While at MRSSM, it does not have neutrinos mass, we then have extended the MRSSM by incorporating right-handed neutrino fields.

The right-handed neutrino field couples with the S field. After the S field acquires a vacuum expectation value, it gives mass to the right-handed neutrinos, and then the seesaw mechanism is employed to generate mass for the left-handed neutrinos [11–15].

Low-energy lepton physics represents a promising frontier for fundamental discoveries in the coming years, as intriguing anomalies and deviations from SM predictions continue to accumulate in lepton-related observables. Among the various lepton-flavor-violating (LFV) processes studied in the literature, the most phenomenologically significant are the radiative decays $\ell^- \rightarrow \ell^- \gamma$ and $\ell^- \rightarrow \ell^- \ell^- \ell^+$, as their branching fractions can be measured with exceptional precision. In Table I, we show the present experimental limits and future sensitivities for the LFV processes. Furthermore, we are also interested here in the LFV of the Higgs boson, $h \rightarrow e\mu$, $h \rightarrow e\tau$, and $h \rightarrow \mu\tau$ [16–20]. We have calculated the μ , τ , and Higgs LFV processes. And we discuss the impact of new parameters.

The paper is organized as follows. In Sec. II, the main ingredients of the minimal R -symmetric supersymmetric Standard Model with a seesaw mechanism (MRSSMSeesaw) are summarized briefly by introducing the superpotential and the general soft-breaking terms. We present the analysis on the decay width of the rare LFV processes and the Higgs LFV in Sec. III. The numerical analyses are given in Sec. IV, and Sec. V gives a summary.

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TABLE I. Present limits and future sensitivities for the branching ratios for the LFV processes.

LFV process	Present limit	Future sensitivity
$\mu \rightarrow e\gamma$	$<4.2 \times 10^{-13}$ [21]	$\sim 6 \times 10^{-14}$ [22]
$\mu \rightarrow 3e$	$<1 \times 10^{-12}$ [23]	$\sim 10^{-16}$ [24]
$\tau \rightarrow e\gamma$	$<3.3 \times 10^{-8}$ [25]	$\sim 10^{-8} - 10^{-9}$ [26]
$\tau \rightarrow 3e$	$<2.7 \times 10^{-8}$ [27]	$\sim 10^{-9} - 10^{-10}$ [28]
$\tau \rightarrow \mu\gamma$	$<4.2 \times 10^{-8}$ [25]	$\sim 10^{-8} - 10^{-9}$ [28]
$\tau \rightarrow 3\mu$	$<2.1 \times 10^{-8}$ [27]	$\sim 10^{-9} - 10^{-10}$ [28]

II. DETAILS OF THE MRSSMSEESAW

The superpotential of the MRSSMSeesaw is structured as follows (see Table II):

$$\begin{aligned}
\mathcal{W}_{MRSSMSeesaw} = & \mu_d \hat{R}_d \hat{H}_d + \mu_u \hat{R}_u \hat{H}_u + \Lambda_d \hat{R}_d \hat{T} \hat{H}_d \\
& + \Lambda_u \hat{R}_u \hat{T} \hat{H}_u + \lambda_d \hat{S} \hat{R}_d \hat{H}_d + \lambda_u \hat{S} \hat{R}_u \hat{H}_u \\
& - Y_d \hat{d} \hat{q} \hat{H}_d - Y_e \hat{e} \hat{l} \hat{H}_d + Y_u \hat{u} \hat{q} \hat{H}_u \\
& + Y_x \hat{\nu} \hat{S} + Y_\nu \hat{l} \hat{H}_u \hat{\nu}. \quad (1)
\end{aligned}$$

The first and second lines represent the superpotential in MRSSM, where $\hat{H}_u$ and $\hat{H}_d$ are the MSSM-like Higgs weak isodoublets, $\hat{R}_u$ and $\hat{R}_d$ are the R -charged Higgs $SU(2)_L$ doublets, and the corresponding Dirac Higgsino mass parameters are denoted as μ_u and μ_d . Y_e , Y_u , and Y_d are the Yukawa couplings of charged lepton, up-type quark, and down-type quark, respectively. λ_u , λ_d , Λ_u , and Λ_d are parameters of Yukawa-like trilinear terms involving the singlet $\hat{S}$ and the triplet $\hat{T}$, which is given by

$$\hat{T} = \begin{pmatrix} \hat{T}^0/\sqrt{2} & \hat{T}^+ \\ \hat{T}^- & -\hat{T}^0/\sqrt{2} \end{pmatrix}.$$

TABLE II. Field content of the MRSSMSeesaw. R charges in the second column correspond to the entire superfield.

Chiral superfield	R charge	Spin 0	Spin $\frac{1}{2}$	$U(1)_Y \otimes SU(2)_L \otimes SU(3)_C$
$\hat{q}$	1	$\tilde{q}$	q	$(\frac{1}{6}, \mathbf{2}, \mathbf{3})$
$\hat{l}$	1	$\tilde{l}$	l	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{H}_d$	0	H_d	$\tilde{H}_d$	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{H}_u$	0	H_u	$\tilde{H}_u$	$(\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{d}$	1	$\tilde{d}_R^*$	d_R^*	$(\frac{1}{3}, \mathbf{1}, \mathbf{3})$
$\hat{u}$	1	$\tilde{u}_R^*$	u_R^*	$(-\frac{2}{3}, \mathbf{1}, \mathbf{3})$
$\hat{e}$	1	$\tilde{e}_R^*$	e_R^*	$(1, \mathbf{1}, \mathbf{1})$
$\hat{S}$	0	S	$\tilde{S}$	$(0, \mathbf{1}, \mathbf{1})$
$\hat{T}$	0	T	$\tilde{T}$	$(0, \mathbf{3}, \mathbf{1})$
$\hat{O}$	0	O	$\tilde{O}$	$(0, \mathbf{1}, \mathbf{8})$
$\hat{R}_d$	2	R_u	$\tilde{R}_d$	$(\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{R}_u$	2	R_d	$\tilde{R}_u$	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{\nu}$	1	$\tilde{\nu}_R^*$	ν_R^*	$(0, \mathbf{1}, \mathbf{1})$
Vector superfield	R charge	Spin $\frac{1}{2}$	Spin 1	$U(1)_Y \otimes SU(2)_L \otimes SU(3)_C$
$\hat{B}$	0	$\tilde{B}^0$	B^0	$(1, \mathbf{1}, \mathbf{1})$
$\hat{W}$	0	$\tilde{W}^\pm, \tilde{W}^0$	$W^\pm, W^0$	$(1, \mathbf{3}, \mathbf{1})$
$\hat{g}$	0	$\tilde{g}$	g	$(0, \mathbf{1}, \mathbf{8})$

In order to generate nonzero masses for neutrinos via the type-I seesaw mechanism, we introduce three generations of neutrino singlets and include the terms $Y_\nu \hat{l} \hat{H}_u \hat{\nu}$ and $Y_x \hat{\nu} \hat{S}$ in the superpotential Eq. (1). The complete mass term for the right-handed neutrino, allowed by the gauge symmetry, are $M_\nu \hat{\nu} \hat{\nu} + Y_x \hat{\nu} \hat{S}$. We argue that the right-handed neutrino mass arises entirely from the vacuum expectation value (VEV) of the $\hat{S}$ field. This is equivalent to setting $M_\nu = 0$.

For the phenomenological studies, we present the soft-breaking terms

$$\begin{aligned}
V_{SB,S} = & m_{H_d}^2 (|H_d^0|^2 + |H_d^-|^2) + m_{H_u}^2 (|H_u^0|^2 + |H_u^+|^2) + (B_\mu (H_d^- H_u^+ - H_d^0 H_u^0) + \text{H.c.}) \\
& + m_{R_d}^2 (|R_d^0|^2 + |R_d^+|^2) + m_{R_u}^2 (|R_u^0|^2 + |R_u^-|^2) + m_T^2 (|T^0|^2 + |T^-|^2 + |T^+|^2) \\
& + m_S^2 |S|^2 + m_O^2 |O|^2 + \tilde{d}_{L,i}^* m_{q,ij}^2 \tilde{d}_{L,j} + \tilde{d}_{R,i}^* m_{d,ij}^2 \tilde{d}_{R,j} + \tilde{u}_{L,i}^* m_{q,ij}^2 \tilde{u}_{L,j} \\
& + \tilde{u}_{R,i}^* m_{u,ij}^2 \tilde{u}_{R,j} + \tilde{e}_{L,i}^* m_{l,ij}^2 \tilde{e}_{L,j} + \tilde{e}_{R,i}^* m_{r,ij}^2 \tilde{e}_{R,j} + \tilde{\nu}_{L,i}^* m_{\nu,ij}^2 \tilde{\nu}_{L,j} + \tilde{\nu}_{R,i}^* m_{\nu,ij}^2 \tilde{\nu}_{R,j}. \quad (2)
\end{aligned}$$

All trilinear scalar couplings involving Higgs bosons to squarks and sleptons are forbidden due to the R symmetry. The soft-breaking terms, which describe the Dirac mass terms for the gauginos and the interaction terms between the adjoint scalars and the auxiliary D fields of the corresponding gauge multiplet, take the form [4]

$$V_{SB,DG} = M_D^B (\tilde{B} \tilde{S} - \sqrt{2} \mathcal{D}_B S) + M_D^W (\tilde{W}^a \tilde{T}^a - \sqrt{2} \mathcal{D}_W^a T^a) + M_D^O (\tilde{g} \tilde{O} - \sqrt{2} \mathcal{D}_g^a O^a) + \text{H.c.}, \quad (3)$$

where $\tilde{B}$, $\tilde{W}$, and $\tilde{g}$ are usually MSSM Weyl fermions and M_D^B , M_D^W , and M_D^O are the mass of the bino, wino, and gluino, respectively.

