

Hadron production through Higgs boson decay at next-to-leading order in the general-mass variable-flavor-number scheme

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It is known that about 60% of all Higgs bosons produced at the CERN-LHC decay into a pair of bottom quarks. Bottoms quickly hadronize, in most cases, into bottom-flavored (B) hadrons before they decay. Therefore, the study of scaled-energy distribution of B mesons in the decay process $H \rightarrow B + \text{Jets}$ can be considered as a channel to search for the Higgs characteristics. In all previous studies, authors have ignored the mass effect of b quarks as well as B mesons by working in the massless scheme. In this work we, for the first time, study the mass effect of b quarks as well as produced mesons on the scaled-energy (x_B) distribution of B mesons by working in the massive scheme or general-mass variable-flavor-number scheme. We find that the meson mass is responsible for a significant enhancement of partial decay width in the low- x_B region while the b quark mass leads to an enhancement of the partial decay rate in the peak region and above.

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I. INTRODUCTION

Observation of the Higgs boson by the CERN-ATLAS and CMS Collaborations in 2012 completed the standard model (SM) of strong and electroweak interactions provided it is compatible with the SM Higgs boson. During the last decades a lot of knowledge has been learned about this key particle, and its discovery was an important breakthrough in our understanding of fundamental interactions. Nevertheless, a basic question is still whether the characteristics of an observed particle is in complete consistency with that predicted by the SM. Consistencies with the SM Higgs boson need to be examined in more detail by measuring its couplings to other SM particles. These are pursued by deriving the related couplings from the study of Higgs boson decays and its production rates at the LHC and will be continued in the future runs. When studying the Higgs boson, particle physicists have to look for its decay products in their detectors, as the Higgs decays very quickly after it is produced. Note that Higgs bosons can decay to different particles, but not all final states are equally likely. For instance, around 60% of all Higgs bosons produced at the LHC decay into a pair of bottom quarks ($H \rightarrow b\bar{b}$), while only 3% decay into a pair of Z

bosons [1]. In recent years, the LHC has achieved great success in discovering the Higgs couplings to charged fermions of third generation as well as heavy vector bosons [2–4]. Note that, in the SM the Higgs couplings to the first and second generation fermions are so weak and very difficult to be measured directly. Nevertheless, due to complicated hadronic backgrounds and the limited Higgs events the precision of these measurements is restricted.

At the CERN-LHC, we expect to have more data collected during Run 3 than the first two runs combined. Moreover, this machine is planned to upgrade to the high-energy and high-luminosity LHC after Run 3, so it is anticipated to have about 1.6×10^8 Higgs events at the energy $\sqrt{s} = 14$ TeV with an integrated luminosity of 3 ab^{-1} and also 2.2×10^9 Higgs events at the energy $\sqrt{s} = 27$ TeV with integrated luminosity of 15 ab^{-1} [5]. Besides that, several new high energy colliders such as the Circular Electron-Positron Collider [6], the e^+e^- Future Circular Collider (FCC-ee) [7], the International Linear Collider (ILC) [8], and the Muon Collider [9] are under consideration so their suitable advantage is to provide a clean environment for the exact measurements of Higgs properties. These future generation colliders allow particle physicists to measure some rare decays of Higgs bosons such as their decays to heavy quarkonia [10–13]. These are necessary to determine the magnitude of Yukawa couplings of Higgs to heavy quark flavors. Decays to heavy quarkonia have distinguished signals which can be detected at the high-energy colliders.

As was mentioned, at the LHC it is observed that around 60% of Higgs bosons decay into a pair of bottom quarks,

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so, in the following, bottom quarks hadronize before they decay. In most cases, bottom quarks hadronize into bottom-flavored mesons B ; however other hadron species are possible to be produced. Therefore, investigating the energy spectrum of B mesons produced through Higgs boson decay in the process $H \rightarrow b\bar{b} \rightarrow B + \text{Jets}$ would be proposed as a channel to look for the Higgs characteristics. Moreover, this proposed channel can also be used to study the production mechanism of B mesons, which is described by the nonperturbative fragmentation functions (non-pFFs). With these motivations, in many papers the energy spectrum of B mesons has been studied by evaluating the quantity $d\Gamma(H \rightarrow BX)/dx_B$ where $x_B (= 2p_B \cdot p_H/p_H^2 = 2E_B/m_H)$ is the scaled-energy of the B meson in the Higgs boson rest frame; see for example Refs. [14–16]. In all previous works, for simplicity, the mass effects of bottom quarks as well as produced mesons have been ignored. This choice is known as the massless scheme or zero-mass variable-flavor-number scheme (ZM-VFNs) where the assumptions $m_b = 0$ and $m_B = 0$ are set from the beginning. Here, we revisit bottom-flavored hadron production from the SM Higgs decay by working in the general-mass variable-flavor-number scheme (GM-VFNs) which provides an ideal theoretical framework to taking into account the effects of quark masses. Being manifestly based on QCD-improved parton model factorization theorem (or Collin's hard-scattering factorization theorem) [17] appropriate for massive quarks, this factorization scheme allows one to resum the large logarithms in m_b , i.e., $\ln(m_H^2/m_b^2)$, to retain the finite- m_b effects and to preserve the universality of the nonperturbative fragmentation functions, whose scaling violations remain to be subject to the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi evolution equations [18]. In this way, the GM-VFNs combine the virtues of fixed-flavor-number scheme (FFNs) and the ZM-VFN scheme, and, at the same time, this scheme avoids their flaws. In fact, the GM-VFN is a suitable tool for global analyses of experimental data on the inclusive production of heavy hadrons, allowing us to transfer nonperturbative information on the hadronization of partons (quarks or gluon) from one type of experiment to another and from one energy scale μ_F to another one, without the restriction $\mu_F \gg m_b$ which is necessary for the ZM-VFNs (more details about the GM-VFNs can be found in Refs. [19,20]). Therefore, our analysis in the GM-VFN scheme is supposed to enhance our previous result in the ZM-VFNs [16] by retaining all nonlogarithmic m_b terms.

