

Correlation function and bound state from the $KD_{s0}^*(2317)$ interaction

Wen-Hao Jia^{1,2}, Hai-Peng Li^{1,2}, Wei-Hong Liang^{1,2,*}, Jing Song^{3,†}, and Eulogio Oset^{1,4,‡}

¹Department of Physics, *Guangxi Normal University*, Guilin 541004, China

²Guangxi Key Laboratory of Nuclear Physics and Technology, *Guangxi Normal University*, Guilin 541004, China

³Center for Theoretical Physics, School of Physics and Optoelectronic Engineering, *Hainan University*, Haikou 570228, China

⁴Departamento de Física Teórica and IFIC, *Centro Mixto Universidad de Valencia-CSIC Institutos de Investigación de Paterna*, 46071 Valencia, Spain



(Received 16 April 2026; accepted 24 July 2026; published 27 August 2026)

In anticipation of the new wave of ALICE experiments on particle-resonance correlation functions, we study the interaction of a kaon with the $D_{s0}^*(2317)$ resonance. Assuming the $D_{s0}^*(2317)$ to be a DK molecular state in isospin $I = 0$, we employ the fixed center approximation to describe the kaon scattering off the DK cluster, and implement elastic unitarity in the $KD_{s0}^*(2317)$ amplitude via an optical potential and the Lippmann-Schwinger equation. We evaluate the scattering length, effective range, and correlation function, which exhibits a shape characteristic of a strongly attractive interaction. Notably, the amplitude develops a narrow resonant peak about 40 MeV below the $KD_{s0}^*(2317)$ threshold, signaling a three-body bound state. We discuss the experimental feasibility of observing this state through the invariant mass distribution of $KD_s^+\pi^0$, and argue that such three-body states, predicted by various theoretical approaches, offer promising targets for future experimental searches, providing valuable insights into the nature of exotic hadronic resonances.

DOI: 10.1103/5qjm-7lvr

I. INTRODUCTION

ALICE collaboration has started a new wave of experiments investigating the correlation functions stemming from the interaction of stable particles with resonances. The first steps in this direction involve the $pf_1(1285)$ system, where the $f_1(1285)$ is detected via the $K\bar{K}\pi$ decay mode [1,2]. The aim of the project is to learn about the nature of many resonances, which are the subject of permanent debate (see review papers on this issue [3–12]). Correlation functions offer a possibility to study the interaction of a particular resonance with many different sources, accumulating novel information that should shed light on the issue of the nature of the resonance investigated. Among multiple resonances to be investigated, some of them play a particular role since they can correspond to exotic states that cannot be cast into the standard $q\bar{q}$ or $3q$ nature for mesons and baryons,

respectively, or correspond to dynamically generated, molecularlike states that emerge from the interaction of other particles, usually within coupled channels [11,12]. This is the case for the $f_1(1285)$ state, which is considered to be a molecular state originating from the $K\bar{K}^* - \bar{K}K^*$ interaction with isospin $I = 0$ [13–18]. A long list of tests supporting this nature is discussed in Ref. [19]. Theoretical work following this new trend has started to appear, and in Ref. [20] the correlation function for the $pf_1(1285)$ interaction was evaluated, finding a significant deviation from unity, which should be contrasted with the coming experimental results [1].

The work of Ref. [20] used the popular fixed center approximation (FCA) to the Faddeev equations [21–30], in which there is an external particle, the proton, interacting with a cluster of two particles, the $f_1(1285)$ as a composite state of $K\bar{K}^* - \bar{K}K^*$, where the cluster is assumed to remain unchanged during the interaction, as is generally assumed when one studies the interaction of particles with nuclei [31–34]. The work of Ref. [20] faced a new challenge when it was found that the FCA, normally used to look for bound states below the threshold of the external particle and the cluster, did not fulfill elastic unitarity at the threshold, a condition necessary to determine the effective range in the effective range expansion of the three-body amplitude and the correlation function. An *ad hoc* solution was found in

*Contact author: liangwh@gxnu.edu.cn

†Contact author: Song-Jing@buaa.edu.cn

‡Contact author: oset@ific.uv.es

Published by the American Physical Society under the terms of the *Creative Commons Attribution 4.0 International* license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

that work by multiplying the amplitude by a factor close to unity, which rendered the approach unitary. The formal solution to this problem was found in Ref. [35] when studying the interaction of a neutron with the $\bar{D}_{s0}^*(2317)$. In that case the $\bar{D}_{s0}^*(2317)$ was also assumed to be generated by the interaction of $\bar{D}\bar{K}$.

In both cases considered in Refs. [20,35], a bound state for the three-body system was also found, providing novel information that calls for an experimental search. These predictions are tied to the nature of the $f_1(1285)$ and $\bar{D}_{s0}^*(2317)$ resonances as dynamically generated from the interaction of more elementary particles, and are quite different from predictions assuming these resonances to be elementary fields. One clear example is shown in the results of Ref. [19], where the $Kf_1(1285)$ interaction was studied along the lines of Ref. [35] and the correlation function was evaluated. The interaction was found to be strong enough to produce a correlation function diverting sizeably from unity, and to generate a bound state close to the threshold. On the other hand, it was found in Ref. [36] that assuming the $f_1(1285)$ as an elementary matter field, the interaction was zero.

More work has followed along these lines, and in Ref. [37], as a complement of the $n\bar{D}_{s0}^*(2317)$ interaction studied in Ref. [35], the correlation functions for the $n\bar{D}_{s1}(2460)$ and $n\bar{D}_{s1}(2536)$ systems were evaluated, with bound states also found. A further application of the new unitary techniques to the three-body interaction was presented in Ref. [38], which predicts a superexotic bound state of $K^{*+}D^{*+}K^{*+}$ nature with total spin $J = 3$.

As a complement of the work done in Ref. [35], in the present work we study the interaction of a kaon with the $D_{s0}^*(2317)$ resonance. The $D_{s0}^*(2317)$ is also assumed to be generated from the interaction of the DK and $D_s\eta$ channels [39–47], mostly DK in isospin $I = 0$, which is also supported by lattice QCD calculations [48–52]. This is a new method to study the nature of the $D_{s0}^*(2317)$ since in

Ref. [35], one is faced with the interaction of the neutron with $\bar{K}$ and $\bar{D}$, while here one is considering the interaction of a kaon with K and D . These interactions are completely different from the dynamical point of view, so the predictions should be very different and will provide complementary information on the nature of the $D_{s0}^*(2317)$ resonance, which is one of the motivations behind the work.

II. FORMALISM

We follow closely the work of Ref. [19], which contains the developments and improvements of the FCA, as implemented in Refs. [35,37,38].¹

We study the interaction of a kaon with the $D_{s0}^*(2317)$ assumed to be a molecular state of DK in isospin $I = 0$. The first step is to sum the diagrams involved in the conventional FCA, which are depicted in Fig. 1.