For the scalar sector, after the electroweak symmetry breaking, (neutral) scalar components of $\hat{H}_d$, $\hat{H}_u$, $\hat{S}$, and $\hat{T}$ acquire VEVs, which we parametrize as

$$\begin{aligned} H_d^0 &= \frac{1}{\sqrt{2}}(v_d + \phi_d + i\sigma_d), & H_u^0 &= \frac{1}{\sqrt{2}}(v_u + \phi_u + i\sigma_u), \\ T^0 &= \frac{1}{\sqrt{2}}(v_T + \phi_T + i\sigma_T), & S^0 &= \frac{1}{\sqrt{2}}(v_S + \phi_S + i\sigma_S). \end{aligned} \quad (4)$$

Since R -Higgs bosons carry R charge 2, they do not develop VEVs. As for the color octet, the masses of the CP -even (O_S) and CP -odd (O_A) components of the sgluon field $O \equiv (O_S + iO_A)/\sqrt{2}$ are split as

$$m_{O_S}^2 = m_O^2 + 4(M_O^D)^2, \quad (5)$$

$$m_{O_A}^2 = m_O^2. \quad (6)$$

The sneutrino slips into CP -even and CP -odd parts:

$$\tilde{\nu}_L = \frac{1}{\sqrt{2}}(\phi_L + i\sigma_L), \quad \tilde{\nu}_R = \frac{1}{\sqrt{2}}(\phi_R + i\sigma_R). \quad (7)$$

After electroweak symmetry breaking, the μ terms result from the VEVs of scalar singlet and triplet, and we define these abbreviations for clarity:

$$\mu_i^{\text{eff}, \pm} = \mu_i + \frac{\lambda_i v_S}{\sqrt{2}} \pm \frac{\Lambda_i v_T}{2}, \quad \mu_i^{\text{eff}, 0} = \mu_i + \frac{\lambda_i v_S}{\sqrt{2}}, \quad i = u, d. \quad (8)$$

The minimization conditions for the scalar potential, or tadpole equations, then read

$$0 = t_d = t_u = t_T = t_S, \quad (9)$$

where the tree-level tadpoles $t_i \equiv \frac{\partial V^{\text{EW}}}{\partial \phi_i}$ are

$$0 = v \cos \beta [4m_Z^2 \cos 2\beta - 8g_1 M_B^D v_S + 8g_2 M_W^D v_T + 4\lambda_D^2 v_S^2 + 4\sqrt{2}\lambda_D v_S(\Lambda_D v_T + 2\mu_D) + 2(\Lambda_D v_T + 2\mu_D)^2 + 8m_{H_d}^2], \quad (10)$$

$$0 = v \sin \beta [-4m_Z^2 \cos 2\beta + 8g_1 M_B^D v_S - 8g_2 M_W^D v_T + 4\lambda_U^2 v_S^2 - 4\sqrt{2}\lambda_U \Lambda_U v_S v_T + 8\sqrt{2}\lambda_U \mu_u v_S + 2\Lambda_U^2 v_T^2 - 8\Lambda_U \mu_u v_T + 8m_{H_u}^2 + 8\mu_u^2], \quad (11)$$

$$\begin{aligned} 0 &= v_T [4(M_W^D)^2 + m_T^2] + [2g_2 M_W^D + \Lambda_d(\sqrt{2}\lambda_d v_S + \Lambda_d v_T + 2\mu_d)] v^2 \cos^2 \beta \\ &\quad - \frac{1}{4} [2g_2 M_W^D + \Lambda_u(\sqrt{2}\lambda_u v_S - \Lambda_u v_T + 2\mu_u)] v^2 \sin^2 \beta, \end{aligned} \quad (12)$$

$$\begin{aligned} 0 &= v_S [4(M_B^D)^2 + m_S^2] + \frac{1}{4} [-2g_1 M_B^D + \lambda_d(2\lambda_d v_S + \sqrt{2}\Lambda_d v_T + 2\sqrt{2}\mu_d)] v^2 \cos^2 \beta \\ &\quad + \frac{1}{4} [2g_1 M_B^D + \lambda_u(2\lambda_u v_S + 2\sqrt{2}\mu_u - \sqrt{2}\Lambda_u v_T)] v^2 \sin^2 \beta. \end{aligned} \quad (13)$$

We introduce the variables $v = \sqrt{v_u^2 + v_d^2}$ and $\tan \beta = v_u/v_d$, analogous to the MSSM. Those equations are then solved analytically to obtain $m_{H_d}^2$, $m_{H_u}^2$, m_S^2 , and m_T^2 .

Using the tadpole equations (9), the pseudoscalar Higgs boson mass matrix in the basis $(\sigma_d, \sigma_u, \sigma_S, \sigma_T)$ has a simple form (in Landau gauge)

$$\mathcal{M}_A = \begin{pmatrix} B_\mu \frac{v_u}{v_d} & B_\mu & 0 & 0 \\ B_\mu & B_\mu \frac{v_d}{v_u} & 0 & 0 \\ 0 & 0 & m_S^2 + \frac{\lambda_d^2 v_d^2 + \lambda_u^2 v_u^2}{2} & \frac{\lambda_d \Lambda_d v_d^2 - \lambda_u \Lambda_u v_u^2}{2\sqrt{2}} \\ 0 & 0 & \frac{\lambda_d \Lambda_d v_d^2 - \lambda_u \Lambda_u v_u^2}{2\sqrt{2}} & m_T^2 + \frac{\Lambda_d^2 v_d^2 + \Lambda_u^2 v_u^2}{4} \end{pmatrix}. \quad (14)$$

Mass for the Z boson is the same as usual, but the mass of the W boson has a contribution from the triplet VEV:

$$m_Z^2 = \frac{g_1^2 + g_2^2}{4} v^2, \quad m_W^2 = \frac{g_2^2}{4} v^2 + g_2^2 v_T^2, \quad \hat{\rho}_{\text{tree}} = 1 + \frac{4v_T^2}{v^2}. \quad (15)$$

The scalar Higgs boson mass matrix in the weak basis $(\phi_d, \phi_u, \phi_S, \phi_T)$ is given by

$$\mathcal{M}_{H^0} = \begin{pmatrix} \mathcal{M}_{\text{MSSM}} & \mathcal{M}_{21}^T \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{pmatrix}, \quad (16)$$

with the submatrices ($c_\beta = \cos \beta$, $s_\beta = \sin \beta$, etc.)

$$\begin{aligned}
\mathcal{M}_{\text{MSSM}} &= \begin{pmatrix} m_Z^2 c_\beta^2 + m_A^2 s_\beta^2 & -(m_Z^2 + m_A^2) s_\beta c_\beta \\ -(m_Z^2 + m_A^2) s_\beta c_\beta & m_Z^2 s_\beta^2 + m_A^2 c_\beta^2 \end{pmatrix}, \\
\mathcal{M}_{22} &= \begin{pmatrix} 4(M_B^D)^2 + m_S^2 + \frac{\lambda_d^2 v_d^2 + \lambda_u^2 v_u^2}{2} & \frac{\lambda_d \Lambda_d v_d^2 - \lambda_u \Lambda_u v_u^2}{2\sqrt{2}} \\ \frac{\lambda_d \Lambda_d v_d^2 - \lambda_u \Lambda_u v_u^2}{2\sqrt{2}} & 4(M_W^D)^2 + m_T^2 + \frac{\Lambda_d^2 v_d^2 + \Lambda_u^2 v_u^2}{4} \end{pmatrix}, \\
\mathcal{M}_{21} &= \begin{pmatrix} v_d(\sqrt{2}\lambda_d \mu_u^{\text{eff},-} - g_1 M_B^D) & v_u(\sqrt{2}\lambda_u \mu_u^{\text{eff},-} + g_1 M_B^D) \\ v_d(\Lambda_d \mu_u^{\text{eff},+} + g_2 M_W^D) & -v_u(\Lambda_u \mu_u^{\text{eff},+} + g_2 M_W^D) \end{pmatrix}.
\end{aligned}$$

The charged Higgs boson mass matrix in the weak basis $(H_d^{-*}, H_u^{+}, T^{-*}, T^{+})$ is given as

$$\mathcal{M}_{H^\pm} = \begin{pmatrix} \mathcal{M}_{\text{MSSM},\pm} & \mathcal{M}_{21,\pm}^T \\ \mathcal{M}_{21,\pm} & \mathcal{M}_{22,\pm} \end{pmatrix}, \quad (17)$$

with the submatrices

$$\begin{aligned}
\mathcal{M}_{\text{MSSM},\pm} &= \begin{pmatrix} m_{H^\pm}^2 c_\beta^2 - 2v_T(g_2 M_W^D + \Lambda_d \mu_d^{\text{eff},0}) & m_{H^\pm}^2 s_\beta c_\beta \\ m_{H^\pm}^2 s_\beta c_\beta & m_{H^\pm}^2 c_\beta^2 + 2v_T(g_2 M_W^D + \Lambda_u \mu_u^{\text{eff},0}) \end{pmatrix}, \\
\mathcal{M}_{22,\pm} &= \begin{pmatrix} 2(M_W^D)^2 + m_T^2 & 2(M_W^D)^2 \\ 2(M_W^D)^2 & 2(M_W^D)^2 + m_T^2 \end{pmatrix} + \begin{pmatrix} \frac{g_2^2 v_T^2 + \Lambda_d^2 v_d^2 c_j^2}{2} - \frac{g_2^2 v^2 \cos 2j}{4} & -\frac{g_2^2 v_T^2}{2} \\ -\frac{g_2^2 v_T^2}{2} & \frac{g_2^2 v_T^2 + \Lambda_u^2 v_u^2 s_j^2}{2} + \frac{g_2^2 v^2 \cos 2j}{4} \end{pmatrix}, \\
\mathcal{M}_{21,\pm} &= \begin{pmatrix} \frac{v_d}{2\sqrt{2}}(2\Lambda_d \mu_d^{\text{eff},-} + 2g_2 M_W^D + v_T g_2^2) & \frac{v_u}{2\sqrt{2}}(2\Lambda_u \mu_u^{\text{eff},-} + 2g_2 M_W^D + v_T g_2^2) \\ \frac{v_d}{2\sqrt{2}}(2\Lambda_d \mu_d^{\text{eff},+} + 2g_2 M_W^D - v_T g_2^2) & \frac{v_u}{2\sqrt{2}}(2\Lambda_u \mu_u^{\text{eff},+} + 2g_2 M_W^D - v_T g_2^2) \end{pmatrix}, \quad (18)
\end{aligned}$$

where the charged Higgs mass parameter $m_{H^\pm}^2 = m_A^2 + \frac{g_2^2 v^2}{4}$ reads as in the MSSM.

The mass matrix of neutral R -Higgs bosons is

$$\mathcal{M}_R = \begin{pmatrix} m_{R_d}^2 & \frac{1}{4}(\Lambda_u \Lambda_d - 2\lambda_u \lambda_d) v_u v_d \\ \frac{1}{4}(\Lambda_u \Lambda_d - 2\lambda_u \lambda_d) v_u v_d & m_{R_u}^2 \end{pmatrix}, \quad (19)$$

with

$$\begin{aligned}
m_{R_d}^2 &= m_{R_d}^2 + (\mu_d^{\text{eff},+})^2 + g_1 M_B^D v_S - g_2 M_W^D v_T + \frac{1}{8}[(g_1^2 + g_2^2)(v_u^2 - v_d^2) + 4\lambda_d^2 v_d^2 + 2\Lambda_d^2 v_d^2], \\
m_{R_u}^2 &= m_{R_u}^2 + (\mu_u^{\text{eff},-})^2 - g_1 M_B^D v_S + g_2 M_W^D v_T + \frac{1}{8}[(g_1^2 + g_2^2)(v_d^2 - v_u^2) + 4\lambda_u^2 v_u^2 + 2\Lambda_u^2 v_u^2],
\end{aligned}$$

which is diagonalized by an orthogonal matrix with mixing angle θ_R .

