

Probing azimuthal alignment in heavy-ion collisions: Clusterization effects

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The influence of kinematic constraints and event selection on the emergence of the alignment phenomenon observed in cosmic-ray experiments is studied within the HYDJET++ model, which is widely used for simulating heavy-ion collisions. It is demonstrated that the high degree of alignment, previously identified for realistic values of the transverse momentum disbalance of the most energetic particles, is also observed at the level of the most energetic clusters. In high-multiplicity heavy-ion collision events, the clustering procedure plays a crucial role in resolving individual particle groups on the detection plane, allowing a more accurate characterization of alignment patterns. These results highlight the combined effects of cluster formation and momentum conservation in shaping the observed azimuthal correlations.

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I. INTRODUCTION

Angular correlations among secondary particles are a well-established feature of multiparticle production at both cosmic and accelerator energies. Among these, the alignment phenomenon represents a particularly striking case, first identified by the Pamir Collaboration in cosmic-ray emulsion experiments using large-area chambers placed at an altitude of about 5 km in the Pamir Mountains [1–5]. In these experiments, the most energetic hadrons and photons, or their clusters, tended to lie approximately along a straight line in the emulsion plane, suggesting a coplanar geometry of the events. Similar effects were later reported in the Capdevielle experiment aboard the Concorde aircraft [6]. Despite considerable theoretical effort [5–13], a universally accepted explanation of this phenomenon has not yet been achieved.

Evidence for alignment in collider experiments remains lacking, even though the collision energies at the Large Hadron Collider (LHC) exceed the effective threshold $\sqrt{s_{\text{eff}}} \gtrsim 4$ TeV observed in cosmic-ray interactions. At the same time, other forms of long-range azimuthal correlations—such as the ridge effect first observed by the

STAR collaboration [14,15] later at RHIC [16] and in high-multiplicity proton-proton collisions at the LHC [17]—have been extensively studied. Several works have attempted to link these effects to alignment [13,18,19], but differences in kinematic conditions and reference frames prevent any direct correspondence. Moreover, the ridge structure can be successfully described within conventional hydrodynamic and flow-based frameworks [20], supporting the interpretation of alignment as a statistical fluctuation and phenomenological models [9,21].

In our previous analyses [22,23], we proposed a geometric interpretation of alignment, demonstrating that pronounced collinearity can naturally arise due to the selection procedure of the most energetic particles, the energy-deposition threshold, and transverse momentum conservation. This concept was later implemented in the realistic heavy-ion event generator HYDJET++ [24], whose statistical model allows for event-by-event conservation of global quantities such as total transverse momentum and net charge [25].

The present study extends this approach by examining the impact of particle clustering in a high-multiplicity environment on the degree of alignment. The remainder of the paper is organized as follows. Section II introduces the essential notation for alignment analysis. Section III provides an overview of the alignment phenomenon and elliptic flow. Section IV describes the simulation of alignment in heavy-ion collisions. Section V discusses the influence of event-by-event transverse momentum conservation. Section VI compares simulations with and without clustering under transverse momentum conservation. Section VII provides concluding remarks. Additional details on modeling statistics are provided in the Appendix.

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II. ESSENTIAL NOTATION FOR ALIGNMENT ANALYSIS

In the Pamir experiment, families were selected and analyzed in which the total energy of γ quanta exceeded a certain threshold and in which at least one hadron was present. The alignment effect becomes pronounced at $\sum E_\gamma > 0.5$ PeV, corresponding to interaction energies of $\sqrt{s} \gtrsim 4$ TeV. These families are produced mainly by protons with energies $\gtrsim 10^4$ TeV interacting at an altitude h ranging from several hundred meters to a few kilometers in the atmosphere above the chamber [1,5]. The collision products are observed within a radial distance $r_{\max}$ up to about 15 mm in the emulsion, with a minimum spot separation $r_{\min} \sim 1$ mm.

The alignment parameter λ_N quantifies the azimuthal correlation among N selected particles or clusters and characterizes their deviation from a straight line, providing a more sensitive measure of asymmetry than other parameters, such as eccentricity or thrust. It is a dimensionless quantity taking values in the interval $[-1/(N-1), 1]$ and is defined as [4],

$$\lambda_N = \frac{\sum_{i \neq j \neq k}^N \cos(2\varphi_{ijk})}{N(N-1)(N-2)}, \quad (1)$$

where φ_{ijk} is the angle between two straight lines connecting the i th spot with the j th and k th spots. The combinatorial factor $N(N-1)(N-2)$ accounts for all triplet combinations and ensures proper normalization. For example, if $N = 3$, an equilateral triangle yields $\lambda_3 = -0.5$. Perfect alignment of all points along a straight line corresponds to $\lambda_N = 1$, independent of N , while deviations from a straight line reduce λ_N , reflecting a lower degree of collinearity.

$$\left[\sqrt{p_{Ti}^2 + m_i^2} \cosh \eta_i, \quad p_{Ti} \cos \phi_i, \quad p_{Ti} \sin \phi_i, \quad \sqrt{p_{Ti}^2 + m_i^2} \sinh \eta_i \right]. \quad (3)$$

The transformation from the center-of-mass system to the laboratory frame amounts to the rapidity shift,

$$\zeta_i = \eta_0 + \eta_i,$$

where η_0, ζ_i are the rapidities of the center-of-mass system and the particle i , respectively, in the laboratory reference frame.

If one neglects the further interactions of the particles propagating through the atmosphere, this provides an upper estimate of the alignment effect, then the needed azimuthal angles (Fig. 1) are calculated over the positions $\mathbf{r}_i$ of the particles in the $(x-y)$ plane in the film,

$$\mathbf{r}_i = \frac{\mathbf{v}_{ri}}{v_{zi}} h, \quad (4)$$

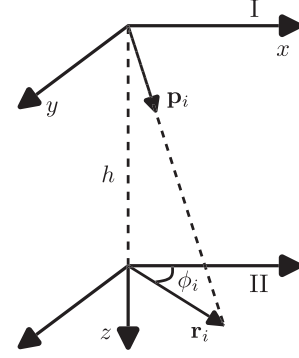


FIG. 1. Kinematics and scheme of the analyzed events in the context of Pamir experiment. Region I represents the Earth's atmosphere, where incoming cosmic-ray protons generate cascades of hadrons and photons whose alignment properties are studied. Region II corresponds to the emulsion film plane. In this region, p_i stands for the momentum of a registered particle (or cluster), r_i denotes its position on the film, ϕ_i is the azimuthal angle, and h indicates the height above the emulsion chamber at which the particle was produced.

The degree of alignment

$$P_N = \frac{l}{L} \quad (2)$$

is defined as the ratio of events l for which $\lambda_N > 0.8$ to the total number of events L and only events with at least N energy centers are counted. [5].

For convenience we parametrize the four-momentum of each particle i under consideration with its transverse momentum p_{Ti} (with respect to the collision axis z), mass m_i , rapidity η_i and azimuthal angle ϕ_i in the center-of-mass system,

where v_{zi} and $\mathbf{v}_{ri}$ are the longitudinal and radial components of particle velocity, respectively. Accounting for the transformation from the center-of-mass frame to the laboratory frame via a rapidity shift, the particle coordinates are given by,

$$\mathbf{r}_i = \frac{\mathbf{p}_{Ti}}{\sqrt{p_{Ti}^2 + m_i^2} \sinh(\eta_0 + \eta_i)} h. \quad (5)$$

Since the size of the observation region is about several centimeters, these radial distances $r_i = |\mathbf{r}_i|$ must obey the following restrictions:

$$r_{\min} < r_i, \quad (6)$$

$$r_i < r_{\max}. \quad (7)$$

The condition (6) simply means that spots are not mixed with the central one formed by the particles that fly close to the collision axis z , predominantly region of incident-hadron fragmentation, and the condition (7) indicates that the particle coordinate does not exceed the observation region.

In this context, it is also interesting and relevant to consider the influence of the applied restrictions $r_{\min} < r_i < r_{\max}$ (the laboratory acceptance criterion) on the particle spectrum selected for the alignment calculation. For particles with sufficiently large transverse momenta p_{Ti} compared to their masses m_i , the conditions (6), (7) reduce mainly to restrictions on the available particle rapidities in the center-of-mass system,

$$r_{\min} < r_i \Rightarrow \eta_i < \eta_{\max} = \ln(r_0/r_{\min}) \simeq 5.98,$$

$$r_i < r_{\max} \Rightarrow \eta_i > \eta_{\min} = \ln(r_0/r_{\max}) \simeq 3.28,$$

since in this case $r_i \simeq r_0/e^{\eta_i}$ for $\eta_0 + \eta_i \gtrsim 1$, where $r_0 = 2h/e^{\eta_0}$. In this estimation, we have set $h = 1$ km, $r_{\min} = 1$ mm, $r_{\max} = 15$ mm, and $\eta_0 = 8.52$ (at $\sqrt{s} = 5.02$ TeV), for the sake of clarity.

Among clusters that satisfy the restrictions (6), (7) one selects the two to seven clusters/particles that are most energetic. After that one calculates the alignment λ_N using the definition above and taking into account the central cluster, i.e., $N - 1 = 2 \dots 7$.

