

Three-family supersymmetric Pati-Salam models from intersecting D6-branes on rigid cycles

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Intersecting D6-brane models without discrete torsion typically suffer from unstabilized open-string moduli arising from D-brane positions and Wilson lines. These moduli generate additional massless adjoint fields, obstructing the realization of negative β functions necessary for asymptotic freedom unless they are decoupled around the string scale. A viable solution involves utilizing rigid cycles, which eliminate these unwanted adjoint fields. In this work, we for the first time present a class of consistent three-family supersymmetric Pati-Salam models from rigid intersecting D6-branes on the factorizable $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}'_2)$ orientifold with discrete torsion. These models satisfy all the known consistency conditions, including $\mathcal{N} = 1$ supersymmetry, K-theory constraints, tadpole cancellation, and recent swampland bounds on the maximal gauge group rank. We provide detailed particle spectra, analyze their phenomenological implications, and discuss the decoupling of exotic states through strong dynamics in the hidden sector.

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I. INTRODUCTION

A central challenge in string phenomenology is the construction of realistic string vacua that yield low-energy supersymmetric Standard Models (SMs) free from exotic particles while simultaneously stabilizing all the moduli fields. Such models can be connected to the observable particle physics through renormalization group equations evolution, allowing for experimental probes at the Large Hadron Collider and future collider experiments. Significant progress has been made in this direction within type IIA string theory compactified on the $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}'_2)$ orientifold whereby the semirealistic supersymmetric Standard-like Models have been extensively developed [1–22]. Among these constructions, Pati-Salam-like models are particularly compelling, as they can naturally

accommodate all the Yukawa couplings [4]. In fact, the first systematic realization of three-family supersymmetric Pati-Salam models was presented in [4]. These models can also be constructed in the heterotic string framework [23,24]. The SM fermion masses and mixings have been explicitly explained in certain intersecting D-brane models [14,19]. However, a common feature of these semirealistic constructions is the presence of three adjoint chiral multiplets for each $U(N)$ gauge symmetry, which arises due to the nonrigidity of the D6-branes. On the other hand, while models based on rigid intersecting D6-branes eliminate such adjoint moduli, they remain far from realistic in terms of reproducing the full chiral and Higgs spectrum [25–32]. Therefore, constructing three-family $\mathcal{N} = 1$ supersymmetric Pati-Salam models using rigid intersecting D6-branes continues to be a major challenge in string model building.

In Ref. [25], it was first demonstrated that global $\mathcal{N} = 1$ chiral models can be constructed using rigid intersecting D6-branes on a factorizable $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}'_2)$ orientifold with discrete torsion. However, engineering consistent three-family models in this setup with factorizable tori has proven to be particularly challenging. In the case of nonrigid branes, we need at least one torus to be tilted to

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generate three-generation models on the mirror orientifold $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ [1–20,22]. In the case of rigid three-cycles, however, no consistent three-family models incorporating tilted tori have been reported. A notable example of a three-generation model based on rigid D6-branes was presented in Ref. [26], but it was realized on nonfactorizable tori. The corresponding model with factorizable tori fails to satisfy the $\mathcal{N} = 1$ supersymmetry conditions. Moreover, that construction featured a significantly enlarged gauge group that exceeded phenomenological expectations.

Recent insights from the swampland program [33] impose additional constraints on string model building by bounding the maximal rank of the gauge group in consistent quantum gravity theories. For theories with 32 supercharges, supersymmetry is strong enough to completely fix all possible configurations. For the next class, with 16 supercharges, it was shown in [34] that in $d = 10$ and in the nonchiral cases, the possibilities are finite because the rank of the gauge group must satisfy $r(G) \leq 26 - d$. For six-dimensional theories with eight supercharges, it was established recently in [35] that the number of tensor multiplets is bounded by $T \leq 193$, and the rank of the gauge group is constrained by $r(V) \leq 480$. However, for lesser supersymmetry, say, for $\mathcal{N} = 1$ Pati-Salam models, the maximal rank was found to be 138 in [36]. For reviews on the swampland program, please see [33,37–40].

In this paper, we for the first time construct a class of consistent three-family $\mathcal{N} = 1$ supersymmetric Pati-Salam models with factorizable tori based on rigid intersecting D6-branes on a factorizable $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}'_2)$ orientifold with discrete torsion.¹ This background naturally accommodates the rigid three-cycles that can be wrapped by D6-branes, enabling the construction of chiral models in which moduli associated with D-brane positions are absent or, more precisely, such would-be moduli acquire masses at the string scale. The construction is particularly tractable, as all three two-tori are taken to be rectangular, with each containing four fixed points. Consequently, rigid D6-branes are restricted to pass through specific pairs of fixed points on each two-torus. A notable feature of this setup is the absence of adjoint matter, which is highly advantageous for realistic model building, as it eliminates unwanted scalar fields.

The construction utilizes semirigid and nonrigid hidden sector branes to ensure the complete cancellation of twisted and untwisted Ramond-Ramond (RR) tadpoles while simultaneously maintaining an odd number of chiral families. The presence of the massless Higgs pairs for the Pati-Salam gauge symmetry breaking is achieved through the controlled recombination of hidden sector branes with the stack supporting the $SU(2)_R$ gauge symmetry, leading to realistic symmetry-breaking patterns without introducing exotic chiral matters.

¹A representative example from this class is discussed in our companion letter [41].

The paper is organized as follows. In Sec. II, we review the model-building rules used to construct three-family Pati-Salam models from configurations of intersecting rigid D6-branes. We also revise the calculation of intersection numbers in this setting, along with the associated consistency conditions arising from $\mathcal{N} = 1$ supersymmetry, K-theory constraints, and the cancellation of RR tadpoles. In Sec. III, after outlining the model-building strategy, we present a number of consistent three-family models with detailed particle spectra and discuss the phenomenological aspects of these models, focusing particularly on the decoupling of exotic particles from the low-energy spectrum via strong dynamics. In Sec. IV, we discuss the realization of asymptotic freedom in all models by evaluating the β functions, which provides yet another consistency check on the viability of the constructed models. Finally, we conclude in Sec. V.

II. PATI-SALAM MODEL BUILDING ON RIGID CYCLES

We consider the type IIA $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}'_2)$ orientifold with the *discrete torsion* turned on. Here, $\mathbb{T}^6$ is a product of three two-tori, with the orbifold group $(\mathbb{Z}_2 \times \mathbb{Z}'_2)$ having the generators θ and ω , respectively, associated with the twist vectors $(1/2, -1/2, 0)$ and $(0, 1/2, -1/2)$ such that their action on complex coordinates z_i is given by

$$\begin{aligned}\theta: (z_1, z_2, z_3) &\rightarrow (-z_1, -z_2, z_3), \\ \omega: (z_1, z_2, z_3) &\rightarrow (z_1, -z_2, -z_3).\end{aligned}\quad (1)$$

Orientifold projection is the gauged $\Omega\mathcal{R}$ symmetry, where Ω is world-sheet parity that interchanges the left- and right-moving sectors of a closed string and swaps the two ends of an open string, while $\mathcal{R}$ acts as complex conjugation on coordinates z^I as

$$\begin{aligned}\Omega: (\sigma_1, \sigma_2) &\mapsto (2\pi - \sigma_1, \sigma_2) \quad (\text{closed}) \\ (\tau, \sigma) &\mapsto (\tau, \pi - \sigma) \quad (\text{open}), \\ \mathcal{R}: z_i &\mapsto \bar{z}_i.\end{aligned}\quad (2)$$

This results in four different kinds of orientifold 6-planes (O6-planes), corresponding to $\Omega\mathcal{R}$, $\Omega\mathcal{R}\theta$, $\Omega\mathcal{R}\omega$, and $\Omega\mathcal{R}\theta\omega$, respectively. These orientifold projections are only consistent with either the rectangular or the tilted complex structures of the factorized two-tori. The wrapping numbers for the rectangular and tilted tori are denoted as $n_a^I[a^I] + m_a^I[b^I]$ and $n_a^I[a^{II}] + m_a^I[b^I]$, respectively, where $[a^{II}] = [a^I] + \frac{1}{2}[b^I]$. Then a generic one-cycle (n_a^I, l_a^I) satisfies $l_a^I \equiv m_a^I$ for the rectangular two-torus and $l_a^I \equiv 2\tilde{m}_a^I = 2m_a^I + n_a^I$ for the tilted two-torus such that $l_a^I - n_a^I$ is even for the tilted tori. The two different bases (n^i, m^i) and (n^i, l^i) are related as

$$l^i = 2^{\beta_i} \left(m^i + \frac{\beta_i}{2} n^i \right), \quad \beta_i = \begin{cases} 0, & \text{rectangular } \mathbb{T}^2, \\ 1, & \text{tilted } \mathbb{T}^2. \end{cases} \quad (3)$$

In contrast to the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold without discrete torsion, where all O6-planes are of type O6^(-,-), the inclusion of discrete torsion ($\eta = -1$) imposes consistency conditions on the cross-cap states requiring the presence of an odd number of exotic O6^(+,+)-planes. This condition arises from the compatibility of the relations

$$\begin{aligned} \langle \Omega \mathcal{R} | e^{-l^i \mathcal{H}_{cl}} | \Omega \mathcal{R} \omega \rangle &= \text{Tr}_\omega (\Omega \mathcal{R} e^{-2\pi i H}), \\ \langle \Omega \mathcal{R} \theta | e^{-l^i \mathcal{H}_{cl}} | \Omega \mathcal{R} \theta \omega \rangle &= \text{Tr}_\omega (\Omega \mathcal{R} \theta e^{-2\pi i H}), \end{aligned} \quad (4)$$

which, along with the choice of discrete torsion $\eta = \pm 1$ in the closed-string sector, enforces the constraint

$$\eta_{\Omega \mathcal{R}} \eta_{\Omega \mathcal{R} \theta} \eta_{\Omega \mathcal{R} \omega} \eta_{\Omega \mathcal{R} \theta \omega} = \eta. \quad (5)$$

Clearly, the choice of discrete torsion, $\eta = -1$, requires an odd number of O6^(+,+)-planes [25].

The discrete torsion gives rise to three $\mathbb{Z}_2$ -twisted sectors with 16 fixed points each. The three-cycles of $\mathbb{T}^6$ will be inherited by the orbifold quotient. However, in order to deal with three-cycles on orbifold spaces, we have to carefully distinguish between three-cycles on the covering space and three-cycles on the actual orbifold. In the case at hand, under the action of $\mathbb{Z}_2 \times \mathbb{Z}_2'$, a factorizable three-cycle on $\mathbb{T}^6$ has three images, all of them with the same wrapping numbers as the initial three-cycle. Therefore, a three-cycle in the bulk of the orbifold space can be identified with $[\Pi_a^B] = 4[\Pi_a^{\mathbb{T}^6}]$. Computing the intersection number, we get

$$\begin{aligned} [\Pi_a^B] \circ [\Pi_b^B] &= 4[\Pi_a^{\mathbb{T}^6}] \circ [\Pi_b^{\mathbb{T}^6}] = 4 \prod_{l=1}^3 (n_a^l \tilde{m}_b^l - \tilde{m}_a^l n_b^l) \\ &= 4 \cdot 2^{-k} \prod_{l=1}^3 (n_a^l l_b^l - l_a^l n_b^l), \\ \therefore k &= \sum_{l=1}^3 \beta^l, \end{aligned} \quad (6)$$

where we have identified the intersection points related by the $\mathbb{Z}_2 \times \mathbb{Z}_2'$ action.

In addition to the untwisted cycles, we have 32 independent collapsed three-cycles for each of the three twisted sectors, θ , ω , and $\theta\omega$. Let us first consider the θ -twisted sector. We denote the 16 fixed points on $(\mathbb{T}_1^2 \times \mathbb{T}_2^2)/\mathbb{Z}_2$ by $[e_{ij}^\theta]$, with $i, j \in \{1, 2, 3, 4\}$. After blowing up the orbifold singularities, these become two-cycles with the topology of $\mathbb{S}^2$. Each such four-dimensional $\mathbb{T}^4/\mathbb{Z}_2$ is an orbifold of $\mathbb{K}3$ before the other elements of the orientifold group are taken into account.

With our choice of discrete torsion, these two-cycles are combined with a one-cycle $(n^3, \tilde{m}^3)$ of $\mathbb{T}_3^2$ in order to form a three-cycle in the θ -twisted sector. Let us denote a basis of such twisted three-cycles as

$$[\alpha_{ij,n}^\theta] = 2[e_{ij}^\theta] \otimes [a^3], \quad [\alpha_{ij,m}^\theta] = 2[e_{ij}^\theta] \otimes [b^3], \quad (7)$$

where the factor of 2 is due to the action of the second $\mathbb{Z}_2$. Analogously, we define the basic twisted three-cycles in the ω and $\theta\omega$ twisted sectors as

$$\begin{aligned} [\alpha_{ij,n}^\omega] &= 2[e_{ij}^\omega] \otimes [a^1], & [\alpha_{ij,m}^\omega] &= 2[e_{ij}^\omega] \otimes [b^1], \\ [\alpha_{ij,n}^{\theta\omega}] &= 2[e_{ij}^{\theta\omega}] \otimes [a^2], & [\alpha_{ij,m}^{\theta\omega}] &= 2[e_{ij}^{\theta\omega}] \otimes [b^2]. \end{aligned} \quad (8)$$

The intersection number between a pair of such cycles is easy to compute knowing that the collapsed two-cycles of the $\mathbb{K}3$ -orbifold have self-intersection number $[e_{ij}] \circ [e_{kl}] = -2\delta_{ik}\delta_{jl}$ in each twisted sector, and that two-cycles of different twisted sectors do not intersect.

For the three-cycles $[\Pi_{ij,a}^g] = n_a^{I_g} [\alpha_{ij,n}] + \tilde{m}_a^{I_g} [\alpha_{ij,m}]$ and $[\Pi_{kl,b}^h] = n_b^{I_h} [\alpha_{kl,n}] + \tilde{m}_b^{I_h} [\alpha_{kl,m}]$, with $g, h = \theta, \omega, \theta\omega$, we find that

$$\begin{aligned} [\Pi_{ij,a}^g] \circ [\Pi_{kl,b}^h] &= 4\delta_{ik}\delta_{jl}\delta^{gh} \left(n_a^{I_g} \tilde{m}_b^{I_g} - \tilde{m}_a^{I_g} n_b^{I_g} \right) \\ &= 4\delta_{ik}\delta_{jl}\delta^{gh} 2^{-\beta^g} \left(n_a^{I_g} l_b^{I_g} - l_a^{I_g} n_b^{I_g} \right), \\ \therefore \tilde{m}_a^{I_g} &\equiv 2^{-\beta^g} l_a^{I_g}. \end{aligned} \quad (9)$$

In this notation, for the sectors twisted by $g = \theta, \omega, \theta\omega$, one has $I_g = 3, 1, 2$, respectively.

To build rigid D6-branes, one considers fractional D6-branes charged under all three twisted sectors of the orbifold. Let us start with a factorizable three-cycle described by three pairs of wrapping numbers $(n_a^l, \tilde{m}_a^l)$. A fractional D6-brane should be invariant under the orbifold action, and hence it must run through four fixed points for each twisted sector. We denote such a set of four fixed points of the element g [each labeled by a pair (i, j)] as S_g^a . It is easy to characterize the sets of pairs of fixed points S_g^a in terms of the wrapping numbers (n, m) as shown in Table I.

Then the entire three-cycle where such a fractional D-brane is wrapping is of the form

TABLE I. Fixed points of a one-cycle on a $\mathbb{T}^2/\mathbb{Z}_2$ in terms of its wrapping numbers.