The external K^0 can interact with the D and the K of the cluster. The DK interaction is attractive, in fact it produces the $D_{s0}^*(2317)$, while the KK interaction is repulsive, but given the large strength of the DK attraction, one expects that this attraction will overcome the repulsion and lead to a three-body bound state of DKK nature.

One of the assumptions in the FCA is that the cluster remains unchanged during the collision with the external particle. In this sense, it is commonly believed that the FCA is good when the external interacting particle is lighter than those of the cluster. While this certainly helps, it is not a necessary condition. The FCA also works if the cluster is much bound, as is the case here with about 45 MeV binding for the DK in the $D_{s0}^*(2317)$ state. A detailed discussion on this issue is done in Sec. 4 of Ref. [55], and we mention some examples that serve to support the scheme in the present case. In Refs. [56,57] the $NK\bar{K}$ system and coupled channels were studied by solving the Faddeev equations, yielding results similar to those of Ref. [58], where Faddeev equations were solved using effective potentials. By applying the FCA in Ref. [59] similar results as in the former works were obtained concerning the bound states of the

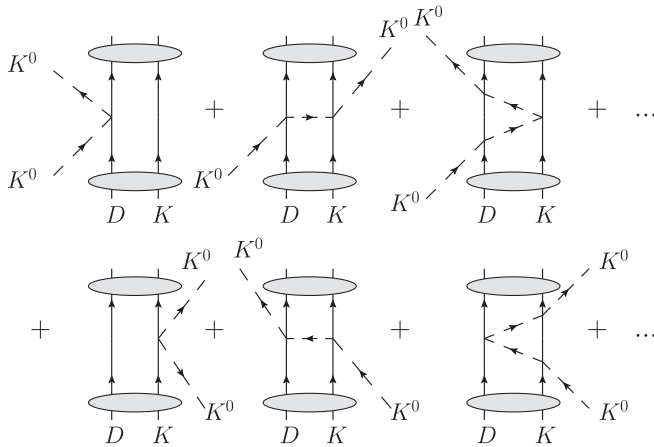


FIG. 1. Diagrams entering the FCA approach for K^0 interacting with the cluster DK .

¹We take advantage here to clarify a discussion made in Ref. [20] concerning the cancellation between off-shell contributions of two-body amplitudes and three-body terms in Faddeev equations when using chiral Lagrangians. This is correct for one representation of the fields in the Lagrangian, the one commonly used [exponential, or polar representation, $U = \exp(i\phi\tau/f)$ in $SU(2)$, with ϕ the pion field], but different ones can be taken in which the exact cancellation would not hold [53] (see also Appendix A of Ref. [54]). In particular, it is shown in Ref. [53] that there is a choice of field representation in which the three-body terms cancel. Physically this would mean that one would have to introduce explicit three-body terms to obtain the same three-body amplitude at the end. What concerns us is that, with the common chiral Lagrangians that we use, the cancellation occurs and it is fair to use on-shell two-body amplitudes, without the off-shell extrapolation that these chiral Lagrangians would provide, and to neglect the three-body terms provided by the same Lagrangians.

three-body system. Other relevant information comes from the study of the $\phi K \bar{K}$ interaction in Ref. [59], with $K \bar{K}$ as a cluster, where the limits of the FCA approximation are shown, reporting that for energies below and not close to the threshold the amplitude is similar to the one obtained from Faddeev equations [60], but the approximation fails for energies above and far away from the threshold. A strong support for the improved FCA that we are using here comes from the recent measurement of the correlation function for the $pf_1(1285)$ in Ref. [61], a system analogous to the one studied here, and the agreement found with the predictions of Ref. [62] (see the comment at the end of the paper).

In the FCA we need the scattering amplitudes of the external particle with the components of the cluster. Let us call t_1 and t_2 the amplitudes for collision of a K^0 with the D and K components of the $D_{s0}^*(2317)$ cluster, respectively. When considering the $I = 0$ structure of the DK cluster, the amplitudes t_1 and t_2 are written as

$$\begin{aligned} t_1 &= \frac{3}{4} t_{DK}^{I=1} + \frac{1}{4} t_{DK}^{I=0}, \\ t_2 &= \frac{3}{4} t_{KK}^{I=1} + \frac{1}{4} t_{KK}^{I=0}. \end{aligned} \quad (1)$$

For reasons of normalization, by referring the final amplitudes to the interaction of K^0 with the cluster as a whole of mass M_C , we define the amplitudes

$$\tilde{t}_1 = \frac{M_C}{m_D} t_1, \quad \tilde{t}_2 = \frac{M_C}{m_K} t_2. \quad (2)$$

The arguments of these amplitudes are given by

$$\begin{aligned} s_1(DK) &= (p_{K^0} + p_D)^2 = m_K^2 + (\xi m_D)^2 + 2\xi m_D q_0, \\ s_2(KK) &= (p_{K^0} + p_K)^2 = m_K^2 + (\xi m_K)^2 + 2\xi m_K q_0, \end{aligned} \quad (3)$$

with q_0 the energy of K^0 in the rest frame of the cluster,

$$q_0 = \frac{s - m_{K^0}^2 - M_C^2}{2M_C}, \quad \xi = \frac{M_C}{m_D + m_K}, \quad (4)$$

which assume that the binding energy of the $D_{s0}^*(2317)$ is shared between the D and K of the cluster proportional to their respective masses.

From Fig. 1 it is useful to separate the terms $\tilde{T}_{ij}$ which include all diagrams where the K^0 interacts first with particle i of the cluster and finishes with particle j . One finds

$$\begin{aligned} \tilde{T}_{11} &= \frac{\tilde{t}_1}{1 - \tilde{t}_1 \tilde{t}_2 G_0^2}, & \tilde{T}_{22} &= \frac{\tilde{t}_2}{1 - \tilde{t}_1 \tilde{t}_2 G_0^2}, \\ \tilde{T}_{12} = \tilde{T}_{21} &= \frac{\tilde{t}_1 \tilde{t}_2 G_0}{1 - \tilde{t}_1 \tilde{t}_2 G_0^2}, \end{aligned} \quad (5)$$

where G_0 is the K^0 propagator from one particle to the other in the cluster, given by

$$\begin{aligned} G_0(\sqrt{s}) &= \int \frac{d^3 q}{(2\pi)^3} \frac{1}{2\omega_K(\mathbf{q})} \frac{1}{2\omega_C(\mathbf{q})} \\ &\times \frac{F_C(\mathbf{q})}{\sqrt{s} - \omega_K(\mathbf{q}) - \omega_C(\mathbf{q}) + i\epsilon} \\ &\times \Theta(q_{\max}^{(1)} - q_1^*) \Theta(q_{\max}^{(2)} - q_2^*), \end{aligned} \quad (6)$$