III. ON ALIGNMENT AND AZIMUTHAL FLOW

After introducing the essential parameters and observables relevant for the characterization of alignment, it is instructive to discuss the strengths and limitations of this method in comparison with the well-established description of azimuthal flow in heavy-ion collisions based on the Fourier harmonic decomposition. Historically, the alignment-based characterization of azimuthal correlations—originally developed in cosmic-ray studies—preceded the now-standard Fourier approach. The method based on the Fourier decomposition of the continuous distribution of outgoing particles over the azimuthal angle was introduced in Ref. [26] and relies on the expression

$$E \frac{d^3 N}{d^3 p} = \frac{d^3 N}{p_T dp_T d\eta d\phi} = \frac{d^2 N}{p_T dp_T d\eta 2\pi} \left[1 + \sum_{n=1}^{\infty} 2v_n \cos n(\phi - \Psi_R) \right], \quad (8)$$

where E denotes the particle energy, p_T the transverse momentum, η the rapidity, ϕ the azimuthal angle of outgoing particles, and Ψ_R the reaction-plane angle. Figure 2(a) schematically illustrates a noncentral collision of two nuclei and the direction of a secondary particle emitted at an azimuthal angle ϕ . Since the reaction-plane angle is difficult to determine experimentally, it is usually replaced by the so-called event-plane angle; for simplicity, in Fig. 2(a) we set $\Psi_R = 0$. The first two coefficients, v_1 and v_2 , correspond to directed and elliptic flow, respectively, while the leading term describes the azimuthally symmetric component of the particle yield. From the orthogonality of the Fourier basis, one obtains

$$v_n(p_T, \eta) = \langle \cos[n(\phi - \Psi_R)] \rangle = \frac{\int_0^{2\pi} d\phi \cos[n(\phi - \Psi_R)] \frac{d^3 N}{p_T dp_T d\eta d\phi}}{\int_0^{2\pi} d\phi \frac{d^3 N}{p_T dp_T d\eta d\phi}}, \quad (9)$$

where the averaging is first performed over all particles in a given event and subsequently over events within a centrality class. The second coefficient, $v_2(p_T, \eta)$, has received the most attention and is the best studied, as it is strongly driven by the ellipticity of the nuclear overlap region and reflects the preference for particle emission along the short axis of the almond-shaped geometry [Fig. 2(a)]. However, there is now substantial evidence for sizeable contributions from higher-order harmonics as well [27–29].

Let us now turn to Fig. 2(b), which schematically illustrates three clusters ($N = 3$, with cluster 1 fixed at the origin for simplicity) in the azimuthal plane used to study alignment. A fundamental difference from the Fourier-harmonic method (8) is that the alignment observable represents a nonflow type of particle correlation. In other words, it does not require particles to share similar p_T , rapidity, or azimuthal angle ϕ . Instead, it focuses on the relative orientation of neighboring or nearby particle

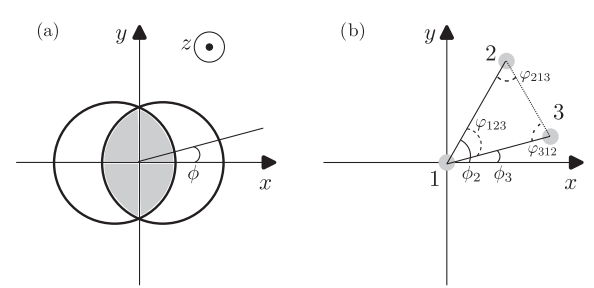


FIG. 2. (a) Schematic illustration of a noncentral collision of two nuclei in the $x - y - z$ plane; ϕ denotes the azimuthal angle of the outgoing secondary particles. (b) Illustration of three clusters in the azimuthal plane used to study the alignment effect. The angles $\phi_1 = 0$, ϕ_2 , and ϕ_3 correspond to the azimuthal positions of clusters 1, 2, and 3, respectively. The quantities ϕ_{123} , ϕ_{213} , and ϕ_{312} represent the angles between the vectors connecting the clusters and are used in the alignment calculation [see Eq. (1)].

clusters, i.e., the angles between the clusters 1–3: φ_{123} , φ_{213} , φ_{312} in Fig. 2(b). In this sense, the alignment procedure may be viewed as a more general characterization of azimuthal structure, as it directly probes the mutual spatial distribution of particle flows in the azimuthal plane.

A natural question arises: is it possible to compare the approaches used to study azimuthal anisotropy through the Fourier coefficient v_n and through particle alignment? Are there any correlations between these two observables, and can they be meaningfully combined in heavy-ion collisions? Strictly speaking, a direct comparison is essentially impossible, since, as noted above, v_2 (and higher harmonics) quantify collective flow, whereas alignment represents a nonflow particle correlation. In this context, it is more reasonable to introduce a nonflow quantity $c_2\{2\}$, often referred to as a two-particle correlator [26] or cumulant,

$$c_2\{2\} = \langle \cos[2(\phi_i - \phi_j)] \rangle = \frac{1}{N_{\text{pairs}}} \sum_{i < j}^N \cos[2(\phi_i - \phi_j)], \quad (10)$$

where $N_{\text{pairs}} = N(N-1)/2$ is the number of particle pairs constructed from N particles, and ϕ_i , ϕ_j denote the azimuthal angles of the i th and j th particles. In this form, $c_2\{2\}$ is the closest nonflow analog of “classical” v_2 and is therefore the most directly comparable to alignment. It is worth noting that the correlator method has been widely used in the analysis of relativistic heavy-ion collisions, as documented, for example, in Refs. [30–35] and $c_2\{2\} = v_2^2$ if particles correlate with the reaction plane only. The methods discussed—the alignment λ , the flow v_n decomposition technique (8), and the correlator-based approach $c_n\{2\}$ —can provide complementary information on azimuthal anisotropies in collisions of relativistic nuclei.

The alignment method is particularly sensitive to nonflow effects and captures the relative geometric arrangement of nearby particle clusters, offering a direct probe of local structures in the azimuthal plane. Its main limitation lies in reduced statistical stability—this effect has not yet been observed in collider experiments, as well as the lack of a direct connection to the global collective flow. In contrast, the Fourier-harmonic decomposition for the elliptic flow coefficient v_2 and the correlator $c_2\{2\}$ are well-established, statistically more robust observables directly related to collective dynamics, although they may mix flow and nonflow contributions unless additional correction procedures are applied. It is important to note that both the flow v_2 and nonflow $c_2\{2\}$ (in the absence of particle correlations) quantities vanish for an isotropic azimuthal distribution. The absence of direct evidence for alignment in collider experiments, coupled with the established observation of elliptic flow through v_2 and $c_2\{2\}$ in the same experiments, strongly motivates the investigation of possible azimuthal alignment in heavy-ion collisions.

IV. SIMULATION OF ALIGNMENT IN HEAVY-ION COLLISIONS

To obtain a clearer picture of how azimuthal alignment may appear under controlled conditions, we extended our earlier analysis [22,25] by defining three centrality selections: 0%–5%, 40%–75%, and the inclusive 0%–75% sample. The most central class (0%–5%) corresponds to near-head-on collisions and yields the largest particle multiplicities. The peripheral class (40%–75%) represents collisions in which anisotropic flow effects are expected to be most pronounced. The inclusive 0%–75% sample combines events over a broad range of collision geometries and centralities and therefore provides a more suitable basis for comparison with cosmic-ray data, in particular with the Pamir experiment. Although such a comparison is necessarily indirect and partly driven by the need to relate collider simulations to cosmic-ray observations, it nevertheless offers a useful reference framework for interpreting possible alignment effects.

Our goal is to examine the possible emergence of alignment structures in simulated heavy-ion reactions while consistently accounting for the system’s space-time evolution and the balance between soft and hard particle production mechanisms. The comparison among different centrality classes also provides a way to study how alignment features may depend on the impact parameter of the initial interaction.

For the simulations, we employ the HYDJET++ event generator [24], a model validated against multiple nucleus-nucleus collision observables at RHIC and LHC energies. Recent developments are summarized in [36–38]. This framework allows one to model particle production in a realistic environment that includes thermal emission as well as jet-related contributions. The simulation accounts for soft particles with relatively low transverse momenta as well as the hard, jet-producing part of the generator, thus providing a comprehensive description of the collision dynamics.

In our analysis, we focus on Pb + Pb collisions at a center-of-mass energy of $\sqrt{s} = 5.02$ TeV per nucleon pair, using the HYDJET++ model with parameters taken from previous calibrations without additional retuning. This system provides a well-established testing ground for studying collective effects in strongly interacting matter. Although Pb + Pb collisions involve a much larger system than those produced in interactions of cosmic nuclei in the atmosphere, such as Fe + O, they can be explored in detail at modern colliders and are reliably described by existing Monte Carlo generators. The alignment phenomenon may, however, originate from nucleus-nucleus interactions at high energies. The collision energy at the LHC far exceeds the energy scale relevant for particle interactions in the Pamir experiment where this effect was observed. Furthermore, the formation of a quark-gluon plasma (QGP) requires surpassing a certain energy density,

which is typically achieved in heavy-ion collisions. Nevertheless, a large atomic number is not a strict requirement for QGP formation; such a dense medium can also be produced in smaller systems, even in high-multiplicity proton-proton collisions [39–41]. Thus, while the consideration of Pb + Pb collisions is not essential for the qualitative features of our findings, this system was chosen for the present study because it has been extensively investigated experimentally and accurately reproduced by the HYDJET++ model without analysis-specific parameter retuning.

A. Brief overview of the HYDJET++ model

HYDJET++ is a Monte Carlo event generator designed for the simulation of relativistic heavy-ion collisions. It combines two main components:

- (i) The soft part represents a thermalized hadronic medium emerging at chemical and thermal freeze out, modeled via relativistic hydrodynamics with parametrized freeze-out conditions. Particle yields are sampled from a Poisson distribution, with the mean proportional to the number of participant nucleons. Collective flow and spatial anisotropies are implemented using the modified FAST MC generator [42,43], while the effective thermal volume accounts for flow profiles and freeze-out geometry, making particle ratios relatively insensitive to detailed freeze-out specifications.
- (ii) The hard part describes high- p_T multiparton production and energy loss in the quark-gluon plasma, following the PYQUEN approach [44], which accounts for both collisional and radiative mechanisms. The number of generated jets is drawn from a binomial distribution, with the mean determined by the number of binary nucleon-nucleon interactions and the hard-process cross section above a minimum transverse momentum $p_T^{\min}$. Partons below this threshold are considered thermalized and included in the soft sector.