To take into account all influencing factors, we also include finite- m_B effects, which modify the relations between hadronic and partonic scaling variables and reduce the available phase space. Their study is essential to fully exploit the enormous statistics of the data from the LHC and future lepton colliders for a high precision determination of the Higgs properties. Our analysis shows that the B meson mass is responsible for a significant enhancement

of energy distribution in the low- x_B region while the bottom quark mass leads to an enhancement in the peak region and above. The masses of hadrons are also responsible for the low- x_B threshold.

The manuscript is organized as follows. In Sec. II, we briefly describe our applied framework to compute the differential decay rates at the parton level and present our analytical results at next-to-leading order (NLO) perturbative quantum chromodynamics (pQCD). In Sec. III we review the factorization theorem in the presence of hadron mass. In Sec. IV, our numerical results at the hadron level are presented, and Sec. V is devoted to the conclusions.

II. PARTON LEVEL RESULTS FOR $H \rightarrow b\bar{b}$

A. Born level rate

In order to obtain the energy distribution of B mesons inclusively produced in Higgs decay, i.e., $H \rightarrow B + \text{Jets}$, we first need to compute the differential decay width of the process $H \rightarrow b\bar{b}$, i.e., $d\tilde{\Gamma}_i(H \rightarrow b\bar{b}(+g))/dx_i$ ($i = b, g$) at each order of perturbation. Here, $x_i (= 2p_i \cdot p_H/p_H^2)$ stands for the scaled energy of b quarks or gluon. In the Higgs rest frame, this is simplified as $x_i = 2E_i/m_H$. The gluon contribution appears in NLO and higher orders. At the lowest-order approximation (Born level), the amplitude for the process $H \rightarrow b\bar{b}$ is parametrized as $M_0 \propto (-im_b/v)v_{\bar{b}}\bar{u}_b$ where $v = (\sqrt{2}G_F)^{-1/2}$ and G_F is the Fermi constant. In the Higgs boson rest frame, the tree-level decay width is given by

$$\tilde{\Gamma}_0 = \frac{N_c}{8\pi v^2} m_b^3 R \beta^3, \quad (1)$$

where $R = (m_b/m_H)^2$, $\beta = \sqrt{1 - 4R}$, and $N_c = 3$ is the color number of flavors. This rate is in complete agreement with the result presented in Refs. [21–24].

In the following, the technical details of our computations for the $\mathcal{O}(\alpha_s)$ QCD radiative corrections to the Born level decay rate of $H \rightarrow b\bar{b}$ are explained using the dimensional regularization approach to regularize all divergences. For this calculation, we apply the FFN scheme where $m_b \neq 0$ is assumed from the beginning.

B. Virtual radiative corrections

Virtual radiative corrections to the $Hb\bar{b}$ vertex include the one-loop vertex contribution and a combination of b -quark mass and wave function renormalization; see Fig. 1(b). These corrections consist of both ultraviolet (UV) and infrared (IR) divergences which arise from the hard and soft virtual gluon radiations, respectively. There are different schemes for the renormalization of physical quantities. In this work we adopt the on shell mass-renormalization scheme, and to regularize all singularities we apply the dimensional regularization approach in which $D = 4 - 2\epsilon$ is adopted for the dimension of space-time

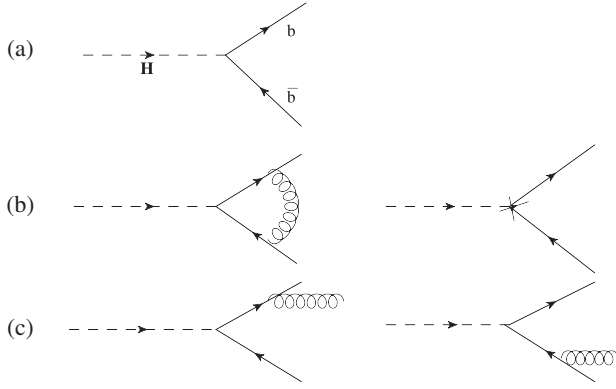


FIG. 1. Higgs decay Feynman diagrams at NLO: (a) tree-level contribution; (b) vertex correction and a combination of mass and wave function renormalization; (c) real radiations.

($\epsilon \ll 1$). Then, all singularities appear as ϵ_{uv} or ϵ_{ir} . To determine the energy spectrum of B mesons produced in $H \rightarrow b\bar{b}(+g) \rightarrow B + X$, where X collectively denotes the unobserved final state particles, we first need to have the partial decay widths $d\tilde{\Gamma}_i/dx_i$ ($i = b, g$) where the scaled energies x_i in the Higgs rest frame are defined as before: $x_i = 2E_i/m_H$. The contribution of virtual gluon radiation into the differential decay width $d\tilde{\Gamma}_b/dx_b$ is computed through the following relation:

$$\frac{d\tilde{\Gamma}_b^{\text{vir}}}{dx_b} = \frac{\beta}{8\pi m_H} |\overline{M^{\text{vir}}}|^2 \delta(1 - x_b). \quad (2)$$

Here, $|\overline{M^{\text{vir}}}|^2 = N_c(m_b/v)^2 \sum_{Spin} (M_0^\dagger M_{\text{loop}} + M_{\text{loop}}^\dagger M_0)$ where M_0 is the amplitude at the Born level, i.e., $M_0 \propto (-im_b/v) v_{\bar{b}} \bar{u}_b$. The renormalized amplitude is written as

$$M_{\text{loop}} = v_{\bar{b}}(\Lambda_{ct} + \Lambda_l) \bar{u}_b, \quad (3)$$

where Λ_{ct} stands for the counterterm and Λ_l arises from the one-loop vertex correction; see Fig. 1(b). The counterterm of the vertex includes a combination of the mass and the wave function renormalization of $b/\bar{b}$ quarks which are expressed as [24–28]

$$\Lambda_{ct} = \delta Z_b + \frac{\delta m_b}{m_b}, \quad (4)$$

where the renormalization constants are [29,30]

$$\begin{aligned} \frac{\delta m_b}{m_b} &= -\frac{\alpha_s(\mu_R)}{4\pi} C_F \left(\frac{3}{\epsilon_{uv}} - 3\gamma_E + 3 \ln \frac{4\pi\mu_F^2}{m_b^2} + 4 \right), \\ \delta Z_b &= -\frac{\alpha_s(\mu_R)}{4\pi} C_F \left(\frac{1}{\epsilon_{uv}} + \frac{2}{\epsilon_{ir}} - 3\gamma_E + 3 \ln \frac{4\pi\mu_F^2}{m_b^2} + 4 \right). \end{aligned} \quad (5)$$