where $\omega_K(q) = \sqrt{q^2 + m_K^2}$, $\omega_C(q) = \sqrt{q^2 + M_C^2}$. The presence of the $\Theta()$ functions in Eq. (6) stems from the $\mathbf{q}, \mathbf{q}'$ dependence of the amplitudes [63],

$$t(\mathbf{q}, \mathbf{q}') = t \Theta(q_{\max} - |\mathbf{q}|) \Theta(q_{\max} - |\mathbf{q}'|), \quad (7)$$

resulting from a separable potential $V(\mathbf{q}, \mathbf{q}') = V \Theta(q_{\max} - |\mathbf{q}|) \Theta(q_{\max} - |\mathbf{q}'|)$. In this case, the cutoff $q_{\max}$ regularizes the meson-meson loop functions entering the evaluation of the Bethe-Salpeter equation (see the Appendix) and leads to a formalism equivalent to the one used in the chiral unitary approach with the on-shell factorization [64]. In Eq. (6), $q_{\max}^{(1)}$ refers to the DK amplitude and $q_{\max}^{(2)}$ refers to the KK amplitude. Furthermore, q_1^* and q_2^* refer to the momenta in the rest frame of the propagating K^0 and the respective particles 1 and 2 of the cluster. A sensible prescription is taken for the internal motion of the particles in the cluster [38], following Refs. [65,66], assuming the momentum transferred to be shared between the initial and final particles of the cluster in a single scattering step, and then we find

$$\mathbf{q}_i^* = \mathbf{q} \left(1 - \frac{1}{2} \frac{m_{K^0}}{m_{K^0} + m_i} \right). \quad (8)$$

In addition, $F_C(\mathbf{q})$ is the form factor of the cluster, which stems from the wave function of the cluster in momentum space [63]

$$\psi(p) = g \frac{\Theta(q_{\max} - |\mathbf{p}|)}{M_C - \omega_D(\mathbf{q}) - \omega_K(\mathbf{q})}, \quad (9)$$

which leads to

$$F_C(\mathbf{q}) = \frac{F(\mathbf{q})}{\mathcal{N}},$$

$$\begin{aligned} F(\mathbf{q}) &= \int_{\substack{|\mathbf{p}| < q_{\max}, \\ |\mathbf{p}-\mathbf{q}| < q_{\max}}} \frac{d^3 p}{(2\pi)^3} \frac{1}{M_C - \omega_D(\mathbf{p}) - \omega_K(\mathbf{p})} \\ &\times \frac{1}{M_C - \omega_D(\mathbf{p}-\mathbf{q}) - \omega_K(\mathbf{p}-\mathbf{q})}, \end{aligned}$$

$$\mathcal{N} = F(0) = \int_{|p| < q_{\max}} \frac{d^3 p}{(2\pi)^3} \left(\frac{1}{M_C - \omega_D(\mathbf{p}) - \omega_K(\mathbf{p})} \right)^2. \quad (10)$$

The cluster in general is a resonance which has a width. Then M_C in Eq. (10) would be complex. We deal with this situation considering that the target has a mass distribution and performing a convolution of the results with this mass distribution. This has been done in the study of the $Kf_1(1285)$ case in Ref. [19] with the result of a softening of the three-body scattering amplitude at the threshold. In the present case, with a cluster width of only 100 keV, the associated effects are negligible.

For later purposes, it is interesting to put $\tilde{T}_{ij}$ of Eq. (5) in the form of a matrix,

$$\tilde{T} = \begin{pmatrix} \tilde{T}_{11} & \tilde{T}_{12} \\ \tilde{T}_{21} & \tilde{T}_{22} \end{pmatrix}. \quad (11)$$

Next we introduce the unitarization in the $K^0 D_{s0}^*(2317)$ system, with the $D_{s0}^*(2317)$ taken as a whole, which proceeds by incorporating the diagrams shown in Fig. 2. The need for the diagrams of Fig. 2 recalls what happens in the interaction of an external particle with a nucleus, where the $t\rho$ (with ρ the density of the nucleus), which would be equivalent to $t_1 + t_2$ in our formalism in the sum of the diagrams of Fig. 1, provides the optical potential, which must be used in the Lippmann-Schwinger equation to obtain the particle-nucleus scattering amplitude.

The sum of diagrams in Fig. 2, including extra iterations in the $K^0 D_{s0}^*(2317)$ propagation, can be represented in terms of the equation

$$\tilde{T}' = [1 - \tilde{T} G_C]^{-1} \tilde{T}, \quad (12)$$

with

$$G_C = \begin{pmatrix} G_C^{(1)} & 0 \\ 0 & G_C^{(2)} \end{pmatrix}, \quad (13)$$

where

$$G_C^{(i)}(\sqrt{s}) = \int \frac{d^3 q}{(2\pi)^3} \frac{1}{2\omega_K(\mathbf{q})} \frac{1}{2\omega_C(\mathbf{q})} \times \frac{[F_C^{(i)}(\mathbf{q})]^2}{\sqrt{s} - \omega_K(\mathbf{q}) - \omega_C(\mathbf{q}) + i\epsilon} \Theta(q_{\max} - q_i^*), \quad (14)$$

with [67]

$$F_C^{(1)}(\mathbf{q}) = F_C \left(\frac{m_K}{m_D + m_K} \mathbf{q} \right), \\ F_C^{(2)}(\mathbf{q}) = F_C \left(\frac{m_D}{m_D + m_K} \mathbf{q} \right). \quad (15)$$

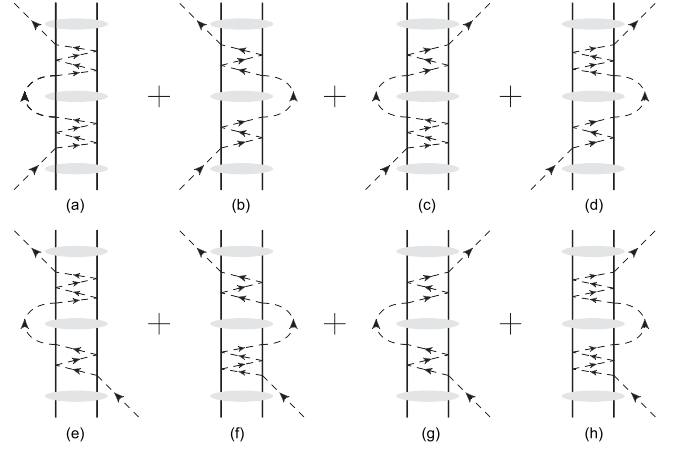


FIG. 2. Diagrams considering the elastic propagation of the K^0 and the cluster $D_{s0}^*(2317)$.