It is important to note that this model does not feature an explicit quark-gluon plasma. Its effects are effectively incorporated through parton hadronization with a characteristic energy scale, which should be understood as approximate rather than strict. This scale reflects the conditions under which the azimuthal alignment phenomenon may emerge, signaling the onset of specific dynamical regimes in high-energy heavy-ion collisions.

B. Modeling approach using secondary-particle clustering

In the initial approach, each particle is treated as an individual cluster with variable size and mutual separation, limited only by the conditions (6) and (7) to prevent overlap with the central region of the detection plane. This simplified treatment does not account for the possibility

that closely spaced particles may merge into a single, compound cluster; results from this procedure were discussed in our previous work [25].

In the present study, we explicitly incorporate clustering of secondary particles and the simulation procedure can be summarized as follows:

- (i) First, an event corresponding to a collision of two nuclei of a given type is generated;
- (ii) For each secondary particle, its position r_i on the emulsion plane is calculated using Eq. (4), and it is verified whether the resulting position satisfies the acceptance criteria in Eqs. (6) and (7);
- (iii) After determining the position of a particle i , the distance to another particle j is computed using the expression,

$$d_{ij} = \sqrt{r_i^2 + r_j^2 - 2r_i r_j \cos(\phi_i - \phi_j)}; \quad (11)$$

- (iv) If the distance d_{ij} satisfies the clustering condition

$$d_{ij} < r_{\text{res}}, \quad (12)$$

with a given resolution parameter r_{res} , the particles are merged into a new cluster, whose coordinates are defined as

$$\mathbf{r}_{ij} = (\mathbf{r}_i E_i + \mathbf{r}_j E_j) / (E_i + E_j). \quad (13)$$

Otherwise, they are retained as separate clusters; and

- (v) Among the accepted clusters the conditions Eqs. (6), (7) are checked again. Then, three to five ($N = 3 - 5$) highest-energy clusters are selected, and their alignment λ_N and degree of alignment P_N are computed according to Eqs. (1) and (2). The most energetic cluster is always fixed at the origin $O(0, 0)$.

This algorithm involves pairwise comparisons of all secondaries within an event and is conceptually analogous to standard jet-finding procedures. The main idea behind pairwise clustering algorithms is the concept of a “nearest neighbor” and the choice of a measure used to define this “nearestness.” In our case, the measure is the ordinary distance in the “emulsion plane” (using the terminology of the Pamir experiment). In collider experiments, typical measures for jet reconstruction include the square of the invariant mass, the transverse momentum of particles relative to the direction defined by the sum of two particles’ momenta, or other more complex variables in phase space.

In the context of the alignment phenomenon in heavy-ion collisions, the particle positions r_i given by Eq. (5), together with the distance and clustering criteria defined in Eqs. (11)–(13), are used to characterize the spatial relations of particles in the azimuthal plane. For clarity, we note the following analogy: in jet physics, a widely used expression for jet selection is

$$\Delta R_{\text{jet}} = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2},$$

where η is the rapidity and ϕ is the azimuthal angle. In our case, a similar expression can be written as

$$\Delta d = r\sqrt{(\Delta\eta)^2 + (\Delta\phi)^2}, \quad (14)$$

which is analogous to Eq. (11) when expressed in terms of η and ϕ .

The Eqs. (5), (14) allows us to establish a relationship between the variation of the distance between clusterlike particles, Δd , in the laboratory frame and the variations of the center-of-mass variables η and ϕ , namely,

$$\Delta d = r\Delta\phi \quad \text{and} \quad \Delta d = -r\Delta\eta,$$

where $r = |\mathbf{r}_i|$. The latter relation can be derived if $\sinh(\eta_0 + \eta_i)$ is approximated by $\exp(\eta_0 + \eta_i)/2$ for sufficiently large $(\eta_0 + \eta_i)$. In this case, the clustering condition (12) can be rewritten as

$$\sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} < r_{\text{res}}/r,$$

where the connection between the ordinary (laboratory-frame) and center-of-mass coordinates becomes more transparent.

It is therefore possible to estimate the effective resolution in the center-of-mass variables for alignment clusters. For example, taking $r_{\text{res}} = 1$ mm and $r = r_{\text{max}} = 15$ mm, the resolution can be estimated as

$$\eta_{\text{res}} = \phi_{\text{res}} = \frac{1}{15}.$$

If instead $r = r_{\text{min}} = 1$ mm, one obtains

$$\eta_{\text{res}} = \phi_{\text{res}} = 1.$$

This estimate is independent of the height h and indicates that the separation between distinguishable clusters in η and ϕ lies in the range

$$\Delta\eta, \Delta\phi \in [1/15, 1],$$

depending on the radial position r_i of the cluster. If the rapidity or azimuthal angle difference between clusters exceeds this resolution scale, they are treated as distinct; otherwise, they are merged into a single cluster.

Such measures are commonly used in modern event generators to identify jets in electron-positron annihilation and hadron collisions. The efficiency (i.e., the number of iterative steps) and the stability of clustering algorithms depend not only on the choice of the measure itself, but also on its numerical value and on the selection of particle pairs from which the clustering procedure is initiated. The clustering algorithm presented in this work demonstrates stability with respect to these criteria. Alignment observables are then calculated using only those clusters (or clusterlike particles) that do not meet the clustering condition in Eq. (12).

C. Simulation outcomes with clustering of secondary particles

The simulation results obtained according to the procedure described above, as well as the data from the Pamir experiment, are presented in Table I.

Table I clearly shows that the results obtained for the alignment degree P_N of three, four, and five ($N = 3, 4, 5$) clusters are not consistent with the experimental data from the Pamir experiment, and the dependence on cluster size r_{res} is rather weak for all centrality classes $c = 0\%-5\%$, $40\%-75\%$, $0\%-75\%$. Furthermore, these results are similar to those obtained in previous studies [22,25], where no clustering procedure for secondary particles was applied.

The reason for this is straightforward: although angular correlations are present in the HYDJET++ model, including correlations with the reaction plane, they are clearly insufficient to generate the correlations required for the emergence of the alignment phenomenon without a dedicated selection procedure. This emphasizes that the angular distribution plays a more significant role than the radial

TABLE I. Simulation results for the alignment degree P_N of three, four, and five ($N = 3, 4, 5$) clusters and experimental data from the Pamir experiment [4]. The size of central cluster $r_{\text{min}} = 1$ mm, the height h is fixed at 1 km throughout this work. The collision energy in the HYDJET++ generator is 5.02 TeV per nucleon pair, and the generation here and below always includes both components of the model—soft and hard.

Alignment degree	P_3			P_4			P_5		
Pamir results [4]	0.83 ± 0.27			0.67 ± 0.33			0.33 ± 0.23		
Centrality									
$r_{\text{res}}, \text{ mm}$	0%–5%	40%–75%	0%–75%	0%–5%	40%–75%	0%–75%	0%–5%	40%–75%	0%–75%
0.5	0.23	0.27	0.26	0.032	0.061	0.053	0.005	0.015	0.011
1	0.23	0.28	0.26	0.037	0.061	0.048	0.006	0.013	0.009
2	0.25	0.28	0.26	0.042	0.058	0.048	0.007	0.013	0.009
5	0.26	0.27	0.26	0.045	0.052	0.046	0.007	0.011	0.009

distribution in the appearance of alignment. Two distinct approaches can be identified. In the geometrical approach [22], the probability density of spots is proportional to r , implying a linear increase with radial distance. In the HYDJET++ model, by contrast, the radial distribution follows $F(r)r$, where $F(r)$ is a rapidly decreasing function of r . Despite the substantial differences in the radial distributions between these two approaches, the insufficiency of angular correlations in both cases underlines the necessity of the selection procedure for reproducing the observed alignment phenomenon.

Furthermore, it is important to note that the comparison with the Pamir experiment is qualitative in nature, since different types of nuclei are involved (light vs heavy), as well as different kinematic regimes. However, as already mentioned, no data on the manifestation of azimuthal alignment of particles in collider experiments currently exist.

V. INFLUENCE OF TRANSVERSE MOMENTUM CONSERVATION ON ALIGNMENT

Since the Pamir Collaboration reported that the alignment phenomenon begins to appear once the detected particle energies exceed a certain threshold, this observation motivated us to attempt to interpret the observed azimuthal correlations in terms of purely kinematic relations, without invoking specific dynamical assumptions or additional mechanisms. We refer to this approach as the transverse momentum disbalance of the most energetic particles and/or clusters, which characterizes the degree to which transverse momentum conservation is preserved in the calculation of alignment.

A. Concept of event-by-event transverse momentum conservation

As discussed above, the observed alignment may be influenced by purely kinematic effects rather than specific dynamical mechanisms. In particular, in statistical models such as HYDJET++, soft particles are produced independently, leading to event-by-event fluctuations of total momentum, energy, and particle number. These quantities are conserved only on average over many events, typically within a limited rapidity interval, rather than exactly in each individual collision. Consequently, the total transverse momentum of all particles vanishes statistically—but not precisely—in a single event, which can affect azimuthal correlations among the most energetic particles. To account for this effect, we introduce after performing the clustering procedure an event-level constraint on the residual transverse momentum,

$$|\mathbf{p}_{T_1} + \mathbf{p}_{T_2} + \cdots + \mathbf{p}_{T_{N-1}}| < \Delta, \quad (15)$$

where $\mathbf{p}_{T_i}$ is the transverse momentum of the i th cluster. The parameter Δ defines the allowed degree of transverse momentum disbalance: smaller Δ values correspond to

stronger event-by-event momentum conservation, while larger ones permit greater deviations. It is important to note that, in applying this approach to the HYDJET++ model, we do not modify its parameters or introduce any changes to the generator. Rather, we work with the particles after generation, i.e., we first select those that satisfy the clustering condition (12), and subsequently the resulting clusters are tested against condition (15).

