

Beautiful exotica

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The chiral quark-soliton model applied to baryons with one heavy quark predicts new exotic states belonging to three $\overline{15}$ SU(3) multiplets. Here, we extend previous analysis of charm exotica to the case of beauty. All model parameters are fixed from the charm sector and from the nonexotic b -baryons. We present predictions for masses of beautiful exotica and discuss the decay widths, which are either very small or relatively large.

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I. INTRODUCTION

Baryons with one heavy quark $Q = b$ or c can be classified according to the SU(3) structure of the light quarks. Both the quark model and the chiral quark-soliton model (χ QSM) [1–4] predict that the light quarks in heavy baryons belong to an SU(3) antitriplet of spin 0 or a sextet of spin 1. Adding a heavy quark results in an antitriplet of spin 1/2 and two hyperfine split sextets of spin 1/2 and 3/2. This structure is fully confirmed by experiment [5].

In the χ QSM the SU(3) structure follows from the collective quantization of the soliton rotations [6–8]. While the resulting representations for the ground states are the same as in the naive quark model, the χ QSM predicts exotic representations of positive parity that cannot be straightforwardly constructed in the quark picture. They correspond to pentaquarks (4 quarks and one antiquark).

In the light sector a prominent example is the elusive $\Theta^+ = uud\bar{s}$ state, the lightest member of $\overline{10}$ SU(3) representation, which is predicted to be light and very narrow [9–12]. It was pointed out in Refs. [2–4] that the lowest lying exotic SU(3) representation in the heavy baryon case is $\overline{15}$.

In fact there exist two exotic $\overline{15}$ light SU(3) multiplets [2]. One, carrying angular momentum $J = 1$, leads to two hyperfine split heavy baryon multiplets, and the second one with $J = 0$, which is 180 MeV heavier [13], leads to a spin 1/2 heavy exotica. In Ref. [13] we asked a question whether there is space in the experimental charm spectra for 45 (or perhaps it is better to say: 18 isospin

submultiplets) exotic states? Here we repeat the same question for b exotica.

Discussing charm sector in Refs. [2–4,13] it was argued that two out five Ω^{c0} states, namely $\Omega^{c0}(3050)$ and $\Omega^{c0}(3119)$, discovered in 2017 by the LHCb Collaboration [14–16], could belong to the exotic $\overline{15}$, mainly due to the very small widths and to the fact that their hyperfine splittings are equal to the ones of the ground state sextet.¹ For different pentaquark interpretations see Refs. [17–20].

As far as the remaining charm pentaquarks are concerned, it was shown in Ref. [13] that the members of multiplets based on $(\overline{15}, J = 1)$ soliton are very narrow (some hint of this behavior was already discussed in Ref. [3]), and — on the contrary — states associated with $(\overline{15}, J = 0)$ multiplet are wide. Both extremes explain why these states are difficult to detect; it is easy to miss a very narrow or very broad resonant signal.

In the present paper we extend the analysis of Ref. [13] to exotica with a b quark. All model parameters are fixed from the spectra of regular baryons both in c and b sector [1–3], and from charm exotica [13]. Therefore, predictions for beautiful pentaquarks do not require any extra input. For decays, we use parameter values from Ref. [3]. The conclusions are basically the same as in the charm sector: pentaquarks in $(\overline{15}, J = 1)$ are very narrow and in $(\overline{15}, J = 0)$ very wide.

The paper is organized as follows. In Sec. II we briefly review the main features of the χ QSM. In Sec. III we first derive analytical formulas for the baryon masses and then fix splitting parameters as functions of the strange moment of inertia $1/I_2$. After constraining $1/I_2$ we compute all pentaquark masses. Next, in Sec. IV, we discuss and compute decays widths, and finally we conclude in Sec. V.

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¹Note that the ground state sextet and exotic $\overline{15}$ belong to the same rotational band, and therefore should have approximately the same value of the hyperfine splitting.

II. BARYONS IN THE CHIRAL QUARK-SOLITON MODEL

It is beyond scope of the present paper to review the χ QSM in detail. We refer the interested reader to the original paper by Diakonov *et al.* [21] and the reviews of Refs. [11,12,22–24] (and references therein).

In the chiral quark model light quarks couple in a chirally invariant way to the Goldstone boson fields that arise from the spontaneous breaking of chiral symmetry. Goldstone bosons are parametrized by an SU(3) matrix $U[\pi, K, \eta]$ with a coupling to quarks being the constituent quark mass $M \sim 350$ MeV. For the vacuum configuration, where $U = 1$, the system reduces to three free quarks of mass M . On the top of that one has to add explicit mass terms related to the Goldstone boson masses (i.e., to the current quark masses). In the present approach this explicit symmetry breaking is treated perturbatively.

The soliton arises when the chiral field U is taken in the form of the so called *hedgehog* Ansatz

$$U_0 = \begin{bmatrix} & & 0 \\ & u_0 & \\ & & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (1)$$

where

$$u_0 = \exp(i\mathbf{n} \cdot \boldsymbol{\tau} P(r)). \quad (2)$$

Here $\mathbf{n} = \mathbf{r}/r$ and the profile function $P(r)$ has to vanish at $r \rightarrow \infty$.

If $P \equiv 0$, the level structure of the corresponding Dirac equation shows a gap for $-M < E < M$. For non zero $P(r)$ the eigenvalue structure changes. The lowest positive energy level (the valence level) sinks into the gap reducing energy, while the sea levels are distorted in a way that energy increases. Such distortion interacts with valence quarks changing their wave functions, which in turn modify the sea until a stable configuration is reached [25]. This configuration is called a *chiral soliton*. Note that this procedure is carried out in a large N_c limit where we have N_c identical copies of the Dirac structure described above.

In practical terms, a soliton corresponds to a configuration, which minimizes the aggregate energy

$$M_{\text{sol}} = N_c \left[E_{\text{val}} + \sum_{E_n < 0} (E_n - E_n^{(0)}) \right], \quad (3)$$

where the sea energy is computed with respect to the vacuum, and the whole sum is suitably regularized. The minimization is performed with respect to the profile function P [25].

