

Reviving the energy-dependent partonic structure of $f_0(980)$ via two-pion distribution amplitudes

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We present a novel framework for analyzing four-body semileptonic weak decays, performing the first high-twist analysis of the $D_s \rightarrow [\pi\pi]_S$ form factors using light cone distribution amplitudes for the isoscalar two-pion state (2π DAs). It is motivated by persistent tensions between phenomenological cascade analyses and QCD-based interpretations of the partonic structure of the f_0 meson. Our study reveals that the twist-three 2π DAs incorporate an asymmetry in the partial-wave expansion, a feature absent in single f_0 distribution amplitudes, that drives a substantial cancellation between twist-two and twist-three contributions to the form factors. As a result, the predicted decay rate falls significantly below the experimental value. These findings indicate that the isoscalar two-pion system near the charm scale is not primarily a $q\bar{q}$ state, reviving the energy-dependent partonic structures of light scalar mesons by highlighting a fundamental limitation of the single-meson light cone distribution amplitude approach in describing such complex systems.

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Introduction. The nature of light scalar mesons remains a long-standing puzzle in hadron physics [1]. Experimental identification of these states is particularly challenging due to their characteristically large widths. From a quantum theory, the light scalar meson is a superposition of all possible Fock states:

$$|S\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle + \cdots \quad (1)$$

The probability amplitudes ψ_n , representing different partonic configurations, depend on the partons' momentum distributions and their spin projections.

Hadron spectroscopy offers clear evidence for such complex configurations [2–4], fueling ongoing theoretical debates about the structure of both conventional resonances [5–9], which may mix with gluonball states [10], and exotic multiparticle states [11–13]. In the case of exotic hadrons, the discussion centers on distinguishing between compact tetraquark states [14–19] and loosely bound molecular states [20–24]. While spectral analysis reveals these exotic

configurations, the intrinsically nonperturbative nature of QCD prevents a direct extraction of the underlying parton dynamics.

Semileptonic heavy meson decays are a powerful probe of scalar meson structure [25–30], where the quark-gluon dynamics are encoded in light cone distribution amplitudes (LCDAs) [31–36]. A key distinction arises between B_s and D_s decays due to the phenomenon of color transparency. In the decay $B_s \rightarrow f_0 \ell^+ \nu_\ell$, the energetic f_0 is dominated by its lowest Fock state [37–40], since the higher-state contributions are doubly suppressed by the large momentum transfer and the strong coupling constant, as depicted in Fig. 1. The $D_s \rightarrow f_0 \ell^+ \nu_\ell$ decay, however, produces a less relativistic quark pair. This makes the hadronization process more susceptible to contributions from higher Fock states, where the creation of additional gluons and $q\bar{q}$ pairs plays a significant role.

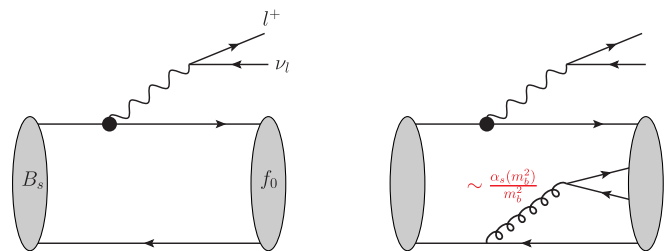


FIG. 1. Illustrative diagrams of the color transparency mechanism in $B_s \rightarrow f_0 e^+ \nu_e$ decay.