The charged R Higgs do not mix because of the R symmetry, and the mass eigenvalues are given as

$$\begin{aligned}
m_{R_1^+}^2 &= m_{R_d}^2 + (\mu_u^{\text{eff},+})^2 + g_1 M_B^D v_S + g_2 M_W^D v_T + \frac{1}{8}[(g_1^2 - g_2^2)(v_u^2 - v_d^2) + 4\Lambda_d^2 v_d^2], \\
m_{R_2^+}^2 &= m_{R_u}^2 + (\mu_d^{\text{eff},-})^2 - g_1 M_B^D v_S - g_2 M_W^D v_T + \frac{1}{8}[(g_1^2 - g_2^2)(v_d^2 - v_u^2) + 4\Lambda_u^2 v_u^2]. \quad (20)
\end{aligned}$$

In the weak basis of eight neutral electroweak two-component fermions: $\xi_i = (\tilde{B}, \tilde{W}^0, \tilde{R}_d^0, \tilde{R}_u^0)$, $\zeta_i = (\tilde{S}, \tilde{T}^0, \tilde{H}_d^0, \tilde{H}_u^0)$ with R charges $+1$ and -1 , respectively, the neutralino mass matrix takes the form

$$m_\chi = \begin{pmatrix} M_B^D & 0 & -\frac{1}{2}g_1 v_d & \frac{1}{2}g_1 v_u \\ 0 & M_W^D & \frac{1}{2}g_2 v_d & -\frac{1}{2}g_2 v_u \\ -\frac{1}{\sqrt{2}}\lambda_d v_d & -\frac{1}{2}\Lambda_d v_d & -\mu_d^{\text{eff},+} & 0 \\ \frac{1}{\sqrt{2}}\lambda_u v_u & -\frac{1}{2}\Lambda_u v_u & 0 & \mu_u^{\text{eff},-} \end{pmatrix}. \quad (21)$$

The transformation to a diagonal mass matrix and mass eigenstates κ_i and ψ_i is performed by two unitary mixing matrices N^1 and N^2 as

$$N^{1,*} m_\chi N^{2,\dagger} = m_\chi^{\text{diag}}, \quad \xi_i = \sum_{j=1}^4 N_{ji}^{1,*} \kappa_j, \quad \zeta_i = \sum_{j=1}^4 N_{ij}^{2,*} \psi_j,$$

and physical four-component Dirac neutralinos are constructed as

$$\chi_i = \begin{pmatrix} \kappa_i \\ \psi_i^* \end{pmatrix}, \quad i = 1, 2, 3, 4. \quad (22)$$

The mass matrix of charginos in the weak basis of eight charged two-component fermions breaks into two (2×2) submatrices. The first, in the basis $(\tilde{T}^-, \tilde{H}_d^-), (\tilde{W}^+, \tilde{R}_d^+)$ of spinors with R charge equal to electric charge, takes the form

$$m_{\chi^+} = \begin{pmatrix} g_2 v_T + M_W^D & \frac{1}{\sqrt{2}}\Lambda_d v_d \\ \frac{1}{\sqrt{2}}g_2 v_d & +\mu_d^{\text{eff},-} \end{pmatrix}. \quad (23)$$

The diagonalization and transformation to mass eigenstates $\lambda_i^\pm$ is performed by two unitary matrices U^1 and V^1 as

$$U^{1,*} m_{\chi^+} V^{1,\dagger} = m_{\chi^+}^{\text{diag}}, \quad \tilde{T}^- = \sum_{j=1}^2 U_{j1}^{1,*} \lambda_j^-, \quad \tilde{H}_d^- = \sum_{j=1}^2 U_{j2}^{1,*} \lambda_j^-, \quad (24)$$

$$\tilde{W}^+ = \sum_{j=1}^2 V_{1j}^{1,*} \lambda_j^+, \quad \tilde{R}_d^+ = \sum_{j=1}^2 V_{2j}^{1,*} \lambda_j^+, \quad (25)$$

and the corresponding physical four-component charginos are built as

$$\chi_i^+ = \begin{pmatrix} \lambda_i^+ \\ \lambda_i^{-*} \end{pmatrix}, \quad i = 1, 2. \quad (26)$$

The second submatrix, in the basis $(\tilde{W}^-, R_u^-), (\tilde{T}^+, \tilde{H}_u^+)$ of spinors with R charge equal to minus electric charge, reads

$$m_{\rho^-} = \begin{pmatrix} -g_2 v_T + M_W^D & \frac{1}{\sqrt{2}}g_2 v_u \\ -\frac{1}{\sqrt{2}}\Lambda_u v_u & -\mu_u^{\text{eff},+} \end{pmatrix}. \quad (27)$$

The diagonalization and transformation to mass eigenstates $\eta_i^\pm$ is performed by U^2 and V^2 as

$$U^{2,*} m_{\rho^-} V^{2,\dagger} = m_{\rho^-}^{\text{diag}}, \quad \tilde{W}^- = \sum_{j=1}^2 U_{j1}^{2,*} \eta_j^-, \quad R_u^- = \sum_{j=1}^2 U_{j2}^{2,*} \eta_j^-, \quad (28)$$

$$\tilde{T}^+ = \sum_{j=1}^2 V_{1j}^{2,*} \eta_j^+, \quad \tilde{H}_u^+ = \sum_{j=1}^2 V_{2j}^{2,*} \eta_j^+, \quad (29)$$

and the corresponding physical four-component charginos are built as

$$\rho_i^- = \begin{pmatrix} \eta_i^- \\ \eta_i^{+*} \end{pmatrix}, \quad i = 1, 2. \quad (30)$$

Moreover, the mass matrix of the CP -odd sneutrino, in basis $(\sigma_L^i, \sigma_R^i), (\sigma_L^i, \sigma_R^i)$, is

$$m_{\nu^j}^2 = \begin{pmatrix} m_{\sigma_L^i \sigma_L^i} & m_{\sigma_R^i \sigma_L^i} \\ m_{\sigma_L^i \sigma_R^i} & m_{\sigma_R^i \sigma_R^i} \end{pmatrix}, \quad (31)$$

$$m_{\sigma_L^i \sigma_L^i} = \frac{1}{2} v_u^2 Y_\nu Y_\nu^T + m_l^2 + \mathbf{1} \left((-g_1 v_s M_D^B + g_2 v_T M_D^W) + \frac{1}{8} (g_1^2 + g_2^2) (-v_u^2 + v_d^2) \right), \quad (32)$$

$$m_{\sigma_R^i \sigma_L^i} = -v_s v_u Y_x Y_\nu, \quad (33)$$

$$m_{\sigma_L^i \sigma_R^i} = -v_s v_u Y_x^T Y_\nu^T, \quad (34)$$

$$m_{\sigma_R^i \sigma_R^i} = \frac{1}{2} v_u^2 Y_\nu^T Y_\nu + m_\nu^2 + 2v_s^2 Y_x^2. \quad (35)$$

This matrix is diagonalized by Z^I :

$$Z^I m_{\nu^j}^2 Z^{I,\dagger} = m_{2,\nu^j}^{\text{dia}}, \quad (36)$$

with

$$\sigma_{Li}^i = \sum_j Z_{ji}^{I,*} \nu_j^i, \quad \sigma_{Ri}^i = \sum_j Z_{ji}^{I,*} \nu_j^i, \quad i, j = 1, 2, 3. \quad (37)$$

The mass matrix for the CP -even sneutrino in basis $(\phi_L^i, \phi_R^i), (\phi_L^i, \phi_R^i)$ is

$$m_{\tilde{\nu}^R}^2 = \begin{pmatrix} m_{\phi_L^i \phi_L^i} & m_{\phi_R^j \phi_L^i} \\ m_{\phi_L^i \phi_R^j} & m_{\phi_R^j \phi_R^j} \end{pmatrix}, \quad (38)$$

$$m_{\phi_L^i \phi_L^i} = \frac{1}{2} v_u^2 Y_\nu Y_\nu^T + m_l^2 + \mathbf{1} \left((-g_1 v_S M_D^B + g_2 v_T M_D^W) + \frac{1}{8} (g_1^2 + g_2^2) (-v_u^2 + v_d^2) \right), \quad (39)$$

$$m_{\phi_R^j \phi_L^i} = -v_s v_u Y_x Y_\nu, \quad (40)$$

$$m_{\phi_L^i \phi_R^j} = -v_s v_u Y_x^T Y_\nu^T, \quad (41)$$

$$m_{\phi_R^j \phi_R^j} = \frac{1}{2} v_u^2 Y_\nu^T Y_\nu + m_\nu^2 + 2v_s^2 Y_x^2. \quad (42)$$

This matrix is diagonalized by Z^R :

$$Z^R m_{\nu^R}^2 Z^{R\dagger} = m_{\nu^R}^{dia}, \quad (43)$$

with

$$\phi_{Li} = \sum_j Z_{ji}^{R,*} \nu_j^R, \quad \phi_{Ri} = \sum_j Z_{ji}^{R,*} \nu_j^R, \quad i, j = 1, 2, 3. \quad (44)$$

Since we have introduced right-handed neutrinos, the neutrino mass matrix is extended to a 6×6 matrix consisting of the blocks given in Eq. (45). The mass matrix in basis $(\nu_L, \nu_R^*), (\nu_L, \nu_R^*)$ can be written as

$$m_\nu = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} v_u Y_\nu \\ \frac{1}{\sqrt{2}} v_u Y_\nu^T & \sqrt{2} v_s Y_x \end{pmatrix}. \quad (45)$$

This matrix is diagonalized by U_ν^V :

$$U_\nu^{V,*} m_\nu U_\nu^{V\dagger} = m_\nu^{dia}, \quad (46)$$

with

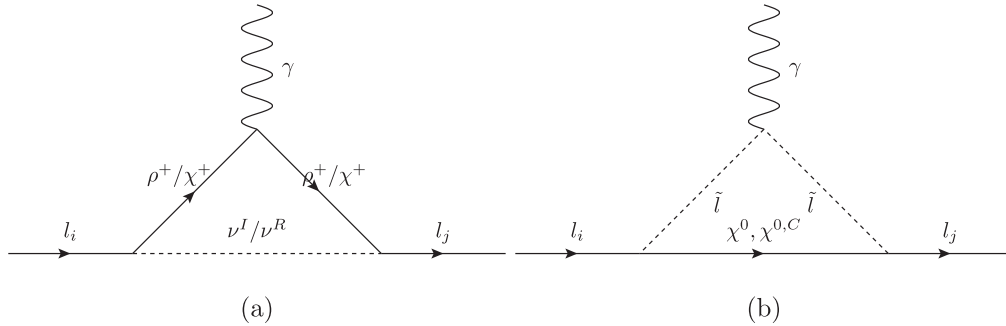


FIG. 1. (a) represents the contributions from charginos and sneutrino loops, and (b) represents the contributions from charged sleptons and neutrino loops.

$$\nu_{L,i} = \sum_j U_{\nu,ji}^{V,*} \lambda_{\nu,j}, \quad \nu_{R,i} = \sum_j U_{\nu,ji}^V \lambda_{\nu,j}^*. \quad (47)$$

The three generations of right-handed neutrinos first acquire Majorana masses through the breaking of the S field. Then, via a Yukawa left-right mixing coupling, the observed small masses for the left-handed neutrinos are produced by the type-I seesaw mechanism.

III. LEPTON-FLAVOR KIT

In this section, we investigate the amplitude and branching ratio of $\ell_i^- \rightarrow \ell_j^- \gamma$, $\ell_i^- \rightarrow \ell_j^- \ell_j^- \ell_j^+$, and $h \rightarrow \ell_i^+ \ell_j^-$.