What happens when one of the N_c light quarks is replaced by a heavy one? The answer can be inferred from Ref. [26] where the authors studied soliton behavior in the

limit where the current quark masses $m \rightarrow \infty$. It turns out that for sufficiently large m the soliton ceases to exist, and the correct heavy quark limit is achieved.

In the present context, Eq. (3) takes the following form:

$$M_{\text{sol}} = (N_c - 1) \left[E_{\text{val}} + \sum_{E_n < 0} (E_n - E_n^{(0)}) \right] + \left[E_{\text{val}}(m_Q) + \sum_{E_n < 0} (E_n(m_Q) - E_n^{(0)}(m_Q)) \right]. \quad (4)$$

As was argued in Ref. [26], for large m_Q , the sum over the sea quarks in the second line of Eq. (4) vanishes, and $E_{\text{val}}(m_Q) \approx m_Q$. One copy of the soliton ceases to exist; however, the remaining $N_c - 1$ quarks still form a stable soliton. The total energy reads therefore

$$M_{\text{sol}} = M'_{\text{sol}} + m_Q, \quad (5)$$

where the prime refers to the soliton constructed from $N_c - 1$ quarks. In the following we will skip the prime.

The soliton constructed in a way described above does not carry any quantum numbers except for the baryon number resulting from the valence quarks. It is therefore often referred to as a *classical baryon*. Spin and SU(3) structures appear when the soliton rotations in space and flavor are quantized [27]. This procedure results in a *collective* Hamiltonian analogous to the one of a quantum mechanical symmetric top, however, due to the Wess-Zumino-Witten term [28,29] and the form of the *hedgehog* Ansatz (1), the allowed Hilbert space is truncated to the representations that contain states of hypercharge $Y' = N_{\text{val}}/3$. For light baryons $N_{\text{val}} = 3$, these are octet and decuplet of ground state baryons [6–8].

In the case of heavy baryons $N_{\text{val}} = 2$ (or rather $N_c - 1$) and the allowed SU(3) representations have to contain states of $Y' = 2/3$. These are $\bar{\mathbf{3}}$, $\mathbf{6}$, and exotic $\bar{\mathbf{15}}$ shown in Fig. 1. They correspond to the rotational excitations of the meson mean field, which is essentially the same as for light baryons.

The soliton mass with one heavy quark including rotational energy takes the following form [30] in the chiral limit:

$$\begin{aligned} \mathcal{M}_{\mathcal{R}, T'}^Q &= M_{\text{sol}} + m_Q \\ &+ \frac{1}{2I_2} \left[C_2(\mathcal{R}) - T'(T' + 1) - \frac{3}{4} Y'^2 \right] \\ &+ \frac{1}{2I_1} T'(T' + 1). \end{aligned} \quad (6)$$

Here $\mathcal{R}$ refers to the allowed SU(3) representation of the $N_c - 1$ light quark sector, $C_2(\mathcal{R})$ stands for the SU(3) Casimir operator. Isospin T' corresponds to the states with hypercharge equal to Y' and is equal to the soliton angular

momentum [6–8], which in the following will be denoted by J . $M_{\text{sol}} \sim N_c - 1$ denotes the classical soliton mass; $I_{1,2} \sim N_c - 1$ are moments of inertia. We see that the two lowest representations correspond to the naive quark model. However, the χ QSM predicts two exotic $\overline{\mathbf{15}}$ representations of spin 1 and 0.

Next, we have to include the explicit symmetry breaking Hamiltonian, which takes the following form [31]:

$$\hat{H}_{\text{sb}} = \alpha D_{88}^{(8)} + \beta \hat{Y} + \frac{\gamma}{\sqrt{3}} \sum_{i=1}^3 D_{8i}^{(8)} \hat{J}_i, \quad (7)$$

where α, β , and γ are proportional to the current strange quark mass (assuming $m_u = m_d = 0$) and are given in terms of the moments of inertia and the pion-nucleon sigma term. We shall treat them as free parameters. It is however worth mentioning that α and β are negative by construction, while γ being phenomenologically negative is in fact given as a difference of two terms of the same order — see Eq. (4) in Ref [1]. Furthermore, α scales as N_c , and β and γ scale as N_c^0 , however, all of them are numerically of the same order [13].

Since the collective wave functions of the rotating soliton are known [10], one can easily compute matrix elements of H_{sb} in the first order of the perturbation theory [13]

$$\begin{aligned} \langle \hat{H}_{\text{sb}} \rangle_{\bar{\mathbf{3}}, J=0} &= \left(\frac{3}{8} \alpha + \beta \right) Y_B \equiv \delta_{\bar{\mathbf{3}}} Y_B, \\ \langle \hat{H}_{\text{sb}} \rangle_{\mathbf{6}, J=1} &= \left(\frac{3}{20} \alpha + \beta - \frac{3}{10} \gamma \right) Y_B \equiv \delta_{\mathbf{6}} Y_B, \\ \langle \hat{H}_{\text{sb}} \rangle_{\overline{\mathbf{15}}, J=1} &= \left(\beta + \frac{17}{144} (\alpha - 2\gamma) \right) Y_B - (\alpha - 2\gamma) \\ &\quad \times \left(\frac{2}{27} - \frac{1}{24} \left(T_B(T_B + 1) - \frac{1}{4} Y_B^2 \right) \right), \\ \langle \hat{H}_{\text{sb}} \rangle_{\overline{\mathbf{15}}, J=0} &= \left(\beta + \frac{1}{48} \alpha \right) Y_B \\ &\quad + \alpha \left(\frac{2}{9} - \frac{1}{8} \left(T_B(T_B + 1) - \frac{1}{4} Y_B^2 \right) \right) \end{aligned} \quad (8)$$

where Y_B and T_B denote the hypercharge and the isospin of a given baryon, respectively.