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While the energy-dependent partonic structure is a QCD result, its practical application is often hindered by the complexity of multiparticle LCDAs. Consequently, in QCD-based approaches like light cone sum rules (LCSRs), the f_0 meson is typically approximated by its lowest Fock state. This simplification is generally reliable for B_s decays, but becomes inadequate in D_s decays where multiparticle Fock states contribute substantially. Surprisingly, recent phenomenological studies using the cascade decay framework [40,41], under the $q\bar{q}$ ansatz, show good agreement with experimental data [42] for the four-body semileptonic decay $D_s \rightarrow [\pi\pi]_S \ell^+ \nu_\ell$ (D_{l4}). These analyses incorporate the $D_s \rightarrow f_0(980)$ form factors obtained from LCSRs with a Flatté parametrization of the $f_0(980)$ resonance, which is subsequently coupled to the isoscalar $\pi\pi$ state. Given the scale dependence observed in the form factor calculations, specifically, the shift in dominance from the nonasymptotic twist-two LCDA for $B_s \rightarrow f_0$ transition to the two-particle twist-three LCDAs for $D_s \rightarrow f_0$ transition [40], the seeming agreement in D_{l4} decays is largely attributed to the cascade framework, where the Flatté parametrization of the intermediate resonance severely disrupts the assessment of color transparency for partonic structures.

In this paper, we propose a novel framework to analyze D_{l4} weak decays by introducing light cone distribution amplitudes for the isoscalar two-pion state (2π DAs). The core concept is to calculate the $D_s \rightarrow [\pi\pi]_S$ form factor directly, bypassing the need for modeling the intermediate f_0 resonance. The apparent agreement within the cascade framework is demonstrated to be an artifact, arising from the symmetric two-particle twist-three LCDAs of the single f_0 resonance. This is clarified by the fact that the corresponding LCDAs of the signal state $[\pi\pi]_S$ are fundamentally asymmetric. Our findings indicate that the $q\bar{q}$ state contributes only modestly to $D_s \rightarrow [\pi\pi]_S \ell \nu$ decay. Consequently, data from charm decays favor a multiparton structure for the isoscalar two-pion system.

2π DAs of the isoscalar two-pion system. The $\pi\pi$ invariant mass spectrum serves as the primary signal channel for observing the f_0 resonance, as the KK channel is strongly suppressed by phase space. 2π DAs provide one of the most general descriptions within QCD for energetic two-pion systems. They were initially developed at leading twist in the reaction $\gamma^* \gamma \rightarrow \pi\pi$ [43–45], and have recently been extended to the twist-three level in the hard exclusive processes [46,47]. Twist is a quantum number that organizes the partonic configurations of the constituent quark and gluon fields.

For an isoscalar two-pion system, 2π DAs of the lowest Fock state are defined by

$$\begin{aligned} \langle \pi(k_1) \pi(k_2) | \bar{q}(0) \gamma^\mu q(x) | 0 \rangle &= \int du e^{i\bar{u}k \cdot x} k^\mu \Phi_{\parallel}, \\ \langle \pi(k_1) \pi(k_2) | \bar{q}(0) q(x) | 0 \rangle &= \int du e^{i\bar{u}k \cdot x} \frac{ik^2(k \cdot x)}{2f_{2\pi}^\perp} \Phi_{\parallel}^{(s)}, \\ \langle \pi(k_1) \pi(k_2) | \bar{q}(0) \sigma^{\mu\nu} q(x) | 0 \rangle &= -\frac{i}{f_{2\pi}^\perp} \int du e^{i\bar{u}k \cdot x} \\ &\quad \left[\frac{k_\mu \bar{k}_\nu - k_\nu \bar{k}_\mu}{2\zeta - 1} \Phi_{\perp} \right. \\ &\quad \left. - k^2 \frac{k_\mu x_\nu - k_\nu x_\mu}{k \cdot x} \Phi_{\parallel}^{(t)} \right]. \end{aligned} \quad (2)$$