The alignment degree P_N , defined in Eq. (2), is evaluated as a function of the transverse momentum disbalance parameter Δ ,

$$P_N(\Delta) = \frac{l^{[\Delta]}}{L^{[\Delta]}}, \quad (16)$$

where $l^{[\Delta]}$ denotes the number of events with $\lambda_N > 0.8$, and $L^{[\Delta]}$ is the number of events satisfying condition (15). This formulation allows us to quantify how the alignment degree $P_N(\Delta)$ depends on the level of transverse momentum conservation for the $N = 3, 4, 5$ most energetic particles and clusters on an event-by-event basis within a limited rapidity range. The total number of generated events used in the analysis is fixed at $L^{[\text{tot}]} = (1 - 3) \times 10^6$ [see Appendix A 1 (Tables III–V), A 2 (Tables VI–VIII), and A 3 (Tables IX–XI) for details].

B. Simulation results with local p_T conservation

After introducing the basic definitions and the transverse momentum disbalance (15) for clusters of secondary particles, we now present the simulation results for the alignment degree P_N as a function of this disbalance Δ . As discussed in the previous Sections, the results are shown for three centrality classes of heavy-ion collisions: 0%–5%, 40%–75%, and 0%–75%.

Figures 3–5 present the dependence of the alignment degree P_N on the transverse momentum disbalance Δ for different values of the cluster size parameter r_{res} but r_{min} unchanged. Figure 3 corresponds to the most central collisions (0%–5%), Fig. 4 to peripheral collisions (40%–75%), and Fig. 5 to the combined sample (0%–75%). For smaller clusters (lower r_{res}), the alignment degree shows a weaker dependence on Δ , whereas for larger clusters the variation of P_N with disbalance becomes more pronounced. This trend reflects the increasing role of collective effects and geometric asymmetries as the effective cluster size grows.

These results indicate that the alignment effect is sensitive both to the event geometry via centrality class and to the scale at which clusters of secondary particles are formed. Taking into account the transverse momentum disbalance in the conservation of clusters momenta (15) leads to an increase in the alignment compared to the case without such consideration, as summarized in the Table I. Overall, for all centrality classes and cluster multiplicities N , our hypothesis is confirmed: the smaller the Δ , the stronger the azimuthal correlation of the clusters, whereas a

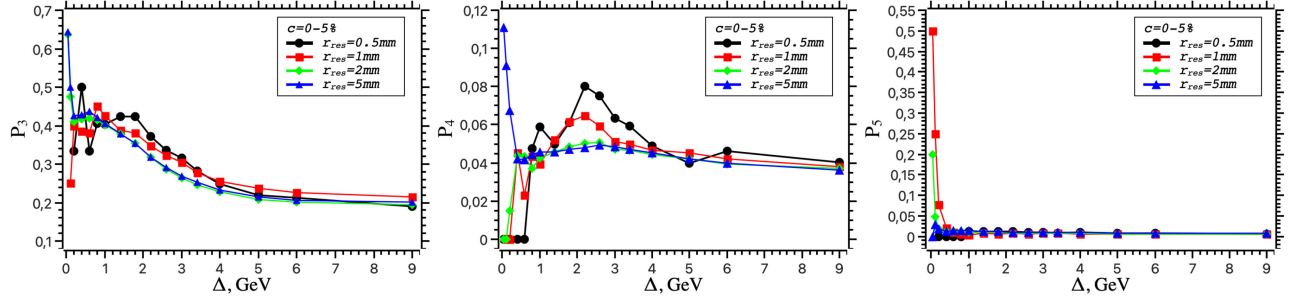


FIG. 3. Degree of alignment P_3 , P_4 , P_5 for the three, four, five clusters as a function of the disbalance Δ at the different values of the resolution parameter $r_{\text{res}} = 0.5, 1, 2, 5$ mm. Centrality class $c = 0\%-5\%$.

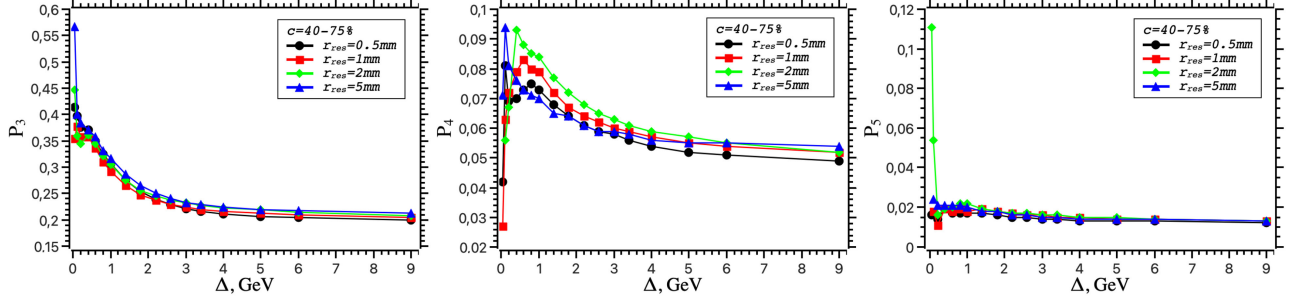


FIG. 4. Degree of alignment P_3 , P_4 , P_5 for the three, four, five clusters as a function of the disbalance Δ at the different values of the resolution parameter $r_{\text{res}} = 0.5, 1, 2, 5$ mm. Centrality class $c = 40\%-75\%$.

larger disbalance corresponds to a weaker alignment. The observed dependence of P_N on missing transverse momentum and the cluster size parameter r_{res} suggests that the interplay between local momentum correlations and global event geometry may play a significant role in shaping the alignment patterns.

C. Results with soft and hard components of HYDJET++ and stable resonances

The HYDJET++ model makes it possible to simulate heavy-ion collisions with only the soft component, only hard jets, or both components simultaneously, as well as to switch resonance decays on or off. This makes it possible to

isolate the role of individual model ingredients in the formation of the alignment effect. In our previous study [25], the influence of jets and the soft component on alignment was investigated separately; however, clustering of secondary particles was not included. In the present analysis, we extend this study by considering the effect of different HYDJET++ components together with the clustering procedure.

Figure 6 presents the alignment degrees P_3 , P_4 , and P_5 as functions of the transverse momentum disbalance Δ for the $0\%-75\%$ centrality class and the resolution parameter $r_{\text{res}} = 1$ mm. The results are shown for events generated with only the soft component, only the hard component, and with both components and resonance decays switched

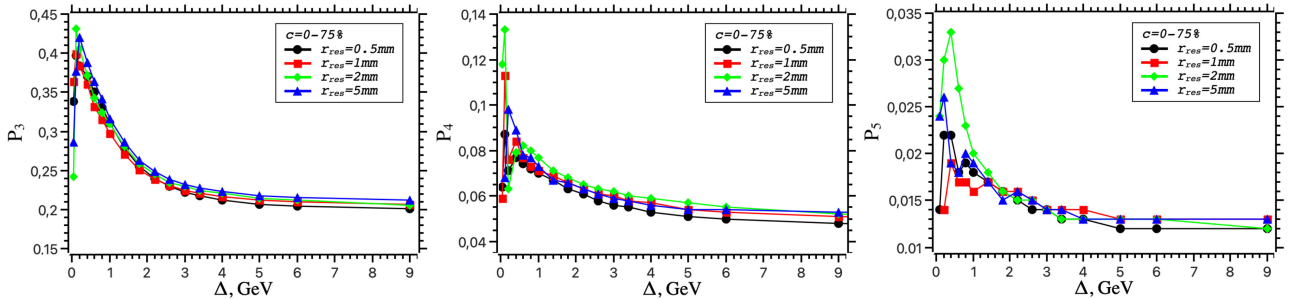


FIG. 5. Degree of alignment P_3 , P_4 , P_5 for the three, four, five clusters as a function of the disbalance Δ at the different values of the resolution parameter $r_{\text{res}} = 0.5, 1, 2, 5$ mm. Centrality class $c = 0\%-75\%$.

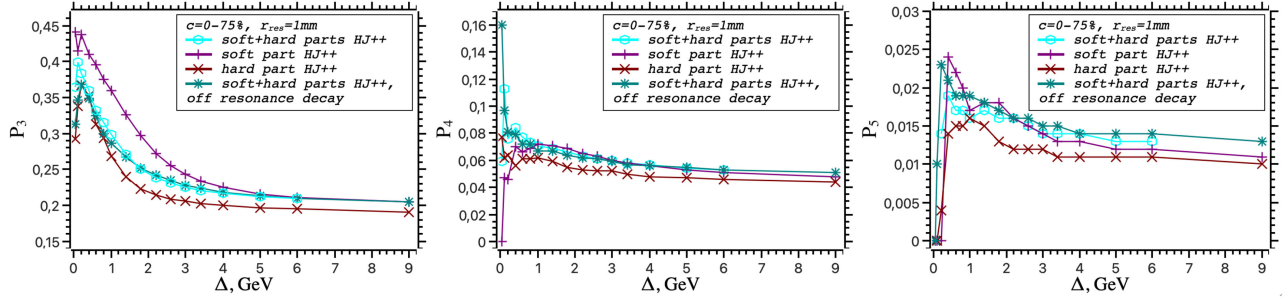


FIG. 6. Comparison of the alignment degrees P_3 , P_4 , and P_5 as functions of the transverse momentum disbalance Δ for the resolution parameter $r_{\text{res}} = 1$ mm and the centrality class $c = 0\% - 75\%$. The results are shown for events generated with only the soft component, only the hard component, and with both components while resonance decays are switched off.

off. For the soft component, the alignment effect appears somewhat stronger than for the hard component, although the difference is quite moderate and noticeable mainly for P_3 . In this case, it seems easier to achieve the required momentum disbalance due to the softer spectrum and potentially anisotropic flow. A noticeable enhancement of the alignment is also observed for P_4 at small Δ in the absence of resonance decays. In all other cases, no significant influence of individual model components on the alignment is seen. Qualitatively similar behavior was previously obtained without applying clustering [25].