There is one more ingredient that we are missing, namely the spin-spin interaction of the heavy quark with the soliton, which is relevant for sextet and ($\overline{\mathbf{15}}, J=1$). To this end we add a phenomenological term corresponding to the chromomagnetic interaction [1] expressed as

$$\hat{H}_{SQ} = \frac{2}{3} \frac{\kappa}{m_Q} \hat{J} \cdot \hat{S}_Q, \quad (9)$$

where κ denotes the anomalous chromomagnetic moment that is flavor independent. The operators $\hat{J}$ and $\hat{S}_Q$ represent spin operators for the soliton and the heavy quark, respectively. This interaction is relevant only for sextet and ($\overline{\mathbf{15}}, J=1$). Therefore we arrive at the following equations for heavy baryon masses

$$\begin{aligned} M_{\mathcal{R}, B, s}^Q &= \mathcal{M}_{\mathcal{R}, J}^Q + \langle \hat{H}_{\text{sb}} \rangle_{\mathcal{R}, J} \\ &\quad + \delta_{J,1} \frac{\kappa}{m_Q} \begin{cases} -2/3 & \text{for } s = 1/2 \\ +1/3 & \text{for } s = 3/2 \end{cases}, \end{aligned} \quad (10)$$

where s denotes spin of a given baryon B , J is the soliton spin, $\mathcal{M}_{\mathcal{R}, J}^Q$ is given by (6), $\langle \hat{H}_{\text{sb}} \rangle_{\mathcal{R}, J}$ by (8) and $\delta_{J,1}$ is the Kronecker delta.

III. HEAVY BARYON MASSES

Before we estimate masses of states in $\overline{\mathbf{15}}$ let us rewrite Eq. (6) for average multiplet masses

$$\begin{aligned} \mathcal{M}_{\overline{\mathbf{15}}, J=0}^Q &= M_{\text{sol}} + m_Q + \frac{5}{2} \frac{1}{I_2}, \\ \mathcal{M}_{\overline{\mathbf{15}}, J=1}^Q &= M_{\text{sol}} + m_Q + \frac{3}{2} \frac{1}{I_2} + \frac{1}{I_1}. \end{aligned} \quad (11)$$

The mass difference

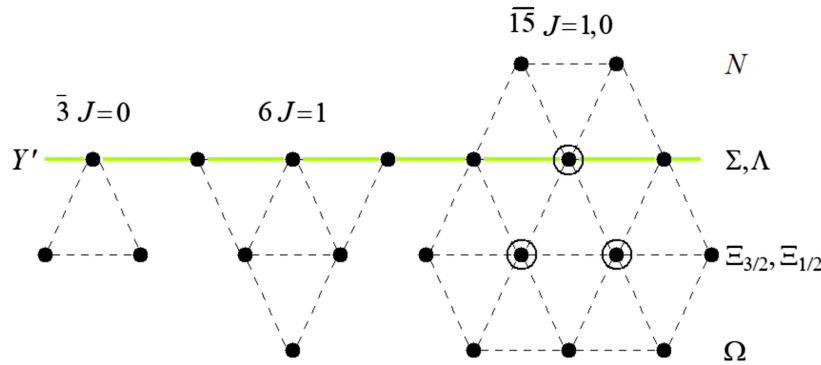


FIG. 1. Rotational band of a soliton with one valence quark stripped off. Soliton spin corresponds to the isospin T' of states on the quantization line $Y' = 2/3$ (green thick line). We show three lowest allowed representations: antitriplet of spin 0, sextet of spin 1, and the lowest exotic representation $\overline{\mathbf{15}}$ of spin 1 or 0. On the right-hand side we display particle names used in the present paper.

$$\Delta_{\overline{15}} = \mathcal{M}_{\overline{15},J=0}^Q - \mathcal{M}_{\overline{15},J=1}^Q = \frac{1}{I_2} - \frac{1}{I_1} \quad (12)$$

is positive, since — from the estimates of the light sector [2,10] — $I_1 \sim (2.5 \div 3) \times I_2$, which means that spin 1 soliton is lighter than the one of spin 0. This was confirmed in the previous study of the charm sector [13].

In principle all model parameters entering Eq. (10) can be calculated in a specific model. However, we follow here the so-called *model-independent* approach introduced in the context of the Skyrme model in Ref. [32], where all parameters are extracted from experimental data. Below we recapitulate the results of Ref. [13].

The model has ten parameters: M_{sol} , m_c , m_b , $1/I_1$, $1/I_2$, α , β , γ and two parameters of the spin splittings κ/m_c and κ/m_b . Since experimental errors of heavy baryon masses are extremely small we fix model parameters from algebraic equations, rather than from the χ^2 fits. Let us first consider spin averaged sextet masses, which are independent of the last two parameters. So the number of parameters is eight.

We use only masses of ground state heavy baryons. It is convenient to define the average multiplet masses, which can be conveniently written the following form

$$\begin{aligned} \mathcal{M}_{\overline{3},J=0}^Q &= m_Q + M_{\text{sol}} + \frac{1}{2I_2}, \\ \mathcal{M}_{\overline{6},J=1}^Q &= \mathcal{M}_{\overline{3}}^Q + \frac{1}{I_1}, \\ \mathcal{M}_{\overline{15},J=1}^Q &= \mathcal{M}_{\overline{6}}^Q + \frac{1}{I_2}, \\ \mathcal{M}_{\overline{15},J=0}^Q &= \mathcal{M}_{\overline{6}}^Q + \frac{2}{I_2} - \frac{1}{I_1} = \mathcal{M}_{\overline{15},J=1}^Q + \Delta_{\overline{15}}, \end{aligned} \quad (13)$$

where for future convenience we have also included the average masses of exotic multiplets. Already here one can see that if we treat $\mathcal{M}_{\overline{3},J=0}^Q$ and $1/I_1$ as new *effective* parameters, we do not need to know m_Q , M_{sol} and $1/I_2$ separately. We need $1/I_2$ only for exotica.