Here $\Phi_{\parallel}$, $\Phi_{\perp}$, and $\Phi_{\parallel}^{(s)}$, $\Phi_{\parallel}^{(t)}$ are leading and subleading twist 2π DAs, respectively. $k = k_1 + k_2$ ($\bar{k} = k_1 - k_2$) denotes the sum (difference) of two pions' momenta, and $f_{2\pi}^\perp$ is the decay constant defined via the local matrix element of the tensor current in the soft-meson limit. 2π DAs are dimensionless functions of three variables: the longitudinal momentum fraction u ($\bar{u}$) carried by the antiquark (quark) field relative to k , the single pion momentum fraction $\zeta = k_1^+/k^+$ along the positive direction, and the invariant mass squared k^2 . They can be expressed formally as a double expansion in Gegenbauer $C_n^{3/2}(2u-1)$ and Legendre polynomials $C_l^{1/2}(2\zeta-1)$, by combining QCD's conformal symmetry with partial wave analysis:

$$\begin{aligned} \Phi_{\parallel,\perp} &= 6u\bar{u}C_1^{3/2} \left[B_{10}^{\parallel,\perp}(k^2, \mu) C_0^{1/2} + B_{12}^{\parallel,\perp}(k^2, \mu) C_2^{1/2} \right], \\ \Phi_{\parallel}^{(s)} &= 6u\bar{u}C_1^{3/2} \left[B_{10}^{\parallel}(k^2, \mu) C_0^{1/2} + B_{12}^{\parallel}(k^2, \mu) C_2^{1/2} \right], \\ \Phi_{\parallel}^{(t)} &= C_1^{3/2} \left[B_{10}^{\parallel}(k^2, \mu) C_0^{1/2} + B_{12}^{\parallel}(k^2, \mu) C_2^{1/2} \right]. \end{aligned} \quad (3)$$

The relation $\Phi(u, \zeta, k^2) = -\Phi(1-u, \zeta, k^2)$, constrained by the C parity, is maintained by the Gegenbauer polynomials with odd number n .

The expansion coefficients B_{nl} weight the universal DAs at a given conformal power n and partial wave l :

$$\begin{aligned} B_{nl}^{\parallel}(\mu, k^2) &= B_{nl}^{\parallel}(0) \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right]^{\frac{\gamma_n - \gamma_0}{2\beta_0}} \text{Exp} \left[k^2 \frac{d \ln B_{nl}^{\parallel}(0)}{dk^2} \right. \\ &\quad \left. + \frac{k^4}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\delta_l(s)}{s^2(s - k^2 - i0)} \right]. \end{aligned} \quad (4)$$

On one hand, these coefficients incorporate the scale dependence through renormalization group evolution [48–50], mirroring the well-established behavior of single-meson LCDAs governed by one-loop anomalous dimensions γ_n [51–53]. On the other hand, they evolve with k^2 , thereby capturing contributions from both intermediate resonant states and the nonresonant QCD background. For the resonant regions, the k^2 dependence can be obtained via a subtracted dispersion relation, guided by