(n_a, m_a)	Fixed points S_g^a
(Odd, odd)	$\{1, 4\}$ or $\{2, 3\}$
(Odd, even)	$\{1, 3\}$ or $\{2, 4\}$
(Even, odd)	$\{1, 2\}$ or $\{3, 4\}$

$$\begin{aligned} \Pi_a = & \frac{1}{4} \Pi_a^B + \frac{1}{4} \sum_{(i,j) \in S_\theta^g} \epsilon_{a,ij}^\theta \Pi_{ij,a}^\theta + \frac{1}{4} \sum_{(j,k) \in S_\omega^g} \epsilon_{a,jk}^\omega \Pi_{jk,a}^\omega \\ & + \frac{1}{4} \sum_{(i,k) \in S_{\theta\omega}^g} \epsilon_{a,ik}^{\theta\omega} \Pi_{ik,a}^{\theta\omega}, \end{aligned} \quad (10)$$

where the signs $\epsilon_{a,ij}^\theta$, $\epsilon_{a,jk}^\omega$, and $\epsilon_{a,ik}^{\theta\omega} = \pm 1$ define the charge of the fractional brane a with respect to the massless fields living at the various fixed points. Geometrically, these numbers indicate the two possible orientations with which the brane can wrap around the blown-up $\mathbb{S}^2$. Clearly, only those fixed points appear in (10), which the D6-brane is passing through. Since the brane is stuck at the orbifold fixed points in all three $\mathbb{T}_7^2$, there are no adjoint scalars appearing in the massless spectrum. It is necessary to use m (and not l) to calculate the fixed points in the standard notation of (10), while $\tilde{m}$ (or, equivalently, $2^{-\beta}l$) is useful for the calculation of intersection numbers, for tadpole cancellation, and for the supersymmetry conditions [42].

A. Spectrum from rigid branes ($\eta = -1$)

The details of the orientifold action prove quite important in order to work out the massless spectrum of the low-energy theory. In particular, they are essential in order to compute the chiral matter content, which, while still described in terms of intersection numbers between three-cycles, now includes chiral fermions transformed in the symmetric and antisymmetric representations of $U(N)$ gauge groups. For simplicity, let us first consider D6-branes wrapping three-cycles not invariant under $\mathcal{R}$ so that the gauge group is of the form $\prod_a U(N_a)$. In this case, we can apply a general rule for determining the massless left-handed chiral spectrum in terms of three-cycle intersection numbers, as presented in Table II.

Given (6) and (9), it is now easy to compute the intersection number between two rigid D6-branes of the form of (10),

$$\begin{aligned} \Pi_a^F \circ \Pi_b^F = & \frac{1}{4} \left(2^{-k} \prod_I (n_a^I l_b^I - l_a^I n_b^I) \right. \\ & \left. + \sum_{g \in \{3,1,2\}} \delta_{ab}^g 2^{-\beta^g} (n_a^g l_b^g - l_a^g n_b^g) \right), \end{aligned} \quad (11)$$

TABLE II. Chiral spectrum for intersecting D6-branes.

Representation	Multiplicity
$(\square_a, \bar{\square}_b)$	$\Pi_a \circ \Pi_b$
$(\square_a, \square_b)$	$\Pi_a \circ \Pi'_b$
$\square_a$	$\frac{1}{2} (\Pi_a \circ \Pi'_a + \Pi_a \circ \Pi_{O6})$
$\bar{\square}_a$	$\frac{1}{2} (\Pi_a \circ \Pi'_a - \Pi_a \circ \Pi_{O6})$

where k is the number of tilted tori and δ_{ab}^g is the number of common fixed points where the branes a and b intersect for each twisted sector $g \in \theta, \omega, \theta\omega$, which can be read from (9) as

$$\delta_{ab}^g \equiv \delta^g(\alpha_{ij}^a, \alpha_{kl}^b) = \delta_{S_i^a, S_k^b}^g \delta_{S_j^a, S_l^b}^g. \quad (12)$$

Assuming that every fractional brane intersects the origin, the above relation simplifies as

$$\delta_{ab}^g = \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 \sum_{l=1}^2 \delta_{S_{1,i}^a, S_{1,k}^b}^g \delta_{S_{2,j}^a, S_{2,l}^b}^g. \quad (13)$$

1. Orientifold action

Let us determine how $\Omega\mathcal{R}$ acts on the various three-cycles. For the untwisted cycles, this is straightforward because the horizontally placed O-planes reflect the vertical axis only,

$$\Omega\mathcal{R}: \begin{cases} [a^I] \rightarrow [a^I], \\ [b^I] \rightarrow -[b^I]. \end{cases} \quad (14)$$

Therefore, the wrapping numbers are mapped as $\Omega\mathcal{R}: (n_a^I, \tilde{m}_a^I) \rightarrow (n_a^I, -\tilde{m}_a^I)$ and $\Omega\mathcal{R}: (n_a^I, m_a^I) \rightarrow (n_a^I, -m_a^I - \beta^I n_a^I)$, respectively.

For the twisted sector three-cycles, the canonical action $\Omega\mathcal{R}$ corresponding to the models without vector structure including the signs $\eta_{\Omega\mathcal{R}}, \eta_{\Omega\mathcal{R}g}$, consistent with (5), reads

$$\Omega\mathcal{R}: \begin{cases} \alpha_{ij,n}^g \rightarrow -\eta_{\Omega\mathcal{R}} \eta_{\Omega\mathcal{R}g} \alpha_{\mathcal{R}(i)\mathcal{R}(j),n}^g, \\ \alpha_{ij,m}^g \rightarrow \eta_{\Omega\mathcal{R}} \eta_{\Omega\mathcal{R}g} \alpha_{\mathcal{R}(i)\mathcal{R}(j),m}^g, \end{cases} \quad (15)$$

where for $\beta^I = 0$ the reflection $\mathcal{R}$ leaves all four fixed points $i \in \{1, 2, 3, 4\}$ invariant, whereas for $\beta^I = 1$ the action $\mathcal{R}$ interchanges the fixed points 3 and 4 while leaving 1 and 2 unchanged,

$$\mathcal{R}: \begin{cases} 1 \rightarrow 1, \\ 2 \rightarrow 2, \\ 3 \rightarrow 3 + \beta^I, \\ 4 \rightarrow 4 - \beta^I, \end{cases} \quad \beta^I \in \{0, 1\}, \quad (16)$$

as can be seen in Fig. 1.

Implementing the $3 \leftrightarrow 4$ interchange (16) coming from the $\Omega\mathcal{R}$ action on the number of common fixed points (12), we get

$$\delta_{a'b}^g \equiv \delta^g(\alpha_{ij}^{a'}, \alpha_{kl}^b) = \delta_{S_i^{a'}, S_k^b}^g \delta_{S_j^{a'}, S_l^b}^g. \quad (17)$$

Again assuming that every fractional brane intersects the origin, the above relation simplifies as

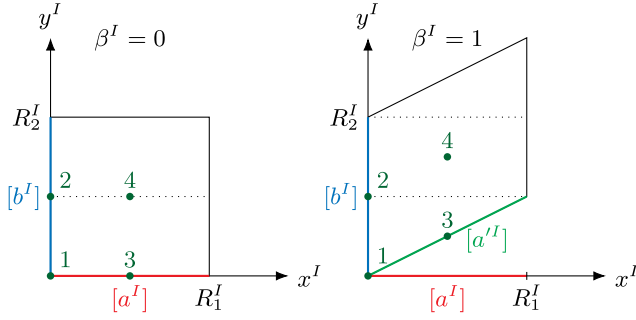


FIG. 1. The $\mathbb{Z}_2$ invariant a -type (left panel) and b -type (right panel) lattices. The $\mathbb{Z}_2$ fixed points $\{1, 2, 3, 4\}$ are shown as dots. The $\mathcal{R}$ invariant x^I axis is along the one-cycle $[a^I] - \frac{\beta^I}{2}[b^I]$, with $\beta^I = 0$ for the a -type lattice and $\beta^I = 1$ for the b -type lattice. $\mathcal{R}$ acts as a reflection along the y^I axis, which is spanned by the one-cycle $[b^I]$. For the a -type lattice, all $\mathbb{Z}_2$ fixed points are invariant under $\mathcal{R}$, whereas for the b -type lattice, only 1 and 2 are invariant, while $3 \leftrightarrow 4$.

$$\delta_{a'b}^g = \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 \sum_{l=1}^2 \delta_{s_{1,i}^a, s_{1,k}^b}^g \delta_{s_{2,j}^a, s_{2,l}^b}^g. \quad (18)$$

When Eqs. (14), (15), and (17) are used, the intersection number of Π_a^F with the $\Omega\mathcal{R}$ image of Π_b^F can be computed as

$$\begin{aligned} \Pi_a^F \circ (\Pi_b^F)' &= \eta_{\Omega\mathcal{R}} \frac{1}{4} \left(-\eta_{\Omega\mathcal{R}} 2^{-k} \prod_I (n_a^I l_b^I + l_a^I n_b^I) \right. \\ &\quad \left. + \sum_{g \in \{\theta, \omega, \theta\omega\}} \eta_{\Omega\mathcal{R}g} \delta_{ab'}^g 2^{-\beta^g} (n_a^g l_b^g + l_a^g n_b^g) \right). \end{aligned} \quad (19)$$

As a special case of (18), setting $a = b$ implies

$$\delta_{aa'}^g = \sum_{h < i} \delta_{aa'}^g \frac{|\epsilon^{ghi}| Q_a^h Q_a^i}{4}, \quad (20)$$

where Q_a^I denotes the number of invariant fixed points on the I th $\mathbb{T}^2$ under the $\Omega\mathcal{R}$ given as

$$Q_a^I \equiv \frac{3}{2} + \frac{(-1)^{\beta^I n_a^I}}{2}, \quad \beta^I \in \{0, 1\}, \quad (21)$$

which is equal to 2 except when $\beta^I = 1$ and n_a^I is odd, where it is equal to 1.

Setting $a = b$ in (19) and using (20), the intersection number of Π_a^F with its own $\Omega\mathcal{R}$ image is

$$\begin{aligned} \Pi_a^F \circ (\Pi_a^F)' &= \eta_{\Omega\mathcal{R}} \left(-\eta_{\Omega\mathcal{R}} 2^{1-k} \prod_I n_a^I l_a^I \right. \\ &\quad + \eta_{\Omega\mathcal{R}\theta} \frac{Q_a^1 Q_a^2}{2} 2^{-\beta^3} n_a^3 l_a^3 \\ &\quad + \eta_{\Omega\mathcal{R}\omega} \frac{Q_a^2 Q_a^3}{2} 2^{-\beta^1} n_a^1 l_a^1 \\ &\quad \left. + \eta_{\Omega\mathcal{R}\theta\omega} \frac{Q_a^1 Q_a^3}{2} 2^{-\beta^2} n_a^2 l_a^2 \right). \end{aligned} \quad (22)$$

For the intersection with the orientifold plane, one obtains

$$\begin{aligned} \Pi_a \circ \Pi_{O6}^F &= 2^{1-k} \left(-\eta_{\Omega\mathcal{R}} \prod_I l_a^I + \eta_{\Omega\mathcal{R}\theta} n_a^1 n_a^2 l_a^3 \right. \\ &\quad \left. + \eta_{\Omega\mathcal{R}\omega} l_a^1 n_a^2 n_a^3 + \eta_{\Omega\mathcal{R}\theta\omega} n_a^1 l_a^2 n_a^3 \right). \end{aligned} \quad (23)$$

Using the expressions in Eqs. (11), (19), (22), and (23), one can determine the chiral spectrum involving symmetric and antisymmetric representations of the gauge group $\prod_a U(N_a)$. In particular, a stack of N_a fractional D6-branes satisfying the condition $\Omega\mathcal{R}\Pi_a^F = \Pi_a^F$ gives rise to a $\text{USp}(2N_a)$ gauge symmetry.

In our construction of Pati-Salam models, we employ four visible sector rigid branes, denoted by a, b, c , and d , in contrast to the usual case, which involves only three branes and does not include twisted sectors. Consequently, the three-family condition is modified and takes the following form:

$$I_{ab} + I_{ab'} = -(I_{ac} + I_{ac'} + I_{ad} + I_{ad'}) = \pm 3, \quad (24)$$

where positive intersection numbers, in our convention, refer to left-chiral supermultiplets.

B. Supersymmetry conditions

Let us define the following products of wrapping numbers:

$$\begin{aligned} A_a &\equiv -n_a^1 n_a^2 n_a^3, & B_a &\equiv n_a^1 l_a^2 l_a^3, \\ C_a &\equiv l_a^1 n_a^2 l_a^3, & D_a &\equiv l_a^1 l_a^2 n_a^3, \\ \tilde{A}_a &\equiv -l_a^1 l_a^2 l_a^3, & \tilde{B}_a &\equiv l_a^1 n_a^2 n_a^3, \\ \tilde{C}_a &\equiv n_a^1 l_a^2 n_a^3, & \tilde{D}_a &\equiv n_a^1 n_a^2 l_a^3. \end{aligned} \quad (25)$$

Preserving $\mathcal{N} = 1$ supersymmetry in four dimensions after compactification from ten dimensions restricts the rotation angle of any D6-brane with respect to the orientifold plane to be an element of $\text{SU}(3)$, i.e.,

$$\theta_1^a + \theta_2^a + \theta_3^a = 0 \pmod{2\pi}, \quad (26)$$

where θ_i is the angle between the D6-brane and the orientifold plane in the i th two-torus and $\chi_i = R_i^2/R_i^1$ are the complex structure moduli for the i th two-torus,

$$\tan \theta_j^a = \chi_j \frac{\tilde{m}_j^a}{n_j^a} = \chi_j \frac{2^{-\beta_j} l_j^a}{n_j^a}. \quad (27)$$

$\mathcal{N} = 1$ supersymmetry conditions are given as

$$x_A \tilde{A}_a + x_B \tilde{B}_a + x_C \tilde{C}_a + x_D \tilde{D}_a = 0, \quad \frac{A_a}{x_A} + \frac{B_a}{x_B} + \frac{C_a}{x_C} + \frac{D_a}{x_D} < 0, \quad (28)$$

where $x_A = \lambda$, $x_B = 2^{\beta_2+\beta_3} \cdot \lambda / \chi_2 \chi_3$, $x_C = 2^{\beta_1+\beta_3} \cdot \lambda / \chi_1 \chi_3$, $x_D = 2^{\beta_1+\beta_2} \cdot \lambda / \chi_1 \chi_2$.

C. Tadpole cancellation

Since D6-branes and O6-orientifold planes are the sources of Ramond-Ramond charges they are constrained by Gauss's law in compact space, implying that the sum of D-brane and cross-cap RR charges must vanish,

$$\sum_a N_a [\Pi_a] + \sum_a N_a [\Pi_{a'}] - 4[\Pi_{O6}] = 0, \quad (29)$$

where the last terms arise from the O6-planes, which have -4 RR charges in D6-brane charge units. RR tadpole constraint is sufficient to cancel the $SU(N_a)^3$ cubic non-Abelian anomaly, while the $U(1)$ mixed gauge and gravitational anomaly or the $[SU(N_a)]^2 U(1)$ gauge anomaly can be canceled by the Green-Schwarz mechanism mediated by untwisted RR fields [43].

The twisted and untwisted tadpole cancellation conditions are given by

$$\begin{aligned} \sum_a N_a n_a^1 n_a^2 n_a^3 &= 16 \eta_{\Omega \mathcal{R}}, \\ \sum_a N_a n_a^i \tilde{m}_a^j \tilde{m}_a^k &= -2^{4-\beta^j-\beta^k} \eta_{\Omega \mathcal{R} i}, \quad i \neq j \neq k \in \{1, 2, 3\}, \\ \sum_a N_a n_a^i (\epsilon_{a,kl}^i - \eta_{\Omega \mathcal{R}} \eta_{\Omega \mathcal{R} i} \epsilon_{a,\mathcal{R}(k)\mathcal{R}(l)}^i) &= 0, \\ \sum_a N_a \tilde{m}_a^i (\epsilon_{a,kl}^i + \eta_{\Omega \mathcal{R}} \eta_{\Omega \mathcal{R} i} \epsilon_{a,\mathcal{R}(k)\mathcal{R}(l)}^i) &= 0, \end{aligned} \quad (30)$$

where N_a denotes the number of D6-branes on stack a and the sum is a sum over all stacks of D6-branes. $\mathcal{R}(k) = k$ with an untilted torus, and $\mathcal{R}(\{1, 2, 3, 4\}) = \{1, 2, 4, 3\}$ with a tilted torus. The twisted charge $\epsilon_{a,ij}^\omega$ is nonzero if and only if $ij \in S_\omega^a$, i.e., if the brane a passes through the fixed point ij in the ω -twisted sector, and so on. The orientifold projection acts on the wrapping numbers and twisted

TABLE III. The wrapping numbers for four O6-planes.