The final amplitude is then given by [37]

$$T^{\text{tot}}(\sqrt{s}) = \sum_{i,j} \tilde{T}'_{ij} \\ = \frac{\tilde{t}_1 + \tilde{t}_2 + (2G_0 - G_C^{(1)} - G_C^{(2)})\tilde{t}_1\tilde{t}_2}{1 - G_C^{(1)}\tilde{t}_1 - G_C^{(2)}\tilde{t}_2 - (G_0 - G_C^{(1)}G_C^{(2)})\tilde{t}_1\tilde{t}_2}. \quad (16)$$

A. Scattering length and effective range

Once we have the total amplitude $T^{\text{tot}}(\sqrt{s})$, one can obtain the $K^0 D_{s0}^*(2317)$ scattering length and effective range. Taking into account the normalization of our amplitude compared to the usual one employed in quantum mechanics, we have

$$-8\pi\sqrt{s}(T^{\text{tot}})^{-1} = (f^{\text{QM}})^{-1} \\ \simeq -\frac{1}{a} + \frac{1}{2}r_0q_{\text{cm}}^2 - iq_{\text{cm}}, \quad (17)$$

where q_{cm} is the K^0 momentum in the $K^0 D_{s0}^*(2317)$ rest frame. The term $-iq_{\text{cm}}$ in Eq. (17) implies the elastic unitarity of the $K^0 D_{s0}^*(2317)$ amplitude, and it was shown analytically in Ref. [37] that the amplitude of Eq. (16) fulfills exactly this unitarity. Then one obtains

$$a = \frac{T^{\text{tot}}}{8\pi\sqrt{s}} \Big|_{\text{th}}, \quad (18)$$

$$r_0 = \frac{1}{\mu} \left[\frac{\partial}{\partial \sqrt{s}} (-8\pi\sqrt{s}(T^{\text{tot}})^{-1} + iq_{\text{cm}}) \right]_{\text{th}}, \quad (19)$$

with μ the $K^0 D_{s0}^*(2317)$ reduced mass.

B. Correlation function

Following Ref. [19] we write the correlation function

$$C_{KD_{s0}^*}(p) = 1 + 4\pi \int_0^\infty dr r^2 S_{12}(r) \times \{|j_0(pr) + TG|^2 - j_0^2(pr)\}, \quad (20)$$

where

$$TG = (\tilde{T}'_{11} + \tilde{T}'_{21})G_1(\sqrt{s}, r) + (\tilde{T}'_{12} + \tilde{T}'_{22})G_2(\sqrt{s}, r) \quad (21)$$

with $S_{12}(r)$ the source function, given by

$$S_{12}(r) = \frac{1}{(4\pi R^2)^{3/2}} \exp(-r^2/4R^2), \quad (22)$$

and R the radius of the source, which is determined in the experiment, or can also be obtained from a fit to the correlation function data, as shown in Ref. [68], and

$$G_i(\sqrt{s}, r) = \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_K(\mathbf{q})} \frac{1}{2\omega_C(\mathbf{q})} \Theta(q_{\max}^{(i)} - q_i^*) \times \frac{j_0(qr)F_C^{(i)}(\mathbf{q})}{\sqrt{s} - \omega_K(\mathbf{q}) - \omega_C(\mathbf{q}) + ic}. \quad (23)$$

III. RESULTS

We first show the results obtained for the scattering length and the effective range. We find

$$a = (0.517 - 0.00016i) \text{ fm}, \quad (24)$$

$$r_0 = (-0.511 + 0.102i) \text{ fm}. \quad (25)$$

The small imaginary part of a reflects the small imaginary part of the DK amplitude arising from the small width of the $D_{s0}^*(2317)$. Both a and r_0 have magnitudes of about half of those of the $pf_1(1285)$ scattering, but the sign of r_0 here is opposite to that found in $pf_1(1285)$. They are, however, much closer to those obtained for the $Kf_1(1285)$ interaction in Ref. [19].

Next we show the results for the T^{tot} matrix as a function of the center-of-mass energy of the $KD_{s0}^*(2317)$ system, $\sqrt{s}$, in Fig. 3. We find a structure typical of a narrow resonance around 40 MeV below the $KD_{s0}^*(2317)$ threshold. The modulus of the imaginary part has a peak around 2777 MeV and the real part goes through zero around that energy. The absolute value of T^{tot} shows a neat and narrow peak. From $|\text{Im} T|$ at half height of its peak we find the width of the state to be around 200 keV. The source of the state found stems from the DK attraction. We have checked the effect of the KK repulsion. If we remove t_{KK} from the

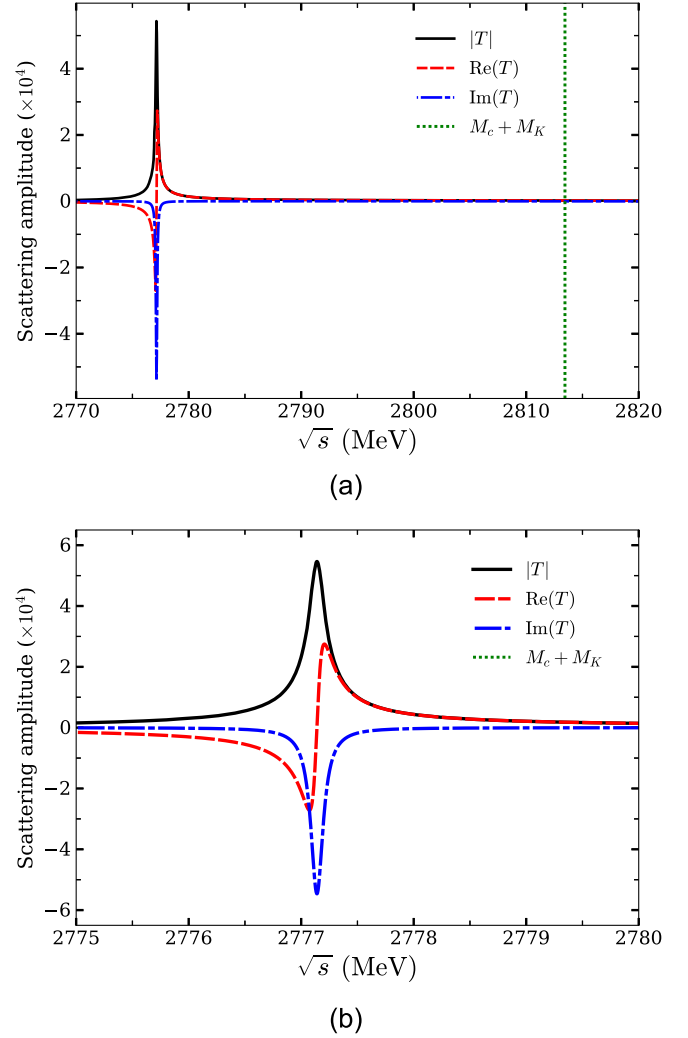


FIG. 3. Results for $T^{\text{tot}}(\sqrt{s})$ as a function of $\sqrt{s}$: (a) relative to the $KD_{s0}^*(2317)$ threshold, and (b) blow up in a narrow energy window.

approach, the state appears at 2771 MeV. The effect of the repulsion has been to move the position of the state 6 MeV closer to the $KD_{s0}^*(2317)$ threshold.