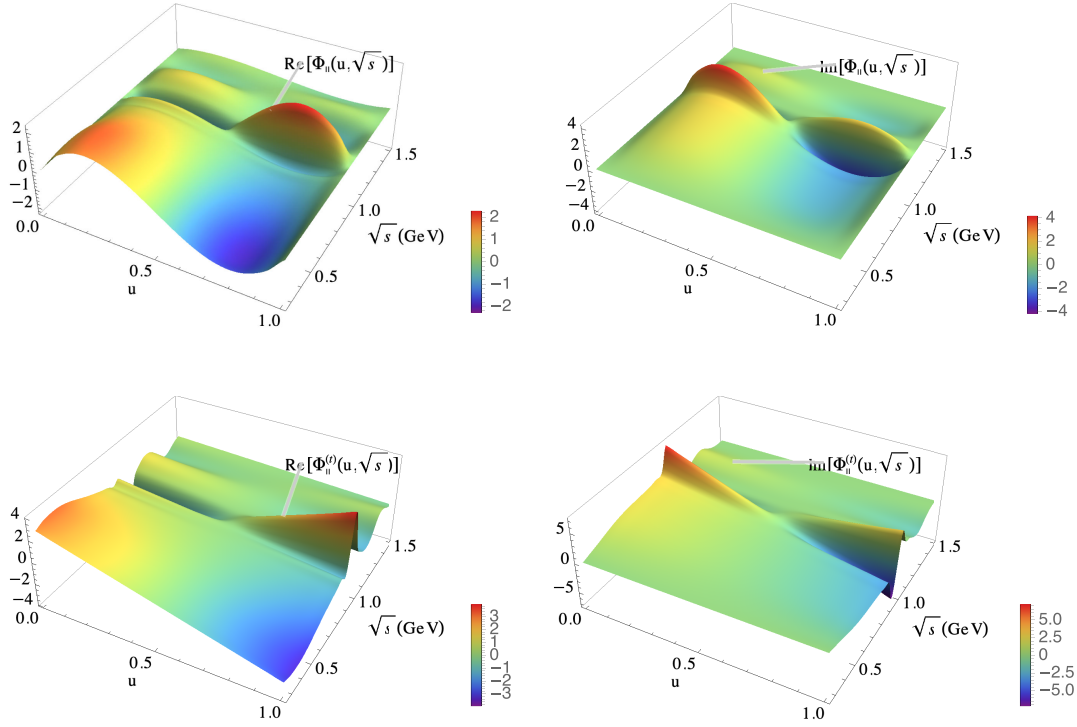


FIG. 2. Twist-two (up) and twist-three (low) isoscalar 2π DAs at the scale $\mu = 1.3$ GeV, by taking the momentum fraction $\zeta = 0.5$.

Watson's theorem [54], and constrained by data on phase shifts δ_l from πN scattering [55,56] and $\pi\pi$ scattering [57,58]. The initial values of the subtracted coefficients in the threshold region are derived from low-energy theorems based on the instanton vacuum [59,60], as provided in the Supplemental Material [61] and the references therein. For the leading-twist 2π DAs, this approach yields the coefficients at the first power: $B_{10}^{\parallel}(0) = -B_{12}^{\parallel}(0) = -0.556$, $d \ln B_{nl}^{\parallel}(0)/dk^2 = 0.413 \text{ GeV}^{-2}$.

In Fig. 2, we present two-dimension plots of the twist-two ($\Phi_{\parallel}$) and twist-three ($\Phi_{\parallel}^{(t)}$) 2π DAs as functions of the parton momentum fraction u and the invariant mass $\sqrt{s} = k$. The pion momentum fraction is fixed at $\zeta = 0.5$, and the renormalization scale is typically set at $\mu = 1.3$ GeV for charm-meson decays. Three key pieces of information can be gleaned from the figure. First, a distinct structural feature, either a peak or a dip, emerges in the $f_0(980)$ region, whereas no comparable signal appears in the $\sigma(500)$ region. This is consistent with the flat phase shift behavior observed in $\pi\pi$ scattering [57,58]. Second, the twist-two and twist-three LCDAs display asymmetry in both their real and imaginary parts with respect to the midpoint of the parton momentum fraction. This contrasts sharply with the LCDAs of a single f_0 meson, for which the twist-three DAs are symmetric about $u = 0.5$ [40,41]. Finally, twist-three LCDA $\Phi_{\parallel}^{(t)}$ displays end-point enhancements, in contrast to the vanishing of the leading-twist LCDA at the boundaries. As we will see, the last two

features, especially the second one, predominantly explain the non- $q\bar{q}$ component of the isoscalar two-pion system in D_{l4} decays.

$D_s \rightarrow [\pi\pi]_S$ form factors and the D_{l4} decays. The transition from a D_s meson to a two-pion system can be described by the matrix element expressed through four orthogonal form factors $F_{i=\parallel,\perp,0,t}$ [72–75],

$$\begin{aligned} & \langle \pi^+(k_1) \pi^-(k_2) | \bar{s} \gamma^\mu (1 - \gamma_5) b | D_s^+(p) \rangle \\ &= i F_{\perp} \frac{\bar{q}_{\perp}^{\mu}}{\sqrt{k^2}} + F_t \frac{q^{\mu}}{\sqrt{q^2}} + F_0 \frac{2\sqrt{q^2} k_0^{\mu}}{\sqrt{\lambda}} + F_{\parallel} \frac{\bar{k}_{\parallel}^{\mu}}{\sqrt{k^2}}. \end{aligned} \quad (5)$$