Here, $\gamma_E = 0.577216 \dots$ is the Euler constant, α_s is the strong coupling constant, and $C_F = (N_c^2 - 1)/(2N_c) = 4/3$ is the color factor. Moreover, μ_R and μ_F are the renormalization and factorization scales, respectively. For our analysis we adopt $\mu_R = \mu_F = \mu$, a choice often made. In addition, for the vertex correction in D dimensions one has

$$\begin{aligned} \Lambda_l &= \frac{\pi\alpha_s C_F}{i(2\pi)^4} \\ &\times \int \frac{d^D p_g}{(2\pi\mu)^{D-4}} \gamma^\mu \frac{(\not{p}_b + \not{p}_g - m_b)(\not{p}_b - \not{p}_g + m_b)}{p_g^2(p_b \cdot p_g)(\bar{p}_b \cdot p_g)} \gamma_\mu, \end{aligned} \quad (6)$$

where, p_g , p_b , and $\bar{p}_b$ are the gluon and $b/\bar{b}$ -quark four-momenta, respectively. Then, the contribution of vertex correction in the mean squared amplitude ($|\overline{M^{\text{vir}}}|^2$) is proportional to the following term:

$$\begin{aligned} &\frac{\alpha_s m_H^2}{\pi} C_F [4R - 1 + 2(1 - 2R)B_0(m_b^2, 0, m_b^2) \\ &- M_H^2(1 - 6R + 8R^2)C_0(m_b^2, m_b^2, m_H^2, m_b^2, 0, m_b^2) \\ &- 4RB_0(m_H^2, m_b^2, m_b^2)], \end{aligned} \quad (7)$$

where, B_0 , and C_0 are the Passarino-Veltman two-point and three-point integrals [31]. The one-loop virtual contribution is purely real as can be understood from an inspection of the one-loop Feynman diagrams. These diagrams do not accept any nonvanishing physical two-particle cut.

The UV divergences are removed when all virtual corrections are summed, but the IR singularities are remaining which are labeled by ϵ from now on. Ignoring more details, the virtual differential decay rate reads as

$$\begin{aligned} \frac{d\tilde{\Gamma}_b^{\text{vir}}}{dx_b} &= \tilde{\Gamma}_0 \frac{\alpha_s(\mu)}{2\pi} C_F \delta(1 - x_b) \left\{ -2 - \frac{4R}{\beta} \ln x_s^2 \right. \\ &- \frac{2(1 - 2R)x_s}{R(1 - x_s^2)} \left[2Li_2(1 - x_s) - Li_2(1 - x_s^2) - \frac{1}{8} \ln^2 x_s^2 \right] \\ &- 2 \left[1 - \frac{(1 - 2R)x_s}{2R(1 - x_s^2)} \ln x_s^2 \right] \left(\ln \frac{4\pi\mu^2}{m_b^2} - \gamma_E + \frac{1}{\epsilon} \right) \Big\}, \end{aligned} \quad (8)$$

where $Li_2(x) = -\int_0^x (dt/t) \ln(1 - t)$ is the Spence function, and we also defined $x_s = (\beta - 1)/(\beta + 1)$.

C. Real gluon radiative corrections (bremsstrahlung)

According to the well-known Bloch-Nordsieck theorem, all singularities cancel exactly to all orders of perturbation theory, leaving a finite radiative correction of order α_s . Then, the real gluon radiative corrections are also needed to remove remaining IR singularities. Considering two Feynman diagrams in Fig. 1(c) for the

process $H(p_H) \rightarrow b(p_b) + \bar{b}(\bar{p}_b) + g(p_g)$, the $\mathcal{O}(\alpha_s)$ real gluon emission amplitude is written as

$$M^{\text{real}} = \frac{-im_b}{v} g_s T^a \bar{u}(p_b, s_b) \left\{ \frac{\gamma^\nu \not{p}_g + 2p_b^\nu}{2p_b \cdot p_g} - \frac{\gamma^\nu \not{p}_g + 2\bar{p}_b^\nu}{2\bar{p}_b \cdot p_g} \right\} v(\bar{p}_b, \bar{s}_b) \epsilon_\nu^*(p_g, r), \quad (9)$$

where the T^a generators are related to the Gell-Mann matrices. Also, $\epsilon(p_g, r)$ stands for the polarization vector of real emitted gluon with the four-momentum p_g and the spin r . Using the dimensional regularization scheme the differential decay rate of process $H \rightarrow b\bar{b}g$ is given by

$$d\tilde{\Gamma}^{\text{real}} = \frac{\mu^{2(4-D)}}{2m_H} |\overline{M^{\text{real}}}|^2 dPS(p_b, \bar{p}_b, p_g, p_H). \quad (10)$$

Here, the phase space element dPS reads as

$$\frac{d^{D-1} \vec{p}_b d^{D-1} \vec{\bar{p}}_b d^{D-1} \vec{p}_g}{2E_b 2\bar{E}_b 2E_g} (2\pi)^{3-2D} \delta^D\left(p_H - \sum_{g,b,\bar{b}} p_f\right). \quad (11)$$