One might be surprised that the binding energies obtained and the width are bigger than those in the DK system itself. This is an effect produced by expressing the results in terms of the invariant mass distribution of particle-cluster, $\sqrt{s}$, rather than particle-particle, $\sqrt{s_1}$, an effect well known in particle-nucleus collisions [69].

Since the $D_{s0}^*(2317)$ decays into $D_s^+\pi^0$, the peak that we obtained should be searched in the invariant mass distributions of $K^0 D_s^+\pi^0$. A detailed discussion on how such invariant mass distributions can be searched in present facilities by LHCb or ALICE collaborations is given at the end of Sec. III in Ref. [38]. Next we turn to the correlation function of the $KD_{s0}^*(2317)$ system. The results are shown in Fig. 4.

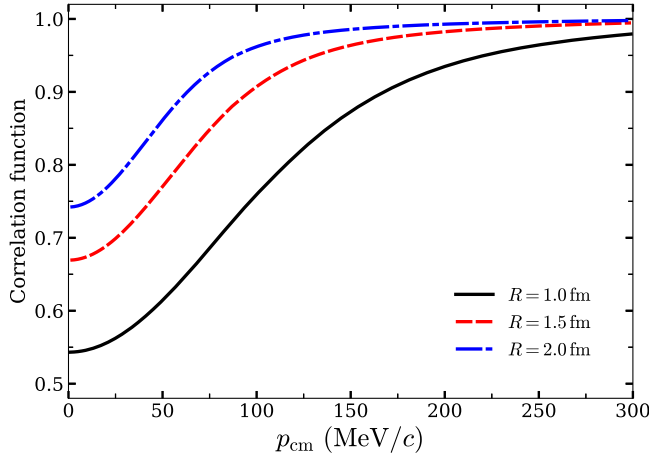


FIG. 4. Correlation function for the $KD_{s0}^*(2317)$ system.

We observe a pattern very similar to the one found for the $pf_1(1285)$ in Ref. [20], and the $Kf_1(1285)$ in Ref. [19], all of which produce a bound state with similar binding energies. In the qualitative picture for correlation functions discussed in Refs. [70,71], it would correspond to the case of a strongly attractive potential. The shapes of the correlation functions in Fig. 4 are also similar to those obtained for the D^0K^+ and D^+K^0 correlation functions in Ref. [72].

As in the case of the $pf_1(1285)$ where the $f_1(1285)$ is identified by a decay channel, the $K\bar{K}\pi$, in the measurement of the $K^0D_{s0}^*(2317)$ correlation function, the $D_{s0}^*(2317)$ would be identified by its $D_s^+\pi^0$ decay, by looking for the peak of the invariant mass distribution of $D_s^+\pi^0$ to identify the peak corresponding to the $D_{s0}^*(2317)$ state, while a K^0 is identified as coming from the same event. With the same experimental machinery, one could look at the invariant mass distribution of $K^0D_s^0\pi^0$ below the threshold to identify the state predicted. We think that there is much to search, and hopefully find, by making a thorough measurement of three-body invariant mass distribution in this and other related cases.

IV. CONCLUSIONS

Anticipating results in the new wave of ALICE experiments on particle-resonance correlation functions, we have carried out the evaluation of observables related to the interaction of a kaon with the $D_{s0}^*(2317)$ resonance. For this purpose we have assumed that the $D_{s0}^*(2317)$ is a molecular state of DK in isospin $I = 0$. Then we have used a version of the fixed center approximation, in which the kaon interacts with the DK cluster without destroying it, and made a mapping onto the conventional interaction of particles with nuclei, which constructs an optical potential that is later used within the Lippmann-Schwinger equation to obtain the particle-nucleus scattering matrix. The FCA provides the “optical potential,” and the propagation of the

elastic channel $KD_{s0}^*(2317)$ is implemented, producing elastic unitarity in the $KD_{s0}^*(2317)$ scattering amplitude, which is essential to evaluate scattering observables at low energies.

We have then evaluated the scattering length, the effective range, and the correlation function, which has a shape corresponding qualitatively to a strongly attractive potential.

As a side product, we observe that the amplitude develops a very narrow resonant shape below the $KD_{s0}^*(2317)$ threshold, corresponding to a three-body state bound by about 40 MeV. We also discuss that while the measurement of this correlation function requires the observation of a kaon and the $KD_{s0}^*(2317)$ from the same event, the $D_{s0}^*(2317)$ being detected via a peak in the $D_s^+\pi^0$ mass distribution, the same experimental framework could look into the invariant mass distribution of three bodies, $KD_s^+\pi^0$ below the threshold, looking for the predicted state. We anticipate that this search, and similar ones of predicted three-body states, are bound to find many three-body states which are predicted by different theoretical approaches (see a review on this issue in Ref. [28]).

We would like to note that recently new results from an ALICE experiment, measuring the correlation function for the $pf_1(1285)$ system, were reported in Ref. [61] and compared with the updated results of Ref. [20], obtained in Ref. [62] prior to the experimental results. The agreement between theory and experiment is quite good and this gives extra support to the predictions made here.

ACKNOWLEDGMENTS

We would like to thank Prof. Natsumi Ikeno for useful information. This work is partly supported by the National Natural Science Foundation of China (NSFC) under Grants No. 12575081 and No. 12365019, by the Natural Science Foundation of Guangxi province under Grant No. 2023JJA110076, and by the Central Government Guidance Funds for Local Scientific and Technological Development, China (No. Guike ZY22096024). This work is also partly supported by the National Key R&D Program of China (Grant No. 2024YFE0105200). J. Song acknowledges support from the National Natural Science Foundation of China (NSFC) under Grants No. 12405089 and No. 12247108, and by the China Postdoctoral Science Foundation under Grants No. 2022M720360 and No. 2022M720359. J.S. would also like to thank the support from the Hainan Provincial Excellent Talent Team under the “Four Talents” Gathering Program of Hainan Province. This work is also partly supported by the Spanish Ministerio de Economía y Competitividad (MINECO) and European FEDER funds under Contracts No. FIS2017-84038-C2-1-PB and No. PID2020-112777 GB-I00, and by Generalitat Valenciana under contract PROMETEO/2020/023. This project has received

funding from the European Union Horizon 2020 research and innovation program under the program H2020-INFRAIA-2018-1, Grant Agreement No. 824093 of the STRONG-2020 project.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX: t_1, t_2 AMPLITUDES