VI. COMPARISON OF SIMULATIONS WITH AND WITHOUT CLUSTERING UNDER TRANSVERSE MOMENTUM CONSERVATION

Since the main motivation and goal of this work is to demonstrate the impact of secondary particle clustering on the alignment phenomenon, it is both appropriate and logical to devote a separate section to this comparison. The simulation results without clustering can be found in our recent work [25]. Figures 7–9 compare the alignment degree simulations with and without clustering for the three centrality classes at a cluster size of $r_{\text{res}} = 1$ mm.

It is evident that, for all three centrality classes, clustering reduces P_3 in a range $\Delta = 0 - 1$ GeV from its maximum value approximately by a factor of 2. This effect can be attributed to the fact that the direction of the cluster's

transverse momentum and its radial position do not necessarily coincide. The cluster momentum is the sum of the momenta of all constituent particles, whereas its position is determined by the energy-weighted average of the particles within it. This discrepancy can lead to a decrease in the observed alignment compared to the case without clustering.

In the case of four clusters, P_4 , the situation is somewhat different: for the most central collisions ($c = 0\% - 5\%$), clustering reduces the alignment over a wide range of the transverse momentum disbalance Δ , whereas the inclusion of peripheral collisions generally increases the alignment, which may reflect the influence of anisotropic azimuthal flow of secondary particles.

For five clusters, P_5 , the results are more unexpected: central collisions exhibit a significant increase in alignment with clustering compared to the case without clustering at small Δ values, although this result has relatively low statistics (see Appendix A 1). Peripheral collisions ($c = 40\% - 75\%$ and $c = 0\% - 75\%$), on the other hand, clearly enhance the alignment relative to the nonclustered case in the range $\Delta > 1$ GeV, which may also be a consequence of the anisotropy in the secondary particle flow.

Overall, by examining Figs. 7–9, we can conclude that the effect of secondary particle clustering on the alignment phenomenon is indeed present. Its manifestation is not uniform and depends on the collision centrality, the number of energetically distinguished clusters under study N , as

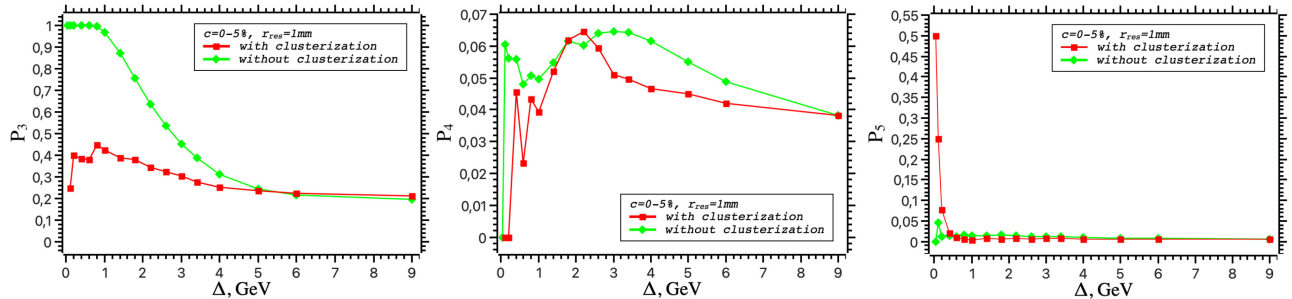


FIG. 7. Comparison of alignment degree P_3 , P_4 , P_5 as a function of the disbalance Δ with and without clustering. The resolution parameter $r_{\text{res}} = 1$ mm. Centrality class $c = 0\% - 5\%$.

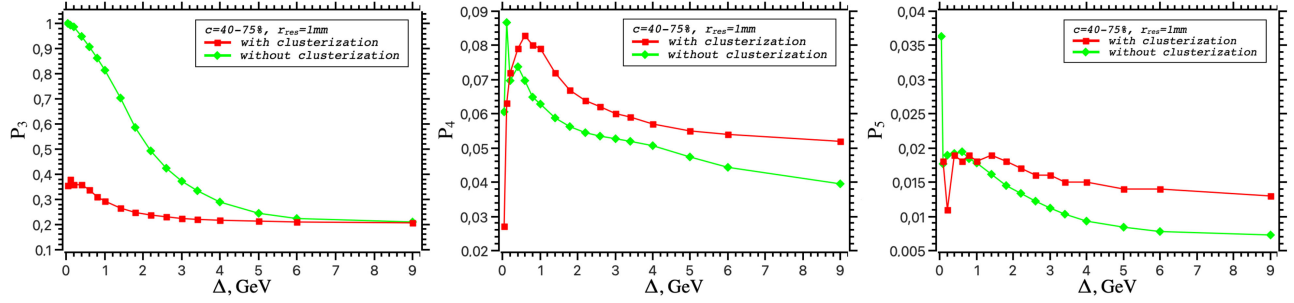


FIG. 8. Comparison of alignment degree P_3 , P_4 , P_5 as a function of the disbalance Δ with and without clustering. The resolution parameter $r_{\text{res}} = 1$ mm. Centrality class $c = 40\%-75\%$.

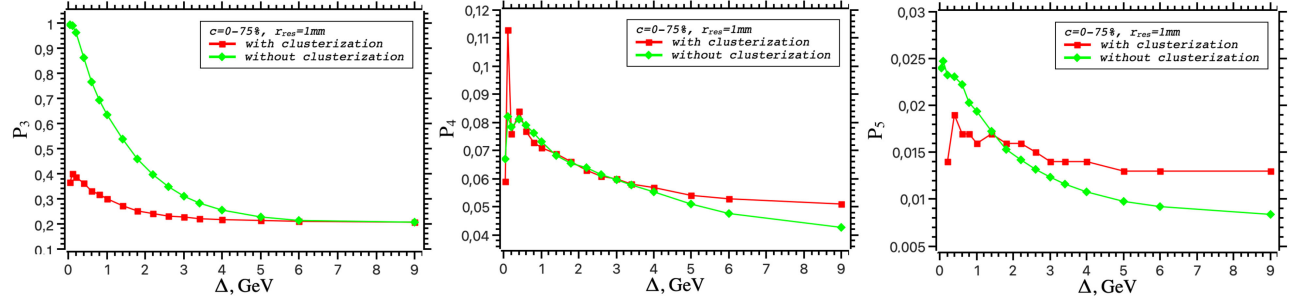


FIG. 9. Comparison of alignment degree P_3 , P_4 , P_5 as a function of the disbalance Δ with and without clustering. The resolution parameter $r_{\text{res}} = 1$ mm. Centrality class $c = 0\%-75\%$.

well as on the event-by-event conservation of the cluster transverse momenta.

VII. DISCUSSION AND CONCLUSIONS

In this work, we present a broader perspective on the alignment phenomenon, extending our general view formulated in Refs. [22,23] and further developing the previously proposed approach to modeling azimuthal alignment in heavy-ion collisions [25] using the HYDJET++ event generator. We demonstrate that the combined effect of particle clustering and event-by-event transverse momentum conservation can significantly affect the observed alignment. Clusters are constructed through an iterative pairwise comparison of particle coordinates on the emulsion “film,” with the cluster position being updated each time a new particle is added, until no pair satisfies the distance condition $d_{ij} > r_{\text{res}}$ (see Sec. IV for details).

The simulations were performed for three centrality classes: 0%–5%, 40%–75%, and 0%–75%. These correspond, respectively, to nearly central collisions with maximal clustering activity, peripheral collisions dominated by anisotropic flow and minimal clustering, and a more realistic mixed scenario that includes all collision types. The simulations of Pb + Pb collisions at $\sqrt{s} = 5.02$ TeV per nucleon pair include both the soft and hard components of the HYDJET++ model, with the height of the primary interaction fixed at $h = 1$ km and the central cluster size set to $r_{\text{res}} = 1$ mm. It is important to note that only a qualitative and, to some extent, forced comparison with the Pamir experiment can be made, since no evidence of the alignment effect has been reported in collider experiments.

Table II shows a comparison of our best simulation results with the Pamir experimental data for three-, four-, and five-cluster configurations. The “Reference” row

TABLE II. Comparison of our best simulation results for alignment with the Pamir experimental data.

Alignment degree	P_3	P_4	P_5
Pamir results [4]	0.83 ± 0.27	0.67 ± 0.33	0.33 ± 0.23
Our results with clustering	0.65	0.13	0.5
Our results without clustering [25]	1.0	0.07	0.04
Reference [22]	0.2	0.04	0.008

corresponds to the alignment degree values for an isotropic particle distribution, consistent with the results of Ref. [22], which provides a purely geometrical interpretation of the alignment phenomenon, and with the recent study [25] employing the HYDJET++ event generator. These findings are also in agreement with the results obtained in the present work (see Table I). Overall, the comparison demonstrates a satisfactory correspondence between the simulated and experimental values, indicating that the implemented model captures the essential features of the observed alignment effect.

The clustering procedure itself does not lead to a significant enhancement of the alignment degree, as can also be seen from Table I. Therefore, a natural next step was to apply the approach developed in our previous works [22,25], which accounts for the missing total transverse momentum of clusters, or the transverse momentum disbalance, i.e., event-by-event transverse momentum conservation (15). This method results in a considerable increase in the simulated alignment (Table II) and, at the same time, does not require introducing or modeling any complex dynamical processes in relativistic nucleus–nucleus collisions.

As seen from Figs. 3–5, the concept of the missing transverse momentum, combined with the selection of the most energetic clusters, provides the highest values of the alignment degree in the disbalance range $\Delta = 0$ –1 GeV. This confirms our hypothesis that minimizing the total transverse momentum of clusters directly affects their azimuthal alignment. Moreover, there is a clear tendency that increasing the cluster size r_{res} leads to a stronger response in the region $\Delta = 0$ –1 GeV for all centrality classes and cluster multiplicities N .