To evaluate the real differential decay rate $d\tilde{\Gamma}_b^{\text{real}}/dx_b$, we fix the b-quark momentum in Eq. (10) and integrate over the gluon energy (E_g). As before, we define $x_g = 2E_g/m_H$ in the Higgs rest frame; then it ranges as

$$\frac{(1-x_b)(2-x_b-S)}{2(1+R-x_b)} \leq x_g \leq \frac{(1-x_b)(2-x_b+S)}{2(1+R-x_b)}, \quad (12)$$

where $S = \sqrt{x_b^2 - 4R}$. When integrating over the phase space of the process, in the rate $d\tilde{\Gamma}_b^{\text{real}}/dx_b$ terms of the

form $(1-x_b)^{-1-2\epsilon}$ appear which are due to the radiation of soft gluon. Note that the limit of $E_g \rightarrow 0$ corresponds to the limit $x_b \rightarrow 1$. Therefore, for a massive b quark where $x_b^{\text{min}} = \eta$ (with $\eta = 2\sqrt{R}$), we apply the following prescription introduced in Ref. [29]:

$$\frac{(x_b - \eta)^{2\epsilon}}{(1-x_b)^{1+2\epsilon}} = -\frac{1}{2\epsilon} \delta(1-x_b) + \frac{1}{(1-x_b)_+} + \mathcal{O}(\epsilon), \quad (13)$$

where the plus description is defined as

$$\begin{aligned} \int_\eta^1 \frac{f(x_b)}{(1-x_b)_+} dx_b \\ = \int_\eta^1 \frac{f(x_b) - f(1)}{1-x_b} dx_b + f(1) \ln(1-\eta). \end{aligned} \quad (14)$$

D. Parton-level results of $d\tilde{\Gamma}_i/dx_i$ in the FFN scheme

Without going into further detail, by summation of all contributions, i.e., the tree-level, virtual, and real gluon radiative contributions, we obtain an analytical result for the differential decay rate of $H \rightarrow b\bar{b}$ at NLO pQCD in the FFN scheme. The final result is free of each singularity. Defining

$$F(x_b) = \ln \frac{x_b - 2R + \sqrt{x_b^2 - 4R}}{x_b - 2R - \sqrt{x_b^2 - 4R}}, \quad (15)$$

the differential decay rate normalized to the Born rate $\tilde{\Gamma}_0$ (1) reads as

$$\begin{aligned} \frac{1}{\tilde{\Gamma}_0} \frac{d\tilde{\Gamma}_b}{dx_b} = & \delta(1-x_b) + \frac{C_F \alpha_s(\mu)}{2\pi} \left(\frac{1-2R}{\beta} \right) \left\{ \delta(1-x_b) \left[\frac{4\beta}{1-2R} \ln \frac{R}{1-\eta} + \ln x_s^2 \left(-\frac{1}{4} \ln x_s^2 + \ln \frac{1+\eta}{1-\eta} - \frac{1+2R}{1-2R} \right) \right. \right. \\ & - 2Li_2(1-x_s^2) + 4Li_2(1-x_s) \left. \right] + \frac{2\beta^{-2}(1-2R)^{-1}}{(1-x_b)_+} \left[(8R^2 - 2R(2x_b+1) + x_b^2) F(x_b) \right. \\ & + \frac{\sqrt{x_b^2 - 4R}}{2(1+R-x_b)} (16R^2 - 10Rx_b + 6R + x_b^2 - x_b) \left. \right] + \frac{\beta^{-2} \sqrt{x_b^2 - 4R}}{2(1+R-x_b)(1-2R)} \left(11R - x_b - 4 + \frac{R(R-1)}{1+R-x_b} \right) \\ & \left. + \frac{\beta^{-2}(1+x_b)}{1-2R} F(x_b) \right\}. \end{aligned} \quad (16)$$

This result, which is presented for the first time, after integrating over x_b (with $2\sqrt{R} \leq x_b \leq 1$) is in complete agreement with the one presented in Refs. [21–24], i.e.,

$$\tilde{\Gamma}(H \rightarrow b\bar{b}) = \tilde{\Gamma}_0 \left\{ 1 + \frac{C_F \alpha_s(\mu)}{\pi} \left(\frac{G(\beta)}{\beta} - \frac{3(1-7\beta^2)}{8\beta^2} + \frac{3+34\beta^2-13\beta^4}{16\beta^3} \ln \frac{1+\beta}{1-\beta} \right) \right\}, \quad (17)$$

where

$$G(\beta) = -3\beta \ln \frac{4}{1-\beta^2} - 4\beta \ln \beta + (1+\beta^2) \left(-2 \ln \beta \ln \frac{1+\beta}{1-\beta} + 3 \ln(1+\beta) \ln \frac{1+\beta}{1-\beta} + 4Li_2 \frac{1-\beta}{1+\beta} + 2Li_2 \frac{\beta-1}{\beta+1} \right). \quad (18)$$

Since the observed B mesons from Higgs decays can also be produced via the fragmentation of real gluons emitted at higher orders, then, in order to have the most accurate energy spectrum of B mesons at NLO, we have to take into account the contribution of gluon fragmentation too. Therefore, we also need to compute the differential decay rate $d\tilde{\Gamma}_g/dx_g$ in the FFN scheme where $x_g (= 2E_g/m_H)$ is the scaled-energy fraction of the real gluon emitted. We will show that the contribution of gluon splitting is important at the low- x_B and decreases the size of partial decay width at the threshold; see also Refs. [32–34].

In order to evaluate the rate $d\tilde{\Gamma}_g/dx_g$, in Eq. (10) we integrate over the b-quark energy by fixing the gluon momentum in the phase space. The scaled energy of the b quark, i.e., $x_b = 2E_b/m_H$, ranges as

$$1 - \frac{x_g}{2}(1+v) \leq x_b \leq 1 - \frac{x_g}{2}(1-v), \quad (19)$$

where $v = \sqrt{1 - 4R/(1-x_g)}$.