The momentum basis is $\bar{q}_{\perp}^{\mu} = 2\epsilon^{\mu\nu\rho\sigma} q_{\nu} k_{\rho} \bar{k}_{\sigma} / \sqrt{\lambda}$, $q^{\mu} = p^{\mu} - k^{\mu}$, $k_0^{\mu} = k^{\mu} - k \cdot q / q^2 q^{\mu}$, $\bar{k}_{\parallel}^{\mu} = \bar{k}^{\mu} - 4(k \cdot q)(\bar{k} \cdot q) \lambda k^{\mu} + 4k^2(\bar{k} \cdot q) / \lambda q^{\mu}$. Form factors $F_{i=\parallel,\perp,0,t}$ are functions of three kinematical variables, saying the momentum transfer squared q^2 , the two-pion invariant mass squared k^2 , and the scalar produce $q \cdot \bar{k} = \beta_{\pi} \cos \theta_{\pi} \sqrt{\lambda}/2$ that specifies the polar angle θ_{π} of the π^+ meson in the two-pion rest frame. In addition, $\lambda = m_B^4 - q^4 - k^4 - 2(m_B^2 q^2 + m_B^2 k^2 + q^2 k^2)$ is the Källén function, and $\beta_{\pi} = (1 - 4m_{\pi}^2/s)^{1/2}$ is the phase space factor.

The LCSRs' derivation of $D_s \rightarrow \pi\pi$ form factors begins with a twofold analysis of the two-point correlation function, defined as the vacuum-to-two-pion matrix element of the time-ordered product of the bilocal currents

TABLE I. The central values of twist-two and twist-three contributions to $D_s \rightarrow [\pi\pi]_S$ form factors at the full recoiled point $q^2 = 0$, in comparing to the $D_s \rightarrow f_0$ form factor.

FFs	$\sqrt{q^2} F_0^{(l=0)}(0)$	$\sqrt{q^2} F_0^{(l=2)}(0)$	$f_+(0)$
Twist two	$0.20 - i0.24$	$0.27 + i0.21$	0.20
Twist three	$-0.41 + i0.51$	$-0.55 - i0.41$	0.41
Total	$-0.21 + i0.27$	$-0.28 - i0.20$	0.61

$j_\mu = \bar{s}\gamma_\mu(1 - \gamma_5)c$ and $j_{D_s} = im_c\bar{c}\gamma_5s$. In the negative half plane of momentum transfers, it can be computed directly via the operator product expansion, where it is expressed as a convolution of perturbative scattering amplitudes with LCDAs, organized by twist. When q^2 crosses into the positive half plane, the dynamics shift to long-distance quark-gluon interactions, and hadronic states begin to form. In this regime, the correlator is instead represented as a sum

$$\begin{aligned} \sqrt{q^2} F_0^{(l'=0,2,\dots)}(q^2, k^2) &= \frac{\sqrt{\lambda}}{m_{D_s}^2 - q^2 - k^2} \sqrt{q^2} F_l^{(l')}(q^2, k^2) \\ &\quad - \frac{im_c k^2 q^2}{2m_{D_s}^2 f_{D_s} f_{2\pi}^\perp} \frac{\sqrt{\lambda}}{m_{D_s}^2 - q^2 - k^2} \int_{u_0}^1 du \left(\frac{1}{u^2} - \frac{m_c^2 - q^2 + u^2 k^2}{u^3 M^2} \right) \Phi^{\parallel,(\text{st})}(u) [B_{10}^\parallel I_{0l'}^\parallel + B_{12}^\parallel I_{2l'}^\parallel] e^{\frac{m_{D_s}^2 - s(u, q^2)}{M^2}} \\ &\quad + \frac{im_c k^2 q^2}{2m_{D_s}^2 f_{D_s} f_{2\pi}^\perp} \frac{\sqrt{\lambda}}{m_{D_s}^2 - q^2 - k^2} \frac{m_c^2 - q^2 + u_0^2 k^2}{u_0(s_0 - q^2)} \Phi^{\parallel,(\text{st})}(u_0) [B_{10}^\parallel I_{0l'}^\parallel + B_{12}^\parallel I_{2l'}^\parallel] e^{\frac{m_{D_s}^2 - s_0}{M^2}}, \end{aligned} \quad (6)$$