Orientifold action	O6-plane	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$
$\Omega \mathcal{R}$	1	$(2^{\beta_1}, 0) \times (2^{\beta_2}, 0) \times (2^{\beta_3}, 0)$
$\Omega \mathcal{R} \omega$	2	$(2^{\beta_1}, 0) \times (0, -2^{\beta_2}) \times (0, 2^{\beta_3})$
$\Omega \mathcal{R} \theta \omega$	3	$(0, -2^{\beta_1}) \times (2^{\beta_2}, 0) \times (0, 2^{\beta_3})$
$\Omega \mathcal{R} \theta$	4	$(0, -2^{\beta_1}) \times (0, 2^{\beta_2}) \times (2^{\beta_3}, 0)$

charges as follows:

$$\begin{aligned} \tilde{m}^I &\rightarrow -\tilde{m}^I, \\ \epsilon_{kl}^i &\rightarrow -\eta_{\Omega \mathcal{R}} \eta_{\Omega \mathcal{R} i} \epsilon_{\mathcal{R}(k)\mathcal{R}(l)}^i. \end{aligned} \quad (31)$$

The cancellation of RR tadpoles requires the introduction of a number of orientifold planes, also called “filler branes,” that trivially satisfy the four-dimensional $\mathcal{N} = 1$ supersymmetry conditions. The filler branes belong to the hidden sector USp group and carry the same wrapping numbers as one of the O6-planes, as shown in Table III. The USp group is hence referred to with respect to the nonzero A , B , C , or D type.

Orientifolds also have discrete D-brane RR charges classified by the $\mathbb{Z}_2$ K-theory groups, which are subtle and invisible by the ordinary homology [44–46], which should also be taken into account [47]. The K-theory conditions are

$$\sum_a \tilde{A}_a = \sum_a N_a \tilde{B}_a = \sum_a N_a \tilde{C}_a = \sum_a N_a \tilde{D}_a = 0 \pmod{4}. \quad (32)$$

D. Gauge couplings from complex structure moduli

Dynamical supersymmetry breaking in D6-brane models derived from type IIA orientifolds was explored in [48]. The Kähler potential is given by

$$K = -\ln(S + \bar{S}) - \sum_{i=1}^3 \ln(U^i + \bar{U}^i). \quad (33)$$

The complex structure moduli U^i can be extracted from the supersymmetry conditions, as shown in [14],

$$U^i = \frac{4i\chi^i + 2\beta_i^2 \chi_i^2}{4 + \beta_i^2 \chi_i^2}, \quad \chi^i \equiv \frac{R_2^i}{R_1^i}. \quad (34)$$

These moduli, expressed in the string theory basis, can be mapped to the field theory basis using $\{s, u^i\}$ as follows [49]:

$$\begin{aligned}
\text{Re}(s) &= \frac{e^{-\phi_4}}{2\pi} \frac{\sqrt{\text{Im}(U^1) \text{Im}(U^2) \text{Im}(U^3)}}{|U^1 U^2 U^3|}, \\
\text{Re}(u^1) &= \frac{e^{-\phi_4}}{2\pi} \sqrt{\frac{\text{Im}(U^1)}{\text{Im}(U^2) \text{Im}(U^3)}} \left| \frac{U^2 U^3}{U^1} \right|, \\
\text{Re}(u^2) &= \frac{e^{-\phi_4}}{2\pi} \sqrt{\frac{\text{Im}(U^2)}{\text{Im}(U^1) \text{Im}(U^3)}} \left| \frac{U^1 U^3}{U^2} \right|, \\
\text{Re}(u^3) &= \frac{e^{-\phi_4}}{2\pi} \sqrt{\frac{\text{Im}(U^3)}{\text{Im}(U^1) \text{Im}(U^2)}} \left| \frac{U^1 U^2}{U^3} \right|. \quad (35)
\end{aligned}$$

The four-dimensional dilaton is related to the moduli via

$$2\pi e^{\phi_4} = (\text{Re}(s)\text{Re}(u^1)\text{Re}(u^2)\text{Re}(u^3))^{-1/4}. \quad (36)$$

The holomorphic gauge kinetic function for a D6-brane stack x wrapping a supersymmetric three-cycle is [50]

$$f_x = \frac{1}{4k_x} \left(n_x^1 n_x^2 n_x^3 s - \frac{n_x^1 l_x^2 l_x^3 u^1}{2\beta_2 + \beta_3} - \frac{l_x^1 n_x^2 l_x^3 u^2}{2\beta_1 + \beta_3} - \frac{l_x^1 l_x^2 n_x^3 u^3}{2\beta_1 + \beta_2} \right), \quad (37)$$

where s and u^i are the four-dimensional dilaton and complex structure moduli, respectively, and k_x is the Kac-Moody level of G_x : $\kappa_x = 1$ for $U(N_x)$ and $\kappa_x = 2$ for $\text{USp}(2N_x)$ or $\text{SO}(2N_x)$ [51,52]. With this convention, the gauge coupling is given by

$$g_x^{-2} = \text{Re}(f_x). \quad (38)$$

When two gauge factors G_c and G_d are Higgsed to their diagonal subgroup G_R , canonical normalization of the gauge kinetic terms implies

$$g_R^{-2} = g_c^{-2} + g_d^{-2} = \text{Re}(f_c) + \text{Re}(f_d). \quad (39)$$

Therefore, the holomorphic gauge kinetic function for the diagonal is simply [53]

$$f_R = f_c + f_d. \quad (40)$$

No factor of $1/2$ appears here: the dependence on the Kac-Moody levels k_x is already included in each f_x through the $1/(4k_x)$ prefactor in its definition (37). For $k_c = k_d = 1$, one finds $k_R = k_c + k_d = 2$ for the diagonal group, but this is automatically encoded in f_R above.

At tree level, the gauge couplings satisfy

$$\begin{aligned}
g_a^2 &= \frac{\text{Re}(f_b)}{\text{Re}(f_a)} g_b^2 = \frac{\text{Re}(f_c) + \text{Re}(f_d)}{\text{Re}(f_a)} g_R^2 \\
&= k_Y \left(\frac{5}{3} \right) g_Y^2 = \gamma \pi e^{\phi_4}, \quad (41)
\end{aligned}$$

where g_a , g_b , and g_Y are the strong, weak, and hypercharge couplings, g_R is the coupling of the diagonal $\text{SU}(2)_R$, k_Y is the effective Kac-Moody level of the canonically normalized hypercharge, and γ is a model-dependent constant fixed by the internal moduli.

III. THREE-FAMILY PATI-SALAM MODELS FROM RIGID BRANES

To construct the three-family Pati-Salam model using rigid, semirigid, and nonrigid branes, we follow the strategy outlined in [26]. In the $\mathbb{Z}_2 \times \mathbb{Z}_2'$ orbifold, fractional branes invariant under $\Omega\mathcal{R}$ are those placed on top of an exotic $\text{O}6^{(+,+)}$ plane that is taken as $\text{O}_{\Omega\mathcal{R}}$ in our choice (5). The adjoint fields from the aa sector do not arise for the rigid branes. All three two-tori are taken to be rectangular.

We begin by outlining the model-building strategy. As previously discussed, obtaining an odd number of chiral families requires that some of the fixed point contributions, denoted by δ_{ab}^g , differ from their maximal values, i.e., $\delta_{ab}^g \neq (4, 4, 4)$, where 4 is the maximum contribution in each entry. Consequently, in order to cancel twisted RR tadpoles at all fixed points, it becomes necessary to introduce additional D6-branes beyond those used in the simpler case where all fixed points are shared among branes, i.e., $\delta_{ab}^g = (4, 4, 4)$ [26].

These additional branes must be introduced carefully so as not to give rise to exotic chiral matter. In general, they are assigned to the hidden sector. However, in order to realize massless grand unified theory (GUT) Higgs pairs in the open-string spectrum, it will be necessary to recombine one of the extra stacks with the stack initially responsible for the $\text{SU}(2)_R$ gauge symmetry of the Pati-Salam model. Additionally, we restrict our attention to models preserving $\mathcal{N} = 1$ supersymmetry, which imposes further constraints on the brane wrapping numbers and leads to partial stabilization of the closed-string moduli.

More specifically, we consider an initial configuration consisting of four rigid D6-branes, labeled $\{a, b, c, d\}$, which constitute the (initially) *visible* sector. These branes share some, but not all, fixed points in various twisted sectors, which generically results in uncanceled twisted tadpoles. To cancel these contributions, an additional (initially) *hidden* sector of branes must be introduced. To minimize the number of required hidden sector branes, the three stacks $\{b, c, d\}$ which generate the $\text{SU}(2)_L \times \text{SU}(2)_R$ gauge symmetry are chosen to share exactly the same set of fixed points, i.e., $\delta_{bc}^g = (4, 4, 4)$. Consequently, their contributions to twisted tadpoles are identical and can be arranged to cancel one another. This leaves only the twisted tadpoles sourced by the stack a .

To cancel the remaining twisted tadpoles, we introduce additional stacks $\{e_1, e_2\}$ such that the total twisted tadpole contributions from the combined set $\{a, b, c, d, e_1, e_2\}$ cancel completely. The semirigid branes $\{e_1, e_2\}$ are

constructed to have identical wrapping numbers and twisted charges with respect to one of the $\mathbb{Z}_2$ orbifold factors, ensuring that their mutual contributions cancel internally.

Finally, the cancellation of untwisted tadpoles is achieved by adding suitable hidden sector branes that do not source additional twisted tadpoles. Branes of the type $\{f_1, f_2, f_3, f_4\}$ can recombine into bulk branes and thereby form nonrigid stacks. In fact, the requirement of unbroken $\mathcal{N} = 1$ supersymmetry restricts all hidden sector branes to be either semirigid or nonrigid.

The anomalies associated with the four global U(1) symmetries embedded in $U(4)_C$, $U(2)_L$, $U(2)_{R_1}$, and $U(2)_{R_2}$ are canceled via the Green-Schwarz mechanism. As a result, the corresponding U(1) gauge bosons acquire masses through linear $B \wedge F$ couplings. Consequently, the effective gauge symmetry is $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2}$. When the fields Δ_i acquire vacuum expectation values (VEVs), the $SU(2)_{R_1} \times SU(2)_{R_2}$ symmetry is broken down to the diagonal subgroup $SU(2)_R$. This leads to the Pati-Salam gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_R$.

We obtain three generations of left-handed fermions F_L^i and three generations of right-handed fermions arising from the combination of F_R^i and $F_R'^i$ according to (24). By assigning suitable VEVs to F_R^{ci} and a linear combination of F_R^i , the Pati-Salam gauge symmetry can be broken down to the Standard Model gauge symmetry. Moreover, this symmetry breaking can preserve four-dimensional $\mathcal{N} = 1$ supersymmetry, provided D-flatness and F-flatness conditions are satisfied.

The Standard Model fermion masses and mixing, as well as vectorlike masses for F_R^{ci} and the two remaining linear combinations of F_R^i , can be generated through the following superpotential:

$$W \supset Y_{ijk} F_L^i F_R^j \Phi^k + Y'_{ijk} F_R^{ci} F_R^j \Delta^k. \quad (42)$$

Furthermore, the $SU(4)$ gauge groups in the hidden sector exhibit negative β functions, which allows for supersymmetry breaking via gaugino condensation.

Following the above outlined model-building strategy, the resulting models are presented in the Appendix in Tables XXIII–XXXII, and their particle spectra are tabulated in Tables IV, V, VII, IX, XI, XIII, XV, XVII, XIX, and XXI, respectively.

Note that in tabulating the full spectrum of each model, we have implemented a specific deformation to eliminate additional massless states in the hidden sector. Specifically, we have displaced the hidden sector D6-branes $e, f, g, \dots$ from the origin of the internal space $\mathbb{T}^2 \times \mathbb{T}^2 \times \mathbb{T}^2$ to alternative orbifold fixed points, as discussed in Table I. This displacement leads to the vanishing of certain intersection numbers between the hidden sector branes and the visible sector branes a, b, c , and d , thereby removing

unwanted massless matter. Alternatively, the same effect can be achieved by introducing nontrivial discrete Wilson lines [5,42].

A. Model r06

The first model presented in Table XXIII is a gauge theory of rank 6, involving a set of four rigid D6-branes labeled $\{a, b, c, d\}$ which share the same fixed points, i.e., $\delta_{ab}^g \neq (4, 4, 4)$. The model has a gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2}$. The particle spectrum, listed in Table IV, categorizes the various matter fields according to their quantum numbers under the gauge symmetry, including left- and right-handed fermions, scalar fields, and Higgs-like particles. Each field is labeled based on its origin from different brane intersections, ab, ac, ad' , etc., corresponding to chiral fermions in the fundamental representations of the gauge factors. The tree-level string scale gauge coupling relation and the complex structure moduli of the three two-tori are provided in the caption of Table XXIII.

The particle spectrum of model r06 is presented in Table IV. The table lists the quantum numbers of the matter fields under this gauge symmetry along with their corresponding field representations. The left-handed fermions, denoted as $F_L^i(Q_L, L_L)$, transform as $(\bar{4}, \bar{2}, 1, 1)$, while the right-handed fermions appear in multiple representations: $F_R^{ci}(Q_R, L_R)$ under $(4, 1, 1, \bar{2})$ and $(4, 1, 1, 2)$, as well as $F_R^i(Q_R, L_R)$ under $(\bar{4}, 1, 2, 1)$ and $(4, 1, 2, 1)$.

Additionally, singlet fields such as S_L^i and S_R^i , which transform under the antisymmetric representations of

TABLE IV. Particle spectrum of model r06 with gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2}$.

Model r06	Quantum numbers	Fields
ab'	$3 \times (\bar{4}, \bar{2}, 1, 1)$	$F_L^{ci}(Q_L, L_L)$
ac	$1 \times (4, 1, \bar{2}, 1)$	$F_R^{ci}(Q_R, L_R)$
ac'	$3 \times (4, 1, 2, 1)$	$F_R^{ci}(Q_R, L_R)$
ad'	$2 \times (4, 1, 1, 2)$	$F_R^{ci}(Q_R, L_R)$
ad	$3 \times (\bar{4}, 1, 1, 2)$	$F_R^i(Q_R, L_R)$
b	$30 \times (1, \bar{1}, 1, 1)$	S_L^i
c	$25 \times (1, 1, \bar{1}, 1)$	S_R^i
d	$20 \times (1, 1, 1, 1)$	S_R^i
a	$9 \times (1, 1, 3, 1)$	T_R^i
d	$6 \times (1, 1, 1, \bar{3})$	T_R^i
cd	$21 \times (1, 1, \bar{2}, 2)$	Δ^i
cd'	$1 \times (1, 1, \bar{2}, \bar{2})$	Δ^i
bc	$12 \times (1, 2, \bar{2}, 1)$	$\Phi^i(H_u, H_d)$
bc'	$24 \times (1, \bar{2}, \bar{2}, 1)$	$\Phi^i(H_u, H_d)$
bd	$10 \times (1, \bar{2}, 1, 2)$	Ξ^i
bd'	$4 \times (1, \bar{2}, 1, \bar{2})$	Ξ^i

$SU(2)_L$, $SU(2)_{R_1}$, and $SU(2)_{R_2}$, are included. The Higgs-like fields $\Phi^i(H_u, H_d)$ appear in the representations $(1, \bar{2}, 2, 1)$ and $(1, \bar{2}, \bar{2}, 1)$. Furthermore, scalar fields Δ^i serve as GUT Higgs fields responsible for breaking the Pati-Salam symmetry with the Standard Model gauge group. In addition, there exist several exotic particle states Ξ^i with quantum numbers $(1, 2, 1, \bar{2})$ and $(1, \bar{2}, 1, \bar{2})$.