A comparison of events generated with only the soft component, only the hard component, and with both components and resonance decays switched off (Fig. 6) suggests that the alignment pattern shows only a moderate sensitivity to the specific particle production mechanism. The alignment is somewhat stronger for the soft component, mainly for P_3 , which may be related to the softer momentum spectrum and possible anisotropic flow facilitating the required transverse momentum disbalance. A modest enhancement of the alignment for P_4 at small Δ appears only when resonance decays are excluded, while in all other cases the influence of individual model ingredients remains limited. This behavior is consistent with previous observations obtained without applying clustering [25] and indicates that the overall alignment signal remains relatively robust with respect to variations in the underlying production dynamics, with resonance decays playing only a minor role in modifying the signal.

In Figs. 7–9 we present the influence of the clustering procedure at a fixed resolution radius $r_{\text{res}} = 1$ mm. For three clusters ($N = 3$), the alignment parameter decreases

substantially in all centrality classes; this effect is attributed to differing directions of the transverse momenta and the radial positions of the clusters (see Sec. VI for details). For four clusters ($N = 4$), the impact of clustering becomes especially noticeable when combined with the anisotropic flow of secondary particles, i.e., for centralities of 40%–75% and 0%–75%. For five clusters ($N = 5$), the case of the most central collisions (0%–5%) stands out: here clustering strongly enhances the alignment at small Δ ; however, this result is based on limited statistics (only a few events, see Appendix A 1). When peripheral collisions are also included, the alignment increases for $\Delta > 1$ GeV.

Referring to Table II, which presents a comparison between our best simulation results for the alignment effect and the data from the Pamir Collaboration, we can see that, in general, our concept provides a reasonable description of the experimental observations. However, we emphasize once again that this comparison should be regarded as mostly qualitative and, to some extent, forced, since no data on the observation of azimuthal alignment of particles at collider experiments are currently available.

Distinctive features of our approach include the type of colliding nuclei, in our case heavy lead nuclei rather than lighter ones such as iron or oxygen. Nevertheless, there exist studies indicating a similarity of the observed effects in systems with both high and low nucleon densities [39–41]. Moreover, in Ref. [10] indications of a significant alignment of particles via jets mechanism production with respect to reference values were obtained in proton-proton collisions at $\sqrt{s} = 14$ TeV using the PYTHIA event generator [45].

The alignment is sensitive to nonflow effects and reflects the relative arrangement of nearby particle clusters, providing a direct probe of local structures in the azimuthal plane. Its limitations are related to reduced statistical stability—i.e., the effect has not been observed in collider experiments within the accessible rapidity range—as well as the lack of a direct connection to the azimuthal collective flow. In contrast, the Fourier decomposition, namely v_2 and the correlator $c_2\{2\}$, are statistically more robust observables directly connected to the collective dynamics of particles, although they may mix flow and nonflow contributions to the anisotropy.

Undoubtedly, we cannot assert with confidence that the alignment phenomenon arises solely from the selection of the most energetic clusters and clusterlike particles together with the conservation of their total transverse momentum. Nevertheless, the results obtained in this work do not exclude the possibility that additional, possibly novel, mechanisms may contribute to the alignment patterns reported in cosmic-ray experiments, thereby motivating dedicated searches for similar types of azimuthal correlations (under appropriate kinematic conditions) in collider

environments. We therefore leave room for further discussion and investigation, as the observed alignment may also be shaped by complex dynamical processes in the quark-gluon plasma, collective effects, and phenomena beyond.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX: STATISTICS OF MODELING

1. Centrality 0%–5%

TABLE III. Statistics of the alignment degree P_3 versus disbalance Δ using clustering with different cluster sizes r_{res} . Centrality class $c = 0\%–5\%$. The alignment degree $P_3(\Delta) = l^{[\Delta]}/L^{[\Delta]}$, and $L^{[\text{tot}]} = 10^6$ denotes the total number of simulated events.

Δ , GeV	$r_{\text{res}} = 0.5$ mm			$r_{\text{res}} = 1$ mm			$r_{\text{res}} = 2$ mm			$r_{\text{res}} = 5$ mm		
	P_3	$l^{[\Delta]}$	$L^{[\Delta]}$	P_3	$l^{[\Delta]}$	$L^{[\Delta]}$	P_3	$l^{[\Delta]}$	$L^{[\Delta]}$	P_3	$l^{[\Delta]}$	$L^{[\Delta]}$
0	...	0	0	...	0	0	...	0	0	...	0	0
0.05	...	0	0	0	0	2	0.636	7	11	0.643	9	14
0.1	...	0	0	0.250	1	4	0.476	10	21	0.500	19	38
0.2	0.333	1	3	0.400	2	5	0.410	48	117	0.426	72	169
0.4	0.500	5	10	0.385	10	26	0.416	179	430	0.427	280	655
0.6	0.333	8	24	0.381	24	63	0.419	335	799	0.437	595	1363
0.8	0.405	17	42	0.450	49	109	0.415	530	1276	0.422	906	2147
1.0	0.404	23	57	0.426	66	155	0.400	699	1747	0.406	1226	3017
1.4	0.423	33	78	0.387	98	253	0.378	1059	2801	0.379	1798	4744
1.8	0.423	41	97	0.380	124	326	0.353	1326	3754	0.354	2255	6378
2.2	0.372	45	121	0.346	142	410	0.318	1489	4689	0.318	2525	7947
2.6	0.336	52	155	0.323	159	492	0.286	1588	5555	0.291	2719	9354
3.0	0.315	57	181	0.304	170	559	0.264	1676	6340	0.269	2872	10659
3.4	0.282	57	202	0.279	178	639	0.246	1735	7063	0.252	2981	11808
4.0	0.249	59	237	0.255	185	726	0.228	1816	7975	0.234	3116	13329
5.0	0.219	62	283	0.236	202	855	0.209	1939	9272	0.214	3301	15429
6.0	0.212	66	312	0.226	217	961	0.202	2036	10091	0.206	3463	16825
9.0	0.190	77	405	0.215	241	1123	0.195	2158	11043	0.201	3681	18298
12.0	0.187	88	471	0.211	255	1209	0.195	2196	11244	0.200	3717	18569
15.0	0.191	102	534	0.210	267	1273	0.195	2216	11342	0.200	3731	18651
18.0	0.190	112	590	0.207	275	1328	0.195	2224	11395	0.200	3737	18685

2. Centrality 40%–75%

TABLE IV. Statistics of the alignment degree P_4 versus disbalance Δ using clustering with different cluster sizes r_{res} . Centrality class $c = 0\%–5\%$. The alignment degree $P_4(\Delta) = l^{[\Delta]}/L^{[\Delta]}$, and $L^{[\text{tot}]} = 10^6$ denotes the total number of simulated events.

Δ , GeV	$r_{\text{res}} = 0.5$ mm			$r_{\text{res}} = 1$ mm			$r_{\text{res}} = 2$ mm			$r_{\text{res}} = 5$ mm		
	P_4	$l^{[\Delta]}$	$L^{[\Delta]}$	P_4	$l^{[\Delta]}$	$L^{[\Delta]}$	P_4	$l^{[\Delta]}$	$L^{[\Delta]}$	P_4	$l^{[\Delta]}$	$L^{[\Delta]}$
0	...	0	0	0	0	0	0	0	0	0	0	0
0.05	0	0	2	0	0	0	0	0	6	0.111	1	9
0.1	0	0	3	0	0	2	0	0	17	0.091	3	33
0.2	0	0	4	0	0	5	0.015	1	67	0.068	9	133
0.4	0	0	8	0.045	1	22	0.044	11	251	0.042	19	453
0.6	0	0	14	0.023	1	43	0.044	23	527	0.042	39	939
0.8	0.048	1	21	0.043	3	69	0.037	34	917	0.044	70	1576
1.0	0.059	2	34	0.039	4	102	0.042	56	1322	0.046	104	2285
1.4	0.050	3	60	0.052	9	173	0.046	100	2182	0.046	174	3808
1.8	0.061	5	82	0.062	16	259	0.049	148	3046	0.047	247	5243
2.2	0.080	9	112	0.065	21	325	0.050	198	3928	0.048	317	6600
2.6	0.075	10	133	0.059	24	404	0.051	242	4788	0.049	391	7922
3.0	0.063	10	158	0.051	24	470	0.047	264	5596	0.048	445	9204
3.4	0.059	11	185	0.050	27	544	0.046	293	6313	0.047	488	10403
4.0	0.049	11	225	0.047	30	642	0.045	323	7256	0.045	542	11969
5.0	0.040	11	275	0.045	35	776	0.042	355	8473	0.042	598	14155
6.0	0.046	15	325	0.042	37	883	0.039	370	9418	0.040	629	15811
9.0	0.040	17	422	0.038	42	1102	0.037	402	10889	0.036	663	18223
12.0	0.040	19	479	0.041	49	1202	0.037	413	11253	0.036	674	18776
15.0	0.041	22	537	0.039	50	1266	0.037	424	11378	0.036	682	18908
18.0	0.039	23	584	0.040	53	1313	0.037	429	11444	0.036	685	18963

TABLE V. Statistics of the alignment degree P_5 versus disbalance Δ using clustering with different cluster sizes r_{res} . Centrality class $c = 0\%–5\%$. The alignment degree $P_5(\Delta) = l^{[\Delta]}/L^{[\Delta]}$, and $L^{[\text{tot}]} = 3 \times 10^6$ denotes the total number of simulated events.