Parameters $\mathcal{M}_{\overline{3},J=0}^Q$ and I_1 are extracted from the nonexotic baryons alone [1]. We have

$$\begin{aligned} \mathcal{M}_{\overline{3},J=0}^Q &= m_Q + M_{\text{sol}} + \frac{1}{2I_2} = 2408.2|_c, \quad 5736.2|_b, \\ \mathcal{M}_{\overline{6},J=1}^Q &= \mathcal{M}_{\overline{3},J=0}^Q + \frac{1}{I_1} = 2579.7|_c, \quad 5906.5|_b, \end{aligned} \quad (14)$$

where the experimental values in MeV from Ref. [1] have been updated [33]. We can compute I_1 from the mass difference of these two multiplets (in MeV)

$$\frac{1}{I_1} = \mathcal{M}_{\overline{6},J=1}^Q - \mathcal{M}_{\overline{3},J=0}^Q = 171.5|_c = 170.4|_b, \quad (15)$$

which, as expected, does not depend on the heavy quark mass. In the following we assume $1/I_1 = 171$ MeV.

Masses in both heavy antitriplets and both sextets are equally split [see Eq. (8)] proportionally to Y_B with the following coefficients

$$\begin{aligned} \delta_{\overline{3}} &= \frac{3}{8}\alpha + \beta = -180 \text{ MeV}, \\ \delta_{\overline{6}} &= \frac{3}{20}\alpha + \beta - \frac{3}{10}\gamma = -121 \text{ MeV}. \end{aligned} \quad (16)$$

Numerical values are taken as the average values from Eqs. (13) and (14) in Ref. [1], where heavy baryon masses have been reproduced with a few MeV accuracy. We see that to fit the data we do not need all three parameters α , β , γ separately, but only their two combinations given above. However, for splittings in exotic $\overline{15}$ multiplets different combinations are necessary (8) and in consequence we need to know α , β , and γ independently.

In summary: in order to predict the masses of heavy pentaquarks, we need additional input data to constrain $1/I_2$, α , β and γ . We follow Ref. [2], where two out of five excited Ω^c hyperons reported by the LHCb Collaboration in 2017 [14] were interpreted as exotic states belonging to $(\overline{15}, J=1)$. Thus, we have an additional input for spin averaged Ω^c masses

$$\bar{M}_{\Omega,(\overline{15},J=1)}^c = 3096 \text{ MeV}. \quad (17)$$

From Eqs. (8), (11), and (13) we obtain that

$$\bar{M}_{\Omega,(\overline{15},J=1)}^c = \mathcal{M}_{\overline{6},J=1}^c + \frac{1}{I_2} - \frac{1}{6}(\alpha + 8\beta - 2\gamma). \quad (18)$$

Equating (17) with (18) together with Eq. (16) gives three independent equations for four parameters α , β , γ , and $1/I_2$. We solve them in function of $1/I_2$ (in MeV)

$$\begin{aligned} \alpha &= 4102.27 - 12 \frac{1}{I_2}, \\ \beta &= -1718.35 + \frac{9}{2} \frac{1}{I_2}, \\ \gamma &= -3273.37 + 9 \frac{1}{I_2}. \end{aligned} \quad (19)$$

We could vary $1/I_2$ in Eq. (19) at will without affecting masses of nonexotic baryons and of the input exotic Ω_c (and also the corresponding Ω_b) states. It is however clear that $1/I_2$ must be limited to some reasonable range. Following Ref. [13] we constrain parameter $1/I_2$ to the region where α , β , and γ are negative and of the same order²: $1/I_2 = 351 \pm 5$ MeV, which we use as an estimate

²See discussion below Eq. (7).

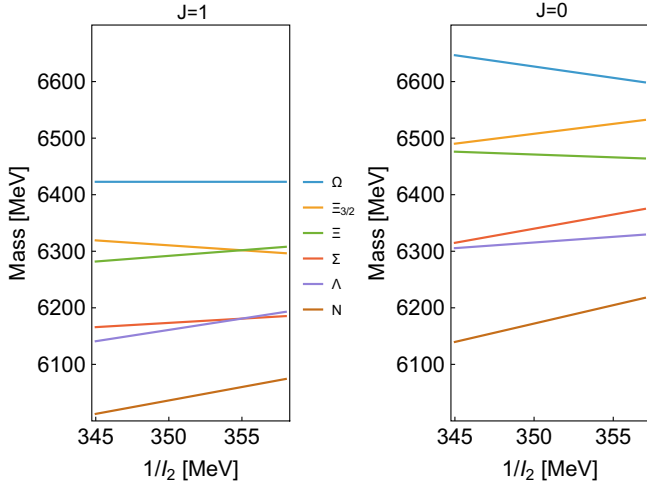


FIG. 2. Spectra of exotic beauty multiplets without spin interaction: $\overline{15}, J = 1$ (left) and $J = 0$ (right) in terms of the inverse moment of inertia $1/I_2$.

of the model's uncertainty. This procedure and the relation of the present model parameters to other calculations is in detail discussed in Ref. [13].

As already explained, due to the very small experimental errors, the χ^2 analysis is not suitable for estimating the error of the procedure discussed above. Instead, we have computed the square root of mean square deviation varying one of the parameters $1/I_1, \alpha, \beta$, or γ with the remaining ones fixed at the values given above (keeping $1/I_2 = 351$ MeV). In each case the deviation was 2 MeV. The uncertainty of at most 2 MeV remains unchanged for the pentaquark masses, and therefore can be neglected compared to the uncertainty due $1/I_2$, which—as we will see—varies between 20 and 50 MeV.

Finally, we have to include the hyperfine interaction (9). This was done in Ref. [13]

$$\frac{\kappa}{m_b} = 19.4|_{\Sigma_b} = 18.8|_{\Xi_b} \quad (20)$$

(in MeV). In what follows we assume 19 MeV.