1. The DK amplitudes

For t_1 we need the DK amplitude in $I = 0, 1$. We take advantage of the recent work done in Ref. [73] where the interactions of the coupled channels, $D^0 K^+, D^+ K^0, D_s^+ \eta$, and $D_s^+ \pi^0$, are considered, which allows one to obtain the strong decay width of the $D_{s0}^*(2317)$ into the isospin forbidden $D_s^+ \pi^0$ channel. By using the local hidden gauge approach [74–77] with the exchange of vector mesons, the transition potentials V_{ij} between these coupled channels are evaluated in Ref. [73], and then the scattering matrix is evaluated with the Bethe-Salpeter equation in matrix form,

$$T = [1 - VG]^{-1}V, \quad (\text{A1})$$

with G the diagonal meson-meson loop function, $G = \text{diag}[G_i]$, with

$$G_i(s) = \int_{|\vec{q}| < q_{\max}} \frac{d^3 q}{(2\pi)^3} \frac{\omega_1^{(i)}(\vec{q}) + \omega_2^{(i)}(\vec{q})}{2\omega_1^{(i)}(\vec{q})\omega_2^{(i)}(\vec{q})} \times \frac{1}{s - [\omega_1^{(i)}(\vec{q}) + \omega_2^{(i)}(\vec{q})]^2 + i\epsilon}, \quad (\text{A2})$$

with $\omega_j^{(i)}(\vec{q}) = \sqrt{\vec{q}^2 + m_j^{(i)2}}$ for $j = 1, 2$, the two mesons of channel i . A pole is obtained for a mass of $M_R = 2317$ MeV and a width of around 100 keV, and the couplings g_i ($i = 1$ to 4) are defined such that the amplitude at the pole is given by

$$T_{ij} = \frac{g_i g_j}{s - M_R^2 + iM_R \Gamma_R}, \quad M_R = 2317 \text{ MeV}, \quad \Gamma_R \simeq 100 \text{ keV}. \quad (\text{A3})$$

The couplings are given in Ref. [73] and from them we can get the $I = 0$ scattering matrix, considering the intrinsic isospin phase convention that we use, with the isospin doublets $(D^+, -D^0), (K^+, K^0)$. The DK isospin states are then

$$|DK, I = 0\rangle = \frac{1}{\sqrt{2}}(D^+ K^0 + D^0 K^+),$$

$$|DK, I = 1, I_3 = 0\rangle = \frac{1}{\sqrt{2}}(D^+ K^0 - D^0 K^+). \quad (\text{A4})$$

Then,

$$t^{I=0} = \frac{1}{2}(t_{D^+ K^0, D^+ K^0} + 2t_{D^+ K^0, D^0 K^+} + t_{D^0 K^+, D^0 K^+})$$

$$= \frac{1}{2s - M_R^2 + iM_R \Gamma_R} \times (g_{D^+ K^0}^2 + 2g_{D^+ K^0} g_{D^0 K^+} + g_{D^0 K^+}^2), \quad (\text{A5})$$

and from Ref. [73],

$$g_{D^+ K^0} = (8129.49 + i 75.70) \text{ MeV},$$

$$g_{D^0 K^+} = (8252.26 - i 69.15) \text{ MeV}. \quad (\text{A6})$$

We cannot use these couplings to calculate the $I = 1$ amplitude since they refer to the $I = 0$ state. Instead we can use the results of Ref. [73] for the potentials in the coupled channels, and taking channel 1 as $D^0 K^+$ and channel 2 as $D^+ K^0$, and we find

$$V^{I=1} = \frac{1}{2}(V_{11} + V_{22} - 2V_{12}) = 0, \quad (\text{A7})$$

where the zero results by taking $m_\omega = m_\rho$. Thus,

$$t_{DK}^{I=1} = 0. \quad (\text{A8})$$

2. The KK amplitudes

These amplitudes are evaluated in Appendix A of Ref. [19] with the result

$$V_{KK}^{I=1} = \frac{4g^2}{M_V^2} \left(\frac{3}{2}s - 2M_K^2 \right), \quad g = \frac{M_V}{2f},$$

$$f = 93 \text{ MeV}, \quad M_V = 800 \text{ MeV}, \quad (\text{A9})$$

and

$$t_{KK}^{I=1} = \frac{V_{KK}^{I=1}}{1 - \frac{1}{2}V_{KK}^{I=1}G}, \quad (\text{A10})$$

with the factor $\frac{1}{2}$ in VG to account for the identity of the two kaons, and G given again in the cutoff form of Eq. (A2) with $q_{\max} = 650$ MeV. Similarly, the isospin $I = 0$ potential $V_{KK}^{I=0}$ is found null and thus