Ignoring more details, the differential decay rate $d\tilde{\Gamma}_g/dx_g$ normalized to the Born rate $\tilde{\Gamma}_0$ (1) in the FFNs reads as

$$\frac{1}{\tilde{\Gamma}_0} \frac{d\tilde{\Gamma}_g}{dx_g} = \frac{C_F \alpha_s(\mu)}{2\pi \beta x_g} \left\{ 4(x_g - 1)v + 2 \left(1 + \beta^2 + \frac{x_g^2}{\beta^2} - 2x_g \right) \ln \frac{1+v}{1-v} \right\}. \quad (20)$$

As was explained, the GM-VFN scheme provides an ideal theoretical framework to study the effects of heavy quark masses. This scheme combines the virtues of the ZM-VFN and FFN schemes and avoids their flaws, at the same time. Through this elaborate scheme the perturbative fragmentation functions enter the formalism via subtraction terms for the hard scattering decay rates, so that the actual FFs are truly nonperturbative. They may be assumed to have some smooth forms which can be specified through global data fits. In contrast to the FFNs, the GM-VFNs do also accommodate FFs for gluons and light flavor quarks, as in the ZM-VFN scheme. In our present study, the GM-VFNs is applied to resum the large logarithms in m_b and to retain the entire nonlogarithmic m_b dependence at the same time. This is reached by introducing suitable

subtraction terms in the $\mathcal{O}(\alpha_s)$ FFN expressions for $d\tilde{\Gamma}_i/dx_i$, so that the $\mathcal{O}(\alpha_s)$ ZM-VFN results are exactly recovered in the limit $R \rightarrow 0 (\equiv m_b/m_H \rightarrow 0)$. These subtraction terms are universal quantities, and so are the FFs in the FFN scheme, as is guaranteed by Collin's hard-scattering factorization theorem [17]. The GM-VFN results for the decay distributions are obtained by matching the FFN results [(16), (20)] to the ZM-VFN ones by subtraction as

$$\frac{1}{\Gamma_0} \frac{d\Gamma_i}{dx_i} \Big|_{\text{GM-VFN}} = \frac{1}{\tilde{\Gamma}_0} \frac{d\tilde{\Gamma}_i}{dx_i} \Big|_{\text{FFN}} - \frac{1}{\Gamma_0} \frac{d\Gamma_i}{dx_i} \Big|_{\text{Sub}}, \quad (21)$$

where the subtraction terms are constructed as

$$\frac{1}{\Gamma_0} \frac{d\Gamma_i}{dx_i} \Big|_{\text{Sub}} = \lim_{m_b \rightarrow 0} \frac{1}{\tilde{\Gamma}_0} \frac{d\tilde{\Gamma}_i}{dx_i} \Big|_{\text{FFN}} - \frac{1}{\hat{\Gamma}_0} \frac{d\hat{\Gamma}_i}{dx_i} \Big|_{\text{ZM-VFN}}. \quad (22)$$

Analytical results for the terms $1/\hat{\Gamma}_0 \times d\hat{\Gamma}_i^{\text{NLO}}/dx_i (i = b, g)$ at the ZM-VFN scheme are given in our previous work [16]. Here, we just quote them as

$$\begin{aligned} \frac{1}{\hat{\Gamma}_0} \frac{d\hat{\Gamma}_b}{dx_b} \Big|_{\text{ZM-VFN}} &= \delta(1-x_b) + \frac{C_F \alpha_s(\mu)}{2\pi} \left\{ \delta(1-x_b) \left(\frac{3}{2} + \frac{2\pi^2}{3} - \frac{3}{2} \ln \frac{\mu^2}{m_H^2} \right) \right. \\ &\quad - \frac{1}{(1-x_b)_+} \left(\frac{3}{2} + (1+x_b^2) \ln \frac{\mu^2}{m_H^2} \right) + 2 \left(\frac{\ln(1-x_b)}{1-x_b} \right)_+ \\ &\quad \left. + \frac{5+x_b}{2} + 2 \left(\frac{1+x_b^2}{1-x_b} \right) \ln x_b - (1+x_b) \ln(1-x_b) \right\}, \end{aligned} \quad (23)$$

and

$$\begin{aligned} \frac{1}{\hat{\Gamma}_0} \frac{d\hat{\Gamma}_g}{dx_g} \Big|_{\text{ZM-VFN}} &= \frac{C_F \alpha_s(\mu)}{2\pi} \left\{ \frac{2(1+(1-x_g)^2)}{x_g} \right\} \left(\frac{x_g^2}{1+(1-x_g)^2} \right. \\ &\quad \left. + \ln(1-x_g) + 2 \ln x_g - \ln \frac{\mu^2}{m_H^2} \right), \end{aligned} \quad (24)$$

where $\hat{\Gamma}_0 = N_c m_H^3 R / (8\pi v^2)$ is the leading order (LO) decay rate in the massless scheme. Taking the limit $m_b \rightarrow 0 (\equiv R \rightarrow 0)$ in Eqs. (16) and (20), we recover Eqs. (23) and (24) up to the terms

$$\begin{aligned} \frac{1}{\Gamma_0} \frac{d\Gamma_i}{dx_i} \Big|_{\text{Sub}} &= \frac{\alpha_s(\mu)}{2\pi} C_F \left\{ \frac{1+x_b^2}{1-x_b} \left[\ln \frac{\mu^2}{m_b^2} - 2 \ln(1-x_b) - 1 \right] \right\}_+, \end{aligned} \quad (25)$$

and

$$\left. \frac{1}{\Gamma_0} \frac{d\Gamma}{dx_g} \right|_{\text{Sub}} = \frac{\alpha_s(\mu)}{2\pi} C_F \frac{2 + 2(1-x_g)^2}{x_g} \left(\ln \frac{\mu^2}{m_b^2} - 2 \ln x_g - 1 \right). \quad (26)$$

As is shown in Ref. [32], for the top quark decay in the SM, i.e., $t \rightarrow bW^+$, Eq. (25) coincides with the perturbative fragmentation function of the transition $b \rightarrow b$. This is in consistency with the Collin's factorization theorem which guarantees that the subtraction terms are universal; see also Refs. [20,35–39].

III. HADRON MASS EFFECTS

Our main purpose is to evaluate the scaled-energy (x_B) distribution of B mesons inclusively produced in the SM Higgs decays at NLO. Here, the scaled-energy fraction of B mesons is defined as $x_B = 2p_B \cdot p_H / p_H^2$ which is simplified as $x_B = 2E_B / m_H$ in the Higgs rest frame.