A. Rare decay $\ell_i^- \rightarrow \ell_j^- \gamma$

At first, the off-shell amplitude for $l_i^- \rightarrow l_j^- \gamma$ is generally written as Eq. (48), where $P_L = (1 - \gamma_5)/2$ and $P_R = (1 + \gamma_5)/2$. The corresponding Feynman diagrams are depicted in Fig. 1.

$$L_{\ell\ell\gamma} = e \bar{\ell}_j [\gamma^\mu (K_1^L P_L + K_1^R P_R) + i m_{\ell_i} \sigma^{\mu\nu} q_\nu (K_2^L P_L + K_2^R P_R)] \ell_i A_\mu + \text{H.c.} \quad (48)$$

Using the amplitude Eq. (48), the decay width for $l_i^- \rightarrow l_j^- \gamma$ can be obtained easily as [29]

$$\Gamma(\ell_i^- \rightarrow \ell_j^- \gamma) = \frac{\alpha m_{\ell_i}^5}{4} (|K_2^L|^2 + |K_2^R|^2), \quad (49)$$

where α is the fine structure constant and the dipole Wilson coefficients $K_2^{L,R}$. The branching ratio for $l_i^- \rightarrow l_j^- \gamma$ is

$$\text{Br}(l_i^- \rightarrow l_j^- \gamma) = \frac{\Gamma(l_i^- \rightarrow l_j^- \gamma)}{\Gamma_{l_i^-}}, \quad (50)$$

where $\Gamma_{l_i^-}$ is the total decay width of the lepton l_i^- . In the numerical calculation, we use $\Gamma_\mu \approx 2.996 \times 10^{-19}$ GeV for the muon and $\Gamma_\tau \approx 2.265 \times 10^{-12}$ GeV for the tauon [30].

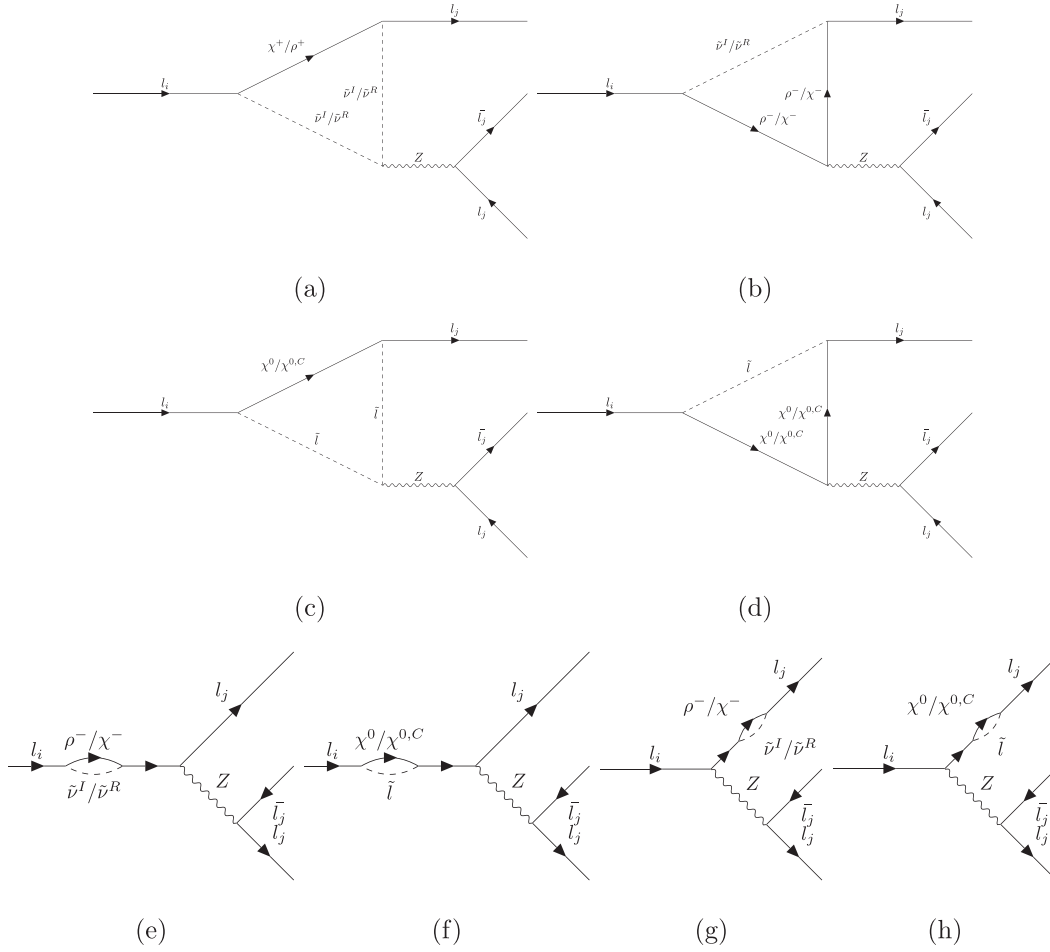


FIG. 2. The Z-penguin diagrams for $\ell_i^- \rightarrow \ell_j^- \ell_j^- \ell_j^+$. (a-d) denote the contributions of vertex diagrams for $\ell_i^- \rightarrow \ell_j^- \ell_j^- \ell_j^+$ from loops. (e-h) denote self-energy diagrams contributing to $\ell_i^- \rightarrow \ell_j^- \ell_j^- \ell_j^+$.

B. Rare decay $\ell_i^- \rightarrow \ell_j^- \ell_j^- \ell_j^+$

For the process $\ell_i^- \rightarrow \ell_j^- \ell_j^- \ell_j^+$, the effective amplitude includes the contributions from penguin-type and box-type diagrams [20,31]. Figures 2 and 3 represent the contributions from the Z boson and Higgs boson, respectively. Moreover, the box-type diagrams of $\ell_i^- \rightarrow \ell_j^- \ell_j^- \ell_j^+$ decay are shown as Fig. 4.

The four-fermion operator can be written as [31,32]

$$L_{4\ell} = \sum_{I=S,V,T|X,Y=L,R} A_{XX}^I \bar{\ell}_j \Gamma_I P_X \ell_i \bar{\ell}_j \Gamma_I P_Y \ell_j + \text{H.c.}, \quad (51)$$

where $I = \{S, V, T\}$, $X, Y = \{L, R\}$, $\Gamma_S = 1$, $\Gamma_V = \gamma_\mu$, and $\Gamma_T = \sigma_{\mu\nu}$. Using Eq. (51), the decay widths are [29,31,32]

$$\begin{aligned} \Gamma(\ell_i^- \rightarrow \ell_j^- \ell_j^- \ell_j^+) &= \frac{m_{\ell_i}^5}{512\pi^3} \left[e^4 (|K_2^L|^2 + |K_2^R|^2) \left(\frac{16}{3} \log \frac{m_{\ell_i}}{m_{\ell_j}} - \frac{22}{3} \right) + \frac{1}{24} (|A_{LL}^S|^2 + |A_{RR}^S|^2) + \frac{1}{12} (|A_{LR}^S|^2 + |A_{RL}^S|^2) \right. \\ &\quad + \frac{2}{3} (|\hat{A}_{LL}^V|^2 + |\hat{A}_{RR}^V|^2) + \frac{1}{3} (|\hat{A}_{LR}^V|^2 + |\hat{A}_{RL}^V|^2) + 6(|A_{LL}^T|^2 + |A_{RR}^T|^2) + \frac{e^2}{3} (K_2^L A_{RL}^{S*} + K_2^R A_{LR}^{S*} + \text{c.c.}) \\ &\quad - \frac{2e^2}{3} (K_2^L \hat{A}_{RL}^{V*} + K_2^R \hat{A}_{LR}^{V*} + \text{c.c.}) - \frac{4e^2}{3} (K_2^L \hat{A}_{RR}^{V*} + K_2^R \hat{A}_{LL}^{V*} + \text{c.c.}) - \frac{1}{2} (A_{LL}^S A_{LL}^{T*} + A_{RR}^S A_{RR}^{T*} + \text{c.c.}) \\ &\quad \left. - \frac{1}{6} (A_{LR}^S \hat{A}_{LR}^{V*} + A_{RL}^S \hat{A}_{RL}^{V*} + \text{c.c.}) \right]. \quad (52) \end{aligned}$$

Here, we have defined

$$\hat{A}_{XY}^V = A_{XY}^V + e^2 K_1^X. \quad (53)$$

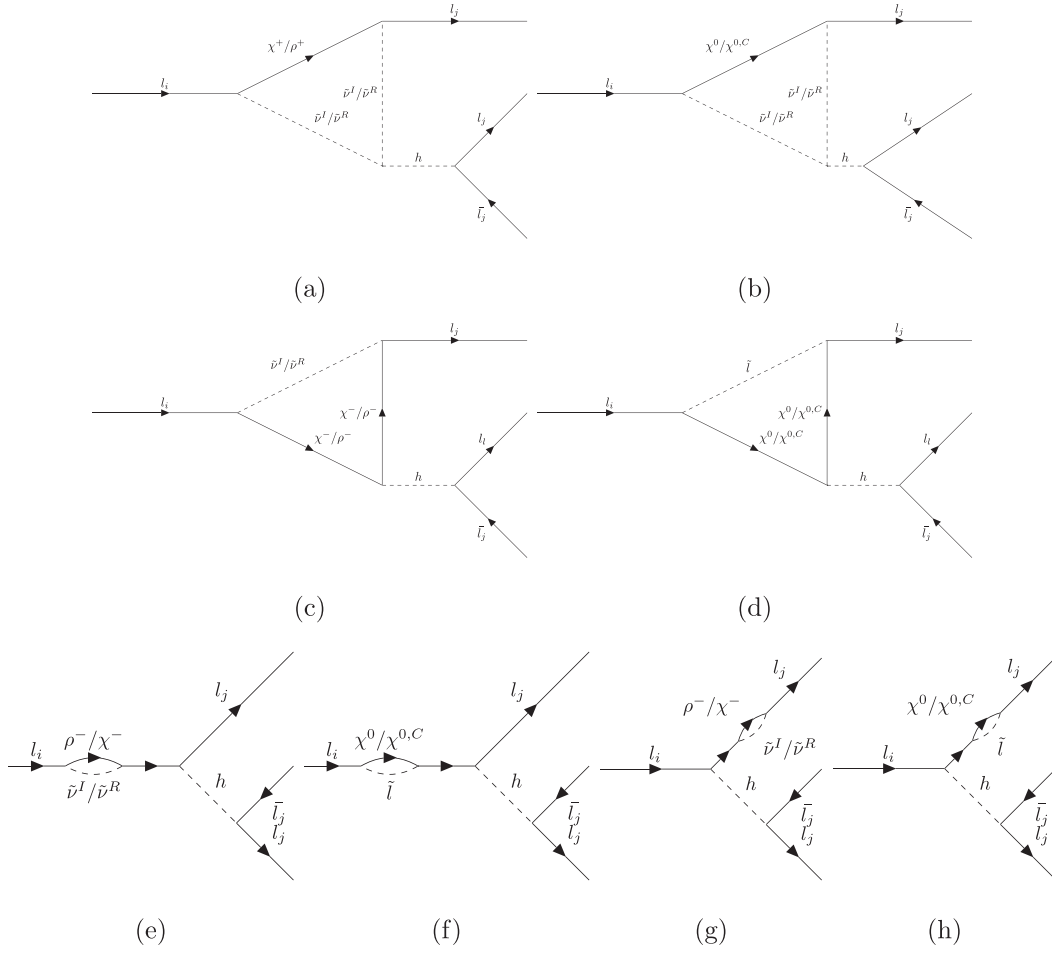


FIG. 3. The H -penguin diagrams for $\ell_i^- \rightarrow \ell_j^- \ell_j^- \ell_j^+$. (a–d) denote the contributions of vertex diagrams for $\ell_i^- \rightarrow \ell_j^- \ell_j^- \ell_j^+$ from loops. (e–h) denote self-energy diagrams contributing to $\ell_i^- \rightarrow \ell_j^- \ell_j^- \ell_j^+$.