where the auxiliary distribution amplitude $\Phi^{\parallel,(\text{st})}(u) = -9u\bar{u}(2u-1) + 6u^2 - 6u$ incorporates the sum of the earlier known twist-two contribution [76–78] and the recently calculated twist-three contribution [46]. The threshold momentum fraction u_0 is the solution of $s_0 = \bar{u}k^2 + (m_c^2 + \bar{u}q^2)/u$. Moreover, the integration over the polar angle θ_π is given by $I_{ll'}^\parallel = \int_{-1}^1 d(\cos\theta_\pi) P_l^{(0)}(\cos\theta_\pi) P_l^{(0)}(\beta_\pi \cos\theta_\pi)$.

We adopt the particle data group values of $m_{D_s} = 1.968$ GeV and $f_{D_s} = 0.250$ GeV [1]. For the LCSR parameters, we choose $M^2 = 5.0 \pm 0.5$ GeV² and $s_0 = 6.0 \pm 0.5$ GeV², in accordance with the earlier study of $D_s \rightarrow f_0$ form factors [40], to ensure optimal stability and convergence of the QCD predictions. Charm quark mass and nonperturbative parameters in 2π DAs are taken from Ref. [46]. The phase shift $\delta_l(k^2)$ incorporated into the expansion coefficients in Eq. (4) is taken from the pion-nucleon reaction $\pi^- p \rightarrow \pi^0 \pi^0 n$ [55,56]. This choice of reaction is motivated by the similarity of its peak structure to that observed for the f_0 produced in D_s decays.

Table I presents the contributions from different twists to the S - and D -wave $D_s \rightarrow [\pi\pi]_S$ form factors at full recoil ($q^2 = 0$), obtained after integrating over the $\pi\pi$ invariant

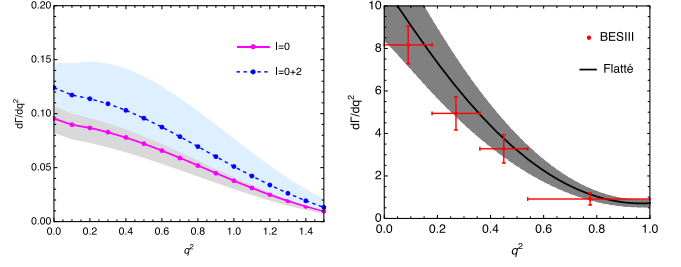


FIG. 3. The differential width $d\Gamma/dq^2$ of $D_{(s)} \rightarrow [\pi\pi]_S e^+ \nu_e$ (left) and $D_s \rightarrow [f_0 \rightarrow \pi\pi] e^+ \nu_e$ decays (right).

over contributions from all possible intermediate hadronic states. The final result is obtained by applying quark-hadron duality where the ground state is separated out from the interpolating current. The longitudinal form factors that contribute to $D_{(s)} \rightarrow [\pi\pi]_S$ transition ultimately read as [46]

mass k^2 . For comparison, the corresponding $D_s \rightarrow f_0$ form factor is also included. While the $D_s \rightarrow f_0$ form factor [40] is real, the $D_s \rightarrow [\pi\pi]_S$ form factors are complex. Furthermore, in contrast to the LCSR result for $D_s \rightarrow f_0$ form factors where twist-two and twist-three contributions both provide enhancement, the direct calculation for the $D_s \rightarrow [\pi\pi]_S$ transition reveals a nontrivial and instructive interplay between them. The origin of this effect lies in the asymmetry of the twist-three 2π DAs exhibited earlier. This contrasts with the symmetric twist-three LCDAs of f_0 , where QCD sum rules dictate that the asymmetric terms vanish, leaving only the symmetric terms associated to the even Gegenbauer polynomial [38–40].