According to the Collin's factorization theorem of the QCD-improved parton model [17], the energy distribution of B mesons is expressed as the convolution of hard scattering decay rates at the parton level, i.e., $d\hat{\Gamma}_a/dx_a (a = b, g)$, with the nonperturbative FFs $D_{a=b,g}^B(z, \mu)$ which describe the transitions $(b, g) \rightarrow B$, i.e.,

$$\frac{d\Gamma}{dx_B}(H \rightarrow BX) = \sum_{a=b,g} \int_{x_a^{\min}}^{x_a^{\max}} \frac{dx_a}{x_a} \frac{d\hat{\Gamma}_a}{dx_a}(x_a, \mu) D_a^B(z, \mu), \quad (27)$$

where $z = x_B/x_a \equiv E_B/E_a$ indicates the energy fraction of partons which is carried away by the B mesons. In Ref. [32], where the process of top quark decays for production of bottom-flavored mesons is studied, i.e., $t \rightarrow bW^+ (+g) \rightarrow B + \text{Jets}$, authors have demonstrated that the above relation is suitable for the massless cases (with $m_b = 0$ and $m_B = 0$). To calculate the quantity $d\Gamma/dx_B$ when passing from the ZM-VFN scheme to the GM-VFN scheme by taking into account the finite- m_B corrections, one should apply the following improved factorization theorem:

$$\begin{aligned} \frac{d\Gamma}{dx_B}(H \rightarrow BX) \\ = \frac{1}{\sqrt{x_B^2 - \rho_B^2}} \sum_{a=b,g} \int_{x_a^{\min}}^{x_a^{\max}} dx_a z \left. \frac{d\Gamma_a}{dx_a} \right|_{\text{GM-VFN}} D_a^B(z, \mu), \end{aligned} \quad (28)$$

where $(d\Gamma_a/dx_a)_{\text{GM-VFN}}$ is obtained via the approach defined in (21). Moreover, the energy fraction z reads as

$$z = \frac{x_B + \sqrt{x_B^2 - \rho_B^2}}{x_a + \sqrt{x_a^2 - \rho_a^2}}, \quad (29)$$

where $\rho_i = m_i/E_b^{\max} = 2m_i/m_H (i = b, g, B)$. Now, the kinematically allowed scaling-variables are

$$\begin{aligned} \frac{1}{2} \left(x_B + \sqrt{x_B^2 - \rho_B^2} + \frac{\rho_b^2}{x_B + \sqrt{x_B^2 - \rho_B^2}} \right) &\leq x_b \leq 1, \\ \frac{1}{2} \left(x_B + \sqrt{x_B^2 - \rho_B^2} \right) &\leq x_g \leq 1 - 4R, \\ \rho_B &\leq x_B \leq 1 - R + \left(\frac{m_B}{m_H} \right)^2 \quad \text{for b-quark transition,} \\ \rho_B &\leq x_B \leq 1 - 4R + \left(\frac{m_B}{m_H} \right)^2 \quad \text{for gluon transition.} \end{aligned} \quad (30)$$

Clearly, if one sets $m_b = 0$ and $m_B = 0$ ($z \rightarrow x_B/x_a$), then Eqs. (27) and (28) coincide by reproducing the familiar factorization formula of the massless parton model.

IV. NUMERICAL ANALYSIS

To perform our phenomenological analysis for the energy spectrum of B mesons inclusively produced in Higgs decays we adopt the following input parameters from Ref. [40]:

$$\begin{aligned} G_F &= 1.16637 \times 10^{-5} \text{ GeV}^{-2}, \\ m_H &= 125.3 \text{ GeV}, \\ m_b &= 4.78 \text{ GeV}, \\ m_B &= 5.279 \text{ GeV}. \end{aligned} \quad (31)$$

We evaluate the strong coupling constant at NLO using the following relation [40]:

$$\alpha_s^{(n_f)}(\mu^2) = \frac{1}{b_0 \log(\mu^2/\Lambda^2)} \left\{ 1 - \frac{b_1 \log[\log(\mu^2/\Lambda^2)]}{b_0^2 \log(\mu^2/\Lambda^2)} \right\}, \quad (32)$$

where n_f is the number of active quark flavors, and b_0 and b_1 are given by

$$b_0 = \frac{33 - 2n_f}{12\pi}, \quad b_1 = \frac{153 - 19n_f}{24\pi^2}. \quad (33)$$

In Eq. (32), Λ is the asymptotic scale parameter for which we adopt $\Lambda_{\overline{\text{MS}}}^{(5)} = 231.0 \text{ MeV}$ adjusted such that $\alpha_s^{(5)} = 0.1184$ for $m_Z = 91.1876 \text{ GeV}$ [40]. Applying this equation one has $\alpha_s(m_H^2) = 0.113$.

According to the factorization theorems in the massless and massive schemes, i.e., Eqs. (27) and (28), we also need the nonperturbative fragmentation functions $D_{a=b,g}^B(z, \mu)$ describing the hadronization processes $(b, g) \rightarrow B$. These functions indicate the probability density of meson production from initial partons at the scale μ . For these

nonperturbative transitions, we use the realistic nonperturbative FFs determined at NLO through an overall fit to pair annihilation data presented by ALEPH [41] and OPAL [42] at CERN LEP1 and by SLD [43] at SLAC SLC. Basically, in comparison with the hadron collisions, the e^+e^- annihilation process provides a clean environment to determine the non-pFFs because, firstly the most precise data exist for these processes, and secondly, one does not deal with the uncertainties induced by the parton distribution functions. In the literature, various phenomenological models have been offered to describe the hadronization mechanism. Among all models the simple power model as $D_b^B(z, \mu^{\text{ini}}) = N z^\alpha (1-z)^\beta$ is well established. In Ref. [44], authors have extracted the free parameters of this model, i.e., N , α , and β , for the fragmentation of all light and heavy flavors as well as gluon into B mesons at the initial scale $\mu^{\text{ini}} = 4.5$ GeV. Their fit for the $b \rightarrow B$ splitting yielded $N = 2575.014$, $\alpha = 15.424$, and $\beta = 2.394$. Furthermore, the gluon fragmentation function is generally assumed to be zero at the initial scale and is generated via the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi evolution equations [18]

$$\frac{d}{d \ln \mu^2} D_i^B(z, \mu) = \frac{\alpha_s(\mu)}{2\pi} \sum_j \int_z^1 \frac{dy}{y} P_{ji}(y, \alpha_s(\mu)) D_j^B\left(\frac{z}{y}, \mu\right), \quad (34)$$

where P_{ji} are the well-known splitting functions.