B. Model r08

The model presented in Table XXIV consists of a set of four rigid D6-branes $\{a, b, c, d\}$ which share the same fixed points, i.e., $\delta_{ab}^g \neq (4, 4, 4)$. However, this leads to the uncanceled twisted tadpoles among themselves. We introduce two semirigid branes e_1 and e_2 related by a $\mathbb{Z}_2$ image from θ twist such that all the twisted and untwisted charges vanish. Table V presents the particle spectrum of model r08, which extends the previous gauge symmetry by an additional $SU(2)^2$, leading to the group $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^2$.

The left-handed fermions $F_L^i(Q_L, L_L)$ transform as $(4, \bar{2}, 1, 1, 1, 1)$, while the right-handed fermions include multiple representations such as $F_R^i(Q_R, L_R)$ in $(4, 1, 1, \bar{2}, 1, 1)$ and $F_R^i(Q_R, L_R)$ in $(\bar{4}, 1, 2, 1, 1, 1)$ and $(4, 1, 2, 1, 1, 1)$. The scalar sector contains multiple Higgs-like fields $\Phi^i(H_u, H_d)$ and Δ^i responsible for gauge symmetry breaking. Several exotic fields X_L^i appear in the representations $(1, \bar{2}, 1, 1, 2, 1)$ and $(1, \bar{2}, 1, 1, 1, 2)$.

TABLE V. Particle spectrum of model r08 with gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^2$.

Model r08	Quantum numbers	Fields
ab	$3 \times (4, \bar{2}, 1, 1, 1, 1)$	$F_L^i(Q_L, L_L)$
ac	$2 \times (4, 1, \bar{2}, 1, 1, 1)$	$F_R^{ci}(Q_R, L_R)$
ad'	$1 \times (4, 1, 1, 2, 1, 1)$	$F_R^{ci}(Q_R, L_R)$
ad	$6 \times (\bar{4}, 1, 1, 2, 1, 1)$	$F_R^{ci}(Q_R, L_R)$
$b \begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$30 \times (1, \bar{1}, \begin{smallmatrix} \square \\ \square \end{smallmatrix}, 1, 1, 1, 1)$	S_L^i
$c \begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$15 \times (1, 1, 1, \begin{smallmatrix} \square \\ \square \end{smallmatrix}, 1, 1, 1)$	S_R^i
$d \begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$20 \times (1, 1, 1, 1, \begin{smallmatrix} \square \\ \square \end{smallmatrix}, 1, 1)$	S_R^i
$d \begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$3 \times (1, 1, \bar{3}, \begin{smallmatrix} \square \\ \square \end{smallmatrix}, 1, 1, 1)$	T_R^i
$d \begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$6 \times (1, 1, 1, \bar{3}, \begin{smallmatrix} \square \\ \square \end{smallmatrix}, 1, 1)$	T_R^i
cd	$1 \times (1, 1, \bar{2}, 2, 1, 1)$	Δ^i
cd'	$15 \times (1, 1, 2, 2, 1, 1)$	Δ^i
bc	$16 \times (1, \bar{2}, 2, 1, 1, 1)$	$\Phi^i(H_u, H_d)$
bd	$6 \times (1, \bar{2}, 1, 2, 1, 1)$	Ξ^i
bd'	$4 \times (1, 2, 1, 2, 1, 1)$	Ξ^i
be_1	$2 \times (1, \bar{2}, 1, 1, 2, 1)$	X_L^i
be_2	$2 \times (1, \bar{2}, 1, 1, 1, 2)$	X_L^i
be'_1	$2 \times (1, \bar{2}, 1, 1, \bar{2}, 1)$	X_L^i
be'_2	$2 \times (1, \bar{2}, 1, 1, 1, \bar{2})$	X_L^i

TABLE VI. The composite particle spectrum for model r08 formed due to strong forces in the hidden sector.

Model r08	$SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^2$		
Force	Intersection	Exotic particle	Confined spectrum
$SU(2)_e$	be_1	$2 \times (1, \bar{2}, 1, 1, 2, 1)$	$(1, \bar{2}^2, 1, 1, 1, 1)$
	be_2	$2 \times (1, \bar{2}, 1, 1, 1, 2)$	
	be'_1	$2 \times (1, \bar{2}, 1, 1, \bar{2}, 1)$	
	be'_2	$2 \times (1, \bar{2}, 1, 1, 1, \bar{2})$	

Table VI describes the composite particle spectrum in model r08 arising from strong interactions in the hidden sector. The presence of the confining force $SU(2)_e$ at the intersections be_1 , be_2 , be'_1 , and be'_2 leads to the formation of composite states. As a result, all exotic particle states X_L^i undergo confinement, giving rise to new bound states $(1, \bar{2}^2, 1, 1, 1, 1)$ which decouple from the low-energy spectrum.

C. Model r10

For model r10, Table VII provides the detailed matter content under the gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^4$. This model features a rich particle spectrum, including multiple generations of fundamental fermions, singlet fields, and higher-dimensional representations. Additionally, several new exotic states X_L^i and X_R^i which transform under extra hidden $SU(2)$ sectors are introduced.

Table VIII outlines the composite spectrum for model r10, describing the effects of confinement in the hidden sector. Exotic particles appearing at intersections such as ce_1 and ce_2 are subject to confinement under $SU(2)_e$, leading to the formation of bound states that are neutral under the hidden gauge group. Similarly, intersections involving bf'_1 and cf_1 interact under the hidden $SU(2)_f$ dynamics, resulting in additional confined composite states. The strong coupling dynamics thus causes the exotic particles to decouple from the low-energy spectrum.

D. Model r19

The particle spectrum of model r19, which details the quantum numbers and associated fields under the gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^5 \times USp(4)^4$, is presented in Table IX.

Table X outlines the composite particle spectrum that emerges due to strong dynamics in the hidden sector. The exotic particles X_L^i and X_R^i undergo confinement under the non-Abelian gauge forces, viz., $SU(2)_e$, $SU(2)_f$, $SU(2)_g$, and $USp(4)_h$. Each row specifies the initial exotic representation under the gauge symmetry and the confined particle spectrum resulting from the strong dynamics.

TABLE VII. Particle spectrum of model r10 with gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^4$.

Model r10	Quantum numbers	Fields
ab	$3 \times (\bar{4}, 2, 1, 1, 1, 1, 1, 1)$	$F_L^i(Q_L, L_L)$
ad'	$2 \times (\bar{4}, 1, 1, \bar{2}, 1, 1, 1, 1)$	$F_R^i(Q_R, L_R)$
ac	$1 \times (\bar{4}, 1, 2, 1, 1, 1, 1, 1)$	$F_R^i(Q_R, L_R)$
ac'	$6 \times (4, 1, 2, 1, 1, 1, 1, 1)$	$F_R^i(Q_R, L_R)$
b	$6 \times (1, 1, \bar{3}, 1, 1, 1, 1, 1)$	S_L^i
c	$60 \times (1, 1, 1, \bar{3}, 1, 1, 1, 1, 1)$	S_R^i
d	$9 \times (1, 1, 1, 1, \bar{3}, 1, 1, 1, 1)$	S_R^i
c	$10 \times (1, 1, 3, \bar{3}, 1, 1, 1, 1, 1)$	T_R^i
d	$1 \times (1, 1, 1, \bar{3}, \bar{3}, 1, 1, 1, 1)$	T_R^i
cd	$3 \times (1, 1, \bar{2}, 2, 1, 1, 1, 1)$	Δ^i
cd'	$17 \times (1, 1, 2, 2, 1, 1, 1, 1)$	Δ^i
bc	$4 \times (1, 2, \bar{2}, 1, 1, 1, 1, 1)$	$\Phi^i(H_u, H_d)$
bc'	$10 \times (1, \bar{2}, \bar{2}, 1, 1, 1, 1, 1)$	$\Phi^i(H_u, H_d)$
bd	$2 \times (1, 2, 1, \bar{2}, 1, 1, 1, 1)$	Ξ^i
bf'_1	$2 \times (1, 2, 1, 1, 1, 1, 2, 1)$	X_L^i
bf'_2	$2 \times (1, 2, 1, 1, 1, 1, 1, 2)$	X_L^i
ce_1	$2 \times (1, 1, 2, 1, \bar{2}, 1, 1, 1)$	X_R^i
ce_2	$2 \times (1, 1, 2, 1, 1, \bar{2}, 1, 1)$	X_R^i
cf_1	$4 \times (1, 1, 2, 1, 1, 1, \bar{2}, 1)$	X_R^i
cf_2	$4 \times (1, 1, 2, 1, 1, 1, 1, \bar{2})$	X_R^i
ce'_1	$2 \times (1, 1, 2, 1, 2, 1, 1, 1)$	X_R^i
ce'_2	$2 \times (1, 1, 2, 1, 1, 2, 1, 1)$	X_R^i
cf'_1	$2 \times (1, 1, 2, 1, 1, 1, 2, 1)$	X_R^i
cf'_2	$2 \times (1, 1, 2, 1, 1, 1, 1, 2)$	X_R^i

TABLE VIII. The composite particle spectrum for model r10 formed due to the strong forces in hidden sector.

Model r10	$SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^4$		
Force	Inter-section	Exotic particle	Confined spectrum
$SU(2)_e$	ce_1	$2 \times (1, 1, 2, 1, \bar{2}, 1, 1, 1)$	$(1, 1, 2^2, 1, 1, 1, 1, 1)$
	ce_2	$2 \times (1, 1, 2, 1, 1, \bar{2}, 1, 1)$	
	ce'_1	$2 \times (1, 1, 2, 1, 2, 1, 1, 1)$	
	ce'_2	$2 \times (1, 1, 2, 1, 1, 2, 1, 1)$	
$SU(2)_f$	bf'_1	$2 \times (1, 2, 1, 1, 1, 1, 2, 1)$	$(1, 2^2, 1, 1, 1, 1, 1, 1)$
	bf'_2	$2 \times (1, 2, 1, 1, 1, 1, 1, 2)$	$(1, 1, 2^2, 1, 1, 1, 1, 1)$
	cf_1	$4 \times (1, 1, 2, 1, 1, 1, \bar{2}, 1)$	
	cf_2	$4 \times (1, 1, 2, 1, 1, 1, 1, \bar{2})$	
	cf'_1	$2 \times (1, 1, 2, 1, 1, 1, 2, 1)$	
	cf'_2	$2 \times (1, 1, 2, 1, 1, 1, 1, 2)$	

TABLE IX. Particle spectrum of model r19 with gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^5 \times USp(4)^4$.

Model r19	Quantum numbers	Fields
ab	$1 \times (4, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$F_L^i(Q_L, L_L)$
ab'	$2 \times (4, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$F_L^i(Q_L, L_L)$
ad	$1 \times (4, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1)$	$F_R^{ci}(Q_R, L_R)$
ac'	$4 \times (\bar{4}, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$F_R^{ci}(Q_R, L_R)$
b	$12 \times (1, \bar{1}, \bar{3}, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	S_L^i
c	$24 \times (1, 1, \bar{1}, \bar{3}, 1, 1, 1, 1, 1, 1, 1, 1)$	S_R^i
d	$24 \times (1, 1, 1, \bar{1}, \bar{3}, 1, 1, 1, 1, 1, 1, 1)$	S_R^i
b	$2 \times (1, 3, \bar{3}, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	T_L^i
cd	$1 \times (1, 1, 2, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1)$	Δ^i
cd'	$12 \times (1, 1, \bar{2}, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1)$	Δ^i
bc	$2 \times (1, \bar{2}, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$\Phi^i(H_u, H_d)$
bc'	$10 \times (1, \bar{2}, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$\Phi^i(H_u, H_d)$
bd	$2 \times (1, 2, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1)$	Ξ^i
bd'	$9 \times (1, \bar{2}, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1)$	Ξ^i
bg_1	$2 \times (1, \bar{2}, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1)$	X_L^i
bg_2	$2 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1)$	X_L^i
bh_1	$1 \times (1, 2, 1, 1, 1, 1, 1, 1, 1, \bar{4}, 1, 1)$	X_L^i
bg'_1	$1 \times (1, \bar{2}, 1, 1, 1, 1, 1, \bar{2}, 1, 1, 1, 1)$	X_L^i
bg'_2	$1 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, \bar{2}, 1, 1, 1)$	X_L^i
bh'_1	$1 \times (1, 2, 1, 1, 1, 1, 1, 1, 1, 4, 1, 1)$	X_L^i
ce	$1 \times (1, 1, \bar{2}, 1, 2, 1, 1, 1, 1, 1, 1, 1)$	X_R^i
cf_2	$1 \times (1, 1, \bar{2}, 1, 1, 1, 2, 1, 1, 1, 1, 1)$	X_R^i
cg_1	$1 \times (1, 1, \bar{2}, 1, 1, 1, 1, 2, 1, 1, 1, 1)$	X_R^i
ch_4	$1 \times (1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, \bar{4})$	X_R^i
ce'	$2 \times (1, 1, \bar{2}, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1)$	X_R^i
cf'_1	$2 \times (1, 1, \bar{2}, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1)$	X_R^i
cg'_2	$1 \times (1, 1, \bar{2}, 1, 1, 1, 1, 1, \bar{2}, 1, 1, 1)$	X_R^i
ch'_4	$1 \times (1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 4)$	X_R^i
de	$1 \times (1, 1, 1, \bar{2}, 2, 1, 1, 1, 1, 1, 1, 1)$	X_R^i
df_1	$2 \times (1, 1, 1, \bar{2}, 1, 2, 1, 1, 1, 1, 1, 1)$	X_R^i
df_2	$2 \times (1, 1, 1, \bar{2}, 1, 1, 2, 1, 1, 1, 1, 1)$	X_R^i
dg_1	$1 \times (1, 1, 1, \bar{2}, 1, 1, 1, 2, 1, 1, 1, 1)$	X_R^i
dg_2	$1 \times (1, 1, 1, \bar{2}, 1, 1, 1, 1, 2, 1, 1, 1)$	X_R^i
dh_2	$1 \times (1, 1, 1, 2, 1, 1, 1, 1, 1, 1, \bar{4}, 1)$	X_R^i
df'_1	$1 \times (1, 1, 1, \bar{2}, 1, \bar{2}, 1, 1, 1, 1, 1, 1)$	X_R^i
df'_2	$1 \times (1, 1, 1, \bar{2}, 1, 1, \bar{2}, 1, 1, 1, 1, 1)$	X_R^i
dh'_2	$1 \times (1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 4, 1)$	X_R^i

TABLE X. The composite particle spectrum for model r19 formed due to the strong forces in the hidden sector.