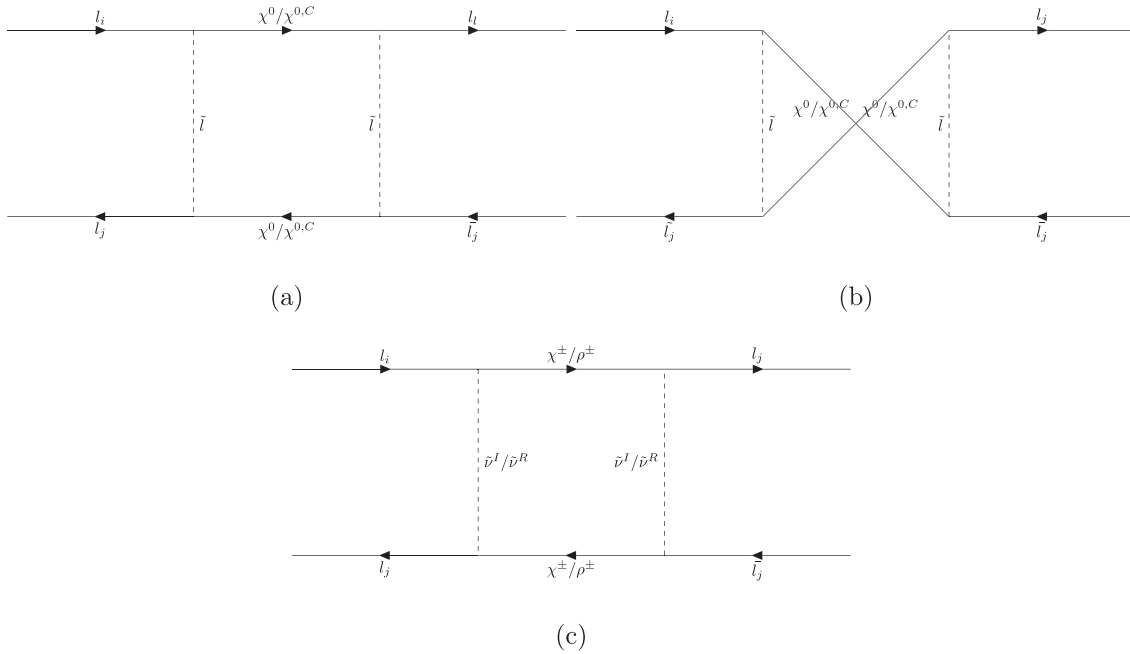


FIG. 4. Box-type diagrams for the process $\ell_i^- \rightarrow \ell_j^- \ell_j^- \ell_j^+$. (a), (b) represent the contributions from neutral fermion neutralinos and charge scalar loop sleptons, respectively, and (c) represents the contributions from charged fermion charginos and neutral scalar sneutrino loops.

The mass of the leptons in the final state has been neglected in this formula, with the exception of the dipole terms $K_2^{L,R}$, where an infrared divergence would otherwise occur due to the massless photon propagator [33]. And the branching ratio for $\ell_i^- \rightarrow \ell_j^- \ell_j^- \ell_j^+$ is

$$\text{Br}(\ell_i^- \rightarrow \ell_j^- \ell_j^- \ell_j^+) = \frac{\Gamma(\ell_i^- \rightarrow \ell_j^- \ell_j^- \ell_j^+)}{\Gamma_{\ell_i^-}}. \quad (54)$$

C. Higgs decay $h \rightarrow \ell_i^+ \ell_j^-$

Then turn to the Higgs LFV decay. The Lagrangian of $h - \ell - \ell$ interaction is given by

$$L_{\ell\ell h} = \bar{\ell}_j (S_1^L P_L + S_1^R P_R) \ell_i h. \quad (55)$$

Then the Higgs LFV decay widths can be written as [34]

$$\begin{aligned} \Gamma(h \rightarrow \ell_i^+ \ell_j^-) &\equiv \Gamma(h \rightarrow \ell_i \bar{\ell}_j) + \Gamma(h \rightarrow \bar{\ell}_i \ell_j) \\ &= \frac{1}{16\pi m_h} \left[\left(1 - \left(\frac{m_{\ell_i} + m_{\ell_j}}{m_h}\right)^2\right) \left(1 - \left(\frac{m_{\ell_i} - m_{\ell_j}}{m_h}\right)^2\right) \right]^{1/2} \\ &\quad \times [(m_h^2 - m_{\ell_i}^2 - m_{\ell_j}^2)(|S_L|^2 + |S_R|^2)_{ij} - 4m_{\ell_i} m_{\ell_j} \text{Re}(S_L S_R^*)_{ij}] + (i \leftrightarrow j). \end{aligned} \quad (56)$$

The branching ratio for $h_i \rightarrow \ell_i^+ \ell_j^-$ is

$$\text{Br}(h_i \rightarrow \ell_i^+ \ell_j^-) = \frac{\Gamma(h_i \rightarrow \ell_i^+ \ell_j^-)}{\Gamma_h}, \quad (57)$$

where Γ_H is the total decay width of Higgs h . In the numerical calculation, we use $\Gamma_h \approx 3.7$ MeV [30]. And Higgs LFV diagrams is shown as plotted in Fig. 5.

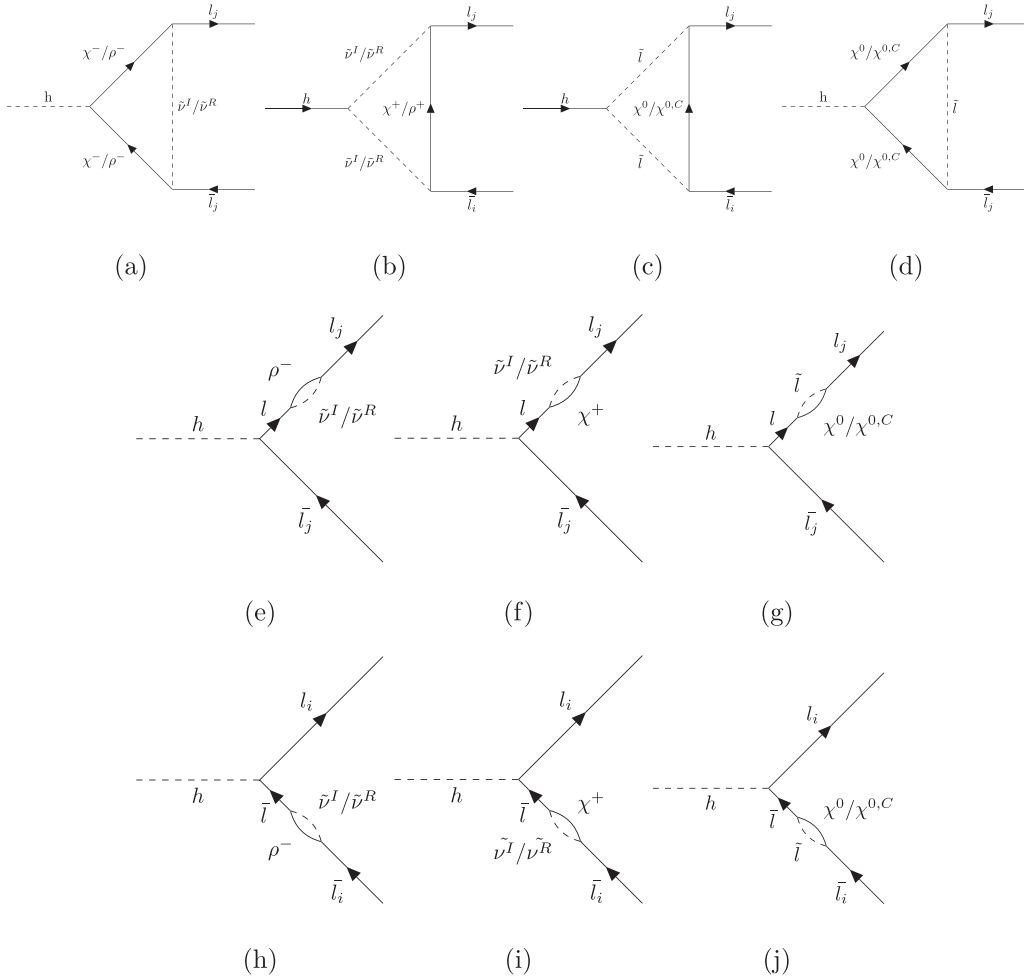


FIG. 5. One-loop diagrams for the Higgs LFV in the MRSSMSeesaw model.

TABLE III. Benchmark points. Dimensionful parameters are given in GeV. The Higgs sector of the benchmark points was checked against existing experimental data using HiggsTools.

$\tan\beta$	λ_D	λ_U	Λ_D	Λ_U	M_D^B	M_D^W	v_T	m_h	m_{χ^0}	S	T	U	HiggsBounds result
39.35	0.15	-0.35	0.66	-2.14	141.44	339.66	-0.52	125.10	146.41	0.05	0.12	-0.002	1

IV. CONSTRAINTS AND NUMERICAL ANALYSIS

To delve into the parameter space of our model, we have encoded MRSSMSeesaw into SARAH [33,35–40]. It results in the generation of a Fortran code that interfaces with the SPheno [41–43] library, enabling the computation of the particle spectrum, decay processes, and flavor processes including rare LFV ones.