$$t_{KK}^{I=0} = 0. \quad (\text{A11})$$

- [1] L. Šerkšnytė, S. Kundu, and M. Korwieser (private communication).
- [2] S. Acharya *et al.* (ALICE Collaboration), Measurement of $f_1(1285)$ production in pp collisions at $\sqrt{s} = 13$ TeV, *Phys. Lett. B* **866**, 139562 (2025).
- [3] N. Brambilla *et al.*, Heavy quarkonium: Progress, puzzles, and opportunities, *Eur. Phys. J. C* **71**, 1534 (2011).
- [4] H.-X. Chen, W. Chen, X. Liu, Y.-R. Liu, and S.-L. Zhu, A review of the open charm and open bottom systems, *Rep. Prog. Phys.* **80**, 076201 (2017).
- [5] Y. Dong, A. Faessler, and V. E. Lyubovitskij, Description of heavy exotic resonances as molecular states using phenomenological Lagrangians, *Prog. Part. Nucl. Phys.* **94**, 282 (2017).
- [6] A. Esposito, A. Pilloni, and A. D. Polosa, Multiquark resonances, *Phys. Rep.* **668**, 1 (2017).
- [7] A. Hosaka, T. Iijima, K. Miyabayashi, Y. Sakai, and S. Yasui, Exotic hadrons with heavy flavors: X, Y, Z, and related states, *Prog. Theor. Exp. Phys.* **2016**, 062C01 (2016).
- [8] R. F. Lebed, R. E. Mitchell, and E. S. Swanson, Heavy-quark QCD exotica, *Prog. Part. Nucl. Phys.* **93**, 143 (2017).
- [9] S. L. Olsen, A new hadron spectroscopy, *Front. Phys. (Beijing)* **10**, 121 (2015).
- [10] S. L. Olsen, T. Skwarnicki, and D. Zieminska, Nonstandard heavy mesons and baryons: Experimental evidence, *Rev. Mod. Phys.* **90**, 015003 (2018).
- [11] E. Oset, W.-H. Liang, M. Bayar *et al.*, Weak decays of heavy hadrons into dynamically generated resonances, *Int. J. Mod. Phys. E* **25**, 1630001 (2016).
- [12] F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, and B.-S. Zou, Hadronic molecules, *Rev. Mod. Phys.* **90**, 015004 (2018); **94**, 029901(E) (2022).
- [13] M. F. M. Lutz and E. E. Kolomeitsev, On meson resonances and chiral symmetry, *Nucl. Phys. A* **730**, 392 (2004).
- [14] L. Roca, E. Oset, and J. Singh, Low lying axial-vector mesons as dynamically generated resonances, *Phys. Rev. D* **72**, 014002 (2005).
- [15] C. Garcia-Recio, L. S. Geng, J. Nieves, and L. L. Salcedo, Low-lying even parity meson resonances and spin-flavor symmetry, *Phys. Rev. D* **83**, 016007 (2011).
- [16] Y. Zhou, X.-L. Ren, H.-X. Chen, and L.-S. Geng, Pseudoscalar meson and vector meson interactions and dynamically generated axial-vector mesons, *Phys. Rev. D* **90**, 014020 (2014).
- [17] L.-S. Geng, X.-L. Ren, Y. Zhou, H.-X. Chen, and E. Oset, S -wave KK^* interactions in a finite volume and the $f_1(1285)$, *Phys. Rev. D* **92**, 014029 (2015).
- [18] P.-L. Lü and J. He, Hadronic molecular states from the $K\bar{K}^*$ interaction, *Eur. Phys. J. A* **52**, 359 (2016).
- [19] W.-H. Jia, J. Song, W.-H. Liang, and E. Oset, Scattering data and correlation function for the $Kf_1(1285)$ interaction, *Eur. Phys. J. C* **86**, 761 (2026).
- [20] P. Encarnación, A. Feijoo, and E. Oset, Correlation function for the $pf_1(1285)$ interaction, *Phys. Rev. D* **111**, 114023 (2025).
- [21] S. S. Kamalov, E. Oset, and A. Ramos, Chiral unitary approach to the K^- deuteron scattering length, *Nucl. Phys. A* **690**, 494 (2001).
- [22] L. L. Foldy, The multiple scattering of waves. 1. General theory of isotropic scattering by randomly distributed scatterers, *Phys. Rev.* **67**, 107 (1945).
- [23] K. A. Brueckner, Multiple scattering corrections to the impulse approximation in the two-body system, *Phys. Rev.* **89**, 834 (1953).
- [24] K. A. Brueckner, The elastic scattering of pions in deuterium, *Phys. Rev.* **90**, 715 (1953).
- [25] R. Chand and R. H. Dalitz, Charge-independence in K^- -deuterium capture reactions, *Ann. Phys. (N.Y.)* **20**, 1 (1962).
- [26] R. C. Barrett and A. Deloff, Strong interaction effects in kaonic deuterium, *Phys. Rev. C* **60**, 025201 (1999).
- [27] A. Deloff, ηd and K^-d zero energy scattering: A Faddeev approach, *Phys. Rev. C* **61**, 024004 (2000).
- [28] A. Martinez Torres, K. P. Khemchandani, L. Roca, and E. Oset, Few-body systems consisting of mesons, *Few Body Syst.* **61**, 35 (2020).
- [29] L. Roca and E. Oset, A description of the $f_2(1270)$, $\rho_3(1690)$, $f_4(2050)$, $\rho_5(2350)$ and $f_6(2510)$ resonances as multi- $\rho(770)$ states, *Phys. Rev. D* **82**, 054013 (2010).
- [30] B. B. Malabarba, K. P. Khemchandani, A. Martinez Torres, and S.-i. Nam, Strangeness +1 light multiquark baryons, *Phys. Rev. D* **111**, 016021 (2025).
- [31] T. E. O. Ericson and W. Weise, *Pions and Nuclei* (Clarendon Press, Oxford, UK, 1988).
- [32] R. Seki and K. Masutani, Unified analysis of pionic atoms and low-energy pion-nucleus scattering: Phenomenological analysis, *Phys. Rev. C* **27**, 2799 (1983).
- [33] J. Nieves, E. Oset, and C. Garcia-Recio, A theoretical approach to pionic atoms and the problem of anomalies, *Nucl. Phys. A* **554**, 509 (1993).
- [34] G. E. Brown and W. Weise, Pion scattering and isobars in nuclei, *Phys. Rep.* **22**, 279 (1975).
- [35] N. Ikeno and E. Oset, Correlation function for the $n\bar{D}_{s0}^*(2317)$ interaction and the issue of elastic unitarity, *Phys. Rev. D* **112**, 094019 (2025).
- [36] M.-J. Yan, J. M. Dias, A. Guevara, F.-K. Guo, and B.-S. Zou, On the $\eta_1(1855)$, $\pi_1(1400)$ and $\pi_1(1600)$ as dynamically generated states and their SU(3) partners, *Universe* **9**, 109 (2023).
- [37] B. Agatão, P. Brandão, A. Martínez Torres, K. P. Khemchandani, L. M. Abreu, and E. Oset, Correlation functions for $n\bar{D}_{s1}(2460)$ and $n\bar{D}_{s1}(2536)$, *Eur. Phys. J. C* **85**, 1136 (2025).
- [38] W.-H. Jia, P.-S. Su, W.-H. Liang, R. Molina, and E. Oset, Superexotic $K^{*+}D^{*+}K^{*+}$ bound state, *Phys. Lett. B* **875**, 140320 (2026).
- [39] E. van Beveren and G. Rupp, Observed $D_s(2317)$ and tentative $D(2100-2300)$ as the charmed cousins of the light scalar nonet, *Phys. Rev. Lett.* **91**, 012003 (2003).
- [40] T. Barnes, F. E. Close, and H. J. Lipkin, Implications of a DK molecule at 2.32 GeV, *Phys. Rev. D* **68**, 054006 (2003).
- [41] Y.-Q. Chen and X.-Q. Li, A Comprehensive four-quark interpretation of $D_s(2317)$, $D_s(2457)$ and $D_s(2632)$, *Phys. Rev. Lett.* **93**, 232001 (2004).
- [42] E. E. Kolomeitsev and M. F. M. Lutz, On heavy light meson resonances and chiral symmetry, *Phys. Lett. B* **582**, 39 (2004).