Δ , GeV	$r_{\text{res}} = 0.5$ mm			$r_{\text{res}} = 1$ mm			$r_{\text{res}} = 2$ mm			$r_{\text{res}} = 5$ mm		
	P_5	$l^{[\Delta]}$	$L^{[\Delta]}$	P_5	$l^{[\Delta]}$	$L^{[\Delta]}$	P_5	$l^{[\Delta]}$	$L^{[\Delta]}$	P_5	$l^{[\Delta]}$	$L^{[\Delta]}$
0	...	0	0	...	0	0	...	0	0	...	0	0
0.05	...	0	0	0.500	1	2	0.200	2	10	0	0	20
0.1	0	0	0	0.250	1	4	0.048	2	41	0.029	2	67
0.2	0	0	3	0.077	1	13	0.018	3	171	0.018	5	285
0.4	0	0	12	0.020	1	51	0.013	8	630	0.013	14	1084
0.6	0	0	31	0.010	1	105	0.012	17	1379	0.015	35	2319
0.8	0	0	56	0.006	1	179	0.011	27	2358	0.015	57	3904
1.0	0.011	1	90	0.004	1	268	0.010	35	3474	0.013	76	5687
1.4	0.012	2	169	0.008	4	472	0.010	60	5877	0.012	118	9752
1.8	0.012	3	243	0.006	4	709	0.010	82	8485	0.011	161	14053
2.2	0.013	4	318	0.008	7	914	0.009	97	11019	0.011	192	18245
2.6	0.010	4	418	0.006	7	1103	0.008	110	13391	0.010	232	22296
3.0	0.010	5	493	0.007	9	1315	0.008	123	15598	0.010	260	26075
3.4	0.009	5	566	0.007	11	1533	0.007	133	17745	0.010	285	29695
4.0	0.009	6	661	0.006	11	1808	0.007	149	20654	0.009	311	34584
5.0	0.007	6	841	0.006	14	2249	0.007	163	24856	0.008	340	41621
6.0	0.007	7	984	0.006	15	2578	0.006	171	28220	0.008	363	47052
9.0	0.006	8	1290	0.006	20	3320	0.006	196	33898	0.007	393	56165
12.0	0.006	9	1505	0.006	21	3718	0.006	202	35684	0.007	399	59098
15.0	0.005	9	1706	0.006	22	3955	0.006	202	36209	0.007	403	59794
18.0	0.006	11	1883	0.006	25	4144	0.006	203	36438	0.007	404	60018

TABLE VI. Statistics of the alignment degree P_3 versus disbalance Δ using clustering with different cluster sizes r_{res} . Centrality class $c = 40\% - 75\%$. The alignment degree $P_3(\Delta) = l^{[\Delta]}/L^{[\Delta]}$, and $L^{[\text{tot}]} = 10^6$ denotes the total number of simulated events.

Δ , GeV	$r_{\text{res}} = 0.5$ mm			$r_{\text{res}} = 1$ mm			$r_{\text{res}} = 2$ mm			$r_{\text{res}} = 5$ mm		
	P_3	$l^{[\Delta]}$	$L^{[\Delta]}$	P_3	$l^{[\Delta]}$	$L^{[\Delta]}$	P_3	$l^{[\Delta]}$	$L^{[\Delta]}$	P_3	$l^{[\Delta]}$	$L^{[\Delta]}$
0	...	0	0	...	0	0	...	0	0	...	0	0
0.05	0.413	52	126	0.354	23	65	0.447	21	47	0.567	17	30
0.1	0.398	208	522	0.378	94	249	0.360	64	178	0.400	46	115
0.2	0.376	742	1972	0.357	350	981	0.345	238	689	0.384	193	502
0.4	0.371	2788	7514	0.358	1361	3800	0.361	932	2585	0.371	696	1878
0.6	0.349	5662	16246	0.336	2810	8367	0.345	1903	5520	0.357	1467	4109
0.8	0.325	9025	27729	0.310	4449	14374	0.322	3008	9346	0.332	2346	7056
1.0	0.307	12684	41312	0.292	6259	21418	0.305	4205	13769	0.317	3270	10311
1.4	0.276	20175	73128	0.265	9989	37639	0.276	6637	24034	0.286	5161	18048
1.8	0.253	27493	108457	0.248	13770	55537	0.256	9031	35259	0.266	7078	26594
2.2	0.239	34312	143754	0.237	17423	73396	0.246	11433	46432	0.251	8774	34913
2.6	0.229	40606	177497	0.229	20754	90458	0.238	13603	57128	0.240	10366	43146
3.0	0.221	45977	207687	0.224	23714	105884	0.232	15570	67140	0.233	11803	50555
3.4	0.216	50671	234395	0.220	26275	119514	0.228	17313	76078	0.229	13140	57335
4.0	0.211	56459	267643	0.216	29581	137048	0.223	19541	87717	0.224	14787	66076
5.0	0.206	63199	306134	0.212	33656	158541	0.219	22463	102746	0.219	17036	77613
6.0	0.204	67154	329705	0.210	36346	173415	0.215	24481	113936	0.217	18802	86612
9.0	0.200	72850	363552	0.205	41253	200845	0.208	28720	138321	0.213	22863	107218
12.0	0.199	75905	381240	0.204	45011	220944	0.203	32300	158871	0.211	26264	124341
15.0	0.198	78361	394912	0.202	48345	238988	0.201	35662	177244	0.211	29463	139876
18.0	0.198	80451	406486	0.201	51422	255416	0.199	38737	194469	0.210	32396	154423

TABLE VII. Statistics of the alignment degree P_4 versus disbalance Δ using clustering with different cluster sizes r_{res} . Centrality class $c = 40\% - 75\%$. The alignment degree $P_4(\Delta) = l^{[\Delta]}/L^{[\Delta]}$, and $L^{[\text{tot}]} = 10^6$ denotes the total number of simulated events.

Δ , GeV	$r_{\text{res}} = 0.5$ mm			$r_{\text{res}} = 1$ mm			$r_{\text{res}} = 2$ mm			$r_{\text{res}} = 5$ mm		
	P_4	$l^{[\Delta]}$	$L^{[\Delta]}$	P_4	$l^{[\Delta]}$	$L^{[\Delta]}$	P_4	$l^{[\Delta]}$	$L^{[\Delta]}$	P_4	$l^{[\Delta]}$	$L^{[\Delta]}$
0	...	0	0	...	0	0	...	0	0	...	0	0
0.05	0.042	3	72	0.027	1	37	0.000	0	15	0.071	1	14
0.1	0.081	25	309	0.063	8	126	0.056	4	71	0.094	6	64
0.2	0.069	86	1238	0.072	39	541	0.067	22	328	0.081	22	272
0.4	0.070	352	4994	0.079	189	2395	0.093	132	1425	0.076	85	1120
0.6	0.073	809	11023	0.083	438	5286	0.088	285	3227	0.073	179	2467
0.8	0.075	1429	19170	0.080	741	9250	0.085	475	5613	0.071	308	4311
1.0	0.073	2115	29004	0.079	1100	13985	0.084	710	8462	0.070	459	6550
1.4	0.068	3578	52409	0.072	1816	25289	0.077	1170	15196	0.065	773	11816
1.8	0.064	5101	79285	0.067	2583	38381	0.072	1668	23078	0.064	1135	17635
2.2	0.061	6594	107853	0.064	3317	52009	0.068	2118	31168	0.061	1453	23831
2.6	0.059	8036	136137	0.062	4067	65710	0.065	2561	39326	0.059	1792	30118
3.0	0.058	9375	162934	0.060	4765	78794	0.063	2980	47274	0.059	2128	36175
3.4	0.056	10485	187775	0.059	5362	91335	0.061	3363	54919	0.058	2419	41973
4.0	0.054	11929	220914	0.057	6149	107922	0.059	3877	65360	0.056	2803	49816
5.0	0.052	13750	263948	0.055	7175	130413	0.057	4557	80319	0.055	3387	61151
6.0	0.051	14909	293625	0.054	7917	147334	0.055	5061	92345	0.055	3861	70027
9.0	0.049	16643	337715	0.052	9366	179403	0.052	6183	118788	0.054	4950	91195
12.0	0.049	17437	358116	0.051	10244	200567	0.050	7031	139818	0.054	5858	108622
15.0	0.048	18006	372763	0.050	11033	218941	0.049	7805	158445	0.054	6699	124122
18.0	0.048	18503	384964	0.050	11752	235955	0.048	8459	175819	0.054	7443	138525

TABLE VIII. Statistics of the alignment degree P_5 versus disbalance Δ using clustering with different cluster sizes r_{res} . Centrality class $c = 40\%–75\%$. The alignment degree $P_5(\Delta) = l^{[\Delta]}/L^{[\Delta]}$, and $L^{[\text{tot}]} = 10^6$ denotes the total number of simulated events.

Δ , GeV	$r_{\text{res}} = 0.5$ mm			$r_{\text{res}} = 1$ mm			$r_{\text{res}} = 2$ mm			$r_{\text{res}} = 5$ mm		
	P_5	$l^{[\Delta]}$	$L^{[\Delta]}$	P_5	$l^{[\Delta]}$	$L^{[\Delta]}$	P_5	$l^{[\Delta]}$	$L^{[\Delta]}$	P_5	$l^{[\Delta]}$	$L^{[\Delta]}$
0	...	0	0	...	0	0	...	0	0	...	0	0
0.05	0.016	1	62	0	0	31	0.111	1	9	0	0	11
0.1	0.017	4	242	0.018	2	112	0.054	3	56	0.024	1	41
0.2	0.014	13	942	0.011	5	436	0.016	4	246	0.021	4	187
0.4	0.019	70	3615	0.019	32	1666	0.019	18	938	0.021	15	721
0.6	0.017	142	8134	0.018	67	3731	0.020	40	2048	0.021	34	1648
0.8	0.017	240	14107	0.019	120	6451	0.022	79	3592	0.021	59	2832
1.0	0.017	372	21526	0.018	179	9841	0.022	119	5480	0.020	85	4256
1.4	0.017	665	39647	0.019	335	17961	0.019	193	9904	0.018	145	7848
1.8	0.016	992	61074	0.018	490	27574	0.018	278	15181	0.018	214	12055
2.2	0.015	1295	84399	0.017	649	37918	0.017	357	21010	0.016	267	16645
2.6	0.015	1625	108193	0.016	804	48914	0.017	457	27205	0.016	332	21379
3.0	0.014	1869	131650	0.016	938	59747	0.016	548	33435	0.015	402	26300
3.4	0.014	2126	154614	0.015	1074	70207	0.016	636	39529	0.015	457	30913
4.0	0.013	2495	186146	0.015	1264	84983	0.015	741	48250	0.014	533	37621
5.0	0.013	2931	228822	0.014	1513	106106	0.015	916	61283	0.014	665	47413
6.0	0.013	3269	260665	0.014	1698	122922	0.014	1047	72540	0.014	783	55860
9.0	0.012	3768	312789	0.013	2038	156987	0.013	1290	98952	0.013	1010	75636
12.0	0.012	3973	336032	0.013	2274	179439	0.012	1477	119982	0.013	1238	92108
15.0	0.012	4091	351362	0.012	2427	198116	0.012	1635	138576	0.013	1450	107582
18.0	0.011	4182	363668	0.012	2587	215213	0.012	1796	155784	0.014	1649	121979

3. Centrality 0%–75%

TABLE IX. Statistics of the alignment degree P_3 versus disbalance Δ using clustering with different cluster sizes r_{res} . Centrality class $c = 0\%–75\%$. The alignment degree $P_3(\Delta) = l^{[\Delta]}/L^{[\Delta]}$, and $L^{[\text{tot}]} = 10^6$ denotes the total number of simulated events.