We use BSMart [44,45] version 1.5, a Python program for the exploration of parameter spaces of the BSM model including SPheno [41–43], HiggsTools [46], and so on. We have designated the unknown parameters as variables for scanning within BSMart [44,45]. The SM input parameters are consistently configured as follows [30]:

$$\begin{aligned}
m_t &= 172.89 \text{ GeV}, & \hat{m}_b^{\overline{\text{MS}}}(\hat{m}_b) &= 4.18 \text{ GeV}, & M_Z &= 91.1887 \text{ GeV}, \\
m_\tau &= 1.776690 \text{ GeV}, & \alpha^{-1} &= 137.035999139, & \alpha_s^{\overline{\text{MS}}}(m_Z) &= 0.1187, \\
G_F &= 1.166370 \times 10^{-5} \text{ GeV}^{-2}, \\
m_e &= 0.511 \times 10^{-3} \text{ GeV}, & m_\mu &= 0.105 \text{ GeV}.
\end{aligned}$$

We fix $M_O^D = 1.5 \text{ TeV}$, $m_O^2 = m_{R_u}^2 = m_{R_d}^2 = (1 \text{ TeV})^2$, $v_S = -1 \text{ TeV}$, $Y_x = -1$, and $Y_\nu = 10^{-6}$, and for soft-breaking slepton mass parameters, we set $m_{\tilde{l}} = m_{\tilde{e}} = m_{\tilde{\nu}} = 1 \text{ TeV}$. We then scan within the ranges

$$\lambda_d, \lambda_u, \Lambda_d, \Lambda_u \in [-3, 3], \quad \tan\beta \in [1, 50], \quad (58)$$

$$M_B^D, M_W^D \in [100, 3000], \quad v_T \in [-4, 4], \quad \mu_u, \mu_d \in [100, 20000]. \quad (59)$$

To obtain the result we implement a Markov chain Monte Carlo scan in BSMart [44,45] using a likelihood function based on the Higgs mass (a Gaussian likelihood peaked at 125.2 GeV with variance 3 GeV); the W boson mass (a Gaussian likelihood peaked at 80.3559 GeV with variance 30 MeV); Higgs data from HiggsTools [46]; and the p value for the SM-like Higgs.

The following constraints are considered in the scanning [30]:

- (1) h_1 is chosen to be the SM Higgs boson with $m_1 = 125.2 \pm 3.0 \text{ GeV}$;
- (2) measurements of the W boson mass with $m_W = 80.3692 \pm 0.003 \text{ GeV}$;
- (3) The STU parameters with $S = -0.04 \pm 0.10$, $T = 0.01 \pm 0.12$, and $U = 0.01 \pm 0.09$;
- (4) All the points must be allowed by HiggsBounds.

Considering these limitations, we adopt the parameters in Table III in the numerical calculation.

Concerning the neutrino mass term we newly introduced, there are two possible neutrino mass orderings (see Table IV): The first is the normal ordering (NO), and the second is the inverted ordering (IO). The effective mass matrix of the left-handed neutrinos is given in the usual seesaw approximation as [15,30,47–51]

$$m_\nu = -\frac{1}{2} v_u^2 Y_\nu^T \cdot (\sqrt{2} Y_x v_s)^{-1} \cdot Y_\nu. \quad (60)$$

Since the neutrino mass matrix is complex symmetric, Eq. (60) is diagonalized by [30]

$$\hat{m}_\nu = U^T \cdot m_\nu \cdot U, \quad (61)$$

where U is the Pontecorvo-Maki-Nakagawa-Sakata matrix [52]. Inverting Eq. (60), Y_ν can be written as [53]

$$Y_\nu = \sqrt{2} \frac{i}{v_u} \sqrt{\sqrt{2} Y_x v_s} \cdot R \cdot \sqrt{\hat{m}_\nu} \cdot U^\dagger, \quad (62)$$

TABLE IV. Neutrino oscillation parameters [30].

Observable	Normal hierarchy	Inverted hierarchy
$\theta_{12}(^{\circ})$	$33.41^{+0.75}_{-0.72}$	$33.41^{+0.75}_{-0.72}$
$\theta_{23}(^{\circ})$	$49.1^{+1.0}_{-1.3}$	$49.5^{+0.9}_{-1.2}$
$\theta_{13}(^{\circ})$	$8.54^{+0.11}_{-0.12}$	$8.57^{+0.12}_{-0.11}$
$\Delta m_{12}^2 (10^{-5} \text{ eV}^2)$	$7.41^{+0.21}_{-0.20}$	$7.41^{+0.21}_{-0.20}$
$\Delta m_{32}^2 (10^{-3} \text{ eV}^2)$	$2.437^{+0.028}_{-0.027}$	$-2.498^{+0.028}_{-0.027}$
$\delta_{CP} (^{\circ})$	197^{+42}_{-25}	286^{+27}_{-32}

where $\hat{m}_\nu$ is the diagonal matrix with m_i eigenvalues and R , in general, is a complex orthogonal matrix. Note that in the special case we set $R = 1$. There are three light neutrino masses in $\hat{m}_\nu$ and the three heavy “right-handed” neutrino masses in $\hat{M}_R = \sqrt{2}Y_\chi v_s$. For U we will use the standard form [30]

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (63)$$

with $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$.

For NO, we set

$$\begin{cases} m_{\nu,1} = 0.01 \text{ eV}, \\ m_{\nu,2} = \sqrt{m_1^2 + \Delta m_{21}^2} \approx 0.0123 \text{ eV}, \\ m_{\nu,3} = \sqrt{m_1^2 + \Delta m_{31}^2} \approx 0.0506 \text{ eV}. \end{cases}$$

For IO, we set

$$\begin{cases} m_{\nu,3} = 0.01 \text{ eV}, \\ m_{\nu,1} = \sqrt{m_3^2 + |\Delta m_{32}^2|} \approx 0.0509 \text{ eV}, \\ m_{\nu,2} = \sqrt{m_1^2 + \Delta m_{21}^2} \approx 0.0516 \text{ eV}. \end{cases}$$

We consider the right-handed neutrino masses to be at the TeV scale. Furthermore, we find that under this assumption the Yukawa coupling Y_ν is constrained to be on the order of 10^{-7} . The newly introduced right-handed neutrinos contribute to LFV processes via Y_ν , rendering the impact of the right-handed neutrino coupling Y_χ on LFV negligible. The corresponding diagrams are shown in Fig. 12. Since Y_χ has a negligible effect, we simply set $v_s = -1$ TeV and $Y_\chi = -1$ for simplicity. Then, using Eq. (62), we get two different Y_ν :

$$Y_\nu^{\text{NO}} = \begin{pmatrix} 4.06 \times 10^{-7} & 3.84 \times 10^{-8} + 0.75 \times 10^{-8}i & -2.92 \times 10^{-7} + 3.25 \times 10^{-8}i \\ 3.84 \times 10^{-8} - 3.75 \times 10^{-8}i & 1.15 \times 10^{-6} & 4.49 \times 10^{-7} - 1.08 \times 10^{-8}i \\ -2.92 \times 10^{-7} - 3.25 \times 10^{-8}i & 4.49 \times 10^{-7} + 1.08 \times 10^{-8}i & 9.52 \times 10^{-7} \end{pmatrix},$$

$$Y_\nu^{\text{IO}} = \begin{pmatrix} 1.53 \times 10^{-6} & 9.46 \times 10^{-8} - 2.80 \times 10^{-8}i & 7.56 \times 10^{-8} - 2.42 \times 10^{-8}i \\ 9.46 \times 10^{-8} + 2.80 \times 10^{-8}i & 1.07 \times 10^{-6} & -4.19 \times 10^{-7} - 2.11 \times 10^{-10}i \\ 7.56 \times 10^{-8} + 2.42 \times 10^{-8}i & -4.19 \times 10^{-7} + 2.11 \times 10^{-10}i & 1.19 \times 10^{-6} \end{pmatrix}.$$

For the soft-breaking slepton mass matrices $M_{\tilde{l}}$ and $M_{\tilde{e}}$, we introduce the slepton-flavor mixings, which take into account the off-diagonal terms:

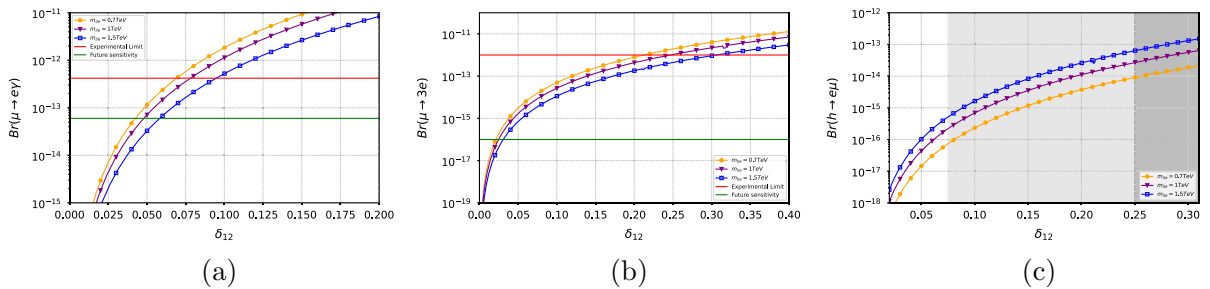


FIG. 6. The influence of δ_{12} and $m_{\tilde{l}/\tilde{e}}$ to $\text{Br}(\mu \rightarrow e\gamma)$, $\text{Br}(\mu \rightarrow 3e)$, and $\text{Br}(h \rightarrow e\mu)$, where the red and green lines denote the present limits and future sensitivities, respectively. (c) Light gray represents the limit for $\mu \rightarrow e\gamma$, while dark gray represents the limit for $\mu \rightarrow 3e$.

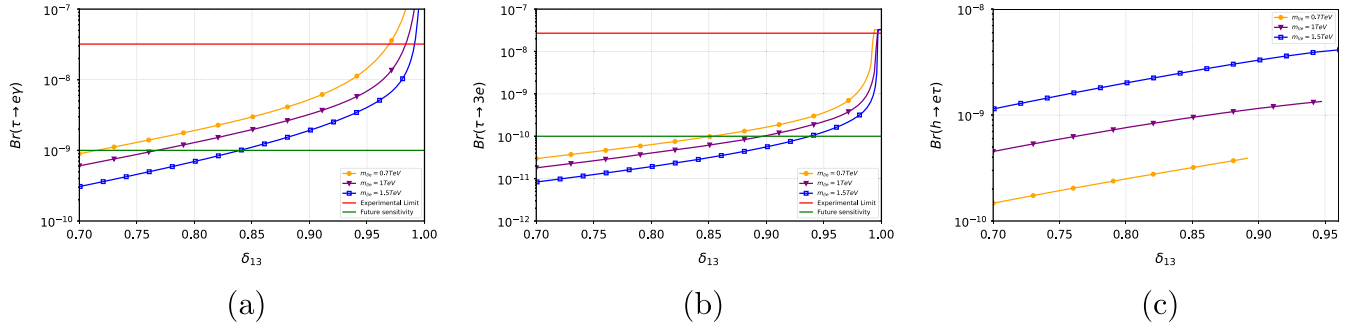


FIG. 7. The influence of δ_{13} and $m_{\tilde{l}/\tilde{e}}$ to $\text{Br}(\tau \rightarrow e\gamma)$, $\text{Br}(\tau \rightarrow 3e)$, and $\text{Br}(h \rightarrow e\tau)$, where the red and green lines denote the present limits and future sensitivities, respectively.