Model r19		$SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^5 \times USp(4)^4$	
Confining force	Intersection	Exotic particle	Confined particle spectrum
$SU(2)_e$	ce	$1 \times (1, 1, \bar{2}, 1, 2, 1, 1, 1, 1, 1, 1, 1)$	$(1, 1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	de	$1 \times (1, 1, 1, \bar{2}, 2, 1, 1, 1, 1, 1, 1, 1)$	$(1, 1, 1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1)$
	de'	$2 \times (1, 1, 1, \bar{2}, \bar{2}, 1, 1, 1, 1, 1, 1, 1)$	
$SU(2)_f$	cf_1	$2 \times (1, 1, \bar{2}, 1, 1, 2, 1, 1, 1, 1, 1, 1)$	$(1, 1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	cf_2	$2 \times (1, 1, \bar{2}, 1, 1, 1, 2, 1, 1, 1, 1, 1)$	$(1, 1, 1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1)$
	cf'_1	$1 \times (1, 1, \bar{2}, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1)$	
	cf'_2	$1 \times (1, 1, \bar{2}, 1, 1, 1, \bar{2}, 1, 1, 1, 1, 1)$	
	df_2	$1 \times (1, 1, 1, \bar{2}, 1, 1, 2, 1, 1, 1, 1, 1)$	
	df'_1	$2 \times (1, 1, 1, \bar{2}, 1, \bar{2}, 1, 1, 1, 1, 1, 1)$	
$SU(2)_g$	bg_1	$2 \times (1, \bar{2}, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1)$	$(1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	bg_2	$2 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1)$	$(1, 1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	bg'_1	$1 \times (1, \bar{2}, 1, 1, 1, 1, 1, \bar{2}, 1, 1, 1, 1)$	$(1, 1, 1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1)$
	bg'_2	$1 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, \bar{2}, 1, 1, 1)$	
	cg_1	$1 \times (1, 1, \bar{2}, 1, 1, 1, 1, 2, 1, 1, 1, 1)$	
	cg_2	$1 \times (1, 1, \bar{2}, 1, 1, 1, 1, 1, 2, 1, 1, 1)$	
	dg_1	$1 \times (1, 1, 1, \bar{2}, 1, 1, 1, 2, 1, 1, 1, 1)$	
	dg'_2	$1 \times (1, 1, 1, \bar{2}, 1, 1, 1, 1, \bar{2}, 1, 1, 1)$	
$USp(4)_h$	bh_1	$1 \times (1, 2, 1, 1, 1, 1, 1, 1, 1, \bar{4}, 1, 1, 1)$	$(1, 2^2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	bh'_1	$1 \times (1, 2, 1, 1, 1, 1, 1, 1, 1, 4, 1, 1, 1)$	$(1, 1, 2^2, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	ch_2	$1 \times (1, 1, 2, 1, 1, 1, 1, 1, 1, 1, \bar{4}, 1, 1)$	$(1, 1, 1, 2^2, 1, 1, 1, 1, 1, 1, 1, 1)$
	ch'_2	$1 \times (1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 4, 1, 1)$	
	dh_4	$1 \times (1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, \bar{4})$	
	dh'_4	$1 \times (1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 4)$	

E. Model r20

The particle spectrum of model r20 is presented in Table XI, which lists the various chiral multiplets and their associated quantum numbers under the gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^6 \times USp(4)^4$.

Table XII provides an overview of the composite particle spectrum that emerges due to confinement in the hidden sector. Several $SU(2)$ and $USp(4)$ gauge groups become strongly coupled, leading to the formation of bound states. For each confining force $SU(2)_e$, $SU(2)_f$, $SU(2)_g$, and $USp(4)_h$, the table specifies the exotic particle content involved in the strong interaction and the corresponding confined spectrum.

F. Model r22

The particle spectrum for model r22 is presented in Table XIII. The gauge symmetry of this model is given by $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times USp(8)^4$. The table lists the various chiral matter fields, along with their corresponding quantum numbers under the gauge group.

Table XIV details the composite particle spectrum arising due to the strong dynamics in the hidden sector.

Specifically, the $USp(8)_e$ gauge interactions confine certain exotic particles, leading to the formation of bound states.

G. Model r25

Table XV presents the chiral spectrum of model r25, characterized by the gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^3 \times USp(8)^4$.

Table XVI details the confined composite states that emerge due to strong gauge dynamics in the hidden sector of model r25. These bound states result from the confining forces associated with $SU(2)_e$, $SU(2)_f$, and $USp(8)_g$.

H. Model r26

The particle spectrum of model r26 with gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times USp(4) \times SU(4)^6$ is presented in Table XVII. The model contains a variety of fields with their corresponding quantum numbers. These fields include left-handed fermions (F_L^i) and right-handed fermions (F_R^i), with the quantum numbers specified for each field under the different gauge groups. Additionally, there are scalar fields like S_L^i and S_R^i , as well as exotic fields such as Δ^i , $\Phi^i(H_u, H_d)$, and Ξ^i . The table also lists other types of fields, such as X_L^i , X_R^i , and various

TABLE XI. Particle spectrum of model r20 with gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^6 \times USp(4)^4$.

Model r20	Quantum numbers	Fields
ab	$2 \times (\bar{4}, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$F_L^{ci}(Q_L, L_L)$
ab'	$1 \times (\bar{4}, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$F_L^{ci}(Q_L, L_L)$
ad'	$4 \times (4, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1)$	$F_R^{ci}(Q_R, L_R)$
ac'	$1 \times (\bar{4}, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$F_R^{ci}(Q_R, L_R)$
$b \begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$8 \times (1, \bar{1}, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	S_L^i
$c \begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$24 \times (1, 1, \bar{1}, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	S_R^i
$d \begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$24 \times (1, 1, 1, \bar{1}, 1, 1, 1, 1, 1, 1, 1, 1)$	S_R^i
cd	$12 \times (1, 1, \bar{2}, 2, 1, 1, 1, 1, 1, 1, 1, 1)$	Δ^i
cd'	$1 \times (1, 1, \bar{2}, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1)$	Δ^i
bc	$2 \times (1, 2, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$\Phi^i(H_u, H_d)$
bc'	$8 \times (1, \bar{2}, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$\Phi^i(H_u, H_d)$
bd	$6 \times (1, \bar{2}, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1)$	Ξ^i
bd'	$2 \times (1, \bar{2}, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1)$	Ξ^i
bg_2	$1 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1)$	X_L^i
bh_3	$1 \times (1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, \bar{4})$	X_L^i
bg'_1	$2 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, \bar{2}, 1, 1, 1)$	X_L^i
bh'_3	$1 \times (1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 4, 1)$	X_L^i
ce_1	$1 \times (1, 1, \bar{2}, 1, 2, 1, 1, 1, 1, 1, 1, 1)$	X_R^i
ce_2	$2 \times (1, 1, \bar{2}, 1, 1, 2, 1, 1, 1, 1, 1, 1)$	X_R^i
cf_1	$1 \times (1, 1, \bar{2}, 1, 1, 1, 2, 1, 1, 1, 1, 1)$	X_R^i
cf_2	$2 \times (1, 1, \bar{2}, 1, 1, 1, 1, 2, 1, 1, 1, 1)$	X_R^i
ch_1	$1 \times (1, 1, 2, 1, 1, 1, 1, 1, 1, \bar{4}, 1, 1)$	X_R^i
cf'_1	$2 \times (1, 1, \bar{2}, 1, 1, 1, \bar{2}, 1, 1, 1, 1, 1)$	X_R^i
cf'_2	$1 \times (1, 1, \bar{2}, 1, 1, 1, 1, \bar{2}, 1, 1, 1, 1)$	X_R^i
cg'_1	$1 \times (1, 1, \bar{2}, 1, 1, 1, 1, 1, \bar{2}, 1, 1, 1)$	X_R^i
cg'_2	$1 \times (1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, \bar{2}, 1, 1)$	X_R^i
ch'_1	$1 \times (1, 1, 2, 1, 1, 1, 1, 1, 1, 4, 1, 1)$	X_R^i
de_1	$2 \times (1, 1, 1, 2, \bar{2}, 1, 1, 1, 1, 1, 1, 1)$	X_R^i
de_2	$1 \times (1, 1, 1, 2, 1, \bar{2}, 1, 1, 1, 1, 1, 1)$	X_R^i
df_1	$1 \times (1, 1, 1, 2, 1, 1, \bar{2}, 1, 1, 1, 1, 1)$	X_R^i
df_2	$2 \times (1, 1, 1, 2, 1, 1, 1, \bar{2}, 1, 1, 1, 1)$	X_R^i
dg_2	$1 \times (1, 1, 1, 2, 1, 1, 1, 1, 1, \bar{2}, 1, 1)$	X_R^i
dh_4	$1 \times (1, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 4)$	X_R^i
de'_1	$1 \times (1, 1, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1)$	X_R^i
de'_2	$2 \times (1, 1, 1, 2, 1, 2, 1, 1, 1, 1, 1, 1)$	X_R^i
dg'_1	$1 \times (1, 1, 1, 2, 1, 1, 1, 1, 2, 1, 1, 1)$	X_R^i
dh'_4	$1 \times (1, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, \bar{4})$	X_R^i

combinations of gauge group components associated with the exotic particles.

Table XVIII describes the composite particle spectrum formed in model r26 due to the strong forces in the hidden sector. The exotic particles arise from different confining forces, each associated with specific intersections. For example, under the $USp(4)_f$ force, composite particles

like $(1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1)$ are formed at the intersection of bf_1 and bf_2 . Similarly, the $SU(4)_g$ and $SU(4)_h$ forces also contribute to the formation of exotic particles, including combinations of different gauge group components. The confined particle spectra are provided for each intersection and detail the specific fields that combine to form the composite states.

I. Model r29

Table XIX presents the particle spectrum of model r29 with the gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^3 \times SU(4)^4 \times USp(4)^4$.

Table XX describes the composite particle spectrum of model r29 formed due to the strong forces in the hidden sector, viz., $SU(2)_e$, $SU(2)_f$, $SU(4)_g$, $SU(4)_h$, and $USp(4)_i$. For each confining force, the table lists the intersection types, the corresponding exotic particles formed, and the spectrum of confined particles.

J. Model r30

The particle spectrum of model r30 is presented in Table XXI. This model features a gauge symmetry group $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times USp(12)^4$.

Table XXII provides the composite particle spectrum of model r30, which arises due to the strong forces in the hidden sector of the model. The table describes the composite particles formed through the confining force $USp(12)_e$.

IV. ASYMPTOTIC FREEDOM

Constructing the Standard Model from rigid cycles, in addition to eliminating adjoint chiral multiplets, is also advantageous for realizing a gauge theory that is asymptotically free, characterized by a negative one-loop β function. This setup facilitates the convergence of the gauge couplings in the minimal supersymmetric Standard Model and also enables gaugino condensation through the non-perturbative superpotential of the form

$$W_{\text{eff}} \sim \frac{M_s \beta_1^g}{32\pi^2} \exp\left(\frac{8\pi^2}{g_{\text{YM}}^2 \beta_1^g}\right), \quad (43)$$

where the gauge couplings depend on the complex structure (or Kähler) moduli in type IIA (or type IIB) string theory. This effective superpotential may, in principle, stabilize some of the closed-string moduli, potentially in combination with other mechanisms such as background fluxes.

In general, the β functions are sensitive to the entire massless spectrum, including additional light nonchiral states that are not captured by intersection numbers or topological invariants. Therefore, it is crucial to have complete control over the full spectrum of the theory.

TABLE XII. The composite particle spectrum for model r20 formed due to the strong forces in the hidden sector.

Model r20		$SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^6 \times USp(4)^4$	
Confining force	Intersection	Exotic particle	Confined particle spectrum
$SU(2)_e$	ce_1	$2 \times (1, 1, 2, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$(1, 1, 2^2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	ce_2	$1 \times (1, 1, 2, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1)$	$(1, 1, 1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	ce'_1	$1 \times (1, 1, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1)$	
	ce'_2	$2 \times (1, 1, 2, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1)$	
	de_1	$1 \times (1, 1, 1, \bar{2}, 2, 1, 1, 1, 1, 1, 1, 1, 1)$	
	de_2	$2 \times (1, 1, 1, \bar{2}, 1, 2, 1, 1, 1, 1, 1, 1, 1)$	
$SU(2)_f$	cf_1	$1 \times (1, 1, 2, 1, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1)$	$(1, 1, 2^2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	cf_2	$2 \times (1, 1, 2, 1, 1, 1, 1, \bar{2}, 1, 1, 1, 1, 1)$	$(1, 1, 1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	df_1	$1 \times (1, 1, 1, \bar{2}, 1, 1, 2, 1, 1, 1, 1, 1, 1)$	
	df_2	$2 \times (1, 1, 1, \bar{2}, 1, 1, 1, 2, 1, 1, 1, 1, 1)$	
	df'_1	$2 \times (1, 1, 1, \bar{2}, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1)$	
	df'_2	$1 \times (1, 1, 1, \bar{2}, 1, 1, 1, \bar{2}, 1, 1, 1, 1, 1)$	
$SU(2)_g$	bg'_1	$2 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, \bar{2}, 1, 1, 1, 1)$	$(1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	bg_2	$1 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1)$	$(1, 1, 2^2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	cg_2	$1 \times (1, 1, 2, 1, 1, 1, 1, 1, 1, \bar{2}, 1, 1, 1)$	$(1, 1, 1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	cg'_1	$1 \times (1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1)$	
	dg'_1	$1 \times (1, 1, 1, \bar{2}, 1, 1, 1, 1, \bar{2}, 1, 1, 1, 1)$	
	dg'_2	$1 \times (1, 1, 1, \bar{2}, 1, 1, 1, 1, 1, \bar{2}, 1, 1, 1)$	
$USp(4)_h$	bh_3	$1 \times (1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, \bar{4}, 1)$	$(1, 2^2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	bh'_3	$1 \times (1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 4, 1)$	$(1, 1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	ch_4	$1 \times (1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1, 4)$	$(1, 1, 1, 2^2, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	ch'_4	$1 \times (1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1, \bar{4})$	
	dh_1	$1 \times (1, 1, 1, 2, 1, 1, 1, 1, 1, 1, \bar{4}, 1, 1)$	
	dh'_1	$1 \times (1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 4, 1, 1)$	

To this end, let us examine the light spectrum arising from fractional branes, which either are constructed by splitting the bulk branes or are otherwise generic rigid branes.

A. Fractional branes from splitting bulk branes

Consider a bulk D-brane a supporting a $U(N)$ gauge group. This brane contains three adjoint chiral multiplets, in addition to other matter from intersections with other D-branes. Neglecting the latter, the one-loop β function is

$$b_1^{U(N)} = -3N + 3 \times N = 0, \quad (44)$$

indicating that bulk D-branes have vanishing or positive β functions.

One might attempt to improve this by splitting the bulk brane into four rigid fractional constituents b_1, b_2, b_3 , and b_4 transforming in the regular representation of $\mathbb{Z}_2 \times \mathbb{Z}_2$, forming, for example, the D-brane stack b . This decomposition yields a gauge group $U(N)^4$ with no massless adjoint fields. However, additional nonchiral matter may arise between pairs of these fractional D-branes, and, in fact, this is generally the case. This spectrum can be computed from the boundary state overlaps,

$$\tilde{A}_{b_i, b_j} = \int_0^\infty dl \langle b_i | e^{-2\pi l H_{cl}} | b_j \rangle + \int_0^\infty dl \langle b_j | e^{-2\pi l H_{cl}} | b_i \rangle, \quad i \neq j, \quad (45)$$

whose loop channel representation is

$$A_{b_i, b_j} = \int_0^\infty \frac{dt}{t} \text{Tr}_{ij+ji} \left(\frac{1 + \theta + \omega + \theta\omega}{4} e^{-2\pi t H_0} \right). \quad (46)$$

The twisted sector projections result in a single massless hypermultiplet composed of scalar states associated with the oscillators $\psi_{-\frac{1}{2}}^I |0\rangle$ and $\tilde{\psi}_{-\frac{1}{2}}^I |0\rangle$ for $I \in \{1, 2, 3\}$. For $i = j$, the noncompact oscillators $\psi_{-\frac{1}{2}}^\mu |0\rangle, \tilde{\psi}_{-\frac{1}{2}}^\mu |0\rangle$ survive the projection, leading to an $\mathcal{N} = 1$ vector multiplet.

Each $U(N)$ factor thereby receives $6N$ chiral supermultiplets in the fundamental representation, resulting in a one-loop β function,

$$b_1^{U(N)} = -3N + 6N \times \frac{1}{2} = 0. \quad (47)$$

Hence, splitting a $U(N)$ bulk brane into fractional branes does not improve the asymptotic behavior of the β function.