- [43] D. Gamermann, E. Oset, D. Strottman, and M. J. Vicente Vacas, Dynamically generated open and hidden charm meson systems, *Phys. Rev. D* **76**, 074016 (2007).
- [44] F.-K. Guo, P.-N. Shen, and H.-C. Chiang, Dynamically generated 1^+ heavy mesons, *Phys. Lett. B* **647**, 133 (2007).
- [45] Z. Yang, G.-J. Wang, J.-J. Wu, M. Oka, and S.-L. Zhu, Novel coupled channel framework connecting the quark model and lattice QCD for the near-threshold D_s states, *Phys. Rev. Lett.* **128**, 112001 (2022).
- [46] M.-Z. Liu, X.-Z. Ling, L.-S. Geng, En-Wang, and J.-J. Xie, Production of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ in B decays as $D^{(*)}K$ and $D_s^{(*)}\eta$ molecules, *Phys. Rev. D* **106**, 114011 (2022).
- [47] W.-H. Liang, Z.-R. Hu, and E. Oset, The $B^{+(0)} \rightarrow \bar{D}^{0(-)}D_{s0}^*(2317)^+$ decays and the molecular structure of $D_{s0}^*(2317)$, *Eur. Phys. J. A* **62**, 104 (2026).
- [48] D. Mohler, C. B. Lang, L. Leskovec, S. Prelovsek, and R. M. Woloshyn, $D_{s0}^*(2317)$ meson and D -meson-kaon scattering from lattice QCD, *Phys. Rev. Lett.* **111**, 222001 (2013).
- [49] C. B. Lang, L. Leskovec, D. Mohler, S. Prelovsek, and R. M. Woloshyn, D_s mesons with DK and D^*K scattering near threshold, *Phys. Rev. D* **90**, 034510 (2014).
- [50] G. S. Bali, S. Collins, A. Cox, and A. Schäfer, Masses and decay constants of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ from $N_f = 2$ lattice QCD close to the physical point, *Phys. Rev. D* **96**, 074501 (2017).
- [51] G. K. C. Cheung, C. E. Thomas, D. J. Wilson, G. Moir, M. Peardon, and S. M. Ryan (Hadron Spectrum Collaboration), $DKI = 0$, $D\bar{K}I = 0$, 1 scattering and the $D_{s0}^*(2317)$ from lattice QCD, *J. High Energy Phys.* **02** (2020) 100.
- [52] A. Martínez Torres, E. Oset, S. Prelovsek, and A. Ramos, Reanalysis of lattice QCD spectra leading to the $D_{s0}^*(2317)$ and $D_{s1}^*(2460)$, *J. High Energy Phys.* **05** (2014) 153.
- [53] E. Ruiz Arriola, Some three-body force cancellations in chiral Lagrangians, *arXiv:1606.07535*.
- [54] J. Nieves and E. Ruiz Arriola, Bethe-Salpeter approach for unitarized chiral perturbation theory, *Nucl. Phys. A* **679**, 57 (2000).
- [55] B. B. Malabarba, K. P. Khemchandani, and A. M. Torres, N^* states with hidden charm and a three-body nature, *Eur. Phys. J. A* **58**, 33 (2022).
- [56] A. Martínez Torres, K. P. Khemchandani, and E. Oset, Solution to Faddeev equations with two-body experimental amplitudes as input and application to $J^*P = 1/2^+, S = 0$ baryon resonances, *Phys. Rev. C* **79**, 065207 (2009).
- [57] A. Martínez Torres and D. Jido, $K\Lambda(1405)$ configuration of the $K\bar{K}N$ system, *Phys. Rev. C* **82**, 038202 (2010).
- [58] E.-W. Jia and H.-R. Pang, $K\bar{K}N$ and anti- K anti- K N molecular states with $I = 1/2, 3/2$ and $J^*P = 1/2^+$ studied with three-body Faddeev calculations, *Chin. Phys. Lett.* **28**, 061401 (2011).
- [59] A. Martínez Torres, E. J. Garzon, E. Oset, and L. R. Dai, Limits to the Fixed Center Approximation to Faddeev equations: the case of the $\phi(2170)$, *Phys. Rev. D* **83**, 116002 (2011).
- [60] A. Martínez Torres, K. P. Khemchandani, L. S. Geng, M. Napsuciale, and E. Oset, The $X(2175)$ as a resonant state of the ϕK anti- K system, *Phys. Rev. D* **78**, 074031 (2008).
- [61] L. Serkšnytė and S. Kundu, First experimental study of axial-vector meson-nucleon interactions using $p-f_1(1285)$ correlations with ALICE, in *Strangeness in Quark Matter Conference* (2026) talk given by Laura Serkšnytė.
- [62] P. Encarnación, A. Feijoo, and E. Oset, Scattering observables and correlation function for $p f_1(1285)$ revisited, *Phys. Rev. D* **113**, L111502 (2026).
- [63] D. Gamermann, J. Nieves, E. Oset, and E. Ruiz Arriola, Couplings in coupled channels versus wave functions: Application to the $X(3872)$ resonance, *Phys. Rev. D* **81**, 014029 (2010).
- [64] J. A. Oller and U. G. Meissner, Chiral dynamics in the presence of bound states: Kaon nucleon interactions revisited, *Phys. Lett. B* **500**, 263 (2001).
- [65] R. C. Carrasco, J. Nieves, and E. Oset, Coherent (γ , π^0) photoproduction in a local approximation to the delta hole model, *Nucl. Phys. A* **565**, 797 (1993).
- [66] S. Boffi, L. Bracci, and P. Christillin, Coherent (γ , π^0) on nuclei, *Nuovo Cimento A* **104**, 843 (1991).
- [67] J. Yamagata-Sekihara, J. Nieves, and E. Oset, Couplings in coupled channels versus wave functions in the case of resonances: application to the two $\Lambda(1405)$ states, *Phys. Rev. D* **83**, 014003 (2011).
- [68] R. Molina and E. Oset, Determination of off-shell ambiguities in correlation functions: Strategies to minimize them, *Phys. Rev. D* **112**, 096006 (2025).
- [69] V. Montesinos, J. Song, W.-H. Liang, E. Oset, J. Nieves, and M. Albaladejo, Study of possible DND^* bound states, *Phys. Rev. D* **110**, 054043 (2024).
- [70] Z.-W. Liu, J.-X. Lu, and L.-S. Geng, Study of the DK interaction with femtoscopic correlation functions, *Phys. Rev. D* **107**, 074019 (2023).
- [71] M.-Z. Liu, Y.-W. Pan, Z.-W. Liu, T.-W. Wu, J.-X. Lu, and L.-S. Geng, Three ways to decipher the nature of exotic hadrons: Multiplets, three-body hadronic molecules, and correlation functions, *Phys. Rep.* **1108**, 1 (2025).
- [72] N. Ikeno, G. Toledo, and E. Oset, Model independent analysis of femtoscopic correlation functions: An application to the $D_{s0}^*(2317)$, *Phys. Lett. B* **847**, 138281 (2023).
- [73] P.-S. Su, W.-T. Lyu, W.-H. Liang, and E. Oset, The $D_{s0}^*(2317)^+$ decay to $D_s^+\pi^0$ and $D_s^{*+}\gamma$, *Eur. Phys. J. C* **86**, 242 (2026).
- [74] M. Bando, T. Kugo, S. Uehara, K. Yamawaki, and T. Yanagida, Is ρ meson a dynamical gauge boson of hidden local symmetry?, *Phys. Rev. Lett.* **54**, 1215 (1985).
- [75] M. Bando, T. Kugo, and K. Yamawaki, Nonlinear realization and hidden local symmetries, *Phys. Rep.* **164**, 217 (1988).
- [76] U. G. Meißner, Low-energy hadron physics from effective chiral Lagrangians with vector mesons, *Phys. Rep.* **161**, 213 (1988).
- [77] H. Nagahiro, L. Roca, A. Hosaka, and E. Oset, Hidden gauge formalism for the radiative decays of axial-vector mesons, *Phys. Rev. D* **79**, 014015 (2009).