Now, when all parameters are fixed, we can easily predict masses of all exotic b -baryons. The results are shown in Fig. 2 and in Table I.

Let us observe that assuming $1/I_2 = 351$ MeV and taking $1/I_1$ from Eq. (15), we obtain that $(\overline{15}, J = 0)$ multiplet is heavier from $(\overline{15}, J = 1)$ multiplet on average by approximately 180 MeV. Indeed

$$\begin{aligned} \mathcal{M}_{\overline{15}, J=1}^b &\simeq 6257.5 \text{ MeV}, \\ \mathcal{M}_{\overline{15}, J=0}^b &\simeq 6437.5 \text{ MeV}. \end{aligned} \quad (21)$$

We can also estimate the average mass of the next exotic representation $(\overline{15}') = (p = 0, q = 4), J = 1)$ to be

TABLE I. Mass predictions in MeV for exotic $\overline{15}$ b -pentaquarks. Since two Ω^c states are taken as input, also two Ω^b states have fixed masses that do not depend on $1/I_2$.

$\overline{15}$	$J = 1$		$J = 0$
	$s = 1/2$	$s = 3/2$	$s = 1/2$
N^b	6005–6052	6024–6071	6146–6211
Λ^b	6132–6172	6151–6191	6307–6328
Σ^b	6155–6170	6174–6189	6320–6370
$\Xi_{1/2}^b$	6271–6291	6290–6310	6475–6465
$\Xi_{3/2}^b$	6305–6287	6324–6306	6494–6529
Ω^b	6410	6429	6643–6603

approximately 6960 MeV, which is indeed substantially heavier than $\overline{15}$.

IV. DECAYS

We compute decay widths of b -exotica to check whether they follow the pattern of charm pentaquarks, namely whether the widths of $(\overline{15}, J = 1)$ are very small, and the widths $(\overline{15}, J = 0)$ are large [13]. We can compute only decays in which the soliton changes its configuration by emitting a pseudo-Goldstone φ boson and the heavy quark is a spectator. Decays into heavy mesons cannot be computed in the present approach. Nevertheless, we try to estimate approximately their influence on the total widths.

We follow exactly the same steps as in Ref. [13], using the decay operator, which has been obtained via the Goldberger-Treiman relation from the collective weak current [3,10],

$$\hat{\mathcal{O}}_\varphi = \frac{1}{2F_\varphi} \left[-\tilde{a}_1 D_{\varphi i}^{(8)} - a_2 d_{ibc} D_{\varphi b}^{(8)} \hat{J}_c - a_3 \frac{1}{\sqrt{3}} D_{\varphi 8}^{(8)} \hat{J}_i \right] p_i. \quad (22)$$

Here $D_{\varphi i}^{(8)}$ denote SU(3) Wigner functions, $\hat{J}_i$ is the soliton spin operator and p_i is the φ meson momentum. Constants $a_{1,2,3}$ that enter Eq. (22) were extracted from the semi-leptonic decays of the baryon octet in Ref. [34]

$$a_1 \simeq -3.509, \quad a_2 \simeq 3.437, \quad a_3 \simeq 0.604. \quad (23)$$

However, due to the fact that a_1 scales as N_{val} , it has been shown in Ref. [3] that in the heavy quark sector a_1 has to be replaced by

$$a_1 \rightarrow \tilde{a}_1 = -2.1596. \quad (24)$$

With this replacement, all decays of charm and bottom sextet and of two exotic Ω^c 's have been successfully

described [3]. For the decay constants F_φ , we take $F_\pi = 93$ MeV and $F_K = F_\eta = 1.2F_\pi = 112$ MeV.

We compute the decays $B_1 \rightarrow B_2 + \varphi$, where $M_{1,2}$ denote masses of the initial and final baryons respectively, and p_i is the c.m. momentum of the outgoing meson of mass m [3,10]

$$|\vec{p}| = p = \frac{\sqrt{(M_1^2 - (M_2 + m)^2)(M_1^2 - (M_2 - m)^2)}}{2M_1}. \quad (25)$$

The decay width is related to the matrix element of $\hat{\mathcal{O}}_\varphi$ squared, summed over the final isospin (but not spin) and averaged over the initial spin and isospin denoted as $[\dots]^2$; see the Appendix of Ref. [10] and Erratum of Ref. [3] for the details of the corresponding calculations,

$$\Gamma_{B_1 \rightarrow B_2 + \varphi} = \frac{1}{2\pi} \overline{\langle B_2 | \hat{\mathcal{O}}_\varphi | B_1 \rangle^2} \frac{M_2}{M_1} p. \quad (26)$$

Because φ meson is in the SU(3) octet, the following decays are possible:

$$\begin{aligned} \overline{\mathbf{15}}_{J=1} &\rightarrow \mathbf{6}_{J=1}, \mathbf{\bar{3}}_{J=0}, \\ \overline{\mathbf{15}}_{J=0} &\rightarrow \overline{\mathbf{15}}_{J=1}, \mathbf{6}_{J=1}. \end{aligned} \quad (27)$$

Direct decays of $\overline{\mathbf{15}}_{J=0}$ to the ground state antitriplet are suppressed.