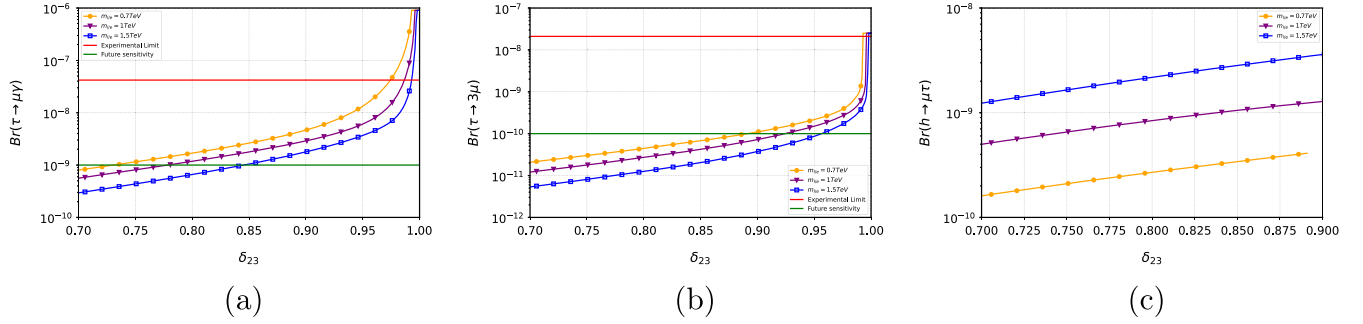


FIG. 8. The influence of δ_{23} and $m_{\tilde{l}/\tilde{e}}$ to $\text{Br}(\tau \rightarrow \mu\gamma)$, $\text{Br}(\tau \rightarrow 3\mu)$, and $\text{Br}(h \rightarrow \mu\tau)$, where the red and green lines denote the present limits and future sensitivities, respectively.

$$M_{\tilde{l}} = \begin{pmatrix} 1 & \delta_{12} & \delta_{13} \\ \delta_{21} & 1 & \delta_{23} \\ \delta_{31} & \delta_{32} & 1 \end{pmatrix} m_{\tilde{l}}, \quad M_{\tilde{e}} = \begin{pmatrix} 1 & \delta_{12} & \delta_{13} \\ \delta_{21} & 1 & \delta_{23} \\ \delta_{31} & \delta_{32} & 1 \end{pmatrix} m_{\tilde{e}}. \quad (64)$$

Then we plot $\text{Br}(\mu \rightarrow e\gamma)$, $\text{Br}(\mu \rightarrow 3e)$, and $\text{Br}(h \rightarrow e\mu)$ versus $\delta_{12} = \delta_{21}$ for $\delta_{13} = \delta_{31} = \delta_{23} = \delta_{32} = 0$ in Fig. 6. In Fig. 7, we picture $\text{Br}(\tau \rightarrow e\gamma)$ and $\text{Br}(\tau \rightarrow 3e)$ versus $\delta_{13} = \delta_{31}$ for $\delta_{12} = \delta_{21} = \delta_{23} = \delta_{32} = 0$, and $\text{Br}(\tau \rightarrow \mu\gamma)$ and $\text{Br}(\tau \rightarrow 3\mu)$ versus $\delta_{23} = \delta_{32}$ for $\delta_{12} = \delta_{21} = \delta_{13} = \delta_{31} = 0$ are drawn in Fig. 8.

It is obvious that the LFV rates increase with the increasing of slepton mixing parameters. Figure 6 shows that the present experimental limit bound of $\text{BR}(\mu \rightarrow e\gamma)$ constrains $\delta_{12} < 0.075$, which also coincides with the present experimental limit bound of $\text{Br}(\mu \rightarrow 3e)$. Because of the constraints of the δ_{12} boundary condition, we obtained Higgs LFV $\text{Br}(h \rightarrow e\mu)$ at 10^{-15} . In addition, from Fig. 7 we can see that $\text{Br}(\tau \rightarrow e\gamma)$ can reach the corresponding present experimental limit bounds, but $\text{Br}(\tau \rightarrow 3e)$ cannot. However, the high future experimental sensitivities still keep a hope to detect $\text{Br}(\tau \rightarrow e\gamma)$ and $\text{Br}(\tau \rightarrow 3e)$, and we get $\text{Br}(h \rightarrow e\tau)$ at 10^{-9} . Moreover, $\text{Br}(\tau \rightarrow \mu\gamma)$ can reach the corresponding present experimental limit bounds, but $\text{Br}(\tau \rightarrow 3\mu)$ cannot reach. And we investigated Higgs LFV $\text{Br}(h \rightarrow \mu\tau)$ at 10^{-9} .

Notice that we must consider the loop contribution by sleptons, neutralinos, and charginos for Higgs mass that the diagrams are shown as Fig. 9.

Furthermore, our analysis reveals that both the flavor mixing parameters δ_{ij} and the slepton mass contribute significantly to LFV observables. We then picture the contribution of LFV observables versus $m_{\tilde{l}/\tilde{e}}$. In Fig. 10, we found that the LFV observables $\text{Br}(\ell_i \rightarrow \ell_j\gamma)$ and $\text{Br}(\ell_i \rightarrow 3\ell_j)$ decrease as the mass of sleptons increases. That is because the mass parameters $m_{\tilde{l}/\tilde{e}}$ will appear in the denominator terms of expression for the decay width. However, we observed that Higgs LFV exhibits the opposite trend; it increases with $m_{\tilde{l}/\tilde{e}}$. As shown in

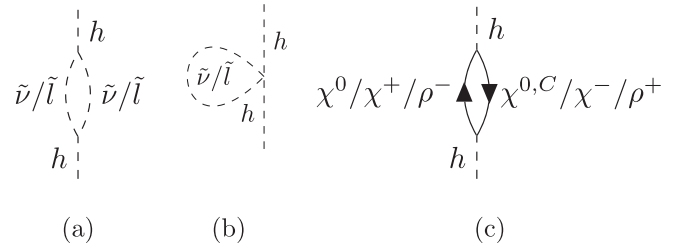


FIG. 9. The influence of charginos, neutralinos, sneutrinos, and sleptons in Higgs self-energies.

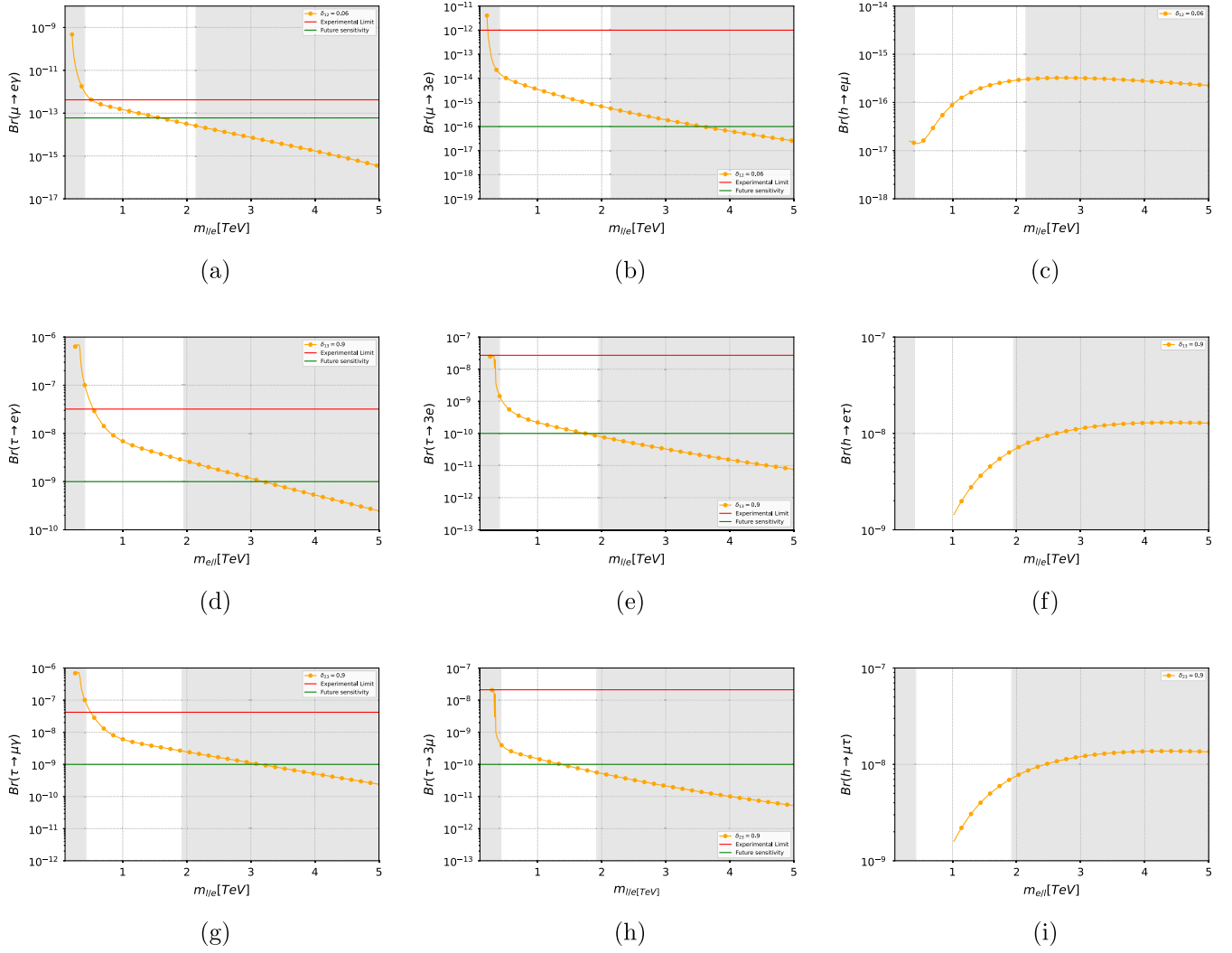


FIG. 10. The influence of slepton mass for LFV where the shadowed area represents exclusion by Higgs mass at 5σ , where the red and green lines denote the present limits and future sensitivities, respectively.

Figs. 9(a) and 9(b), the mass of sleptons can affect the Higgs one-loop self-energy. As the slepton mass increases, the Higgs mass corrections from loop diagrams decrease. Using Eq. (57), since the Higgs mass appears in the denominator, this leads to an increase in the $\text{Br}(h \rightarrow \ell_i^+ \ell_j^-)$.

We then illustrated the impact of the MRSSM parameters on these LFV processes which are shown in Fig. 11. We find that, within the allowed Higgs mass range, M_D^B has no significant impact on these nine LFV processes. That is because the term $g_1 v_S M_D^B$ appears in the scalar particle mass matrix. Because of the constraints of v_S at 1 TeV and the tadpole equation, the range of M_D^B is significantly restricted. But, in contrast, the parameter range of M_D^W is larger than that of M_D^B . This is because M_D^W is coupled to the T field, and, due to the smallness of the T field's VEV, the contribution of M_D^W to the Higgs mass is not significant. They also appear in the tree-level neutralino chargino

sleptons and squark mass matrices. Since M_D^B and M_D^W affect the masses of particles, they can influence the LFV processes by altering these particles in the decay loops. We see that in Figs. 11(d)–(f) the LFV decreases as M_D^W increases.

Yukawa-like parameters λ and Λ appear in the Higgs neutralino and chargino mass matrix. So these parameters will also affect the self-energy correction of neutralino, chargino, sneutrino, and sleptons, thereby influencing LFV. The impact of these parameters would be complex. Figures 11(g)–(i) show consistency with our analysis. In the Higgs mass matrix, the parameter λ_D appears in terms like $\lambda_D v_S$, which can significantly impact the tree-level Higgs mass and, consequently, influence the Higgs LFV.