Δ , GeV	$r_{\text{res}} = 0.5$ mm			$r_{\text{res}} = 1$ mm			$r_{\text{res}} = 2$ mm			$r_{\text{res}} = 5$ mm		
	P_3	$l^{[\Delta]}$	$L^{[\Delta]}$	P_3	$l^{[\Delta]}$	$L^{[\Delta]}$	P_3	$l^{[\Delta]}$	$L^{[\Delta]}$	P_3	$l^{[\Delta]}$	$L^{[\Delta]}$
0	...	0	0	...	0	0	...	0	0	...	0	0
0.05	0.338	48	142	0.364	24	66	0.242	8	33	0.286	8	28
0.1	0.397	215	541	0.399	119	298	0.431	78	181	0.377	58	154
0.2	0.384	751	1958	0.384	404	1051	0.406	274	675	0.420	237	564
0.4	0.370	2745	7418	0.360	1410	3920	0.372	961	2580	0.388	785	2023
0.6	0.350	5637	16126	0.332	2789	8393	0.343	1862	5432	0.364	1542	4240
0.8	0.331	9097	27520	0.315	4534	14377	0.324	2993	9229	0.342	2457	7187
1.0	0.311	12900	41413	0.298	6412	21485	0.310	4230	13665	0.316	3339	10578
1.4	0.279	20431	73155	0.271	10272	37846	0.281	6711	23877	0.286	5218	18245
1.8	0.255	27579	107983	0.251	13956	55563	0.259	9059	34915	0.263	6983	26575
2.2	0.240	34353	143110	0.239	17533	73377	0.246	11302	45937	0.249	8713	34966
2.6	0.230	40563	176589	0.231	20820	90325	0.235	13328	56651	0.239	10270	42887
3.0	0.222	45980	206933	0.225	23781	105723	0.230	15278	66402	0.232	11699	50368
3.4	0.218	50760	233367	0.221	26420	119554	0.225	16987	75348	0.228	13009	57135
4.0	0.212	56520	266037	0.217	29633	136767	0.221	19120	86707	0.223	14742	65981
5.0	0.207	62975	304101	0.212	33574	158092	0.215	21881	101667	0.218	16938	77586
6.0	0.205	67050	327704	0.210	36293	172806	0.212	24005	112986	0.216	18681	86611
9.0	0.201	72672	361356	0.207	41386	200390	0.206	28409	137800	0.212	22728	107161
12.0	0.200	75800	379143	0.204	45063	220707	0.203	32003	157919	0.210	26144	124636
15.0	0.199	78204	392852	0.203	48320	238545	0.200	35276	176282	0.209	29279	140393
18.0	0.199	80355	404720	0.202	51443	254959	0.199	38332	192999	0.208	32225	154945

TABLE X. Statistics of the alignment degree P_4 versus disbalance Δ using clustering with different cluster sizes r_{res} . Centrality class $c = 0\%–75\%$. The alignment degree $P_4(\Delta) = l^{[\Delta]}/L^{[\Delta]}$, and $L^{[\text{tot}]} = 10^6$ denotes the total number of simulated events.

Δ , GeV	$r_{\text{res}} = 0.5$ mm			$r_{\text{res}} = 1$ mm			$r_{\text{res}} = 2$ mm			$r_{\text{res}} = 5$ mm		
	P_4	$l^{[\Delta]}$	$L^{[\Delta]}$	P_4	$l^{[\Delta]}$	$L^{[\Delta]}$	P_4	$l^{[\Delta]}$	$L^{[\Delta]}$	P_4	$l^{[\Delta]}$	$L^{[\Delta]}$
0	...	0	0	...	0	0	...	0	0	...	0	0
0.05	0.064	3	47	0.059	1	17	0.118	2	17	0	0	15
0.1	0.087	18	208	0.113	11	97	0.133	11	83	0.068	5	74
0.2	0.071	55	777	0.076	30	395	0.063	20	316	0.098	25	256
0.4	0.077	233	3027	0.084	132	1566	0.079	88	1113	0.089	86	962
0.6	0.074	485	6593	0.077	265	3424	0.082	194	2356	0.078	156	1997
0.8	0.072	822	11406	0.073	425	5826	0.080	315	3937	0.077	262	3384
1.0	0.070	1209	17372	0.071	621	8722	0.077	452	5899	0.073	368	5057
1.4	0.067	2113	31563	0.069	1090	15880	0.071	754	10563	0.067	605	9006
1.8	0.063	3004	47437	0.066	1567	23680	0.068	1066	15755	0.066	875	13323
2.2	0.061	3898	64361	0.063	2026	32066	0.065	1369	21116	0.063	1118	17875
2.6	0.058	4704	81030	0.061	2453	40252	0.063	1658	26453	0.061	1358	22347
3.0	0.056	5430	96816	0.060	2879	48186	0.062	1957	31804	0.059	1592	26779
3.4	0.055	6091	111625	0.058	3261	55865	0.060	2227	36912	0.058	1785	30911
4.0	0.053	6951	130924	0.057	3748	65838	0.059	2564	43783	0.056	2052	36579
5.0	0.051	7944	155853	0.054	4316	79277	0.057	3034	53631	0.054	2407	44329
6.0	0.050	8631	173045	0.053	4787	89503	0.055	3387	61278	0.054	2722	50618
9.0	0.048	9610	199445	0.051	5598	109072	0.052	4088	78198	0.053	3424	64477
12.0	0.047	10092	212627	0.050	6143	122667	0.050	4585	91434	0.053	3973	75361
15.0	0.047	10411	222183	0.049	6595	134554	0.049	5039	103116	0.052	4439	85212
18.0	0.047	10740	230606	0.048	7011	145341	0.048	5500	114024	0.052	4909	94301

TABLE XI. Statistics of the alignment degree P_5 versus disbalance Δ using clustering with different cluster sizes r_{res} . Centrality class $c = 0\%–75\%$. The alignment degree $P_5(\Delta) = l^{[\Delta]}/L^{[\Delta]}$, and $L^{[\text{tot}]} = 10^6$ denotes the total number of simulated events.

Δ , GeV	$r_{\text{res}} = 0.5$ mm			$r_{\text{res}} = 1$ mm			$r_{\text{res}} = 2$ mm			$r_{\text{res}} = 5$ mm		
	P_5	$l^{[\Delta]}$	$L^{[\Delta]}$	P_5	$l^{[\Delta]}$	$L^{[\Delta]}$	P_5	$l^{[\Delta]}$	$L^{[\Delta]}$	P_5	$l^{[\Delta]}$	$L^{[\Delta]}$
0	...	0	0	...	0	0	...	0	0	...	0	0
0.05	0	0	33	0	0	16	0	0	7	0	0	13
0.1	0.014	2	142	0	0	68	0.024	1	41	0.024	1	42
0.2	0.022	13	581	0.014	4	277	0.030	5	166	0.026	4	156
0.4	0.022	49	2267	0.019	21	1089	0.033	20	609	0.019	12	621
0.6	0.018	86	4834	0.017	39	2289	0.027	38	1415	0.018	25	1363
0.8	0.019	159	8590	0.017	69	4024	0.023	57	2475	0.020	48	2365
1.0	0.018	231	13042	0.016	99	6143	0.020	75	3782	0.019	67	3557
1.4	0.017	398	24104	0.017	187	11330	0.018	130	7070	0.017	108	6337
1.8	0.016	578	36961	0.016	271	17218	0.016	179	10850	0.015	151	9743
2.2	0.015	748	50688	0.016	370	23724	0.015	225	14960	0.016	206	13184
2.6	0.014	920	64853	0.015	452	30210	0.015	280	19096	0.015	252	16780
3.0	0.014	1067	78751	0.014	525	36910	0.014	323	23447	0.014	287	20364
3.4	0.013	1225	92033	0.014	617	43303	0.013	371	27587	0.014	326	23809
4.0	0.013	1425	110252	0.014	721	52200	0.013	435	33321	0.013	368	28705
5.0	0.012	1682	135371	0.013	874	64871	0.013	552	41950	0.013	458	35924
6.0	0.012	1863	153818	0.013	982	75050	0.013	635	49516	0.013	539	41782
9.0	0.012	2152	184339	0.013	1214	95679	0.012	811	66346	0.013	709	55202
12.0	0.011	2266	198635	0.012	1342	109683	0.012	952	79797	0.013	847	65910
15.0	0.011	2370	208770	0.012	1449	121802	0.012	1055	91605	0.012	942	75743
18.0	0.011	2443	217326	0.012	1560	132835	0.011	1144	102445	0.012	1049	84793

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