After averaging over the initial spin and isospin and summing over the final isospin and over final spin third component m_2 , we obtain [13]

$$\begin{aligned} \Gamma_{B_1 \rightarrow B_2 + \varphi} &= \frac{p^3}{24\pi F_\varphi^2} \frac{M_2}{M_1} \frac{\dim \mathcal{R}_2}{\dim \overline{\mathbf{15}}} \gamma_{J_1 \rightarrow J_2}(s_1 \rightarrow s_2) \\ &\times \left(\sum_\mu \mathcal{G}_{\overline{\mathbf{15}}_{J_1} \rightarrow \mathcal{R}_2}^{(\mu)} \begin{bmatrix} 8 & \mathcal{R}_2 \\ \varphi & B_2 \end{bmatrix} \overline{\mathbf{15}}_\mu \begin{bmatrix} \mathbf{15} \\ B_1 \end{bmatrix} \right)^2. \end{aligned} \quad (28)$$

Here the square bracket stands for the pertinent SU(3) isoscalar factor and $\mathcal{G}_{\overline{\mathbf{15}}_{J_1} \rightarrow \mathcal{R}_2}^{(\mu)}$ are the decay couplings.³ The sum over μ is relevant only for $\mathcal{R}_2 = \overline{\mathbf{15}}$ in the final state. We have⁴ [13]

$$\begin{aligned} \mathcal{G}_{\overline{\mathbf{15}}_1 \rightarrow \mathbf{\bar{3}}_0} &= \sqrt{\frac{1}{2}} \left(-\tilde{a}_1 - \frac{1}{2} a_2 \right) = 0.312, \\ \mathcal{G}_{\overline{\mathbf{15}}_1 \rightarrow \mathbf{6}_1} &= -\sqrt{\frac{1}{3}} \left(-\tilde{a}_1 - \frac{1}{2} a_2 - a_3 \right) = 0.094, \end{aligned} \quad (29)$$

³Recall that the soliton in SU(3) $\overline{\mathbf{15}}$ can be quantized as spin $J_1 = 1$ or $J_1 = 0$.

⁴Present definitions of the decay constants contain pertinent SU(3) *spin* isoscalar factors, which have not been included in definitions of Ref. [3].

and

$$\begin{aligned} \mathcal{G}_{\overline{\mathbf{15}}_0 \rightarrow \mathbf{6}_1} &= \frac{1}{2} \left(-\tilde{a}_1 - \frac{3}{2} a_2 \right) = -1.498, \\ \mathcal{G}_{\overline{\mathbf{15}}_0 \rightarrow \overline{\mathbf{15}}_1}^{(\mu=1)} &= \sqrt{\frac{1}{366}} \left(-\tilde{a}_1 + \frac{41}{2} a_2 \right) = 3.796, \\ \mathcal{G}_{\overline{\mathbf{15}}_0 \rightarrow \overline{\mathbf{15}}_1}^{(\mu=2)} &= -\sqrt{\frac{81}{122}} \left(-\tilde{a}_1 + \frac{1}{6} a_2 \right) = -2.226. \end{aligned} \quad (30)$$

Here, we adopt the de Swart conventions for the SU(3) phase factors [35] and label the representations as in the numerical code of Ref. [36].

Factors γ take care of the spin dependence for the soliton angular momenta $J = 1$ or 0 (see Erratum in Ref. [3]):

$$\begin{aligned} \gamma_{1 \rightarrow 1}(1/2 \rightarrow 1/2) &= 2/3, & \gamma_{1 \rightarrow 1}(1/2 \rightarrow 3/2) &= 1/3, \\ \gamma_{1 \rightarrow 1}(3/2 \rightarrow 1/2) &= 1/6, & \gamma_{1 \rightarrow 1}(3/2 \rightarrow 3/2) &= 5/6, \\ \gamma_{0 \rightarrow 1}(1/2 \rightarrow 1/2) &= 1/3, & \gamma_{0 \rightarrow 1}(1/2 \rightarrow 3/2) &= 2/3, \\ \gamma_{1 \rightarrow 0}(1/2 \rightarrow 1/2) &= 1, & \gamma_{1 \rightarrow 0}(3/2 \rightarrow 1/2) &= 1. \end{aligned} \quad (31)$$

Note that

$$\sum_{s_2} \gamma_{J_1 \rightarrow J_2}(s_1 \rightarrow s_2) = 1. \quad (32)$$

We see from Eq. (29) that decay constants of $(\overline{\mathbf{15}}, J = 1)$ are very small; in fact they vanish in the large N_c limit [37]. On the contrary, decay constants of $(\overline{\mathbf{15}}, J = 0)$ are almost an order of magnitude larger (30), so we expect the corresponding decay widths to be large (also the phase space factor is larger than in the $J = 1$ case).

Numerical results are presented in Tables II and III. Decays not included in the tables are not allowed by energy or isospin conservation. Note that these widths are pure predictions based on the light sector values of the decay parameters, except for rescaling (24). In Table IV we list all possible strong decays to heavy mesons and indicate whether they are allowed by energy conservation.

Table II shows that all states in $\overline{\mathbf{15}}_{J=1}$, have very small widths, in most cases not exceeding 2 MeV. For all states in $\overline{\mathbf{15}}_{J=1}$, decay channels to light baryons and heavy mesons are closed. We therefore conclude that exotic b -pentaquarks from $\overline{\mathbf{15}}_{J=1}$ can be found only in dedicated searches in high resolution experiments. Interestingly, the lightest member of $\overline{\mathbf{15}}_{J=1}$, namely the nucleonlike pentaquark, is stable with respect to two body strong decays.

The situation is very different in the case of $\overline{\mathbf{15}}_{J=0}$ widths listed in Table III. Here all decay widths are within 30–220 MeV range. The only exception is again the lightest nucleonlike pentaquark, which can decay only to the N^b state in $\overline{\mathbf{15}}_{J=1}$, which is stable against strong decays. Furthermore,

TABLE II. Decay widths of exotic pentaquarks in the $SU(3)$ representation $\overline{\mathbf{15}}$ and $J = 1$. Uncertainties correspond to the mass ranges from Table I (two Ω^c states are used as the input; therefore, also Ω^b masses and decay widths are not subject to such uncertainties).