The impact of the newly introduced parameters, Y_x and $m_{\tilde{\nu}}$, on the LFV process is not very significant, as shown in Fig. 12. This is because the Yukawa coupling Y_ν is small.

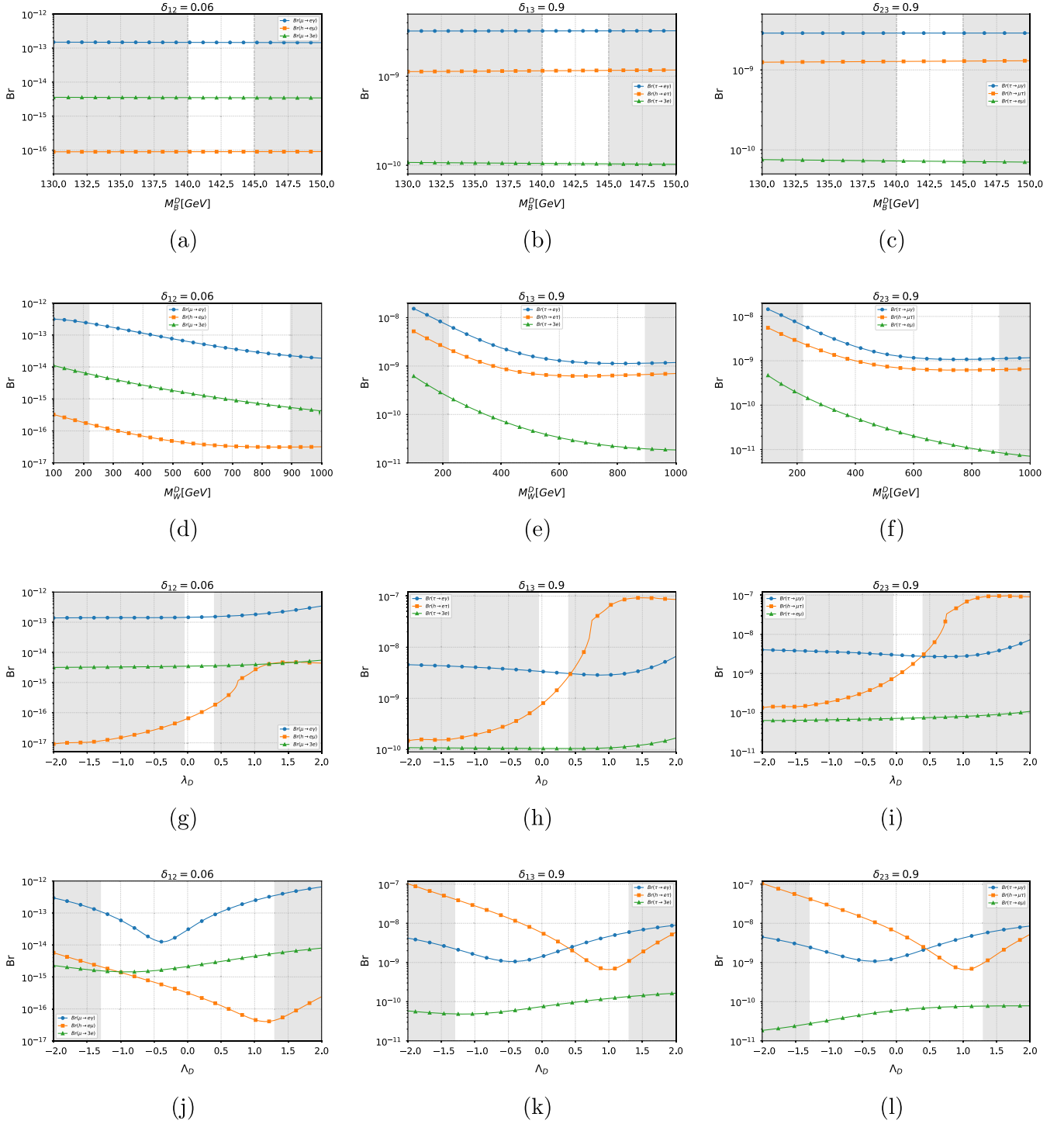
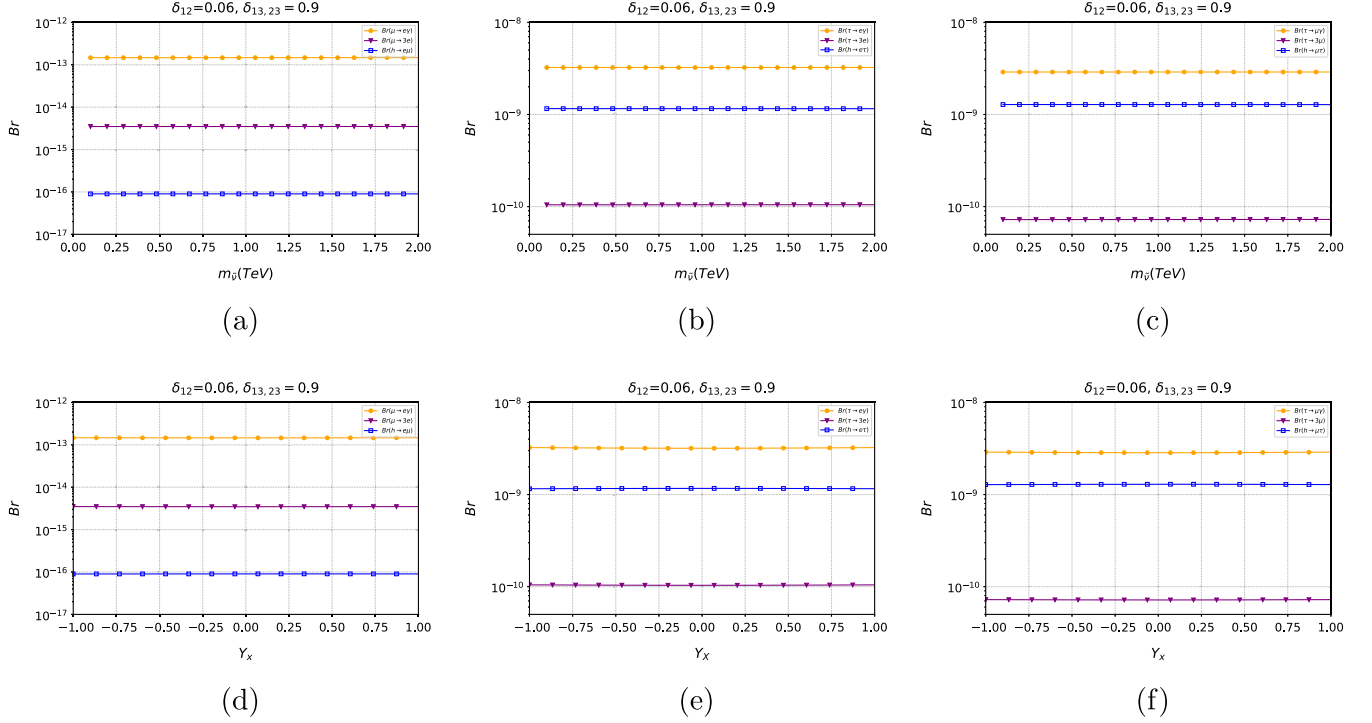
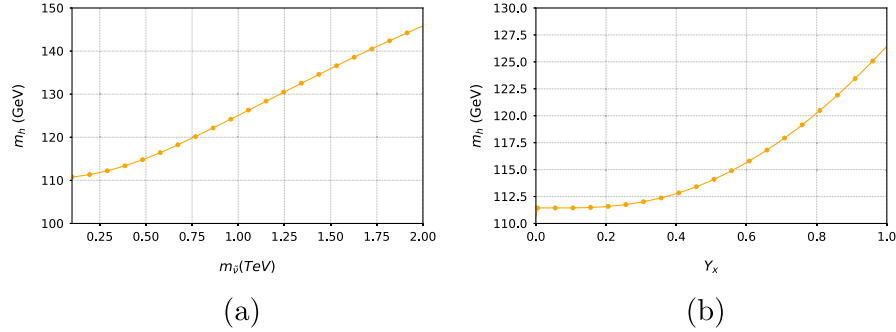


FIG. 11. The influence of Yukawa-like parameter λ_D , Λ_D , and Dirac gaugino parameters M_B^D and M_W^D where the shadowed area represents exclusion by Higgs mass at 5σ .

The right-handed neutrinos, whose masses are influenced by Y_x , participate in the LFV processes through the coupling Y_ν to the left-handed neutrinos. Therefore, their contribution is suppressed by the smallness of Y_ν .

The impact of the right-handed sneutrino mass parameter $m_{\tilde{\nu}}$ on LFV processes is still negligible. That is because the contribution from right-handed sneutrino vertices to LFV is suppressed by the coupling Y_ν .

FIG. 12. The influence of new parameters Y_x and $m_{\tilde{\nu}}$ to LFV; δ_{ij} is defined as in Eq. (64).FIG. 13. The influence of new parameters Y_x and $m_{\tilde{\nu}}$ to the Higgs mass.

Since sneutrinos appear in the Higgs self-energy correction, as shown in Fig. 13, these two new parameters do have a slightly significant impact on the Higgs mass.

V. CONCLUSION

In this work, we analyzed the MRSSMSeesaw model, which extends the MRSSM by incorporating right-handed neutrino field to generate neutrino masses via the type-I seesaw mechanism. We focus on LFV processes, including rare decays such as $\ell_i \rightarrow \ell_j \gamma$, $\ell_i \rightarrow 3\ell_j$, and Higgs decays $h \rightarrow \ell_i \ell_j$. We derived the mass matrices for Higgs bosons, neutralinos, charginos, sleptons, neutrinos, and sneutrinos.

Our numerical analysis showed that LFV decay rates are highly sensitive to the slepton mixing parameters δ_{ij} .

The experimental limit on $\text{Br}(\mu \rightarrow e\gamma)$ constrains $\delta_{12} < 0.075$, while Higgs LFV decay rate $\text{Br}(h \rightarrow e\mu)$ up to $\mathcal{O}(10^{-15})$ and $\text{Br}(h \rightarrow e\tau)$ and $\text{Br}(h \rightarrow \mu\tau)$ reach up to $\mathcal{O}(10^{-9})$. We found that $\text{Br}(\ell_i \rightarrow \ell_j \gamma)$ and $\text{Br}(\ell_i \rightarrow 3\ell_j)$ decrease with increasing slepton masses, while Higgs LFV decay rates increase due to suppressed loop corrections to the Higgs mass. Because M_B^W , M_D^W , and Yukawa-like parameters λ and Λ appear at the tree-level mass matrix of Higgs neutralinos, charginos, sneutrinos, and sleptons, they will have significant impact on the considered LFV processes. Because of the smallness of Y_ν , the new parameters Y_x and $m_{\tilde{\nu}}$ negligibly affect LFV processes but slightly influence the Higgs mass via sneutrino loops.

In conclusion, our study provides insights into the MRSSMSeesaw model's phenomenology; through the

analysis of LFV mixing parameters, we find that the nondiagonal elements which correspond to the generations of the initial lepton and final lepton are the main sensitive parameters and LFV sources. Most of the parameters are able to break the experimental upper limit, providing new ideas for the search for new physics.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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