TABLE XIII. Particle spectrum of model r22 with gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times USp(8)^4$.

Model r22	Quantum numbers	Fields
ab'	$3 \times (4, 2, 1, 1, 1, 1, 1)$	$F_L^i(Q_L, L_L)$
ad'	$3 \times (4, 1, 1, 2, 1, 1, 1)$	$F_R^{c'i}(Q_R, L_R)$
ad	$2 \times (\bar{4}, 1, 1, 2, 1, 1, 1)$	$F_R^i(Q_R, L_R)$
ac	$3 \times (\bar{4}, 1, 2, 1, 1, 1, 1)$	$F_R^i(Q_R, L_R)$
ac'	$1 \times (\bar{4}, 1, \bar{2}, 1, 1, 1, 1)$	$F_R^i(Q_R, L_R)$
$b \begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$4 \times (1, 1, \begin{smallmatrix} \square \\ \square \end{smallmatrix}, 1, 1, 1, 1)$	S_L^i
$c \begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$15 \times (1, 1, 1, \begin{smallmatrix} \square \\ \square \end{smallmatrix}, 1, 1, 1, 1)$	S_R^i
$d \begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$96 \times (1, 1, 1, 1, \begin{smallmatrix} \square \\ \square \end{smallmatrix}, 1, 1, 1, 1)$	S_R^i
$d \begin{smallmatrix} \square \\ \square \\ \square \end{smallmatrix}$	$3 \times (1, 1, \bar{3}, \begin{smallmatrix} \square \\ \square \\ \square \end{smallmatrix}, 1, 1, 1, 1)$	T_R^i
$d \begin{smallmatrix} \square \\ \square \\ \square \\ \square \end{smallmatrix}$	$24 \times (1, 1, 1, 3, \begin{smallmatrix} \square \\ \square \\ \square \\ \square \end{smallmatrix}, 1, 1, 1, 1)$	T_R^i
cd	$7 \times (1, 1, 2, \bar{2}, 1, 1, 1)$	Δ^i
cd'	$33 \times (1, 1, 2, 2, 1, 1, 1)$	Δ^i
bc	$9 \times (1, 2, \bar{2}, 1, 1, 1, 1)$	$\Phi^i(H_u, H_d)$
bc'	$3 \times (1, \bar{2}, \bar{2}, 1, 1, 1, 1)$	$\Phi^i(H_u, H_d)$
bd	$6 \times (1, \bar{2}, 1, 2, 1, 1, 1)$	Ξ^i
bd'	$6 \times (1, \bar{2}, 1, \bar{2}, 1, 1, 1)$	Ξ^i
ce_4	$1 \times (1, 1, \bar{2}, 1, 1, 1, 8)$	X_R^i
ce'_4	$1 \times (1, 1, \bar{2}, 1, 1, 1, \bar{8})$	X_R^i
de_1	$1 \times (1, 1, 1, \bar{2}, 8, 1, 1)$	X_R^i
de_3	$1 \times (1, 1, 1, \bar{2}, 1, 1, 8)$	X_R^i
de'_1	$1 \times (1, 1, 1, \bar{2}, \bar{8}, 1, 1)$	X_R^i
de'_3	$1 \times (1, 1, 1, \bar{2}, 1, 1, \bar{8})$	X_R^i

TABLE XIV. The composite particle spectrum for model r22 formed due to the strong forces in the hidden sector.

Model r22	$SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times USp(8)^4$		
Force	Inter-section	Exotic particle	Confined spectrum
$USp(8)_e$	ce_1	$1 \times (1, 1, \bar{2}, 1, 8, 1, 1)$	$(1, 1, \bar{2}^2, 1, 1, 1, 1)$
	ce_3	$1 \times (1, 1, \bar{2}, 1, 1, 1, 8)$	$(1, 1, 1, \bar{2}^2, 1, 1, 1)$
	ce'_1	$1 \times (1, 1, \bar{2}, 1, \bar{8}, 1, 1)$	
	ce'_3	$1 \times (1, 1, \bar{2}, 1, 1, 1, \bar{8})$	
	de_4	$1 \times (1, 1, 1, \bar{2}, 1, 1, 8)$	
	de'_4	$1 \times (1, 1, 1, \bar{2}, 1, 1, \bar{8})$	

Consequently, rigid D-branes obtained through this method cannot yield asymptotically free gauge groups.

B. Generic rigid fractional branes

Let us now consider a rigid D-brane a that supports a $U(4)$ gauge group and investigate whether the corresponding $SU(4)$ subgroup is asymptotically free. Owing to the

TABLE XV. Particle spectrum of model r25 with gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^3 \times USp(8)^4$.

Model r25	Quantum numbers	Fields
ab'	$3 \times (4, 2, 1, 1, 1, 1, 1, 1)$	$F_L^i(Q_L, L_L)$
ad	$1 \times (\bar{4}, 1, 1, 2, 1, 1, 1, 1)$	$F_R^i(Q_R, L_R)$
ac'	$2 \times (\bar{4}, 1, \bar{2}, 1, 1, 1, 1, 1)$	$F_R^i(Q_R, L_R)$
$b \begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$10 \times (1, 1, \begin{smallmatrix} \square \\ \square \end{smallmatrix}, 1, 1, 1, 1, 1)$	S_L^i
$c \begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$40 \times (1, 1, 1, \begin{smallmatrix} \square \\ \square \end{smallmatrix}, 1, 1, 1, 1)$	S_R^i
$d \begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$40 \times (1, 1, 1, 1, \begin{smallmatrix} \square \\ \square \end{smallmatrix}, 1, 1, 1)$	S_R^i
cd	$24 \times (1, 1, \bar{2}, 2, 1, 1, 1, 1)$	Δ^i
cd'	$4 \times (1, 1, \bar{2}, \bar{2}, 1, 1, 1, 1)$	Δ^i
bc	$4 \times (1, 2, \bar{2}, 1, 1, 1, 1, 1)$	$\Phi^i(H_u, H_d)$
bc'	$4 \times (1, \bar{2}, \bar{2}, 1, 1, 1, 1, 1)$	$\Phi^i(H_u, H_d)$
bd	$2 \times (1, \bar{2}, 1, 2, 1, 1, 1, 1)$	Ξ^i
be	$2 \times (1, \bar{2}, 1, 1, 2, 1, 1, 1)$	X_L^i
bf_1	$2 \times (1, \bar{2}, 1, 1, 1, 2, 1, 1)$	X_L^i
bf_2	$2 \times (1, \bar{2}, 1, 1, 1, 1, 2, 1)$	X_L^i
bg_4	$1 \times (1, \bar{2}, 1, 1, 1, 1, 1, 8)$	X_L^i
be'	$2 \times (1, \bar{2}, 1, 1, \bar{2}, 1, 1, 1)$	X_L^i
bf'_1	$2 \times (1, \bar{2}, 1, 1, 1, \bar{2}, 1, 1)$	X_L^i
bf'_2	$2 \times (1, \bar{2}, 1, 1, 1, 1, \bar{2}, 1)$	X_L^i
bg'_4	$1 \times (1, \bar{2}, 1, 1, 1, 1, 1, \bar{8})$	X_L^i
cf_1	$2 \times (1, 1, \bar{2}, 1, 1, 2, 1, 1)$	X_R^i
cf_2	$2 \times (1, 1, \bar{2}, 1, 1, 1, 2, 1)$	X_R^i
cg_2	$1 \times (1, 1, 2, 1, 1, 1, 1, \bar{8})$	X_R^i
ce'	$2 \times (1, 1, \bar{2}, 1, \bar{2}, 1, 1, 1)$	X_R^i
cg'_2	$1 \times (1, 1, 2, 1, 1, 1, 1, 8)$	X_R^i
de	$2 \times (1, 1, 1, 2, \bar{2}, 1, 1, 1)$	X_R^i
df_1	$4 \times (1, 1, 1, 2, 1, \bar{2}, 1, 1)$	X_R^i
df_2	$4 \times (1, 1, 1, 2, 1, 1, \bar{2}, 1)$	X_R^i
dg_3	$1 \times (1, 1, 1, \bar{2}, 1, 1, 1, 8)$	X_R^i
de'	$4 \times (1, 1, 1, 2, 2, 1, 1, 1)$	X_R^i
df'_1	$2 \times (1, 1, 1, 2, 1, 2, 1, 1)$	X_R^i
df'_2	$2 \times (1, 1, 1, 2, 1, 1, 2, 1)$	X_R^i
dg'_3	$1 \times (1, 1, 1, \bar{2}, 1, 1, 1, \bar{8})$	X_R^i

rigidity of the brane, no adjoint fields appear in the non-chiral spectrum. However, from the ad' sector, one hypermultiplet (i.e., two chiral multiplets) in the antisymmetric representation of $SU(4)$ arises [25].

Potential extra massless states charged under $SU(4)$ may arise from bifundamental vectorlike pairs in the ab sectors, where b is another fractional D-brane. These states must be computed by accounting for twisted sector projections in the loop channel. Effectively, this involves computing the spectrum in the unorbifolded theory and then applying the orbifold projection.

TABLE XVI. The composite particle spectrum of model r25 formed due to the strong forces in the hidden sector.

Model r25	$SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^3 \times USp(8)^4$		
Force	Sector	Exotic particle	Confined spectrum
$SU(2)_e$	be	$2 \times (1, \bar{2}, 1, 1, 2, 1, 1, 1, 1, 1, 1)$	$(1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	be'	$2 \times (1, \bar{2}, 1, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1)$	$(1, 1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1)$
	ce'	$2 \times (1, 1, \bar{2}, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1)$	$(1, 1, 1, 2^2, 1, 1, 1, 1, 1, 1, 1)$
	de	$2 \times (1, 1, 1, 2, \bar{2}, 1, 1, 1, 1, 1, 1, 1)$	
	de'	$4 \times (1, 1, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1)$	
$SU(2)_f$	bf_1	$2 \times (1, \bar{2}, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1)$	$(1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	bf_2	$2 \times (1, \bar{2}, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1)$	$(1, 1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1)$
	bf'_1	$2 \times (1, \bar{2}, 1, 1, 1, 1, \bar{2}, 1, 1, 1, 1, 1)$	$(1, 1, 1, 2^2, 1, 1, 1, 1, 1, 1, 1)$
	bf'_2	$2 \times (1, \bar{2}, 1, 1, 1, 1, 1, \bar{2}, 1, 1, 1, 1)$	
	cf_1	$2 \times (1, 1, \bar{2}, 1, 1, 2, 1, 1, 1, 1, 1, 1)$	
	cf_2	$2 \times (1, 1, \bar{2}, 1, 1, 1, 2, 1, 1, 1, 1, 1)$	
	df_1	$4 \times (1, 1, 1, 2, 1, \bar{2}, 1, 1, 1, 1, 1, 1)$	
	df_2	$4 \times (1, 1, 1, 2, 1, 1, \bar{2}, 1, 1, 1, 1, 1)$	
	df'_1	$2 \times (1, 1, 1, 2, 1, 2, 1, 1, 1, 1, 1, 1)$	
	df'_2	$2 \times (1, 1, 1, 2, 1, 1, 2, 1, 1, 1, 1, 1)$	
$USp(8)_g$	bg_4	$1 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1, 8)$	$(1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	bg'_4	$1 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1, \bar{8})$	$(1, 1, 2^2, 1, 1, 1, 1, 1, 1, 1, 1)$
	cg_2	$1 \times (1, 1, 2, 1, 1, 1, 1, 1, 1, \bar{8}, 1, 1)$	$(1, 1, 1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1)$
	cg'_2	$1 \times (1, 1, 2, 1, 1, 1, 1, 1, 1, 8, 1, 1)$	
	dg_3	$1 \times (1, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 8, 1)$	
	dg'_3	$1 \times (1, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, \bar{8}, 1)$	

TABLE XVII. Particle spectrum of model r26 with gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times USp(4) \times SU(4)^6$.

Model r26	Quantum numbers	Fields
ab'	$1 \times (\bar{4}, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$F_L^{ci}(Q_L, L_L)$
ab	$4 \times (4, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$F_L^i(Q_L, L_L)$
ad	$1 \times (\bar{4}, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1)$	$F_R^{ii}(Q_R, L_R)$
ac	$2 \times (\bar{4}, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1)$	$F_R^i(Q_R, L_R)$
$b_{\square}$	$48 \times (1, \bar{1}_{\square}, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	S_L^i
$a_{\square}$	$2 \times (1, 1, 1_{\square}, 1, 1, 1, 1, 1, 1, 1, 1)$	S_R^i
$d_{\square}$	$4 \times (1, 1, 1, \bar{1}_{\square}, 1, 1, 1, 1, 1, 1, 1)$	S_R^i
$b_{\square\square}$	$4 \times (1, \bar{3}_{\square\square}, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	T_L^i
cd'	$2 \times (1, 1, \bar{2}, \bar{2}, 1, 1, 1, 1, 1, 1, 1)$	Δ^i
bc	$4 \times (1, 2, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1)$	$\Phi^i(H_u, H_d)$
bc'	$3 \times (1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1)$	$\Phi^i(H_u, H_d)$
bd	$3 \times (1, 2, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1)$	Ξ^i
bf_1	$2 \times (1, \bar{2}, 1, 1, 1, 4, 1, 1, 1, 1, 1)$	X_L^i
bf_2	$1 \times (1, \bar{2}, 1, 1, 1, 1, 4, 1, 1, 1, 1)$	X_L^i
bg_1	$1 \times (1, \bar{2}, 1, 1, 1, 1, 1, 4, 1, 1, 1)$	X_L^i
bg_2	$2 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, 4, 1, 1)$	X_L^i
bh_1	$1 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 4, 1)$	X_L^i
bh_2	$2 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 4)$	X_L^i

(Table continued)

TABLE XVII. (Continued)

Model r26	Quantum numbers	Fields
bf'_1	$1 \times (1, \bar{2}, 1, 1, 1, \bar{4}, 1, 1, 1, 1, 1)$	X_L^i
bf'_2	$2 \times (1, \bar{2}, 1, 1, 1, 1, \bar{4}, 1, 1, 1, 1)$	X_L^i
bg'_1	$4 \times (1, \bar{2}, 1, 1, 1, 1, 1, \bar{4}, 1, 1, 1)$	X_L^i
bg'_2	$2 \times (1, \bar{2}, 1, 1, 1, 1, 1, \bar{4}, 1, 1)$	X_L^i
bh'_1	$2 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, \bar{4}, 1)$	X_L^i
bh'_2	$1 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, \bar{4})$	X_L^i
ch_1	$1 \times (1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 4, 1)$	X_R^i
cg'_1	$2 \times (1, 1, 2, 1, 1, 1, 1, 4, 1, 1, 1)$	X_R^i
cg'_2	$1 \times (1, 1, 2, 1, 1, 1, 1, 1, 4, 1, 1)$	X_R^i
ch'_2	$1 \times (1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, \bar{4})$	X_R^i
df_2	$2 \times (1, 1, 1, \bar{2}, 1, 1, 4, 1, 1, 1, 1)$	X_R^i
df'_1	$1 \times (1, 1, 1, \bar{2}, 1, \bar{4}, 1, 1, 1, 1, 1)$	X_R^i
dg'_1	$1 \times (1, 1, 1, 2, 1, 1, 1, 4, 1, 1, 1)$	X_R^i

TABLE XVIII. The composite particle spectrum of model r26 formed due to the strong forces in the hidden sector.