Decay		Γ [MeV]		
B_1	$B_2 + \varphi$	$s_2 = \frac{1}{2}$	$s_2 = \frac{3}{2}$	Σ_{s_2}
$\Omega^b(\overline{\mathbf{15}}_1^{1/2})$	$\Xi^b(\overline{\mathbf{3}}_0) + \bar{K}$	0.813	...	0.813
	$\Omega^b(\mathbf{6}_1) + \pi$	0.079	0.030	0.109
	Total			0.922
$\Omega^b(\overline{\mathbf{15}}_1^{3/2})$	$\Xi^b(\overline{\mathbf{3}}_0) + \bar{K}$	1.027	...	1.027
	$\Omega^b(\mathbf{6}_1) + \pi$	0.024	0.089	0.113
	Total			1.140
$\Xi_{3/2}^b(\overline{\mathbf{15}}_1^{1/2})$	$\Xi^b(\overline{\mathbf{3}}_0) + \pi$	2.599–2.899	...	2.599–2.899
	$\Xi^b(\mathbf{6}_1) + \pi$	0.036–0.042	0.015–0.018	0.051–0.060
	Total			2.650–2.959
$\Xi_{3/2}^b(\overline{\mathbf{15}}_1^{3/2})$	$\Xi^b(\overline{\mathbf{3}}_0) + \pi$	2.916–3.237	...	2.916–3.237
	$\Xi^b(\mathbf{6}_1) + \pi$	0.011–0.013	0.045–0.053	0.056–0.066
	Total			2.972–3.303
$\Xi_{1/2}^b(\overline{\mathbf{15}}_1^{1/2})$	$\Xi^b(\mathbf{3}_0) + \pi$	0.147–0.167	— — —	0.147–0.167
	$\Lambda^b(\mathbf{3}_0) + \bar{K}$	0.455–0.554	— — —	0.455–0.554
	$\Xi^b(\mathbf{6}_1) + \pi$	0.047–0.058	0.020–0.024	0.067–0.082
	Total			0.669–0.803
$\Xi_{1/2}^b(\overline{\mathbf{15}}_1^{3/2})$	$\Xi^b(\mathbf{3}_0) + \pi$	0.166–0.187	...	0.166–0.187
	$\Lambda^b(\mathbf{3}_0) + \bar{K}$	0.549–0.656	...	0.549–0.656
	$\Xi^c(\mathbf{6}_1) + \pi$	0.014–0.017	0.060–0.073	0.074–0.091
	Total			0.789–0.934
$\Sigma^b(\overline{\mathbf{15}}_1^{1/2})$	$\Lambda^b(\mathbf{3}_0) + \pi$	1.653–1.796	...	1.653–1.796
	$\Sigma^b(\mathbf{6}_1) + \pi$	0.032–0.037	0.013–0.015	0.045–0.052
	Total			1.698–1.848
$\Sigma^b(\overline{\mathbf{15}}_1^{3/2})$	$\Lambda^b(\mathbf{3}_0) + \pi$	1.835–1.986	...	1.835–1.986
	$\Sigma^c(\mathbf{6}_1) + \pi$	0.010–0.011	0.040–0.046	0.049–0.057
	Total			1.884–2.043
$\Lambda^b(\overline{\mathbf{15}}_1^{1/2})$	$\Lambda^b(\mathbf{3}_0) + \eta$	0.000–0.007	...	0.000–0.007
	$\Sigma^b(\mathbf{6}_1) + \pi$	0.019–0.028	0.007–0.012	0.026–0.040
	Total			0.026–0.047
$\Lambda^b(\overline{\mathbf{15}}_1^{3/2})$	$\Lambda^b(\mathbf{3}_0) + \eta$	0.000–0.054	...	0.000–0.054
	$\Sigma^b(\mathbf{6}_1) + \pi$	0.006–0.008	0.023–0.035	0.029–0.043
	Total			0.029–0.097

as seen from Table IV, all states in $\overline{\mathbf{15}}_{J=0}$ (except for N^b) have one open channel to the decays to light baryons and heavy mesons. We are not able to compute these widths within the present approach. However, since the available phase space is comparable to the decays listed in Table III, we may expect that the total decay widths will double with respect to the estimates given in Table III.

One should also note, that all decays of $\overline{\mathbf{15}}_{J=0}$ lead to either $\mathbf{6}$ or $\overline{\mathbf{15}}_{J=1}$, which decay further to $\mathbf{6}$ and $\overline{\mathbf{3}}$.

We conclude therefore, that pentaquarks from $\overline{\mathbf{15}}_{J=0}$ multiplet are very wide and — similarly to the charm case discussed in Ref. [13] — may be interpreted as a background, rather than as a signal. Therefore, they could have been missed in general purpose experiments.

TABLE III. Decay widths of exotic b -pentaquarks in the SU(3) representation $\overline{\mathbf{15}}$ and $J = 0$. Uncertainties correspond to the mass ranges from Table I.