Assuming $\mu = m_H$ and $m_B = 0$, in Fig. 2 we study the NLO energy distribution of B mesons in Higgs decays using the GM-VFN scheme (with $m_b \neq 0$). For this aim, we consider the normalized rate $(1/\tilde{\Gamma}_0)(d\Gamma/dx_B)$ where $\tilde{\Gamma}_0$ is given in Eq. (1). For this prediction, we investigate the size

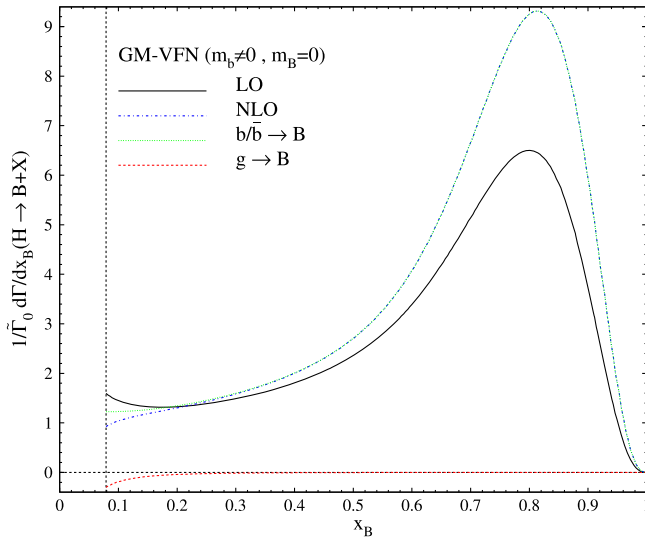


FIG. 2. $1/\tilde{\Gamma}_0 \times d\Gamma(H \rightarrow B + X)/dx_B$ as a function of x_B in the GM-VFN scheme. The NLO result (blue dot-dashed line) is compared to the LO one (black solid line) and broken up into the contributions due to $b \rightarrow B$ (green dotted line) and $g \rightarrow B$ (red dashed line) fragmentation. Here we set $m_B = 0$.

of NLO corrections by comparing the LO (black solid line) and NLO (blue dot-dashed line) results, and the relative importance of the $b \rightarrow B$ (green dotted line) and $g \rightarrow B$ (red dashed line) fragmentation channels at NLO. To expose the size of NLO radiative corrections at the parton level we evaluated the LO result using the same NLO FFs. We see that the NLO improvements lead to an important enhancement of partial decay width in the peak region ($x_B \approx 0.8$) and above by as much as 46%, at the expense of a reduction in the lower- x_B range. Moreover, the peak position of energy spectrum is shifted toward higher amounts of x_B . It is also observed that the contribution of $g \rightarrow B$ is throughout negative and appreciable only in the low- x_B region ($x_B < 0.3$). For higher- x_B the NLO result is practically exhausted by the contribution of $b \rightarrow B$, as we expect. Furthermore, the finite- m_b corrections are responsible for the appearance of a threshold.

To study the mass effects of b quark and B meson, separately, in Fig. 3 we study the energy spectrum of B mesons in three cases: (a) full massless or ZM-VFN scheme where one sets $m_b = 0 = m_B$ (black solid line), (b) massive or GM-VFN scheme (blue dot-dashed line), and (c) GM-VFN scheme with finite- m_B corrections (red dashed curve). It is observed that the finite- m_B corrections are responsible for the appearance of a threshold at $x_B = 0.093$. The finite mass corrections, generally, lead to an enhancement in size throughout the whole x_B range allowed and, specifically the finite- m_B corrections lead to the considerable enhancement in the region $0.093 \leq x_B \leq 0.25$. Enhancements in other regions ($0.25 \leq x_B \leq 1$) are due to the finite- m_b correction which is more important than the effect of B meson mass.

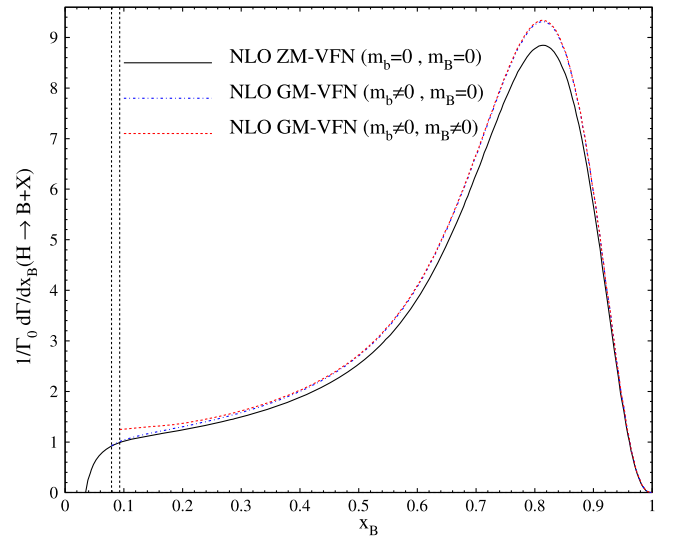


FIG. 3. $1/\Gamma_0 \times d\Gamma(H \rightarrow B + X)/dx_B$ as a function of x_B at NLO. The GM-VFN ($m_b \neq 0$) results with (red dashed) and without finite- m_B corrections (blue dot-dashed) are compared to the ZM-VFN ($m_b = 0$) result for $m_B = 0$ (black solid line).