Model r26	$SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times USp(4) \times SU(4)^6$		
Confining force	Intersection	Exotic particle spectrum	Confined particle spectrum
$USp(4)_f$	bf_1	$2 \times (1, \bar{2}, 1, 1, 1, 4, 1, 1, 1, 1, 1)$	$(1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	bf_2	$1 \times (1, \bar{2}, 1, 1, 1, 1, 4, 1, 1, 1, 1)$	$(1, 1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1)$
	bf'_1	$1 \times (1, \bar{2}, 1, 1, 1, \bar{4}, 1, 1, 1, 1, 1)$	
	bf'_2	$2 \times (1, \bar{2}, 1, 1, 1, 1, \bar{4}, 1, 1, 1, 1)$	
	cf_2	$2 \times (1, 1, \bar{2}, 1, 1, 1, 4, 1, 1, 1, 1)$	
	cf'_1	$1 \times (1, 1, \bar{2}, 1, 1, \bar{4}, 1, 1, 1, 1, 1)$	
$SU(4)_g$	bg_1	$1 \times (1, \bar{2}, 1, 1, 1, 1, 1, 4, 1, 1, 1)$	$(1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	bg_2	$2 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, 4, 1, 1)$	$(1, 1, 2^2, 1, 1, 1, 1, 1, 1, 1, 1)$
	bg'_1	$4 \times (1, \bar{2}, 1, 1, 1, 1, 1, \bar{4}, 1, 1, 1)$	$(1, 1, 1, 2^2, 1, 1, 1, 1, 1, 1, 1)$
	bg'_2	$2 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, \bar{4}, 1, 1)$	
	cg'_1	$1 \times (1, 1, 2, 1, 1, 1, 1, 4, 1, 1, 1)$	
	dg'_1	$2 \times (1, 1, 1, 2, 1, 1, 1, 4, 1, 1, 1)$	
$SU(4)_h$	dg'_2	$1 \times (1, 1, 1, 2, 1, 1, 1, 1, 4, 1, 1)$	
	bh_1	$1 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 4, 1)$	$(1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	bh_2	$2 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 4)$	$(1, 1, 1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1)$
	bh'_1	$2 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, \bar{4}, 1)$	
	bh'_2	$1 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, \bar{4})$	
	dh_1	$1 \times (1, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 4, 1)$	
	dh'_2	$1 \times (1, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, \bar{4})$	

TABLE XIX. Particle spectrum of model r29 with gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^3 \times SU(4)^4 \times USp(4)^4$.

Model r29	Quantum numbers	Fields
ab	$3 \times (4, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$F_L^i(Q_L, L_L)$
ac	$1 \times (4, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$F_R^{ci}(Q_R, L_R)$
ad	$2 \times (4, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$F_R^{ci}(Q_R, L_R)$
ac'	$6 \times (\bar{4}, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$F_R^i(Q_R, L_R)$
$b \begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$4 \times (1, \bar{1}, \begin{smallmatrix} \square \\ \square \end{smallmatrix}, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	S_L^i
$c \begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$60 \times (1, 1, \bar{1}, \begin{smallmatrix} \square \\ \square \end{smallmatrix}, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	S_R^i
$d \begin{smallmatrix} \square & \square \end{smallmatrix}$	$6 \times (1, 1, \bar{3}, \begin{smallmatrix} \square & \square \end{smallmatrix}, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	T_R^i
cd'	$3 \times (1, 1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	Δ^i
bc	$3 \times (1, \bar{2}, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$\Phi^i(H_u, H_d)$
bc'	$5 \times (1, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$\Phi^i(H_u, H_d)$
bd'	$1 \times (1, 2, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	Ξ^i
bf_1	$1 \times (1, 2, 1, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1)$	X_L^i
bg_2	$2 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, 4, 1, 1, 1, 1, 1)$	X_L^i
be'	$1 \times (1, 2, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	X_L^i
bf'_2	$1 \times (1, 2, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1)$	X_L^i
bg'_1	$2 \times (1, \bar{2}, 1, 1, 1, 1, 1, \bar{4}, 1, 1, 1, 1, 1, 1)$	X_L^i
ce	$1 \times (1, 1, \bar{2}, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	X_R^i
cf_1	$2 \times (1, 1, \bar{2}, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1)$	X_R^i
cf_2	$1 \times (1, 1, \bar{2}, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1)$	X_R^i
cg_1	$4 \times (1, 1, \bar{2}, 1, 1, 1, 1, 4, 1, 1, 1, 1, 1, 1)$	X_R^i
cg_2	$2 \times (1, 1, \bar{2}, 1, 1, 1, 1, 1, 4, 1, 1, 1, 1, 1)$	X_R^i
ch_1	$2 \times (1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 4, 1, 1, 1, 1)$	X_R^i
ch_2	$2 \times (1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 4, 1, 1, 1)$	X_R^i
ci_2	$1 \times (1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, \bar{4}, 1, 1)$	X_R^i
ce'	$2 \times (1, 1, \bar{2}, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	X_R^i
cf'_1	$1 \times (1, 1, \bar{2}, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1)$	X_R^i
cf'_2	$2 \times (1, 1, \bar{2}, 1, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1)$	X_R^i
cg'_1	$2 \times (1, 1, \bar{2}, 1, 1, 1, 1, \bar{4}, 1, 1, 1, 1, 1, 1)$	X_R^i
cg'_2	$4 \times (1, 1, \bar{2}, 1, 1, 1, 1, 1, \bar{4}, 1, 1, 1, 1, 1)$	X_R^i
ch'_1	$2 \times (1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, \bar{4}, 1, 1, 1, 1)$	X_R^i
ch'_2	$2 \times (1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, \bar{4}, 1, 1, 1)$	X_R^i
ci'_2	$1 \times (1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 4, 1, 1)$	X_R^i
dh_2	$1 \times (1, 1, 1, 2, 1, 1, 1, 1, 1, 1, \bar{4}, 1, 1, 1, 1)$	X_R^i
dg'_1	$1 \times (1, 1, 1, \bar{2}, 1, 1, 1, \bar{4}, 1, 1, 1, 1, 1, 1)$	X_R^i
dg'_2	$1 \times (1, 1, 1, \bar{2}, 1, 1, 1, 1, \bar{4}, 1, 1, 1, 1, 1)$	X_R^i
dh'_1	$1 \times (1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 4, 1, 1, 1, 1)$	X_R^i

The resulting formula for the $SU(4)$ β function is given by

$$\beta^{SU(4)} = -3 \times 4 + N_a^{\text{chiral}} \times \frac{1}{2} + 2 \times 1, \quad (48)$$

where N_a^{chiral} is twice the number of chiral multiplets arising from the intersection of the $SU(4)$ brane with other visible

sector branes,

$$N_a^{\text{chiral}} = 2(|I_{ab}| + |I_{ab'}| + |I_{ac}| + |I_{ac'}| + |I_{ad'}| + |I_{ad''}|). \quad (49)$$

Applying the relation (48) to our set of models, we obtain the following β function coefficients for the $SU(4)$

TABLE XX. The composite particle spectrum of model r29 formed due to the strong forces in the hidden sector.

Model r29		$SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^3 \times SU(4)^4 \times USp(4)^4$	
Confining force	Intersection	Exotic particle	Confined particle spectrum
$SU(2)_e$	be'	$1 \times (1, 2, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$(1, 2^2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	de	$1 \times (1, 1, 1, \bar{2}, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$(1, 1, 1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	de'	$2 \times (1, 1, 1, \bar{2}, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	
$SU(2)_f$	bf_1	$1 \times (1, 2, 1, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1)$	$(1, 2^2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	bf'_2	$1 \times (1, 2, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1)$	$(1, 1, 1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	df_1	$2 \times (1, 1, 1, \bar{2}, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1)$	
	df_2	$1 \times (1, 1, 1, \bar{2}, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1)$	
	df'_1	$1 \times (1, 1, 1, \bar{2}, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 1)$	
	df'_2	$2 \times (1, 1, 1, \bar{2}, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 1)$	
$SU(4)_g$	bg_2	$2 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, 1, 4, 1, 1, 1, 1, 1)$	$(1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	bg'_1	$2 \times (1, \bar{2}, 1, 1, 1, 1, 1, 1, \bar{4}, 1, 1, 1, 1, 1, 1)$	$(1, 1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	cg'_1	$1 \times (1, 1, \bar{2}, 1, 1, 1, 1, 1, \bar{4}, 1, 1, 1, 1, 1, 1)$	$(1, 1, 1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	cg'_2	$1 \times (1, 1, \bar{2}, 1, 1, 1, 1, 1, \bar{4}, 1, 1, 1, 1, 1, 1)$	
	dg_1	$4 \times (1, 1, 1, \bar{2}, 1, 1, 1, 4, 1, 1, 1, 1, 1, 1, 1)$	
	dg_2	$2 \times (1, 1, 1, \bar{2}, 1, 1, 1, 1, 4, 1, 1, 1, 1, 1, 1)$	
	dg'_1	$2 \times (1, 1, 1, \bar{2}, 1, 1, 1, \bar{4}, 1, 1, 1, 1, 1, 1, 1)$	
	dg'_2	$4 \times (1, 1, 1, \bar{2}, 1, 1, 1, 1, \bar{4}, 1, 1, 1, 1, 1, 1)$	
$SU(4)_h$	ch_2	$1 \times (1, 1, 2, 1, 1, 1, 1, 1, 1, 1, \bar{4}, 1, 1, 1, 1)$	$(1, 1, 2^2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	ch'_1	$1 \times (1, 1, 2, 1, 1, 1, 1, 1, 1, 4, 1, 1, 1, 1)$	$(1, 1, 1, \bar{2}^2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	dh_1	$2 \times (1, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 4, 1, 1, 1, 1)$	$(1, 1, 1, 2^2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	dh_2	$2 \times (1, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, 4, 1, 1, 1)$	
	dh'_1	$2 \times (1, 1, 1, \bar{2}, 1, 1, 1, 1, 1, \bar{4}, 1, 1, 1, 1)$	
	dh'_2	$2 \times (1, 1, 1, \bar{2}, 1, 1, 1, 1, 1, 1, \bar{4}, 1, 1, 1)$	
$USp(4)_i$	di_2	$1 \times (1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, \bar{4}, 1, 1)$	$(1, 1, 1, 2^2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$
	di'_2	$1 \times (1, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 1, 4, 1, 1)$	

TABLE XXI. Particle spectrum of model r30 with gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times USp(12)^4$.

Model r30	Quantum numbers	Fields
ab	$3 \times (4, \bar{2}, 1, 1, 1, 1, 1, 1)$	$F_L^i(Q_L, L_L)$
ad	$2 \times (\bar{4}, 1, 1, 2, 1, 1, 1, 1)$	$F_R^i(Q_R, L_R)$
ad'	$3 \times (4, 1, 1, 2, 1, 1, 1, 1)$	$F_R^i(Q_R, L_R)$
ac	$1 \times (\bar{4}, 1, 2, 1, 1, 1, 1, 1)$	$F_R^i(Q_R, L_R)$
ac'	$3 \times (\bar{4}, 1, \bar{2}, 1, 1, 1, 1, 1)$	$F_R^i(Q_R, L_R)$
$b \begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$30 \times (1, \bar{1}, \begin{smallmatrix} \square \\ \square \end{smallmatrix}, 1, 1, 1, 1, 1)$	S_L^i
$\alpha \begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$20 \times (1, 1, 1, \begin{smallmatrix} \square \\ \square \end{smallmatrix}, 1, 1, 1, 1, 1)$	S_R^i
$d \begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$40 \times (1, 1, 1, 1, \begin{smallmatrix} \square \\ \square \end{smallmatrix}, 1, 1, 1, 1)$	S_R^i
$d \begin{smallmatrix} \square & \square \end{smallmatrix}$	$6 \times (1, 1, \bar{3}, \begin{smallmatrix} \square & \square \end{smallmatrix}, 1, 1, 1, 1, 1)$	T_R^i
cd'	$21 \times (1, 1, 2, 2, 1, 1, 1, 1)$	Δ^i
bc	$21 \times (1, \bar{2}, 2, 1, 1, 1, 1, 1)$	$\Phi^i(H_u, H_d)$
bc'	$6 \times (1, 2, 2, 1, 1, 1, 1, 1)$	$\Phi^i(H_u, H_d)$
bd	$19 \times (1, \bar{2}, 1, 2, 1, 1, 1, 1)$	Ξ^i

(Table continued)

TABLE XXI. (Continued)

Model r30	Quantum numbers	Fields
bd'	$2 \times (1, \bar{2}, 1, \bar{2}, 1, 1, 1, 1)$	Ξ^i
be_3	$1 \times (1, 2, 1, 1, 1, 1, \bar{12}, 1)$	X_L^i
be'_3	$1 \times (1, 2, 1, 1, 1, 1, 12, 1)$	X_L^i
ce_2	$1 \times (1, 1, \bar{2}, 1, 1, 12, 1, 1)$	X_R^i
ce'_2	$1 \times (1, 1, \bar{2}, 1, 1, \bar{12}, 1, 1)$	X_R^i
de_1	$1 \times (1, 1, 1, \bar{2}, 12, 1, 1, 1)$	X_R^i
de'_1	$1 \times (1, 1, 1, \bar{2}, \bar{12}, 1, 1, 1)$	X_R^i

TABLE XXII. The composite particle spectrum of model r30 formed due to the strong forces in the hidden sector.

Model r30	$SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times USp(12)^4$		
Force	Sector	Exotic particle	Confined spectrum
$USp(12)_e$	be_3	$1 \times (1, 2, 1, 1, 1, 1, \bar{12}, 1)$	$(1, 2^2, 1, 1, 1, 1, 1, 1)$
	be'_3	$1 \times (1, 2, 1, 1, 1, 1, 12, 1)$	$(1, 1, \bar{2}^2, 1, 1, 1, 1, 1)$
	ce_2	$1 \times (1, 1, \bar{2}, 1, 1, 12, 1, 1)$	$(1, 1, 1, \bar{2}^2, 1, 1, 1, 1)$
	ce'_2	$1 \times (1, 1, \bar{2}, 1, 1, \bar{12}, 1, 1)$	
	de_1	$1 \times (1, 1, 1, \bar{2}, 12, 1, 1, 1)$	
	de'_1	$1 \times (1, 1, 1, \bar{2}, \bar{12}, 1, 1, 1)$	

gauge group:

$$\begin{aligned}
\text{Model r06: } \beta^{SU(4)} &= -3 \times 4 + 24 \times \frac{1}{2} + 2 \times 1 = 2, \\
\text{Model r08: } \beta^{SU(4)} &= -3 \times 4 + 24 \times \frac{1}{2} + 2 \times 1 = 2, \\
\text{Model r10: } \beta^{SU(4)} &= -3 \times 4 + 24 \times \frac{1}{2} + 2 \times 1 = 2, \\
\text{Model r19: } \beta^{SU(4)} &= -3 \times 4 + 16 \times \frac{1}{2} + 2 \times 1 = -2, \\
\text{Model r20: } \beta^{SU(4)} &= -3 \times 4 + 16 \times \frac{1}{2} + 2 \times 1 = -2, \\
\text{Model r22: } \beta^{SU(4)} &= -3 \times 4 + 24 \times \frac{1}{2} + 2 \times 1 = 2, \\
\text{Model r25: } \beta^{SU(4)} &= -3 \times 4 + 12 \times \frac{1}{2} + 2 \times 1 = -4, \\
\text{Model r26: } \beta^{SU(4)} &= -3 \times 4 + 16 \times \frac{1}{2} + 2 \times 1 = -2, \\
\text{Model r29: } \beta^{SU(4)} &= -3 \times 4 + 24 \times \frac{1}{2} + 2 \times 1 = 2, \\
\text{Model r30: } \beta^{SU(4)} &= -3 \times 4 + 24 \times \frac{1}{2} + 2 \times 1 = 2. \quad (50)
\end{aligned}$$

From these results, we observe that models r19, r20, r25, and r26 yield negative β function coefficients, indicating that the corresponding $SU(4)$ gauge groups are asymptotically free. In contrast, the remaining models have positive β functions and thus do not exhibit asymptotic freedom.