The differential width of $D_s \rightarrow [\pi\pi]_S e^+ \nu_e$ decay on the momentum transfers is quoted as [46]

$$\frac{d\Gamma}{dq^2} = \int_{4m_\pi^2}^{k_{\text{max}}^2} dk^2 \frac{G_F^2 |V_{cs}|^2 \beta_\pi \sqrt{\lambda} q^2}{3(4\pi)^5 m_{D_s}^3} \sum_l |F_0^{(l)}(q^2, k^2)|^2. \quad (7)$$

Here the integral limit k_{max}^2 is a q^2 function related to the kinematics. We employ the simplified z -series expansion parametrization [79] to extend the LCSR prediction in the large-recoil region, approximately $0 \lesssim q^2 \lesssim 0.2$ GeV², across the entire physical range. Figure 3 depicts the result.

TABLE II. Decay widths (in unit of 10^{-4}) obtained from direct D_{I4} framework and cascade decay framework.

$D_s \rightarrow [\pi\pi]_S e^+ \nu_e$	$D_s \rightarrow [f_0 \rightarrow \pi\pi] e^+ \nu_e$ [40]	Data [42]
$0.81^{+0.34}_{-0.14}$	$18.8^{+4.5}_{-3.8}$	17.2 ± 1.6

In the left panel, the S - and D -wave contributions to the D_{I4} decay are displayed with iteration. The right panel compares BESIII data [42] to the prediction derived under the cascade framework [40] where the decay $D_s \rightarrow f_0 e^+ \nu_e$ is followed by $f_0 \rightarrow \pi\pi$, employing a Flatté parametrization for the resonance. Table II provides the decay widths obtained by integrating q^2 within these two frameworks under the $q\bar{q}$ ansatz.

While the cascade decay analysis can reproduce the BESIII data, it critically depends on two artificial assumptions: a marked sensitivity to the parametrization of the resonance, and the theoretically unsound use of a single-meson LCDA for the f_0 . The observed agreement therefore appears to arise from a fine-tuned, and presumably coincidental, delicate interplay between these methodological choices. In contrast, the result of a direct D_{I4} analysis using $D_s \rightarrow [\pi\pi]_S$ form factors is roughly 2 orders of magnitude smaller than the BESIII data. Such a significant discrepancy reveals that, unlike in analogous B -meson decays [46], the isoscalar two-pion system at the charm scale is not primarily a $q\bar{q}$ state. This result provides strong support for the partonic configuration interpretation governed by a color transparency mechanism.

Conclusions. Introducing 2π DAs to describe the partonic structure of the isoscalar two-pion state, the visible signal channel of the f_0 meson, we propose a novel framework for

analyzing D_{I4} decays through the direct calculation of $D_s \rightarrow [\pi\pi]_S$ form factors. We observe a significant cancellation between the contributions from twist-two and twist-three 2π DAs, leading to a predicted decay width significantly smaller than the BESIII measurement. This outcome aligns with QCD-based expectations from the color transparency mechanism, supporting the view that the f_0 meson at the charm mass scale is not dominated by the lowest $q\bar{q}$ Fock state. These findings emphasize a fundamental limitation of the traditional single-particle LCDA framework when it comes to characterizing the structure of the light scalar meson. In such cases, the asymptotic term vanishes, and twist-three distribution amplitudes become dominant. These higher-twist amplitudes encode an asymmetry in the partial-wave expansion that the single-particle picture fails to capture. Ultimately, our results reveal the rich and scale-dependent structure of f_0 in exclusive processes, underscoring the necessity of a multiparticle, channel-sensitive approach to describe resonances in QCD.

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Data availability. The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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