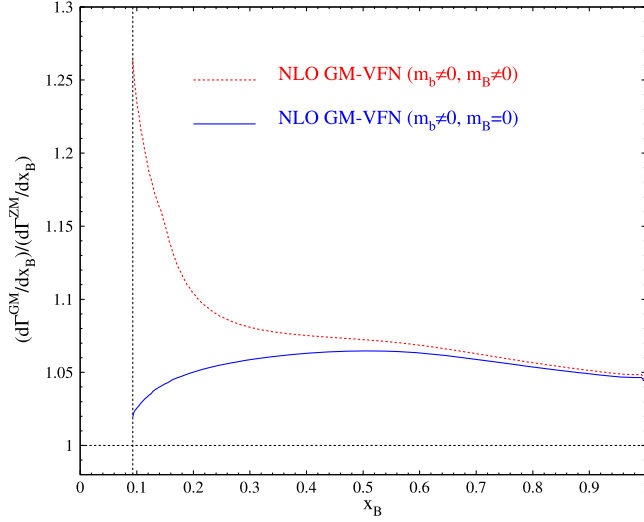


FIG. 4. $1/\Gamma_0 \times d\Gamma(H \rightarrow B + X)/dx_B$ as a function of x_B at NLO in the GM-VFN ($m_b \neq 0$) scheme with (red dashed line) and without (blue solid line) finite- m_B corrections. Both results are normalized to the ZM-VFN result ($m_b = 0$) for $m_B = 0$. Here, we set $\mu = m_H$.

For a more quantitative interpretation of Fig. 3, we consider in Fig. 4 the GM-VFN results with (red dashed line) and without (blue solid line) finite- m_B corrections both normalized to the full ZM-VFN result for $m_B = 0$. We observe that the inclusion of finite- m_B and m_b corrections leads to an enhancement of the partial decay width in size throughout the whole x_B range allowed. Specifically, the finite- m_B corrections leads to a significant enhancement of the partial decay width in the lower- x_B range so that it increases the size of partial decay rate about 27% at the

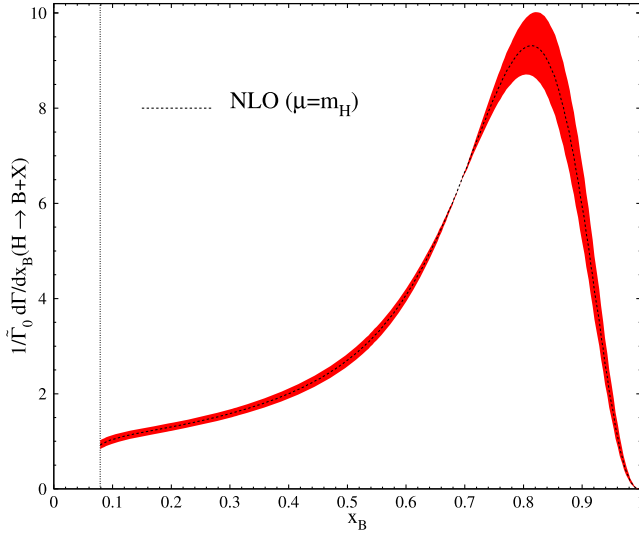


FIG. 5. $1/\tilde{\Gamma}_0 \times d\Gamma(H \rightarrow B + X)/dx_B$ as a function of x_B at NLO in the GM-VFN scheme considering various values for the scale μ , i.e., $m_H/2 \leq \mu \leq 2m_H$. The result for $\mu = m_H$ is also shown (black dashed line).

threshold. For higher values of x_B , the m_b effect is governed. For example, the size of partial decay width increases about 6% at $x_B \approx 0.55$.

As was previously mentioned, in Eqs. (27) and (28) we can use different values for the scale μ ; however, in most literature to avoid large logarithms $\ln(\mu^2/m_H^2)$ in partonic differential decay rates a choice often made consists of setting $\mu = m_H$ (as we assumed in Figs. 2–4). Here, for studying the variation effect of the scale μ , in Fig. 5 we consider different choices for that as $m_H/2 \leq \mu \leq 2m_H$. For comparison, the result for $\mu = m_H$ is also shown (black dashed line). As is seen, the effect of scale variation is considerable in the peak region and above.

The formalism elaborated here is also usable for production of hadron species other than B mesons, e.g., protons, pions, D mesons, etc., through Higgs decay. The corresponding FFs $(b, g) \rightarrow \pi/P/D^+$ are given in Refs. [45–48].

V. SUMMARY AND CONCLUSIONS

Since the discovery of Higgs boson in 2012 at the CERN-LHC, a lot of knowledge has been learned about this fundamental particle. However, it is still required to check whether its characteristics, like couplings, are completely consistent with those predicted by the SM so that any deviation from the SM predictions may be a signal of new effects. Its weak Yukawa coupling to the light fermions leaves space for theories of new physics beyond the SM and has attracted a lot of interest. The high-luminosity LHC as well as the high-energy LHC will allow physicists to perform precise measurements that are impossible otherwise. This motivates particle physicists to make a more detailed study of Higgs bosons, specifically via their decay processes. Over 80% of Higgs boson decays are fully hadronic, of which around 60% decay into bottom quark pairs ($H \rightarrow b\bar{b}$). Bottom quarks hadronize before they decay and produce, in most cases, bottom-flavored hadrons (B mesons). Therefore, in the present work we studied the process of B meson production in the decays of Higgs bosons at NLO pQCD, i.e., $H \rightarrow b\bar{b}(+g) \rightarrow B + \text{Jets}$. For this study we evaluated the distribution in the scaled-energy of B mesons which would be of particular interest at the LHC and future colliders. In all previous works, for simplicity, the masses of bottom quarks as well as produced B mesons have been ignored. In this work we, for the first time, preserved the bottom quark mass by working in the GM-VFN scheme and computed an analytic expression for the NLO radiative corrections to the differential decay width of Higgs bosons at the parton level, i.e., $d\tilde{\Gamma}_i^{\text{NLO}}(H \rightarrow \bar{b}b(+g))/dx_i (i = b, g)$ where $x_i = 2E_i/m_H$ is the scaled energy of partons in the Higgs rest frame. In the following, using the nonperturbative FFs of $(b, g) \rightarrow B$ the x_B spectrum is predicted. We also analyzed the size of finite- m_B effects on the energy spectrum as well as the theoretical uncertainty due to the freedom in the choice of

factorization scale μ . Our results show that finite masses are responsible for the low- x_B threshold, and a significant enhancement of the partial decay rate occurs in the low- x_B due to m_B -finite mass while the b-quark mass leads to enhancements of the partial decay width in the peak region and above. The achievements of this research are suitable for more accurate studies of Higgs bosons in the high-energy and high-luminosity colliders such as the LHC and future colliders like FCC-ee and ILC.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [40].

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