Decay		Γ [MeV]		
B_1	$B_2 + \varphi$	$s_2 = \frac{1}{2}$	$s_2 = \frac{3}{2}$	Σ_{s_2}
$\Omega^b(\overline{\mathbf{15}}_0)$	$\Omega^b(\mathbf{6}_1) + \pi$	38.58–47.41	65.23–81.30	103.81–128.71
	$\Xi^b(\mathbf{6}_1) + \bar{K}$	7.41–10.48	12.50–18.22	19.91–28.71
	$\Omega^b(\overline{\mathbf{15}}_1) + \pi$	0.71–1.88	0.69–2.51	1.40–4.39
	Total			125.12–161.81
$\Xi_{3/2}^b(\overline{\mathbf{15}}_0)$	$\Xi^b(\mathbf{6}_1) + \pi$	19.46–23.31	35.49–42.79	54.95–66.10
	$\Sigma^b(\mathbf{6}_1) + \bar{K}$	16.56–22.11	27.41–37.67	43.97–59.78
	$\Xi_{3/2}^b(\overline{\mathbf{15}}_1) + \pi$	10.40–37.18	5.19–51.26	15.59–88.44
	$\Xi_{1/2}^b(\overline{\mathbf{15}}_1) + \pi$	1.40–4.31	1.54–6.23	2.95–10.54
	Total			117.46–224.86
$\Xi_{1/2}^b(\overline{\mathbf{15}}_0)$	$\Xi^b(\mathbf{6}_1) + \pi$	25.92–27.42	46.97–49.80	72.88–77.21
	$\Sigma^b(\mathbf{6}_1) + \bar{K}$	0.78–0.87	1.25–1.40	2.03–2.27
	$\Xi_{3/2}^b(\overline{\mathbf{15}}_1) + \pi$	0.46–1.78	0.00–1.60	0.46–3.38
	$\Xi_{1/2}^b(\overline{\mathbf{15}}_1) + \pi$	0.03–0.09	0.02–0.10	0.05–0.20
	Total			75.43–83.06
$\Sigma^b(\overline{\mathbf{15}}_0)$	$\Sigma^b(\mathbf{6}_1) + \pi$	14.41–19.13	25.46–34.28	39.87–53.41
	$\Sigma^b(\mathbf{6}_1) + \eta$	0.00–0.12	— — —	0.00–0.12
	$\Sigma^b(\overline{\mathbf{15}}_1) + \pi$	0.94–20.26	0.00–24.56	0.94–44.82
	$\Lambda^b(\overline{\mathbf{15}}_1) + \pi$	0.10–4.58	0.00–6.22	0.10–10.80
	Total			40.91–109.15
$\Lambda^b(\overline{\mathbf{15}}_0)$	$\Sigma^b(\mathbf{6}_1) + \pi$	9.98–11.33	17.56–20.08	27.55–31.41
	$\Sigma^b(\overline{\mathbf{15}}_1) + \pi$	0.00–2.20	0.00–1.24	0.00–3.44
	Total			27.55–34.85
$N^b(\overline{\mathbf{15}}_0)$	$N^b(\overline{\mathbf{15}}_1) + \pi$	0.00–18.57	0.00–21.03	0.00–39.60
	Total			0.00–39.60

V. SUMMARY AND CONCLUSIONS

TABLE IV. Decays of $\overline{\mathbf{15}}_{J=1,0}$ to the ground state octet baryons and B mesons (there is no distinction between particles and antiparticles since only the masses matter for thresholds).

$\overline{\mathbf{15}}_J$	Final	$J = 1$	$J = 0$
Ω^b	$\Xi + B$	No	Yes
$\Xi_{3/2}^b$	$\Sigma + B$	No	Yes
$\Xi_{1/2}^b$	$\Xi + B_s$	No	No
	$\Lambda + B$	No	Yes
	$\Sigma + B$	No	No
Σ^b	$\Sigma + B_s$	No	No
	$N + B$	No	Yes
Λ^b	$\Lambda + B_s$	No	No
	$N + B$	No	Yes
N^b	$N + B_s$	No	No

In the present paper we have closely followed Ref. [13] on charm $\overline{\mathbf{15}}$ pentaquarks extending it to the case of beauty. We have used the χ QSM where all model parameters were extracted from the heavy baryon spectra including charm exotica and from the negativity of splitting parameters α , β , and γ (7). In this way we in fact tested the underlying *hedgehog* SU(3) symmetry rather than a particular implementation of the model. We did not present numerical support for the model mass formulas (8), (10), and (13) and other regularities that were at length discussed in Ref. [13].

We computed masses of b pentaquarks with uncertainties of the order ~ 50 MeV, which are grouped in three $\overline{\mathbf{15}}$ multiplets spanned in a range 6000–6600 MeV. Question arises, can one observe these states experimentally?

Before answering this question let us comment on four Ω^{b-} states reported in Ref. [38] by the LHCb Collaboration at CERN. These states are very narrow with masses within

6316 ÷ 6350 MeV range. A natural explanation of these states is that they are copies of analogous charm resonances [14–16] (actually five not four) reported by LHCb and Belle Collaborations in 2017 with masses within 3000 ÷ 3119 MeV range. Indeed, the heavy quark symmetry implies that to a good approximation, b baryon spectra are copies of the c baryon spectra shifted by the $m_b - m_c \simeq 3330$ MeV [39] mass difference.⁵ This means that we expect excited Ω^{b-} states within the range of 6330 ÷ 6450 MeV rather than 6316 ÷ 6350 MeV. For exotic multiplets the predicted range is a few tens of MeV higher. Moreover, their mass differences are much smaller than the Ω^c mass splittings in the charm sector, which is strange since the mass splittings in all known cases are basically independent of the heavy quark mass (neglecting spin-spin interaction). The low masses of the newly discovered Ω^b states are still a matter of debate and some models can accommodate them in the excited sextet [40,41]. This may be due to the fact that they are attributed to the so called λ modes [40], which in the present approach are suppressed in the large N_c limit [4]. So the LHCb Ω^{b-} states are not easy to accommodate in the present model. The pentaquark Ω^b states discussed in this paper are heavier and either very

narrow, $\Gamma \sim 1$ MeV, or very wide, $\Gamma \sim 125 \div 160$ MeV (see below).

Finally, we computed strong decay widths using the same approach as Ref. [13]. We have found that the decay pattern is very similar to the charm case. Pentaquarks belonging to the $\overline{\mathbf{15}}_{J=1}$ SU(3) multiplet are very narrow having widths of the order of ~ 2 MeV, while the remaining states from the $\overline{\mathbf{15}}_{J=0}$ SU(3) multiplet are wide, in most cases of the order of ~ 100 MeV or more. Moreover, all these decays lead to the unstable resonances. For all pentaquarks in $\overline{\mathbf{15}}_{J=1}$ decay channels to heavy mesons are closed, and on the contrary, $\overline{\mathbf{15}}_{J=0}$ have open channels to the final states containing B mesons, which can double decay widths computed in the χ QSM. Multipurpose searches can easily miss narrow or wide exotic states, so dedicated studies are needed to confirm/reject our predictions.

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DATA AVAILABILITY

No data were created or analyzed in this study.

⁵This mass difference comes from the comparison of the $\bar{\mathbf{3}}$ and $\mathbf{6}$ heavy baryon spectra.

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