V. CONCLUSION

We have presented a class of consistent three-family $\mathcal{N} = 1$ supersymmetric Pati-Salam models constructed from rigid intersecting D6-branes on the factorizable $T^6/(\mathbb{Z}_2 \times \mathbb{Z}'_2)$ orientifold with discrete torsion. These models are the first of their kind to be built entirely on a factorizable compactification lattice consisting of rectangular two-tori with frozen moduli. A key feature of the setup is the use of rigid three-cycles, which stabilize the open-string moduli associated with D-brane positions and Wilson lines. This eliminates unwanted adjoint matter and allows for negative β functions in certain models, thereby ensuring asymptotic freedom. All models contain suitable GUT Higgs fields to ensure that the Pati-Salam gauge symmetry can be spontaneously broken down to that of the Standard Model.

The models satisfy the full set of consistency conditions, including $\mathcal{N} = 1$ supersymmetry, K-theory constraints, RR tadpole cancellation, and the recently proposed swampland bounds on the maximal rank of the gauge group. We have provided detailed chiral spectra and have discussed the decoupling of exotic states via strong dynamics in the hidden sector. A complete phenomenological analysis, including the computation of Yukawa couplings and the structure of soft supersymmetry breaking terms, is left for future work.

It would be interesting to explore variations of this model-building framework that exclude the d -stack of rigid D6-branes while still enabling the construction of GUT

Higgs fields responsible for breaking the Pati-Salam gauge symmetry. Such constructions may yield Standard Model-like spectra after appropriate symmetry breaking. Although our analysis has been confined to models with rectangular tori, the framework presented in Sec. II remains applicable to tilted tori as well. However, to date, no three-family model with tilted tori and suitable GUT Higgs fields has been found within this framework. It would be valuable to investigate the origin of this obstruction more thoroughly, or ideally to identify a rigorous argument that rules out such constructions altogether.

A particularly compelling direction for future work involves a statistical exploration of the landscape of such vacua in string theory. In this context, it would be highly interesting to perform a systematic classification of all possible vacua using modern machine learning techniques, along the lines of [54].

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX: THREE-FAMILY PATI-SALAM MODELS ON RIGID CYCLES

In this appendix, we present a list of the ten independent three-family $\mathcal{N} = 1$ supersymmetric Pati-Salam models

that arise from rigid intersecting D6-branes on a type IIA $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}'_2)$ orientifold. The models are named according to the rank of their gauge group and are sorted in ascending order. Additionally, we provide the relevant two-torus complex structure moduli and the gauge coupling relations in the captions.

TABLE XXIII. Model r06 with the gauge group $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2}$, the torus moduli $\chi_1 = \sqrt{13}$, $\chi_2 = \frac{12}{\sqrt{13}}$, and $\chi_3 = \frac{16}{\sqrt{13}}$, and the gauge coupling relation $g_a^2 = \frac{308}{39} g_b^2 = \frac{700}{39} g_R^2 = \frac{51040}{27241} \frac{5g_Y^2}{3} = \frac{16\pi e^{\phi_4}}{\sqrt{3}\sqrt{13}}$.

Model r06	N	$(n^1, m^1) \times (n^2, m^2) \times (n^3, m^3)$
a	4	$(0, 1) \times (0, -1) \times (1, 0)$
b	2	$(1, 1) \times (-4, -1) \times (2, 1)$
c	2	$(-4, -1) \times (0, -1) \times (-4, -1)$
d	2	$(3, -1) \times (4, -1) \times (0, 1)$

TABLE XXIV. Model r08 with the gauge group $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^2$, the torus moduli $\chi_1 = \sqrt{5}$, $\chi_2 = \frac{12}{\sqrt{5}}$, and $\chi_3 = \frac{8}{\sqrt{5}}$, and the gauge coupling relation $g_a^2 = \frac{28}{5} g_b^2 = \frac{44}{3} g_R^2 = \frac{1680}{947} \frac{5g_Y^2}{3} = \frac{8\sqrt{5}\pi e^{\phi_4}}{\sqrt{5}}$.

Model r08	N	$(n^1, m^1) \times (n^2, m^2) \times (n^3, m^3)$
a	4	$(0, 1) \times (0, -1) \times (1, 0)$
b	2	$(-1, -1) \times (4, 1) \times (2, 1)$
c	2	$(-2, 1) \times (0, 1) \times (-4, 1)$
d	2	$(3, -1) \times (-4, 1) \times (0, -1)$
e_1	2	$(1, 0) \times (0, -1) \times (0, 1)$
e_2	2	$(-1, 0) \times (0, 1) \times (0, 1)$

TABLE XXV. Model r10 with the gauge group $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^4$, the torus moduli $\chi_1 = 8$, $\chi_2 = 2$, and $\chi_3 = 2$, and the gauge coupling relation $g_a^2 = \frac{5}{4} g_b^2 = \frac{41}{4} g_R^2 = \frac{500}{323} \frac{5g_Y^2}{3} = 2\sqrt{2}\pi e^{\phi_4}$.

Model r10	N	$(n^1, m^1) \times (n^2, m^2) \times (n^3, m^3)$
a	4	$(0, 1) \times (0, -1) \times (1, 0)$
b	2	$(4, -1) \times (-1, 0) \times (-1, -1)$
c	2	$(-4, 1) \times (3, -2) \times (1, -1)$
d	2	$(0, 1) \times (-2, 1) \times (-2, 1)$
e_1	2	$(0, -1) \times (1, 0) \times (0, 1)$
e_2	2	$(0, 1) \times (-1, 0) \times (0, 1)$
f_1	2	$(0, -1) \times (0, 1) \times (1, 0)$
f_2	2	$(0, -1) \times (0, -1) \times (-1, 0)$

TABLE XXVI. Model r19 with the gauge group $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^5 \times USp(4)^4$, the torus moduli $\chi_1 = \sqrt{6}$, $\chi_2 = \frac{9\sqrt{3}}{2}$, and $\chi_3 = \sqrt{6}$, and the gauge coupling relation $g_a^2 = 5g_b^2 = \frac{70}{9}g_R^2 = \frac{175}{124}\frac{5g_Y^2}{3} = \frac{82^{3/4}\pi e^{\phi_4}}{3\sqrt{3}}$.

Model r19	N	$(n^1, m^1) \times (n^2, m^2) \times (n^3, m^3)$
a	4	$(0, 1) \times (0, -1) \times (1, 0)$
b	2	$(-3, -1) \times (0, -1) \times (-2, -1)$
c	2	$(2, 1) \times (-3, -1) \times (1, 1)$
d	2	$(1, 1) \times (3, 1) \times (-2, -1)$
e	2	$(0, 1) \times (0, -1) \times (1, 0)$
f_1	2	$(1, 0) \times (0, -1) \times (0, 1)$
f_2	2	$(-1, 0) \times (0, 1) \times (0, 1)$
g_1	2	$(0, -1) \times (1, 0) \times (0, 1)$
g_2	2	$(0, 1) \times (-1, 0) \times (0, 1)$
h_1	2	$(1, 0) \times (1, 0) \times (1, 0)$
h_2	2	$(-1, 0) \times (-1, 0) \times (1, 0)$
h_3	2	$(1, 0) \times (-1, 0) \times (-1, 0)$
h_4	2	$(-1, 0) \times (1, 0) \times (-1, 0)$

TABLE XXVII. Model r20 with the gauge group $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^6 \times USp(4)^4$, the torus moduli $\chi_1 = \sqrt{3}$, $\chi_2 = \sqrt{3}$, and $\chi_3 = 9\sqrt{3}$, and the gauge coupling relation $g_a^2 = 4g_b^2 = \frac{56}{9}g_R^2 = \frac{14}{11}\frac{5g_Y^2}{3} = \frac{8\pi e^{\phi_4}}{3\sqrt{3}}$.

Model r20	N	$(n^1, m^1) \times (n^2, m^2) \times (n^3, m^3)$
a	4	$(1, 0) \times (0, 1) \times (0, -1)$
b	2	$(-1, -1) \times (3, 1) \times (0, 1)$
c	2	$(-2, -1) \times (-1, -1) \times (-3, -1)$
d	2	$(1, -1) \times (-2, 1) \times (3, -1)$
e_1	2	$(1, 0) \times (0, -1) \times (0, 1)$
e_2	2	$(-1, 0) \times (0, 1) \times (0, 1)$
f_1	2	$(0, -1) \times (1, 0) \times (0, 1)$
f_2	2	$(0, 1) \times (-1, 0) \times (0, 1)$
g_1	2	$(0, -1) \times (0, 1) \times (1, 0)$
g_2	2	$(0, -1) \times (0, -1) \times (-1, 0)$
h_1	2	$(1, 0) \times (1, 0) \times (1, 0)$
h_2	2	$(-1, 0) \times (-1, 0) \times (1, 0)$
h_3	2	$(1, 0) \times (-1, 0) \times (-1, 0)$
h_4	2	$(-1, 0) \times (1, 0) \times (-1, 0)$

TABLE XXVIII. Model r22 with the gauge group $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times USp(8)^4$, the torus moduli $\chi_1 = 2$, $\chi_2 = 8$, and $\chi_3 = 4$, and the gauge coupling relation $g_a^2 = \frac{5}{2}g_b^2 = \frac{45}{2}g_R^2 = \frac{250}{127}\frac{5g_Y^2}{3} = 4\pi e^{\phi_4}$.

Model r22	N	$(n^1, m^1) \times (n^2, m^2) \times (n^3, m^3)$
a	4	$(0, 1) \times (0, -1) \times (1, 0)$
b	2	$(-1, 0) \times (4, -1) \times (-2, -1)$
c	2	$(4, -1) \times (0, 1) \times (2, -1)$
d	2	$(-3, 2) \times (-4, 1) \times (-2, 1)$
e_1	4	$(1, 0) \times (1, 0) \times (1, 0)$
e_2	4	$(-1, 0) \times (-1, 0) \times (1, 0)$
e_3	4	$(1, 0) \times (-1, 0) \times (-1, 0)$
e_4	4	$(-1, 0) \times (1, 0) \times (-1, 0)$

TABLE XXIX. Model r25 with the gauge group $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^3 \times USp(8)^4$, the torus moduli $\chi_1 = 2\sqrt{611}$, $\chi_2 = \sqrt{\frac{47}{13}}$, and $\chi_3 = \sqrt{\frac{47}{13}}$, and the gauge coupling relation $g_a^2 = \frac{6120}{47}g_b^2 = \frac{9840}{47}g_R^2 = \frac{4100}{1687}\frac{5g_Y^2}{3} = \frac{8\sqrt{213^{3/4}}\pi e^{\phi_4}}{\sqrt[4]{47}}$.

Model r25	N	$(n^1, m^1) \times (n^2, m^2) \times (n^3, m^3)$
a	4	$(1, 0) \times (0, 1) \times (0, -1)$
b	2	$(-2, -1) \times (-4, 1) \times (1, -1)$
c	2	$(4, 1) \times (1, 1) \times (-3, -1)$
d	2	$(-4, 1) \times (3, -1) \times (1, -1)$
e	2	$(0, 1) \times (0, -1) \times (1, 0)$
f_1	2	$(0, -1) \times (0, 1) \times (1, 0)$
f_2	2	$(0, -1) \times (0, -1) \times (-1, 0)$
g_1	4	$(1, 0) \times (1, 0) \times (1, 0)$
g_2	4	$(-1, 0) \times (-1, 0) \times (1, 0)$
g_3	4	$(1, 0) \times (-1, 0) \times (-1, 0)$
g_4	4	$(-1, 0) \times (1, 0) \times (-1, 0)$

TABLE XXX. Model r26 with the gauge group $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times USp(4) \times SU(4)^6$, the torus moduli $\chi_1 = \frac{2\sqrt{10}}{3}$, $\chi_2 = 2\sqrt{10}$, and $\chi_3 = 3\sqrt{10}$, and the gauge coupling relation $g_a^2 = \frac{121}{10}g_b^2 = \frac{247}{60}g_R^2 = \frac{2695}{3548}\frac{5g_Y^2}{3} = \frac{42^{3/4}\pi e^{\phi_4}}{\sqrt{5}}$.

Model r26	N	$(n^1, m^1) \times (n^2, m^2) \times (n^3, m^3)$
a	4	$(0, 1) \times (1, 0) \times (0, -1)$
b	2	$(-3, -1) \times (2, 1) \times (3, 1)$
c	2	$(1, 0) \times (-2, -1) \times (-3, 1)$
d	2	$(1, -1) \times (-3, -1) \times (-1, 0)$
e	2	$(1, 0) \times (1, 0) \times (1, 0)$
f_1	4	$(1, 0) \times (0, -1) \times (0, 1)$
f_2	4	$(-1, 0) \times (0, 1) \times (0, 1)$
g_1	4	$(0, -1) \times (1, 0) \times (0, 1)$
g_2	4	$(0, 1) \times (-1, 0) \times (0, 1)$
h_1	4	$(0, -1) \times (0, 1) \times (1, 0)$
h_2	4	$(0, -1) \times (0, -1) \times (-1, 0)$

TABLE XXXI. Model r29 with the gauge group $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times SU(2)^3 \times SU(4)^4 \times USp(4)^4$, the torus moduli $\chi_1 = 8$, $\chi_2 = 4$, and $\chi_3 = 4$, and the gauge coupling relation $g_a^2 = \frac{5}{4}g_b^2 = \frac{55}{8}g_R^2 = \frac{125}{182}\frac{5g_Y^2}{3} = 2\sqrt{2}\pi e^{\phi_4}$.

Model r29	N	$(n^1, m^1) \times (n^2, m^2) \times (n^3, m^3)$
a	4	$(0, 1) \times (1, 0) \times (0, -1)$
b	2	$(-4, -1) \times (-2, 1) \times (1, 0)$
c	2	$(4, 1) \times (2, 1) \times (-3, -1)$
d	2	$(-1, 0) \times (-2, 1) \times (2, 1)$
e	2	$(1, 0) \times (0, 1) \times (0, -1)$
f_1	2	$(1, 0) \times (0, -1) \times (0, 1)$
f_2	2	$(-1, 0) \times (0, 1) \times (0, 1)$
g_1	4	$(0, -1) \times (1, 0) \times (0, 1)$
g_2	4	$(0, 1) \times (-1, 0) \times (0, 1)$
h_1	4	$(0, -1) \times (0, 1) \times (1, 0)$
h_2	4	$(0, -1) \times (0, -1) \times (-1, 0)$
i_1	2	$(1, 0) \times (1, 0) \times (1, 0)$
i_2	2	$(-1, 0) \times (-1, 0) \times (1, 0)$
i_3	2	$(1, 0) \times (-1, 0) \times (-1, 0)$
i_4	2	$(-1, 0) \times (1, 0) \times (-1, 0)$

TABLE XXXII. Model r30 with the gauge group $SU(4)_C \times SU(2)_L \times SU(2)_{R_1} \times SU(2)_{R_2} \times USp(12)^4$, the torus moduli $\chi_1 = 3\sqrt{\frac{7}{2}}$, $\chi_2 = 10\sqrt{\frac{2}{7}}$, and $\chi_3 = 4\sqrt{\frac{2}{7}}$, and the gauge coupling relation $g_a^2 = \frac{26}{7}g_b^2 = \frac{118}{15}g_R^2 = \frac{18525}{13192}\frac{5g_Y^2}{3} = \frac{82^{3/4}}{\sqrt[4]{7}\sqrt{15}}\pi e^{\phi_4}$.

Model r30	N	$(n^1, m^1) \times (n^2, m^2) \times (n^3, m^3)$
a	4	$(0, 1) \times (0, -1) \times (1, 0)$
b	2	$(-1, -1) \times (4, 1) \times (2, 1)$
c	2	$(4, -1) \times (0, -1) \times (-3, 1)$
d	2	$(-3, 1) \times (-4, 1) \times (-1, 1)$
e_1	6	$(1, 0) \times (1, 0) \times (1, 0)$
e_2	6	$(-1, 0) \times (-1, 0) \times (1, 0)$
e_3	6	$(1, 0) \times (-1, 0) \times (-1, 0)$
e_4	6	$(-1, 0) \times (1, 0) \times (-1, 0)$

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